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Neutrosophic Soft e-Compact Spaces and Application Using Entropy Measure

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Abstract

In this paper, the concept of neutrosophic soft e-compactness is presented on neutrosophic soft topological spaces using the definition of e-open cover and its types. In addition, neutrosophic soft e-compactness and neutrosophic soft e-separation axioms are associated. Also, the concept of neutrosophic soft locally e-compactness is introduced in neutrosophic soft topological spaces and some of its properties are discussed. Added to that, an application in decision making problem is given using entropy.

Keywords: Neutrosophic soft e-open cover; Neutrosophic soft e-closed cover; Neutrosophic soft e-compact space and neutrosophic soft locally e-compact space

MSC 2010 No.: 03E72, 54D10, 54D15, 54D45, 54E45

1. Introduction and Preliminaries

Zadeh (1965) introduced the fuzzy set and Chang (1968) developed its topological structure. Intuitionistic fuzzy set was established by Atanassov (1986) and Coker (1997) developed its topological structure. Molodtsov (1999) introduced the soft set theory. The soft topological spaces were established by Shabir and Naz (2011). The neutrosophic set was introduced by Smarandache (2005) and Salama and Alblowi (2012) developed its topological structure. The neutrosophic soft set (in short, N_sSs) was defined by Maji (2013), modified by Deli and Broumi (2015) and Bera and Mahapatra (2017) developed its topological structures. The concept of δ-open sets was given by Saha (1987) in fuzzy topological spaces, Vadivel et al. (2021a), Vadivel and John Sundar (2021b, 2021c) in neutrosophic topological spaces and Acikgoz and Esenbel (2019) in neutrosophic soft topological spaces. The e-open sets were introduced and also studied by Ekici (2007, 2008a, 2008b, 2008c, 2008d, 2009) in a general topology, Seenivasan and Kamala (2014) in fuzzy topological spaces, Chandrasekar et al. (2018) in intuitionistic fuzzy topological spaces, Vadivel et al. (2021b, 2021c) in neutrosophic topological spaces and Revathi et al. (2021, 2022) in neutrosophic soft topological spaces (in short, N_sSeos). Aras et al. (2019) introduced separation axioms, Khattak et al. (2019) developed soft b-separation axioms, and Acikgoz and Esenbel (2020) introduced pre-separation axioms in neutrosophic soft topological spaces. Recently, soft e-separation axioms in neutrosophic soft topological spaces were introduced by Revathi et al. (2021, 2022). Ozturk et al. (2021) studied neutrosophic soft compact spaces. Arockiarani (2017) and Vadivel and John Sundar (2021c) introduced entropy measures in neutrosophic soft sets and studied its application in multi attribute decision making.

The aim of this paper is to introduce e-open cover and its types in neutrosophic soft topological space (in short, N_sSts), thereby defining neutrosophic soft e-compact spaces. Also, some properties of neutrosophic soft e-compact spaces related to neutrosophic soft e-separation axioms are discussed. Furthermore, the concept of neutrosophic soft locally e-compactness and its related properties are discussed. In addition, an application in decision making problem is given using entropy measure.

The basic definitions, properties and theorems of neutrosophic soft (in short N_sS) topological spaces needed in this paper are shown in Deli and Broumi (2015), Bera and Mahapatra (2017), Maji (2013), Ozturk et al. (2019), Acikgoz and Esenbel (2019), Revathi et al. (2021, 2022), Aras et al. (2019), Ozturk et al. (2021) and Arockiarani (2017).

2. Neutrosophic soft e-compact spaces

In this section, we define N_sSe-compact space on a N_sSts and its related properties are discussed.

Definition 2.1.

Let (U, τ, Λ) be a N_sSts over U and ϑ be a N_sS cover (in short, N_sSc) of $1_{(U,\Lambda)}$. If each element of the cover ϑ is a N_sSe-open (e-closed) in (U, τ, Λ), then ϑ is said to be a N_sSe-open (e-closed) cover (in short, N_sSeoc (N_sSecc)).

Definition 2.2.

If there is a N_sSe-nbd that intersects only the finite number of of the cover ϑ of each $\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in N_sSs$ on U, ϑ is said to be a N_sSe-locally finite cover.

Remark 2.1.

If the N_sSc's are e-open, each N_sS star with finite cover is N_sSe-locally finite and each N_sSe-locally finite cover is N_sS finite.

Definition 2.3.

Let (U, τ, Λ) be a N_sSts over U and $(\tilde{\Phi^*}, \Lambda)$ be a N_sSs on U.

(i) If every N_sSeoc of (U, τ, Λ) has a finite N_sS subcover, then (U, τ, Λ) is said to be a N_sSe-compact space (in short, N_sSeCOS).
(ii) If $((\tilde{\Phi^*}, \Lambda), \tau_{(\tilde{\Phi^*},\Lambda)}, \Lambda)$ is a N_sSeCOS, then $(\tilde{\Phi^*}, \Lambda)$ is said to be a N_sSe-compact set (in short, N_sSeCos) in (U, τ, Λ).

Example 2.1.

Let $U = \{\phi_{11}, \phi_{22}, \phi_{33}\}$ be an initial universe set, $\Lambda = \{\theta_1, \theta_2\}$ be a set of parameters and $\tau = \{0_{(U,\Lambda)}, 1_{(U,\Lambda)}, (\tilde{\Phi^*}_{11}, \Lambda), (\tilde{\Phi^*}_{22}, \Lambda), (\tilde{\Phi^*}_{33}, \Lambda)\}$ where $(\tilde{\Phi^*}_{11}, \Lambda), (\tilde{\Phi^*}_{22}, \Lambda)$ and $(\tilde{\Phi^*}_{33}, \Lambda)$ are defined as

$$(\tilde{\Phi^*_{11}}, \Lambda) = \begin{Bmatrix} \theta_1 = \langle\phi_{11}, (1,0,1)\rangle, \langle\phi_{22}, (0,1,1)\rangle, \langle\phi_{33}, (0,0,1)\rangle \\ \theta_2 = \langle\phi_{11}, (0,0,1)\rangle, \langle\phi_{22}, (1,0,1)\rangle, \langle\phi_{33}, (1,1,0)\rangle \end{Bmatrix},$$
$$(\tilde{\Phi^*_{22}}, \Lambda) = \begin{Bmatrix} \theta_1 = \langle\phi_{11}, (0,1,0)\rangle, \langle\phi_{22}, (1,1,0)\rangle, \langle\phi_{33}, (1,1,0)\rangle \\ \theta_2 = \langle\phi_{11}, (1,1,0)\rangle, \langle\phi_{22}, (1,1,0)\rangle, \langle\phi_{33}, (0,1,0)\rangle \end{Bmatrix},$$
$$(\tilde{\Phi^*_{33}}, \Lambda) = \begin{Bmatrix} \theta_1 = \langle\phi_{11}, (0,0,1)\rangle, \langle\phi_{22}, (0,1,1)\rangle, \langle\phi_{33}, (0,0,1)\rangle \\ \theta_2 = \langle\phi_{11}, (0,0,1)\rangle, \langle\phi_{22}, (1,0,1)\rangle, \langle\phi_{33}, (0,1,0)\rangle \end{Bmatrix}.$$

