Incremental Analysis and Opportunity Costs

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Businessmen may be reluctant to offer goods at reduced prices even if incremental revenues exceed incremental costs. However, they are usually more than willing to follow the incremental approach in cost reduction programs. But even here there are risks—

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THE LITERATURE of cost analysis abounds with illustrations attesting to the validity of the incremental approach in decision making. A frequently used illustration is that of a company operating below capacity and faced with the choice between acceptance and rejection of an order to sell an incremental quantity of goods at a reduced price. Following a disclaimer as to any effect on current or future sales or negative reactions in the market place, the solution offered suggests a matching of incremental costs (usually but not always restricted to variable costs) with incremental revenue and a recommendation for acceptance if incremental revenue exceeds incremental cost.

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The cost analyst is often disturbed when management rejects this approach. In fact, he believes that the business executive does not understand the concept. The analyst also fails to understand the executive's frequent observation that if the incremental analysis is adopted, the resulting decisions extended over time could drive the business "into the ground." The little story told by a business executive on "An Approach to Winter Bear Hunting" may better illustrate his attitude.

The first step requires digging a small hole in a frozen lake. Next one sharpens both edges of a knife razor keen, inserts it into the hole with the point up, and waits until it is frozen into place. Then the hunter pricks his finger on the top of the knife, permitting some blood to drip on the knife. The hunter now leaves the scene. Pretty soon the bear, attracted by the scent of human blood, approaches the knife and proceeds to lick the blood. In so doing, he cuts his tongue and thus begins the slow process of bleeding to death.

We suggest that the business executive understands the incremental approach quite well but cannot accept the oversimplified assumptions regarding the impact on current and future prices and markets. The risk involved and the opportunity costs associated with the risk are not quantified in the incremental approach, and in the absence of such quantification the decision maker adopts the full cost approach since he believes that this approach will minimize risk. Essentially then, the typical incremental analysis is incomplete because of a failure to incorporate a measure of risk and is, therefore, frequently rejected in decisions of the type described.

By contrast, at least in the authors' experience, the business executive is inclined to accept the incremental approach in the typical cost reduction type of decision. Here incremental costs for alternatives are compared under the reasonable assumption that gross revenues will not be affected and any cost savings will be reflected as increased profits.

However, although they may be of a different order, elements of risk occur in these latter types of decision. Failure to consider these elements could well result in decisions that will not achieve maximum utilization of resources and associated profits. Attempts to treat risk by the injection of average estimates of one or more future occurrences are inadequate because they ignore the statistical nature of the elements of risk.

This paper attempts to illustrate the importance of explicitly recognizing the statistical elements present when an incremental analysis is applied to a series of make-or-buy decisions in a maintenance department. Specifically, the example illustrates how the resources of a maintenance department can be misallocated in the long run if the usual incremental analysis is applied to a series of decisions that deal with the problem of whether the company's maintenance department should perform other work during periods of unscheduled time or whether the company should contract the project work to outside contractors.

Statement of problem

The maintenance department of Company X employs a relatively fixed labor force of 75 men. Management established the 75-man force through trial and error, and it is satisfied that the department is adequate for the needs of the plant. However, because of the varying demands of breakdown work there are frequent periods when the time of all 75 men cannot be scheduled. The practice is not to lay off these skilled craftsmen; rather, it is for the foremen to assign them to small jobs that are, for the most part, of the "make work" variety or of low productivity.

Within the past year, the company attempted to improve the utilization of unscheduled time in the maintenance department through the establishment of a backlog of nonemergency jobs. Each of the jobs selected would make a contribution to fixed cost and profit in contrast to the "make work" which is readily justified by maintenance people but difficult to evaluate objectively. The jobs were assigned to unscheduled time in order of descending contribution to fixed cost and profit.

There was little doubt that the system reduced the amount of time assigned to nonproductive work, but a review of the work performed in the electrical shop cast some doubt as to the adequacy of the analysis employed by the company.

For example, during the period of June 1 - July 15 three electricians were assigned to the Warehouse Construction Project. The decision to assign the three men to the project was based on the data and analysis in Exhibit 1, page 15.

