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Proposal for Condensing Diverse Accounting Procedures

A. Wayne Corcoran

Traditional accounting has different techniques for almost every field in which it deals. Yet the use of simple matrix algebra could make all such procedures almost uniform—and simpler, as well—

A PROPOSAL FOR CONDENSING DIVERSE ACCOUNTING PROCEDURES

by A. Wayne Corcoran
University of Connecticut

WHENEVER a person proceeds from one accounting area to another, he encounters what seems to be an entirely new set of inputs, rules, definitions, and procedures. As traditionally presented, such diverse accounting areas as partnerships, process cost accounting, liquidation statements, consolidated financial statements, variance analysis, determining overhead absorption rates, and preparing depreciation lapse schedules—to mention but a few—seem to be virtually unrelated. In 1953 A. C. Littleton recognized this problem when he wrote:

"In actual historical evolution, accounting principles have been slowly distilled out of accounting actions. That is to say, accounting rules, having first been the fruits of tentative actions, grew in significance until they became guides to predetermined actions. As these accounting particulars grew increasingly diverse and complex, so did accounting actions and the accompanying rules, customs, practices. And as this diversity of particulars falls under more and more critical consideration, it becomes increasingly advisable to decide whether there are elements of order, sequence, interrelation within the mass."[1]

Not only is this lack of interrelationship annoying, bewildering, and time-consuming, but it is also unnecessary. This article advocates the use of the mathematical tool of matrices to interrelate diverse accounting areas from a procedural viewpoint. It shows how just a few, simple matrix manipulations may be used as substitutes for the myriad procedures now employed to accomplish allocation.

Accounting procedure structure

Much of traditional accounting procedure involves the acquisition, valuation, and allocation of input

BASIC STEPS IN ACCOUNTING PROCEDURE

Process Cost Reports		Liquidation Statement
The listing of material, labor, and overhead components	Acquisition	The listing of all available assets
The determining of historical cost outlays of components	Valuation	The determining of realizable values of assets
The distributing of valued cost components to output designations	Allocation	The distributing of valued assets to various types of creditors and owners

EXHIBIT I

data. Concentrating on these processes makes it possible to interrelate diverse accounting areas. Let us illustrate this idea by referring to two accounting areas that perhaps, at first glance, seem related only in that money and accounting are concerned. These areas are the preparation of process cost reports and the preparation of liquidation statements. These areas may be viewed in terms of their acquisition, valuation, and allocation phases as shown in Exhibit 1 at the top of this page.

The similarities between these areas are now more apparent. Both involve listing a set of inputs (acquisition phase), determining appropriate values for these inputs (valuation phase), and distributing the valued inputs to output destinations (allocation phase). Likewise, the differences between the two areas are evident: The inputs in process costing are data on materials, labor, and overhead while those involved in liquidation are data on all available assets. The values assigned to inputs in process costing are historical cost outlays while those in liquidation are realizable values. The output destinations in process costing are product costs while those in liquidation are claimants' equities.

Because these two accounting areas are most similar to each other in the allocation phase, it would seem that their interrelationship could best be accomplished by concentrating on allocation processes. The inputs and outputs in the various accounting areas differ, and so do the methods of input valuation. Thus, the acquisition and valuation processes are not likely to lead to extensive interrelation. This leaves us with allocation processes as the most promising avenue. We seek, therefore, the answer to the question, "Can the allocation of inputs to outputs be standardized so that diverse accounting areas may be interrelated?"

In mathematics the framework for allocation problems is found in vector spaces, and the allocation process itself is carried out by transformation matrices. A matrix may be defined as something that consists of rows and columns of numbers. These rows and columns of numbers are referred to as vectors, and a matrix consists of one or more vectors. This is a row vector: (1, 3, –1, 4); this is a column vector: $\begin{bmatrix} 8 \\ 2 \\ 0 \end{bmatrix}$. An example of a matrix containing more than a single vector is $\begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 5 \end{bmatrix}$.

Vectors and matrices may be added and subtracted element by element, provided they have the same dimensions. For instance:

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 5 & 2 \\ 6 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 2 \\ 7 & -3 & 5 \end{bmatrix}$$

Vectors and matrices may be multiplied, provided the number of columns in the lefthand matrix equals the number of rows in the righthand matrix. The exact procedure for multiplication is expressed in the formula:

$$c_{ik} = \sum_{j=1}^{n} a_{ij}\, b_{jk} = a_{i1}\, b_{1k} + \dots + a_{in}\, b_{nk}$$

where: $i = 1, 2, \dots, m.$
$j = 1, 2, \dots, n.$
$k = 1, 2, \dots, r.$

EXHIBIT 2

DEPRECIATION MATRIX

b (depreciable bases)	r (rates per time period)	L (lapse schedule)				Totals
60,000	.4, .3, .2, .1	24,000	18,000	12,000	6,000	60,000
80,000		32,000	24,000	16,000	8,000	80,000
20,000		8,000	6,000	4,000	2,000	20,000
	Totals	64,000	48,000	32,000	16,000	160,000

