Management Services: A Magazine of Planning, Systems, and Controls

Volume 2 | Number 3 Article 6

5-1965

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Recommended Citation

Gargiulo, Granville R. (1965) "Use of Statistical Methods in Determining Cost Differentials," *Management Services: A Magazine of Planning, Systems, and Controls*: Vol. 2: No. 3, Article 6.
Available at: https://egrove.olemiss.edu/mgmtservices/vol2/iss3/6

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Price differentials based on the costs of serving different customer groups are economically sound—but only when they are based on careful cost analyses. Yet record keeping systems that provide detailed data are prohibitively expensive. A possible solution is— ..

THE USE OF STATISTICAL METHODS IN DETERMINING COST DIFFERENTIALS

by Granville R. Gargiulo
Arthur Andersen & Co.

MANY COMPANIES' price schedules are veritable mazes of quantity, cash, and other discounts. For the most part, these price differentials are assumed to reflect cost differentials. In practice, unfortunately, this assumption is not always valid, as a number of companies have found when they attempted to defend themselves against charges of price discrimination brought under the Robinson-Patman Act. All too many price differentials are based on trade custom, historical practice, or pure intuition rather than on thorough analysis of the actual costs of serving different customer groups.

Price differentials that reflect differences in the cost of manufacture, sale, or delivery resulting from differing methods of delivery or varying quantities sold to customers are both legal and economically sound. It might seem obvious that the time to analyze and justify these cost differences is at the time the prices are set, not after the prices have been attacked under the Robinson-Patman Act or after they have caused noticeable erosion of profits.

Yet few companies actually do analyze their costs in this way. The primary reason lies in the prohibitive cost of maintaining a record keeping system that would provide detailed enough data to support this kind of cost analysis.

There is, however, an alternative. As in quality control and other comparable areas of business, statistical sampling and related analytical procedures offer a reliable substitute for 100 per cent verification in validating the allocations and estimates needed to identify the costs applicable to different customers or classes of customers.

This article explains how statistical techniques may be used to establish the quantitative measures of direct and indirect cost variations per unit of product that are needed to evaluate cost differentials. A case example is used throughout to explain these techniques and to demonstrate their application to cost and price differentials.

Case background

A distributor of a standard machine replacement part has a price schedule offering quantity dis-

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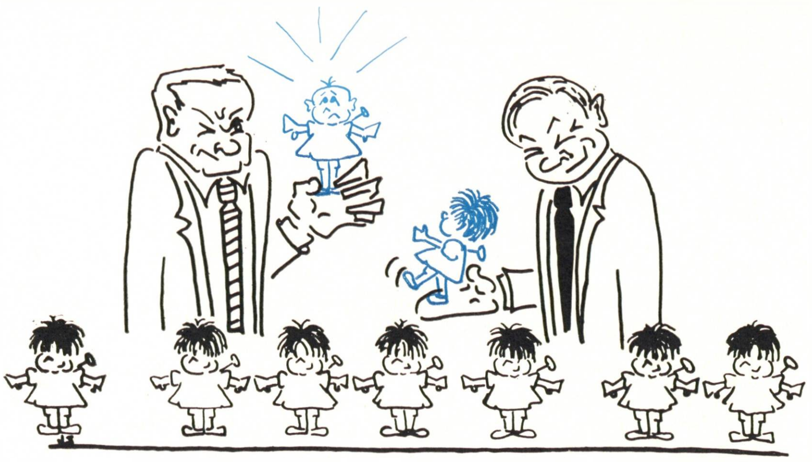
Published by eGrove, 1965



Statistical sampling offers a reliable substitute for 100 per cent verification.

counts. This schedule purports to reflect the cost reduction he realizes in delivering larger amounts of the part, which is packed in standard containers of one to three equivalent units.

The cost accounting system used by the distributor is soundly constructed and provides an appropriate cost picture for management purposes. However, the detail and continuity of record keeping are insufficient to generate historical costs that would substantiate the existing price schedule on a continuing basis. On the other hand, it would be uneconomical to initiate and maintain more detailed record keeping as a routine procedure. The distributor therefore decides to gather data on a selective basis to demonstrate the cost differences associated with serving accounts of different size.

Sales and deliveries are handled on a route basis. There are 100 routes, each serving five accounts. Each of the 500 customers is visited once a week, with a sale (and delivery) made about 50 per cent of the time. On an annual total of 5,000 route trips, 25,000 customer visits are made.

