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Computation of Earned Discounts

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By George W. Wilde

One of the most remarkable developments of modern business is the increasingly large volume and range of sales on the instalment plan. There are few commodities of some permanent value that can not be acquired by means of one or another of the many forms of deferred payments. The scope of these transactions is constantly expanding and affords an ever-widening field for the exercise of the functions pertaining to the financing of such operations. This has resulted in a large growth of organizations variously known as mortgage companies, automotive finance corporations, discount and acceptance corporations and the like, which, by whatever name known, are established for the common purpose of financing transactions involving the purchase and sale of real or personal property on the instalment plan. It is with such organizations that we are at present concerned, and particularly with the manner in which their income is earned.

The income of these companies is derived from two main sources, namely, (a) a consideration paid by the borrower for the privilege of obtaining the funds required, known as discount, commission, bonus or advance interest, and (b) interest on the loan. Incidentally, other income is derived from insurance, fees and miscellaneous sources, involving no special problem not commonly encountered in the ordinary course of business.

Upon reflection it becomes apparent that the consideration paid for obtaining the loan is nothing but a form of interest paid in advance, for which reason the title "advance interest" describes it better, perhaps, than any other term. However, for convenience, and to distinguish this sum from the additional charge which is often made for interest at a fixed rate on the nominal amount of the loan, we shall refer to it as discount.

The particular problem confronting the management of these companies and the professional accountants engaged to advise them is the proper method of determining when and in what proportions this discount is earned. It can not be emphasized too strongly that the problem is one requiring a practical solution productive of results as close to the precision of actuarial exactitude as can be obtained, capable of ready application to a large

number and variety of loans. However desirable mathematical accuracy may be, concessions necessarily must be made to conditions as they exist, one of which is that employees of commercial organizations generally are not students of actuarial science. Moreover, even if they were, it is questionable whether the time consumed and expense involved in making precise calculations would be justified, if an alternative and much more rapid method were available which would ensure approximately accurate results that could be regarded as satisfactory for all practical purposes.

We shall endeavor to explain and illustrate some of these alternative methods and, by comparison with results on the actuarial basis, to indicate the degree of their inaccuracy, thus furnishing definite information upon which to base conclusions as to the desirability or otherwise of adopting any of them.

It may be assumed that in all the types of loan hereinafter discussed the amount to be repaid by the borrower is greater than the cost of the loan to the lender (i. e., the actual sum lent) by the amount of the discount. Stated concretely, for an actual loan of $900 the borrower may agree to repay $1,000, the difference of $100 representing the discount, in addition to which he may be required to pay interest at a fixed rate per cent. per annum on the nominal principal of the loan, or $1,000.

A description of the several types of loans most frequently made is given hereunder:

(a) Loans repayable in full at a specified date, interest at a fixed rate being payable periodically during the term of the loan.

(b) Loans repayable in regular periodical instalments of equal amount, without interest.

(c) Loans repayable in regular periodical instalments of unequal amount, without interest.

(d) Loans repayable in regular periodical instalments of equal amount, plus interest at a fixed rate on the outstanding balances.

(e) Loans repayable in regular periodical instalments of unequal amount, plus interest at a fixed rate on the outstanding balances.

(f) Loans repayable in regular periodical instalments of equal amount, including interest at a fixed rate on the nominal principal.

(g) Loans repayable partly in regular periodical instalments of equal amount, including interest, and the balance in one payment at the end of the term of the instalment period.

On loans in class "a" the interest is received as earned, but the discount is not received until the expiration of the term of the loan. Hence, the amounts of discount earned periodically should be regarded as successive additions to the actual loan, and as the amount of the principal increases throughout the term of the loan, the periodical earnings will increase correspondingly. However, little fault can be found with the common practice of crediting income periodically with an aliquot part of the discount, for the reason that, ordinarily, the error introduced by this method is negligible.

Alternative methods of computing earned discount applicable to loans in class "b" are illustrated in tables I and II, the example used being a loan of $1,500, discounted $150, repayable in five periods in equal instalments of $300 each.

