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# Revisiting the limits of photon momentum based optical power measurement method, employing the case of multi-reflected laser beam

# Revisiting the limits of photon momentum based optical power measurement method, employing the case of multi-reflected laser beam

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## Abstract

In this work, we review the viability and precision of the photon-momentum-based optical power measurement method that employs an amplification effect caused by a multi-reflected laser beam trapped in an optical cavity. Measuring the total momentum transfer of the absorbed and re-emitted photons from a highly reflective surface (reflection of the laser beam from an optical mirror) as a force provides the possibility of measuring the optical power with direct traceability to SI units. Trial measurements were performed at two different metrology laboratories: the laboratory for mass/force at the Technical University of Ilmenau, and the clean room laser radiometry laboratory at PTB, with a portable force measurement setup consisting of two electromagnetic force compensation balances. We compared the results of the optical power measurements performed with the force measurement setup, via the photon-momentum-based method, with those performed using a calibrated reference standard detector traceable to PTB's primary standard for optical power, the cryogenic radiometer. The comparison was carried out for an optical power range between 1 W and 10 W at a wavelength of 532 nm, which corresponds to a force of approximately 2000 nN at the upper limit, yielding approximately 2.3% relative standard uncertainty in the case of 33 reflections. Thus, conflating the high-precision force metrology technique at μN to nN levels with the optical setup required to achieve specular multi-reflection configuration of the laser beam, where a macroscopic optical cavity with ultra-high reflective mirrors (>99.995%) can adjustably be suspended from the force sensors, depending on required geometry of reflections, we show that the uncertainty of the optical power measurements upon further increase of the nominally applied optical power, the number of laser beam reflections, or the reflectivity coefficient of the mirrors can be markedly reduced.

## 1. Introduction

The use of photon momentum to determine the optical radiant power or to generate precision/calibration small forces [1–3] has made significant progress in mass/force and optical metrology fields in recent years, especially for measurements of the optical power of a laser at kilowatt levels [4]. The measuring principle is based on the measurement of the force exerted by the transfer of the photon momentum when reflecting the radiant power upon a highly reflective mirror. A measurement device developed by *Williams et al* uses this measurement technique, with which has been claimed relative expanded measurement uncertainties up to 1.6% for optical power levels between 1 kW and 50 kW [4]. In the core of the device is a force sensor, which consists of a commercial off-the-shelf electromagnetic force compensation (EMFC) weighing balance and a mirror with high reflectivity ($R = 0.9998$) attached to it. The measurement of optical power of less than 1 kW using the photon momentum generated by a single reflection is quite challenging, as the force sensor needs to resolve forces at least in the nN-range. Moreover, there are not traceable reference mass standards for this force range, to date. To resolve this problem, *Vasilyan et al* [1, 2] developed a device with two force sensors adapted for differential force measurements, by which the noise level was reduced by one order of magnitude (from below the 1 μN level for a single sensor to less than 100 nN for a differential signal). Here, a multi-reflection configuration of a laser beam (approximately 1 W) achieved in an optical cavity was demonstrated for the first time to generate a calibration force at the currently existing lowest end of the small force standard, from 10 nN up to 10 μN, which are yet connected and are routinely being calibrated in relation to the mass standards, 1 μg up to 1 mg, respectively. Under the multi-reflection configuration, the total net force was amplified by at least an order of magnitude in comparison with the single reflection configuration. Furthermore, using single- and multi-reflection configurations, a possible standard for the force calibration, or the inverse, a standard for optical (laser) power calibration with direct traceability to the recently renewed SI base units, has already been proposed [2, 5–7].

In comparison with the thermal detectors traditionally used for optical power measurements from W to kW levels, the photon-momentum approach is based on an entirely different physical concept. Theoretically, photon-momentum-based optical power detectors may directly quantify the absolute magnitude of the non-absorbed portion of the optical field energy and are practically only limited by the accuracy of the force measurements and the quality of the reflective surface of a mirror.