Consider this problem:

$$\underset{(2 \times 3)}{\overset{\textbf{A}}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}}} \underset{(3 \times 4)}{\overset{\textbf{B}}{\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix}}} = \underset{(2 \times 4)}{\begin{bmatrix} C \end{bmatrix}}$$

a_{ij} represents any element from Matrix A; the subscript i indicates the row number and the subscript j indicates the column number.

b_{jk} represents any element from Matrix B; the subscript j indicates the row number while the subscript k indicates the column number.

Let us substitute arbitrary numerical values and see what Matrix C looks like.

$$\overset{\textbf{A}}{\begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 5 \end{bmatrix}} \overset{\textbf{B}}{\begin{bmatrix} -2 & 3 & -1 & 4 \\ 1 & 8 & 1 & 0 \\ 4 & 0 & 2 & 4 \end{bmatrix}} =$$

$$\overset{\textbf{C}}{\begin{bmatrix} -3 & 14 & -1 & 8 \\ 16 & -13 & 7 & 24 \end{bmatrix}}$$

To see how an element of Matrix C is determined, let us apply the formula to determine c_{23}.

$$c_{23} = \sum_{j=1}^{3} a_{2j}\, b_{j3} = 1(-1) - 2(1) + 5(2) = 7.$$

Depreciation application

Perhaps the simplest accounting application of matrix multiplication is to be found in preparing depreciation lapse schedules. Here the accountant is concerned with allocating portions of the depreciable bases of assets—the inputs of the problem—to appropriate time periods—the output designations of the problem. This problem is illustrated in Exhibit 2 on page 16.[2]

Note that Matrix L arrays inputs (assets) according to outputs (time periods). This form of schedule clearly depicts allocation and is easily understood. It can be made to result from other types of matrix multiplication, but the important thing is that the more widely used the matrix schedule is the more interrelation among accounting areas will exist.

Process cost application

Let us return now to the preparation of process cost reports and statements of affairs and see how matrices may be used to further interrelate these accounting areas.

PROCESS COST MATRICES: AVERAGE METHOD, SINGLE PRODUCT

Equivalent Production Computation:

A

Inputs \ Outputs	T	E	L
"Preceding"	1	1	1
"Materials"	1	f_M	g_M
"Conversion"	1	f_C	g_C

$$\overset{\textbf{b}}{\begin{bmatrix} T \\ E \\ L \end{bmatrix}} = \begin{bmatrix} T \\ T \\ T \end{bmatrix} + \begin{bmatrix} E \\ f_M E \\ f_C E \end{bmatrix} + \begin{bmatrix} L \\ g_M L \\ g_C L \end{bmatrix} = \overset{\textbf{e}}{\begin{bmatrix} E_P \\ E_M \\ E_C \end{bmatrix}}$$

Unit cost formula:

$$U_i = \sum_{j=1}^{2} I_j \div E_i$$

Cost Allocation:

$$\overset{\textbf{U}}{\begin{bmatrix} U_P & O & O \\ O & U_M & O \\ O & O & U_C \end{bmatrix}} \overset{\textbf{D}}{\begin{bmatrix} T & E & L \\ T & f_M E & g_M L \\ T & f_C E & g_C L \end{bmatrix}} = \overset{\textbf{R (cost report)}}{\begin{bmatrix} U_P T & U_P E & U_P L \\ U_M T & U_M f_M E & U_M g_M L \\ U_C T & U_C f_C E & U_C g_C L \end{bmatrix}}$$

KEY:
- A = Matrix containing proportions of each output quantity appearing in each input category. Note that the rows (labelled) show the input categories while the columns show the output designations.
- b = *A vector showing the total quantities in each of the three output designations (T, E, L).*
- e = A vector that shows the equivalent production (E_i) for each type of input.
- T = Units transferred.
- E = Units in ending inventory.
- L = Units lost.
- f_i = Fraction of ending inventory completed in terms of input i.
- g_i = Fraction of lost units completed in terms of input i.
- U_i = Unit cost if input i; i = P (Preceding department's transferred production costs), M (Direct materials), C (Conversion costs).
- I_j = Total cost of input I (I' = P, M, C, as defined above under index i) appearing in opening inventory (j = 1) or in the costs incurred during the present period (j = 2).
- E_i = Equivalent production of i.
- D = Matrix composed of the equivalent production vectors.

EXHIBIT 3

Exhibit 3 on this page contains a generalized presentation of a matrix approach to preparing a process cost report. The dashed lines in Matrix A and Vector b indicate partitioning. Wherever the partitions are drawn, the usual procedure of multiplication of column and row elements and the summing of individual products must be halted, and the results to that point must be entered in separate vectors.

