Weierstraß-Institut
für Angewandte Analysis und Stochastik
Leibniz-Institut im Forschungsverbund Berlin e. V.

Preprint ISSN 2198-5855

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submitted: December 18, 2019

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No. 2661
Berlin 2019

2010 *Mathematics Subject Classification.* 78A40,78A60, 35Q60.

2010 *Physics and Astronomy Classification Scheme.* 42.25.Bs, 42.65.Sf, 42.65.Wi, 42.81.Dp.

Key words and phrases. Nonlinear fibers, pulse propagation, pulse manipulation, controlling light by light, solitons, Raman scattering.

U.B. acknowledges support by the German Research Foundation in the framework of the Collaborative Research Center 787 "Semiconductor Nanophotonics" under project B4. Sh.A. acknowledges support of the German Research Foundation under Project 389251150.

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Abstract

We prove theoretically, that the evolution of optical solitons can be dramatically influenced in the course of nonlinear interaction with much smaller group velocity matched pulses. Even weak pump pulses can be used to control the solitons, e.g., to compensate their degradation caused by Raman-scattering.

1 Introduction

Optical data transmission at high bitrates requires robust ultra-short optical pulses that propagate along fibers in a stable manner. An idealized example is given by fundamental solitons, i.e., stable solutions of the the integrable nonlinear Schrödinger equation (NLSE) for which fiber nonlinearity and group velocity dispersion precisely compensate each other [1]. However, solitons in the real-world fibers inevitably suffer from various higher-order effects, such as Cherenkov radiation, self-steepening, and Raman interpulse scattering [21]. In what follows we review a method to “recover” an optical soliton by applying a specially chosen pump pulse, e.g., to change soliton's frequency and amplitude by choice. In this scenario, the usual cross phase modulation (XPM) of pulses is utilized in the so-called *optical event horizons* regime [24, 33, 38]. The key feature is that a low-power pump pulse can be employed to realize the concept of controlling light by light [13]. This study belongs to a general framework of *front-induced transitions* [16].

2 Numerical model

To model propagation of ultra-short pulses we employ the generalized nonlinear Schrödinger equation (GNLSE) for the complex pulse envelope $\psi(z,\tau)$ [6]

$$i\partial_z\psi + \hat{\mathcal{D}}\psi + \frac{n_2}{c}(\omega_0 + i\partial_\tau)(\mathcal{I}\psi) = 0,$$

where

$$\mathcal{I} = f_K|\psi(z,\tau)|^2 + f_R\int_0^\infty R(\tau')|\psi(z,\tau-\tau')|^2 d\tau',$$

where z is measured along the fiber, $\tau = t - z/v_g$ denotes the delay, v_g refers to the group velocity at the carrier frequency ω_0, the Kerr parameter n_2 quantifies nonlinearity, $R(\tau)$ is a causal unit-area Raman response function, f_K and f_R relate contributions of the Kerr and Raman effects ($f_K + f_R = 1$).

The dispersion operator $\hat{\mathcal{D}}$ is yielded by the polynomial

$$\mathcal{D}(s) = \sum_{j=2}^{J}\beta_j\frac{s^j}{j!} \qquad \text{with} \qquad \hat{\mathcal{D}} = \mathcal{D}(i\partial_\tau),$$

where β_j are dispersion coefficients. In some cases the use of a rational function $\mathcal{D}(s)$, which yields a non-local operator $\hat{\mathcal{D}}$, is preferable [2] and one should separate positive- and negative-frequency components of the nonlinear term [3, 8].

DOI 10.20347/WIAS.PREPRINT.2661 Berlin 2019

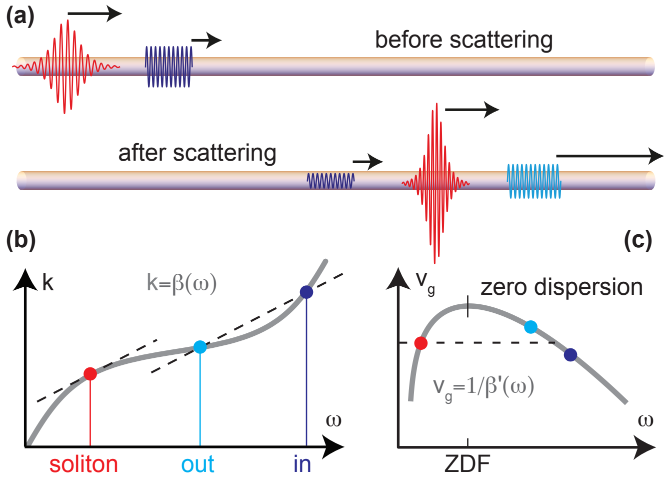

Figure 1: (a) Schematic representation of a pump pulse (dark blue) that is scattered at a fiber soliton (red). A new frequency down-shifted pulse (light blue) appears after reflection. (b) A schematic dispersion law and geometric representation of how ω_{out} depends on soliton frequency and ω_{in}. (c) Group velocity dispersion profile is schematically shown. Only group-velocity matched pulses, which co-propagate with slightly different velocities, are effectively scattered.

