Original Research Article: full paper

(2022), «EUREKA: Physics and Engineering»
Number 6

ANALYTICAL STUDY OF THE VELOCITY OF THE LUBRICATING FLUID IN THE HYDRODYNAMIC JOURNAL BEARING WITH THE EFFECT OF CENTRIFUGAL FORCE FOR SHORT BEARING TYPE

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Abstract

The paper aims to obtain the expression of the velocity of lubricating fluid in the hydrodynamic journal bearings by analytical method. In the classical short-bearing theory, the fluid flow was studied by ignoring the effect of centrifugal force of the lubricating fluid film. However, the self-oscillation of the shaft of high-power motors does not follow the rules in classical hydrodynamic lubrication theory. To explain this phenomenon, a modified form of Reynolds equation, in which the influence of centrifugal force of the lubricant is not ignored, is established. The study aims to establish the modified Reynolds equation by including the effect of centrifugal forces for the case of the short bearing type. Integration of the Navier-Stokes equations, yield the expressions for the components of velocity of the lubricating fluid in the gap. The oil's pressure in the hydrodynamic journal bearing is obtained by solving the modified Reynolds equation. The numerical results are considered in the case of the stable equilibrium position of the motion of the shaft, i.e. the symmetry axis of the shaft does not move. The plots of the velocity components in the tangential axial direction are displayed too. The theoretical results of a parabolic velocity distribution similar to that of a Newtonian fluid, derived from a Bingham plastic flow model. The flow in the gap, which are obtained by analytical method, are completely consistent with its boundary conditions and its physical properties. Further, with these results, the paper as a lemma to solve the dynamics problem in floating ring bearing with the influence of the centrifugal force of the lubricant.

Keywords: centrifugal force of the lubricating fluid, short bearing, floating ring bearing, velocity of lubricant, analytical method, hydrodynamic journal bearings.

DOI: 10.21303/2461-4262.2022.001962

1. Introduction

One of the problems of lubrication theory is that the self-oscillation of the shaft, which is mainly caused by the nonlinear hydrodynamic force of the lubricating oil film. To investigate the dynamics as well as the stability of motion of the shaft, it is necessary to calculate these hydrodynamic forces. These forces are obtained by integral the pressure distribution function of the oil film over the surface of the rotating shaft. So, the core of this problem is investigation of the oil's flow and establishing the equation of the oil's pressure – the Reynolds equation. In classical hydrodynamic lubrication theory [1, 2], the Reynolds equation is established by using the system of hypothesis. In which the author pays attention to the hypothesis about ignoring the centrifugal force of lubricating oil film. However, the phenomenon of self-oscillation and the dynamic instability of the shafts of high-power motors do not follow the rules in classical hydrodynamic lubrication theory. In industry, the angular velocity of the shaft is very large, so the influence of the centrifugal force of the lubricating oil film cannot be ignored. To explain this phenomenon, the author proposes a plan to establish a modified form of Reynolds equation, in that the influence of centrifugal force of the lubricating oil film is not ignored [3, 4]. The investigation of dynamics of the shaft can be launched by this equation.

In the classical theory of hydrodynamic lubrication for the short bearings, the object of study is a shaft rotating in a fixed bearing. The narrow gap between the shaft and the bearing is filled by a thin film of oil [1, 2, 5]. However, when considering the dynamic of the shaft in hydrodynamic bearings, two types of bearings are classified: common hydrodynamic bearing (two-solid body model) and hydrodynamic bearing with floating ring (three-solid body model) [6, 7], **Fig. 1, *b***.

To approach these two types of bearings simultaneously when researching the dynamics of the shaft, let's consider the general model, **Fig. 1, *a*** [3, 4]. The shaft is an absolute rigid cylinder 1, which rotates free with angular speed ω_1 inside the bearing modeled as a cylindrical shell 2 rotating in the same direction with angular speed ω_2. The rotating axis of the cylindrical shell is fixed. The narrow gap between two solid bodies is filled with the pre-stressed oil with constant dynamic viscosity.

