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CONTROLLABILITY AND STABILITY CAPACITY OF A ROLL PAIR MOTION RESEARCH

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Abstract

The process of deriving the equation of the perturbed motion of a roll pair during the processing of leather material is discussed in the article, taking into account the influence of dynamic factors. It is shown that one of the reasons for the unstable stress state on the contact surfaces in the roller mechanism are dynamic factors arising from inaccuracies in the manufacture of its individual parts, assembly defects, and the occurrence of an oscillatory process in the roller mechanism, as well as due to the non-uniform thickness of the processed material at its gripping during starting and stopping the machine. Methods for determining optimal controls are shown; they provide asymptotic stability of the unperturbed motion of a roll pair and the torque applied to the upper roll as a function of generalized coordinates.

It is shown that the width of the contact strip of the clamp, which depends on the radii of the rolls and the hardness of the coatings, has a significant effect on the efficiency of the rolls. The larger the shaft radius, the lower the actual pressure per unit contact area.

It is shown that the squeezing efficiency increases with the improvement of the conditions for the removal of the squeezable liquid from the rolls (with their horizontal arrangement), with an increase in its temperature and a decrease in viscosity. Efficiency decreases with increasing material speed and thickness.

It is shown that the width of the contact strip of the clamp, which depends on the radii of the shafts and the stiffness of the coatings, has a significant impact on the efficiency of the shafts. The larger the shaft radius, the lower the actual pressure per unit contact area.

Keywords: processing, spreading, pressure, shafts, clamping, skin, stability, humidity, deformation, technology.

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1. Introduction

Roller mechanisms are widely used due to the simplicity of design, the possibility of a continuous technological operation when processing the material and the combination of several functions. The variety of operations performed by roller mechanisms has not made it possible to create a unified system for their design, due to the difference in technological tasks and phenomena occurring in the contact zone of the rollers.

Most roller mechanisms perform two or more functions at the same time: active and auxiliary ones. The latter most often refers to the feeding the leather material; the squeezing mechanism can be wringing, pressing and deforming one.

Now consider the dynamic factors affecting the operation of the roll pair. One of the reasons for the unstable stress state on the contact surfaces in the roller mechanism is a dynamic factor arising from inaccuracies in the manufacture of its individual parts, assembly defects, and the occurrence of an oscillatory process in the roller mechanism, as well as due to the non-uniform thickness of the processed material, during its capture, machine start and stop operations.

The reason for the occurrence of an oscillatory process in the roller mechanism may be inaccuracies in the manufacture of machine parts. Let, for example, the geometric axis of the roller do not coincide with its axis of rotation. Then there is a kinematic excitation of harmonic nature, which leads to vibrations and resonance in the system. Vibrations in the roller mechanism are transmitted to the machine frame and require a definition of the inertial and elastic characteristics of its elements [1, 2].

With the effective use of a biocatalytic modifier, the stiffness index of the semi-finished leather product is reduced by two times compared to the semi-finished leather product developed according to the existing technology, while maintaining its mechanical strength [3]. It should be noted that this paper does not consider the removal of excess fluid from the semi-finished leather product.

The articles [4–6] present the results of studies on the control of technological parameters of the operation to extract excess liquid from multilayer moisture-saturated fibrous materials using leather as an example. Mathematical dependences of the amount of moisture removed for each layer of wet leather on the feed rate between the rotating working rollers and their pressing force were obtained.

However, a critical analysis of the operation of the experimental roller stand used in these works shows that when processing a multilayer package of wet leathers, the control of their feed rate between the rotating working shafts is of great importance. Consequently, the greater the thickness of the leather package, the correspondingly lower the feed rate should be, but the authors of these works did not take this into account when conducting experiments. From this it should be noted that by controlling the feed rate of processed wet skins between the working shafts, it would be possible to achieve a significant increase in the quality of their processing.

There are publications devoted to the development and improvement of leather machines. In [7], the results of studies on determining the ratio of forces in the process of feeding a semi-finished leather product to the working area of a multi-operational machine by a conveying device are presented, and in [8] a solution to the problems of ensuring the stable motion of the feeding mechanism of multi-operation machines is proposed. These studies were carried out for manual work, where the issues of process control and the stability of the movement of the roll pair, depending on the uneven thickness of the processed material, were not considered.

The study in [9] is devoted to the solution of contact interaction in the roll pair. Mathematical models of the contact stresses distribution patterns are developed. Research is underway aimed at solving the problems of using tooth-lever differential gears in technological roller machines [10, 11]. However, these works do not take into account the dynamic factors of the process of contact interaction in roller pairs with a leather semi-finished product.

2. Materials and methods

The aim of the study is to determine the optimal controls that ensure the asymptotic stability of the unperturbed motion of the roll pair and the torque applied to the upper roll as a function of generalized coordinates.

Roll pairs are widely used in many industries. When studying the dynamics of a roll pair, it is necessary to proceed from the forces acting on the rolls during operation. The magnitude and direction of the acting forces during the capture of the processed material and in the steady state are different. They also depend on many parameters and factors, i.e. from the diameters of the rolls, which may be equal or different, from the kinematic connection between the rolls, which may be rigid or one of the rolls is free, which will rotate due to fiction; from the installation of rolls that can be installed horizontally or obliquely, one above the other with the location of their axes of rotation on a vertical or inclined plane, also vertically; at the same time, to create a clamp between the rolls, the upper, lower or both rolls can be movable; swaths can be with hard or elastic coatings, which can be moisture-permeable or impervious, depending on the technology, combinations of them can be selected. In addition, one of the rolls or both rolls may be composite. In all cases, the action of force, movement must be continuous and stable. Therefore, it is important to ensure the stability of the process of pressing leather materials.