Here, $1_{(U,\Lambda)} = \bigcup_{i=1}^{3} (\tilde{\Phi^*_{ii}}, \Lambda)$. Hence $\{(\tilde{\Phi^*}_{11}, \Lambda), (\tilde{\Phi^*}_{22}, \Lambda), (\tilde{\Phi^*}_{33}, \Lambda)\}$ is a N_sSeoc of (U, τ, θ) as $(\tilde{\Phi^*}_{11}, \Lambda), (\tilde{\Phi^*}_{22}, \Lambda)$ and $(\tilde{\Phi^*}_{33}, \Lambda)$ are e-open sets. Also, $1_{(U,\Lambda)} = (\tilde{\Phi^*}_{11}, \Lambda) \cup (\tilde{\Phi^*}_{22}, \Lambda)$. Therefore, (U, τ, Λ) is N_sSeCOS.

Theorem 2.1.

Let (U, τ, Λ) be N_sSts over U and $(\tilde{\Phi^*}, \Lambda)$ be a N_sS's on U. Then, $(\tilde{\Phi^*}, \Lambda)$ is a N_sSeCos if and only if every N_sS eoc of $(\tilde{\Phi^*}, \Lambda)$ has a finite N_sS subcover in (U, τ, Λ).

Proof:

Let $(\tilde{\Phi^*}, \Lambda)$ be a N_sSeCos and the family $\vartheta = \{(\tilde{\Phi^*}, \Lambda)_t : t \in I\}$ is a N_sSeoc of $(\tilde{\Phi^*}, \Lambda)$ in (U, τ, Λ). Then, $(\tilde{\Phi^*}, \Lambda) \subseteq \bigcup_{t \in I} (\tilde{\Phi^*}, \Lambda)_t$. That is, $(\tilde{\Phi^*}, \Lambda) = \bigcup_{t \in I} ((\tilde{\Phi^*}, \Lambda) \cap (\tilde{\Phi^*}, \Lambda)_t)$. As for every $t \in I$, $((\tilde{\Phi^*}, \Lambda) \cap (\tilde{\Phi^*}, \Lambda)_t) \in \tau_{(\tilde{\Phi^*},\Lambda)}$ and so the N_sS family $\{(\tilde{\Phi^*}, \Lambda) \cap (\tilde{\Phi^*}, \Lambda)_t\}_{t \in I}$ is a N_sSeoc

of $(\tilde{\Phi^*}, \Lambda)$. Since $((\tilde{\Phi^*}, \Lambda), \tau_{(\tilde{\Phi^*},\Lambda)}, \Lambda)$ is N_sSeCOS, there exist $t_1, t_2, ..., t_n$ such that $(\tilde{\Phi^*}, \Lambda) = \bigcup_{j=1}^{n} ((\tilde{\Phi^*}, \Lambda) \cap (\tilde{\Phi^*}, \Lambda)_{tj}) \subseteq \bigcup_{j=1}^{n} (\tilde{\Phi^*}, \Lambda)_{tj}$, that is, the family $\{(\tilde{\Phi^*}, \Lambda)_{tj}\}_{j=1,2...,n}$ is a N_sS finite subcover of $(\tilde{\Phi^*}, \Lambda)$.

Conversely, let the family $\{(\tilde{\Psi}, \Lambda)_t \ : \ t \ \in \ I\}$ be a N_sSeoc of $((\tilde{\Phi^*}, \Lambda), \tau_{(\tilde{\Phi^*},\Lambda)}, \Lambda)$. Since $(\tilde{\Psi}, \Lambda)_t \in \tau_{(\tilde{\Phi^*},\Lambda)}$ for each $t \in I$, there exists $(\tilde{\Phi^*}, \Lambda)_t \in \tau$ such that $(\tilde{\Psi}, \Lambda)_t = (\tilde{\Phi^*}, \Lambda) \cap (\tilde{\Phi^*}, \Lambda)_t$. Hence the family $\{(\tilde{\Phi^*}, \Lambda)_t \ : \ t \ \in \ I\}$ is a N_sSeoc of $(\tilde{\Phi^*}, \Lambda)$ in (U, τ, Λ) and there exists $(\tilde{\Phi^*}, \Lambda)_{i1}, (\tilde{\Phi^*}, \Lambda)_{i2}, ..., (\tilde{\Phi^*}, \Lambda)_{in}$ such that $(\tilde{\Phi^*}, \Lambda) \subseteq \bigcup_{j=1}^{n} (\tilde{\Phi^*}, \Lambda)_{tj}$ which implies

$(\tilde{\Phi^*}, \Lambda) = (\tilde{\Phi^*}, \Lambda) \cap \big(\bigcup_{j=1}^{n} (\tilde{\Phi^*}, \Lambda)_{tj} \big) = \bigcup_{j=1}^{n} (\tilde{\Phi^*}, \Lambda) \cap (\tilde{\Phi^*}, \Lambda)_{tj} = \bigcup_{j=1}^{n} (\tilde{\Psi}, \Lambda)_{tj}$ by the hypothesis. ■

Theorem 2.2.

Let (U, τ, Λ) be a N_sSts over U. $1_{(U,\Lambda)}$ is N_sSeCOS if and only if every family of N_sSecs's with empty intersection in (U, τ, Λ) has a finite subfamily with empty intersection.

Proof:

Let (U, τ, Λ) be a N_sSeCOS and the intersection of the family $\vartheta = \{(\tilde{\Phi^*}, \Lambda)_t : t \in I\}$ is the N_sSecs's family which is empty. Hence, the family, $\vartheta = \{(\tilde{\Psi}, \Lambda)_t = 1_{(U,\Lambda)} \setminus (\tilde{\Phi^*}, \Lambda)_t\}_{t\in I}$ is the N_sSeos's family and we get, $\bigcup_{t\in I} (\tilde{\Psi}, \Lambda)_t = \bigcup_{t\in I} \big(1_{Y,\theta} \setminus (\tilde{\Phi^*}, \Lambda)_t\big) = 1_{(U,\Lambda)} \setminus \big(\bigcap_{t\in I} (\tilde{\Phi^*}, \Lambda)_t\big) = 1_{(U,\Lambda)} \setminus 0_{(U,\Lambda)} = 1_{(U,\Lambda)}$.

