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Transient radiation from a circular string of dipoles excited at superluminal velocity

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Abstract

This paper discusses the features of transient radiation from periodic one-dimensional resonant medium excited by ultrashort pulses. The case of circular geometry is considered for a harmonic distribution of the density of the particles along the circle. It is shown that a new frequency component arises in the spectrum of the scattered radiation in addition to the resonance frequency of the medium. The new frequency appears both in the case of linear and nonlinear interaction, its value depends on the velocity of excitation pulse propagation and on the period of spatial modulation. In addition, the case when the excitation moves at superluminal velocity and Cherenkov radiation arises is also studied.

1 Introduction

In a recent study of radiation of a linear string of harmonic oscillators with spatially modulated density excited by a moving point source, we found several non-trivial features of the spectrum associated with the transient processes [1,2]. Namely, in addition to the radiation typical for resonance systems at the eigenfrequency of the oscillator, a spectral component is excited that is specific to the spatial distribution of the density of oscillators. In particular, new quasi monochromatic components appear in the radiation spectrum of the medium in the case of periodic modulation of the oscillators density. Moreover, the frequencies of these components depend on the period of modulation and the direction of observation. Our main focus lies on the situation when the velocity of propagation of the excitation exceeds the velocity of light in vacuum c, which is accompanied by the emission of waves characteristic of the Cherenkov effect. The theory of relativity does not allow the movement of a physical object at a speed larger than the speed of light in vacuum when the transfer of the signal (information) occurs. However, in physics, and in particular in optics, the possibility of superluminal motion of a localized perturbation without signal transmission was well-known before the appearance of the theory of relativity [3–5]. Throughout the 20th century, several aspects of superluminal motion were studied. In particular, in the 1960s-70s, the problem of existence of tachyons, i. e. the particles moving at velocities greater than the velocity of light, was considered [6, 7]. In the 1930s, the Cherenkov radiation arising when electric charge moves through a medium at a velocity exceeding the phase velocity of light in the medium was discovered [8–14]. This radiation is associated with the movement of the local polarization of the medium [15], which has an instantaneous response to an external excitation. Various sources of electromagnetic radiation moving at superluminal velocity are considered in Refs. [16–18]. In [19] the radiation of a charge moving faster than light is studied. Recently, a broad class of papers appeared [20] in which the occurrence of Cherenkov radiation in the form of the second harmonic with respect to the frequency of the incident light wave in the media with periodic distribution of the susceptibility (nonlinear photonic crystals) was demonstrated. Radiation induced by the motion of charged particles along the periodic structure was also considered in [21–23]. This radiation is known as the Purcell-Smith radiation and it was demonstrated experimentally in [21] for the first time.

In optics, the examples of objects moving at superluminal velocity are well-known. In the 1960s-70s, after the appearance of first lasers, the possibility of superluminal propagation of short powerful light pulses in an amplifying medium due to nonlinear amplification of pulse was demonstrated experimentally and theoretically by Basov et al. [24–28]. Superluminal motion can also be observed in the situation when a light pulse is incident obliquely on a flat surface. In this case the intersection of the pulse and the screen (light spot) moves along the screen with the velocity $V = c/\sin\beta > c$, where β is the angle of the incidence, see Fig. 1 [17, 29]. In particular, in Refs. [1, 2, 30] the scattering of ultrashort pulse propagating at superluminal velocity on a flat surface with a spatially modulated density of scattering centers located along a straight line or plane was studied. The case of instant relaxation of scattering centers was considered. The resulting time dependence of the amplitude of the scattered light describes the variation of the density of scattering centers, and the time scale of the corresponding signal depends on the angle of observation.

However, the motion of a light spot on a remote screen from rotating searchlight at a constant angular velocity (or even a pulsar) is most often mentioned in the literature as an example of superluminal excitation agent [16–18]. Modern methods of circular scan of light or electron beams management allow one to obtain the values of frequency of rotation of the order of 10^{10} Hz and above. Accordingly, the linear velocity of the circular movement of the excitation zone formed at the intersection of a "pencil-form" rotating beam from the emitter and a plane orthogonal to the axis of the cone can be varied in a very wide range from subluminal to superluminal by simple displacement of the plane along the axis of the cone. The dynamic polarization of the medium created by the incident beam of photons or electrons is the source of radiation in all cases described above. In Refs. [1, 2], an interesting feature of the emission of a periodic linear string consisting of identical harmonic oscillators was reported. Namely, the generation of coherent Cherenkov-type radiation at a frequency that can be different from the resonance frequency of the medium was observed. The appearance of a new frequency in the medium emission spectrum is related to the fact that the scattered waves arriving at the observation point contain the transient process. The properties of this transient process are related to the time delay for radiation to come to the point of observation from different parts of the medium. Such a delay occurs both at superluminal and at subluminal velocity of the excitation propagation. The new frequency occurring in the scattered field spectrum does not depend on the resonance frequency of the oscillators, but depends on the angle of observation α (see Fig. 1), on the spatial period of the oscillator distribution along the z axis Λ_z and on the velocity of the exciting ultrashort pulse V [1, 2]:

$$\Omega_1 = 2\pi \frac{V/\Lambda_z}{|\frac{V}{c}\cos\alpha - 1|}. \tag{1}$$

We emphasize that the relation (1) corresponds to the case when oscillators are arranged along a straight line and nonlinear effects are not taken into account. In the present paper, we consider the cases when the medium formed of identical oscillators is arranged along a circle. Of special methodological and possibly practical interest is, in our opinion, the study of interaction of ultrashort pulses of light with a resonant medium under conditions when the area of its impact on the medium moves faster than the velocity of light in vacuum. The optical response of the medium can be used to detect such superluminal motions. This response may have interesting properties that differ from the properties of the "classical" Cherenkov radiation [1,2] arising due to the motion of physical objects with the velocity exceeding the phase velocity of light in the medium, but is not greater than the velocity of light in vacuum.

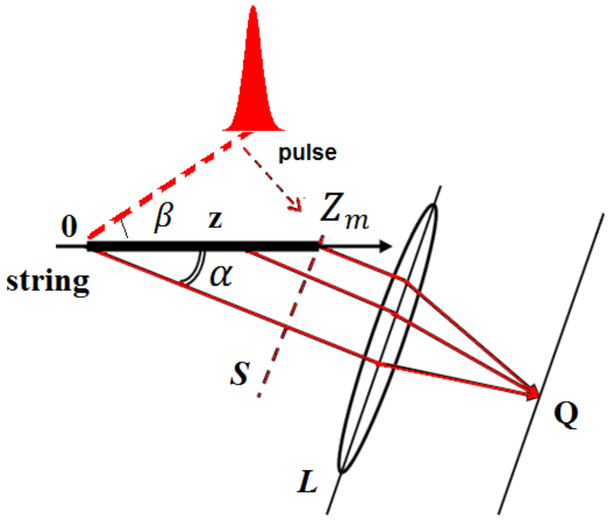

Figure 1: Excitation of a string by a pulse train propagating at superluminal velocity. The intersection point of the beam and the medium moves along the string at the velocity $V = c/\sin\beta > c$. The observer is placed far away from the string or in the focus of a lens L which collects the radiation of the string propagating at the angle α with respect to the z-axis.

In the presented work, we examine the situation when the medium oscillators are arranged along the circle and their density is modulated harmonically along the circle. We focus on two cases when the emitted radiation is measured in the center of the circle or at the circle. We consider both cases of linear and nonlinear interaction of the excitation field with the medium particles. We demonstrate that the medium radiation that occurs in the system under the excitation by a short pulse contains both a resonance frequency of the oscillators and the new frequency. This new frequency arises in the transient regime; this is why we used the term "transient" in the title of paper. The generated emission is shown to be different from the resonance frequency of the medium in wide range of the excitation conditions. The possibility of the experimental demonstration and practical application of the considered phenomena is briefly discussed.

2 Radiation in the center of the circle

In this paper we consider the scattering medium with a one-dimensional ring geometry (see Fig. 2). We suppose that oscillators are arranged along a circle of radius R, and the distribution

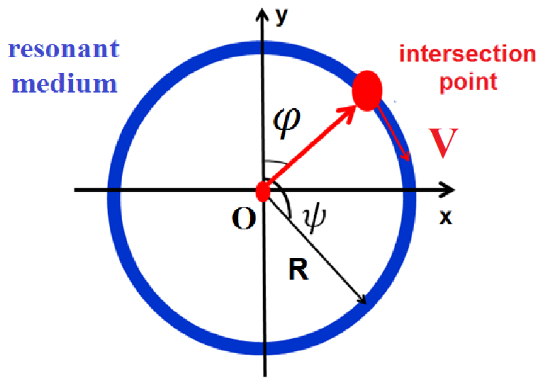

Figure 2: Circular geometry of the string. The source of an ultrashort pulse with a broad spectrum is located in the center of the circle and rotates quickly. The cross-section of the pulse and medium (red dot) moves at the velocity V along the resonant string of dipoles (blue circle).