To determine the value of the basic parameters for controlling the motion of a roll pair, let's first compose a differential equation of motion in the Lagrange form with holonomic servo constraints, obtained in [12]:

$$\frac{d}{dt}\cdot\frac{\partial\tilde{T}}{\partial\dot{q}_i}-\frac{\partial\tilde{T}}{\partial q_i}=\tilde{Q}_i+\sum_j \mu_j a_{j\tau}. \tag{1}$$

Here

$$\tilde{T} = \tilde{T}(q_j, \dot{q}_j t),$$

is the kinetic energy of the system;

$$\tilde{Q}_j = \sum_i Q_i \frac{\partial \tilde{q}_i}{\partial q_j},$$

are the generalized forces referred to coordinates q_j;

$$a_{j\tau} = \sum_i A_{i\tau} \frac{\partial \tilde{q}_i}{\partial q_j},$$

are the known functions of time and q_j.

In the center of the fixed roll, let's install a fixed system of coordinates $Oxyz$, and the x axis is directed along the roll axis, and the y axis in the direction of leather motion and the z axis is directed vertically upwards. The origin of the moving coordinate system $O_1x_1y_1z_1$ is set to the center of mass of the moving roller. Let's direct the x_1 and y_1 axes parallel to the x and y axes, respectively, and let's direct the z axis along the z axis.

To determine the positions of the fixed roll, semi-finished leather product and the upper movable roll, five independent parameters must be set. Let's take the following values as generalized coordinates: $\tilde{q}_1 = \varphi_1$ – angle of rotation of the fixed roll; $\tilde{q}_2 = \varphi_2$ – angle of rotation of the movable roll around the x_1 axis; $\tilde{q}_3 = y$ – ordinates of leather; $\tilde{q}_4 = z_c$ – applicate of the center of mass of the movable roll; $\tilde{q}_5 = \psi$ – deviation of the movable roll from the x axis.

At the end of the movable roll, let's install special devices that at each time point provide for the following conditions:

$$\vartheta = \vartheta_M = \vartheta_N, \tag{2}$$

where ϑ_M and ϑ_N are the linear velocities of the point lying on the rim of the movable and fixed rolls (**Fig. 1**).

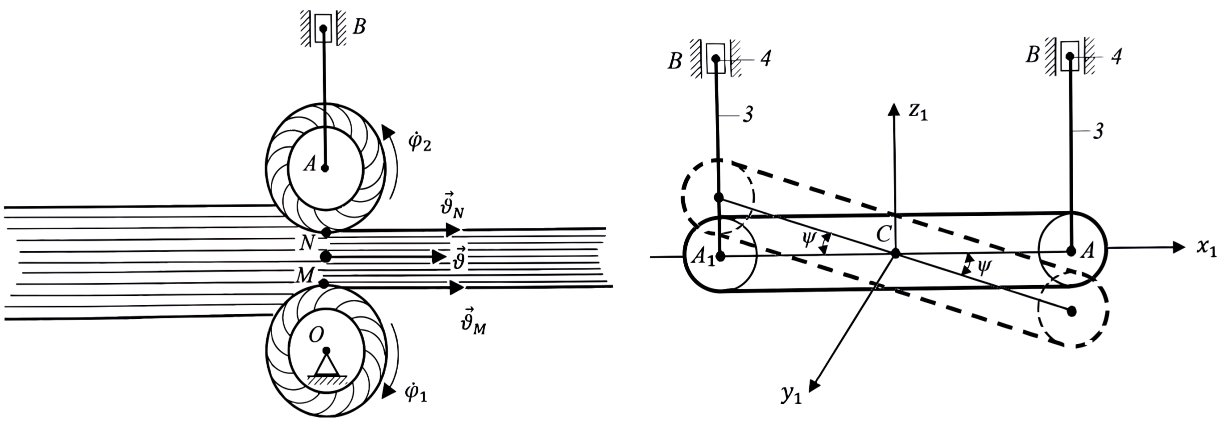

Fig. 1. Side view of a roll: *a* – side view of a roll pair; *b* – deviation of the upper roll around the center of mass

In order for condition (2) to be fulfilled, the velocity of a point lying on a movable roll relative to the x_1 axis must be zero. In addition, to fulfill condition (2) at each time point (if the system moves from a state of rest), the following conditions should be fulfilled:

$$\dot{\varphi}_1 = \frac{\vartheta}{R_1},$$

or

$$\dot{\varphi}_2 = \frac{\vartheta}{R_2},$$

$$\dot{\varphi}_1 = \frac{1}{R_1} y, \ \dot{\varphi}_2 = \frac{1}{R_2} y. \quad (3)$$

Since angular velocities of the movable and fixed rolls, and the leather feed rate are small, the fulfillment of condition (2) could be done using special equipment.