Hence, the family $\vartheta = \{(\tilde{\Psi}, \Lambda)_t\}_{t\in I}$ is a N_sSeoc of $1_{(U,\Lambda)}$. As (U, τ, Λ) is a N_sSeCOS, there exists $(\tilde{\Psi}, \Lambda)_{i1}, (\tilde{\Psi}, \Lambda)_{i2}, ..., (\tilde{\Psi}, \Lambda)_{in}$ such that $1_{(U,\Lambda)} = \bigcup_{j=1}^{n} (\tilde{\Psi}, \Lambda)_{tj}$. Now, the intersection of the finite subfamily $\{(\tilde{\Phi^*}, \Lambda)_{tj}\}_{j=1,2...,n}$ is the family $\vartheta = \{(\tilde{\Phi^*}, \Lambda)_t\}_{t\in I}$ and we get, $\bigcap_{j=1}^{n} (\tilde{\Phi^*}, \Lambda)_{tj} = \bigcap_{j=1}^{n} (1_{(U,\Lambda)} \setminus (\tilde{\Psi}, \Lambda)_{tj}) = 1_{(U,\Lambda)} \setminus \big(\bigcup_{j=1}^{n} (\tilde{\Psi}, \Lambda)_{tj}\big) = 1_{(U,\Lambda)} \setminus 1_{(U,\Lambda)} = 0_{(U,\Lambda)}$.

Conversely, let the family $\vartheta = \{(\tilde{\Psi}, \Lambda)_t\}_{t\in I}$ be a N_sSeoc of $1_{(U,\Lambda)}$. The intersection of the N_sSecs's family $\vartheta = \{(\tilde{\Phi^*}, \Lambda)_t = 1_{(U,\Lambda)} \setminus (\tilde{\Psi}, \Lambda)_t\}_{t\in I}$ is empty. Really, $\bigcap_{t\in I} (\tilde{\Phi^*}, \Lambda)_t = \bigcap_{t\in I} \big(1_{(U,\Lambda)} \setminus (\tilde{\Psi}, \Lambda)_{tj}\big)$ $= 1_{(U,\Lambda)} \setminus \big(\bigcup_{t\in I} (\tilde{\Psi}, \Lambda)_{tj}\big) = 1_{(U,\Lambda)} \setminus 1_{(U,\Lambda)} = 0_{(U,\Lambda)}$.

Then by the hypothesis, there exist $(\tilde{\Phi^*}, \Lambda)_{i1}, (\tilde{\Phi^*}, \Lambda)_{i2}, ..., (\tilde{\Phi^*}, \Lambda)_{in}$ such that $\bigcap_{j=1}^{n} (\tilde{\Phi^*}, \Lambda)_{tj} = 0_{(U,\Lambda)}$. Therefore, $\bigcup_{j=1}^{n} (\tilde{\Psi}, \Lambda)_{tj} = \bigcup_{j=1}^{n} (1_{(U,\Lambda)} \setminus (\tilde{\Phi^*}, \Lambda)_{tj}) = 1_{(U,\Lambda)} \setminus \big(\bigcap_{j=1}^{n} (\tilde{\Phi^*}, \Lambda)_{tj}\big) = 1_{(U,\Lambda)} \setminus 0_{(U,\Lambda)} = 1_{(U,\Lambda)}$. Hence, the N_sS finite subcovering of $\vartheta = \{(\tilde{\Phi^*}, \Lambda)_t\}_{t\in I}$ is obtained. As a result, (U, τ, Λ) is a N_sSeCOS. ■

Theorem 2.3.

Let (U,τ,Λ) be a N_sSts over U. (U,τ,Λ) is a N_sSeCOS if and only if the intersection of all the sets of every N_sS centered e-closed sets family is different from empty in $1_{(U,\Lambda)}$.

Proof:

Let (U,τ,Λ) be a N_sSeCOS over U and $\vartheta=\{(\tilde{\Phi^*},\Lambda)_t\}_{t\in I}$ be N_sS centered e-closed sets family. Suppose $\bigcap\limits_{t\in I}(\tilde{\Phi^*},\Lambda)_t=0_{(U,\Lambda)}$. Then, the family $\vartheta=\{(\tilde{\Psi},\Lambda)_t=1_{(U,\Lambda)}\setminus(\tilde{\Phi^*},\Lambda)_t\}_{t\in I}$ is N_sSeoc of $1_{(U,\Lambda)}$. Since (U,τ,Λ) is a N_sSeCOS, there exist $(\tilde{\Psi},\Lambda)_{i1},(\tilde{\Psi},\Lambda)_{i2},...,(\tilde{\Psi},\Lambda)_{in}$ such that $1_{(U,\Lambda)}=\bigcup\limits_{j=1}^{n}(\tilde{\Psi},\Lambda)_{tj}$. In this case, $\bigcap\limits_{j=1}^{n}(\tilde{\Phi^*},\Lambda)_{tj}=\bigcap\limits_{j=1}^{n}(1_{(U,\Lambda)}\setminus(\tilde{\Psi},\Lambda)_{tj})=1_{(U,\Lambda)}\setminus\big(\bigcup\limits_{j=1}^{n}(\tilde{\Psi},\Lambda)_{tj}\big)$ $=1_{(U,\Lambda)}\setminus 1_{(U,\Lambda)}=0_{(U,\Lambda)}$.

This contradicts with ϑ is a N_sS centered family. So, the assumption is wrong and so $\bigcap\limits_{t\in I}(\tilde{\Phi^*},\Lambda)_t\neq 0_{(U,\Lambda)}$.

Conversely, let the theorem is true, but (U,τ,Λ) is not N_sSeCOS. Then, $\vartheta=\{(\tilde{\Psi},\Lambda)_t\}_{t\in I}$ is not a finite N_sS subcovering of $1_{(U,\Lambda)}$. Hence, for any $(\tilde{\Psi},\Lambda)_{i1},(\tilde{\Psi},\Lambda)_{i2},...,(\tilde{\Psi},\Lambda)_{in}$, the finite N_sS subfamily $\bigcup\limits_{j=1}^{n}(\tilde{\Psi},\Lambda)_{tj}\neq 1_{(U,\Lambda)}$ is obtained for the N_sS family ϑ. Consider the family of N_sSecs's $\vartheta=\{(\tilde{\Phi^*},\Lambda)_t=1_{(U,\Lambda)}\setminus(\tilde{\Psi},\Lambda)_t\}_{t\in I}$. Here, because $\bigcap\limits_{j=1}^{n}(\tilde{\Phi^*},\Lambda)_{tj}=\bigcap\limits_{j=1}^{n}(1_{(U,\Lambda)}\setminus(\tilde{\Psi},\Lambda)_{tj})=1_{(U,\Lambda)}\setminus$ $\big(\bigcup\limits_{j=1}^{n}(\tilde{\Psi},\Lambda)_{tj}\big)\neq 0_{(U,\Lambda)}$, the family ϑ is a N_sS centered e-closed sets family and the condition of the theorem is $\bigcap\limits_{t\in I}(\tilde{\Phi^*},\Lambda)_t\neq 0_{(U,\Lambda)}$. Then, $1_{(U,\Lambda)}=\bigcup\limits_{t\in I}(\tilde{\Psi},\Lambda)_t=\bigcup\limits_{t\in I}\big(1_{(U,\Lambda)}\setminus(\tilde{\Phi^*},\Lambda)_{tj}\big)=$ $1_{(U,\Lambda)}\setminus\big(\bigcap\limits_{t\in I}(\tilde{\Phi^*},\Lambda)_t\big)\neq 1_{(U,\Lambda)}$ which implies the assumption is wrong. Hence, the theorem. ■

Theorem 2.4.