The principal costs of the route deliveries are wages paid for the time of the delivery men in loading trucks, driving the route, and unloading the parts. The output of this effort is the number of containers delivered to customers. Consequently, the cost reduction passed on to large-quantity buyers is believed to be directly related to the time saved per container in servicing such customers as opposed to smaller accounts. Thus, analysis must be conducted in terms of time, representing cost, and containers of parts delivered, representing the unit of work measurement.

The distributor decides to use statistical sampling for this study. For the benefit of those readers who are not already familiar with the basic principles of statistical sampling, this technique will now be briefly explained.

Basic concepts

The aggregate or entirety of items about which information is desired is commonly referred to as a "universe" or "population"; in the case of the distributor it may be defined as 5,000 route trips. Sampling is the process of selecting a portion of this specified population in order to draw inferences about the population from it.

Statistical sampling embraces three distinct steps: (1) the determination of sample size based on a statement of the reliability requirements of the study, (2) the selection of the sample by completely objective methods, and (3) the evaluation of the results. Knowledge of the following basic terms is essential to an understanding of statistical sampling:

Random selection — Statistical sampling depends on the principle of random selection. Random selection means selection governed wholly by the laws of probability, where each item in a population being sampled has an equal chance for inclusion in the sample.

Sampling error — Random sampling methods make it possible to estimate in advance the sampling error that will result solely from the use of a sample. This estimate is an indication of how close, with determinable probability, the sample characteristic being measured will be to the actual characteristic of the population.

Reliability statement — The extent to which the difference between the sample result and the population value is controlled is expressed in a reliability statement. The degree of sampling precision, represented by sampling error, is a specific value added to and subtracted from the sample result. The range created by this addition and subtraction is called a confidence level and is one part of the reliability statement.

For example, suppose that the distributor desired to determine the average order size for a class of customers. If the sampling precision is estimated at 2 per cent and the average order size for a randomly selected sample of customer orders is 500 containers, then the average order size for all store accounts is, with measurable probability, within the confidence interval of 490 and 510 containers.

The other part of the reliability statement is the degree of certainty or probability that the population characteristic will lie within the confidence interval. When we speak of a 95 per cent assurance that the average order size for all accounts will lie within 490 and 510 containers, we mean that there is a 5 per cent chance, or one chance in twenty, that the average will actually fall outside this interval. Consequently, the selection of the assurance level used in the reliability statement reflects the risk assumed of having the population value lie outside the confidence level.

The reliability statement is the key to the interpretation and use of facts derived from sampling. The level of assurance and degree of precision required should be decided by considering what the resulting confidence interval means in terms of its effect on the acceptability of the results. In interpreting the confidence interval, however, the interdependence of the assurance level and the confidence level in the reliability statement must be recognized.

By way of illustration, consider the previously mentioned sample, which provided a 2 per cent confidence interval and used a 95 per cent assurance level. For the same sample, other assurance levels would result in different confidence intervals as follows:

Assurance Level	Confidence Interval
80%	±1.3%
90%	±1.7%
99%	±2.6%
99.7%	±3.0%
99.99%	±4.0%

As this example shows, for the same sample the higher the assurance level taken the wider the associated confidence interval will be. Wide confidence intervals, however, may be considered detrimental to a demonstration of true cost differentials. It is possible, of course, to increase the reliability by increasing the size of the sample taken. Yet to reduce the confidence interval by one-half, the size of the sample must be quadrupled; to reduce the confidence interval to one-fourth its original size requires about a sixteenfold increase in sample size. This relationship must be kept in mind in evaluating the cost and feasibility of larger samples.

Other sampling techniques

Statistical sampling is not, of course, the only means of taking a sample. It is, however, by far the best if results of measurable validity are sought.

Nonstatistical methods of sampling are of two varieties: judgment sampling and quasi-scientific sampling. Judgment sampling involves the selection of a subgroup of the population that is considered to be representative of the total population on the basis of the best available information. The distributor, for example, finding it impractical to extend the analysis over a year's time, might select a shorter time period that he considered completely typical—free of seasonal, cyclical, or accidental variances in volume of business, characteristics of distribution operations, or incidence of expense.

The need for applying such judgment points up one of the major shortcomings of this approach, namely, the need for considerable knowledge of the population and the subgroup selected—or for strong assumptions about them. The validation of such assumptions is likely to require research beyond the scope of most cost analyses. Another serious limitation of judgment samples is that the representativeness of the data from them cannot be supported through the application of acceptable statistical testing procedures. Nor can the reliability of the sample results, in terms of

Statistical sampling depends on a wholly random selection.