Let's compose the equations of motion (1) taking into account relation (3) obtained in [13]:

$$2a_1 \ddot{y} = a_5 + a_6 \mu_1, \quad (4)$$

$$2a_2 \ddot{z}_c + a_3 \ddot{\psi} \cdot \cos\psi - a_3 \sin\psi \cdot \dot{\psi}^2 = a_7 - 2cz_c + \mu_2, \quad (5)$$

$$a_3 \cos\psi \cdot \ddot{z}_c + 2a_4 \cos^2\psi \cdot \ddot{\psi} - 2a_3 \sin\psi \cdot \dot{z}_c \dot{\psi} - 3a_4 \sin 2\psi \cdot \dot{\psi}^2 = a_8 \cos\psi - a_9 \sin 2\psi + \mu_3, \quad (6)$$

where

$$a_1 = \frac{1}{2}\left(J_1 \frac{1}{R_1^2} + J_2 \frac{1}{R_2^2} + M_3 \right), \ a_2 = \frac{1}{2}(M_2 + M_4 + M_5), \ a_3 = \frac{l}{2}(M_4 - M_5),$$

$$a_4 = \frac{l^2}{8}(M_4 + M_5), \ a_5 = (M_1^a - M_1^*)\frac{1}{R_1} + (M_2^a - M_2^*)\frac{1}{R_2},$$

$$a_6 = \frac{R_1^2 + R_2^2 + R_1^2 R_2^2}{R_1^2 R_2^2}, \ a_7 = g(M_5 - M_2 - M_4), \ a_8 = -(m_4 + m_5) \cdot g\frac{l}{2}, \ a_9 = \frac{cl^2}{4}.$$

The resulting equations (4)–(6), together with the constraint equations (2), completely describe the motion of the squeezing machine shown in **Fig. 1**.

3. Results and discussion

Next, let's consider the problems of controllability of technological processes during the pressing of a semi-finished leather product.

When implementing technological processes, the parameters that characterize these processes must change according to certain laws (or be constant).

The purpose of the control is to ensure the necessary velocity between the various nodes of the system or to achieve the maximum power acting to a certain part of the squeezing machine. The processes that must change according to certain laws are velocity and power under the influence of external sources, realized by signals.

Now let's compose the equation of the perturbed motion of a roll pair. For the unperturbed motion, let's take the motion defined by equations (4)–(6) and assume that these equations have a particular solution [14, 15]:

$$y = y_0, \ \psi = 0, \ z_c = \frac{a_7}{2c}, \ \mu_1 = -\frac{a_5}{a_6}, \ \mu_2 = 0, \ \mu_3 = -a_8.$$

Under perturbed motion there are:

$$y = y_0 + x_1, \ z_c = \frac{a_7}{2c} + x_2, \ \psi = x_3, \ \mu_1 = -\frac{a_5}{a_6} + u_1, \ \mu_2 = u_2, \ \mu_3 = -a_8 + u_3,$$

where x_1, x_2 and x_3 are the deviations of the perturbed motion from the unperturbed motion.

Then the equations of perturbed motion have the following form:

$$2a_1\ddot{x}_1 = a_6 u_1, \tag{7}$$

$$2a_2\ddot{x}_2 + a_3\ddot{x}_3 \cdot \cos(x_3) - a_3 \sin(x_3) \cdot \dot{x}_3^2 = -2cx_2 + u_2, \tag{8}$$

$$\begin{aligned} &a_3 \cos x_3 \ddot{x}_2 + 2a_4 \cos^2(x_3) \cdot \ddot{x}_3 - 2a_3 \sin x_3 \dot{x}_2 \dot{x}_3 - 3a_4 \sin x_3 \dot{x}_3^2 = \\ &= a_8(\cos x_3 - 1) - a_9 \sin 2x_3 + u_3. \end{aligned} \tag{9}$$

It is easy to see that the right-hand sides of these equations vanish under conditions. $x_1 = x_2 = x_3 = u_1 = u_2 = u_3 = 0$. Expanding the right-hand sides into series in powers of x_1, x_2, x_3 and restricting ourselves to terms of the first order of smallness, let's obtain the equations of the first approximation:

$$2a_1\ddot{x}_1 = a_6 u_1,$$

$$2a_2\ddot{x}_2 + a_3\ddot{x}_3 + 2cx_2 = u_2 + R_2,$$

$$a_3\ddot{x}_2 + 2a_4\ddot{x}_3 + 2a_9 x_3 = u_3 + R_3, \tag{10}$$

where symbols R_2 and R_3 denote the terms the measurement of which in $x, \dot{x}_2, x_3, \dot{x}_3$ is higher than the first making the necessary calculations, let's write these equations in normal form. To do this, introducing notation, $x_{2i-1} = y_i$, $x_{2i} = \dot{y}_i$ $(i = 1, 2, 3)$ and doing the necessary calculations, let's obtain:

$$\dot{y}_1 = y,\ \dot{y}_2 = \frac{a_6}{2a}u_1,\ \dot{y}_3 = y_4,$$

$$\dot{y}_4 = -\frac{c(a_3^2 + a_4)}{a_2 \cdot a_{10}} y_3 + \frac{2a_3 a_9}{a_{10}} y_5 + \frac{(a_3^2 + a_{10})}{2a_2 \cdot a_{10}} u_2 - \frac{a_3}{a_{10}} u_3 + R_3,\ \dot{y}_5 = y_6,$$

$$\dot{y}_6 = \frac{2a_3 c}{a_{10}} y_3 - \frac{4a_2 a_9}{a_{10}} y_5 - \frac{2a_3}{a_{10}} u_2 + \frac{2a_2}{a_{10}} u_3 + R_4, \tag{11}$$

where

$$a_{10} = 4a_2\ a_4 - a_3^2 = \frac{l^2}{4}(M_2 M_4 + M_2 M_5 + 4M_4 M_5).$$

The system of equations (11) in a matrix formulation takes the following form or in a short form:

$$\dot{y} = Ay + Bu + W, \tag{12}$$

where W denotes that the terms the dimension of which in y_1, y_2,..., y_6 and $\dot{y}_1, \dot{y}_2, ..., \dot{y}_6$ is higher than the first.