of number density of oscillators depends harmonically on the polar angle as

$$N_{\varphi}(\varphi) = \frac{1}{2}\left[1 + \cos\left(\kappa\varphi\right)\right], \tag{2}$$

where $\kappa = \frac{2\pi}{\Lambda_{\varphi}}$ is the dimensionless frequency of the angular distribution of the medium oscillators, Λ_{φ} is the angular period of this distribution. The medium is excited by some physical object (for example, a beam of light, or an electron beam, or ultrashort laser pulses, etc.) running along the circle only once at constant linear velocity V. Among the possible examples of oscillators, nanoantennas [31–33], two-level atoms, quantum dots [34–37] or dye molecules deposited onto the concave diffraction grating can be considered. In this section we consider the case where the observer detects the radiation in the center of the circle, see Fig. 2.

2.1 Linear response of oscillators

In the case when the exciting pulse is weak and the dipole response is linear, its response to the excitation pulse $\mathbf{E}_{exc}(t)$ is described by the polarization $P(t)$:

$$\ddot{P} + \gamma\dot{P} + \omega_0^2 P = gE_{exc}(t), \tag{3}$$

where g is the coupling strength to the field.

We also assume, as in [2], that the interaction time of the exciting beam with the excitation zone of the circle has a duration smaller than the period of oscillations of the medium $T_0 = 2\pi/\omega_0$ (or comparable with it), and the pulse spectrum of interaction is flat and broad enough to include the resonance frequency of the oscillator. Optical pulses with these characteristics can be obtained, for example, in the terahertz range upon excitation of the gaseous medium, see reviews [38–40]. In this case the response of the oscillators can be described to a high degree of precision by the following expression for the polarization [2, 41]:

$$P(t) \approx e^{-\gamma t}\cos(\omega_0 t)\Theta(t), \tag{4}$$

where $\Theta(t)$ is the Heaviside step-function, ω_0 is the resonance frequency of the medium oscillators and γ is the decay constant of the electric field. Thus, the radiation field of an oscillator located on the circle at a point which corresponds to the polar angle φ is proportional the expression (except for a constant factor):

$$E(\varphi, t) \sim \exp\left[-\frac{\gamma}{2}\left(t - \frac{R\varphi}{V}\right)\right] \cos\left[\omega_0\left(t - \frac{R\varphi}{V}\right)\right] \Theta\left[t - \frac{R\varphi}{V}\right]. \tag{5}$$

Relation (5) takes into account the fact that the oscillator located at the point φ starts to radiate at the moment of arrival of the excitation to the point with coordinate $\frac{R}{V}\varphi$. The total field in the center of the circle is obtained by the integration of the Eq. (5) over the circle taking into account the spatial density distribution of the oscillators Eq. (2) and the time of propagation of the radiation from the circle to its center R/c. This field is given by the expression [2]:

$$E(t) \sim \int_0^{2\pi} N_\varphi(\varphi) \exp\left[-\frac{\gamma}{2}\left(t - \frac{R\varphi}{V} - \frac{R}{V}\right)\right] \cos\left[\omega_0\left(t - \frac{R\varphi}{V} - \frac{R}{V}\right)\right] \Theta\left[t - \frac{R\varphi}{V} - \frac{R}{V}\right] d\varphi. \tag{6}$$

It can be easily shown that for $\gamma \to 0$ the new frequency component Ω_2 appears in the spectrum of the registered emission, see Ref. [2]:

$$\Omega_2 = 2\pi \frac{V/\Lambda_\varphi}{R} = k\frac{V}{R}, \tag{7}$$

which depends on the angular frequency of the oscillators density distribution $1/\Lambda_\varphi$, on the radius of the circle R, and on the velocity of excitation propagation V. Eq. (7) can be rewritten as

$$\Omega_2 = k\Omega_{exc}, \tag{8}$$

where Ω_{exc} is the angular velocity of the excitation pulse propagation.

Under the condition of resonance:

$$\frac{V}{c} = \frac{\Lambda_\varphi R}{\lambda_0}, \tag{9}$$

Eq. (7) gives $\Omega_2 = \omega_0$, i.e. the proper medium frequency coincides with the new frequency and the observer located at the center will register the radiation at the resonance frequency ω_0. Note that the case considered above qualitatively coincides with the case considered in our recent works [1, 2] in the case of linear topology of the medium when the radiation is detected at the direction perpendicular to the string.