Every N_sSec subset of a N_sSeCOS is N_sSeCos.

Proof:

Let (U,τ,Λ) be a N_sSeCOS and N_sS sets family $\vartheta=\{(\tilde{\Phi^*},\Lambda)_t\}_{t\in I}$ be a N_sSeoc of $(\tilde{\Phi^*},\Lambda)$. In this case, $(\tilde{\Phi^*},\Lambda)\subseteq\bigcup\limits_{t\in I}(\tilde{\Phi^*},\Lambda)_t$ and $(\tilde{\Phi^*},\Lambda)$ is N_sSecs. Hence, the set $1_{(U,\Lambda)}\setminus(\tilde{\Phi^*},\Lambda)$ is N_sSeos. On the other hand, it can also be written as, there exist $(\tilde{\Phi^*},\Lambda)_{i1},(\tilde{\Phi^*},\Lambda)_{i2},...,(\tilde{\Phi^*},\Lambda)_{in}$ such that $\big(\bigcup\limits_{j=1}^{n}(\tilde{\Phi^*},\Lambda)_{tj}\big)\bigcup\big(1_{(U,\Lambda)}\setminus(\tilde{\Phi^*},\Lambda)_{tj}\big)=1_{(U,\Lambda)}$. Hence, we obtain $(\tilde{\Phi^*},\Lambda)\subseteq\bigcup\limits_{j=1}^{n}(\tilde{\Phi^*},\Lambda)_{tj}$. Then, $(\tilde{\Phi^*},\Lambda)$ is N_sSeCos from Theorem 2.1. ■

Corollary 2.1.

Let $(\tilde{\Phi^*},\Lambda)_{i1},(\tilde{\Phi^*},\Lambda)_{i2},...,(\tilde{\Phi^*},\Lambda)_{in}$ be the family of N_sSecs's of (U,τ,Λ). Then

$(\tilde{\Phi^*}, \Lambda) \subseteq \bigcup_{j=1}^{n} (\tilde{\Phi^*}, \Lambda)_t$ is a N_sSeCos if and only if $\forall\, t = 1,\ 2, \ldots,\ n$, $(\tilde{\Phi^*}, \Lambda)_t$ is a N_sSeCos.

Theorem 2.5.

Every N_sSeCos in a N_sSe-Hausdorff topological space is a N_sSecs.

Proof:

Let (U, τ, Λ) be a N_sSe-Hausdorff space and the N_sS $(\tilde{\Phi^*}, \Lambda)$ on U be a N_sSeCos. To prove that it is e-closed, it is enough to prove that the set $1_{(U,\Lambda)} \setminus (\tilde{\Phi^*}, \Lambda)$ is e-open. ${}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in 1_{(U,\Lambda)} \setminus (\tilde{\Phi^*}, \Lambda)$ is not equal to any $\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')} \in (\tilde{\Phi^*}, \Lambda)$. Then, since (U, τ, Λ) is a N_sSe-Hausdorff space, for N_sSp's ${}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \neq \psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')} \in 1_{(U,\Lambda)}$, there exist $(\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)},\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')}}, (\tilde{B}, \Lambda)_{\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')},{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \in \tau$ such that ${}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)},\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')}}, \psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')} \in (\tilde{B}, \Lambda)_{\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')},{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$ and $(\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)},\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')}} \cap (\tilde{B}, \Lambda)_{\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')},{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} = 1_{(U,\Lambda)}$.

If $\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')}$ walk in all the N_sSs's $(\tilde{\Phi^*}, \Lambda)$, the family $\{(\tilde{B}, \Lambda)_{\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')},{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}\}_{\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')} \in (\tilde{\Phi^*},\Lambda)}$ of $(\tilde{\Phi^*}, \Lambda)$ is N_sSeoc at $1_{(U,\Lambda)}$. Since $(\tilde{\Phi^*}, \Lambda)$ is a N_sSeCos, there exist $(\tilde{B}, \Lambda)_{{}^1\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')},{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}, \ldots, (\tilde{B}, \Lambda)_{{}^n\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')},{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$ such that that $(\tilde{\Phi^*}, \Lambda) \subseteq \bigcup_{t=1}^{n} (\tilde{B}, \Lambda)_{{}^i\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')},{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$.

Consider the N_sSe-*nbd*'s $(\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)},{}^i\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')}}$, $t = 1,\ 2, \ldots,\ n$ that provide the condition $(\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)},{}^i\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')}} \cap (\tilde{B}, \Lambda)_{{}^i\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')},{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} = 0_{(U,\Lambda)}$ of ${}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}$ corresponding to the sets $(\tilde{B}, \Lambda)_{{}^i\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')},{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$, $t = 1, 2..., n$.

The set $(\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} = \bigcap_{t=1}^{n} (\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)},{}^i\psi^{\theta'}_{(\mathfrak{O}_1{}',\mathfrak{O}_2{}',\mathfrak{O}_3{}')}}$ is a N_sSe-open neighbourhood of ${}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}$ and

$$
\begin{aligned}
&(\tilde{\Phi^*}, \Lambda) \cap (\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \\
&\qquad = (\tilde{\Phi^*}, \Lambda) \cap \Big(\bigcap_{t=1}^{n} (\tilde{A}, \theta)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}, {}^i\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}} \Big) \\
&\qquad \subseteq \Big(\bigcup_{t=1}^{n} (\tilde{B}, \Lambda)_{{}^i\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}, {}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \Big) \cap \Big(\bigcap_{t=1}^{n} (\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}, {}^i\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}} \Big) \\
&\qquad = \bigcup_{t=1}^{n} \Big((\tilde{B}, \Lambda)_{{}^i\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}, {}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \cap \bigcap_{t=1}^{n} (\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}, {}^i\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}} \Big) \\
&\qquad \subseteq \bigcup_{t=1}^{n} \Big((\tilde{B}, \Lambda)_{{}^i\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}, {}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \cap (\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}, {}^i\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}} \Big) \\
&\qquad = 0_{(U,\Lambda)}.
\end{aligned}
$$

Here, ${}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \subseteq 1_{(U,\Lambda)} \setminus (\tilde{\Phi^*}, \Lambda)$ is obtained. Hence, $1_{(U,\Lambda)} \setminus (\tilde{\Phi^*}, \Lambda)$ is N_sSeos and so $(\tilde{\Phi^*}, \Lambda)$ is N_sSecs. ∎

Theorem 2.6.