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \\ \dot{y}_5 \\ \dot{y}_6 \end{bmatrix} = \begin{Vmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{c(a_3^2 + a_{10})}{a_2 \cdot a_{10}} & 0 & \frac{2a_3 a_9}{a_{10}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{2a_3 c}{a_{10}} & 0 & -\frac{4a_2 a_9}{a_{10}} & 0 \end{Vmatrix} \begin{matrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{matrix} + \begin{Vmatrix} 0 & 0 & 0 \\ \frac{a_6}{2a_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{a_3^2 + a_{10}}{2a_2 \cdot a_{10}} & -\frac{a_3}{a_{10}} \\ 0 & 0 & 0 \\ 0 & -\frac{a_3}{a_{10}} & \frac{2a_2}{a_{10}} \end{Vmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + W,$$

Now consider the matrix [14–18]:

$$K=\left\{B,AB,A^2B,A^3B,A^4B,A^5B\right\}. \quad (13)$$

Based on the system controllability criterion, i.e. in order for the system described by (12) to be completely controllable on segment $[t_1,\ t_2]$, it is necessary and sufficient that the rank of matrix K be equal to 6.

Let's now compose matrix K:

$$K=\begin{Vmatrix} 0 & 0 & 0 & \frac{1}{a_1k^2} & 0 & 0 & -\frac{a_3}{a_1^2k^2} & 0 & 0 & \frac{a_3^2}{a_1^3k^2} & 0 & 0 & \frac{a_3^2}{a_1^3k^2} & 0 & 0 & \frac{a_3^4}{a_1^5k^2} & -\frac{c_4}{m_4^2} & 0 \\ \frac{1}{a_1k^2} & 0 & 0 & -\frac{a_3}{a_1^2k^2} & 0 & 0 & \frac{a_3^2}{a_1^3k^2} & 0 & 0 & -\frac{a_3^2}{a_1^4k^2} & 0 & 0 & -\frac{a_3^4}{a_1^5k^2} & -\frac{c_4}{M_4^2} & 0 & -\frac{a_3^5}{a_1^6k^2} & -\frac{c_4}{M_4^2\cdot a_1} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{M_4} & 0 & 0 & 0 & 0 & 0 & -\frac{c_4}{M_4^2} & 0 & 0 & 0 & 0 & 0 & -\frac{c_4^2}{M_4^3} & 0 \\ 0 & \frac{1}{M_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{c_4^2}{M_4^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{M_5} & 0 & 0 & 0 & 0 & 0 & -\frac{c_5}{M_6M_5} & 0 & 0 & 0 & 0 & 0 & \frac{c_5^2}{M_5M_6^2} \\ 0 & 0 & \frac{1}{M_5} & 0 & 0 & 0 & 0 & 0 & -\frac{c_5}{M_5M_6} & 0 & 0 & 0 & 0 & 0 & \frac{c_5^2}{M_6^2M_5} & 0 & 0 & 0 \end{Vmatrix}.$$

Let's take the first 6 columns of matrix K:

$$(B,AB)=\begin{Vmatrix} 0 & 0 & 0 & 1 & \frac{a_6}{2a_1} & 0 \\ \frac{a_6}{2a_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{a_3^2+a_{10}}{2a_2\cdot a_{10}} & -\frac{a_3}{a_{10}} \\ 0 & \frac{a_3^2+a_{10}}{2a_2\cdot a_{10}} & -\frac{a_3}{a_{10}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2a_3}{a_{10}} & \frac{2a_2}{a_{10}} \\ 0 & -\frac{a_3}{a_{10}} & \frac{2a_2}{a_{10}} & 0 & 0 & 0 \end{Vmatrix}.$$

Given that, the determinant does not change if any linear combination of other rows is added to one of its rows, then:

$$(B,AB)=4a_2a_4(2a_4-a_3)-a_3^2(2a_4-a_3)=(4a_2a_4-a_3^2)\cdot(2a_4-a_3),$$

where

$$2a_4-a_3=\frac{l^2}{4}(M_4+M_5)-\frac{l^2}{2}(M_4-M_5)\neq 0,$$

$$4a_2a_4-a_3^2=4\frac{1}{2}(M_2+M_4+M_5)\cdot\frac{l^2}{8}(M_4+M_5)-\frac{l^2}{4}(M_4^2-2M_4M_5+M_5^2)=$$
$$=\frac{l^2}{4}(M_2+M_4+M_5)\cdot(M_4+M_5)-\frac{l^2}{4}(M_4^2-2M_4M_5+M_5^2)=\frac{l^2}{4}M_2(M_4+M_5)\neq 0.$$

Thus, the rank of matrix K is 6, and, consequently, the system (7)–(9) is completely controllable.

To solve the stabilization problem, let's use the following theorem [19]: Controlled system (12) can always be stabilized with respect to the manifold defined by constraints (2).

Let's choose values of u_j for the nonlinear system (12) as control actions:

$$u_j^0 = \gamma_{lj} \cdot y_j,$$

or in expanded form:

$$u_1^0 = \gamma_{11}y_1 + \gamma_{21}y_2 + \gamma_{31}y_3 + \gamma_{41}y_4 + \gamma_{51}y_5 + \gamma_{61}y_6,$$

$$u_2^0 = \gamma_{12}y_1 + \gamma_{22}y_2 + \gamma_{32}y_3 + \gamma_{42}y_4 + \gamma_{52}y_5 + \gamma_{62}y_6,$$

$$u_3^0 = \gamma_{13}y_1 + \gamma_{23}y_2 + \gamma_{33}y_3 + \gamma_{43}y_4 + \gamma_{53}y_5 + \gamma_{63}y_6, \tag{14}$$

where γ_{ij} are the constants, ($i = 1, 2,..., 6$); ($j = 1, 2, 3$).