The analysis of the problem consisted of selecting the alternate that produced the highest contribution to fixed cost and profit. Contribution is simply the difference between the bid of an outside contractor and the raw material cost, the latter being the incremental cost. Labor is considered a fixed cost in this case since the size of the labor force is to remain unchanged.

Inspection of the work available during the period June 1 - July 15 reveals that under the usual incremental analysis furnished, the Warehouse Construction Project would be scheduled for the 840 man-hours of unscheduled time for the June 1-July 15 period. The contribution of the Warehouse Construction Project is $400 greater than the contribution of the combination of Jobs 2 and 3.

This analysis implicitly assumes that there is no alternate utilization of the unscheduled 840 man-hours, and it does not recognize alternate utilization (i.e., other than the incremental project work) of the 840 man-hours because it assumes with certainty that the 840 hours will remain unscheduled. Thus, the opportunity costs associated with alternate utilization of the time are ignored, and the time is assigned a zero cost. However, the assumption is not always valid. Therefore, the possibility of utilizing the unscheduled time should be examined before the assumption is accepted.

The statistical analysis

The statistical analysis explicitly recognizes the risk present in a problem by developing the average or expected contribution that unscheduled time can make to fixed costs and profit. To calculate the expected contribution, it is necessary to determine the expected hours to be spent on emergency work over the period and to multiply these hours by the estimated contribution per hour of the emergency work. The estimates required in the calculations are obtained from operating and cost data that have been grouped into the following frequency distributions: (1) arrival of emergency breakdown work, (2) contribution of breakdown work to fixed costs and profits (cost of the work if performed by an outside contractor less the costs that would be avoided if the job were not performed by the department), and (3) time spent on individual emergency jobs. If data are not available, it is necessary to estimate the distributions from knowledge of emergency breakdown work.

Assume that the following data have been supplied:

1. *Repair time consumed on past emergency breakdowns.* An analysis of job cards revealed that the distribution of time spent on emergency breakdowns could be closely approximated by a normal curve with a mean of 50 man-hours and a standard deviation of two man-hours. Because the standard devia-

tion was only two man-hours, the variation in the time spent on emergency breakdowns was ignored in the analysis. Of course if the variation in time spent on jobs is significant, it would have to be incorporated into the analysis by working with the distribution of time spent on the breakdowns in addition to the distribution of arrivals of breakdowns.

2. *Arrival of breakdowns.* Analysis revealed that the distribution of the arrival of breakdowns was closely approximated by a Poisson distribution with $m = .4t$ where t is measured in weeks. [A Poisson distribution is a measure of the probability of an occurrence in a unit of space or time.]

3. *Incremental contribution of past emergency work.* A study of past breakdowns revealed that the incremental contribution of individual jobs was closely approximated by a normal curve with a mean of $20/man-hour.[1]

Examination of the unscheduled time available and the list of incremental work available for the period reveals that there are five alternate uses of the idle time:

1. Reserve the entire 840 man-hours for assignment to emergency breakdown work.
2. Assign 700 man-hours to the Warehouse Construction Project, and leave 140 man-hours of residual unscheduled time free for assignment to emergency breakdowns.
3. Assign 240 man-hours to Overhaul of Switch Gear and leave 600 man-hours of residual unscheduled time available for emergency breakdowns.
4. Assign 360 man-hours to Remodeling and Rewiring of Production Area A, and leave 480 man-hours of residual unscheduled time for emergency breakdowns.
5. Assign 600 man-hours to the Switch Gear and Production Area A projects, and leave 240 man-hours of residual unscheduled time for emergency breakdown work.

[1]It is assumed that the variation of the incremental contribution of past emergency work is insignificant in this problem. Again, if the variation is significant it would have to be incorporated into the analysis by working with the distribution of the incremental contribution as well as the distribution of arrivals of breakdowns, and in some cases with the distribution of time spent on breakdowns.