Every N_sSe-compact Hausdorff space is N_sSe-normal space.

Proof:

Let (U, τ, Λ) be a N_sSe-compact Hausdorff space and $(\tilde{\Phi^*}_{11}, \Lambda)$ & $(\tilde{\Phi^*}_{22}, \Lambda)$ be two distinct N_sSecs's. From Theorem 2.4, these sets are N_sSeCos's. For each $\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{\Phi^*}_{11}, \Lambda)$ and $\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')} \in (\tilde{\Phi^*}_{22}, \Lambda)$, $\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \neq \psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}$. Since (U, τ, Λ) is a N_sSe-Hausdorff space, there exist

$$(\tilde{A}, \Lambda)_{\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}, \psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}}, (\tilde{B}, \Lambda)_{\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}, \phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \in \tau$$

such that

$$\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{A}, \Lambda)_{\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}, \psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}}, \ \psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')} \in (\tilde{B}, \Lambda)_{\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}, \phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$$

and $(\tilde{A}, \Lambda)_{\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}, \psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}} \cap (\tilde{B}, \Lambda)_{\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}, \phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} = 0_{(U,\Lambda)}$ can be written for $\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \neq \psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')} \in 1_{(U,\Lambda)}$. Let ${}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{\Phi^*}_{11}, \Lambda)$ be constant.

Then, for each $\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')} \in (\tilde{\Phi^*}_{22}, \Lambda)$ and ${}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{\Phi^*}_{11}, \Lambda)$, there exist $(\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}, \psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}}, (\tilde{B}, \Lambda)_{\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}, {}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \in \tau$ such that

${}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}, \psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}}$, $\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')} \in (\tilde{B}, \Lambda)_{\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}, {}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$ and

$(\tilde{A}, \Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}, \psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}} \cap (\tilde{B}, \Lambda)_{\psi^{\theta'}_{({\mathfrak{O}_1}',{\mathfrak{O}_2}',{\mathfrak{O}_3}')}, {}^0\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} = 0_{(U,\Lambda)}$. It is clear that the family $\{(\tilde{B},\Lambda)_{\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')},{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}\}_{\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')}\in(\tilde{\Phi^*}_{22},\Lambda)}$ of $(\tilde{\Phi^*}_{22},\Lambda)$ is N_sSeoc at $1_{(U,\Lambda)}$. Since $(\tilde{\Phi^*}_{22},\Lambda)$ is a N_sSeCos, there exist

$$(\tilde{B},\Lambda)_{{}^1\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')},{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}},...,(\tilde{B},\Lambda)_{{}^n\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')},{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}$$

such that $(\tilde{\Phi^*}_{22},\Lambda)\subseteq\bigcup_{t=1}^{n}(\tilde{B},\Lambda)_{{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')},{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}$.

Consider the N_sSe-*nbd*'s $(\tilde{A},\Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')}}$, $t=1,\ 2,\ldots,\ n$ that provide the condition $(\tilde{A},\Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')}}\cap(\tilde{B},\Lambda)_{{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')},{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}=0_{(U,\Lambda)}$ of ${}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}$ corresponding to the sets $(\tilde{B},\Lambda)_{{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')},{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}$, $t=1,2...,n$.

Thus, the sets

$$(\tilde{A},\Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},(\tilde{\Phi^*}_{22},\Lambda)}=\bigcap_{t=1}^{n}(\tilde{A},\Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')}}$$

and

$$(\tilde{B},\Lambda)_{(\tilde{\Phi^*}_{22},\Lambda),{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}=\bigcup_{t=1}^{n}(\tilde{B},\Lambda)_{{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')},{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}$$

are N_sSe-open neighbourhoods of ${}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}$ and $(\tilde{\Phi^*}_{22},\Lambda)$, respectively.

Hence,

$$\begin{aligned}
&(\tilde{A},\Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},(\tilde{\Phi^*}_{22},\Lambda)}\cap(\tilde{B},\Lambda)_{(\tilde{\Phi^*}_{22},\Lambda),{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}\\
&=\Big(\bigcap_{t=1}^{n}(\tilde{A},\Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')}}\Big)\cap\Big(\bigcup_{t=1}^{n}(\tilde{B},\Lambda)_{{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')},{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}\Big)\\
&=\bigcup_{t=1}^{n}\Big(\Big(\bigcap_{t=1}^{n}(\tilde{A},\Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')}}\Big)\cap(\tilde{B},\Lambda)_{{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')},{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}\Big)\\
&\subseteq\bigcup_{t=1}^{n}\Big((\tilde{A},\Lambda)_{{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')}}\cap(\tilde{B},\Lambda)_{{}^i\psi^{\theta'}_{(\mathfrak{D}_1',\mathfrak{D}_2',\mathfrak{D}_3')},{}^0\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}\Big)\\
&=0_{(U,\Lambda)}.
\end{aligned}$$

Thus, each $\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}$ and $(\tilde{\Phi^*}_{22},\Lambda)$ have N_sSe-*nbd*'s so that for each $\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}\in(\tilde{\Phi^*}_{11},\Lambda)$ and $(\tilde{\Phi^*}_{22},\Lambda)$, $(\tilde{A},\Lambda)_{\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},(\tilde{\Phi^*}_{22},\Lambda)}\cap(\tilde{B},\Lambda)_{(\tilde{\Phi^*}_{22},\Lambda),\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}=0_{(U,\Lambda)}$ is provided. Hence, the family $\{(\tilde{A},\Lambda)_{\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},(\tilde{\Phi^*}_{22},\Lambda)}\}_{y(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)\in(\tilde{\Phi^*}_{11},\Lambda)}$ of $(\tilde{\Phi^*}_{11},\Lambda)$ has a N_sSeoc in $1_{(U,\Lambda)}$. Since $(\tilde{\Phi^*}_{11},\Lambda)$ is N_sSeCos, there exist $(\tilde{A},\Lambda)_{{}^1\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},(\tilde{\Phi^*}_{22},\Lambda)},...,(\tilde{A},\Lambda)_{{}^k\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},(\tilde{\Phi^*}_{22},\Lambda)}$ such that

$$(\tilde{\Phi^*}_{11},\Lambda)\subseteq\bigcup_{t=1}^{k}(\tilde{A},\Lambda)_{{}^i\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},(\tilde{\Phi^*}_{22},\Lambda)},\ (\tilde{\Phi^*}_{22},\Lambda)\subseteq(\tilde{B},\Lambda)_{(\tilde{\Phi^*}_{22},\Lambda),{}^i\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}},\ t=1,2...,k,$$

and

$$(\tilde{A},\Lambda)_{{}^i\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)},(\tilde{\Phi^*}_{22},\Lambda)}\cap(\tilde{B},\Lambda)_{(\tilde{\Phi^*}_{22},\Lambda),{}^i\phi^{\theta}_{(\mathfrak{D}_1,\mathfrak{D}_2,\mathfrak{D}_3)}}=0_{(U,\Lambda)}.$$