For thermal detectors, the optical power is determined by measuring the relative change of the passively dissipated heat resulting from the absorbed energy of the optical field. Traditionally, flat or cavity-based thermal detectors are used as reference standards, which are directly traceable to electrical SI units (volt, ohm), or indirectly, through a primary standard for low optical power (cryogenic radiometer) [8–10]. Although the existing reference standard thermal detectors achieve relative expanded measurement uncertainties of approximately 0.2% ($k = 2$) [11] for optical power measurements, for example at 1 W, this values increase up to 1%–2% for power measurements in the kW range, additionally becoming a nontrivial technological task to implement because their measurement capability and accuracy strongly depend on the absorbance and heat capacity of the cavity used as a sensor. Here, the cavity size (total heat capacity) increases proportionally with the maximum laser power to be measured, and more thermal mass translates to a slower measurement response time.

Thus, in this paper, we aim to review the viability of the photon-momentum-based optical power measurement method via the multi-reflected laser beam approach, which is potentially traceable to the kilogram (already to be considered for Planck's constant, meter and second). The measurement results are compared with those performed using a calibrated reference detector traceable to PTB's primary standard for optical power, the cryogenic radiometer.

In section 2, the traceability routes of the optical power measurements used in this work and the interpretation of the photon-momentum-based method are presented. In addition, the advantages and underlying limiting factors of the photon-momentum-based method are briefly described. The operating principle of the force measurement setup using the photon momentum for optical power measurements is described in subsection 3.1. In subsections 3.2–3.4, the specially tailored optical power measurement setup and the procedure used for comparison, including the operational characteristics of the thermal detectors, are described. Section 4 presents the comparison measurements and the uncertainty evaluation of the measurement performed by both methods; furthermore, options for improving the traceability with reduced measurement uncertainties under the redefined SI are discussed.

## 2. Traceability routes

### 2.1. Reference detector (thermopile and Si-diode)

The traceability chart of the reference detector (integrating sphere with attached thermopile and Si photodiode detector) used in this work for optical power measurements is shown in figure 1 (left). Both detector configurations (integrating sphere—thermopile and integrating sphere—Si photodiode)
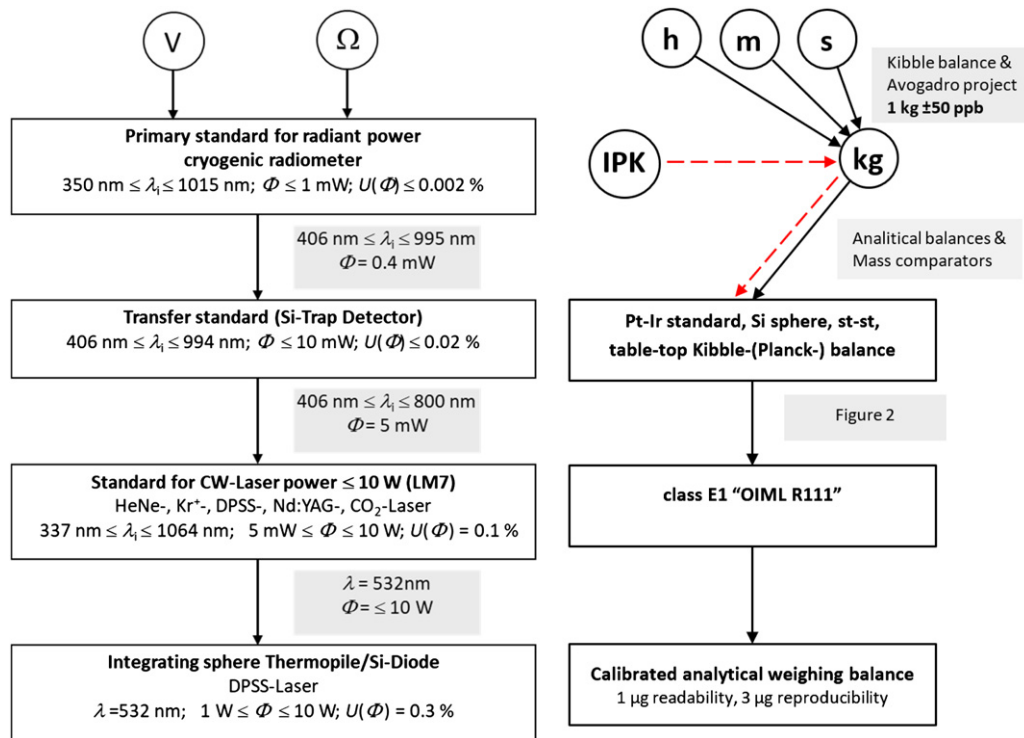
**Figure 1.** Traceability charts. (*left*) Integrating sphere with attached thermopile and Si photodiode detector used for optical power measurement. (*right*) Photon-momentum-based force measurement method for optical power calibration.

