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To Obtain Sinking-fund Payments from Compound-interest Tables

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In a paper read by A. Lowes Dickinson to the old Illinois Association in February, 1902, he stated:

> "INTEREST CALCULATIONS
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> "Many public accountants have, and all should have, a thorough knowledge of this subject. Such calculations as sinking funds for redemption of leases, the instalments payable on car-trust obligations, amounts required to redeem bond issues or repay loans over a series of years and other kindred points are only a few of those matters of every-day occurrence in ordinary business affairs upon which a public accountant is generally qualified to advise his clients."

To most junior accountants, and perhaps to many seniors, the problem of computing sinking-fund payments is wrapped in mystery. They consider the subject beyond their depth and as requiring much painful study of what is known as actuarial science. The reason for this feeling, in the writer's opinion, lies in the fact that most textbooks and treatises on the subject delve at once into what, to the uninitiated, looks like a maze of words, figures, formulæ and symbols. While a knowledge of actuarial science or of certain formulæ derived therefrom is of course necessary for the solution of the more intricate problems, the purpose of this paper is to show that with the aid of a compound-interest table any schoolboy can solve those of usual occurrence once the method is explained, even if the reasoning lying behind the method is not understood. We shall therefore give first a diagram and follow it with an explanation.

In order to show the simplicity of the problem take this elementary question: "What is the annual sinking-fund necessary to repay a loan of $100,000.00 due 10 years hence, the sinking-fund payments to be invested at 5% interest, compounded annually?" The answer is $100,000.00 divided by 12.57789, the divisor being calculated from the figure opposite 10 (periods) in the 5% column of a compound-interest table. If the payments had been made and the interest compounded at more frequent intervals the answer would have been just as simple. For instance, 20 semi-annual payments with interest compounded semi-annually at 2½% per half year would each be $100,000.00 divided by 25.54465, the divisor being calculated from the figure opposite 20 (periods) in the 2½% column of the table. Leaving the explanation until later we shall show how the above figure of 12.57789 was obtained.

Compound-interest tables are composed of columns of figures with various rates per cent. along the top and have usually from 1 to 100 periods of time down the side. They are based upon an investment of 1. The column headed 5% would begin thus:

End of period	5%
0	1.000 000
1	1.050 000
2	1.102 500
3	1.157 625
4	1.215 506
5	1.276 281
6	1.340 095
7	1.407 100
8	1.477 455
9	1.551 328
10	1.628 894

The tables do not always show the zero period line at the top with the original investment of 1 but it is here included to make the table complete. If omitted it can be inserted if desired.

The divisor of 12.57789 above referred to can be obtained from this 5% table in two ways: (1) It is the footing of the column from line 0 to line 9 inclusive, that is one line for each period of the problem beginning with the zero line. (2) From the tenth line deduct 1 (the original investment) and divide the result by .05 (the rate per cent. on 1). There is naturally a slight difference in the last decimal figures owing to fractions.

The figure of 25.54465 in the case of half-yearly payments with interest of 2½% per half-yearly period, the loan being payable at the end of 20 half-years, was obtained similarly: (1) The footing of the 2½% column from line 0 to line 19 inclusive. (2) From the 20th line deduct 1 and divide the result by .025.

This is the solution of any such elementary problem as that given above. Having shown the method we may now discuss the meaning of these figures for those who have no table available or desire to follow the subject further. Thereafter we shall discuss as simple a method of finding the purchase price of a leasehold or similar "terminable annuity."

It should first be mentioned that simple interest is seldom used in dealing with large amounts, as the interest payments are in themselves large enough to be invested and so increase the return on the original capital. This, of course, results in compound interest.

Throughout this paper 5% per annum compounded annually will be the interest rate used unless otherwise stated. The method is the same for all rates of interest so long as the interest is compounded at the same time as the payments are made, the payments are made at equal intervals of time and the interest rate corresponds to that interval. For instance, if the payments were made every six months for ten years and the interest rate were 2½% every six months, compounded every six months, the table used would be 2½% for 20 periods. In the latter case it is customary to speak of the interest rate as being "5% per annum compounded semi-annually." This actually produces more than 5% per annum, the annual rate on this basis being 5.0625% and on a quarterly basis 5.0945%. These, incidentally, are known as the "effective" annual rates. If on the other hand the effective annual rate were given as 5% the corresponding semi-annual rate would be the square root of 1.05 deducting 1 from the answer, and the quarterly rate would be the square root of the square root of 1.05 deducting 1 from the answer.

