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What is management accounting information? Surely, more than just a collection of data. Rather, it should reduce uncertainty to the minimum, demand action, and indicate the appropriate action.

MEASURING THE VALUE OF INFORMATION— AN INFORMATION THEORY APPROACH

by Norton M. Bedford, University of Illinois
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How much and what kind of information does an executive really need for a given decision? Up to what point does the collection of additional information produce more return than the cost incurred in the collection?

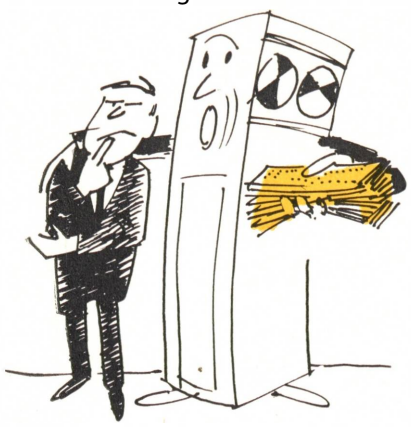
This has always been a problem for accounting executives, but it has not been an overwhelming one. The cost of expanding the traditional information provided by accounting for planning purposes is ordinarily not great, nor is it particularly difficult to determine.

With the growing emphasis on scientific techniques in management planning, however, this problem is becoming more acute. The new accounting planning models,[1] based on such techniques as regression analysis and linear programing, do seem to provide quantitative data closer to the underlying state of nature than that provided by the traditional accounting process. These models, however, are expensive to develop, and the data that go into them are costly to gather. The accountant needs quantitative measurements of the "amount" and "value" of the information provided by the model.

A possible way of locating the point at which the cost of data collection outweighs the value of the information gained is provided by the application of "information theory." This article outlines recent progress in information theory and indicates how it can be applied to accounting measurement and communication.

Information theory is a body of knowledge that has been developed in recent years. Its uses in different disciplines are increasing widely. Although a precise definition of information theory has not yet been established,[2] its basic concepts are generally accepted. There are three basic concepts that should be discussed: the information concept, the concept of measuring the "amount" of information, and the concept of the "value" of information.[3]

Information is conventionally defined as obtained knowledge, facts, data, or news, even though it has



When a number of choices are equally probable, information indicating the one best action can often become almost invaluable.

more precise definitions.[4] The main inadequacy of the conventional definition is that distinctions exist among data, information, and knowledge.

To illustrate, accounting data customarily are assumed to represent material that may be used as a basis for inference but is not evaluated for its worth to a specific individual for a specific situation. Such data are "collections of signs and characters generally arranged in some orderly way to represent facts — any facts that are a matter of direct observation."[5]

Information, on the other hand, represents accounting data evaluated for a specific use. To an accountant, information is the fundamental material upon which intelligent action is based. It has the connotation of significant data. Significance has been defined as a measure of the net value obtained from matching the needs of a specific problem with appropriate elements of data.[6]

The distinction between information and data is that information is concerned with the use of evaluated data for a specific problem and for a certain individual at a certain time to achieve a definite goal. As problems vary, persons change, or time passes, the value of information differs. That is, the value of information is not detached and permanent in itself. Its value is a function of its uses.

Knowledge, the third common section of information, represents data evaluated or information accumulated for general use in the future. Its value does not depend upon its use by a particular person.

The concept of "information" may be clarified by relating it to the decrease in ignorance that is experienced rather than to the amount of knowledge obtained.[7] There are advantages to the view that information is a process of ignorance reduction rather than a process of knowledge creation. The need to reduce ignorance seems to motivate and stimulate accounting research and study. If the accountant concentrates on what is known, he may tend to become satisfied with the status quo and may exert less effort to acquire further information.

Regardless of the manner in which information is viewed, its function is to reduce the amount or range of uncertainty under which decisions are made. In this sense, the more information there is supporting an accounting estimate of probable costs and revenues of possible plans the more accurate is the estimate and the smaller the range of possible errors. An accounting estimate is improved by reducing the uncertainty under which it is made. Thus, the "amount of information" is measured by the reduction of ignorance and uncertainty and not by the addition of knowledge.[8]

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The conventional definition of information as knowledge, facts, and data may have been adequate for the recording of economic events. The second definition of information will be more useful for the management accountant when applying information theory to the planning function.