Next, we consider numerical examples. We assume that the excitation point propagates over the circle only one time and take the following parameters of the problem: $c = 1$, $R = 1$, $\frac{\Lambda_\varphi R}{\lambda_0} = 2$, $\omega_0 = 20$, $\gamma = 0.95$, $V/c = 4.1$, $\Omega_2 = 41$. The result of the numerical solution of the integral Eq. (6) and the spectrum of the registered field are illustrated in Fig. 3a and Fig. 3b, respectively.

Analysis of the obtained solution shows that the radiation at the point O begins at the time moment that coincides with the time moment of the arrival at the observation point O of the field emitted by the first oscillator at the circle. Afterwards, the radiation from the other oscillators of the string arrives at this point. As a result, a complicated transient process is formed which leads to the appearance of the new frequency in the radiation spectrum, see Fig. 3b. After the transient process ends, the observer at the point O registers decaying oscillations of the emitted field at the resonance frequency, see Fig. 3a.

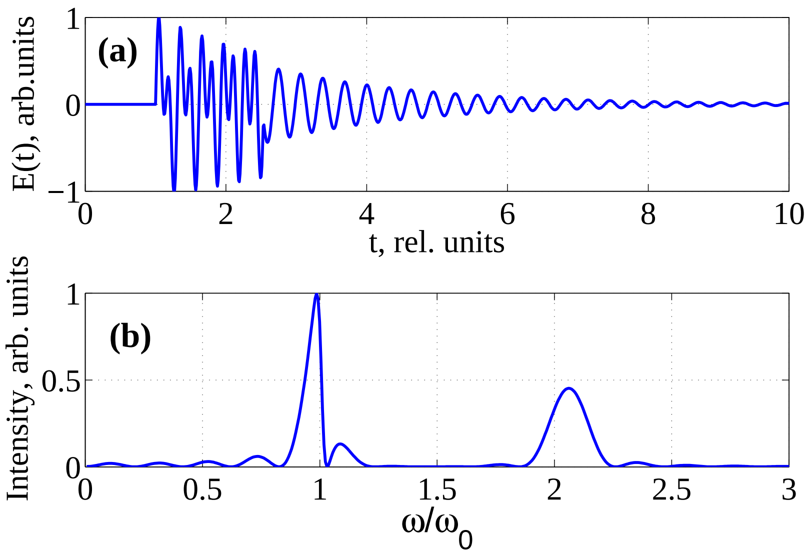

Figure 3: (a) Time dependence of the electric field $E(t)$ excited by the string and (b) the corresponding intensity spectrum $I(\omega)$ in the center of the circle for the circular scheme depictured in Fig. 2 and the parameters $c = 1$, $R = 1$, $\frac{\Lambda_\varphi R}{\lambda_0} = 2$, $\omega_0 = 20$, $\gamma = 0.95$, $V/c = 4.1$, $\Omega_2 = 41$.

The consideration given above is valid for arbitrary values of the velocity of excitation V. If the velocity of excitation is subluminal or equal to the velocity of light c the observer at the point O registers the radiation at the frequency given by Eq. (7).

2.2 Nonlinear response

A two-level atomic particle can be taken as an example of an oscillator, and the consideration presented above is valid if the pumping pulse is of significantly small power so that nonlinear effects arising when short pulse interacts with resonant medium can be neglected. However, if the pumping pulse is strong enough the medium response becomes nonlinear. For the calculation of the dynamics of the medium polarization $P(t,\varphi) = d_{12}N_0\left[u(t,\varphi)\cos\omega t + v(t,\varphi)\sin\omega t\right]$ and the population difference $N(t,\varphi) = N_0 w(t,\varphi)$ under the excitation with strong pulse, we use the system of optical Bloch equations as in Ref. [2] for the two-level atoms in the dimensionless coordinates [26, 42]:

$$\frac{du(t,\varphi)}{dt} = -\Delta\omega v(t,\varphi) - \frac{1}{T_2}u(t,\varphi), \tag{10}$$

$$\frac{dv(t,\varphi)}{dt} = -\Delta\omega u(t,\varphi) - \frac{1}{T_2}v(t,\varphi) + \Omega_R(t,\varphi)w(t,\varphi), \tag{11}$$

$$\frac{dw(t,\varphi)}{dt} = -\frac{1}{T_1}(w(t,\varphi)+1) - \Omega_R(t,\varphi)v(t,\varphi). \tag{12}$$