Substituting (14) into equations (12), let's obtain the nonlinear equations of perturbed motion:

$$\dot{y}_1 = y_2,\ \dot{y}_2 = c_{21}y_1 + c_{22}y_2 + c_{23}y_3 + c_{24}y_4 + c_{25}y_5 + c_{26}y_6,$$

$$\dot{y}_3 = y_4,\ \dot{y}_4 = c_{41}y_1 + c_{42}y_2 + c_{44}y_3 + c_{46}y_6 + R_2^1,$$

$$\dot{y}_5 = y_6,\ \dot{y}_6 = c_{61}y_1 + c_{62}y_2 + c_{63}y_3 + c_{64}y_4 + c_{65}y_5 + c_{66}y_6 + R_3^1, \tag{15}$$

where

$$c_{21} = \frac{a_6}{2a_1}\gamma_{11},\ c_{22} = \frac{a_6}{2a_1}\gamma_{21},\ c_{23} = \frac{a_6}{2a_1}\gamma_{31},\ c_{24} = \frac{a_6}{2a_1}\gamma_{41},\ c_{25} = \frac{a_6}{2a_1}\gamma_{51},$$

$$c_{26} = \frac{a_6}{2a_1}\gamma_{61},\ c_{41} = \frac{(a_3^2 + a_{10})\cdot\gamma_{12} - 2a_2a_3\cdot\gamma_{13}}{2a_2\cdot a_{10}},\ c_{42} = \frac{(a_3^2 + a_{10})\cdot\gamma_{22} - 2a_2a_3\cdot\gamma_{23}}{2a_2\cdot a_{10}},$$

$$c_{43} = \frac{(a_3^2 + a_{10})\cdot\gamma_{32} - 2c(a_3^2 + a_{10}) - 2a_2a_3\cdot\gamma_{33}}{2a_2\cdot a_{10}},\ c_{44} = \frac{(a_3^2 + a_{10})\cdot\gamma_{42} - 2a_2a_3\cdot\gamma_{43}}{2a_2\cdot a_{10}},$$

$$c_{45} = \frac{4a_2a_3a_9 + (a_3^2 + a_{10})\cdot\gamma_{52} - 2a_2a_3\cdot\gamma_{53}}{2a_2\cdot a_{10}},\ c_{46} = \frac{(a_3^2 + a_{10})\cdot\gamma_{62} - 2a_2a_3\cdot\gamma_{63}}{2a_2\cdot a_{10}},$$

$$c_{61} = \frac{2a_2\cdot\gamma_{13} - a_3\cdot\gamma_{12}}{a_{10}},\ c_{62} = \frac{2a_2\cdot\gamma_{23} - a_3\cdot\gamma_{22}}{a_{10}},\ c_{63} = \frac{2a_3c - a_3\cdot\gamma_{32} + 2a_2\cdot\gamma_{33}}{a_{10}},$$

$$c_{64} = \frac{2a_2\cdot\gamma_{43} - a_3\cdot\gamma_{42}}{a_{10}},\ c_{65} = \frac{2a_2\cdot\gamma_{53} - a_3\cdot\gamma_{52} - 4a_9a_2}{a_{10}},\ c_{66} = \frac{2a_2\cdot\gamma_{63} - a_3\cdot\gamma_{62}}{a_{10}}.$$

Without violating the constancy of coefficients γ_{ij}, let's assume that:

$$\gamma_{13} = \frac{a_3}{2a_2}\gamma_{12},\ \gamma_{23} = \frac{a_3}{2a_2}\gamma_{22},\ \gamma_{32} = 2c,\ \gamma_{43} = \frac{a_3}{2a_2}\gamma_{42},$$

$$\gamma_{52} = 0,\ \gamma_{53} = 2a_9,\ \gamma_{63} = \frac{a_3}{2a_2}\gamma_{62}.$$

Then equation (15) takes the following form:

$$\dot{y}_1 = y_2,\ \dot{y}_2 = c_{21}y_1 + c_{22}y_2 + c_{23}y_3 + c_{24}y_4 + c_{25}y_5 + c_{26}y_6,$$

$$\dot{y}_3 = y_4,\ \dot{y}_4 = c_{41}y_1 + c_{42}y_2 + c_{44}y_3 + c_{46}y_6,$$

$$\dot{y}_5 = y_6,\ \dot{y}_6 = c_{63}y_3 + c_{65}y_5. \tag{16}$$

Let's construct a characteristic determinant for system (15):

$$\begin{vmatrix} -\lambda & 1 & 0 & 0 & 0 & 0 \\ c_{21} & c_{22}-\lambda & c_{23} & c_{24} & c_{25} & c_{26} \\ 0 & 0 & -\lambda & 1 & 0 & 0 \\ c_{41} & c_{42} & c_{43} & c_{44}-\lambda & 0 & c_{46} \\ 0 & 0 & 0 & 0 & -\lambda & 1 \\ 0 & 0 & c_{63} & 0 & c_{65} & -\lambda \end{vmatrix} = 0. \tag{17}$$