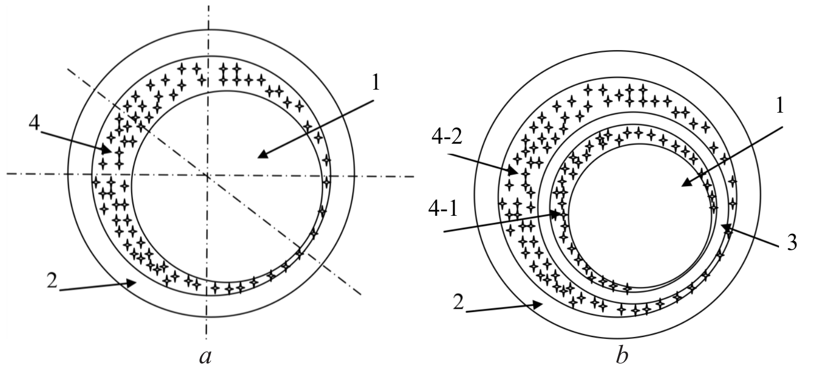


Fig. 1. Model of hydrodynamic bearing: *a* – case of two solid bodies, *b* – case of three solid bodies: 1 – shaft; 2 – fixed bearing; 3 – hard floating ring; 4 – lubricating fluid film (4-1 – inner lubricating fluid film, 4-2 – outer lubricating fluid film)

With this general model, the research as a lemma to solve the dynamic problem for hydrodynamic bearings with floating ring. The dynamic investigation of this bearing type has been carried out elaborately in the works [8–10]. Hence, the experimental results of the velocity distribution are obtained in [11, 12] and the static, dynamic characteristics of high speed journal floating ring hybrid bearing compensated by interior restrictor under laminar flow and turbulent flow respectively [13].

2. Materials and methods

The research was carried out in Newtonian fluid film filling the narrow gap between two solid bodies. The outer cylinder rotates at an angular speed $\omega_2 = 5{\cdot}10^2$ rad/s. The shaft rotates at an angular speed $\omega_1 = 10^3$ rad/s. The inner radius of the outer cylinder: $R_2 = 5{\cdot}10^{-2}$ m. The radius of the rotating shaft is $R_1 = 4.996{\cdot}10^{-2}$ m, hence the local oil film thickness is $0 \le h \le 80{\cdot}10^{-6}$ m = 80 μm. The oil SAE-40, at a temperature of 20 °C, its density is $\rho = 0.9{\cdot}10^3$ kg/m^3. The dynamic viscosity – $\mu = 0.319$ Pa·s.

The motion of the lubricating fluid film in the bearing is determined by the balance of linear momentum [1]:

$$\nabla \cdot \tau - \rho \dot{v} = 0, \tag{1}$$

inwhich τ – stress tensor and ρ – mass density of the fluid.

The velocity $\mathbf{v}$ and the Hamiltonian operator ∇ in the cylindrical coordinate system are:

$$\mathbf{v} = \mathbf{e}_r u + \mathbf{e}_\varphi v + \mathbf{e}_z w,\ \nabla = \mathbf{e}_r \frac{\partial}{\partial r} + \mathbf{e}_\varphi \frac{1}{r}\frac{\partial}{\partial \varphi} + \mathbf{e}_z \frac{\partial}{\partial z}, \tag{2}$$

inwhich u, v, w – the velocity components of the fluid flow in the radial, tangential and axis directions, respectively.

Assuming that, the flow in the bearing is real flow (turbulent flow), the gap is very narrow (in practice the width of the gap is micrometer) and the bearing is a short bearing type. By ignoring infinitesimal quantities, let's obtain a system of three equations of the oil's pressure and the components of the flow's velocity [2, 3]:

$$\begin{cases} \dfrac{\partial p}{\partial r} = \rho \dfrac{v^2}{r}; \\ \dfrac{1}{\mu}\dfrac{\partial p}{\partial \varphi} = \dfrac{\partial}{\partial r}\left(r\dfrac{\partial v}{\partial r}\right) - \dfrac{v}{r}; \\ \dfrac{1}{\mu}\dfrac{\partial p}{\partial z} = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial w}{\partial r}\right), \end{cases} \quad (3)$$

inwhich μ – the kinematic viscosity.

This is a popular system of equations of the theory of hydrodynamic lubrication. However in the classical theory, the right hand side of the first equation in this system of (3) is zero [1, 5, 6]. That is, the equation of motion of the fluid – the Reynolds equation will be obtained with ignoring the centrifugal force of the lubricating fluid film. But the system of (3) shows that, when the velocity of the shaft is large, the quantity on the right hand side of it is not small. Therefore, for bearings, in which the shaft rotates at high speed, the influence of the centrifugal force of the lubricating fluid film cannot be ignored.