Published by eGrove, 1965

bias and variability, be scientifically determined or controlled.

Some of these deficiencies are overcome by the employment of quasi-scientific sampling. For example, while the period for study (which in our example provides a limited population consisting of all trips in that period) may be selected on the basis of judgment or conjecture, the actual sample observations (less than the limited population) are drawn by the same objective procedures used in statistical sampling. This makes it possible to calculate a measure of reliability of the sample once it has been drawn from within this limited population. Thus, quasi-scientific sampling shares with statistical sampling the advantage of objectivity. Proper selection procedures assure that the test will bring to light a reasonable cross section of the area of cost difference being examined.

The purpose of using any sampling procedure, of course, is to obtain the needed information with a minimum expenditure of time and money. Such economy is of questionable value, however, if the results obtained are unreliable or if their reliability is unknown. No measure of reliability is available in judgment sampling. In quasi-scientific sampling the reliability is indeterminate until the sample is drawn.

Statistical sampling, on the other hand, permits measurement of the reliability and degree of assurance that can be placed on the results. By providing, in advance, such measures obtainable with varying sample sizes, statistical sampling makes it possible to select the smallest sample that will yield the reliability needed to justify cost allocations and the resultant price differentials.

Application

The distributor, now armed with his new-found knowledge of sampling theory, is still confronted with some practical considerations. In order to keep the costs of his investigation within reason and to have better control over the conduct of the study, he selects one regional warehouse (Warehouse A) for analysis. This warehouse has 20 routes. He also decides to extend the study over a full year's operations. The consequence of these decisions is that a smaller population is defined, namely, "1,000 trips from Warehouse A" (20 routes/week × 50 weeks).

Without prior experience in analyzing areas of cost differentials, the distributor is unable to estimate in advance the variation and, consequently, the sampling errors that can be expected in any sample evidence. Therefore, he cannot apply any statistical formula for determining the sample size that will give him the reliability he requires in his sample results. As an alternative, he proceeds in the following way:

1. He takes a random sample of route trips and customer visits to obtain observations for some portion of four weeks' activity of Warehouse A.
2. He performs a statistical analysis of the resultant data and calculates the variation in the sample data.
3. Based on the assurance level desired, he evaluates how much he needs to cut down on the potential errors in the sample evidence to meet his criteria for acceptable cost differential statements.

The distributor may arbitrarily decide to obtain 20 observations of route trips. Warehouse A normally has 80 trips in four weeks. A table of random numbers is used to select the first 20 numbers under 80. These numbers represent the trips to be observed, e.g., 10th, 64th, 3rd, 8th, 30th, etc. (A sample of customer visits can be obtained by randomly selecting, from each trip selected, any number from 1 to 5 representing the number of accounts serviced per trip. Since the distributor is interested in unloading time, he can choose two random numbers from 1 to 5 for each trip, based on his experience that only 50 per cent of the time will a visit include a delivery of a part.)

The type of statistical analysis performed by the distributor is discussed in the following paragraphs. This analysis is performed on both this limited sample and the total sample eventually taken. The number of additional observations to be taken beyond this limited sample is determined as a part of the last step previously outlined.

Correlation analysis

The distributor's objective is to use the sampling procedure to generate observations of different factors which might have some direct or indirect bearing on cost or the measure of effort representing cost (in this case, time). Some elements of time, when analyzed in terms of output of effort, are clearly assignable to the customer being served. For example, the time of a delivery man taken to unload ten containers of parts at a customer location is directly allocable to that customer. The cost differentials may result from the fact that the time requirement per case can be expected to decline (at perhaps a diminishing rate) with larger volumes per delivery, thereby lowering the cost of servicing high-volume customers.

In servicing customers, there are many other elements of time incurred by the delivery man that cannot be attributed to specific customers. However, since these costs constitute part of the total cost differential they must be applied on appropriate bases to the classes of customers under consideration. The statistical technique for relating observations of other factors or variables in delivery activity to the sample observations of indirect time elements involves the use of correlation analysis.

Basically, this analysis involves the establishment of some assumption as to which variables might affect the indirect time elements, a subsequent statistical test of the significance of these assumed relationships, and a quantification of the relationships as a basis for allocating costs. As an example, the distributor may logically believe there is sufficient relationship between the time spent by the delivery man in loading his truck and the number of containers he loads to justify using volume as a basis for allocating loading time (and related cost) to customers. For each trip in the sample, observations are obtained on the total number of containers loaded for that trip and the total time required to load the truck.