Expanding the determinant (17) and transforming, let's obtain the characteristic equation in the following form:

$$\lambda^6 + (-c_{22} - c_{44})\cdot\lambda^5 + (c_{22}c_{44} - c_{24}c_{42} - c_{21})\cdot\lambda^4 + (c_{21}c_{44} - c_{43} - c_{24}c_{41})\cdot\lambda^3 +$$
$$+ (c_{43}c_{22} - c_{63}c_{46} - c_{42}c_{23} + c_{21}c_{46}c_{63} - c_{23}c_{41})\cdot\lambda^2 + (c_{22}c_{63}c_{46} - c_{42}c_{63}c_{26} -$$
$$- c_{41}c_{63}c_{26})\cdot\lambda - c_{42}c_{63}c_{25} - c_{41}c_{25}c_{63} = 0,$$

or

$$b_0\lambda^6 + b_1\lambda^5 + b_2\lambda^4 + b_3\lambda^3 + b_4\lambda^2 + b_5\lambda + b_6 = 0, \tag{18}$$

where

$$b_0 = 1,\ b_1 = -c_{22} - c_{44},\ b_2 = c_{22}c_{44} - c_{24}c_{42} - c_{21},\ b_3 = c_{21}c_{44} - c_{43} - c_{24}c_{41},$$

$$b_4 = c_{43}c_{22} - c_{63}c_{46} - c_{42}c_{23} + c_{21}c_{46}c_{63} - c_{23}c_{41},$$

$$b_5 = c_{22}c_{63}c_{46} - c_{42}c_{63}c_{26} - c_{41}c_{63}c_{26},\ b_6 = -c_{42}c_{63}c_{25} - c_{41}c_{25}c_{63}.$$

Let's construct the following matrix, the so-called Hurwitz matrix [20] from coefficients $b_0, b_1, ..., b_6$ of equation (18):

$$\begin{Vmatrix} b_1 & b_3 & b_5 & 0 & 0 & 0 \\ b_0 & b_2 & b_4 & b_6 & 0 & 0 \\ 0 & b_1 & b_3 & b_5 & 0 & 0 \\ 0 & b_0 & b_2 & b_4 & b_6 & 0 \\ 0 & 0 & b_1 & b_3 & b_5 & 0 \\ 0 & 0 & b_0 & b_2 & b_4 & b_6 \end{Vmatrix}. \tag{19}$$

It is known that if for $b_0 > 0$ all principal diagonal Hurwitz minors $\Delta_1, \Delta_2, \ldots, \Delta_6$ are positive, then the unperturbed motion is asymptotically stable, regardless of the terms higher than the first order of smallness. Therefore, let's determine the coefficients of equation (14) in such a way that the Hurwitz conditions are satisfied [20]:

$$\Delta_1 = b_1 = \frac{2}{3},\ \Delta_2 = b_1b_2 - b_3b_0 = \frac{2}{3},\ \Delta_3 = b_3\cdot\Delta_2 - b_1^2b_0b_4 + b_0b_1b_5 = \frac{2}{9}\frac{a_2a_9}{a_{10}} > 0,$$

since $a_2 > 0$, $a_9 > 0$ and $a_{10} > 0$.

$$\Delta_4 = b_4 \cdot \Delta_3 - b_2 b_5 \cdot \Delta_2 + b_1^2 b_2 b_6 + b_0 b_1 b_4 b_5 - b_0^2 b_5^2 - b_0 b_1 b_3 b_6 = \frac{2a_2 a_9 a_{10} - a_2^2 a_9^2}{9a_{10}^2} + 0.004 > 0,$$

since $a_{10} > a_2 a_9$.

$$\Delta_5 = b_5 \Delta_4 - b_6 \Delta_3 + b_6 b_1 b_5 \Delta_2 - b_6^2 b_1^3 = \frac{2a_2^2 a_9^2 a_{10} - a_2^3 a_9^3 + 0.009 a_2 a_9 a_{10}^2}{27 a_{10}^3} - 0.000029 > 0,$$

$$\Delta_6 = \Delta_5 \cdot b_6 = \Delta_5 \cdot 0.01 > 0,$$

where the values of the coefficients of the control action (14) are determined by the following formulas:

$$\gamma_{11} = -\frac{16a_1}{7a_6},\ \gamma_{12} = 2a_3,\ \gamma_{13} = \frac{a_3^2}{a_2},\ \gamma_{21} = \frac{a_1}{a_6},\ \gamma_{22} = -\frac{205}{42},\ \gamma_{23} = \frac{205a_3}{84a_2},$$

$$\gamma_{31} = \frac{84a_1a_2a_3}{(205-84a_3)a_6},\ \gamma_{32} = 2c,\ \gamma_{33} = \frac{a_{10}}{c},\ \gamma_{41} = \frac{2a_1a_2}{a_6},\ \gamma_{42} = -\frac{7}{3}a_2,$$

$$\gamma_{43} = -\frac{7}{6}a_3,\ \gamma_{51} = \frac{0.84a_1c}{a_6(205-84a_1a_3)},\ \gamma_{52} = 0,\ \gamma_{53} = 2a_9,$$

$$\gamma_{61} = \frac{28(3a_2a_9 + 7a_{10})}{3a_{10}(205a_6 - 168a_3a_6)},\ \gamma_{62} = -\frac{14}{9}c,\ \gamma_{63} = -\frac{7a_3c}{9a_2}. \quad (20)$$

Let's substitute the values of the coefficients (20) into equation (14):

$$u_1^0 = -\frac{16a_1}{7a_6}y_1 + \frac{a_1}{a_6}y_2 + \frac{84a_1a_2a_3}{(205-84a_3)a_6}y_5 + y_3\frac{2a_1a_2}{a_6}y_4 + \frac{0.84a_1c}{a_6(205-84a_1a_3)}y_5 + $$
$$+\frac{28(a_2a_9+7a_{10})}{3a_{10}(205\cdot a_6 - 168a_3a_6)}y_6,$$

$$u_2^0 = 2a_3y_1 - \frac{205}{42}y_2 + 2cy_3 - \frac{7}{3}a_2y_4 - \frac{14}{9}cy_6,$$

$$u_3^0 = \frac{84a_1a_2a_3}{(205-84a_3)a_6}y_1 + y_3\frac{205a_3}{84a_2}y_2 + \frac{a_{10}}{c}y_3 - \frac{7}{6}a_3y_4 + 2a_9y_5 - \frac{7a_3c}{9a_2}. \quad (21)$$