Integrating the system of three equations above for the general case shown in **Fig. 1, *a***, let's apply boundary conditions [3]:

$$\begin{gathered} r = R: u = 0, v = R_2\omega_2, w = 0; \\ r = R - h: u = -\dot{h}, v = (R_2 - h)\omega_1, w = 0, \end{gathered} \quad (4)$$

inwhich h – the local thickness of the lubricating oil film; ω_2, R_2 – the angular velocity and the inner radius of the rigid cylinder; ω_1, R – the angular velocity and radius of the shaft.

Integration of the first equation in (3) yields:

$$p = \int \rho \frac{v^2}{r} \mathrm{d}r + P(\varphi, z), \quad (5)$$

inwhich p, $P(\varphi, z)$ – the oil's pressure and its axial and tangential directions components, respectively. The components of velocity in the tangential and axial directions are:

$$v = \frac{1}{4}\frac{1}{\mu}\frac{\partial P}{\partial \varphi} f(r) + g(r), \quad (6)$$

$$w = \frac{1}{4}\frac{1}{\mu}\frac{\partial P}{\partial z} q(r), \quad (7)$$

inwhich:

$$f(r) = 2\frac{r^2 R_2^2 \ln\left(\dfrac{r}{R_2}\right) + r^2 (R_2 - h)^2 \ln\left(\dfrac{R_2 - h}{r}\right)}{r\left[R_2^2 - (R_2 - h)^2\right]} + \frac{R_2^2 (R_2 - h)^2 \ln\left(\dfrac{R_2}{R_2 - h}\right)}{r\left[R_2^2 - (R_2 - h)^2\right]};$$

$$g(r) = \frac{\omega_2 R_2^2 \left[r^2 - (R_2 - h)^2\right]}{r\left[R_2^2 - (R_2 - h)^2\right]} + \frac{\omega_1 (R_2 - h)^2 \left[R_2^2 - r^2\right]}{r\left[R_2^2 - (R_2 - h)^2\right]};$$

$$q(r) = r^2 + \frac{(R_2 - h)^2 \ln\left(\dfrac{r}{R_2}\right) + R_2^2 \ln\left(\dfrac{R_2 - h}{r}\right)}{\ln\left(\dfrac{R_2}{R_2 - h}\right)}.$$

The radial velocity cannot be determined in the framework of the present analysis. For this reason, let's assume a linear distribution satisfying the boundary conditions:

$$u(r) = -\frac{\dot{h}}{h}(R_2 - r). \quad (8)$$

From (6), (7) it is possible to see that, to determine the flow of the lubricating oil in the bearing, it is necessary to determine the oil's pressure. The oil's pressure in the bearing is determined through the modified Reynolds equation. Integration of the continuity equation of the fluid flow in the bearing yields the modified Reynolds equation examining the effect of the centrifugal force of lubricating oil film [2, 3]:

$$\frac{h^3}{6}\frac{\partial^2 P}{\partial z^2}+\frac{h^3}{6}\frac{1}{R_2^2}\left[\frac{\partial^2 P}{\partial \varphi^2}+\frac{3}{h}\frac{\partial P}{\partial \varphi}\frac{\partial h}{\partial \varphi}\right]=\mu\left[2\frac{\partial h}{\partial t}+\left(\omega_1+\omega_2\right)\frac{\partial h}{\partial \varphi}\right]. \tag{9}$$

In this equation, $P(\varphi, z)$ is the axial and tangential pressure component in (6), (7). To solve the (9), let's simplify the new modified Reynolds equation to the form by considering of the case of short bearings:

$$\frac{h^3}{6}\frac{\partial^2 P}{\partial z^2}=\mu\left[2\frac{\partial h}{\partial t}+\left(\omega_1+\omega_2\right)\frac{\partial h}{\partial \varphi}\right]. \tag{10}$$

This is a quadratic differential equation that can be solved by analytical method. The two integration constants in (10) can be determined by the boundary condition and the symmetry condition [2]:

$$z=0,\ r=R_2:\ p=p^{in};\ p(z)=p(-z),\ \forall z\in\left[0,\ L/2\right]. \tag{11}$$

Integration of the equation (10) yields:

$$P(\varphi,z)=\frac{6\mu}{h^3}\left[2\frac{\partial h}{\partial t}+(\omega_1+\omega_2)\frac{\partial h}{\partial \varphi}\right]\left(\frac{1}{2}z^2+C\right), \tag{12}$$

inwhich:

$$h=h_0-e\cos\varphi;\ h_0=R_2-R_1. \tag{13}$$

The constant C in this expression is completely determined based on the first condition (11). To simplify, let's take the constant $C=1$. Substituting the expression (12) of the oil's pressure into the expressions (6), (7) let's obtain the analytical expression for the flow of the fluid.