were individually calibrated against PTB's reference standard (LM7) for optical powers between 1 W and 10 W at a wavelength of 532 nm. The LM7 reference standard is a cavity-based (cone-shaped) thermal detector [12, 13] whose traceability to the cryogenic radiometer (primary standard) is established via a Si-trap detector (transfer standard) [14].

The responsivity of the reference detectors (thermopile and Si photodiode) is calibrated with a relative expanded uncertainty of 0.3%. It should be noted that here the only primary standard detector that is directly traceable to the SI unit ampere through the volt and the ohm is the cryogenic radiometer.

### 2.2. Interpretation of the traceability route of the photon-momentum-based method

As mentioned in the previous section, the photon-momentum-based method may directly relate the measure of the force that is produced by the momentum transfer of absorbed and re-emitted photons from a highly reflective mirror with the measure of the optical power of a laser beam. For the case where the angle of incidence of the laser beam is parallel (the angle of incidence is $\theta \approx 0$, $\cos \theta = 1$) to the normal of the surface of an ultra-high reflective mirror, $R_L \rightarrow 1$ (negligible transmission and absorption), and for the given optical power, the relationship is

$$F = \frac{\text{Power}}{c}(1 + R_L). \quad (1)$$

This force can be equated with the gravitational force acting on the mass piece by
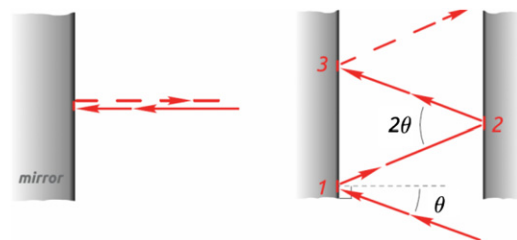
$$F = mg, \quad (2)$$



**Figure 2.** Geometry of the laser beam and flat surface mirrors. (*left*) Single reflection, where the incidence angle of the laser beam is parallel to the normal of the surface; (*right*) two quasi-parallel mirrors with relative angle $\alpha$, where the incidence angle is $\theta$.

where $c$ is the speed of the light, $m$ is the mass of an object, and $g$ is the gravitational acceleration. Because of technical limitations and the absence of reliably manufactured small calibration mass pieces, this kind of direct comparison is currently possible only at laser powers on the kW level with low resolution. To overcome this obstacle, a method employing a multi-reflected laser beam configuration has been introduced [1, 2, 6, 7] which has been further been adapted in [3]. Considering a cavity-like system with two ultra-high reflective mirrors, the laser beam path could be folded within; thereby, the same optical power can accumulate a higher total net force produced by such a configuration because the remaining portion of the reflected laser power after each reflection can still be measured as a force. The total sum of forces is described by

$$\sum_{i=1}^{N} F_i = \frac{1 + R_L}{c} \sum_{i=1}^{N} \text{Power}_i \quad (3a)$$

$$\sum_{i=1}^{N} \text{Power}_i = \text{Power}_1 \sum_{i=1}^{N} R_L{}^{i-1}, \tag{3b}$$

where $i = 1, 2, \ldots, N$ is an integer value showing the number of reflections (see figure 2), $\text{Power}_1$ and $\text{Power}_i$ are the laser power at the input (first reflection, hence $\text{Power}_1 = \text{Power} = \text{Power}_{\text{input}}$) and at the $i$th instance of the reflection (at $i = N + 1$, $\text{Power}_{N+1} = \text{Power}_{\text{output}}$), respectively. It follows that the optical power of the laser can be measured with traceability directly connected to the mass standard.