Table I

Based upon the time for which one unit or its equivalent is outstanding

Period	Balance of nominal principal	Discount earned			Amortization of loan	Balance of actual loan
		Basis	Percentage of total	Amount		
						$1,350.00
1.....	$1,500.00	5/15	33.33	$50.00	$250.00	1,100.00
2.....	1,200.00	4/15	26.67	40.00	260.00	840.00
3.....	900.00	3/15	20.00	30.00	270.00	570.00
4.....	600.00	2/15	13.33	20.00	280.00	290.00
5.....	300.00	1/15	6.67	10.00	290.00	.....
	$4,500.00	15/15	100.00	$150.00	$1,350.00	.....

Table II

Based on the effective interest rate (3.618% per period)

Period	Payments	Discount		Amortization of loan	Balance of actual loan
		Percentage of total	Amount		
					$1,350.00
1.....	$300.00	32.56	$48.84	$251.16	1,098.84
2.....	300.00	26.51	39.76	260.24	838.60
3.....	300.00	20.23	30.34	269.66	568.94
4.....	300.00	13.72	20.58	279.42	289.52
5.....	300.00	6.98	10.48	289.52	.....
	$1,500.00	100.00	$150.00	$1,350.00	.....

In table I, amounts ranging from $1,500 to $300 each are outstanding for one period, equivalent to a unit of $300 outstanding for terms of five periods to one period respectively, together equivalent to one unit outstanding for fifteen periods. The total discount is divided on the basis of the time equivalent of the amount outstanding at the beginning of each period. It will be noted that, in relation to the balances of nominal principal, the percentage of discount taken up as earned is constant (3.33%), but that in relation to the outstanding balances of the actual loan the percentage gradually declines, as shown hereunder.

Period	Balance of actual loan	Discount earned	
		Amount	Percentage
1.....	$1,350.00	$50.00	3.7037
2.....	1,100.00	40.00	3.6363
3.....	840.00	30.00	3.5714
4.....	570.00	20.00	3.5088
5.....	290.00	10.00	3.4483

A comparison of these tables discloses the fact that the earnings on outstanding balances for corresponding periods vary from an overstatement of .0857 to an understatement of .1697 per cent., although the greatest variation is in respect of the smallest balance outstanding, thus modifying its effect in dollars and cents. The differences in periodical earnings vary from 77/100ths to 32/100ths of one per cent. of the total discount, which surely may be regarded as meeting the requirement of approximate accuracy satisfactory for all practical purposes, especially when it is borne in mind that the term used in the example given is extremely short, and that it can be demonstrated that the longer the term of the loan the smaller the gradation of differences. Admittedly, this method results in a slight overstatement of income in the initial stages of operations, which, however, is automatically adjusted in due course as to current profits by the overlapping of the periods of successive loans.

It is obvious that in table I the same result would be obtained by using as a basis the percentage of the outstanding balances of nominal principal to the aggregate of such balances. The further application of this basis will be illustrated hereafter.

Tables III, IV, V and VI indicate alternative methods applicable to loans in class "c", the example used being a loan of $5,095.74,

discounted $416.85, repayable in twelve periods. While the example apparently exaggerates the form of loan repayable in unequal instalments, it is submitted as an actual case of the kind of loan giving rise to the subject under discussion.

Table III

Taking up as earned an aliquot part of the discount each period

Period	Payments	Discount	
		Percentage of total	Amount
1	$606.00	8.333	$34.74
2	606.00	8.333	34.74
3	595.34	8.333	34.74
4	551.60	8.333	34.74
5	518.35	8.333	34.74
6	483.67	8.333	34.74
7	436.92	8.333	34.74
8	414.44	8.333	34.74
9	401.07	8.333	34.74
10	304.18	8.333	34.73
11	167.42	8.333	34.73
12	10.75	8.333	34.73
	$5,095.74	100.00	$416.85