A rich set of the analytic GNLSE solutions is available for $f_R = 0$, see [1, 4, 5]. To analyze XPM interactions of pulses we will employ both the numerical GNLSE solutions and a semi-analytical theory of solitons with slowly varying parameters [18, 27].

3 Mechanism

An optical pulse that propagates in a Kerr medium, creates a moving localized nonlinear perturbation δn of the refractive index [21]. Among other things, the perturbation can manifest itself by scattering other optical pulses [39, 34] as schematically shown in Fig. 1a, see [38, 14, 36, 37] for recent experimental realizations. The scattered wave attains a frequency shift according to the rule [35, 31]

$$\omega_{\text{in}} \mapsto \omega_{\text{out}},$$

such that

$$\beta'(\omega_s) = \frac{\beta(\omega_{\text{out}}) - \beta(\omega_{\text{in}})}{\omega_{\text{out}} - \omega_{\text{in}}},$$

where ω_s refers to the initial soliton frequency and $k = \beta(\omega)$ is the dispersion law, see Fig. 1b. A special case $\omega_{\text{in}} = \omega_{\text{out}}$ corresponds to the group velocity matched pulses. We will be mostly dealing with slightly different incoming and outgoing frequencies.

A frequency down-shift of the pump pulse in Fig. 1 indicates an energy transfer to the soliton, which experiences an induced frequency up-shift and reshaping. This is the manipulation mechanism we are interested for.

4 Remarks

Figure 1a shows how the soliton overtakes the pump pulse, an opposite scenario is also possible. The transmitted pulse and the reflected one move in the same direction in the lab frame, which is *forward reflection* [32]. Note, that the XPM interactions of optical pulses are often studied employing two coupled NLSEs, one for each

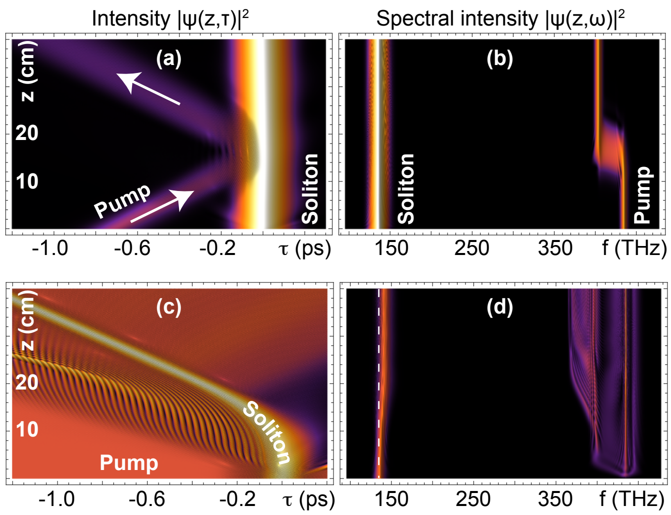

Figure 2: (a) A nearly perfect scattering of a small-amplitude pump pulse ($0.67\,\mu$m) at an approximately group-velocity matched fundamental soliton ($2.2\,\mu$m, FWHM 70 fs). (b) The soliton and the frequency down-shifted pump pulse are shown in the spectral domain. (c) The pump pulse is replaced by a continuous wave. The soliton is now compressed and changes its velocity. (d) A small up-shift of the soliton carrier frequency is observed (the initial carrier is labelled by the dashed line) together with a pronounced frequency down-shift of the pump wave in the spectral representation.

pulse, see [30] for the study on forward reflection. However, the simplified approach does not describe the energy exchange between the pulses, this is why we keep the shock term $i\omega_0^{-1}\partial_\tau(\mathcal{I}\psi)$ in the GNLSE. See [25] for example calculations with and without the shock term, see [20] for influence of the self-frequency shift.

Another important remark is that the nonlinear perturbation of the refractive index is small, e.g., $\delta n \approx 10^{-4}$ for a 3-cycle (half-maximum) soliton in fused silica at $1.55\,\mu$m. Only a small portion of the pump pulse is then reflected, the pulses tend to go through each other without noticeable changes. An exception is given by the (approximately) group velocity matched pulses, which always exist on the opposite sides of the zero dispersion frequency (ZDF), Fig. 1c. Here a nearly perfect reflection becomes possible [7], see Fig. 2(a,b).