For instance, without partitioning we would determine the elements in a product matrix, C, as was described previously, that is

$$c_{ik} = \sum_{j=1}^{n} a_{ij}\, b_{jk}.$$

Suppose now that Matrix A is partitioned after Columns 3 and 7 and hence Matrix B is correspondingly partitioned after Rows 3 and 7. There would be three matrices resulting from the multiplication of the separate partitioned matrices,

$$\sum_{j=1}^{3} a_{ij}\, b_{jk}, \quad \sum_{j=4}^{7} a_{ij}\, b_{jk},$$

$$\text{and } \sum_{j=8}^{n} a_{ij}\, b_{jk}.$$

The separate vectors may then

PROCESS COST PROBLEM

Key: (P, M, C) = portion of production done during present month for P (goods received from preceding department), M (departmental materials), C (departmental conversion costs).

QUANTITY DATA

Opening inventory (0, 1/4, 1/2)	40,000
Received from preceding department during period	360,000
Units added by present department	100,000
	500,000
Transferred out	320,000
Ending Inventory (1, 2/3, 1/2)	150,000
Lost units (normal loss occurring gradually during processing; no provision in overhead rate: 1, 2/5, 1/3)	30,000
	500,000

COST DATA

Opening inventory:	
Preceding department's costs	$ 120,000
Departmental material costs	60,000
Departmental conversion costs	60,160
Costs during month:	
Preceding department's costs	1,380,000
Departmental material costs	804,000
Departmental conversion costs	1,187,240
TOTAL	$3,611,400

EXHIBIT 4

Another advantage of the use of a diagonalized matrix in multiplication is that it results in an input-output-type matrix. . . . Such a matrix arrays inputs according to outputs, and, after all, this is what allocation is all about.

be added to obtain the total equivalent production vector e—which, parenthetically, could have been obtained by ignoring the partitioning and performing the multiplication Ab. The elements E_i in Vector e are used in the computation of the unit costs, U_i. The unit costs are then entered in Matrix U, and the cost report results from the multiplication UD. Exhibit 3 essentially reduces to a system of equations for solving process cost problems under the average method.

The form of Matrix U in Exhibit 3 deserves further comment. In this form—that is, with non-zero numbers on the main diagonal of the matrix and zeros everywhere else—the matrix is called a diagonalized matrix. A diagonalized matrix has a number of properties, the most interesting of which for present purposes is that the elements of Matrix R, the cost report, can be obtained by multiplying the elements of U and D in a distributive manner (that is, so to say, straight-across multiplication) rather than by observing the ordinary rules of matrix multiplication (which would generate the same results—but in a more complicated way). In a nutshell—a diagonalized matrix simplifies matrix multiplication.

Another advantage of the use of a diagonalized matrix in multiplication is that it results in an input-output-type matrix such as shown in Matrix R. Such a matrix arrays inputs according to outputs, and, after all, this is what allocation is all about. No other form for re-

A. WAYNE CORCORAN, Ph.D., CPA, is associate professor of accounting in the School of Business Administration of the University of Connecticut. In the past, he served with the accounting offices of the Grange League Federation and with Ernst & Ernst and taught at St. John Fisher College, Rochester, New York, and at the University of Buffalo. Dr. Corcoran is the author of **Mathematical Applications in Accounting** and has contributed numerous articles to professional publications.

porting allocations is as appealing as the input-output form. No other report format shows correspondence of inputs to outputs as well. No other report format is as easy to understand. No other report format is as simple. We shall use a numerical example to make this argument more concrete.

Exhibit 4 on page 18 presents the data for an illustrative problem. The problem deals with several of the usual complicating features of process costing, including opening inventories, incomplete products received from a previous department, units "gained" through adding departmental materials, lost units, and the reallocation of lost-unit costs.

The matrix solution to the problem appears in Exhibit 5, shown below. Exhibit 5 traces the generalized presentation of Exhibit 3. Three inputs—costs from preceding department, departmental materials, and departmental conversion costs—have been allocated to three designations—units transferred, units in ending inventory, and units lost.[3] The reallocation of lost-unit costs to the transferred- and ending-inventory designations has been done in the proportion these output designations have in the equivalent production of conversion.