Table IV

Based upon the percentage of each periodical payment to the nominal principal

Period	Payments	Discount	
		Percentage of total	Amount
1	$606.00	11.892	$49.57
2	606.00	11.892	49.57
3	595.34	11.683	48.70
4	551.60	10.825	45.13
5	518.35	10.172	42.40
6	483.67	9.492	39.57
7	436.92	8.574	35.74
8	414.44	8.133	33.90
9	401.07	7.871	32.81
10	304.18	5.969	24.88
11	167.42	3.286	13.70
12	10.75	.211	.88
	$5,095.74	100.00	$416.85

Table V

Based upon the percentage of the balance of the nominal principal outstanding at the beginning of each period to the aggregate of such balances

Period	Balance of nominal principal	Payments	Discount: Percentage of total	Discount: Amount	Amortization of loan	Balance of actual loan	Per cent earned
						$4,678.89	1.7261
1....	$5,095.74	$606.00	19.375	$80.76	$525.24	4,153.65	1.7132
2....	4,489.74	606.00	17.071	71.16	534.84	3,618.81	1.7011
3....	3,883.74	595.34	14.767	61.56	533.78	3,085.03	1.6894
4....	3,288.40	551.60	12.503	52.12	499.48	2,585.55	1.6778
5....	2,736.80	518.35	10.406	43.38	474.97	2,110.58	1.6659
6....	2,218.45	483.67	8.435	35.16	448.51	1,662.07	1.6546
7....	1,734.78	436.92	6.596	27.50	409.42	1,252.65	1.6421
8....	1,297.86	414.44	4.935	20.57	393.87	858.78	1.6302
9....	883.42	401.07	3.359	14.00	387.07	471.71	1.6196
10....	482.35	304.18	1.834	7.64	296.54	175.17	1.6156
11....	178.17	167.42	.678	2.83	164.59	10.58	1.6068
12....	10.75	10.75	.041	.17	10.58	.....	.....
	$26,300.20	$5,095.74	100.00	$416.85	$4,678.89	.....	.....

Table VI

Based upon the effective interest rate (1.6915% per period)

Period	Payments	Discount: Percentage of total	Discount: Amount	Amortization of loan	Balance of actual loan
					$4,678.89
1.....	$606.00	18.985	$79.14	$526.86	4,152.03
2.....	606.00	16.848	70.23	535.77	3,616.26
3.....	595.34	14.674	61.17	534.17	3,082.09
4.....	551.60	12.506	52.13	499.47	2,582.62
5.....	518.35	10.479	43.68	474.67	2,107.95
6.....	483.67	8.555	35.66	448.01	1,659.94
7.....	436.92	6.736	28.08	408.84	1,251.10
8.....	414.44	5.076	21.16	393.28	857.82
9.....	401.07	3.481	14.51	386.56	471.26
10.....	304.18	1.912	7.97	296.21	175.05
11.....	167.42	.710	2.96	164.46	10.59
12.....	10.75	.038	.16	10.59	.....
	$5,095.74	100.00	$416.85	$4,678.89	.....

The basis reflected in table III can be defended only on the ground of expediency. Obviously it results in a material misstatement of earnings throughout the entire term of the loan. It is submitted for comparative purposes as a method that in some cases has been adopted.

Table IV is based upon the fallacy that the discount is in the nature of a capital sum recoverable only if and when payments are made. The degree of its inaccuracy becomes apparent by comparison with the succeeding tables.

Table V is similar in construction to table I, with the necessary elimination of the unit column on account of the unequal payments.

Table VI is self-explanatory.

A comparison of these last two tables discloses differences of a nature similar to those existing between tables I and II. Again it is apparent that the amounts of discount earned periodically vary so little that the method indicated by table V might reasonably be adopted without involving any material degree of inaccuracy.

We come now to the somewhat more complex types of loans upon which, in addition to the discount included in the nominal principal, interest is paid at a fixed rate.

That type of loan involving payment of interest on the outstanding balance in addition to the repayment of the nominal principal presents no problem in the computation of the earned discount different from those heretofore discussed, so that for loans of class "d" we might adopt the method indicated in table I, and for class "e" that used in table V. When we come to class "f", however, we are confronted with a variation not so readily handled.

