CASE STUDY

Engaging with Maths Online - teaching mathematics collaboratively and inclusively through a pandemic and beyond

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Abstract

This case study details several concrete approaches to integrating the use of student-loaned iPads in the teaching of mathematics in Higher Education. Although there is a scarcity of rigorous studies into the efficacy of tablet devices for improved educational outcomes, previous case studies have argued that tablet devices, if used, should be integrated into the whole learning experience. The mathematics teaching team at Middlesex University have developed an inclusive digital pedagogy over the last five years that enabled us to effectively respond to the remote teaching imposed by the COVID-19 pandemic by loaning iPads to all students on specialist mathematics programmes. As we begin the return to campus, we continue to integrate these devices into our teaching to address the observed "digital divide" in Generation-Z students which is characterised not by access to smart devices but by the digital skills to use them as effective learning tools. This is particularly relevant at Middlesex University which is disproportionately affected by digital poverty amongst its student population. We discuss the use of virtual whiteboard apps, the necessity of handwritten mathematics, the rich integration of multimedia content, persistent collaborative "problem solving spaces", and how a common hardware platform allows for varied and equitable inclusive assessment. We also report the results of students' surveys of iPad use during the remote-only 2020-21 academic year.

Keywords: Inclusive assessment, digital poverty, technology enhanced learning, iPad, virtual whiteboard.

1. Introduction and background

For several years mathematics lecturers have been using technology enhanced learning to develop the student learning experience on the specialist mathematics programmes at Middlesex University. The university invested in a number of tablets and styluses for use by staff for teaching and feedback in 2017-18. This meant that when the UK went into lockdown in March 2020 due to the global pandemic the teaching team seamlessly moved to online teaching having previously prepared much of our resources and assessment electronically (see Figure 1 for an example of pre-pandemic lecture delivered from a staff iPad). Like many other HEIs, issues around the motivation and attendance at online classes soon began to appear. Speaking with students, it soon became clear that one significant reason was digital poverty, a problem that disproportionately affects students at Middlesex University. Around 58% of Middlesex Students are from the most deprived areas (2019-20 Index of Multiple Deprivation quintiles 1-2) and although this geographic data doesn't necessarily reflect students' individual circumstances, the university does have the seventh highest proportion of undergraduates who were eligible for free school meals at Key Stage 4 across the whole English Higher Education sector (Office for Students – Access and Participation Data Dashboard), and the second highest proportion across non-private universities (see Figure 2). We found that many students were unable to access the internet from their homes or didn't have the requisite hardware to access online content.

It became clear that the sector's default Bring Your Own Device (BYOD) approach to online teaching was not suitable for our students. As a result, the university decided to expand its investment in tablets and styluses to provide hardware for all specialist mathematics students from the start of the 2020-21 academic year. A total of 32 devices were distributed at a cost of approximately £400 per device including accessories. The devices are expected to have a 5-year life-span.
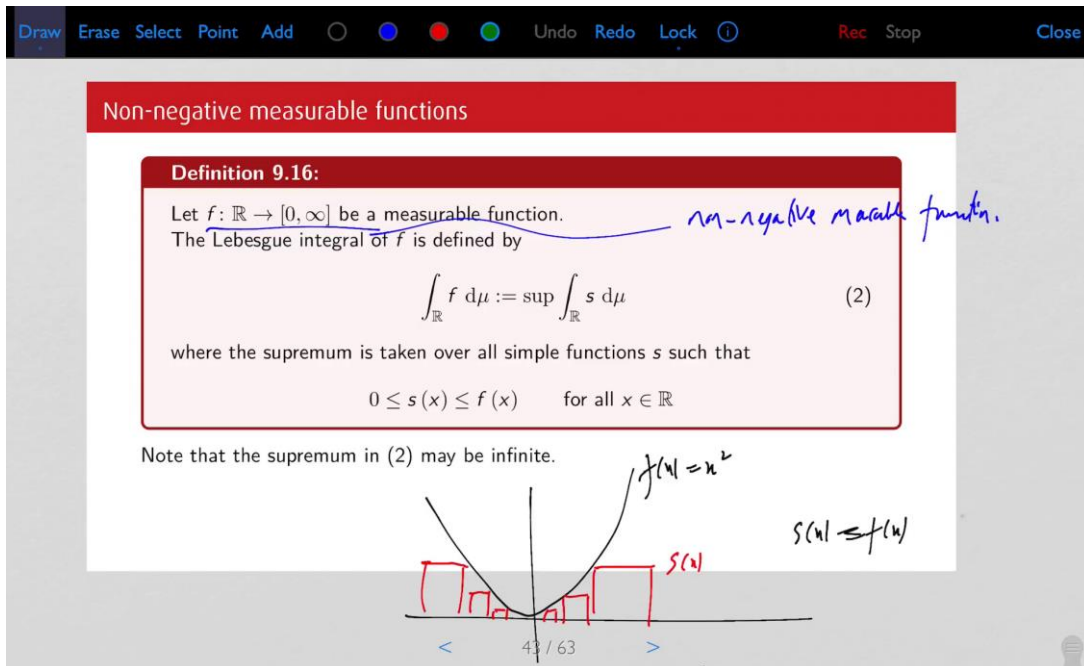