Assessment of the measurement uncertainty can be provided with the following simplified numerical example based on equations (1)–(3) and basic specifications of the measurement components; for the 1 kW laser power, after the first reflection, the force exerted on the ultra-high reflective mirror with 99.995% reflectivity is 6.673 116 µN, whereas after 33 reflections, this value is 220.036 76 µN. Standard practices known from mass/force metrology can be used to perform force measurements at this level. In the case that one uses a commercially available precision weighing balance with a reproducibility (standard deviation) of 3 µg for measurements of the weight pieces, the equivalent force calculated with equation (2) is 29.4 nN, assuming the value for the local gravitational acceleration is 9.812 502 900(20) m s$^{-2}$ at the site of the measurements. Thus, in this case, we can measure the optical-power-generated forces after the first reflection with a relative standard deviation of 0.440%, and that after 33 reflections with 0.013%. However, because the force standard at such small force levels (nN) has not yet been established (refer figure 1 (right)), the implementation of traceable optical power measurements using the photon-momentum approach can only be verified in connection with the measurement capabilities of a certain class of apparatuses. As an example, consider an EMFC weighing balance traceable down to 10 µN, and from there on down to approximately 10 nN, only accounting for the reproducibility or in general the type B uncertainty evaluation (an alternative could be the development of a uniquely designed custom-made instrument reaching calibration of lower force values with well-characterized and SI-traceable calibrated certificate whose performance additionally could be verified by an independent party). Generally, the balances are calibrated by employing the dead weight effect exerted on standard weight pieces in accordance with equation (2). Conversely, the optical power can be determined by combining equation (1) with equation (2) for single reflection and equation (1) with equation (3) for multiple reflections in the following simplified form:

$$\text{Power} = c \frac{mg}{1 + R_L} \tag{4}$$

The relative measurement uncertainty of using an apparatus, as presented in [1, 2, 6, 7] for mass/force determination, can roughly be estimated by equation (5), which can be derived from equation (4) by the standard uncertainty propagation. The uncertainty of the optical power measurements primarily depends on the relative measurement uncertainties of the mass, gravitational acceleration, and reflectivity value of the

ultra-high reflective mirror.

$$\frac{u(\text{Power})}{\text{Power}} = \sqrt{\left(\frac{u(m)}{m}\right)^2 + \left(\frac{u(g)}{g}\right)^2 + \left(\frac{u(R_L)}{R_L}\right)^2} \tag{5}$$

The value of the gravitational acceleration at the measurement site is roughly 9.8 125 029 m s$^{-2}$ and can be determined by means of a (free-fall) absolute gravimeter to approximately 0.2 ppm and below, similar to velocity measurements using a laser interferometer and a frequency standard. Furthermore, for more accurate gravity value determination, it is necessary to measure its gradient in the laboratory near the setup with a relative gravimeter. The value typically can be obtained to better than 0.01 ppm.

The values of the reflectivity for the ultra-high reflective mirrors, to the best of our knowledge, can be estimated only indirectly by measurement of the losses, typically given as the optical transmission curve. In accordance with most datasheets provided by different manufacturers, it varies in the range of 10 ppm to 70 ppm. Here, we will choose a rather conservative value of 70 ppm [15].

The uncertainty values of the mass vary depending on the nominally used discrete set of standard mass pieces, against which the nominally applied magnitude of the optical power should be compared (the full description is given in 'uncertainties of the weights of the classes E1, E2, F1, and F2 according to OIML R111' [16]). In accordance with a set of standard mass pieces of the class E1, for the 1 mg piece, the expanded relative uncertainty is 1000 ppm, in case of the 20 mg it is 50 ppm, and in the case of the 1 g it is 3.3 ppm; see figure 3 (left).

