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On the complexity of the economic lot-sizing problem with rework of defectives

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Abstract. In this paper, we will show that the economic lot-sizing problem with rework of defectives is NP-hard. Therefore, we reduce it to the well-known PARTITION problem. This is in line with the findings for similar models that investigate lot-sizing with remanufacturing.

Keywords: lot-sizing, rework, complexity.

1 Introduction

We consider a planning problem based on the dynamic lot-sizing problem that was first published by [1]. This seminal work has been extend by many researchers and led to various models to capture different practical situations [2]. The addressed model in this paper faces an imperfect production process that generates a fraction of defective items that can be reworked to serve the same demand as initially perfect quality items.

Different from remanufacturing with external product returns, models that consider rework of defectives have only rarely been considered. [3] introduced a remanufacturing model including a joint and a separate setup case for production and remanufacturing. [4] showed that both models are NP-hard. [5] investigated internal returns in a multi-product model with limited capacity and rework of defectives.

This paper continues with the problem definition, including the MIP formulation in section 2. Afterward, section 3 demonstrates that the model presented here is NP-hard, and section 4 provides concluding remarks.

2 Problem definition and formulation

The imperfect production process that is the basis of our planning problem produces perfect quality items, called serviceables, but also a fraction of β defectives items. These units can be reworked to serve the same demand as initially perfect quality items. Consequently, all goods satisfy the same demand d_t in every period. Backordering of demand and disposal of defectives are not allowed. There are inventories of serviceables $I_{s,t}$, and of defectives $I_{d,t}$, at the end of each period. h_s denotes the costs associated with holding a serviceable and h_d with holding a defective item for one period. At the beginning and at the end of the finite planning horizon T, all inventories must be empty. The setup cost R_p incur when there is production p_t at one period and R_r when there is rework r_t; each indicated by a binary variable y_t and z_t respectively. Similar to other models, we assume that the unit production costs are negligible for both processes, see [6].

Finally, the objective of the planning problem is to minimise the sum of the total cost, which comprises of all holding and all setup costs. The MIP formulation for our model reads as follows:

$$\min \sum h_s I_{s,t} + h_d I_{d,t} + y_t R_p + z_t R_r \tag{1}$$

subject to

$$I_{s,t} = I_{s,t-1} + (1-\beta)p_t + r_t - d_t \quad \text{for all } 1,\dots,T \tag{2}$$

$$I_{d,t} = I_{d,t-1} + \beta p_t - r_t \quad \text{for all } 1,\dots,T \tag{3}$$

$$p_t \leq y_t \cdot d_{t,T} \quad \text{for all } 1,\dots,T \tag{4}$$

$$r_t \leq z_t \cdot d_{t,T} \quad \text{for all } 1,\dots,T \tag{5}$$

$$I_{s,0} = I_{d,0} = I_{s,T} = I_{d,T} = 0 \tag{6}$$

$$p_t, r_t, I_{s,t}, I_{d,t} \geq 0 \quad \text{for all } 1,\dots,T \tag{7}$$

$$y_t, z_t \in \{0;1\} \quad \text{for all } 1,\dots,T \tag{8}$$

The objective function (1) minimizes the sum of the inventory and the setup costs for production and rework. The inventory balance equations for serviceables and defectives are listed in (2) and (3). The formulation for the binaries for production and rework can be found in (4) and (5). As stated above, all inventories are zero at the beginning and the end of the planning horizon (6). The variable domains are specified by (7) and (8).

Indices	
t	Actual period
T	End of the planning horizon
k	Production period
l	End of the planning horizon
Parameters	
d_t	Demand at period t
$d_{k,l}$	Demand from period k to l
β	Defective rate at production
h_s	Holding cost per serviceable
h_d	Holding cost per defective
R_p	Setup cost for production
R_r	Setup cost for rework
Variables	
p_t	Production amount in period t
r_t	Rework amount in period t
$I_{s,t}$	Inventory of serviceables at the end of period t
$I_{d,t}$	Inventory of defectives at the end of period t
y_t	Binary variable for production setup
z_t	Binary variable for rework setup
C	Total cost

Table 1. Notation

3 The proof of NP-hardness

We will show now that our problem of the single-item dynamic lot-sizing with rework of defectives is NP-hard. It can be reduced to the well-known PARTITION problem, which can be found in [7]:

> Problem PARTITION: For the given positive integers $a_1, \dots, a_n$, does there exist a set $S \subset N = \{1, \dots, n\}$ such that $\sum_{i \in S} a_i = \sum_{i \in N \setminus S} a_i$.

Moreover, our proof is based on [4], and we will use the following parameters:

$$\beta = 0.5 \qquad R_p = 1 \qquad R_r = 1 \qquad h_s = 3 \qquad h_d = 0$$

Due to $\beta = 0.5$, half of the demand is fulfilled by production directly and the other half by rework: $\beta \sum p = \sum r$. As the setup costs are equal $\left(R_p = R_r = 1\right)$ and defectives are stored at no cost $(h_d = 0)$, it is not reasonable to store one serviceable for just one period $(h_s = 3)$. Even the non-optimal approach of producing and reworking simultaneously in one period would lead to a better result than storing one serviceable for one period.

A few parameters are set due to technical purposes:

$$T = \quad n$$

$$d_t = \quad a_t \quad \text{for all } 1, \dots, T$$
$$a_t > \quad 0 \quad \text{for all } 1, \dots, T$$
$$a_t \leq \quad \beta \sum_{i=1}^{t-1} a_i \quad \text{for all } 2, \dots, T$$

Hereby, the condition $a_t \leq \beta \sum_{i=1}^{t-1} a_i$ is required so that there are enough defective items available for rework at any time. While the production option is always possible, rework needs a sufficient stock of defective items to satisfy the demand. Without this condition, there must probably be production to satisfy a_t as there may not be enough defective items available for the rework option. For the remanufacturing model, a similar assumption exists, and all returns are already available in $t = 1$ to guarantee that the remanufacturing option is always available.

Based on the parameters, the optimal solution for the planning problem generates the total costs of $C = n$. Consequently, there is either production or rework to satisfy demand but not both. Likewise, there are no serviceables stored at all. Let us assume that there is production and rework for one period while there is only one or the other for the rest. As all demands are positive for every period ($a_t > 0$), the total cost would exceed $n = T$. Let us further assume that one serviceable is stored for one period. If this item is transferred to the next period, the total costs are reduced by at least 2. Hence, there should not be storage of serviceables at all. Finally, we end up in a situation in which there is either production or rework at one period, but not both. Also, there is no storage of serviceables at any time.

Our planning problem presented in this paper can be reduced to the PARTITION problem. The answer to PARTITION is positive, which means there is a solution to the PARTITION problem if and only if there are n positive integers a_n with $A = a_1, a_2, \dots, a_n$ that form two subsets $S \subset N$ and $N \setminus S$ for which $\sum_{i \in S} a_i = \sum_{i \in N \setminus S} a_i$. One of the subsets contains all periods where production is used to meet demand, while the other subset contains all periods where only rework is used and there is no storage of serviceables at all.

Contrary, if the subsets do not match, there must be at least one period in which production and rework are necessary, and the total cost would exceed $n = T$. Conversely, this instance will have no solution to the PARTITION problem.

4 Concluding remarks

We investigated on the complexity of our model for an imperfect production process where there is a rework of defectives. Various models for remanufacturing have already proven to be NP-hard. To the best of our knowledge, this paper is the first to address this for a model with a rework of defectives. We could show that our problem can be reduced to the well-known PARTITION problem, which is NP-hard.

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