A single investment of $1,000.00 will produce in one year at 5% $50.00. If this $50.00 be also invested at the same rate the second year's income will be 5% on $1,050.00 or $52.50 and so on. The investment will increase therefore by years as follows:

Original investment	$1,000.00
Interest thereon	50.00
End of first year...............	$1,050.00
Interest thereon	52.50
End of second year............	$1,102.50
Interest thereon	55.125
End of third year..............	$1,157.625
Interest thereon	57.88125
End of fourth year..............	$1,215.50625

Any other amount invested at 5% per annum will increase, of course, in the same proportion. If the amount invested were only one dollar, for instance, it would accumulate thus, the figures being those already shown as composing the 5% table:

Original investment	$1.000 000
One year later.................	1.050 000
Two years later................	1.102 500
Three years later...............	1.157 625
Four years later................	1.215 506

As compound-interest tables are all based upon an investment of 1, the amount to which any other investment will accumulate at the given rate in the given time is simply the amount to which 1 will accumulate under these conditions multiplied by the amount of the investment, whether it be dollars, cents, pounds, francs or any other currency. As an investment of 1 accumulates in one year to 1.05, or 1×1.05, so an investment of 1.05 will amount in one year to 1.05×1.05 or $(1.05)^2$ or 1.102 500 (see above), and 1.102 500 will amount in one year to $1.102\ 500 \times 1.05$ or $(1.05)^3$ or 1.157 625 (see above), which means that an investment of 1 at 5% compound interest will amount in 1 year to $(1.05)^1$, in 2 years to $(1.05)^2$, in three years to $(1.05)^3$ and so on. This fact is the foundation of all formulæ in actuarial science as it applies to all rates of interest and to all periods of time. If the accumulation at the end of, say, the 10th period is desired it is necessary only to raise to the 10th power 1 plus the rate of interest which, if 5% per period, is of course .05 on an investment of 1, or $(1.05)^{10}$ equal to 1.628 894.

As it is difficult for the beginner to accustom himself to the use of symbols in place of dollars and cents they are studiously avoided in this treatise with very few exceptions, and these are only parenthetical. In order to state the above theory in its briefest form let i represent the rate of interest per unit invested per period and let n represent the number of periods. The amount to which 1 will accumulate at compound interest in n periods is therefore $(1+i)^n$.

That the two methods of finding the divisor referred to above are the same (that is (1) the footing of lines 0 to $(n-1)$ inclusive or (2) 1 deducted from line n and the result divided by the rate per cent) can easily be proved arithmetically as the items to be added are in geometric progression. The fact can be seen, however, from the above table. The tenth line for example is

1.628 894; deduct 1, leaving .628 894; divide this by .05 (the rate per cent on 1) and you have 12.577 880 which is the footing to the end of line 9. This in formula form is $\frac{(1+i)^n-1}{i}$.

As .628 894 is obviously the compound interest on 1 for ten periods this formula can be stated in words as the amount of compound interest on 1 for the whole number of periods divided by the rate of interest per period.

Compound-interest tables give columns showing, as before stated, the amount to which 1 will accumulate at various rates of interest in from 1 to perhaps 100 periods. If the table is condensed and does not contain the number of periods desired, say 63, this amount may be obtained by *multiplying* together the amounts of say the 50th and 13th lines on the arithmetical theory of powers, that $a^{50} \times a^{13}$ equals a^{50+13} equals a^{63}.

There is a still shorter method of finding the divisor required. The table of which a fragment is given above is known as the "Amount of 1" table, that is the amount to which an investment of 1 will accumulate. A second table, derived therefrom but not always available, is the "Amount of an annuity of 1." That is the amount to which an annuity of 1 per period will accumulate with compound interest. The word annuity is used as applying to equal payments made at equal intervals of time although these intervals may be of any duration and do not necessarily mean years as the word itself implies.