3. Results and discussion

To build the graph of the oil flow, let's apply the geometrical and kinematical parameters of the bearing, **Fig. 2**.

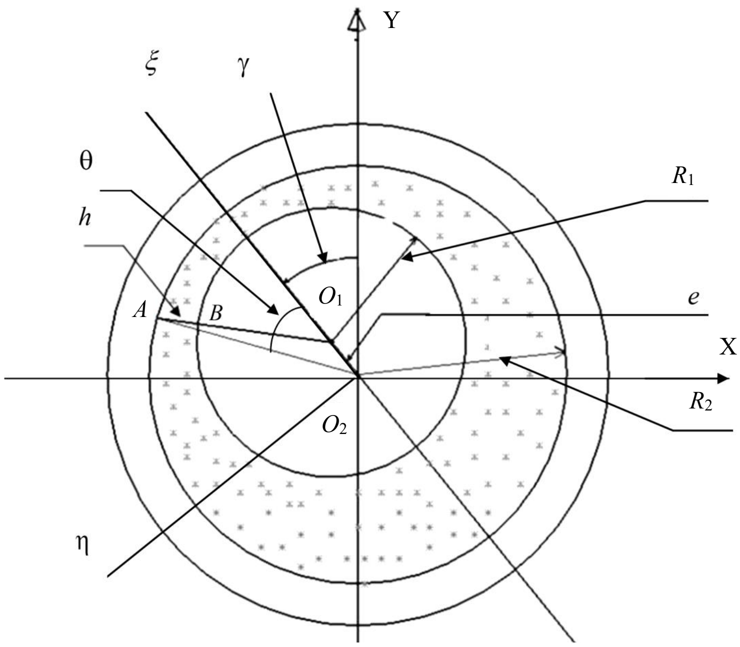


Fig. 2. The schema of the bearing in the case of two cylinders

For simplicity, let's consider the case of the equilibrium position of the motion of the shaft is stable, i.e. the axis of the shaft does not move. Then, the eccentricity $O_1O_2 = e(t)$ and the angle of rotation $\gamma(t)$ will be constants, **Fig. 2**. From (8), (13) it is easy to see that the radial velocity component $u(r) = 0$.

3. 1. The case ofzero eccentricity (*e* = 0)

The schema of the bearings in the case of the eccentricity $e = 0$, **Fig. 3**. The graphs of the velocity components in the tangential direction, **Fig. 4, *a*** and axial direction, **Fig. 4, *b*** of the velocity of the lubricating fluid film in the bearing.