Here N_0 is the concentration of two-level atoms in the string, $u(t,\varphi)$, $v(t,\varphi)$ are the components of the polarization that are in phase and quadrature components of the medium polarization per unit atom, respectively, $w(t,\varphi)$ is the population difference of the single atom, $\Delta\omega$ is the frequency detuning between pumping field and resonance frequency of the medium, d_{12} is the transition dipole moment, T_1 is the population difference relaxation time, T_2 is the polarization relaxation time, Ω_R is the Rabi frequency of the pumping field. In our consideration we assume that the pumping pulse has a Gaussian shape $\varepsilon(t) = E_0 \exp\left(\frac{-t^2}{\tau_p^2}\right)$. In the particular case of vanishing detuning $\Delta\omega = 0$ and sufficiently large relaxation times, $T_1 = T_2 = \infty$, one can obtain the following analytical solution for the material equations (10)-(12) [26, 42]:

$$N(t,\varphi) = N_0 w(t,z) = N_0 \cos\Phi(t,\varphi), \tag{13}$$

$$P(t,\varphi) = d_{12}N_0 v(t,z) = d_{12}N_0 \sin\Phi(t,\varphi)\sin\omega_0 t, \tag{14}$$

where Φ is so-called pulse area:

$$\Phi(t,\varphi) = \frac{d_{12}}{\hbar}\int_{-\infty}^{t} \varepsilon(t',\varphi)dt', \tag{15}$$

The system of optical Bloch equations (10)-(12) has a simple physical interpretation [26, 42]. The dependence of N and P on time can be represented as the rotation of a unit Bloch vector $\rho = (v,w)^t$ in the (v,w) plane. In this case the function Φ given by (15) is the angle of rotation of this vector. For example, $\Phi = \pi$ corresponds to a rotation of the Bloch vector at angle π, which is followed by the complete transition of the atom to the upper excited level (π pulse), and $\Phi = 2\pi$ corresponds to an excitation of the atom to the excited level, which is followed by the return to the ground level (2π pulse). Taking into account Eq. (6) and Eq. (14) we obtain that the electric field at the point O in the nonlinear case is proportional to:

$$E(t) \sim \int_0^{2\pi} N_\varphi(\varphi) P\left[t - \frac{R\varphi}{V} - \frac{R}{V}\right] \sin\left[\omega_0\left(t - \frac{R\varphi}{V} - \frac{R}{V}\right)\right] d\varphi. \tag{16}$$

An example of the nonlinear dynamics of the electric field and its spectrum is presented in Fig. 4a,b. ($c = 1$, $R = 1$, $\frac{\Lambda_\varphi R}{\lambda_0} = 2$, $\omega_0/\gamma = 22.2$, $V/c = 2.3$, $\Omega_2/\omega_0 = 1.15$, $\tau_p = 2T_0$, where T_0 is the period of proper atomic oscillations $T_0 = 2\pi/\omega_0$, total pulse area is

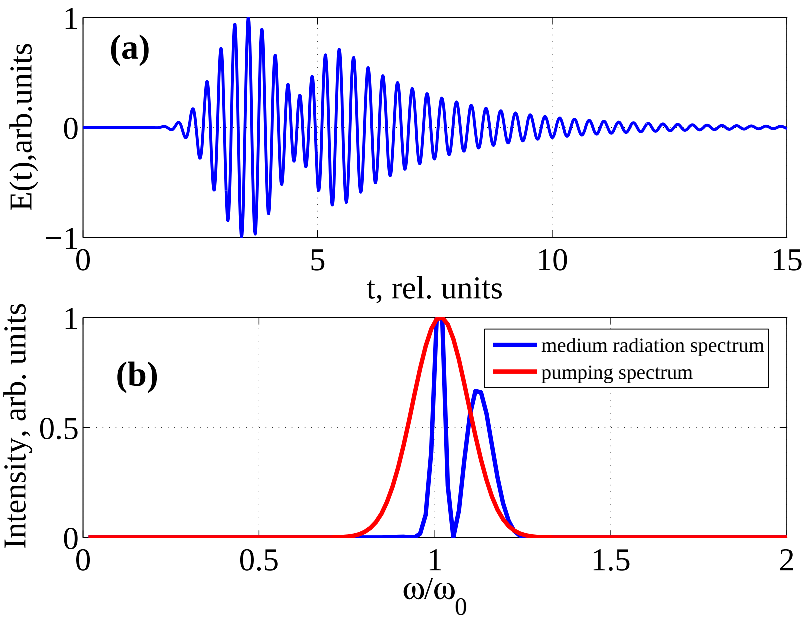

Figure 4: (a) Time dependence of the electric field $E(t)$ excited by the string and (b) the corresponding intensity spectrum $I(\omega)$ in the center of the circle for the circular scheme depictured in Fig. 2 and the parameters $c = 1$, $R = 1$, $\frac{\Lambda_\varphi R}{\lambda_0} = 2$, $\omega_0/\gamma = 22.2$, $V/c = 2.3$, $\Omega_2/\omega_0 = 1.15$, $\tau_p = 2T_0$.