Exhibit 6 on page 20 presents a conventional cost report treatment of this same process cost report. The purpose of presenting this exhibit is merely to provide something to compare with the input-

EXHIBIT 5

MATRIX SOLUTION TO PROCESS COST PROBLEM

Equivalent Production

A

Inputs \ Outputs	T	E	L
"Preceding"	1	1	1
"Materials"	1	2/3	2/5
"Conversion"	1	1/2	1/3

b, e

$$\begin{bmatrix}320{,}000\\150{,}000\\30{,}000\end{bmatrix} = \begin{bmatrix}320{,}000\\320{,}000\\320{,}000\end{bmatrix} + \begin{bmatrix}150{,}000\\100{,}000\\75{,}000\end{bmatrix} + \begin{bmatrix}30{,}000\\12{,}000\\10{,}000\end{bmatrix} = \begin{bmatrix}500{,}000\\432{,}000\\405{,}000\end{bmatrix}$$

Unit Costs:

$U_P = (\$120{,}000 + \$1{,}380{,}000) \div 500{,}000 = \$3.00$

$U_M = (\$\ 60{,}000 + \$\ 804{,}000) \div 432{,}000 = \$2.00$

$U_C = (\$\ 60{,}160 + \$1{,}187{,}240) \div 405{,}000 = \$3.08$

Cost Allocation:

$$\begin{array}{c} U \\ 3.00 \quad 2.00 \quad 3.08 \end{array} \qquad \begin{array}{c} D \\ \begin{bmatrix}320{,}000 & 150{,}000 & 30{,}000\\320{,}000 & 100{,}000 & 12{,}000\\320{,}000 & 75{,}000 & 10{,}000\end{bmatrix} \end{array} = R$$

R
COST REPORT

	Transferred	Ending Inventory	Lost	Costs to be Accounted for
Preceding department's costs	$ 960,000	$450,000	$ 90,000	$1,500,000
Departmental material costs	640,000	200,000	24,000	864,000
Departmental conversion costs	985,600	231,000	30,800	1,247,400
Totals	$2,585,600	$881,000	$144,800	$3,611,400
Reallocation of lost costs	117,306	27,494	(144,800)	—0—
Costs accounted for	$2,702,906	$908,494	$ —0—	$3,611,400

Reallocation: $\$144{,}800 \left(\frac{320{,}000}{395{,}000};\ \frac{75{,}000}{395{,}000}\right) = (\$117{,}306;\ \$27{,}494)$

Note: The totals surrounding the basic matrix, R, have been obtained merely by adding and cross adding. The multiplication UD did not produce these totals. Similarly, UD had nothing to do with reallocation.

CONVENTIONAL COST REPORT

	Total Cost	Unit Cost
Costs to be accounted for:		
Cost from preceding department:		
Opening inventory	$ 120,000	—
Costs during period	1,380,000	$3.00000
Departmental costs:		
Opening inventory		
Departmental material costs	60,000	—
Departmental conversion costs	60,160	—
Costs during period:		
Departmental material costs	804,000	2.00000
Departmental conversion costs	1,187,240	3.08000
	$3,611,400	$8.08000
Adjustment for lost units	—0—	.36658
TOTAL COST TO BE ACCOUNTED FOR	$3,611,400	$8.44658
Costs accounted for:		$2,702,906
Transferred (320,000 x $8.44658)		
Ending inventory:		
Preceding department costs (150,000 x $3.00)	$ 450,000	
Departmental material costs (100,000 x $2.00)	200,000	
Departmental conversion costs (75,000 x $3.08)	231,000	
Adjustment for lost units (75,000 x $.36658)	27,494	908,494
TOTAL COST ACCOUNTED FOR		$3,611,400

Additional computations:

Unit costs:

Preceding department costs: ($120,000 + $1,380,000) ÷ 500,000 = $3.00

Departmental material costs: ($ 60,000 + $ 804,000) ÷ 432,000 = $2.00

Departmental conversion costs: ($ 60,160 + $1,187,240) ÷ 405,000 = $3.08

Adjustment for lost units:

(30,000($3.00) + 12,000($2.00) + 10,000($3.08)) ÷ 395,000 = $.36658

EXHIBIT 6

output format of the cost report. It seems probable that only the initiated could follow the traditional cost report. The allocation of inputs to outputs is much more clearly presented in matrix format.

To expedite the discussions ahead, we introduce a form of matrix shorthand, shown in Exhibit 7 below.

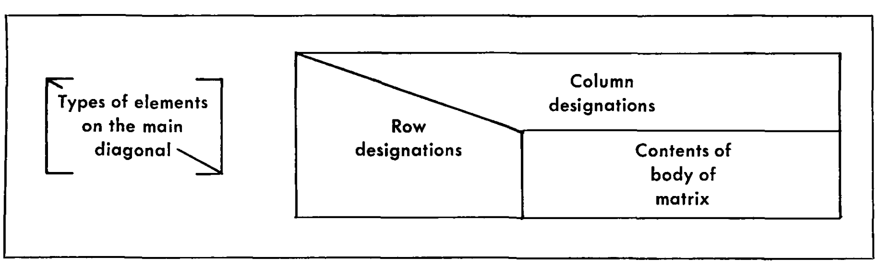
We could use this shorthand to summarize the matrices U, D, and R in Exhibit 3 as shown in Exhibit 8 on page 21.