A common class of loans repayable in equal periodical instalments including interest is that comprising loans secured by mortgages on real estate. Invariably, these are long-term loans of, say, from three to ten years, usually repayable in monthly instalments. In practice, the instalments are frequently determined in the following manner:

The term of the loan is divided into periods of six months each. The periodical payments necessary to liquidate the loan with interest at the fixed rate are then computed from annuity tables. These amounts are divided by six to determine the monthly payments, which payments, as they are recorded, are apportioned

to principal and interest. The amounts of interest decrease and the amounts of principal increase periodically, so that we have the type of loan in which periodical payments of principal are unequal. Thus, we may revert to the use of table V if a tabulation of the loan is made, but this involves so much work for a large number of long-term loans that recourse is sometimes taken to the method indicated in table I.

An illustration will indicate the degree of variation between different methods. The figures used are typical of thousands of similar loans which vary in amount, rate of discount and term, but embody principles common to all.

A loan of $4,000, discounted $400, is repayable in ten years in equal monthly instalments including interest at the rate of 8 per cent. per annum. For convenience, the term will be divided into twenty periods of six months each. The annuity necessary to repay the loan in the given time at the given rate is fractionally less than $294.33, and from table VII we find that the actual amount paid by the borrower is $5,886.54, of which $1,886.54 is interest and $400 discount. It remains to be determined how this discount shall be taken up as earned. The amortization of nominal principal in table VII includes amortization of the discount, which is segregated in table VIII upon the basis of a constant periodical rate ascertained in this case to be .9089 per cent., applied to the outstanding balances of the actual loan, and the resultant earnings will be compared with those derived from the methods used in tables I and V, and the actuarial basis.

The sum actually lent was $3,600, so that instead of a nominal rate of 4 per cent. per period, the theoretically effective rate determined by the actuarial method is 5.22163 per cent. per period (table IX). As previously indicated, however, circumstances preclude the adoption of the actuarial basis involving the determination of the effective rate. Therefore, admitting its superiority from the standpoint of accuracy, we shall use the table only for comparative purposes without further discussion of its merits.

The compilation of table X enables us to compare the earnings from table IX with the interest received plus the discount earned on the bases of tables I, V and VIII respectively.

Table VII

Showing balance of nominal principal after applying each periodical payment (interest computed at nominal rate on balance of nominal principal)

Period	Payments	Interest	Amortization of nominal principal	Balance of nominal principal
				$4,000.00
1	$294.33	$160.00	$134.33	3,865.67
2	294.33	154.63	139.70	3,725.97
3	294.32	149.04	145.28	3,580.69
4	294.33	143.23	151.10	3,429.59
5	294.33	137.18	157.15	3,272.44
6	294.32	130.90	163.42	3,109.02
7	294.33	124.36	169.97	2,939.05
8	294.33	117.56	176.77	2,762.28
9	294.32	110.49	183.83	2,578.45
10	294.33	103.14	191.19	2,387.26
11	294.33	95.49	198.84	2,188.42
12	294.32	87.54	206.78	1,981.64
13	294.33	79.27	215.06	1,766.58
14	294.33	70.66	223.67	1,542.91
15	294.32	61.72	232.60	1,310.31
16	294.33	52.41	241.92	1,068.39
17	294.33	42.73	251.60	816.79
18	294.32	32.67	261.65	555.14
19	294.33	22.20	272.13	283.01
20	294.33	11.32	283.01	.....
	$5,886.54	$1,886.54	$4,000.00	.....

Table VIII

Showing amortization of discount on the basis of nominal principal as in table VII.