Thus, the N_sSeos's

$$(\tilde{A},\Lambda)_{(\tilde{\Phi^*}_{11},\Lambda)} = \bigcup_{t=1}^{k} (\tilde{A},\Lambda)_{{}^i\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)},(\tilde{\Phi^*}_{22},\Lambda)},$$

and

$$(\tilde{B},\Lambda)_{(\tilde{\Phi^*}_{22},\Lambda)} = \bigcap_{t=1}^{k} (\tilde{B},\Lambda)_{(\tilde{\Phi^*}_{22},\Lambda),{}^i\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}},$$

contain the sets $(\tilde{\Phi^*}_{11},\Lambda)$ and $(\tilde{\Phi^*}_{22},\Lambda)$, respectively. Also,

$$\begin{aligned}
(\tilde{A},\Lambda)_{(\tilde{\Phi^*}_{11},\Lambda)} \cap (\tilde{B},\Lambda)_{(\tilde{\Phi^*}_{22},\Lambda)} &= \Big(\bigcup_{t=1}^{k}(\tilde{A},\Lambda)_{{}^i\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)},(\tilde{\Phi^*}_{22},\Lambda)}\Big) \cap \Big(\bigcap_{t=1}^{k}(\tilde{B},\Lambda)_{(\tilde{\Phi^*}_{22},\Lambda),{}^i\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}\Big)\\
&= \bigcup_{t=1}^{k}\Big((\tilde{A},\Lambda)_{{}^i\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)},(\tilde{\Phi^*}_{22},\Lambda)} \cap \big(\bigcap_{t=1}^{k}(\tilde{B},\Lambda)_{(\tilde{\Phi^*}_{22},\Lambda),{}^i\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}\big)\Big)\\
&\subseteq \bigcup_{t=1}^{n}\Big((\tilde{A},\Lambda)_{{}^i\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)},(\tilde{\Phi^*}_{22},\Lambda)} \cap (\tilde{B},\Lambda)_{(\tilde{\Phi^*}_{22},\Lambda),{}^i\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}\Big)\\
&= 0_{(U,\Lambda)}.
\end{aligned}$$

Thus, the proof is completed. ■

3. Neutrosophic soft locally e-compact spaces

In this section, N_sS locally e-compactness is defined which is weaker than N_sSe-compactness.

Definition 3.1.

Let (U,τ,Λ) be a N_sSts over U. (U,τ,Λ) is said to be a N_sS locally e-compact or locally countable e-compact space (in short, $N_sSleCOS$) if for each $\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in N_sS(U,\theta)$, there exists a N_sSe-nbd $(\tilde{A},\Lambda) \in \tau$ of $\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}$ with $N_sSecl(\tilde{A},\Lambda)$ is N_sSe-compact.

When (U,τ,Λ) is also N_sSe-Hausdorff, the property of N_sSe-local compactness becomes much stronger as stated below.

Theorem 3.1.

Let (U,τ,Λ) be a $N_sSleCOS$ and N_sSe-Hausdorff space. If $(\tilde{\Phi^*},\Lambda)$ is a N_sSeCos in U and $(\tilde{\Phi^*},\Lambda) \subseteq (\tilde{B},\Lambda)$, $(\tilde{B},\Lambda) \in \tau$. Then, there exists $(\tilde{A},\Lambda) \in \tau \ni$ the $N_sSecl(\tilde{A},\Lambda)$ is N_sSe-compact and $(\tilde{\Phi^*},\Lambda) \subseteq (\tilde{A},\Lambda) \subseteq N_sSecl(\tilde{A},\Lambda) \subseteq (\tilde{B},\Lambda)$.

Proof:

As (U,τ,Λ) is a N_sSe-Hausdorff space, there exists a N_sSe-nbd for each $\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{\Phi^*},\Lambda) \subseteq (\tilde{B},\Lambda)$ such that $\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{B},\Lambda)_{\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \subseteq N_sSecl(\tilde{B},\Lambda)_{\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \subseteq (\tilde{B},\Lambda)$. Since (U,τ,Λ) is $N_sSleCOS$, each $\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{\Phi^*},\Lambda)$ has a $N_sSecl(\tilde{W},\theta)_{\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$ which is N_sSe-compact neighbourhood. Therefore, for N_sSe-nbd $(\tilde{A},\Lambda)_{\phi^\theta_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} =$

$(\tilde{B},\Lambda)_{\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \cap (\tilde{W},\theta)_{\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$ of $\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}$, $ecl(\tilde{A},\Lambda)_{\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$ is N_sSe-compact since $N_sSecl(\tilde{A},\Lambda)_{\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}} \subseteq N_sSecl(\tilde{W},\theta)_{\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$. Since $(\tilde{\Phi^*},\Lambda)$ is N_sSeCos, the N_sSe-covering of $(\tilde{\Phi^*},\Lambda)$, $\{(\tilde{A},\Lambda)_{\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}\}_{\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}\in(\tilde{\Phi^*},\Lambda)}$ has a finite N_sS subcover $(\tilde{A},\Lambda)_{^1\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}, ..., (\tilde{A},\Lambda)_{^n\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$.

Here, for N_sSeos $(\tilde{A},\Lambda) = \bigcup_{t=1}^{n} (\tilde{A},\Lambda)_{^i\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}$,

$$\begin{aligned}(\tilde{\Phi^*},\Lambda) \subseteq (\tilde{A},\Lambda) &\subseteq ecl(\tilde{A},\Lambda)\\ &= \bigcup_{t=1}^{n} ecl(\tilde{A},\Lambda)_{^i\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}\\ &\subseteq \bigcup_{t=1}^{n} ecl(\tilde{B},\Lambda)_{^i\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}}\\ &\subseteq (\tilde{B},\Lambda).\end{aligned}$$

■

Theorem 3.2.

Let (U,τ,Λ) be a $N_sSleCOS$ and N_sSe-Hausdorff space. If $(\tilde{\Phi^*},\Lambda)$ is a N_sSecs (N_sSeos) in U, then $\big((\tilde{\Phi^*},\Lambda),\tau_{(\tilde{\Phi^*},\Lambda)},\Lambda\big)$ is $N_sSleCOS$.