Now let us turn our attention to Exhibit 9 on page 21, which contains an illustrative statement of affairs. Exhibit 9 presents the traditional format of this report, which again is probably understood only by the initiated. Exhibit 10 on page 22 shows how this report would look in input-output format. The matrix format emphasizes the distribution of inputs (types of assets) to output designations (types of claimants). With the exception of the row and column totals which were obtained by addition, the matrix report results from the multiplication shown in Exhibit 11 on page 23.

How well have matrices succeeded in further interrelating the process costing and statement of affairs areas? The matrix approach in both cases employed diagonalized matrices. The transformation matrices were composed of either quantities or proportions depending on whether the non-zero elements in the diagonalized matrices were dollars per unit or total dollars. Hence, the procedures of allocation in these areas are very similar under the matrix approach. The reports that resulted from matrix allocation are identical in format, and this is significant.

EXHIBIT 7

MATRIX SHORTHAND



When process costing and statements of affairs are first encountered, perhaps the single most time-consuming chore is to understand the separate report formats. Under the matrix approach only one, easy-to-understand report format is necessary.

Many accounting areas can be approached in exactly this same manner, that is, by the formulation of a diagonalized matrix and a transformation matrix to obtain an input-output matrix report.[4] The trick is to recognize data inputs and outputs as such and to determine the accounting criteria that govern the allocation. Usually, the accounting criteria can be reduced to simply measuring ownership or to reflecting usage. If any difficulty is encountered, it is likely to be not so much in recognizing inputs as in recognizing output designations.

Bonus-tax computations

There are other types of matrices that are important in accounting allocations. One of these is the inverse matrix. Although it would take too long to develop matrix inversion in full here, the broad concepts can be presented briefly if we restrict ourselves to systems in which there are two unknowns and two equations.

Consider the situation where it is necessary to calculate simultaneously an executive bonus based on profits after tax and a tax of some sort:

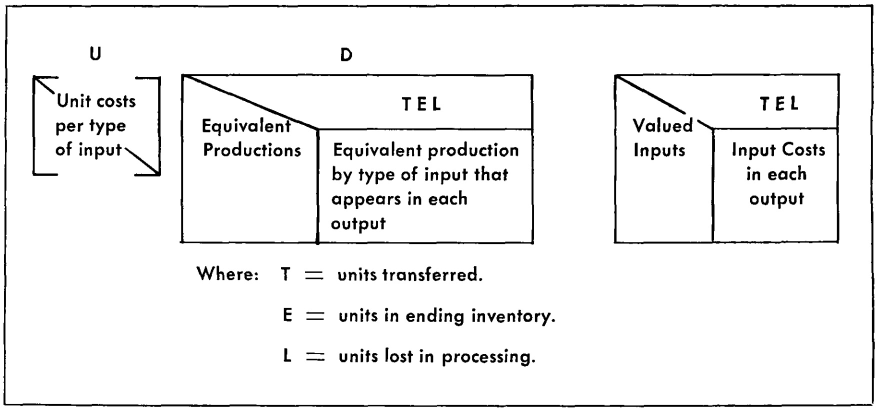
Key: B = Bonus
T = Tax
$90,000 = Profits before B and T
B = .20($90,000 — T)
T = .50 ($90,000 — B)

This system of equations can be restated and put into matrices as follows:

$$B + .2T = \$18{,}000$$
$$.5B + T = \$45{,}000$$

$$\underset{A}{\begin{bmatrix} 1 & .2 \\ .5 & 1 \end{bmatrix}} \underset{x}{\begin{bmatrix} B \\ T \end{bmatrix}} = \underset{b}{\begin{bmatrix} 18{,}000 \\ 45{,}000 \end{bmatrix}}$$

It is always wise to check the



PROCESS COST MATRICES

EXHIBIT 8

EXHIBIT 9

ILLUSTRATIVE STATEMENT OF AFFAIRS

Book Value			Expected to Realize
	Assets pledged with fully secured creditors:		
$25,000	Land and buildings:		
	Estimated value	$25,500	
	Less mortgage payments—contra	15,000	$11,500
	Assets pledged with partially secured creditors:		
3,000	Bonds of X Company—deducted contra		
	Estimated value	$ 3,200	
	Free assets:		
300	Cash		300
9,000	Accounts receivable:		
	$8,000 Good		8,000
	$1,000 Doubtful		600
	$9,000		
18,700	Merchandise		19,200
	Total free assets		$39,600
	Deduct liabilities having priority—per contra		600
$56,000			$39,000

Book Value			Expected to Rank
	Liabilities having priority:		
$ 600	Accrued wages—deducted contra		
	Fully secured liabilities:		
15,000	Mortgage payable—deducted contra		
	Partially secured liabilities:		
5,000	Notes payable	$ 5,000	
	Less bonds of X Company	3,200	$ 1,800
	Unsecured liabilities:		
23,000	Accounts payable		23,000
	Net worth per books:		
12,000	Capital stock		
400	Retained earnings		
	Total unsecured liabilities		$24,800
	Excess of net free assets over unsecured liabilities		14,200
$56,000			$39,000

An example of a case where matrix manipulation is useful is secondary overhead allocation . . .

matrix set-up by mentally performing the matrix multiplication $Ax = b$ to see that the original equations are obtained.