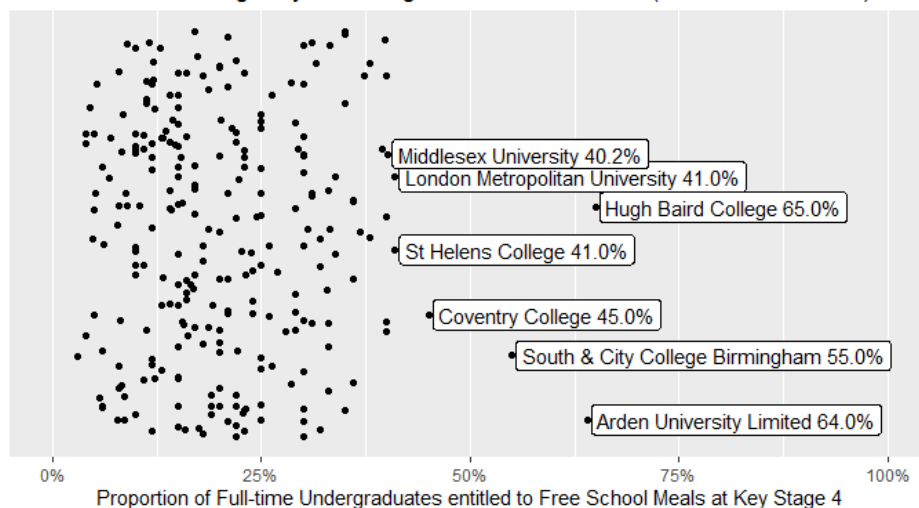Figure 1. Screenshot of iPad app "Vittle Pro" used to record and mark-up slides during lectures.

Figure 2. Proportion of full-time undergraduates eligible for Free School Meals at Key Stage 4 amongst 255 English HE providers. The vertical direction is jittered to prevent overplotting. Providers are labelled when proportion is greater than 40%. R code available at https://github.com/nicholassharples/OFSdata

In this paper we provide a case study for the approaches used in developing teaching, learning and assessment strategies based on the premise that students have a common hardware platform. Overwhelmingly we found that the use of the common hardware platform meant that we were able to leverage the wealth of software available that allowed us to develop a rich learning experience – in many cases improving on what was possible in a face-to-face setting, especially surrounding collaborative learning.

In the rest of this paper, we will describe and evaluate the novel pedagogic approaches to mathematics, enabled by the common hardware and software that the Middlesex University mathematics team have developed and will retain:

- **Courses are built around tablets to achieve true blended learning**
  Students use their tablets to work on collaborative, persistent whiteboards before, during, and after scheduled lecture times. All work can be completed, and feedback given, using the tablets to provide equality of access.

- **Collaborative "problem-solving spaces" create learning communities**
  Students contribute at times convenient for them, working with peers to build solutions to complex problems over an extended period.

- **Equitable access to technology allows for inclusive assessment**
  Students can choose their preferred submission format without disadvantaging others.

2. Discussion

Since the release of the iPad in 2010 the potential applications of these devices in education settings have been the subject of much qualitative work and case study. However, in a systematic literature review, Nguyen, Barton and Nguyen (2015) reported that studies were at an "early exploratory stage" and showed that "the iPad was found to enhance the learning experience but not necessarily lead to better learning outcomes" critically noting that "it is not clear how best to align and integrate it within the academic programmes". Haßler, Major and Hennessy (2016) describe a similar "scarcity of rigorous studies" in the primary and secondary education sectors. In a more recent systematic review, Svela et al. (2019) look specifically at the use of tablets in maths education but conclude that "almost eight years into this generation of tablet technology, the body of knowledge … is still limited".