'Amount' of information

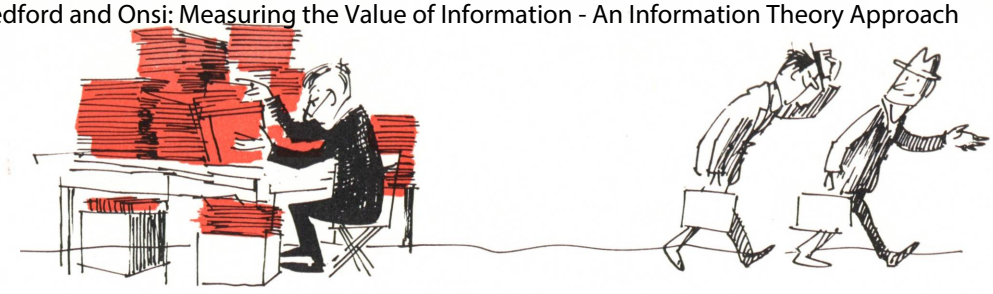
The "amount" of information is equal to the reduction of uncertainty. Stated in other terms, the amount of information is a function of the "unexpectedness of an event."[9] The assumption underlying this definition has been expressed in the following manner: "If the message to be transmitted is known in advance to the recipient, no information is conveyed to him by it. There is no initial uncertainty or doubt to be resolved; the ensemble of a priori possibilities shrinks to a single case and hence has zero uncertainty. The greater the intial uncertainty, the greater the amount of information conveyed when a definite choice is made."[10]

According to information theory, the more uncertain the decision maker is in selecting a certain course of action the greater is the amount of information supplied by useful data. If, before new informamation is supplied, a possible "choice" by a decision maker is practically certain (has a probability near one) and the other possible choices have probabilities of being selected near zero, the amount (and value) of any additional information are low. On the other hand, if all possible courses of future action have an equal probability of being selected, any data that would reveal which choice should be made would represent a large amount of information.

Two propositions follow:

1. When the number of choices is fixed, the amount of information in data is greater when the probabilities of the various choices are nearly equal.

2. If all choices are equally likely, the more choices there are



Information is valuable only when it results in gain; it should be gathered only to the point where incremental cost of additional information equals the profit that can be earned through having it.

the greater the amount of information. There is more information in data if a decision maker has to make a choice from a set of fifty possible actions than if he selects one out of a set of twenty-five. If there are sixteen alternative choices, data will provide more information if, by its use, the alternative choices are reduced to four rather than to eight.[11]

'Value' of information

The value of information implies its effectiveness and usefulness. Information conveyed to a receiver should induce the desired conduct. Usefulness implies a relationship to a purpose or a goal, and the value of information is determined by the changes caused through the use of the information in the pursuit of a purpose. The value of information is measured by comparing the outcome of the actions of the decision maker before and after the receipt of the information. Ackoff suggests three ways in which information may change the actions of a decision maker. A message *informs* if it changes the probability of choice or potential course of action of an individual. A message *instructs* if it indicates a basis of choice from among potential courses of action. A message *motivates* if it changes the value of the outcome of a course of action.[12]

Fundamentally, information may have value for one person but not for another, depending on its relevance for potential action by an individual in a certain environment and time. A message may have a large amount of data without being "informative" in the sense of saying something useful or even true. The value of information must be measured by the receiver in terms of its uses to him.

The flexibility of information usage influences its value. There are two dimensions to the problem of determining the worth of flexible information in decision making. The first indicates that in the same environment one piece of information has a wider usage if it can be used for more than one potential decision. The other dimension indicates that one piece of information has wider usage if it can potentially be used in several environments. The difficulty, however, is that the accountant cannot anticipate all desirable uses of the information that could be or is provided to the different departments in the organization. This makes it difficult for the accountant to assign a probability for the use of specific items of information. However, this need not handicap the accountant in developing from past experience a type of subjective probability of the uses of information.