Thus, with measurements conducted at the 1 kW optical power level for the single reflection case, the combination of uncertainty contributions using equation (5) leads to an estimated standard uncertainty of optical power measurements of approximately $u(\text{Power})/\text{Power} = 1002.4$ ppm; for the multi-reflection configuration with 33 reflections, the value is 86 ppm. In figure 3 (right), the full estimation curves of relative measurement uncertainties in the optical power obtained by equation (5) are presented.

This method of establishing the traceability of optical power measurements through the use of measurements of the photon-momentum generated forces and their further inter-comparisons with the standards and procedures known from mass/force metrology is theoretically more direct and simplistic, which may further improve and lower the measurement uncertainties, at least for continuous wave (CW) optical power measurement.

## 3. Measurement setup

### 3.1. Photon-momentum based force measurement system

The system is developed with the use of two EMFC high-precision weighing balances. These weighing balances are a special class of state-of-the-art systems used in mass metrology for measurements, calibrations, and cross-comparisons of weight pieces ranging from 1 µg up to several kg. These
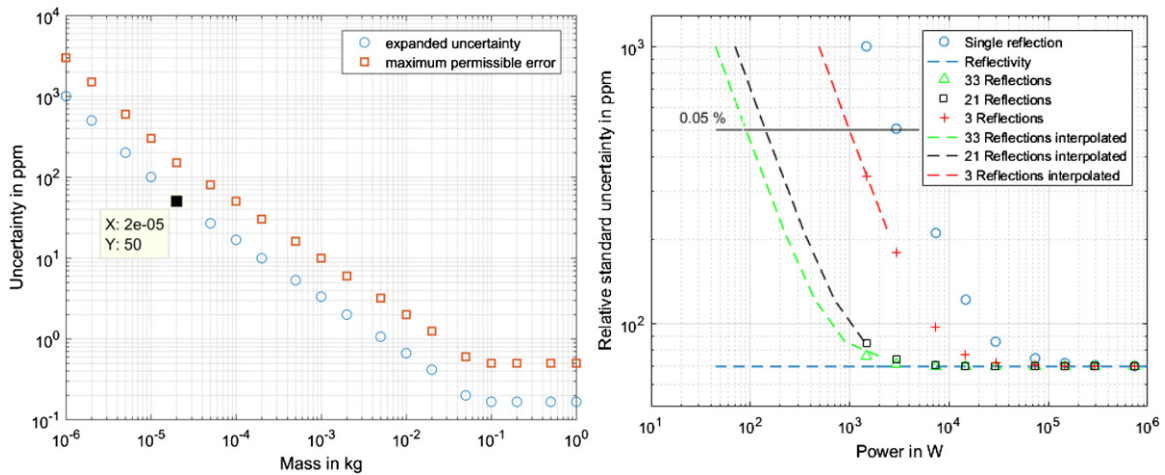
**Figure 3.** (*left*) Expanded relative uncertainties and maximum permissible errors associated with the weights of the class E1 'OIML R111' [16]. (*right*) Estimated relative standard uncertainty of the optical power realization by the equations (4) and (5) as a function of the nominal magnitude of the applied optical power, in four different (1, 3, 21, 33) reflection configuration cases with interpolated data as an extended estimate for power levels below 1 kW; the red dashed line presents the lower limiting factor as a result of the reflectivity coefficient of the mirrors.