Proof:

Let (U,τ,Λ) be a $N_sSleCOS$ and N_sSe-Hausdorff space. Let $(\tilde{\Phi^*},\Lambda)$ be a N_sSecs in U. Consider a N_sSe-compact neighbourhood $N_sSecl(\tilde{A},\Lambda)$ of each $\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{\Phi^*},\Lambda)$ in (U,τ,Λ). For N_sSe-open neighbourhood $(\tilde{A},\Lambda)\cap(\tilde{\Phi^*},\Lambda)$ of $\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}$ in $\big((\tilde{\Phi^*},\Lambda),\tau_{(\tilde{\Phi^*},\Lambda)},\theta\big)$, $N_sSecl\big((\tilde{A},\Lambda)\cap(\tilde{\Phi^*},\Lambda)\big)$ is N_sSeCos because $N_sSecl\big((\tilde{A},\Lambda)\cap(\tilde{\Phi^*},\Lambda)\big) = N_sSecl(\tilde{A},\Lambda)\cap(\tilde{\Phi^*},\Lambda) \subseteq N_sSecl(\tilde{A},\Lambda)$. Therefore, $\big((\tilde{\Phi^*},\Lambda),\tau_{(\tilde{\Phi^*},\Lambda)},\theta\big)$ is a $N_sSleCOS$. When the theorem is proved for N_sSecs, (U,τ,Λ) need not to be a N_sSe-Hausdorff space.

Now, consider that $(\tilde{\Phi^*},\Lambda)$ is a N_sSeos in U. As the single-point set in the N_sSe-Hausdorff space is N_sSecs, there exists $(\tilde{A},\Lambda)\in\tau$ such that $\{\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}\} \subseteq (\tilde{A},\Lambda) \subseteq N_sSecl(\tilde{A},\Lambda) \subseteq (\tilde{\Phi^*},\Lambda)$, $(\tilde{A},\Lambda)$ is N_sSeCos for each $\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{\Phi^*},\Lambda)$ by Theorem 3.1. Therefore, $(\tilde{A},\Lambda)\cap(\tilde{\Phi^*},\Lambda)$ in $\big((\tilde{\Phi^*},\Lambda),\tau_{(\tilde{\Phi^*},\Lambda)},\theta\big)$ is a N_sSe-nbd of $\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)}$ and $N_sSecl\big((\tilde{A},\Lambda)\cap(\tilde{\Phi^*},\Lambda)\big) = N_sSecl(\tilde{A},\Lambda)\cap(\tilde{\Phi^*},\Lambda) \subseteq N_sSecl(\tilde{A},\Lambda)$ is N_sSeCos. Hence, $\big((\tilde{\Phi^*},\Lambda),\tau_{(\tilde{\Phi^*},\Lambda)},\theta\big)$ is a $N_sSleCOS$. ■

Theorem 3.3.

If (U,τ,Λ) is a N_sSe-Hausdorff space and $(\tilde{\Phi^*},\Lambda)$ is a N_sS locally e-compact subspace in U, then $(\tilde{\Phi^*},\Lambda)$ is a N_sSeos in $N_sSecl(\tilde{\Phi^*},\Lambda)$ and $(\tilde{\Phi^*},\Lambda)$ can be written as $(\tilde{\Phi^*},\Lambda) = (\tilde{A},\Lambda)\cap(\tilde{B},\Lambda)$, where $(\tilde{A},\Lambda)$ and $(\tilde{B},\Lambda)$ are N_sSecs and N_sSeos in (U,τ,Λ) respectively.

Proof:

As $\big((\tilde{\Phi^*}, \Lambda), \tau_{(\tilde{\Phi^*},\Lambda)}, \Lambda\big)$ is $N_sSleCOS$, a N_sSe-nbd $(\tilde{P}, \Lambda) \in \tau_{(\tilde{\Phi^*},\Lambda)}$ of $\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{\Phi^*}, \Lambda)$ can be obtained such that $N_sSecl(\tilde{P}, \Lambda) \cap (\tilde{\Phi^*}, \Lambda)$ is N_sSe-compact in $(\tilde{\Phi^*}, \Lambda)$. Since, (U, τ, Λ) is a N_sSe-Hausdorff space, $ecl(\tilde{P}, \Lambda) \cap (\tilde{\Phi^*}, \Lambda)$ is N_sSecs in (U, τ, Λ). For $(\tilde{P}, \Lambda) \subseteq N_sSecl(\tilde{P}, \Lambda) \cap (\tilde{\Phi^*}, \Lambda)$, we have $N_sSecl(\tilde{P}, \Lambda) \subseteq N_sSecl(\tilde{P}, \Lambda) \cap (\tilde{\Phi^*}, \Lambda) \subseteq (\tilde{\Phi^*}, \Lambda)$. We get, $(\tilde{P}, \Lambda) = (\tilde{\Phi^*}, \Lambda) \cap (\tilde{W}, \theta)$, $(\tilde{W}, \theta) \in \tau$ for $(\tilde{P}, \Lambda) \in \tau_{(\tilde{\Phi^*},\Lambda)}$. Hence, we consider that $(\tilde{\Phi^*}, \Lambda)$ is N_sS dense set in $N_sSecl(\tilde{\Phi^*}, \Lambda)$. Then, $\phi^{\theta}_{(\mathfrak{O}_1,\mathfrak{O}_2,\mathfrak{O}_3)} \in (\tilde{W}, \theta) \subseteq N_sSecl(\tilde{W}, \theta) = N_sSecl\big((\tilde{\Phi^*}, \Lambda) \cap (\tilde{W}, \theta)\big) = N_sSecl(\tilde{P}, \Lambda) \subseteq (\tilde{\Phi^*}, \Lambda)$. Therefore, $(\tilde{\Phi^*}, \Lambda)$ is N_sSeos. Hence, $N_sSecl(\tilde{\Phi^*}, \Lambda) = N_sSecl(\tilde{\Phi^*}, \Lambda) \cap (\tilde{\Phi^*}, \Lambda) = (\tilde{A}, \Lambda) \cap (\tilde{B}, \Lambda)$. ■

4. Application using Entropy Measure

In this section, an application of the N_sSs theory is given in a decision making problem using Entropy which is used for measuring uncertain information. If the evaluation has less uncertainty, then there is larger possibility to select that evaluation in order to get optimal.

Example 4.1.

