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Edward Fraser

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Some Problems in Loan Valuation Simplified

By Edward Fraser

MONTHLY PAYMENT LOANS

Text-books on elementary actuarial science, such as accountants use, deal generally, for the sake of explicit examples, with loans which mature exactly in a certain number of equal payments, as in this manner the correctness of the solution given is easily proved. In practice, however, the final payment is often an odd amount, as it is usually desirable to have the monthly or periodic payments in round figures.

Where the equal monthly payment includes interest on the unpaid balance the accountant is frequently called upon to calculate the balance unpaid at a certain date. In the case of a loan extending over a long period with a final payment of an unknown odd amount this might be a tedious matter if each payment had to be calculated separately, but a short formula is as follows, P being the amount of each equal periodic payment, i the periodic rate of interest and m the number of payments made:

$$\text{Unpaid balance} = \frac{P}{i} - \left(\frac{P}{i} - \text{original loan}\right)(1+i)^m$$

For instance, if the original loan were \$1,291.23, the monthly payments \$20.00 each, the rate of interest one-half of one per cent per month and 41 payments had been made, the unpaid balance would be:

$$\frac{20}{.005} - \left(\frac{20}{.005} - 1{,}291.23\right)(1.005)^{41}$$
$$= 4{,}000 - (4{,}000 - 1{,}291.23)\ 1.226898 = 676.61 \text{ (plus)}$$

Only one multiplication has to be made if a compound-interest table is available.

This formula is explained as follows. Consider the loan as originally made to have been the purchase price of two investments, the first an annuity of P for m periods, costing $P\ a_{\overline{m}|}$, and the second the present value of a certain sum of money, X, due at the end of m periods, X representing the present value at that time of the balance of the annuity, an unknown amount.

As the total original cost is known, being the amount of the loan, the formula becomes:

$$\begin{aligned} \text{Original loan} &= P\, a_{\overline{m|}} + X\, v^m \\ X\, v^m &= \text{loan} - P\, a_{\overline{m|}} \\ X &= (\text{loan} - P\, a_{\overline{m|}})(1+i)^m \\ &= \left(\text{loan} - \frac{P}{i}\left(1 - \frac{1}{(1+i)^m}\right)\right)(1+i)^m \\ &= \left(\text{loan} - \frac{P}{i} + \frac{P}{i(1+i)^m}\right)(1+i)^m \\ &= \left(\text{loan} - \frac{P}{i}\right)(1+i)^m + \frac{P}{i} \\ &= \frac{P}{i} - \left(\frac{P}{i} - \text{loan}\right)(1+i)^m \end{aligned}$$

The result can of course be obtained from the first lines of the formula if the $a_{\overline{n|}}$ table is available but as more calculations have to be made the liability to err is greater.

ANNUITIES DUE

Where the payment is made in advance in place of at the end of the period the present value or amount of such an annuity is found by multiplying $a_{\overline{n|}}$ or $s_{\overline{n|}}$ by $(1+i)$ or, if m and p are different, by $\left(1+\frac{j}{m}\right)^{\frac{m}{p}}$, j being the nominal annual rate of interest, m the number of times per annum the interest is to be compounded and p the number of times per annum the payments are made. This is a simpler formula than that usually given and there is less chance of error in interpreting the symbols.

That this method is correct can be easily shown by writing down a few terms of the geometrical progression composing an annuity certain and the corresponding terms for an annuity due. Each of the latter will be seen to be each of the former multiplied by $(1+i)$ or by $\left(1+\frac{j}{m}\right)^{\frac{m}{p}}$ as the case may be.

DEFERRED ANNUITIES

Two methods are usually given for ascertaining the present worth of deferred annuities: $a_{\overline{t+n|}} - a_{\overline{t|}}$ and $v^t\, a_{\overline{n|}}$. As the present value of an annuity is the same as the present value of its amount another formula is $v^{t+n}\ s_{\overline{n|}}$.

A question in the Institute examination of May, 1925, required the sinking fund at 4% necessary to meet an annuity of $10,000.00 per annum for ten years deferred ten years. This could be answered either by dividing the present value of the deferred annuity by $a_{\overline{20|}}$ or by dividing the amount by $s_{\overline{20|}}$.