Nevertheless, students are increasingly using digital devices to access education materials independent of universities. The Ipsos MORI (2018) report acknowledges the ubiquity of digital communication technology for Generation-Z students, but describes a significant digital divide in their use: students from lower incomes families may not be using the technology for learning and therefore are less likely to develop certain key skills as a result. Universities have an important role in addressing this gap: we can encourage the development of digitals skills by ensuring technology is integrated into courses. Elphick (2018) describes how embedding iPad use in HE courses can help improve student perceptions of their digital capabilities.

In one case study Mang and Wardley (2012) describe a trial of loaning iPads to 47 HE students across three summer courses to provide tablet-format lecture notes, electronic reading materials, electronic quizzes and a social platform to discuss lectures. The authors conclude their experiences with six recommendations including "Make the tablet an integral component of your class" to fully realise the potential benefits and prevent unstructured and distracting use.

Our case study therefore describes a variety of concrete ways to integrate iPads into mathematics teaching with a view to conduct a rigorous evaluation of learning experience and learning outcomes from this approach.

3. Virtual whiteboards and handwritten mathematics

Before the pandemic iPads were used by staff as virtual whiteboards (see Figure 1). Handwriting over prepared slides (digital ink) is a powerful way to distinguish mathematical content (such as typeset definitions) with pedagogic commentary (such as handwritten calculations). After lectures, marked-up slides are shared as PDF files as well as full-length audio/video capture of the whiteboard, which helps to improve the accessibility of the handwritten content.

When we loaned iPads and Apple Pencils to students a priority was to integrate digital ink into the learning environment: we designed lecture notes with spaces for annotations and encouraged working directly on the iPad rather than scanning or photographing work on paper. One approach was to use shared folders in Microsoft OneDrive that removed the need to submit work for feedback: once a student finished editing a document lecturers would be automatically informed.

Later we adopted the Miro platform (see Figure 3) as a more fully featured unbounded virtual whiteboard. This web application allows for slides and lecture notes to be easily imported, annotated and recorded but also allows a large number of multimedia resources to be directly embedded onto the whiteboard using HTML iFrames or third-party plugins. For example, YouTube videos, files through cloud storage services, or interactive graphing applications such as Demos or Geogebra can all be made available on a single whiteboard. Miro whiteboards themselves can be embedded in Virtual Learning Environments. We tried a variety of virtual whiteboard platforms, including Microsoft Whiteboard and Mural, but are continuing with Miro as they currently offer free educational licenses.

An immediate benefit with unbounded whiteboard space is the ability to present a non-linear discussion of mathematics (see Figure 3) so that dependencies and relationships between mathematical ideas are clear and spontaneous discussions can be appropriately placed. Common structures (such as definition/example/non-example) can be saved as templates to reduce preparation time. We also encouraged students to browse the virtual whiteboards using their own iPads rather than relying on projectors or screen sharing. Each student has acursor visible to all whiteboard users which indicates their current focusmaking it easy to judge engagement and respond to students' attention even during remote lectures. These cursors can be hidden or labelled with student names or pseudonyms.
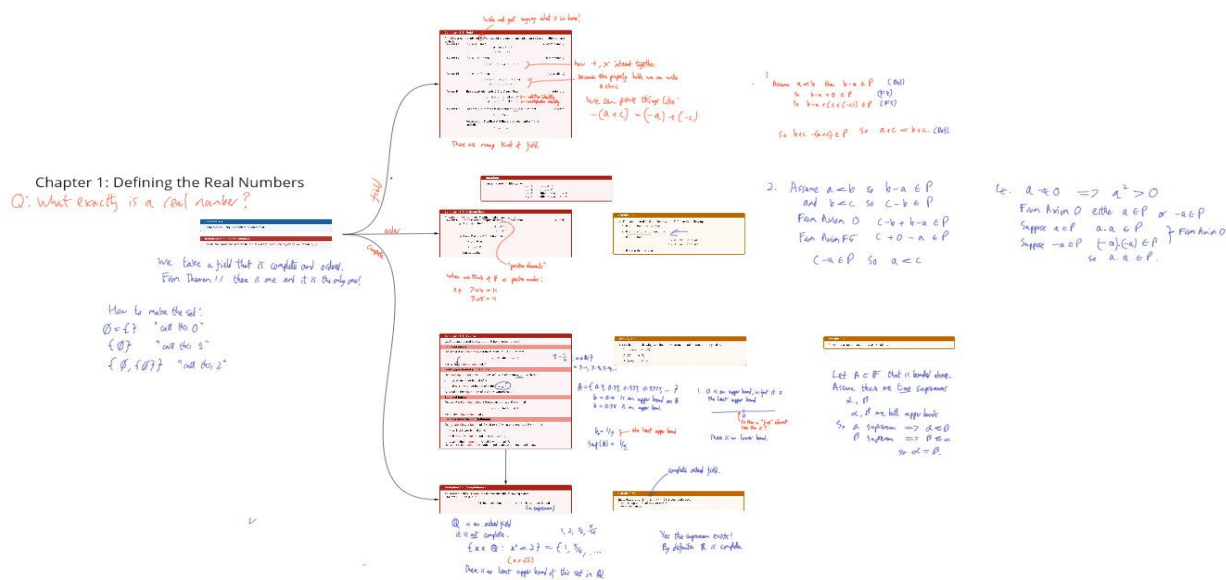
Figure 3. Non-linear virtual whiteboard with Miro.

