Improving Sonic Rarefactions in Elastic Vessels: Application to the Tourniquet Manoeuvre

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Summary

Elastic vessels like arteries and specially veins are prone to sharp changes in their area or external pressure. These phenomena create discontinuities due to a sudden jump on mechanical properties, which is a challenge when trying to simulate unsteady blood flow circulation. This work compares numerical methods to face this challenge in the context of a finite volume model.

Introduction

Blood flow in the human circulatory system is extremely difficult to simulate in detail. Its pulsatile nature causes transitory states to be more present and the elasticity of vessels introduces another complication. The mechanics of veins in particular is highly nonlinear in their pressure-area relation, making their precise numerical resolution an open problem. Moreover, while flow in arteries is mainly driven by the pumping of the heart, flow in veins is gravity-driven and thus, very susceptible to outside effects such as postural changes, which makes this problem very relevant during surgeries [1].

In the event of a vein collapse, which can ~~be~~ easily occur during surgery, the flow is affected by a sudden increase in pressure whose magnitude can provoke a sonic flow change. Indeed, supersonic shock waves in blood flow with harmful effect over surgery patients have been recorded but their numerical modelling is still very lacklustre.

The~~ basic~~ model used to simulate vessel flow is a 1D approach [2] in terms of conserved variables (*U*), their fluxes (*F*) and geometric source terms (*S*), leading to non-linear, hyperbolic~~s~~ system~~s~~ of equations.

$$\partial_t \mathbf{U} + \partial_x \mathbf{F}(\mathbf{U}) = \mathbf{S} \qquad (1)$$

Using

$$\mathbf{U} = \begin{bmatrix} A \\ Q \end{bmatrix} \qquad \mathbf{F}(\mathbf{U}) = \begin{bmatrix} Q \\ k\frac{Q^2}{A} \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} 0 \\ \bar{s} \end{bmatrix} \qquad (2)$$

Numerical methods

These equations are solved numerically using Godunov's method, in which the spatial 1D domain is divided in cells and the numerical flux is calculated using Roe's approximation [3]. This method works well capturing shock waves but not rarefaction waves, which can only be represented as a succession of small shocks. Moreover, if the rarefaction is such that the flow becomes supersonic at some point inside of it, the usual Roe method fails to capture it correctly. It is at this point that an entropy correction is necessary. Although there are many studies about this topic and the most common entropy corrections were developed in the 1980s [4], the inclusion of a source term like the external vessel's wall pressure offers a new challenge.

Another added complication is the fact that the source term *S* has a complicated expression with dependence on the cross-sectional vessel area *A*, the pressure gradient, and gravitational forces. Additionally, a pressure-area relation closes the system, but in veins this expression is non-linear. In order to salvage this complication, time-linearization is applied, and the source term is approximated by a constant value $\bar{s}$ in each timestep. Energy conservation is imposed to find the linearized source term value. However, this approximation needs monitoring because, of course, the vein's cross-section area cannot be smaller than zero and therefore new corrections are needed to avoid this numerical error.

This work implements this method for blood flow in veins with attention to detail and seeks to examine each correction's effect on the results. The numerical test cases selected include sonic rarefactions, which can occur during and after a tourniquet manoeuvre. Indeed, when a vein that was collapsed is released, the pressure source term stopping the flow decreases rapidly, possibly inducing supersonic acceleration on the blood.

The crucial point to the numerical correction is splitting the shock wave that is used to represent the rarefaction wave into two waves whose celerity is calculated using the source term value, which needs to be corrected. This means that both corrections feed on each other and therefore the order in which they are done is important. Two ways to proceed are shown. In the first one, the calculations are carried precisely (Method 1) while in the second one (Method 2), a more approximated approach is taken.

Conclusions

The numerical results of this work, of which some examples are shown in Figure 1 show that the numerical corrections introduced to an otherwise very popular method help decrease the numerical anomalies. Moreover, it is shown that applying the corrections in a more approximated manner might sometimes fail by giving a false positive sonic rarefaction, but otherwise works well, especially with certain choices of source term linearization.

Future lines of work

This work opens the door to more refined solutions in critical blood flow by further examination on the interactions between linearization of the source term and the treatment of rarefactions. In turn, a better treatment of these flows would allow to extend this kind of calculation to more complex systems, like vessel junctions, of which the cardiovascular system has many. Another natural follow up to this work is the calculation of the stress exerted on the venous walls by sonic flows.

References

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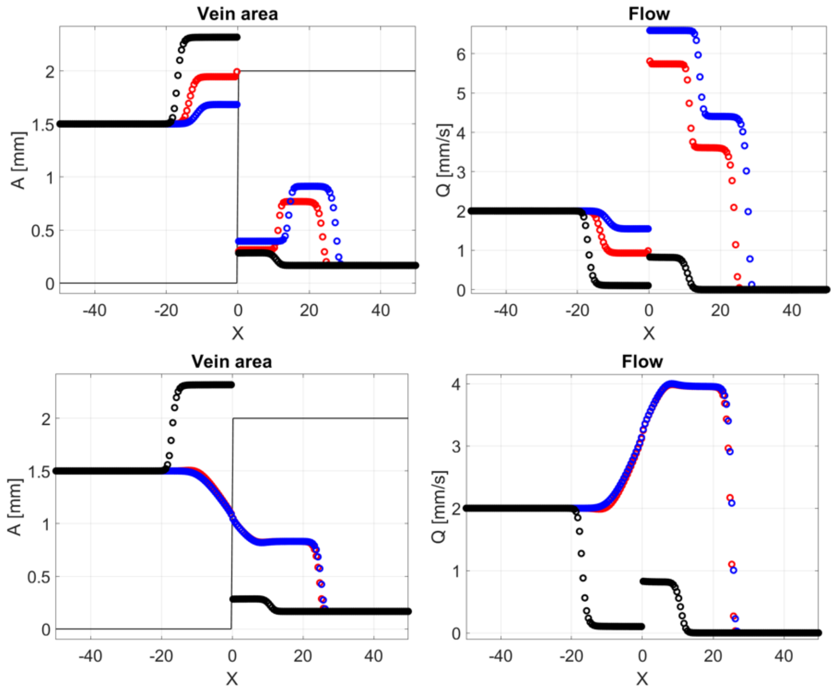

Figure 1: Example of a discontinuity of pressure (solid line) on a vein. Top graphs show results using the detailed correction (Method 1) while bottom graphs show results using the approximate correction (Method 2). Distinct colours show different source term linearization.