If the masses of hydraulic servomotors 8 and 9 are considered equal, i.e. $m_4 = m_5$ then the optimal control actions (14) take the form:

$$u_1^0 = -\frac{16a_1}{7a_6}y_1 + \frac{a_1}{a_2}y_2 + \frac{2a_1a_2}{a_6}y_4 + \frac{0.84a_1c}{205a_6}y_5 - \frac{14}{9}cy_6,$$

$$u_2^0 = -\frac{205}{42}y_2 + 2cy_3 - \frac{7}{3}a_2y_4 - \frac{14}{9}cy_6,$$

$$u_3^0 = -\frac{a_{10}}{c}y_3 + 2a_9y_5.$$

When designing a squeezing machine, all parameters of the machine must satisfy condition (20).

Equations (21) are the sought-for laws of guidance of control parameters u_1, u_2, u_3.

Let's substitute relation (21) into equation (10) and obtain:

$$2a_1\ddot{x}_1 = a_6\left(-\frac{16a_1}{7a_6}x_1 + \frac{a_1}{a_2}\dot{x}_1 + \frac{2a_1a_2}{a_6}\dot{x}_2 + \frac{0.84a_1c}{205a_6}x_3 - \frac{14}{9}c\dot{x}_3\right),$$

$$2a_2\ddot{x}_2 + 2cx_2 = -\frac{205}{42}\dot{x}_1 + 2cx_2 - \frac{7}{3}a_2\dot{x}_2 - \frac{14}{9}c\dot{x}_3,$$

$$2a_4\ddot{x}_3 + 2a_9x_3 = -\frac{a_{10}}{c}x_2 + 2a_9x_3,$$

or

$$\ddot{x}_1 = -\frac{8}{7}x_1 + \frac{a_6}{2a_2}\dot{x}_1 + 2a_1a_2\dot{x}_2 + \frac{0.84a_1c}{205}x_3 - \frac{14}{9}a_6c\dot{x}_3,$$

$$\ddot{x}_2 = -\frac{205}{42\cdot 2a_2}\dot{x}_1 - \frac{7}{6}\dot{x}_2 - \frac{14}{9\cdot 2a_2}c\dot{x}_3,\ \ddot{x}_3 = -\frac{a_{10}}{2c\cdot a_4}x_2. \quad (22)$$

From (**Fig. 2)** let's determine:

$$\psi = \frac{z_2 - z_1}{l},\ z_c = -\frac{z_1 l_2}{l} + z_2\frac{l_1}{l} \text{ or } l\cdot\dot{\psi} = \dot{z}_2 - \dot{z}_1,\ \dot{z}_c = -\frac{1}{2}\dot{z}_1 + \frac{1}{2}\dot{z}_2.$$

From this equation, let's obtain:

$$l\cdot\dot{\psi} = 2\dot{z}_c \text{ or } \dot{x}_2 = \frac{1}{2}\dot{x}_3,$$

integrating, let's obtain:

$$x_2 = \frac{1}{2}x_3 + c_1^*,$$

where c_1^* is the integration constant.

From equation (22), let's obtain:

$$\ddot{x}_3 = -\frac{a_{10}}{2c\cdot a_4}\cdot\frac{1}{2}(x_3 + c_1^*) \text{ or } \ddot{x}_3 + k^2x_3 = -\frac{a_{10}c_1^*}{4c\cdot a_4}, \quad (23)$$

where

$$k^2 = \frac{a_{10}}{4c\cdot a_4}.$$

The general solution to equation (23) has the following form:

$$x_3 = c_2^*\cos kt + c_3^*\sin kt - \frac{a_{10}\cdot c_1}{4c\cdot a_4}\cdot\frac{4c\cdot a_4}{a_{10}}$$

or

$$x_3 = c_2^*\cos kt + c_3^*\sin kt - c_1^*, \quad (24)$$

where c_1^*, c_2^* and c_3^* are the constants of integration, determined from the initial conditions: $t = 0$;

$$x_3 = \psi_0 = 0,\ \dot{x}_3 = 0,\ x_2 = \frac{a_7}{2c},\ \dot{x}_2 = 0,\ x_1 = y_0,\ \dot{x}_1 = \dot{y}_0. \quad (25)$$

Using equations (25), let's determine:

$$c_1^* = \frac{a_7}{2c},\ c_2^* = 0,\ c_3^* = \frac{a_7}{2c}.$$

The law of changing the angle of rotation of the roll from the x axis is:

$$x_3 = \frac{a_7}{2c}\sin kt - \frac{a_7}{2c}.$$

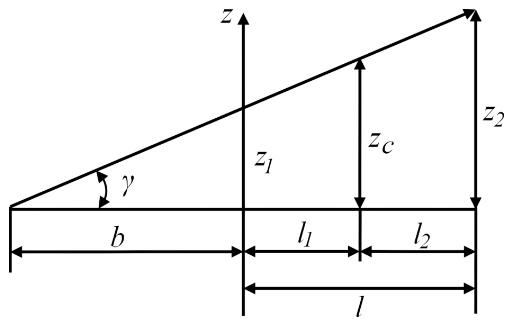

Fig. 2. Design scheme of the definition of independent coordinates of spreading roll pair