4. Problem-solving spaces

Virtual whiteboard apps such as Miro also allow hosts and invitees to collaborate synchronously. Students can write to whiteboards, work in teams on problems or discuss material live in class. Importantly this can be done online so that students do not need to be in the same physical environment. A typical example of the output from a learning session is given in Figure 4. On the left is part of the whiteboard and the right is a closer look at a section where students have collaborated on a problem.

Influenced by approaches to teaching from other areas of the Design Engineering and Mathematics department, especially from colleagues from the product design and design engineering subject teams, the mathematics team have also developed Problem-Solving spaces for students in emulation of the "studio culture" of these other disciplines. Our approach is to use virtual whiteboards to build on lectures but give students their own space to interact with their peers outside the confines of timetabled sessions. These Problem-Solving spaces help develop a sense of a learning community: students contribute at times convenient for them, working with peers to build solutions to complex problems over an extended period, and can provide feedback and ask questions of their peers' contributions.
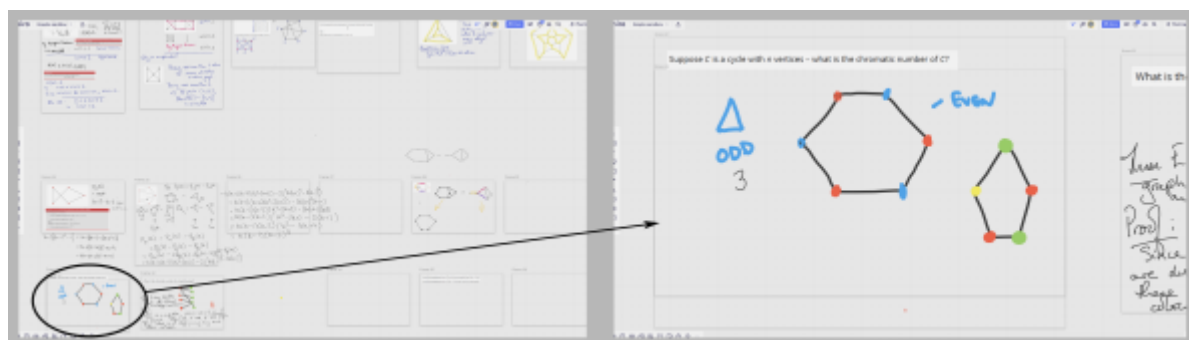

Figure 4. Student collaboration in a "problem solving space" using iPads in the Miro app.
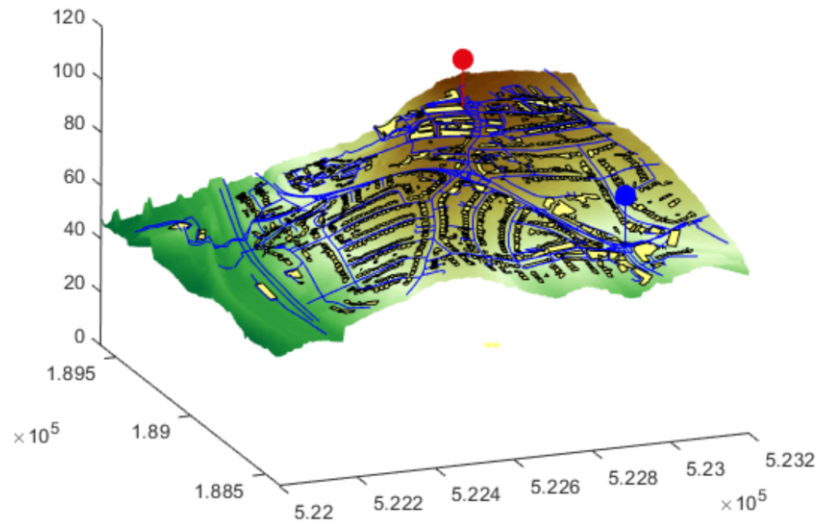
5. Inclusive assessment design