Recipient important, too

The value of information depends not only on its potential use to the recipient but also on his reception and interpretation of the message. This type of effectiveness is influenced by psychological factors of behavior. By regarding a human being as an information-receiving device, several psychological experiments have thrown light upon an individual's ability to discern meanings.[13]

The value of information, from an economic point of view, is measured by the gain achieved from using such information. Marschak notes that "to an economist it seems natural to call value of information the average amount earned with the help of that information."[14] The underlying premise is that "information should be gathered up to the point where the incremental cost of additional information is equal to the incremental profit that be earned having it."[15]

The psychological and economic dimensions of information are not in conflict. The psychological dimension is concerned with the response of the receiver to information, while the economic dimension is concerned with the outcome of using such information. Management accounting, as a quantitative discipline, is concerned with both dimensions, since it deals with both people and economic events.

Planning uses

The use of information theory in management accounting is based on the assumption that "any organism is held together in action by the possession of means for the acquisition, recording, preservation, transmission, and use of information."[16] Management accounting information is informative if it tells the receiver something not already known. Information is gained only about matters in which there is some degree of uncertainty. The amount of accounting information

supplied for planning is equal to the reduction of uncertainty that it provides. Once the accountant is able to measure uncertainty, he can also measure accounting information in similar terms. Accordingly, management accounting information has to meet these requirements:

1. It must be informative in the sense of decreasing the amount of uncertainty.
2. It must demand action. The collected information is worthwhile if, and only if, it indicates action relevant for the achievement of a certain goal.
3. It must motivate an appropriate action. Unsatisfactory results must be prevented by its use.

Planning under conditions of certainty generates no information, since there is no reduction in uncertainty. Planning under conditions of risk and uncertainty generates information which can be measured according to the concepts of information theory. Information, in these cases, can be measured if the number of all possible relevant events and the probability of each occurring are given. The management accountant is concerned with those events whose presence or absence would have a direct effect on decisions. By a study of past experience or by sampling, an accountant can determine the kinds of patterns existing in a sequence of past activities by placing them in the form of a frequency distribution. Information theory measures the amount of information generated by detecting these patterns and determining the way in which one outcome has followed another.

Planning under conditions of certainty generates no information, since there is no reduction in uncertainty. Planning under conditions of risk and uncertainty generates information that can be measured.

The impact of information theory on concepts underlying the accounting budget depends on the validity of the premise that "the 'modern entrepreneur' is probabilistic omniscient: that he knows the probability distribution of outcomes from all alternatives."[17] The idea of probability distribution and the concept of "expected income" are, therefore, appropriate notions for accounting planning under conditions of uncertainty. By developing accounting planning models on a probability basis, information theory may be used to provide a measurement of the amount of information generated and its value for a certain use. The following illustrations indicate how this could be done:

1. *The probabilistic sales budget.* Probability is introduced into a sales forecast by treating probable sales as a frequency distribution or as having multiple values, rather than as a single-valued estimate. It is hypothesized that unless the sales forecast is defined in terms of probability, it is impossible to develop realistic sales budgets.[18] A sales forecast represents a probability distribution of expected sales constructed by applying either time series analysis or modern decision theory. The probability distribution of expected sales may be normal or skewed, depending on the enterprise situation. The implication of the probabilistic budget is that the accounting sales budget is not a single value but a multiple of values. The decision level, which represents the sales level for the next period, indicates the number of units that should be produced to maximize expected profit. This is achieved when the expected profit from producing an additional unit is equal to the expected loss of not selling it.

2. *The probabilistic cost budget.* Variable cost and fixed cost per unit of output may be random variables. Neither fixed nor variable cost per unit of output can be determined precisely as a single value. While such costs may not be known with certainty, a probability distribution of their possible values for the forthcoming output is either known or can be reasonably approximated.