Period	Amortization of nominal principal	Amortization of discount	Amortization of actual loan	Balance of actual loan
				$3,600.00
1	$134.33	$32.72	$101.61	3,498.39
2	139.70	31.80	107.90	3,390.49
3	145.29	30.82	114.47	3,276.02
4	151.09	29.78	121.31	3,154.71
5	157.15	28.67	128.48	3,026.23
6	163.43	27.51	135.92	2,890.31
7	169.96	26.27	143.69	2,746.62
8	176.77	24.96	151.81	2,594.81
9	183.84	23.58	160.26	2,434.55
10	191.18	22.13	169.05	2,265.50

Table VIII (Contiuued)

Period	Amortization of nominal principal	Amortization of discount	Amortization of actual loan	Balance of actual loan
11	$198.84	$20.59	$178.25	$2,087.25
12	206.79	18.97	187.82	1,899.43
13	215.05	17.26	197.79	1,701.64
14	223.67	15.47	208.20	1,493.44
15	232.61	13.57	219.04	1,274.40
16	241.91	11.58	230.33	1,044.07
17	251.60	9.49	242.11	801.96
18	261.66	7.29	254.37	547.59
19	272.12	4.98	267.14	280.45
20	283.01	2.56	280.45	.....
	$4,000.00	$400.00	$3,600.00	.....

Table IX

Showing result by actuarial method (effective rate 5.22163 per cent. per period)

Period	Payments	Interest	Amortization of loan	Balance of loan
				$3,600.00
1	$294.33	$187.98	$106.35	3,493.65
2	294.33	182.43	111.90	3,381.75
3	294.32	176.58	117.74	3,264.01
4	294.33	170.43	123.90	3,140.11
5	294.33	163.96	130.37	3,009.74
6	294.32	157.16	137.16	2,872.58
7	294.33	149.99	144.34	2,728.24
8	294.33	142.46	151.87	2,576.37
9	294.32	134.53	159.79	2,416.58
10	294.33	126.18	168.15	2,248.43
11	294.33	117.40	176.93	2,071.50
12	294.32	108.17	186.15	1,885.35
13	294.33	98.45	195.88	1,689.47
14	294.33	88.22	206.11	1,483.36
15	294.32	77.46	216.86	1,266.50
16	294.33	66.13	228.20	1,038.30
17	294.33	54.22	240.11	798.19
18	294.32	41.68	252.64	545.55
19	294.33	28.49	265.84	279.71
20	294.33	14.62	279.71	.....
	$5,886.54	$2,286.54	$3,600.00	.....

Table X

Comparison of results by actuarial methods with those arrived at by use of tables already given herein.

	Interest and discount earned			
		Tables		
	IX	VII and I	VII and V	VII and VIII
	1	2	3	4
1	$187.98	$198.10	$193.92	$192.72
2	182.43	190.82	187.41	186.43
3	176.58	183.33	180.64	179.86
4	170.43	175.61	173.60	173.01
5	163.96	167.66	166.27	165.85
6	157.16	159.47	158.65	158.41
7	149.99	151.03	150.73	150.63
8	142.46	142.32	142.49	142.52
9	134.53	133.35	133.92	134.07
10	126.18	124.09	125.01	125.27
11	117.40	114.54	115.74	116.08
12	108.17	104.68	106.10	106.51
13	98.45	94.51	96.08	96.53
14	88.22	83.99	85.64	86.13
15	77.46	73.15	74.80	75.29
16	66.13	61.93	63.52	63.99
17	54.22	50.35	51.79	52.22
18	41.68	38.38	39.60	39.96
19	28.49	26.01	26.91	27.18
20	14.62	13.22	13.72	13.88
	$2,286.54	$2,286.54	$2,286.54	$2,286.54

The significance of the differences in the amounts of interest and discount earned will be more readily comprehended by submitting them in the form of percentages.

Percentage of periodical earnings to total

	Table X Column			
	1	2	3	4
First period	8.22	8.66	8.48	8.43
Eighth period	6.23	6.22	6.23	6.23
Aggregate to eighth period	58.21	59.84	59.20	59.02
Last period	.64	.58	.60	.61

Using the actuarial method (column 1) as a basis, it will be noted that for the first period the overstatements of earnings by the three other methods are respectively only 44/100ths, 26/100ths and 21/100ths of 1 per cent. of the total earnings. These overstatements decrease periodically until the eighth period, when the

earnings by any method are substantially the same. The aggregate earnings to the end of the eighth period are overstated only 1.63%, .99% and .81% respectively. Thereafter the earnings are understated by varying percentages which, in the last period are 6/100ths, 4/100ths and 3/100ths of 1 per cent. respectively.