systems consist of a complete monolithic realized from a single piece of material mechanism, with a flexure hinges, an adjustable or fixed counterweight, parallelogram guidance, and a transmission lever (figure 4). Later is considered as a simple beam with a proportional lever arm structure supported by the flexure hinges. The lever couples the mechanical system to the EMFC system, similar to those known as voice coil actuators used in loudspeakers. Additionally, they are equipped with an opto-electronic absolute one degree-of-freedom (1DOF) position measuring sensor, whose output electrical signal together with an analog or digital controller completes the guided measurement process in various operational regimes including, among others, the open-loop, closed-loop, static, and dynamic regime. One of the prominent examples of their use is in mass comparators, where the comparison of two 1 kg weights can be realized at a standard deviation level of approximately 0.1 µg [17], that is, with a relative standard deviation of $10 \times 10^{-10}$. Extensive research on their complex technical behavior is presented elsewhere, on the basis of which apparatuses such as the Planck balance [6, 19] have already been developed. After the redefinition of the SI unit of mass, based on the fixed numerical value of the Planck constant $h$ at $6.62607015 \times 10^{-34}$ when expressed in the unit J s, which is equal to kg m$^2$ s$^{-1}$, such devices (Kibble balances and their table-top versions) will replace traditional mass comparators; however, they have very similar technical and mechanical realization concepts [21, 22]. Furthermore, additional developments with this weighing balance are presented in [6, 10], and its functionality is also cross-checked in other non-orthodox orientations differing from its common usage [18, 20].

The precision of the weight measurements using these balances depend on the actual application and the value of the measurand. A detailed description of the balance and the functional characteristics of the differential force measurement setup by which we conducted photon-momentum-based force
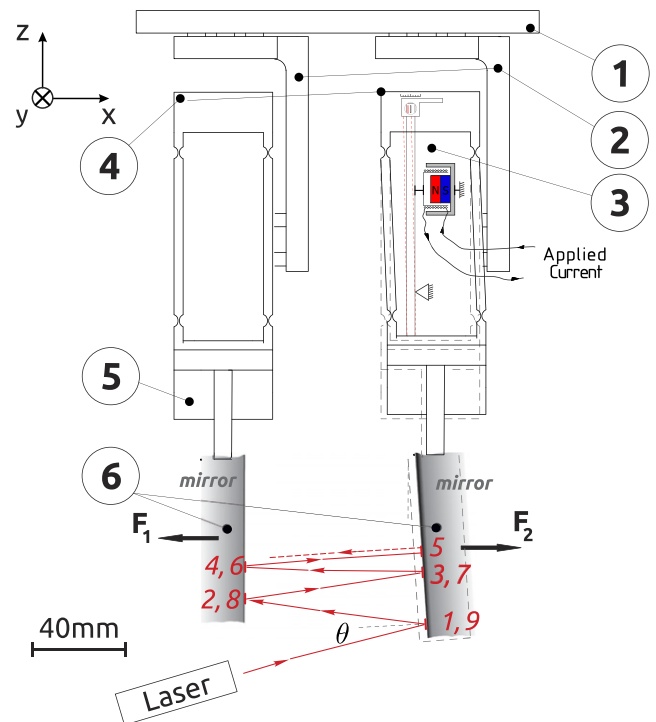


**Figure 4.** Functional diagram and geometrical configuration of the FMS, mirrors, and the laser source. Dashed line in the right EMFC balance indicates the initial state. (1) and (2) common bearing plate and separate mechanical supports for adjustment of EMFC balances in horizontal plane, (3) internal assembly of monolithic EMFC balance joining the three main internal components, i.e., the internal voice coil actuator, proportional lever arm, and positioning sensor, (4) EMFC balance, (5) load carrier, (6) 2″ mirrors adjusted from load carrier (the size is exaggerated for better visibility).