The first step is to obtain a measure of the relationship between these two variables. If the distributor's intuition is sound, we would expect to find that as more containers per trip are loaded, the delivery man spends more time in the loading activity. These two variables would be said to be positively correlated. The calculated measure of this relationship is referred to as the "correlation coefficient," which expresses the degree of association in terms that are independent of the units of the original data. The correlation coefficient can have a maximum value of 1 (perfect positive correlation) and a minimum value of −1 (perfect negative correlation). Zero represents perfect independence in the two variables.

If the correlation coefficient between loading time and volume loaded was +0.95, this would indicate that there is a high degree of association between the paired variables and that as one variable increased the other would also increase. Since the correlation coefficient is a relative measure, one coefficient can be compared with any other. Consequently, it is possible to check on other variables that might have an association with loading time to give assurance that the selected variable is the best (has the closest association) we can obtain as a basis for cost allocation.

Testing the correlation

Having determined the value of the correlation coefficient, it must be determined whether or not the apparent relationship between the paired values, as developed from a sample, is not attributable solely to chance and is truly indicative of the relationship of these variables in the population. In other words, we must determine how likely such an estimate is to be obtained if the true population coefficient has some specified value. Methods are available to test the significance, at a stated probability level, of the correlation coefficient. The essence of the test is as follows:

1. Make two suppositions: first, that there is no correlation in the population and, second, that there is a correlation in the population. (The first supposition is called the "null hypothesis" since it is this hypothesis that the testing procedure is designed to nullify or support.)
2. State the significance level of the test, that is, the acceptable chances of getting a correlation coefficient as calculated from the sample if, in fact, there were no correlation in the population.
3. Select from the appropriate statistical table the value of the correlation coefficient associated with a given significance level and sample size.
4. If the sample correlation co-

The distributor selects one regional warehouse for close analysis.

Published by eGrove, 1965

The more containers loaded, the longer the loading time. The two variables are positively correlated.

efficient is less than or equal to the one from the table, then the null hypothesis is accepted, that is, the correlation coefficient is not significant. If the calculated correlation coefficient is greater than the coefficient from the table, then the null hypothesis is rejected (the alternative accepted).

Suppose the distributor, from a sample of ten route trips, computes a correlation coefficient of .72 between quantity loaded and loading time. He decides to test the significance of this calculated coefficient at the .05 level (five chances in 100 that .72 would result from a sample when there is no correlation between these two variables in the population). The value from the table indicates a correlation coefficient of .63. Since the calculated coefficient .72 is greater than the table value .63, he rejects the supposition that no correlation exists in the universe and concludes that there is evidence of a significant correlation.

Estimating equations

Once satisfied that volume loaded has a significant effect on the loading time and that no other variable has an association with loading time, the distributor needs to describe the relationship between these two variables in a way suitable as a basis for allocating costs to customers. The calculations may be performed in a number of ways, but the end product is a regression or estimating equation, representing a given curve, which permits us to make estimates of the dependent variable (load time) for specified values of the independent variable (volume loaded). In our example this might take on the form of a straight line relationship expressed as:

Load time (estimated) = 10 minutes + 1.2 minutes $\times$ number of containers

This equation indicates that 10 minutes were required for activities not related to volume (placing truck at dock, general paper work, etc.) and that load time varied (in the sample) in relation to volume at the rate of 1.2 minutes per case. The average number of containers for a given customer class can then be substituted in the equation as a means for estimating the average time required to service a customer of given size on the basis of 1.2 minutes per case plus an appropriate share of the fixed load time of 10 minutes.

The use of estimating equations is not limited to indirect time elements where cost allocations must be made. As a practical matter, price differentials for different customer volume levels are based on average volume and average costs over a period of time. Consequently, it is desirable to develop estimating equations for directly attributable cost elements (e.g., time spent in unloading containers for a given customer) which would yield results more reliable over time than those directly observed in any study period of limited duration.

In the case of unloading time, the estimating equation is itself a measure of the average relationship of all sample observations of unloading time versus quantity delivered. The equation yields for practical use the most likely unloading time required for delivery of specified quantities. By specifying delivery quantities, the equation can be used to compute the loading time; this computed time represents a characteristic of the population which reflects the separation of systematic factors affecting unloading time in a regular and predictable way from chance factors that are irregular and unpredictable and distort studies based upon restricted samples.