It is of interest to note the variation in the percentages of the periodical earnings to the corresponding periodical balances in table VII.

Table X

	Column 2	Column 3	Column 4
First period	4.9525%	4.848%	4.818%
Eighth period	4.8424%	4.848%	4.849%
Last period	4.6712%	4.848%	4.904%

Thus it will be seen that by using the method indicated in table V, applied to the balances in table VII, the periodical earnings are a constant percentage of the corresponding periodical nominal balances, which furnishes a strong argument in favor of the adoption of this basis whenever it is feasible to do so. However, the fact must not be overlooked that it will involve the construction of an additional table for each loan, and that this work can be eliminated by the adoption of the method indicated in table I. While this latter method results in a slightly larger overstatement of earnings in the earlier periods, from a practical point of view consideration must be given to the relative importance of this condition and the additional cost of securing more nearly accurate results by any alternative method.

There remains to be dealt with only that type of loan described as class "g". In practice, the treatment is quite simple. The loan is divided into two parts — one, that portion to be paid in instalments, and the other, the balance payable at the end of the term. The discount is divided in the proportions of these two parts to the whole, and that part applicable to the instalment loan is taken up on the basis of table I or V, the balance being prorated equally over the term of the loan. In this relation reference is made to the discussion of the treatment of discount on class "a" loans.

The application of the method indicated in table I to discounts on loans repayable over lengthy periods is facilitated by the use of tables in the following form which may be compiled very rapidly with the aid of a calculating machine.

Term—5 periods			Term—10 periods		
Period	Unit	Percentage	Period	Unit	Percentage
1.....	5/15	33.33	1.....	10/55	18.18
2.....	4/15	26.67	2.....	9/55	16.36
3.....	3/15	20.00	3.....	8/55	14.55
4.....	2/15	13.33	4.....	7/55	12.73
5.....	1/15	6.67	5.....	6/55	10.91
			6.....	5/55	9.09
			7.....	4/55	7.27
			8.....	3/55	5.45
			9.....	2/55	3.64
			10.....	1/55	1.82

Tables may be prepared for any number of periods desired in such form as will render them instantly available for application to loans for any term.

The discount earned each period is readily ascertained by multiplying the total discount by the percentage for that period.

It may be well to describe briefly the steps to be taken in recording the earned discount when it is ascertained.

The earnings on each loan are entered in a columnar record with a separate column for each successive period (usually months), and the totals of these columns are posted periodically to earned-discounts account. Upon repayment of a loan prior to its maturity the whole of the discount not previously credited to income becomes earned and is taken up at that time. Such loans are then entered in another and similar record, and amounts corresponding to the discounts unearned at the date the loan is paid off are distributed to the appropriate columns. Periodically the totals of these columns are transferred to and deducted from the totals of the corresponding columns in the earned-discounts record, leaving a net amount to be credited to earned-discounts account.

Installing this system for an organization commencing business is a comparatively simple process, but for a company which has been operating for some time the inception involves considerable labor. In spite of the volume of work entailed there seems to be no more satisfactory method for all purposes than listing the outstanding loans as of a specific date, computing and tabulating the unearned discounts at that date by the new method, and making the necessary adjustments of the accounts affected. Thereafter, the saving in time will more than offset the time consumed in effecting the change, while additional satisfaction will be derived

from the knowledge that the accounts accurately reflect the result of operations.

One other point deserves attention. Where it appears to be necessary to effect a change in the method of computing earned discounts it is not unlikely that the assertion will be made that the margin of inaccuracy resulting from the method in use is not sufficient to warrant any change, and it may prove somewhat difficult to persuade the responsible officers or employees to open their minds to the reception of the suggestion that there may be other and better bases for the computation than the one in use. Hence, the accountant must be prepared to demonstrate the fact to their complete satisfaction, and to furnish an answer to the inevitable question, "What is the proper method of computing earned discounts?" The foregoing presentation of the salient features of the subject will have served its purpose if it furnishes a guide to the solution of the problems ordinarily encountered.