Now, as matrix algebra is ordinarily put forth, division by a matrix is undefined, that is, one could not solve for x by performing x = b divided by A as one would solve 5x = 20 by performing x = 20 divided by 5. Instead one must use an inverse matrix; this corresponds to solving 5x = 20 by performing x = 20(.2). Recognize that the multiplication of a number by its inverse yields the number 1 (for example, since the inverse of 5 is 1 divided by 5 = .2, we have 5(.2) = 1). So it is with matrices; the multiplication of a matrix A by its inverse A^{-1} yields the identity matrix, I. I has the property that when it multiplies another matrix the product of the multiplication is the other matrix. Note that this is the same result produced when we multiply the number 1 by some other number, for example, 1 x 5 = 5.

The procedure for solving our bonus-tax problem is as follows:

$$\begin{aligned} Ax &= b \\ (A^{-1}A)x &= A^{-1}b \\ (Ix) &= A^{-1}b \\ x &= A^{-1}b \end{aligned}$$

We may form A^{-1} by interchanging the main diagonal elements of A, putting minus signs next to the cross diagonal elements, and dividing the resulting elements by the product of the main diagonal elements minus the product of the cross diagonal elements (in our example: 1(1) – .5(.2) = .9). The solution to this example is shown in Exhibit 12 on page 23.

Secondary overhead allocation

Another example of a case in which this kind of matrix manipulation is useful[5] is secondary overhead allocation. Here primary overhead costs (such as indirect labor, repairs, depreciation, insurance, heat, light, power, and so forth) have been distributed to both service and production departments, and it remains necessary to

EXHIBIT 10

STATEMENT OF AFFAIRS—MATRIX FORMAT

	Liabilities Having Priority	Fully Secured Liabilities	Partially Secured Liabilities	Unsecured Liabilities	Owners	Totals
Assets: Pledged in full security (Land)	$—0—	$15,000	$ —0—	$ —0—	$ —0—	$15,000
Pledged in partial security (Bonds owned)	—0—	—0—	3,200	—0—	—0—	3,200
Free (See note)	600	—0—	1,800	23,000	14,200	39,600
	$ 600	$15,000	$5,000	$23,000	$14,200	$57,800

Note: Free assets include:		
	Cash	$ 300
	Accounts receivable	8,600
	Merchandise	19,200
	Land & buildings ($26,500 — $15,000)	11,500
		$39,600

DEFICIENCY ACCOUNT

Owners' equity per books	$12,400
Gains on realization:	
Land and buildings	1,500
Bonds of X Company	200
Merchandise	500
	$14,600
Loss on realization:	
Accounts receivable	400
Amount payable to owners in liquidation	$14,200

. . . where primary overhead costs have already been distributed.

reallocate service department costs to service-consuming departments (secondary allocation) so that overhead absorption rates may be determined. Deciding the percentages of services consumed involves the accountant in estimating potential and actual usage of departmental services.

Let us consider a simple illustration. Assume that the percentages reflecting usage have already been determined and are as shown in Exhibit 13 on page 24.

There are two approaches to be considered: (1) the traditional approach, whereby the primary costs of the service-rendering departments are first augmented by the costs these departments are responsible for as service consumers and then the new totals are allocated to the production departments and (2) the "linked" approach, whereby the intermediate stage is omitted since it serves no purpose.

Under the traditional approach, augmenting the service department primary costs is accomplished by solving the following system of equations:

$$S_1 = 90 + .25S_2$$
$$S_2 = 180 + .40S_1$$

The system may be stated in matrices as follows:

$$\begin{array}{ccc} A & x & b \\ \begin{bmatrix} 1 & -.25 \\ -.40 & 1 \end{bmatrix} & \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = & \begin{bmatrix} 90 \\ 180 \end{bmatrix} \end{array}$$