The mathematics team continue to embed the use of iPads within all learning activities and now are working more collaboratively with students by designing inclusive assessments that provide multiple equivalent options for demonstrating learning outcomes e.g., written work, video, audio, multimedia blogs. This aims to improve equality of opportunity, and be more inclusive, reducing the need for reasonable adjustment. Traditionally, assessment of mathematics is based on written coursework, timed assessment such as examinations and tests and project work. Often students are asked to communicate arguments or calculations and are assessed on the clarity of their exposition.

Although the programme assessment strategy had built in elements of choice, feedback from students indicated that digital poverty limited their engagement with it. Now that students have a common hardware platform and consistent access to specific software means that the mathematics team has been able to develop a more modern and inclusive approach to this type of assessment, removing barriers to learning (see Figure 5). Instead of asking students to write a report we now also give students the option to make a video commentary. Instead of asking students to show their working, we allow them to add an audio commentary to their work, or to record themselves working through the solution to a problem. While it is not unusual to assess students using audio and video submissions (these are used extensively in our Faculty of Arts and Creative Industries) this form of assessment is less common in the sciences. Our approach goes further by giving students the opportunity to choose the method they might employ to communicate their work whilst retaining the essence of the learning being demonstrated. The Faculty of Science and Technology at Middlesex have funded an evaluation into this radical form of assessment, which we expect to roll out further once successfully completed. It is hoped that the work will feed into the national conversation around authentic forms of assessment of mathematics and their integrity following the recent joint statement (London Mathematical Society, 2021) of mathematics learned societies concerning assessment adaptations necessitated by lockdown. The societies are concerned that open-book assessments don't provide the desired academic integrity and urge caution in the removal of traditional exams. We believe that technology-supported coursework provides greater opportunity for authentic real-world assessment, the connectivity afforded by this digital platform can improve academic integrity, and using technology to provide options to students is the best way to gauge their learning.

6. Evaluation

In order to evaluate the October 2020 roll-out of iPads for students we surveyed the recipients in February 2021 using a webform. The questions were a combination of number-of-star ratings and free-text responses, and an archived copy is available (Jones, Megeney and Sharples, 2021). All teaching provision had been done remotely during this period. At the time of this survey, during the 3rd national lockdown in the UK, students were using their iPads primarily to access live-streamed lectures and teaching resources, and to handwrite mathematics for feedback. Students were mainly submitting work asynchronously with some limited synchronous contributions during lectures and office hours. As detailed above we have since further integrated iPads into our teaching and the return to campus has enabled us to support and encourage students' use of iPads to a much greater degree. A follow up survey will be conducted at the end of the autumn term 2021 to continue this evaluation.

4. From first principles (i.e. working directly from Definition 3.3 and without using other theorems) prove that f is differentiable at the point **p**.   5 marks

5. In fact **all** multivariable polynomials are differentiable at every point in the domain. Sketch an argument to show this. Either

   · write a formal argument,
   · draw an illustration,
   · record an audio explanation or
   · record a video explanation.

   Imagine you're trying to convince a mathematician of this fact.

   > **Pro-tip:**
   >
   > You may recall last year we demonstrated all single variable polynomials $f : \mathbb{R} \to \mathbb{R}$ were differentiable.