EXHIBIT I

WORK ASSIGNMENT ANALYSIS
Work Schedule for June 1 - July 15

Scheduled Work (Preventative Maintenance and Routine)		7.5 men
Trouble Call*		4.0
Unscheduled (equivalent to 840 man-hours)		3.5
	Total Accounted For	15.0 men
Planned Labor Force over the June 1-July 15 period		15.0 men**

*Trouble Call work is breakdown and set-up work that requires less than two man-hours. Craftsmen assigned to this work patrol specific work areas.
**The electrical department has maintained a staff of 15 men for the past year, and the budget for the current year has provided for the same 15-man crew.

Incremental Work Available During the June 1 - July 15 Period

Job	Bid	Estimated Man-Hours Required	Raw Material Cost	Estimated Contribution to Profit & Fixed Cost
(1) Warehouse Construction Project	$16,200	700	$5,000	$11,200
(2) Overhaul Switch Gear	5,920	240	2,000	3,920
(3) Remodel & Rewire Production Area A	9,880	360	3,000	6,880

With the above information, it is possible to determine the expected return from the emergency work that can be handled with each of the alternates. The expected return on emergency work is added to the contribution of the incremental work that has been assigned to each of the alternates to determine the expected contribution to fixed costs and profits of the alternate.

To compute the expected contribution of emergency work, it is necessary first to determine the number of emergency jobs that can be handled in the residual unscheduled time that is available with each of the alternates. These jobs are determined by dividing the residual time by the average time consumed on an emergency breakdown (50 hours). In this case, the number of jobs that can be handled by each of the alternates has been defined in whole number values by dropping the fractions in the quotients. [Dropping the fractions in the example can be justified since the probability of finding a breakdown job that would consume the fractions dropped is small enough to be neglected.][2]

Once the expected number of emergency jobs that can be handled by each of the alternates has been determined, it is possible to assign the Poisson probabilities that these jobs will arrive in the time that the alternate provides for emergency work. The product of the probability of arrival of the number of jobs provided by each of the alternates and the expected hours required to complete the emergency jobs (50 hours per job) is, by definition, the expected hours to be spent on emergency work of the alternate. The expected hours to be spent on emergency work are multiplied by the average hourly contribution of emergency work ($20 per man-hour) to determine the expected contribution of unscheduled time of the alternate. These calculations are carried out in Exhibit 2, page 16.

Similar computations for the re-

[2]The largest fraction dropped is 0.8 in Alternates 1, 2, and 5. However, for all practical purposes, the probability of finding a breakdown job that will consume 40 hours or less is zero.

EXHIBIT 2

COMPUTATION OF EXPECTED CONTRIBUTION FROM EMERGENCY BREAKDOWN AND INCREMENTAL PROJECT WORK OF THE ALTERNATES

Alternate 1:

Residual Unscheduled Time = 840 man-hours

Expected number of emergency jobs that can be handled in 840 man-hours = 840/50 = 16

Expected Hours of Emergency Work With 840 Man-Hours of Residual Time:

r (# of Breakdown Jobs)	P_{Po} (r)	×	Expected Hours to Complete r Jobs	=	Expected Hours on r Emergency Jobs
0	.091		0		0
1	.218		50		10.9
2	.261		100		26.1
3	.209		150		31.4
4	.125		200		25.0
5	.060		250		15.0
6	.024		300		7.2
7	.009		350		3.2
8	.002		400		.8
9	.001		450		.4
10 or more	0		500		0
			Total Expected Hours on Emergency Work =		120.0 man-hours

Expected Contribution to Fixed Costs & Profit:

Expected Contribution of Breakdown Work: 120 Man-Hours × $20/Man-Hour =	$2,400
Expected Contribution of Incremental Project Work =	0
Total Expected Contribution . . .	$2,400

Alternate 2:

Residual Unscheduled Time = 840 — 700 = 140 man-hours.

Expected number of emergency jobs that can be handled in 140 man-hours = 2

Expected Hours on Emergency Work With 120 Man-Hours of Residual Time:

r (# of Breakdown Jobs)	P_{Po} (r)	×	Expected Hours to Complete r Jobs	=	Expected Hours on r Emergency Jobs
0	.091		0		0
1	.218		50		10.9
2 or more	.691		100		69.1
			Total Expected Hours on Emergency Work =		80.0 man-hours

Expected Contribution to Fixed Costs & Profits:

Expected Contribution of Breakdown Work: 80.0 Man-Hours × $20/Man-Hour =	$ 1,600
Expected Contribution of Incremental Project Work =	11,200
Total Expected Contribution . . .	$12,800

maining alternates would reveal the results shown in Exhibit 3. Table 1 compares the results for all alternates.