From the second equation of (22), let's obtain:

$$\ddot{x}_2 = -\frac{205}{42\cdot 2a_2}\dot{y} - \left(\frac{7}{6} + \frac{7}{9a_2}c\cdot 2\right)\cdot \dot{x}_2.$$

Considering that the squeezing process $\dot{y}$ has constant values, let's obtain:

$$\frac{d\dot{x}_2}{dt} = b_1\dot{y} + b_2\dot{x}_2$$

or

$$\frac{d\dot{x}_2}{dt} = b_2\left(\frac{b_1}{b_2}\dot{y} + \dot{x}_2\right),$$

where

$$b_1 = -\frac{205}{84a_2},\ b_2 = \frac{7}{6} + \frac{14}{9a_2}c.$$

Integrating the last equation, let's obtain:

$$\ell n\left|\frac{b_1}{b_2}\dot{y} + \dot{x}_2\right| = b_2 t + \ell n c_4^*,$$

where $\ell n c_4^*$ is the constant of integration.

From this equation, let's determine $\dot{x}_2$:

$$\dot{x}_2 = c_4^* \cdot e^{l_2 t} - \frac{b_1}{b_2}\dot{y}.$$

Integrating again, let's determine:

$$x_2 = c_4^* \cdot \frac{e^{\ell_2 t}}{b_2} - \frac{b_1}{b_2}\dot{y}\cdot t + c_5^*.$$

With initial conditions (25), let's determine c_4^* and c_5^*:

$$c_4^* = \frac{b_1}{b_2}\dot{y},\ c_5^* = \frac{a_7 b_2 l_2 - 2cb_1\dot{y}}{2cb_2 l_2}.$$

Thus, the law of change of the center of mass of the roll has the following form:

$$x_2 = \frac{b_1}{b_2 l_2}\dot{y}\cdot e^{\ell_2 t} - \frac{b_1}{b_2}\dot{y}\cdot t + \frac{b_2 a_7 l_2 - 2cb_1\dot{y}}{2cb_2 l_2}$$

or

$$x_2 = \dot{y}\left(\frac{b_1}{b_2 l_2}\cdot e^{\ell_2 t} - \frac{b_1}{b_2}t - \frac{b_1}{b_2 l_2}\right) + \frac{a_7 b_2}{2cl_2}.$$

Further, setting $\dot{y} = \text{const}$ and $\ddot{y} = 0$, from equation (4), let's determine:

$$a_5 = \frac{16a_1}{7}x_1 - \frac{a_1\cdot a_6}{a_2}\dot{x}_1 - 2a_1a_2\dot{x}_2 - \frac{0.84a_1\cdot c}{205}x_3 + \frac{14a_6\cdot c}{9}\dot{x}_3,$$

or given the values of a_5, let's determine the torque M_1^a:

$$M_1{}^a = M_1^* + M_2^*\frac{R_1}{R_2} + \frac{16a_1R_1}{7}x_1 - \frac{a_1a_6R_1}{a_2}\dot{x}_1 - 2a_1a_2R_1\dot{x}_2 - \frac{0.84a_1cR_1}{205}x_3 + \frac{14a_6cR_1}{9}\dot{x}_3,$$

where

$$a_1 = \frac{1}{2}\left(\frac{J_1}{R_1^2} + \frac{J_2}{R_2^2} + m_3\right),\ a_2 = \frac{1}{2}(m_1 + 2m_4),\ a_6 = \frac{(R_1+R_2)^2 - 2R_1R_2\left(1 - \frac{1}{2}R_1R_2\right)}{(R_1\cdot R_2)^2}.$$

When determining M_1^a, it is assumed that one roll of the pair (a drive roll), the material being processed and the second roll (a non-drive roll) receive motion due to the force of friction on the contact surfaces.

If both rolls are driven, i. e. are connected with a rigid kinematic constraint and a drive, the material being processed is set in motion due to the friction force on the contact surfaces with the rolls, then it is recommended that the radii of both rolls and the monchons (fabric tires, that absorb and remove moisture well) should be the same. In this case, M_1^a and M_2^a will be equal.

Therefore, for the dynamic stability of the movement of a roll pair, the coefficients of equation (14) must satisfy equalities (20).

Without compiling the equation of perturbed motion and the requirement that the condition of asymptotic stability of the process under consideration be satisfied, this method loses its meaning. In order to apply the results obtained in practice, using the initial data (satisfying the conditions of the control function), let's substitute equalities (20) into the equations of motion and, with the exact implementation of the servo constraint, determine the reaction forces of the constraints and the kinematic characteristics of motion.

To develop this method in the future, it is necessary to select servomotors, sensors and create programs that describe all technological processes of machining; to develop a wringing machine operating in automatic mode.

4. Conclusions

It is substantiated that the primary task that needs to be solved when designing automatic control systems is the construction of a kinematic diagram and mathematical model of the control object. It is shown that the basic aspect in the theory of automatic control is precisely the formulation of a mathematical description of the control object functioning, its properties and relationships, which makes it possible to evaluate (predict) information about the change in the state of the object when external impacts are applied to it.

It is shown that one of the reasons for the unstable stress state on the contact surfaces in the roller mechanism is dynamic factors arising from inaccuracies in the manufacture of its individual parts, assembly defects, the occurrence of an oscillatory process in the roller mechanism, and the non-uniform thickness of the processed material during its capture, starting and stopping the machine.

Optimal controls are determined that ensure the asymptotic stability of the unperturbed motion of the roll pair and the torque applied to the upper roll as a function of generalized coordinates.

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