Consider U as a set of 3 mobile phone models which is denoted by $U = \{\phi_{11}, \phi_{22}, \phi_{33}\}$. There are 3 customers A, B and C who have to make the decision on which mobile phone can be purchased based on their evaluation. Let Λ be the set of 5 parameters $\Lambda = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\}$ = {Price, Look, Memory capacity, Camera, Processor}. Based on the mobile phone reviews by the experts in the website, the N_sSs's $(\tilde{\Phi^*}_{11}, \Lambda)$, $(\tilde{\Phi^*}_{22}, \Lambda)$ and $(\tilde{\Phi^*}_{33}, \Lambda)$ describe the evaluation of the customers A, B and C, respectively.

$$(\tilde{\Phi^*_{11}}, \Lambda) = \left\{\begin{array}{l} \theta_1 = \langle\phi_{11}, (0.6, 0.4, 0.8)\rangle, \langle\phi_{22}, (0.5, 0.2, 0.4)\rangle, \langle\phi_{33}, (0.2, 0.5, 0.2)\rangle \\ \theta_2 = \langle\phi_{11}, (0.4, 0.9, 0.3)\rangle, \langle\phi_{22}, (0.3, 0.5, 0.6)\rangle, \langle\phi_{33}, (0.7, 0.2, 0.5)\rangle \\ \theta_3 = \langle\phi_{11}, (0.4, 0.6, 0.7)\rangle, \langle\phi_{22}, (0.5, 0.5, 0.8)\rangle, \langle\phi_{33}, (0.6, 0.4, 0.7)\rangle \\ \theta_4 = \langle\phi_{11}, (0.8, 0.2, 0.4)\rangle, \langle\phi_{22}, (0.6, 0.1, 0.5)\rangle, \langle\phi_{33}, (0.5, 0.3, 0.6)\rangle \\ \theta_5 = \langle\phi_{11}, (0.5, 0.8, 0.1)\rangle, \langle\phi_{22}, (0.4, 0.5, 0.5)\rangle, \langle\phi_{33}, (0.7, 0.4, 0.6)\rangle \end{array}\right\},$$

$$(\tilde{\Phi^*_{22}}, \Lambda) = \left\{\begin{array}{l} \theta_1 = \langle\phi_{11}, (0.2, 0.5, 0.8)\rangle, \langle\phi_{22}, (0.4, 0.4, 0.6)\rangle, \langle\phi_{33}, (0.3, 0.5, 0.7)\rangle \\ \theta_2 = \langle\phi_{11}, (0.9, 0.4, 0.2)\rangle, \langle\phi_{22}, (0.7, 0.6, 0.3)\rangle, \langle\phi_{33}, (0.8, 0.2, 0.4)\rangle \\ \theta_3 = \langle\phi_{11}, (0.6, 0.5, 0.3)\rangle, \langle\phi_{22}, (0.5, 0.6, 0.4)\rangle, \langle\phi_{33}, (0.7, 0.4, 0.5)\rangle \\ \theta_4 = \langle\phi_{11}, (0.7, 0.2, 0.6)\rangle, \langle\phi_{22}, (0.4, 0.5, 0.7)\rangle, \langle\phi_{33}, (0.6, 0.2, 0.5)\rangle \\ \theta_5 = \langle\phi_{11}, (0.2, 0.6, 0.7)\rangle, \langle\phi_{22}, (0.3, 0.7, 0.8)\rangle, \langle\phi_{33}, (0.5, 0.6, 0.6)\rangle \end{array}\right\},$$

$$(\tilde{\Phi^*_{33}}, \Lambda) = \left\{\begin{array}{l} \theta_1 = \langle\phi_{11}, (0.7, 0.2, 0.5)\rangle, \langle\phi_{22}, (0.8, 0.3, 0.6)\rangle, \langle\phi_{33}, (0.6, 0.4, 0.5)\rangle \\ \theta_2 = \langle\phi_{11}, (0.8, 0.3, 0.4)\rangle, \langle\phi_{22}, (0.9, 0.1, 0.5)\rangle, \langle\phi_{33}, (0.5, 0.7, 0.3)\rangle \\ \theta_3 = \langle\phi_{11}, (0.5, 0.6, 0.2)\rangle, \langle\phi_{22}, (0.6, 0.3, 0.8)\rangle, \langle\phi_{33}, (0.7, 0.6, 0.1)\rangle \\ \theta_4 = \langle\phi_{11}, (0.2, 0.4, 0.8)\rangle, \langle\phi_{22}, (0.4, 0.6, 0.4)\rangle, \langle\phi_{33}, (0.3, 0.7, 0.5)\rangle \\ \theta_5 = \langle\phi_{11}, (0.5, 0.7, 0.3)\rangle, \langle\phi_{22}, (0.7, 0.5, 0.2)\rangle, \langle\phi_{33}, (0.3, 0.7, 0.4)\rangle \end{array}\right\}.$$

Using the entropy definition, we get

$\mathfrak{h}_1(\tilde{\Phi^*}_{11}, \Lambda) = 0.2255$, $\mathfrak{h}_2(\tilde{\Phi^*}_{11}, \Lambda) = 0.1624$, $\mathfrak{h}_3(\tilde{\Phi^*}_{11}, \Lambda) = 0.2393$, $\mathfrak{h}_4(\tilde{\Phi^*}_{11}, \Lambda) = 0.1515$,

$\mathfrak{h}_5(\tilde{\Phi^*}_{11}, \Lambda) = 0.2037.$

$\mathfrak{h}_1(\tilde{\Phi^*}_{22}, \Lambda) = 0.2072$, $\mathfrak{h}_2(\tilde{\Phi^*}_{22}, \Lambda) = 0.1439$, $\mathfrak{h}_3(\tilde{\Phi^*}_{22}, \Lambda) = 0.2381$, $\mathfrak{h}_4(\tilde{\Phi^*}_{22}, \Lambda) = 0.1951$, $\mathfrak{h}_5(\tilde{\Phi^*}_{22}, \Lambda) = 0.175.$

$\mathfrak{h}_1(\tilde{\Phi^*}_{33}, \Lambda) = 0.1984$, $\mathfrak{h}_2(\tilde{\Phi^*}_{33}, \Lambda) = 0.1407$, $\mathfrak{h}_3(\tilde{\Phi^*}_{33}, \Lambda) = 0.1709$, $\mathfrak{h}_4(\tilde{\Phi^*}_{33}, \Lambda) = 0.1852$, $\mathfrak{h}_5(\tilde{\Phi^*}_{33}, \Lambda) = 0.1810.$

Then, we have $\mathfrak{h}(\tilde{\Phi^*}_{11}, \Lambda) = 0.1965$, $\mathfrak{h}(\tilde{\Phi^*}_{22}, \Lambda) = 0.1919$ and $\mathfrak{h}(\tilde{\Phi^*}_{33}, \Lambda) = 0.1752.$

Hence, $\mathfrak{h}(\tilde{\Phi^*}_{33}, \Lambda) < \mathfrak{h}(\tilde{\Phi^*}_{22}, \Lambda) < \mathfrak{h}(\tilde{\Phi^*}_{11}, \Lambda).$

Therefore, the customer C has the larger possibility to make the correct decision than the customers A and B. Hence the customer C has the larger possibility to buy the better model mobile phone than the customers A and B.

5. Conclusion

In this article, neutrosophic soft e-compactness is introduced and its basic properties are discussed. We have also defined and studied neutrosophic soft locally e-compact spaces. We further investigated relationship between neutrosophic soft e-compactness and e-separation axioms. Also, an application in decision making problem is given using entropy measure which can be used for measuring uncertain information. In future, this work can be developed to neutrosophic soft e-connectedness, neutrosophic soft contra e-continuous functions and neutrosophic soft contra e-open maps.

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