In essence, the statistical analysis centers on the assignment of a realistic opportunity cost to the residual unscheduled time. The expected contribution of residual time represents the net savings over fixed costs that the company can expect to realize from emergency breakdown work in the long run if the same causal system operates and if the analysis and decision rule is applied to the problem of allocating unscheduled time to incremental work projects. In the long run the correct amount of unscheduled time is allocated to both the incremental work and to free time since the alternates with the highest total contributions from both elements are chosen.

The typical incremental cost analysis does not afford long-run protection against the misallocation of the department's resources, because it fails to recognize the contribution of residual time but assumes with certainty that no breakdown work will arrive (see Work Schedule in Exhibit 1, page 15). Hence, rational protection against the long-run misallocation of resources can only be obtained through the statistical recognition of the contribution that residual time can make to the department by enabling it to service emergency breakdowns as they occur.

Schedule based on averages

It is important to note that the statistical analysis of the previous section did not schedule emergency work on the basis of the average hours that would be spent on emergency work over the six-week period. Because of the variability in the distribution of arrivals of breakdown jobs, the use of the average number of arrivals over the six-week period would lead to an incorrect decision—the decision to schedule the Warehouse Construction Project.

If the average number of arrivals had been used as a decision parameter, the Warehouse Construction Project would have been selected on the basis of the analysis in Exhibit 4.

If it is correct to accept the average number of hours to be spent on emergency work and to schedule on the basis of the average, then it would be correct to assign the 720 hours of unscheduled time to the Warehouse Construction Project since (1) the contribution of emergency work to fixed costs and profit would no longer be pertinent to the evaluation of the contribution of unscheduled time, and (2) the Warehouse Construction Project would make the highest contribution of all or any combination of the remaining incremental projects.

However, as the statistical analysis of the previous section has shown, the assignment of unscheduled time to the Warehouse Construction Project would be incorrect since the total contribution from the project and the 140 hours of residual time would be lower than the total contribution of Alternate 5. The scheduling technique based on averages will not produce the correct decision in this problem because there is too much variability in the arrival distribution. Because of this variability, the estimates based on the average number of arrivals are not precise enough for use as a decision parameter in the problem. Hence, in the maintenance problem and in many other problems where the variability is significant the analysis cannot always rely on average values but must take into account the variability of the data by working from the distribution.

EXHIBIT 3

TOTAL EXPECTED CONTRIBUTIONS

Alternate 3:

Expected Contribution of Breakdown Work	$2,400
Expected Contribution of Project Work	3,920
Total	$6,320

Alternate 4:

Expected Contribution of Breakdown Work	$2,400
Expected Contribution of Project Work	6,880
Total	$9,280

Alternate 5:

Expected Contribution of Breakdown Work	$ 2,252
Expected Contribution of Project Work	10,800
Total	$13,052

Summary of Results:

Alternate	Total Expected Contribution
1	$ 2,400
2	12,800
3	6,320
4	9,280
5	13,052*

*Decision Rule: Select the alternate with the highest Total Expected Contribution.

TABLE I
COMPARISON OF RESULTS

	Expected Total Contribution	
Alternate	Traditional Analysis	Statistical Analysis
1	Not Considered	$ 2,400
2	$11,200*	12,800
3	3,920	6,320
4	6,880	9,280
5	10,800	13,052*

*Action recommended by analysis

EXHIBIT 4

ANALYSIS BASED ON AVERAGE ARRIVALS

Average number of emergency jobs to arrive in the six-week period:

$m = .4t = 4 \times 6 = 2.4$ jobs

Average number of hours to be spent on emergency work over the six-week period:

2.4 jobs x 50 man-hours/job = 120 hours

Schedule with the Provision for the Average Hours to be Spent on Emergency Work:

Scheduled	7.5 men
Trouble Call	4.0
Emergency Breakdowns	0.5
Unscheduled	3.0
Total Men Scheduled	15.0

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