measurements is presented in [1, 2]. With a single weighing balance integrated in our particular system, it is possible to perform weight measurements of up to 5.5 g with a resolution of approximately 3 µg and a standard error of 7 µg. According to the specifications of the balance (model series
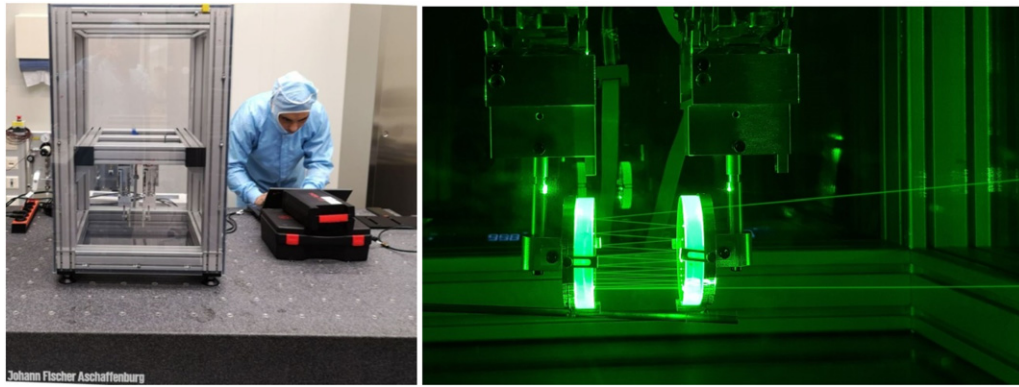
**Figure 5.** Photographs (*left*) of the force measurement setup placed on a vibration-isolated table in a temperature-stabilized clean room for laser radiometry at PTB, Braunschweig, and (*right*) the optical cavity with ultra-high reflective 2″ mirrors (>99.995%), adjustably suspended from the EMFC weighing balances at the configuration where 21 reflections occur. The lower laser beam is the beam entering into the optical cavity, which has an optical power of approximately 8.5 W, while the upper laser beam is the beam exiting the optical cavity, which has an optical power of approximately 8.3 W.
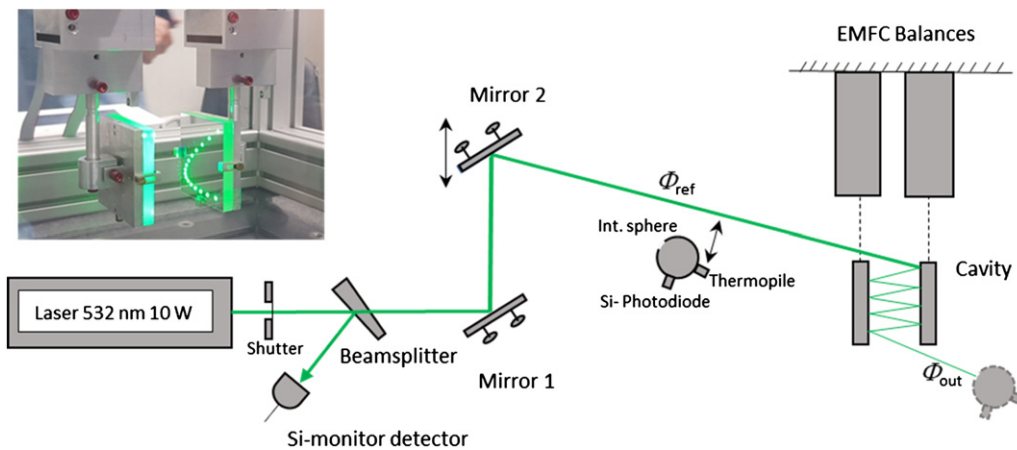


**Figure 6.** Setup for measuring optical power via photon-momentum-generated forces, (*inset upper left*) photograph showing the case with 33 reflections in a cavity with square-shaped highly reflective mirrors (>99.5 %); the optical power of the input laser beam is approximately 15 mW.

WZA with 1 μg readability from Sartorius—AG) provided by the manufacturer, the reproducibility of the balance is given as the standard deviation of the weight measurements, which is 3 μg, whereas the readability of the balance is given as 1 μg, which is the minimal incremental change that the balance can show. However, with two weighing balances, the differential measurement setup is adapted for measurement of a horizontally directed force, where an order of magnitude noise reduction is achieved in comparison with a single weighing balance. The force measurements are tested only up to a 100 μN range at normal atmospheric pressure, reaching resolutions between 10 nN and 30 nN, which strongly depends on the surrounding environment and the filtering method adapted for the particular measurement in that environment. According to cross-checking measurements performed for vertical and horizontal force measurement directions below 10 μN, we found that the stiffness of the system differs by several percent. In both cases, an externally adjusted electromagnetic voice coil actuator is used as a reference force actuator. Comparison of the linearity slopes of the force measurements against externally applied reference forces with the electromagnetic voice coil actuator showed a difference of approximately 5% at the level of several 100 nNs (≈10 μg). This further decreases for higher force levels to less than 1%. This difference is attributed mostly to the use of an external voice coil actor whose force factor changes because of the influence of temperature on the magnet system and because of the misalignment of the coil in relation to the magnet system. However, the results obtained by our setup for the photon-momentum-based force measurements in the horizontal direction show reproducible results when using the internal built-in voice coil actuator to measure the photon momentum forces directly. In general, according to the standard practice of using calibrated weight pieces to calibrate force values, the calibration of force values at 10 μN can reliably be performed with an error of no better than 0.3%, which is derived and connected to the maximum permissible error ±0.003 mg of the 1 mg E1 class standard weight piece. Using this practice for the calibration of the force values at the 10 nN level, the existing artifacts (1 μg weight pieces) already have approximately 100% error.