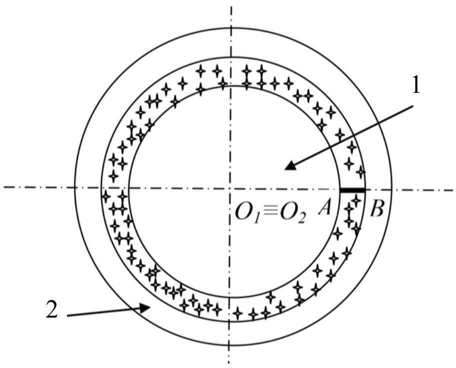


Fig. 3. The schema of the bearings in the case of the eccentricity $e = 0$

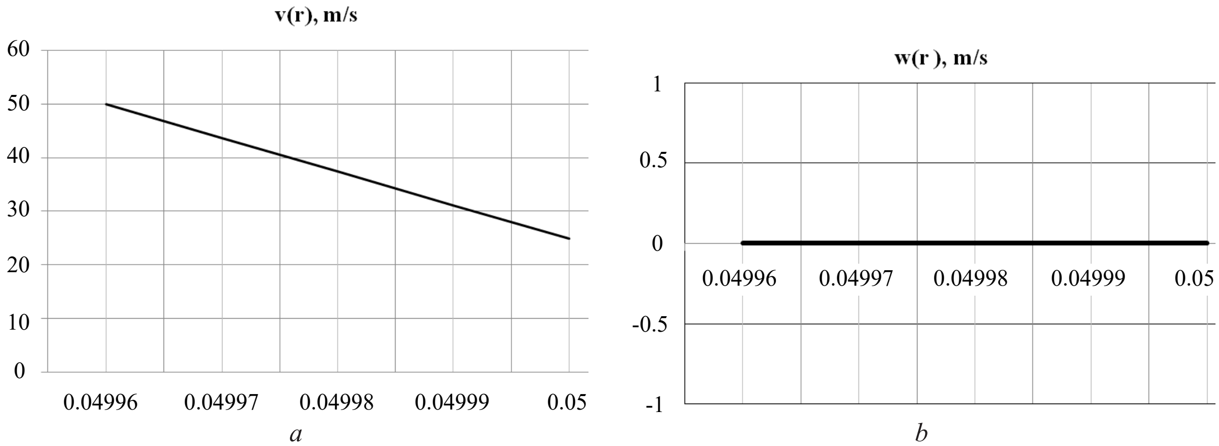


Fig. 4. The velocity components of lubricating fluid film on segment *AB* in: *a* – $v(r)$; *b* – $w(r)$

3. 2. The case of non-zero eccentricity ($e = h_0$; $\gamma = \pi/2$)

The schema of the bearings in the case of the eccentricity $e = h_0$; $\gamma = \pi/2$, **Fig. 5**.

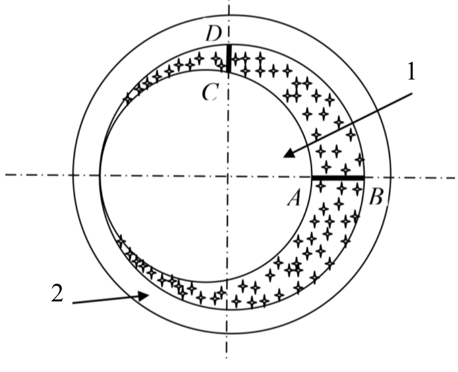
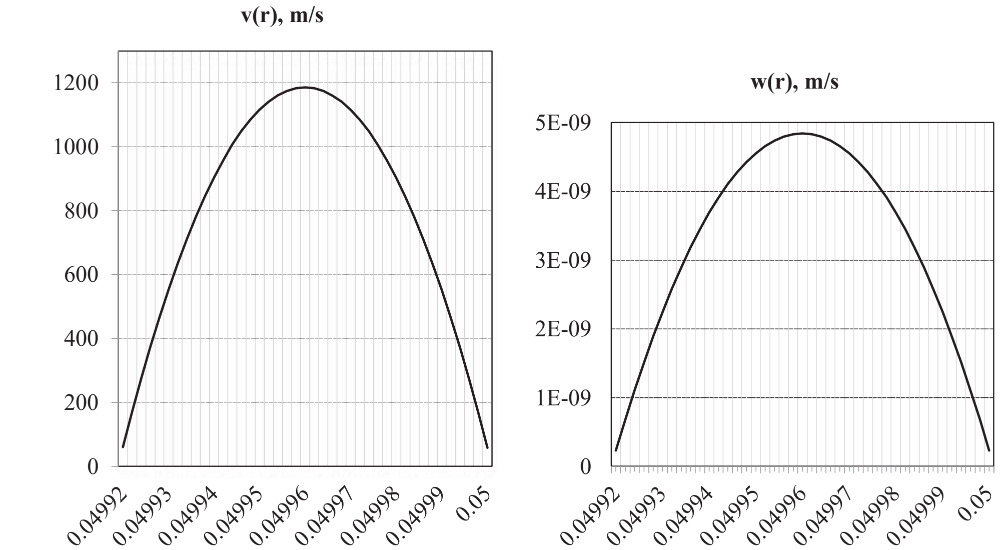


Fig. 5. The schema of the bearings in the case of the eccentricity $e = h_0$; $\gamma = \pi/2$