An annuity of 1 per annum for ten years, saved and invested at 5% interest compounded annually, represents naturally a series of 10 investments of 1 each, earning 5% interest compounded annually, each successive payment earning interest for one year less than its predecessor thus:

	Periodical Investments				*Total*
	First	*Second*	*Third*	*Fourth*	*investment*
First period	1.000 000				1.000 000
Interest	.050 000				.050 000
Second period	1.050 000	1.000 000			2.050 000
Interest	.052 500	.050 000			.102 500
Third period	1.102 500	1.050 000	1.000 000		3.152 500
Interest	.055 125	.052 500	.050 000		.157 625
Fourth period	1.157 625	1.102 500	1.050 000	1.000 000	4.310 125

And so on. The total column above is that shown in the tables as the "Amount of an annuity of 1," or some such title.

It will be observed from the above totals that if each payment be made at the end of the period the amount to which an annuity of 1 will accumulate at the end of the fourth period, when the fourth payment has just been made, is 1 plus $(1.05)^1$ plus $(1.05)^2$ plus $(1.05)^3$, or the footing of the 5% column in the compound-interest table from the zero line to line 3 inclusive. Similarly, had the annuity been payable for 10 periods, the total including the tenth payment would have been equal to the total of lines 0 to 9 inclusive—12.577 890. Therefore, if a table of the "Amount of an annuity" be available, this divisor can be obtained without the trouble either of footing the column or of deducting 1 from the nth line and dividing the result by the rate per cent. We have thus three methods of ascertaining sinking-fund payments from compound-interest and annuity tables as a sinking fund is simply an annuity.

A word of caution is necessary. The first payment of a sinking fund is usually made at the end of the first period, not in advance, while the "Amount of an annuity" table may be based on an advance payment when it is technically known as an "annuity due." The first line of the table will show the basis. If the first line (end of the first period) is 1.000 000 and the second 2.050 000, the above explanation holds good. If the first line is 1.050 000 and the second 2.152 500, the table is based on an advance payment and the adjustment will have to be made by using one period less and adding 1 to the total shown.

Similarly, if the payment has to be made in advance and the table is constructed on the other basis, take one period more and deduct 1 from the figure shown.

Having discussed the methods of ascertaining the periodic payments to be made let us now study their application.

We have shown that a deposit of 1 per period earning interest at the rate of 5% per period will in 10 periods be sufficient to pay off a loan of 12.577 890. In the meantime interest at the rate of 5% or .628 894 per period is being paid on the loan, the total payment made by the borrower each period being therefore 1.628 894. This it will be observed is the amount of 1 for 10 periods.

Loans may be paid off in various ways including the following:

(1) By the payment of one sum at maturity, interest being paid periodically on the whole loan meantime.

(2) By equal periodic payments, part of which are in reduction of principal and part in payment of interest on the decreasing balance.

(3) By equal periodic payments on account of principal, the periodic interest payments reducing accordingly.

(4) By the payment of one sum at maturity covering principal and compound interest for the entire period.

The cost to the borrower is exactly the same in all cases if we assume that the periods of time used in the four methods do not differ, that the interest is compounded once each period, that the sinking fund earns interest at the same rate as the loan bears and that the borrower is entitled to credit for interest at the same rate if he make advance payments.

The following table shows the annual result of the sinking-fund method, where a loan of 12.577 890 is repaid in one sum at maturity, interest being paid annually.

Sinking-Fund Method

End of year	*Annual sinking-fund deposits*	*Interest earned on sinking fund*	*Total in sinking fund*	*Annual interest paid on bonds*	*Total annual payments*
1	1.000 000		1.000 000		
2	1.000 000	.050 000	2.050 000		
3	1.000 000	.102 500	3.152 500		
4	1.000 000	.157 625	4.310 125		
5	1.000 000	.215 506	5.525 631	.628 894	1.628 894
6	1.000 000	.276 281	6.801 912		
7	1.000 000	.340 095	8.142 007		
8	1.000 000	.407 100	9.549 107		
9	1.000 000	.477 455	11.026 562		
10	1.000 000	.551 328	12.577 890		

If the whole 1.628 894 had been paid annually to the lender instead of placing 1.000 000 in the sinking fund and paying the lender .628 894 for interest, the lender would apply it thus, the loan being exactly repaid by the end of the tenth year.