VALUE OF AN ANNUITY WHERE A SINKING FUND IS TO BE ACCUMULATED AT A SMALLER RATE THAN IS TO BE EARNED ON THE INVESTMENT

Where the rates of interest are the same it is obvious to the student of actuarial science that the cost of an annuity of 1 per period will be refunded by providing a sinking fund of $(1-i\ a_{\overline{n|}})$ therefore $a_{\overline{n|}}=s_{\overline{n|}}\ (1-i\ a_{\overline{n|}})$. This same formula can be used to ascertain $a_{\overline{n|}}$, an unknown amount, by calculating $s_{\overline{n|}}$ at the given sinking-fund rate, i being the rate to be earned on the investment. By multiplying the items within the bracket by $s_{\overline{n|}}$, as calculated, $a_{\overline{n|}}$ is arrived at and is then multiplied by the amount of the annuity. This method is much more easily understood than Hoskold's complicated formula.

For example, an investor desiring to earn 7% on his investment, but able to earn only 4% in a sinking fund, wishes to purchase a lease having 20 years to run with a net rental income of $10,000.00 per annum. What could he pay for the lease?

Formula: $a_{\overline{n|}}=s_{\overline{n|}}\ (1-i\ a_{\overline{n|}})$

$s_{\overline{n|}}$ for 20 years at 4% is 29.7780786

$$a_{\overline{n|}}=29.7780786\ (1-.07\ a_{\overline{n|}})$$

$$a_{\overline{n|}}=29.7780786-2.0844655\ a_{\overline{n|}}$$

$$3.0844655\ a_{\overline{n|}}=29.7780786$$

$$a_{\overline{n|}}=9.6542103$$

multiply by $10,000, cost equals $96,542.103.

Interest at 7% on the investment would be $6,757.9472 per annum and a sinking fund of the annual excess received—$3,242.0528—invested at 4% would produce $96,542.10 in 20 years.

Where the rates of interest differ, the excess received over the interest earned is not applied in direct reduction of the investment and the amount of interest earned annually is maintained unchanged during the entire period. An investor might have considerable trouble in proving to a field examiner that the interest earned on the sinking fund was not taxable income!

PURCHASE PRICE OF A BOND REDEEMABLE AT MATURITY AT A PREMIUM

Bond tables do not reflect premiums and Makeham's formula is easily forgotten, in addition to which j does not represent the actual interest collected on par (unity), which is confusing to the student, "par" in this case being the principal ultimately receivable including the premium.

Simply add to the value shown by the bond table the present value of the premium to be received. This is obvious, as the bond-table figure is the present value of the interest coupons receivable plus the present value of par (unity), both calculated at the rate of interest desired.

Where neither bond nor $a_{\overline{n}|}$ tables are available and where the rate of interest desired is one to be found in a compound interest table, a simple means of ascertaining the purchase price of a bond is to multiply the amount receivable at maturity, including premium if any, by $\left(v^n+\frac{j}{i}\right)-\left(v^n\times\frac{j}{i}\right)$, j being the nominal rate received on the principal including premium and v^n of course being calculated at rate i. This is an adaptation of Makeham's formula.

For example a \$1,000 bond bearing $4\frac{1}{2}\%$ interest is due in 20 years at a premium of 20%. The purchaser desires to earn 5%. On a basis of \$1,200 j is not $4\frac{1}{2}\%$ but $3\frac{3}{4}\%$; v^n at 5% is .3768895. The purchase price is therefore:

$$1200\left(.3768895+\frac{.0375}{.05}-.3768895\times\frac{.0375}{.05}\right)$$
$$=1200\,(.3768895+.75 \quad -.3768895\times .75)$$
$$=1200\,(1.1268895 \quad -.2826671)$$
$$=1200\times .8442224=1013.06688$$

Where an $a_{\overline{n}|}$ table is available the present worth at 5% of the coupon annuity of \$45.00 per annum for 20 years is shown to be \$560.7995 and the present worth at 5% of \$1,200.00 due in 20 years is \$452.2674, a total of \$1,013.0669 as above.

The "yield" rate given in the usual bond table is the nominal rate per annum, not the effective rate: that is to say a yield rate of 5% means an effective rate of $2\frac{1}{2}\%$ per half year where the coupon rate given is stated to be payable semi-annually.