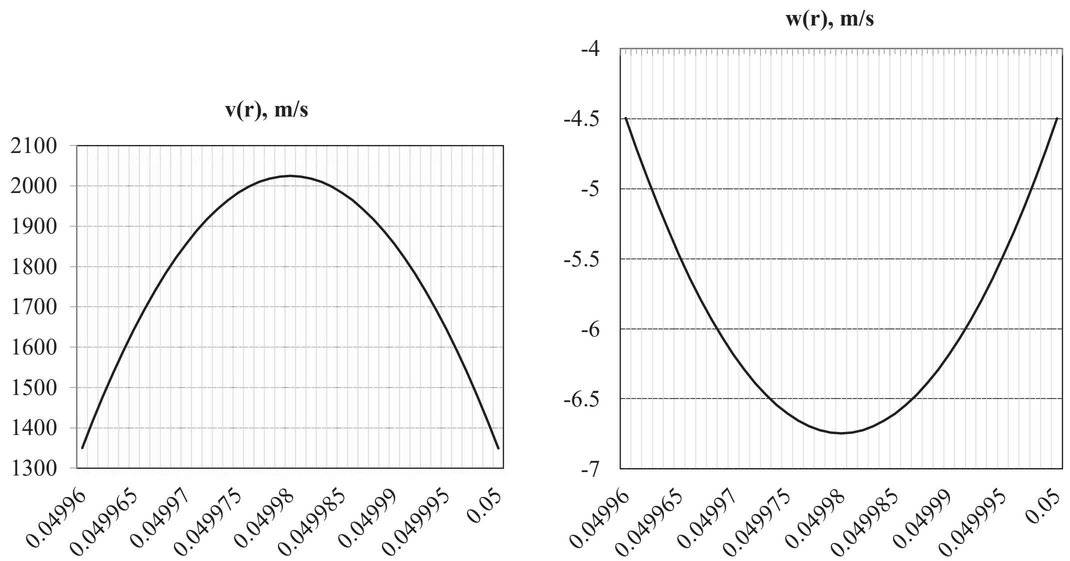
The graphs of the velocity components in the tangential direction, **Fig. 6, *a*** and axial direction, **Fig. 6, *b*** of the velocity of the lubricating fluid film on segment *AB*. While the graphs of the velocity components in the tangential direction, **Fig. 7, *a*** and axial direction, **Fig. 7, *b*** of the velocity of the lubricating fluid film on segment *CD*.

Let's consider the case of the motion of the shaft is stable then the radial component of velocity is zero.



a *b*

Fig. 6. The flow components of lubricating fluid film on segment *AB* in: *a* – *v*(*r*); *b* – *w*(*r*)



a *b*

Fig. 7. The flow components of lubricating fluid film on segment *CD* in: *a* – *v*(*r*); *b* – *w*(*r*)

3. 3. Thecomparison and discussion

The article has established the analytical expressions of the lubricating oil flow in the hydrodynamic bearing (6)–(8). The oil's pressure in these formulas is determined via the modified Reynolds equation for the case of short bearing approach (10) with boundary condition (11). The analytical results of the oil's pressure are shown in (12). **Fig. 4, *a*** shows the tangential velocity component, which is obtained in equation (6) in case of the eccentricity is zero. It is possible to see that this component reaches its maximum value at the shaft's surface and it decreases linearly to the inner surface of the bearing. **Fig. 4, *b*** shows that, the axial flow is absent.

Fig. 6, *a,* 7, *a* showthe plots of the tangential velocity in the case of steady motion with non-zero eccentricity. Here, it is possible to see that, this velocity distributes in a parabolic shape and reaches its maximum value at the middle of the gap. Similarly, **Fig. 6, *b,* 7, *b*** are plots of the axial flow. This axial flow reaches its maximum value also at the middle of the gap, and it reaches the minimum value is zero at the surface of the shaft and bearing. This is fully consistent with liquid flow in a fixed channel. The theoretical results of a parabolic velocity distribution similar to that of a Newtonian fluid, derived from a Bingham plastic flow model. Theseresults of the flow compare very well with the experimental results of the grease shear flows in a concentric cylinder configuration [11, 12].

The experimental part was carried out on the hydrodynamic journal bearing with the locations of pressure and temperature sensors, **Fig. 8.**

The oil's pressure in these formulas of the flow is determined via the modified Reynolds equation for the case of short bearing approach (10). The comparison of the obtained analytical results (12) with experimental data for difference case of load Fr, rotation N_1 and temperature T are shown on **Fig. 9**. The convergence of experimental and analytical results can testify to the reliability of the obtainedresults.The results of this study would be more accurate if the approximations for the short bearing were ignored. The case of the long bearing must be considered or the solution of the equation (9) for any type of bearing must be continued in further studies. In addition, the paper as a lemma to solve the dynamics problem in floating ring bearing with the influence of the centrifugal force of the lubricant.