Repayment Method

End of year	*Annual payment of $1.628 894 distributed thus* — *Int. on bal. of loan*	*Principal repaid*	*Balance of loan*
0			12.577 890
1	.628 894	1.000 000	11.577 890
2	.578 894	1.050 000	10.527 890
3	.526 394	1.102 500	9.425 390
4	.471 269	1.157 625	8.267 765
5	.413 388	1.215 506	7.052 259
6	.352 613	1.276 281	5.775 978
7	.288 799	1.340 095	4.435 883
8	.221 794	1.407 100	3.028 783
9	.151 439	1.477 455	1.551 328
10	.077 566	1.551 328	.000 000

It will be noted that the annual interest paid less interest earned on the sinking fund in the first tabulation above equals the annual interest paid in the second tabulation and that the principal repaid annually in the second tabulation increases on the basis of the $(1+i)$ column.

In the third case, where equal payments of principal are to be made annually, as in serial bonds, the result is the same although the similarity is not quite so obvious.

Serial-Bond Method

End of year	*Bonds repaid annually*	*Interest paid annually*	*Total annual payments*	*Serial bonds outstanding*
0				12.577 890
1	1.257 789	.628 894	1.886 683	11.320 101
2	1.257 789	.566 005	1.823 794	10.062 312
3	1.257 789	.503 116	1.760 905	8.804 523
4	1.257 789	.440 226	1.698 015	7.546 734
5	1.257 789	.377 337	1.635 126	6.288 945
6	1.257 789	.314 447	1.572 236	5.031 156
7	1.257 789	.251 558	1.509 347	3.773 367
8	1.257 789	.188 668	1.446 457	2.515 578
9	1.257 789	.125 779	1.383 568	1.257 789
10	1.257 789	.062 889	1.320 678	.000 000

Here the total annual payments decrease from 1.886 683 in the first year to 1.320 678 in the tenth. If these decreasing payments be calculated with compound interest the result will be exactly the same as the equal annual payments of 1.628 894 similarly calculated.

If no payments of either principal or interest were made until the maturity of the loan the result would be the same to the borrower and lender as if paid annually.

(1) As a loan of 1.000 000 would accumulate with 5% compound interest in 10 years to 1.628 895, a loan of 12.577 890 would accumulate on that basis to 12.577 890×1.628 895.

(2) As 10 annual payments of 1.000 000 amount with 5% compound interest to 12.577 890, ten annual payments of 1.628 895 under that method would amount to 12.577 890× 1.628 895, which is the same as the result of (1).

It is not always possible to obtain as high a rate of interest on the sinking-fund as is paid on the loan. In that case the payments to the sinking fund would be greater, but this involves a more complicated calculation than should be covered in this paper.

Up to this point we have spoken of the repayment of a loan in so many periods of time. The first and second methods above mentioned apply of course to the amortization of a leasehold when the cost is known. Let us look at the subject now from another angle: What can I afford to invest in a lease running for ten years paying me an annual rental of $1,000.00 so that my investment will be repaid by the end of the period with 5% interest? This is a sum in simple proportion. We have already shown that ten annual payments of 1.628 894 will repay a loan of 12.577 890 in ten years including 5% interest, so that ten annual payments of 1 would repay a loan of 7.721 734. As the annual payments are $1,000.00 each the answer is $7,721.73.

A shorter method, however, is to use the table of present values which is usually incorporated with a compound-interest table. This table is also based upon 1 but, in place of showing the amount to which 1 will accumulate it shows the amount which will accumulate to 1. The reasoning and proportionate results are exactly the same but questions like that just given can be answered with less work through the present-value table.

As 1.000 000 will accumulate to 1.050 000 at 5% interest in one year, 1.000 000 is the present value of 1.050 000 due one year hence. Similarly 1.050 000 is the present value of 1.102 500 due one year hence therefore 1.000 000 is the present value of 1.102 500 due two years hence. As each succeeding term in an "Amount of 1" table at 5% is the prior figure multiplied by 1.05, so each amount in the table is the succeeding amount divided by 1.05. For example, the third term is $(1.05)^3$ so that the second term is $(1.05)^3$ divided by 1.05 or $(1.05)^2$. The present value of $(1.05)^3$ due three years hence is 1, that is $(1.05)^3$ divided by $(1.05)^3$. To obtain the present value therefore of any amount due n years hence we divide it by $(1+i)^n$. The following fragment of the table of present values of 1 at 5% represents 1 divided by $(1.05)^1$, $(1.05)^2$, $(1.05)^3$ and so on. In order to save repetition the "Present value of an annuity of 1" is here given in conjunction, this table being formed by addition as in the amount-of-annuity table previously explained:

Periods	*Present value of 1*	*Present value of annuity of 1*
1	.952 381	.952 381
2	.907 029	1.859 410
3	.863 838	2.723 248
4	.822 702	3.545 950
5	.783 526	4.329 476
6	.746 215	5.075 692
7	.710 681	5.786 373
8	.676 839	6.463 212
9	.644 609	7.107 821
10	.613 913	7.721 734

And so on.

