Journal of Accountancy

Volume 39 | Issue 1 Article 6

1-1925

Formula in Compound Interest

E. J. Oglesby

A Formula in Compound Interest

By E. J. Oglesby

Several texts on investment accounting give an approximate formula for determining the time necessary for money to double itself at compound interest. This formula is

$$n=\frac{.693}{r}+.35$$

where n is the number of periods and r is the rate per period. For example, if we want to find the time for doubling at 6%, compounded semi-annually, $r=.03$ and we have

$$n=\frac{.693}{.03}+.35=23.45 \text{ periods,}$$

or 11.72 years, approximately.

We shall extend this formula to include the general problem of finding the time necessary for money to be multiplied by any factor, k.

We have the relation,

$$(1+r)^n=k,$$

which we must solve for n. Taking the logarithms of each side of this equation, we have

$$n \log_e (1+r)=\log_e k$$

where the subscript, e, indicates that we are using natural logarithms, base $e=2.71828$.

This gives

$$n=\frac{\log_e k}{\log_e (1+r)}.$$

Now

$$\log_e (1+r)=r-\frac{r^2}{2}+\frac{r^3}{3}-\frac{r^4}{4}+\ \ldots$$

(See Hall and Knight, *Higher Algebra*, page 191.)

Therefore, we have

$$n=\frac{\log_e k}{r-\frac{r^2}{2}+\frac{r^3}{3}-\frac{r^4}{4}+\ \ldots}$$

which becomes, by division,

$$n=\log_e k\left(\frac{1}{r}+\frac{1}{2}-\frac{r}{12}+\ \ldots\right).$$

Taking the first two terms of this series, we have

$$n = \log_e k\left(\frac{1}{r} + \frac{1}{2}\right)$$

the general formula for n in terms of k and r.

From a table of natural logarithms, we have

k	$\log_e k$
2	.6931
3	1.0986
4	1.3863
5	1.6094
6	1.7918
7	1.9459
8	2.0794
9	2.1972
10	2.3026

For $k=2$, we get, by substituting in the general formula,

$$n = .6931\left(\frac{1}{r} + \frac{1}{2}\right) = \frac{.6931}{r} + .3465$$

or $n = \frac{.6931}{r} + .35$, the well-known formula for doubling the principal.

Similarly

$$k=3,\ n = 1.0986\left(\frac{1}{r} + \frac{1}{2}\right) = \frac{1.0986}{r} + .5493$$

$$n = \frac{1.0986}{r} + .55$$

$$k=4,\ n = \frac{1.3863}{r} + .69$$

$$k=5,\ n = \frac{1.6094}{r} + .80$$

$$k=6,\ n = \frac{1.7918}{r} + .90$$

$$k=7,\ n = \frac{1.9459}{r} + .97$$

$$k=8,\ n = \frac{2.0794}{r} + 1.04$$

$$k=9,\ n = \frac{2.1972}{r} + 1.10$$

$$k=10,\ n = \frac{2.3026}{r} + 1.15$$

From a table of natural logarithms we can form an approximate rule for any value of k. If a table of natural logarithms is not available, we can use the relation

$$\log_e k = 2.3025851 \log_{10} k.$$

For example, suppose we want a formula for $k=1.75$ and have no table of natural logarithms

$$\begin{aligned}\log_e 1.75 &= 2.30259 \log_e 1.75\\ &= (2.30259)\ (.24304)\\ &= .5596\end{aligned}$$

whence, for $k=1.75$,

$$n=\frac{.5596}{r}+.28.$$

Suppose we test the accuracy of our general formula,

$$n=\log_e k\left(\frac{1}{r}+\frac{1}{2}\right).$$

This was derived by taking the first two terms of a series of ascending powers of r. Since r is always small, these terms decrease in value very rapidly. The error made by taking only the first two terms is less than the first discarded term,

$$\left(\frac{-r}{12}\right)\log_e k.$$

For $k=100$ and $r=.05$, we have

$$\begin{aligned}\left(\frac{-r}{12}\right)\log_e k &= -\left(\frac{.05}{12}\right)\log_e 100 = \frac{(-.05)\ (2.30259)}{12}\log_{10} 100\\ &= \frac{(-.05)\ (2.30259)\ (2.00000)}{12} = -.02\end{aligned}$$

That is, the result given by our formula will be greater than the true result by less than .02.

For $k=100$ the formula is

$$\begin{aligned}n &= \log_e 100\left(\frac{1}{r}+\frac{1}{2}\right)\\ &= (2.3025851)\log_{10} 100\left(\frac{1}{r}+\frac{1}{2}\right)\\ &= 4.605170\left(\frac{1}{r}+\frac{1}{2}\right)\\ n &= 4.6052\left(\frac{1}{r}+\frac{1}{2}\right)=\frac{4.6052}{r}+2.30\end{aligned}$$

For $r=.05$ this gives $n=94.40$.

Whereas the correct value is $94.387+$.