Information on the behavior of production costs may be obtained from an analysis of past data. If the probability of various quantities of inputs used to produce a unit of output is known and a standard price is set, a frequency distribution of various unit costs of output can be developed. The frequency distribution of expected

cost may be normal or skewed. An accounting cost budget no longer consists of one value but of a multiple of values. For example, for a certain level of output, the expected unit cost may be 50¢ with a probability of .75 or 52¢ with a probability of .25. For each sales level, there is an equivalent production level with different values, each with a certain assigned probability.

3. *The probabilistic profit budget.* A probability distribution of profit for each course of action can be developed. Assuming that demand and total cost are probabilistic variables and that the distribution of expected sales is independent of the distribution of costs, the distribution of profits may be developed as the joint distribution of revenues and costs. For each level of output, a probability distribution of profit may be developed and expected income calculated. Accordingly, it is no longer valid to say that the accounting profit budget is a residual. After developing the accounting budget in probabilistic terms, the amount of information generated by the model can be measured.

'Amount' measurement

According to information theory, information contained in the occurrence of a particular event (i.e., sales, cost, profit) depends on its probability of occurrence. If an event is certain to occur, the actual occurrence of that event contains no information; the information generated is: $H = \log_2 1 = 0$ bits.[19] The smaller the probability of occurrence of a certain event the higher the amount of information generated. If the probability is ¼, the information generated is: $H = \log_2 ¼ = 2$ bits.

If there is a set of sales values $X_1, X_2, X_3, \ldots X_n$, and the probability of occurrence assigned to each is $P_1X_1, P_2X_2, \ldots P_nX_n$, the information generated from the occurrence of a particular event, i.e., X_1, is equal to: $H = -\log_2 P_1(X_1)$. However, the average information received from the occurrence of one event in a set of sales values, $H(X)$, is:

$$H(X) = \sum_{i=1}^{n} P(X_1)\ 1(X_1);$$

$1(X_1) = -\log_2 P_1(X)$

= the summation of the self-information of each sales event X_1 weighted by its probability of occurrence.

Proceeding in applying information theory, assume that: $P_1(X_1)$ is the initial or a priori probability of sales (X_1) occurring and $P_2(X_2)$ is the posteriori probability of such a sales value (X_1) occurring after the first evidence is received. The information generated, assuming such a sales value occurs before receiving the first evidence, is:

$$H = [-\log_2 P_1(X)] - [-\log_2 P_2(X)]$$

$= \log_2$ Probability of sales after receiving message, i.e., posteriori prob., $P_2(X) \div$ Probability of sales before receiving message, i.e., a priori prob., $P_1(X)$

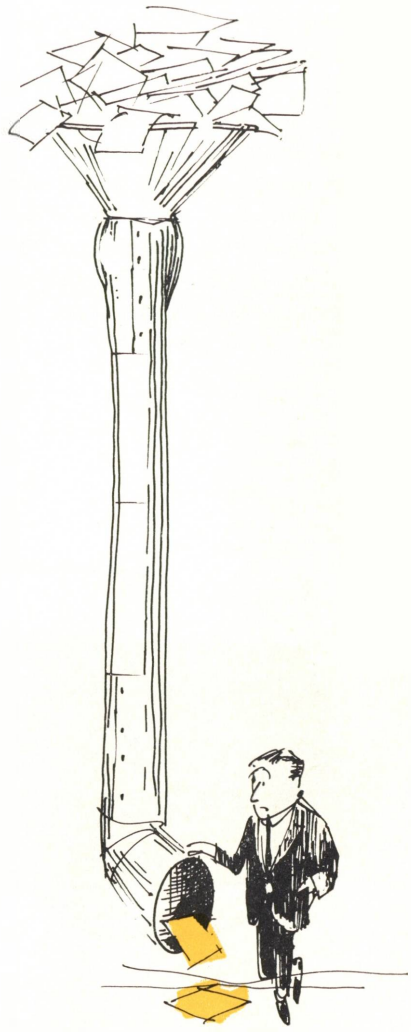
To measure the amount of information generated in cases of independent events, assume that sales $(X_1, \ldots, X_n)$ and cost $(Y_1, \ldots, Y_n)$ are two independent sets of events. Assuming that a certain sale value (X_1) and cost (Y_1) may occur, the information of the joint occurrence of the sale value (X_1) and the cost value (Y_1) is equal to summation of the information of each:

$$H(X_1, Y_1) = (X_1) + (Y_1)$$

For example, if the probability of expected sales (X_1) as 20,000 units is ¼ and the probability of expected unit cost (Y_1) of producing such quantity as $10 is 1/16, the amount of information generated by the occurrence of these two independent events is:[20]

$$H(X_1, Y_1) = (-\log_2 ¼) + (-\log_2 1/16) = 2 + 4 = 6 \text{ bits}$$

Through estimating the bits of information generated from ac-



An organization cannot function smoothly if the amount of information to be sent through any channel exceeds the capacity of that channel. . . .

counting data, the determination of the kind and size of information network is possible. Knowledge of the amount of information transmitted from one department to another or from one level to another is used in determining such factors as the channel capacity or kind of channels. The underlying premise is that "an organization will not function smoothly if the amount of information to be sent on any channel or processed at any center exceeds the capacity of that channel or center."[21] The flow of information between any two levels of management or between two departments can be compared with the maximum attainable flow of information to indicate the inefficient channels. The problem may be solved by (a) increasing or decreasing accounting channel capacity — the number of accounting employees, machines, or types of reports supplied; (b) decreasing the redundancy of reports by developing codes that can be used for different purposes of analysis; (c) routinizing reports indirectly via underutilized channels; or (d) reassigning some functions of overloaded centers to decrease noise in accounting reports.

Management accounting, as an information center, is concerned with the measurement of the amount of information and the efficiency of its communication within the organization. This concern means that accounting must inevitably be concerned with information theory.

'Value' measurement

Not only must the amount of information be measured but the expected value of such information should also be specified. The relevant question is how far the management accountant should go in an attempt to gather information. What weights should be placed on various sources and types of information that differ in reliability? It may be believed that to reduce uncertainty, the accountant has to gather and synthesize more data about the future. There is, however, a limit on resources to be expended for research and a point of diminishing returns on the information-gathering process. The problem is to what extent the managerial accountant should replace less reliable expectations at lower costs with more reliable expectations at higher costs.

Using the expected value model, it is possible to quantify the value of additional information. If the outcome of the best act under uncertainty is calculated (EMV) and if the expected profit under certainty is obtained, the difference represents the expected value of perfect information (EVPI). The expected value of perfect information is equal to the expected opportunity loss of the optimum act but not equal to the expected opportunity loss (EOL) of any other act, if such an act is chosen. The EOL may be decreased by choosing the optimum act without any need for additional information at

The relevant question is: What weights should the management accountant place on the various types of data that differ in reliability?

. . . the expected value of such information should also be specified.

the moment, but the EOL of the optimal act can be decreased only by obtaining new information and decreasing the uncertainty involved. The value of new information is measured relative to the optimal action, given the information already available.

To indicate the way expected value of information increases with uncertainty, assume that the distribution of expected sales has a mean, E (u), equal to 1,000 units and a standard deviation of 150 units. The breakeven point may be more, equal to, or less than the expected mean of sales. (The method of analysis can be applied in each situation.) Assume that the breakeven point is equal to the sale of 775 units. If the marginal profit per unit is $2, the expected profit line (see the illustration on this page) may be fitted in a probability density function, assuming that the Y axis measures two scales: the height of the density function and the expected profit.