The present value of an annuity, that is the amount which could be paid or lent now, to be returned by ten annual payments of $1,000.00 each which would include 5% interest on the loan, would be (1) the footing of lines 1 to 10 inclusive of the first column, (2) line 10 deducted from 1 and the result divided by the rate per cent or (3) the item on line 10 of the second column. In this case the result would be multiplied by $1,000.00, giving the answer computed by proportion at the beginning of this discussion.

If the first payment of the annuity is to be made in advance take one period less and add one to the result. The formula for the first table is $\frac{1}{1+i^n}$ and for the second $\frac{1-\frac{1}{1+i^n}}{i}$,

the latter symbol representing the compound discount on 1 for the entire term of n periods divided by the rate per cent per period.

As 7.721 734 is the present value at 5% of an annuity of 1 for ten years the present value at that rate of an annuity of any other amount for ten years would be that amount multiplied by 7.721 734.

Similarly, if the amount of the loan be known the equal annual payments which would pay it off with interest in 10 years would be the loan divided by 7.721 734. If, for instance, the loan be 12.577 890 the annual payments would be 12.577 890 divided by 7.721 734, which equals 1.628 894, as already shown by the table previously given.

As in amounts, the above rules apply to all rates of interest and to all periods of time so long as the interest is compounded once each period at the stated rate per period.

It sometimes happens that although a payment is made now the annuity will not begin for some years. For instance, an annuity might be purchased from a life insurance company by a man aged 45 to begin when he was 65. This is called a deferred annuity. To eliminate life contingencies, however, let us assume that in consideration of a payment made now an investment company is to pay to some charitable institution a sum of $1,000.00 per annum for 10 years but that the payments are to be deferred for 5 years. This would mean from an actuarial point of view that the first payment instead of being paid at the end of the first year would be paid at the end of the sixth, not fifth, year. So long as the investment company expects to be able to earn the same rate of interest during the first five years as during the next ten there are two methods of determining the purchase price: (1) From the present value of an annuity of 15 years deduct the present value of one for 5 years. (2) Find the present value of an annuity for 10 years and divide by $(1+i)^5$ which will give the present value of that sum.

If the company expects to be able to earn 6% during the first five years and 5% thereafter, the first method cannot be used and the divisor in the second method will be $(1.06)^5$.

If the student has reached this point and has understood the reasoning he has mastered what most beginners seem to find the hardest part of the textbooks on actuarial science, the application of the formulæ given to everyday subjects. The writer has not tried to invade the province of Finney, Sprague, Glen or other authors of excellent textbooks on the subject, but has tried to reach the student who considers these textbooks beyond his depth through having perhaps forgotten the small amount of algebra required. He hopes, however, that this foundation will lead beginners to a study of these textbooks which they will find exceedingly interesting as well as instructive.

Appendix

While all such problems may be worked out arithmetically this method is very cumbersome particularly if the payments are made and the interest is compounded at more frequent intervals than one year or if the interest is compounded at intervals more frequent than the payments. The following formulæ which are those used by Glen, whose textbook was suggested by the Institute board of examiners before Mr. Finney's book was published, are accordingly given and do not require to be understood to be used if the explanation given below is followed. They are based on an investment of 1.

Explanation of Symbols

i	actual interest earned per unit of time (usually a year) per unit invested (5% equals .05 on an investment of 1)
j	nominal annual interest rate where interest is compounded more frequently than once a year e.g. 6% per annum compounded semi-annually: j is .06 but i is .0609 or $(1+\frac{j}{2})^2-1$
n	number of units of time, usually years
m	number of times interest is compounded per unit of time
p	number of times payments of principal are made per unit of time (each payment is $\frac{1}{p}$)
$(1+i)^n$	amount to which 1 will accumulate in n periods with compound interest at rate i per period
$(1+i)^n-1$	amount of compound interest on 1 for n periods at rate i per period
$(1+\frac{j}{m})^{mn}$	amount to which 1 will accumulate in n periods with compound interest at nominal rate j per period compounded m times per period (multiply m by n)
$\frac{1}{p}$ (power)	the p root of the amount stated (1/2 or 2/4 would therefore mean square root)
$s_{\overline{n}\rceil}$	amount to which an annuity of 1 will accumulate in n periods at rate i. The first payment to be made at the end of the first period
$s_{\overline{n+1}\rceil}-1$	amount of an annuity due—similar to above except that the first payment is made at the beginning of the first period