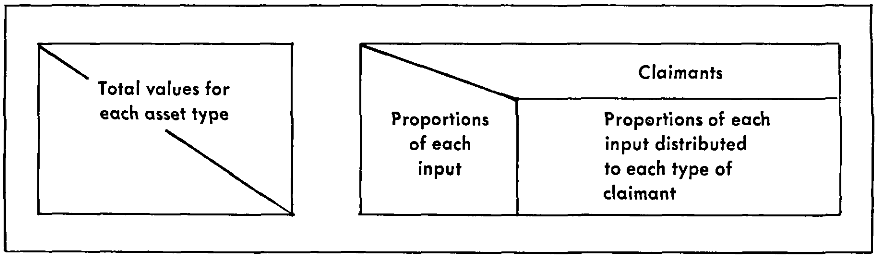
TOTAL =	270

The solution is:

$$\begin{array}{cccc} x & = & A^{-1} & b \\ \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} & = & \begin{bmatrix} 1/.9 & .25/.9 \\ .40/.9 & 1/.9 \end{bmatrix} & \begin{bmatrix} 90 \\ 180 \end{bmatrix} = \end{array}$$

$$\begin{array}{c} x \\ \begin{bmatrix} 150 \\ 240 \end{bmatrix} \end{array}$$

Vector x contains the augmented service department costs.



MULTIPLICATION FOR STATEMENT OF AFFAIRS—MATRIX FORMAT

EXHIBIT 11

SOLUTION TO BONUS—TAX PROBLEM

$$\begin{array}{ccccc} x & = & A^{-1} & b & \\ \begin{bmatrix} B \\ T \end{bmatrix} & = & \begin{bmatrix} 1/.9 & -.2/.9 \\ -.5/.9 & 1/.9 \end{bmatrix} & \begin{bmatrix} 18,000 \\ 45,000 \end{bmatrix} & = \begin{bmatrix} 10,000 \\ 40,000 \end{bmatrix} \end{array}$$

This says that B = \$10,000 and T = \$40,000.

EXHIBIT 12

Now the amounts in Vector x must be allocated to the production departments. Accordingly, we form Matrix P by transposing the percentages shown under the P_i and use this matrix to obtain our ultimate amounts for redistribution (shown in Vector r).

$$\begin{array}{ccc} P & x & r \\ \begin{bmatrix} .10 & .45 \\ .30 & .20 \\ .20 & .10 \end{bmatrix} & \begin{bmatrix} 150 \\ 240 \end{bmatrix} = & \begin{bmatrix} 123 \\ 93 \\ 50 \end{bmatrix} \end{array}$$

TOTAL =	270

The amounts in Vector r must then be added to the primary allocation amounts for the production departments (say, Vector d) to obtain the total overhead costs (Vector t) for each production department.

$$\begin{array}{ccc} r & d & t \\ \begin{bmatrix} 123 \\ 93 \\ 54 \end{bmatrix} + & \begin{bmatrix} 377 \\ 307 \\ 246 \end{bmatrix} = & \begin{bmatrix} 500 \\ 400 \\ 300 \end{bmatrix} \end{array}$$

The amounts in Vector t would next be divided by the respective estimated standard machine hours to obtain the desired overhead absorption rates of \$500 divided by 200 = \$2.50, \$400 divided by 50 = \$8.00, and \$300 divided by 150 = \$2.00.

The alternate or "linked" approach recognizes the uselessness of the augmented service department totals of \$150,000 and \$240,000 (shown in Vector x). Control over the reallocated portions of these totals (that is, over \$150,000 – \$90,000 and \$240,000 – \$180,000) is typically achieved by the "departmental cross charges" of responsibility accounting. Hence, for product costing purposes the intermediate augmented service department totals may be bypassed, provided the effects of these totals are provided for.

Since matrices may be multiplied and added, it is possible to "link up" several stages of allocation. In our secondary overhead allocation

SECONDARY OVERHEAD ALLOCATION PROBLEM

Key: P_i = Service department i.
S_j = Production department j.

Consumers / Renderers	S_1	S_2	P_1	P_2	P_3
S_1	0	40%	10%	30%	20%
S_2	25%	0	45%	20%	10%

	S_1	S_2	P_1	P_2	P_3
Primary overhead allocation totals (000 omitted)	$90	$180	$377	$307	$246
Standard Machine hours (estimated, 000 omitted)			200	50	150

EXHIBIT 13

Besides organizing the calculation of variances and aggregating inputs to aid in determining the overall significance of the respective variances, the matrix approach permits ready calculation of the significance of individual input variances.

example, for instance, we could proceed as follows:

$$t = d + PA^{-1} b$$

Let us first form PA^{-1}. It would always make sense to do this where the departmental interrelationships can be expected to remain stable —as they might for planning purposes.

$$\underset{P}{\begin{bmatrix} .10 & .45 \\ .30 & .20 \\ .20 & .10 \end{bmatrix}} \underset{A^{-1}}{\begin{bmatrix} 1/.9 & .25/.9 \\ .40/.9 & 1/.9 \end{bmatrix}} = \underset{PA^{-1}}{\begin{bmatrix} .3111 & .5278 \\ .4222 & .3055 \\ .2667 & .1667 \end{bmatrix}}$$