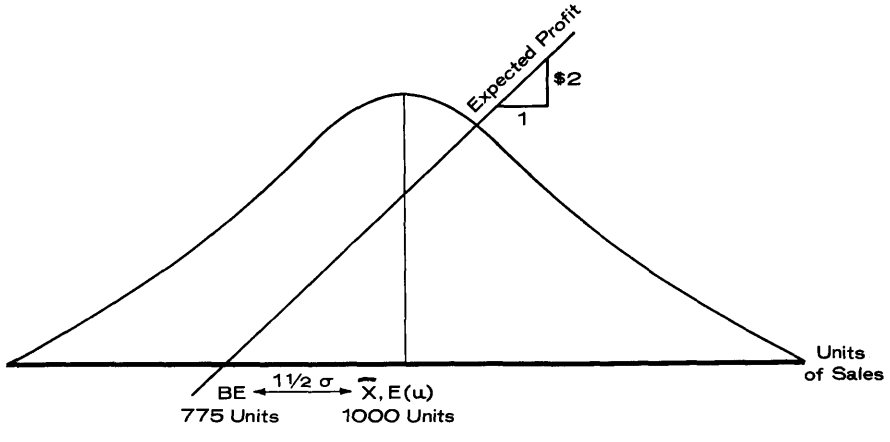
Criterion of acceptance

If the submitted project is at least equal to the breakeven point, it is accepted; otherwise it is rejected. To express an opinion, the accountant may gather information before the final decision is made. In making a decision, he will be concerned with the expected value of gathering new information. The accountant faces many alternatives. First, if the actual event (X) will occur at or above the breakeven point (i.e., X>BE), no decision will be changed. The conditional value of the information is zero (CVPI = 0), since it does not change the decision to be made. Second, if the event (X) will occur at less than the breakeven point (i.e., X<BE), the decision will be changed. The conditional value of perfect information, accordingly, will be positive and will increase with the value of the actual event. The conditional opportunity loss (equal to EVPI) is equal to the profit margin per unit multiplied by the difference of the breakeven point from the actual point; i.e. [C(BE-X)].

The expected value of perfect information in this case is calculated by multiplying the conditional value of perfect information by the probability distribution. The expected value of information will be little or nothing if the mean of sales is certain, having a probability of occurrence near 1.

From such investigation, two concepts emerge. The first is the "uncertainty cost" concept. The uncertainty cost is equal to the difference between the expected profit under certainty and the maximum expected profit under certainty. In other words, it is equal to the expected opportunity loss of the best decision under a given probability distribution (i.e., minimum opportunity loss). Such an uncertainty cost represents a motive to gather new information to reduce uncertainty. This concept will assist the management accountant in deciding when to stop collecting new evidence. If the cost of collecting additional information is higher than the uncertainty cost, the accountant will stop collecting new evidence.

The second concept is the "irrationality cost" concept. It represents the amount by which the expected opportunity loss of the chosen decision exceeds the cost of uncertainty according to the given probability distribution. Such a cost does not represent the uncertainty of the conditions under which the decision is made but represents the degree of irrationality on the part of the decision maker. The management account-



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ant is concerned with the irrational cost. It is his responsibility to direct the attention of management toward the costs which could have been avoided.

Conclusions

The application of information theory to accounting measurement and communication is promising. It adds new concepts and techniques to the accountant's knowledge that refine and supply new dimensions to his data. However, the scope of the application of information theory in accounting is not broad, nor does it promise solutions to every problem. Information theory will not indicate from where data can be collected to solve a problem, nor will it determine the decision that has to be made in a certain problem. To apply the information theory, the accountant has to have reliable and relevant data indicating the whole ensembles of events in a certain situation, expressed in their relative frequencies.

A distinction has been made between the amount and value of information. Information value and information amount do not necessarily correspond. The ranking of information structures according to their value is a subjective matter, depending on its usefulness for a given user and the payoff function. On the other hand the amount of information is independent of the payoff function and it reaches its maximum when the messages are equiprobable and, as such remove the highest degree of uncertainty.

The emphasis on measuring the amount of information in the system as a whole and at each level of reporting is a result of developing an information flow matrix for the entire organization. Within this matrix, not only the periodicity of information flow and time-lag problems but also the amount of information and the redundancy of information can be manipulated. A simulated model of the amount of information that can be sent over a given channel(s) with a certain noise can be developed.

It is our belief that continuing research will increase the reliability and significance of accounting planning and control systems.

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[1] Richard Mattessich, "Budgeting Models and System Simulation," *Accounting Review*, July, 1961, pp. 384-397; Andrew C. Stedry, *Budget Control and Cost Behavior*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1960, pp. 113-143.