$\Phi_\infty = \frac{\pi}{2}$). Under these parameters the spectrum of the registered oscillations is presented in Fig. 4b using the blue solid line and the spectrum of pumping pulse is given by the red line. The new frequency can be easily seen in the medium radiation spectrum. Fig. 5 shows the dependence of the secondary emission spectrum on the total pulse area Φ. The pulse area was changed via variation of the Rabi frequency (pulse amplitude), keeping the pulse duration constant. In Fig. 5, one can observe two branches corresponding to the resonance frequency ω_0 and to the shifted frequency Ω_2. The analysis of Fig. 5 shows that, as in the case of straight string, in strongly nonlinear regime when the pulse area is large, the radiation at the resonance frequency ω_0 has smaller intensity than that at the frequency Ω_2. A slight shift of the maximum of the radiation at the resonance frequency ω_0 is caused by oscillator damping and does not depend on the magnitude of the external field. The frequency shift of the resonant oscillations is not observed at all values of the excitation pulse area Φ. Note the absence of radiation at the resonance frequency at the pulse areas multiple to 2π. These points correspond to both observed amplitude maxima of the shifted frequency.

The value of the new frequency excited by the external field coincides with the result of the calculation in the weak-field approximation when the pulse area is small. Increasing the field amplitude leads to approximately linear increase of the frequency shift, and this process has a periodic sawtooth character. From Fig. 4 one can see that in the strongly nonlinear regime when the pulse area is large, the intensity of radiation at the new frequency Ω_2 exceeds the intensity of the radiation at the fundamental frequency ω_0 . The spectrum of the two-level system response to an external field, contains the spectral component at the frequency $\omega_+ = \omega_0 + \Omega_R$. In the

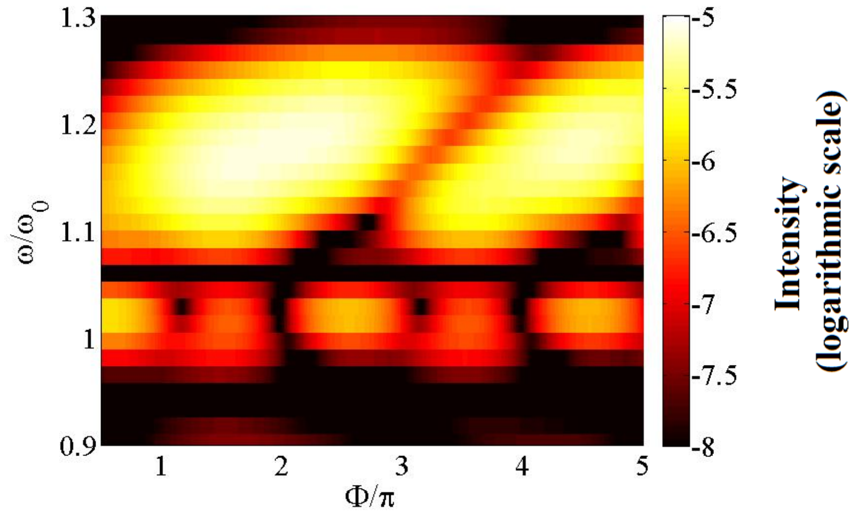

Figure 5: Dependence of the radiation spectrum on the pulse area Φ when $V/c = 2.3$, $\frac{\Lambda_\phi R}{\lambda_0} = 2$, $\omega_0/\gamma = 22.2$, $\Omega_2/\omega_0 = 1.15$, $\tau_p = 2T_0$.

example illustrated in Fig. 5 the greater the amplitude of pump field the more accurate is fulfilled the equality $\omega_+ \approx \Omega_2$. With the increase of the amplitude of the pump pulse the linear increase of the intensity of radiation at frequency ω_+ occurs, see Fig. 5. The periodic structure observed in Fig. 5 can be explained by the periodic dependence of the polarization of the medium on the pulse area (the term $\sin\Phi$ in the expression (14) for the polarization P). We note that a similar dependence was observed in our recent paper [2] in the case of the linear geometry of the medium.