Last but not least, a perfect reflection for $\beta'(\omega_{\text{in}}) \approx \beta'(\omega_s)$ does not indicate the optimal energy transfer to the soliton because in this regime $\omega_{\text{in}} - \omega_{\text{out}}$ vanishes (Fig. 1b). A small increase of ω_{in} improves the energy transfer, yet decreases the reflection coefficient. The optimal ω_{in}, for which the velocity of the pump pulse is not too far and not to close to that of the soliton, was quantified in [26].

5 Soliton manipulation

The pump pulse, which is reflected by the soliton in an example calculation in Fig. 2(a,b), is too small to cause a noticeable soliton change. Such reflections follow the standard rules of the geometric optics [29]. To modify the soliton one can increase either the amplitude or duration of the pump pulse. We go for the second approach.

Figure 2(c,d) shows how the soliton is affected by a small-amplitude continuous pump wave. We see an up-shift of the soliton frequency (cf. the white dashed line in Fig. 2d). The frequency up-shift changes the delay, i.e., trajectory of the soliton in Fig. 2c becomes curved. Note, that even a small up-shift of the soliton frequency may lead to a considerable decrease of its duration for a sufficiently steep profile of the group velocity dispersion. Soliton compression affects the reflection coefficient and at $z \approx 20$ cm in Fig. 2c the soliton suddenly becomes transparent for the pump. Soliton interaction with the pump wave is thereafter reduced, the soliton trajectory is now a straight line.

The most exotic application of the pulse interactions depicted in Fig. 1,2 is an ongoing search for the analogue

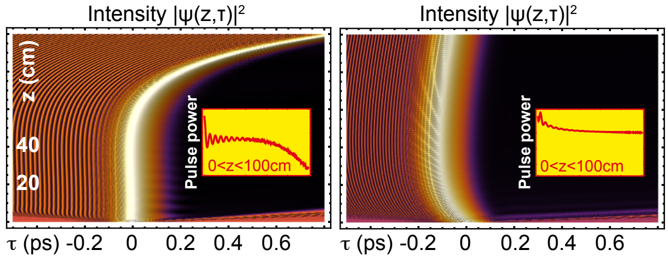

Figure 3: Two attempts to remove the SSFS effect for a 70 fs (half-maximum) fundamental soliton at $2.8\,\mu$m in fused silica. Left: the compensation is unstable and SSFS wins the competition with the pump wave at $z \approx 60$ cm. Right: after a proper shift of the pump frequency the compensation becomes stable, see [28].

Hawking radiation, this is where the "optical event horizons" come from [24, 15]. From the practical side, a perfect reflection of an optical pump pulse at the velocity matched soliton generates a new pulse with a new and predictable frequency [34, 14]. On the other hand, one can employ a proper pump wave to trap the soliton [17] or to "switch" it on and off in a kind of all-optical transistor [13]. Moreover, a nearly tenfold increase in the soliton's peak power is possible, which provides us with a new possibility to generate the so-called *champion solitons*, see [11].

The XPM interactions we are interested in naturally appear in all generic situations, where solitons coexist with dispersive waves and a ZDF is present, such interactions contribute then to the rogue events [9, 10]. In some cases the frequency lines, which initially correspond to the transmitted/reflected pump waves and to the soliton, experience considerable broadening. They can even merge with each other and create a coherent supercontinuum state [12]. Scattering of the pump waves at higher-order and dark solitons is also possible, see [22, 23].

6 Raman effect

The pump-induced up-shift of the soliton carrier frequency (Fig. 2d) competes with the soliton self-frequency shift (SSFS), which is a natural down-shift of the soliton frequency due to the Raman scattering [21]. To keep it simple, the SSFS was first ignored (f_R was set to zero) in the example calculations in Fig. 2. Actually, scattering of a short pump pulse at a soliton, like the one in Fig. 2a, is not affected much by the Raman term. It must be kept in mind that the velocity matching condition is required only when the XPM interaction takes place, which makes the choice of the initial soliton frequency more involved. Yet the scattering is the same.

Scattering of the continuous pump, like the one in Fig. 2c, is more sophisticated in the presence of the Raman term. The slow down-shift of the soliton frequency should violate the velocity matching condition at some point. Let us consider the following problem. Can one precisely compensate the SSFS by applying a proper pump wave?

Figure 3 shows two example calculations with the full GNLSE. Two pump waves with slightly different frequencies are scattered by the same fundamental soliton. The pump amplitude in both cases was chosen by trial and error to remove the SSFS. As we see, the compensation may be either unstable (Figure 3 left) or stable (Figure 3 right) depending on the pump frequency. To explain this behavior and to provide a stable compensation we need a more deep understanding of the scattering at solitons.