[2] Information theory is sometimes defined as a branch of the mathematical theory of probability. Another definition is that information theory is concerned with the problem of measuring changes in knowledge. See S. Kullback, *Information Theory and Statistics*, John Wiley and Sons, Inc., New York, 1959, p. vii, and Donald M. Mackey, "The Nomenclature of Information Theory," in *Cybernetics*, ed. Heinz Von Forester, Josiah Macy, Jr., Foundation, New York, 1952, p. 222.

[3] Information theory can be discussed on three levels: (1) the technical level, (2) the semantic level, and (3) the effectiveness level. The other concepts of information theory are the "noise," the "redundancy," and the "channel capacity" concepts.

[4] In philosophy, information is the process by which the form of an object of knowledge is impressed upon the apprehending mind to bring about a state of knowledge. In logic, according to *Webster's New International Dictionary of the English Language*, 2nd ed., G. & C. Merrian Co., Springfield, Massachusetts, 1945, p. 276, information is a "logical quantity belonging to propositions and arguments. . . ."

[5] Hector R. Anton, "Some Aspects of Measurement and Accounting," *Journal of Accounting Research*, 2, Spring, 1964, p. 3.

[6] Adrian M. McDonough, *Information Economics and Management Systems*, McGraw-Hill Book Company, Inc., New York, 1963, p. 76.

[7] Frederick A. Ekeblad, *The Statistical Method in Business*, John Wiley and Sons, Inc., New York, 1962, p. 37.

[8] For a comparison and measure of ignorance and uncertainty, see: T. F. Schouten, "Ignorance, Knowledge and Information," in *Information Theory*, ed. Colin Cherr, Academic Press, Inc., New York, 1956, pp. 37-47.

[9] Donald M. Mackey, "Information Theory and Human Information Systems," *Impact of Science on Society*, 8, No. 2, 1957, p. 88.

[10] Jerome Rothstein, "Information, Measurement, and Quantum Mechanics," *Science*, 113, August, 1951, p. 172.

[11] Information from a choice in a given set is measured, by Hartley, as the logarithm to the base 2 of the total number of choices available in the set. This assumes that the choice of one event from a set is as likely as any other. However, this assumption in general is not true. Shannon, accordingly, measures information in terms of probability and not in terms of choice. His equation measures the average amount of information per event of all the events in a set as follows: $H_N = \sum_{i=1}^{N} P_i \log P_i$. See: Claude E. Shannon and Warren Weaver, *The Mathematical Theory of Communication*, University of Illinois Press, Urbana, 1963, p. 106.

[12] Russell L. Ackoff, "Towards A Behavioral Theory of Communication," *"Management Science*, 4, April, 1958, p. 220.

[13] George Miller discusses the ability of individuals to distinguish between stimuli in, "The Magical Number Seven Plus or Minus Two," *Psychological Review*, 73, March, 1956, p. 81-97.

[14] J. Marschak, "Remarks on the Economics of Information," in *Contributions to Scientific Research in Management*, University of California, Los Angeles, January, 1959, p. 80.

[15] Herbert A. Simon, "Theories of Decision-Making in Economics and Behavioral Science," *The American Economic Review*, 49, June, 1959, p. 270.

[16] Francis Bello, "The Information Theory," *Fortune*, 48, December, 1953, p. 137.

[17] Richard M. Cyert and James G. March, *A Behavioral Theory of the Firm*, Prentice Hall, Inc., Englewood Cliffs, New Jersey, 1963, p. 45.

[18] R. W. Crawford, "The Probability Budget — A New Management Concept," in *Operations Research Applied*, American Management Association, Special Report, No. 17, American Management Association, New York, 1957, p. 67.

[19] The amount of information, in information theory, is measured in terms of bits, from "binary digit." Accordingly, the common logarithms of the base 10 are converted to the base 2 by multiplying the common log by 3.32.

[20] Harold Bierman, Jr., L. E. Fouraker, and R. A. Jaedicke, *Quantitative Analysis for Business Decisions*, Richard D. Irwin, Inc., Homewood, Illinois, 1961, p. 310.

[21] Robert Dorfman, "Operations Research," *The American Economic Review*, 50, September, 1960, p. 586.