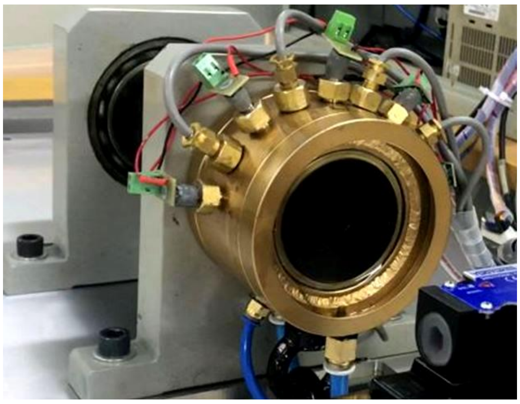
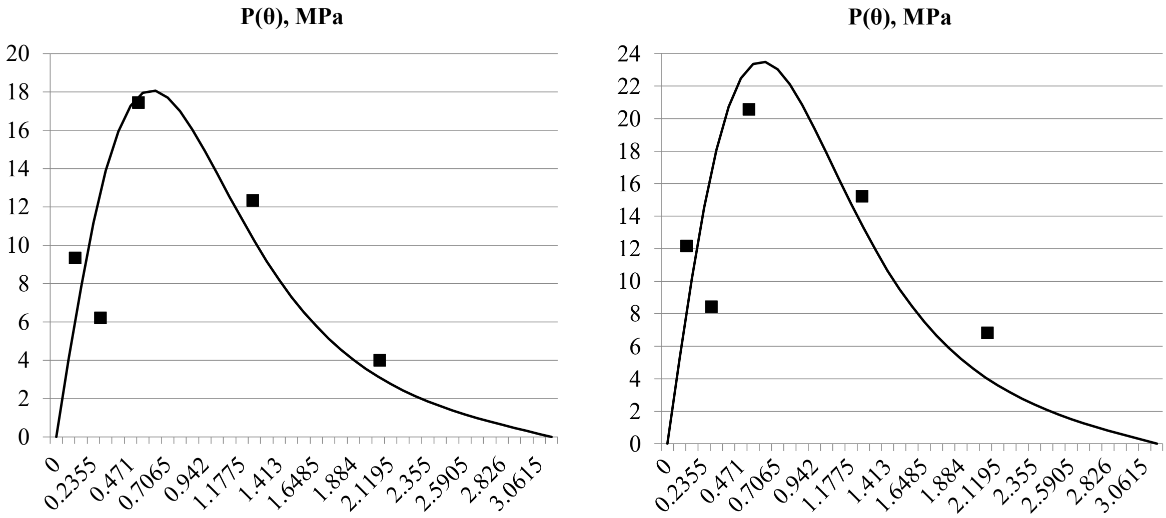


Fig. 8. The hydrodynamic journal bearing



a *b*

Fig. 9. The comparison of the pressure distribution in hydrodynamic journal bearing:
— – analytical method; ■ – experimental method: *a* – Fr = 300 *N*, N_1 = 1000 rpm, T = 29 °C;
b – Fr = 500 *N*, N_1 = 2000 rpm, T = 31 °C

4. Conclusions

This paper obtains the expression of the velocity of lubricating fluid in the hydrodynamic journal bearings by analytical method. The oil's pressure is obtained by solving the modified Reynolds equation in the case of the short bearing type approach. In this equation, the effect of centrifugal force of the lubricating oil film is mentioned.The numerical results are considered in the case of the stable equilibrium position of the motion of the shaft, i.e. the symmetry axis of the shaft does not move. The parameters of hydrodynamic journal bearing in numerical estimates were taken from the experimental part. The plots of the velocity of lubricating fluid in the gap are completely consistent with the boundary conditions and physical properties of the flow.The convergence of experimental data of the pressure distribution and analytical results can testify to the reliability of the new information obtained.

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Received date 27.07.2021
Accepted date 10.05.2022
Published date

How to cite: *Thang, N. V. (2022). Analytical study of the velocity of the lubricating fluid in the hydrodynamic journal bearing with the effect of centrifugal force for short bearing type. EUREKA: Physics and Engineering, 3, 00–00. doi: https://doi.org/10.21303/2461-4262.2022.001962*