Inasmuch as the methods of statistical sampling are used to generate the observations of the paired variables upon which estimating or regression equations are developed, the reliability of the relationship is determinable and may be interpreted in the following way:

Sample observations of randomly selected customer visits are used to develop a regression equation relating unloading time (T) and quantities delivered (Q). This equation is $T_{est.} = 15$ minutes + 1.5 minutes $\times$ Q.

A measure of the random sampling errors in using the regression equation as an estimator of the population average unloading time for any quantity delivered is developed. From the sample this measure equals 10 minutes at a 95 per cent confidence level.

For a delivery of 20 containers we substitute in the above equation

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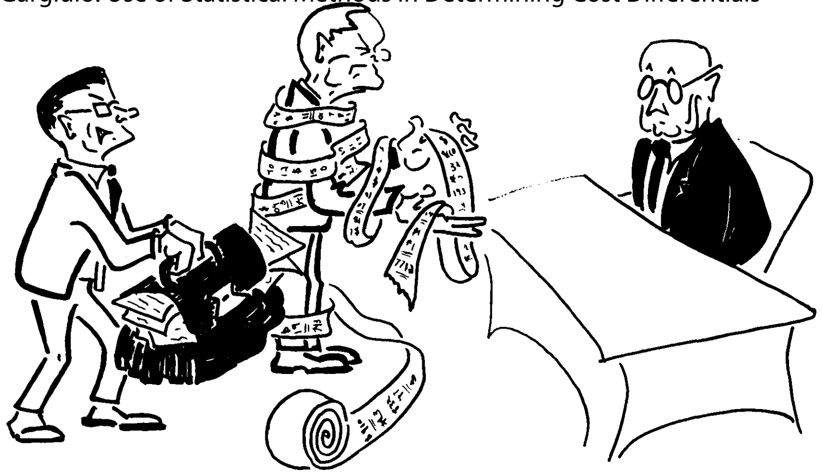

Federal Trade Commission proceedings in Robinson-Patman Act cases seem to indicate that statistical techniques are clearly recognized as a basis for cost analysis.

to obtain an estimate of 45 minutes as the average amount of time required to unload this quantity.

From our knowledge of the sampling error, we can make statements of the following type (similar statements could be developed for different confidence levels):

1. "The probability is 95 per cent that the interval 35-55 minutes will include the population value of average unloading time for 20 containers."
2. "There are 5 chances in 100 that the true average unloading time for 20 containers will be greater than 55 minutes or less than 35 minutes."

One further use of the estimating equation might be mentioned. In the case study previously outlined, the distributor conducted his analysis on the operations of Warehouse A. However, his price schedule is the same for all customers (excluding freight costs from the manufacturer to the warehouse). To ensure the universal application of his basis of cost allocation in all distribution regions, the distributor could take a restricted (and much less costly) statistical sample of delivery activity at other warehouses. Actual quantities loaded and delivered could then be substituted in the regression equations developed from the study at Warehouse A to obtain estimates of time to carry out this activity. These estimates could subsequently be compared with the actual times observed. The measurable variation between estimated and actual times required could serve two purposes: first, to further confirm the validity of the estimating equations developed as a basis for cost allocations and, second, to demonstrate the general applicability of the equations, thus enabling the distributor to avoid the incurrence of re-incurrence of the high cost of conducting such studies in every distribution region.

Conclusion

Occasional special studies of cost differentials executed in accordance with accepted statistical and other analytical principles can provide management with useful pricing guidelines at a cost well below that of maintaining detailed records on a continuing basis. Only the management that knows the cost of serving each group of its customers is in a position to set prices that truly reflect cost differentials.

Furthermore, the company that conducts such studies has a good head start if the need should ever arise to justify its price differentials under the Robinson-Patman Act. Substantially all cost justification defenses in court proceedings since the Robinson-Patman Act was passed have involved the use of sampling in the cost data presented. In certain cases the cost defense was weakened, not because the supporting statistical techniques were unsound but because it was not proved that the samples selected were truly representative of the seller's operations.

Because of their demonstrable objectivity and lack of personal bias and because of their measurable reliability consistent with reasonable expense, statistical sampling, correlation analyses, and regression or estimating equations provide a reliable basis for allocations and estimates. While the acceptance of statistical techniques as legal evidence is currently unclear, a perusal of Federal Trade Commission proceedings in Robinson-Patman Act cases seems to indicate that the techniques as such are clearly recognized and accepted by the authorities as a basis for cost analyses and a means of specifying the reliability of the analyses in statistical terms. Consequently, there should be no hesitation to expand the use of statistical techniques to lend support to the data being presented in cost justification defenses.

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