                                                                                                                  3 marks

6. We are using a polynomial to approximate the altitude of terrain near Middlesex University. In general would you expect the true altitude of terrain to be differentiable? If not, provide a mathematical description of a non-differentiable geographic feature. Either

   · write an explanation,
   · draw an illustration,
   · record an audio explanation or
   · record a video explanation

   to justify your answer.   4 marks

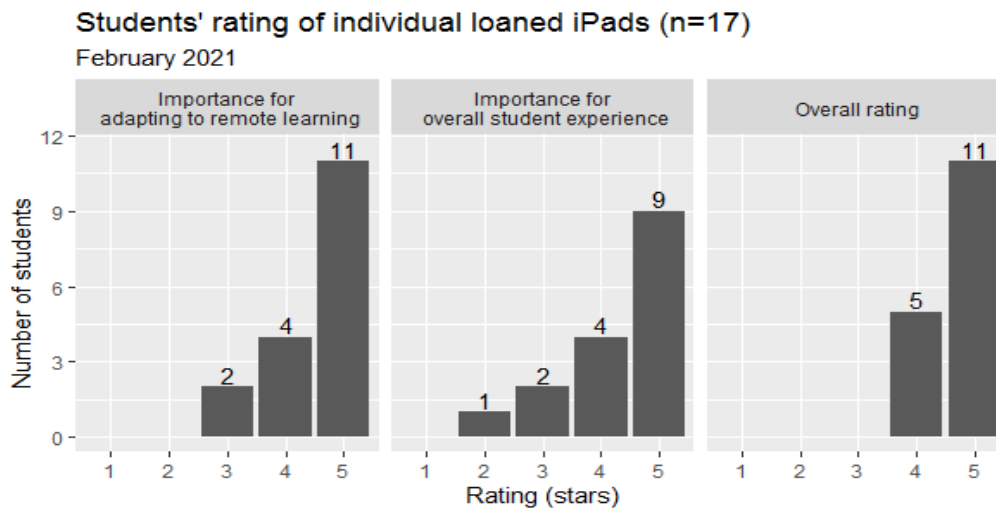Figure 5. Example coursework with inclusive submission options for students.

Figure 6. Results of student survey (Feb. 2021) following iPad loans (Oct. 2020).

Table 1. Response rate of student survey.

Programme	Year	Students enrolled	Responses	Response rate
MSc Financial Mathematics	1	2	1	50%
BSc Mathematics	1	6	3	50%
	2	8	6	75%
	3	10	3	30%
BSc Mathematics with Computing	1	2	1	50%
	2	1	1	100%
	3	3	2	67%
Totals:		32	17	53%

In Figure 6 we report the result of the numerical rating questions. Table 1 shows the response rate, indicating a fairly uniform sampling across the mathematics cohort. A comparison across programmes and year groups wasn't conducted due to small sample sizes. Perhaps unsurprisingly the students gave a high overall rating to the free loan of an iPad. However, we also see that students regard the iPads as being an important tool in adapting to remote learning, but have a more mixed view on the importance for the overall student experience.

The free-text survey questions sought to identify the challenges students faced that iPads could resolve, to investigate how students were using the iPads and the effect it had on their learning, how we could continue using iPads on the return to campus, and whether the devices could improve staff and student interactions and equality of experience. The responses were evaluated and the following themes identified.

Theme 1: iPads facilitated better communication in online sessions.
14/17 respondents reported that "writing" or "whiteboard" usage had been helped by the iPads, including the ability to "scribble down quick proofs" and "writing simultaneously with [the] lecturer". 6/17 respondents reported that iPads improved "interactivity" or enabled "interaction".

Theme 2: iPads enabled easier collaboration.

7/17 respondents reported that "sharing", "collaboration" or "working closely [with] others" was improved.

Theme 3: iPads enabled quicker, more relevant feedback.

4/17 respondents mentioned a benefit with "feedback". Staff have been using iPads for a number of years to give feedback on summative assessment. The novelty here perhaps is that students are completing formative tasks in a format they can more easily share to get detailed feedback.

Theme 4: iPads enabled students to be more organised, work faster and more efficiently and reduce stress.

5/17 respondents made reference to time saved through using the iPad compared to traditional approaches, in one case "shortening my courseworks by hours". 5/17 respondents mentioned increased efficiency, organisation or ease in working. 2/17 respondents mention a reduction in stress due to iPad use in learning.

Theme 5: a universal iPad loan scheme encourages equality of access.

2/17 respondents explicitly mentioned that the learning experience was more equitable. 7/17 respondents reported that the iPad loan scheme helped them overcome technical issues that had previously restricted their access to learning such as slow laptops, malfunctioning cameras and microphones and the use of small screens (such as mobile phones) as their primary device.