$a_{\overline{n}\rceil}$	present value of an annuity of 1 for n periods at rate i, the first payment to be made at the end of the first period
$a_{\overline{n-1}\rceil}+1$	present value of an annuity due.

Formulæ for Sinking Funds

To find the amount to be deposited annually in a sinking fund earning rate i to meet a loan of x maturing in n years use the following formulæ, which show the sum to which an annual payment of 1 would amount, and divide x by the result. The amount to be provided for annual interest on the loan itself is in addition:

If payments are made and interest is compounded annually.

$$(1) \qquad s_{\overline{n}\rceil} = \frac{(1+i)^n-1}{i}$$

If payments are made and interest is compounded more frequently but p and m are the same:

(2) Using the nominal rate j:

$$s_{\overline{n}\rceil} = \frac{(1+\frac{j}{m})^{mn}-1}{j}$$

(3) Using the actual or effective annual rate i (if given):

$$s_{\overline{n}\rceil} = \frac{(1+i)^n-1}{p\{(1+i)^{\frac{1}{p}}-1\}}$$

If p and m are different:

(4) Using the nominal rate j:

$$s_{\overline{n}\rceil} = \frac{(1+\frac{j}{m})^{mn}-1}{p\{(1+\frac{j}{m})^{\frac{m}{p}}-1\}}$$

(5) Using the effective annual rate i:

$$s_{\overline{n}\rceil} = \frac{(1+i)^n-1}{p\{(1+i)^{\frac{1}{p}}-1\}}$$

In formulæ 2, 3, 4 and 5 multiply the result by p to obtain the divisor for the periodic payment.

Example. Required the amount to be deposited annually in a sinking fund earning 5% to meet a loan of \$100,000.00 maturing in 10 years. Divide \$100,000.00 by the result of the appropriate formula.

Formula 1. i is .05 and n is 10. Result 12.57789

Formula 2. If made half yearly j is .05, m is 2 and mn is 20. Result 12.77232

Formula 3. If effective rate per annum is given as 5% i is .05 and p is 2 and $\frac{1}{p}$ here means square root of 1.05. Result 12.73318

Formula 4. If p is 4 and m is 2. Result 12.85169

Formula 5. If p is 4. Result 12.81132

The results of 2, 3, 4 and 5 would as above stated be multiplied by 2, 2, 4 and 4 to obtain the divisors, but the basic amounts are stated to show more clearly the small differences in the results of more frequent compounding and payment.

Formulæ for Repayment of Loans by Equal Instalments including Interest

The formulæ are very similar to the above and show the amount of a present loan which would be repaid with interest by payments of 1 annually (including interest) for n years. The annual payments necessary to repay any stated amount would be the stated amount divided by the result of the formula. Using the same conditions and numbers as in sinking funds above:

$$(1) \qquad a_{\overline{n}|} = \frac{1-\frac{1}{(1+i)^n}}{i}$$

$$(2) \qquad a_{\overline{n}|} = \frac{1-\frac{1}{(1+\frac{j}{m})^{mn}}}{j}$$

$$(3) \qquad a_{\overline{n}|} = \frac{1-\frac{1}{(1+i)^n}}{p\{(1+i)^{\frac{1}{p}}-1\}}$$

$$(4) \qquad a_{\overline{n}|} = \frac{1-\frac{1}{(1+\frac{j}{m})^{mn}}}{p\{(1+\frac{j}{m})^{\frac{m}{p}}-1\}}$$

$$(5) \qquad a_{\overline{n}|} = \frac{1-\frac{1}{(1+i)^n}}{p\{(1+i)^{\frac{1}{p}}-1\}}$$

If $s_{\overline{n}|}$ be known $a_{\overline{n}|}$ can be obtained by dividing $s_{\overline{n}|}$ by $(1+i)^n$ or by $(1+\frac{j}{m})^{mn}$ as the case may be.