3 Radiation at the circle

In this section we assume that the radiation propagates along the circle and it is measured at some point with polar coordinate ψ (Fig.1) at the circle. We assume that the medium radiation can propagate only along a circle as in whispering gallery mode lasers [43–45], and not along the chord. In this case, it is easy to obtain an expression for the electric field at the point ψ:

$$E(t,\psi) = \int_0^{2\pi} N_\varphi(\varphi) \cos\left[\omega_0 f_\psi\left(t,\psi\right)\right] \Theta\left[f_\psi\left(t,\psi\right)\right] d\varphi. \tag{17}$$

Here $f_\psi\left(t,\psi\right) = t - \frac{R\varphi}{V} - \frac{R(\psi-\varphi)}{c}$.

Since we assume that the light can propagate only along the circle, the term $\frac{R(\psi-\varphi)}{c}$ in (17) corresponds to the time of the radiation propagation from the oscillator located at a point with polar angle φ to the observation point ψ. The calculation of this integral shows that the spectrum of the transient process also contains a new frequency in addition to the fundamental frequency of the medium (see Appendix):

$$\Omega_3 = 2\pi \frac{V/\Lambda_\varphi}{|1-\frac{V}{c}|R}. \tag{18}$$

Unlike the previous case, the new frequency depends on the ratio $\frac{V}{c}$.

We now discuss the physical sense of (18). The numerator of this formula is the frequency of the incident pulse excitation of the system of oscillators periodically arranged along the circle. For the observer located at the point ψ, the process would seem to have a frequency Ω_3. Under the condition of resonance (9) formula (18) becomes

$$\Omega_{3D} = \frac{\omega_0}{|1-\frac{V}{c}|}, \tag{19}$$

and coincides with the formula for the frequency shifted due to the Doppler effect when the source is moving towards a stationary receiver.

In order to illustrate the dependence of Ω_3 on the parameters of system, we present the radiation spectrum in dependence on V (Fig. 6a) and R (Fig. 6b). As can be easily seen from (18), the new frequency increases with V when $V/c < 1$ and decreases with an increase of V when $V/c > 1$. Also V decreases with the increase of R.

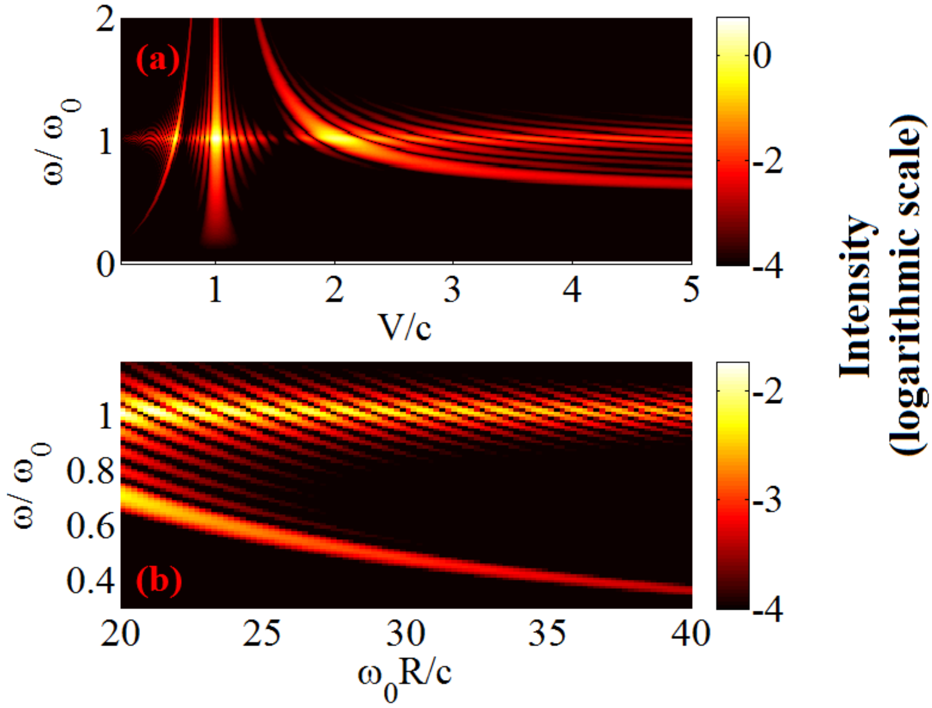

Figure 6: (a) Dependence of the radiation spectrum on the normalized propagation speed V of the excitation and the radius of the circle R (b) for $\psi = 2\pi$.