We see that the equation for t holds.[6]

$$\underset{t}{\begin{bmatrix} 500 \\ 400 \\ 300 \end{bmatrix}} = \underset{d}{\begin{bmatrix} 377 \\ 307 \\ 246 \end{bmatrix}} + \underset{PA^{-1}}{\begin{bmatrix} .3111 & .5278 \\ .4222 & .3055 \\ .2667 & .1667 \end{bmatrix}} \underset{b}{\begin{bmatrix} 90 \\ 180 \end{bmatrix}} = \begin{bmatrix} 377 \\ 307 \\ 246 \end{bmatrix} + \begin{bmatrix} 123 \\ 93 \\ 54 \end{bmatrix}$$

Other applications

Matrices may be helpful in price-level work and traditional variance analysis. Let us consider the analysis of labor variances. Here the inputs involve wage rates for different categories of labor; transformation involves labor hours, and the outputs are the standard costs and variances. An example is shown in Exhibit 14 on page 25.

Individual input calculation

Besides organizing the calculation of variances and aggregating inputs to aid in determining the overall significance of the respective variances, the matrix approach permits ready calculation of the significance of individual input variances. For instance, since the vector of standard wage rates is arrayed on top of the rate changes vector, it would be an easy matter to determine percentages of change (for example, –.25 divided by 3.00 = –8 1/3 per cent, 1 divided by 4 = 25 per cent, etc.). Then those percentages that exceed a stipulated amount can be further investigated. Similarly, calculations could easily be made for changes in hours. In this way, the matrix approach could be used to implement statistical "quality" control techniques.

Conclusions

This review of some of the rudiments of matrix algebra and its ap-

LABOR VARIANCE ANALYSIS

Given data:

Labor type A:
Standard = 600 hours at \$3 per hour
Actual = 640 hours at \$2.75 per hour

Labor type B:
Standard = 1000 hours at \$4 per hour
Actual = 900 hours at \$5 per hour

Labor type C:
Standard = 800 hours at \$2 per hour
Actual = 1000 hours at \$2.50 per hour

Matrix solution:

Key:
P = standard wage rate
ΔP = change in wage rate
$P+\Delta P$ = actual wage rate
Q = standard hours
ΔQ = change in standard hours
$Q+\Delta Q$ = actual hours

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} P \\ \Delta P \end{array} & \begin{bmatrix} 3 & 4 & 2 \\ -.25 & 1 & .50 \end{bmatrix} \end{array} \quad \begin{array}{cc} & \begin{array}{cc} Q & \Delta Q \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 600 & 40 \\ 1000 & -100 \\ 800 & 200 \end{bmatrix} \end{array} =$$

Standard	Net Efficiency Variance
\$7,400	\$120
\$1,250	−\$ 10
Net Wage Variance	Net Mixed Variance

Note: The signs attached to the net variances may be interpreted as follows: — indicates a favorable variance; + indicates an unfavorable variance.

EXHIBIT 14

plications to the field of financial accounting offers a basis for putting forth the following claims:

1. With matrix algebra, inputs and outputs in the various accounting areas can be more easily recognized as such.

2. Matrix algebra can be accepted as a basic way of accomplishing the allocation of inputs to outputs.

3. Matrix algebra may be considered as offering one or two procedures to accomplish allocation instead of the myriad of procedures presently in use.

4. The input-output form of report may be recognized as being superior to most other forms. This is true not only because it is readily understood but also because it is of significant help in the interrelation of a number of diverse accounting areas.

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[1] A. C. Littleton, *Structure of Accounting Theory,* Monograph Number 5, American Accounting Association, 1953, p. 123.

[2] Note that the multiplication of the vectors would yield only the body of Matrix L; the rim totals have merely been obtained by addition. Such addition could be accomplished in matrix algebra by use of sum vectors, that is, vectors all elements of which are ones. However, this use of sum vectors would only be a mathematical nicety and would needlessly complicate our example.

[3] These outputs exhaust the set of possibilities; units can still be in process, or they can be completed, or they can be lost in some way—nothing else can take place. The matrix approach accords lost units full status as an output designation. This logical view of lost units is not found in most cost accounting texts, but it is ably put forth in Charles T. Horngren, *Cost Accounting—A Managerial Emphasis,* Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1962.

[4] Some of the other accounting areas that can be treated this way include job-order costing, standard costing, period budgeting, primary overhead allocation, and responsibility accounting.

[5] A third example involving an inverse matrix occurs in consolidated financial statements. Here the inputs are intercompany profits in inventory, fixed assets, and bonds that are made by each constituent company. The outputs are the majority and minority interests. When the intercompany relationships are entered in a matrix and adjusted to reflect effective interests, the resulting transformation matrix may be used to determine the adjusting entries to correct the various retained earnings accounts.

[6] Further discussion of this type of transformation may be found in Neil Churchill, "Linear Algebra and Cost Allocation: Some Examples," *The Accounting Review,* October, 1964.