7. Conclusions and recommendations

Supporting the recommendations of Mang and Wardley (2012) we argue that it's not enough to simply give students access to the technology and the ability to collaborate, learning sessions must be designed so that we are integrating these digital pedagogies seamlessly into the learning. This approach is best realised with a common hardware and software platform such as iPads otherwise problems of inclusivity and equality of access become apparent.

We recommend that learning sessions be designed so that:

- Significantly less time is spent on presentation of content, ideally in small 10–20 minute chunks at a time. The collaborative virtual whiteboards and student iPads use make it easy to punctuate sessions with problem-based activities to consolidate learning: students following the lecture on their iPad simply start writing on the device next to the question while enjoying immediate access to the lecture material. Even remote sessions can be delivered in this way if students have equal access to tablet devices.
- Lecture slides and/or notes are available before and during the session. Tablet devices allow lectures to be given directly ontop of these notes through annotation. Students can use iPads to review and contribute to the notes in the same format they were presented.
- Time is given to students to collaborate on activities. This can be effectively managed using the virtual whiteboard timer tool and breakout rooms to separate group discussion if the session is remote. Students can use their iPads to immediately share their ideas as well as highlighting and commenting on each others' work, which helps to stimulate discussion.
- Separate areas of a common whiteboard are devoted for groups to work on, both during and after scheduled sessions. Each group can then see the lecture material as well as the work of other groups for inspiration. Individual student iPads ensure that everyone has effective access to group work outside of scheduled lecture time.

- Whiteboards are persistent – we use one per learning session and do not delete it afterwards so that students can come back later to see a record of the work done.

This approach allows the team to teach collaboratively in a way that isn't possible even in a face-to-face session where teamwork can be more stilted and less integrated in teaching, and interaction between groups can be difficult to harness. That students and lecturers are working on the same whiteboard provides an invaluable and unique resource beyond the classroom (virtual or physical) and ensuring persistence of the whiteboard means students have a record of their learning and their peers' learning.

We plan to continuing to offer iPad loans to all specialist mathematics undergraduates but will incorporate more sessions to familiarise students with the devices to help develop the necessary digital skills.

8. References

Elphick, M., 2018. The impact of embedded iPad use on student perceptions of their digital capabilities. *Education Sciences,* 8(3), 102. https://doi.org/10.3390/educsci8030102

Haßler, B., Major, L. and Hennessy, S., 2016. Tablet use in schools: a critical review of the evidence for learning outcomes. *Journal of Computer Assisted Learning*, 32(2), pp.139-156. https://doi.org/10.1111/jcal.12123

Ipsos MORI, 2018. Ipsos Thinks: Beyond Binary: The lives and choices of Generation Z. Available at: https://www.ipsos.com/ipsos-mori/en-uk/ipsos-thinks-beyond-binary-lives-and-choices-generation-z [Accessed 22 March 2022].

Jones, M.M., Megeney, A. and Sharples, N., 2021. Mathematics iPad project student survey. Available at: https://maths.mdx.ac.uk/wp-content/uploads/2021/11/Mathematics-iPad-Project-Student-Survey-Microsoft-Forms.pdf [Accessed 22 March 2022].

London Mathematical Society, 2021. Statement on methods of assessment in mathematical sciences. Available at: https://www.lms.ac.uk/node/1740 [Accessed 22 March 2022].

Mang, C.F. and Wardley, L.J., 2012. Effective adoption of tablets in post-secondary education: recommendations based on a trial of iPads in university classes. *Journal of Information Technology Education: Innovations in Practice,* 11(1), pp.301-317. https://www.learntechlib.org/p/71156/

Nguyen, L., Barton, S.M. and Nguyen, L.T., 2015. iPads in higher education – Hype and hope. *British Journal of Educational Technology*, 46(1) pp.190-203. https://doi.org/10.1111/bjet.12137

Office for Students. Access and Participation Data Dashboard. Available at: https://www.officeforstudents.org.uk/data-and-analysis/access-and-participation-data-dashboard/ [Accessed 22 March 2022].

Sharples, N. 2022. Github repository: Office for Students Access and Participation Data Analysis. Available at: https://github.com/nicholassharples/OFSdata [Accessed 22 March 2022].

Svela, A., Nouri, J., Viberg, O. and Zhang, L. 2019. A systematic review of tablet technology in mathematics education. *International Journal of Interactive Mobile Technologies*, 13(8). https://doi.org/10.3991/ijim.v13i08.10795