If the excitation propagates at the velocity c, the denominator in (18) for the new frequency is equal to zero. In this case the radiation from all points on the circle converges to point ψ simultaneously, and the transient process is absent. Accordingly, there will not be any new frequencies in the spectrum. We performed an elementary consideration of this type of propagation which does not take into account spatial nonlinear effects that may occur when a short pulse propagates in the resonant medium. In general case, it is necessary to perform calculations using a more rigorous semiclassical theory of the interaction of light with the resonant medium [42]. This topic is the subject of a separate study and will not be considered in this paper. However, one can qualitatively understand what happens if the ultrashort pulse propagates along the circle considering the results of the study of straight resonant medium. We suppose that the short pulse with the duration smaller than the relaxation times of the medium T_1 and T_2 moves at some velocity (for example, superluminal, as in [1,2]). We assume that the medium is optically dense. As is known [46], a short pulse propagating in such a medium leaves the "tail" of induced resonance polarization behind it. This polarization emits an electromagnetic field - "coherent optical ringing" after the excitation pulse comes out. Since the medium is optically dense, one can expect the appearance of cooperative effects, i.e., complex periodic exchange of the energy between field and polarization can occur. This process takes place during time intervals shorter than the relaxation time of the polarization of the medium T_2 [47–51]. This will lead to the appearance of new frequencies in the spectrum of medium response.

The consideration we presented in Ref. [2] for the case of linear topology of the string shows that in the nonlinear case the occurrence of the frequency determined by the expression (1) is also possible.

4 Conclusion

Finally, we have demonstrated the possibility of generation of new frequencies in the resonant medium by considering different medium topologies and space-distribution parameters. These new frequencies can be observed in the absence of nonlinear effects and in the presence of them, as well. In the case of linear string topology, the new frequency depends on the velocity of excitation, on the period of the spatial density distribution of the oscillators, and on the angle of observation [2]. In the case of the circular string topology the expressions for the frequencies strongly depend on the position of the observation point.

Our calculations show that in the linear case when the power of the pump pulse is small, the radiation intensity at the resonance frequency is always greater than the radiation intensity at the new frequency. However, in the nonlinear case, when the power of the pump pulse is high, the situation is vice versa. The intensity of radiation at the new frequency can exceed the emission intensity at the resonance frequency. We have observed the similar situation in the case of linear geometry of the medium in [2].

The studied phenomena can be used for detection of superluminal motions in a resonant medium, frequency conversion, and determination of the spatial structure of the scattering system using the spectrum of the scattered signal. The occurrence of the transient process can also be used for the generation of radiation pulse profiled in time. The technique of the analysis presented here can be applied for the study of nonstationary diffraction on photonic crystals.

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5 Appendix: Analytical solution of Eq. (17).

In the case when radiation is measured at the point ψ at the circle one can obtain for the transient regime ($\frac{R\psi}{V} < t < \frac{R\psi}{V} + \frac{R\psi}{c}$ and if $V > c$):

$$E(t) = \int_{W_0 t'''/R}^{2\pi} N(\phi) \cos\left[\omega_0 \left(t''' - \frac{R\phi}{W_0}\right)\right] d\phi = -\frac{W_0}{R\omega_0} \sin\left(\omega_0 (t''' - \frac{2\pi R}{W_0})\right) + \\ + \frac{W_0^2 \nu_\phi \sin(2\pi\nu_\phi) \cos\left[\omega_0 (t''' - \frac{2\pi R}{W_0})\right]}{\nu_\phi^2 W_0^2 - R^2\omega_0^2} + \frac{R\omega_0 W_0 \cos(2\pi\nu_\phi) \sin\left[\omega_0 (t''' - \frac{2\pi R}{W_0})\right]}{\nu_\phi^2 W_0^2 - R^2\omega_0^2} - \\ - \frac{W_0^2 \nu_\phi \sin(\Omega_3 t''')}{\nu_\phi^2 W_0^2 - R^2\omega_0^2}. \tag{20}$$

Here $t''' = t - \frac{R\psi}{c}$ and $\frac{1}{W_0} = \frac{1}{V} - \frac{1}{c}$. This expression contains terms oscillating with the frequencies ω_0 and Ω_3.

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