7 Adiabatic theory

Here we outline a qualitative theory of the group velocity matched soliton-pump interactions, as given in [27, 28]. Let ω_a be the initial soliton frequency and ω_b be the group velocity matched frequency, such that we have the

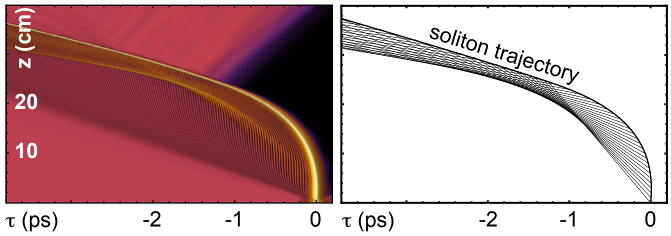

Figure 4: An example calculation how a pump wave ($0.525\,\mu$m) is scattered at a fundamental soliton (70 fs at half-maximum, $2.8\,\mu$m) in fused silica. Left: full GNLSE solution in the space-time domain. Right: soliton trajectory from the adiabatic ODEs (black curved line) and geometric rays for the reflected portions of the pump wave (thin straight lines). Adiabatic approach reveals formation of the caustic. For the further details see [27].

same delay variable

$$\tau = t - \beta'(\omega_a)z = t - \beta'(\omega_b)z,$$

for both the soliton and the pump. Soliton frequency is then denoted by $\omega_a + \nu$, where ν is the self-organized offset induced by the interaction with the pump. The pump frequency is denoted by $\omega_b + \Omega$, where the offset parameter Ω can be freely chosen, e.g., to optimize the energy transfer to the soliton. While the standard GNLSE is perfectly suited for the numerical solutions presented above, the theory is based on two XPM-coupled GNLSEs centered at ω_a and ω_b respectively. We adopt the following trial function for the soliton

$$\psi_a^{\text{trial}} = \sqrt{\frac{|\beta''(\omega_a+\nu)|c}{(\omega_a+\nu)n_{2a}}}\;\frac{e^{i\Theta}}{\sigma\cosh(\frac{\tau-\mathcal{T}}{\sigma})},$$

where the offset $\nu(z)$, the duration $\sigma(z)$, the delay $\mathcal{T}(z)$, and the phase $\Theta(z)$ are yet unknown functions. In the simplest situation when the GNLSE for the soliton is reduced to the NLSE, we have [1]

$$\frac{d\nu}{dz} = 0, \qquad \frac{d\sigma}{dz} = 0, \qquad \frac{d\mathcal{T}}{dz} = \beta'(\omega_a+\nu) - \beta'(\omega_a),$$

where the missing ODE for $\Theta(z)$ can be ignored because the XPM interactions depend on terms like $|\psi_a|^2\psi_b$. The above set of ODEs is now modified because of two reasons. The first reason is the difference in the NLSE and the GNLSE, i.e., presence of the higher-order dispersion, Raman effect, and self-steepening. The corresponding ODEs for the soliton parameters are well known [18]. The second reason is the pump wave. Here we first employ the GNLSE for the pump and quantify the reflected and transmitted waves in a full analogy with the scattering theory for the $\cosh^{-2}$ potential in Quantum Mechanics [19]. Thereafter the solution for the pump wave is used to quantify the pump effect on the soliton in the spirit of the soliton perturbation theory [18].

The resulting set of three adiabatic equations for the soliton parameters (not shown, see [27, 28]) looks very complicated and contains several hypergeometric integrals, yet it is just a set of ODEs. Therefore the numerical solution of the adiabatic equations is much faster than that of the original GNLSE, allowing, e.g., a quick search of the optimal pump parameters. What is more important, the resulting solutions are easy to interpret. For instance, the dispersive wave that compensates the SSFS corresponds to the stationary point of the adiabatic dynamical system. Stability of the stationary point is investigated by standard means and yields the amplitude of the pump wave and the interval of the offsets Ω for the stable compensation.

8 Conclusion

In conclusion, we presented a review of the recent results dealing with the XPM scattering of the pump waves at the velocity matched solitons. This scattering process has many intriguing applications ranging from the exotic analogue of the event horizons to the real-live soliton control, e.g., robust cancellation of the soliton self-frequency shift. The interaction between the soliton and the pump is quantified by an adiabatic approach, which provides a deep understanding of the underlying processes. For instance, the adiabatic equations reveal existence of the caustic structures in the scattered optical field, as shown in Fig. 4.

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