**Table 1.** Measurement settings.

| Parameter | | | | |
|---|---|---|---|---|
| Input optical power, W | 2 | 4 | 6.5 | 8.6 |
| Wavelength, nm | | | 532.5 | |
| Cross-section of the laser beam, mm | | | 4 | |
| Cavity (mirror reflectivity, %) | | 99.5 | | 99.995 |
| Number of reflections | | 21 | | 33 |

The weighing balances are arranged such that their combined and simultaneous operation leads to a continuous 1-DOF force measurement signal described by the following equation,

$$F_{\text{Total}} = F_{\text{Net2}} - F_{\text{Net1}} = (F_2 + F_{\text{err}}) - (-F_1 + F_{\text{err}}) \quad (6)$$

$$F_{\text{Total}} = F_1 + F_2 \quad (7)$$

where $F_1$ and $F_2$ are force signals measured separately along the *x*-axis as a function of time, $F_{\text{net}}$ represents the measured raw signal from the balance, and $F_{\text{err}}$ is the force measurement noise. All the other measurement deviations at defined measurement time scales, such as …, are either eliminated or after the tests are found to be insignificant to within the value of the floor-noise of the setup (refer to [1, 2] given earlier, 10 nN to 30 nN, which varies depending on environmental conditions). Thus, the total measurement force is taken as a combination of two measurement forces, as shown in equation (7), each of which is obtained as an equivalent mass value measurement in closed-loop operating mode.

The weighing balances can be used in an open-loop operation mode (alternatively, this can be considered as the '*velocity mode*' in Kibble balances [3, 6, 23]), where typically the traceable measurements are conducted to test and compensate for material- or non-material-based effects (such as for hysteresis) and to allow a direct traceability to the meter and second. In our case, however, for the measurements through the closed-loop control for the position keeping it at the zero point, the measure of the position does not enter the general measurement equation (similar to the '*force mode*' in Kibble balances [23]). For such off-the-shelf ready-to-use precision weighing balances operating on the basis of the compensation principle, during the factory-based calibration, the traceability to the meter is not necessary. Instead, it requires traceability to the kilogram and electrical quantities within the range and the precision of the measurement interest. For more details on the recommendations for traceable mass determination and the uncertainty calculation, see [16, 24].

### 3.2. Description of the optical cavity and laser

The measurements were carried out with two different optical cavities. In one case, the cavity was created using conventional highly reflective 2-inch square-shaped plane mirrors with $R = 99.5\%$ reflectivity, and for the other case, by 2-inch round ultra-high reflective plane mirrors with $R = 99.995\%$ reflectivity; in both cases, the reflective surface is optimized for a wavelength of $\lambda = 532$ nm by a multi-layer dielectric coating on a synthetic fused silica substrate that has a surface flatness

of less than $\lambda/10$. The mirror reflectivity values were assumed to be as provided by the datasheets of the manufacturers. In the case of the mirror with 99.995% reflectivity, the value was measured at the center point with an angle of incidence of $0°$ on the separate witness sample using the cavity ring down method, with losses typically of approximately 20 ppm. The clear aperture parameter of these mirrors' reflective surfaces is yet to be studied and will be defined more rigorously in the future.

In figure 5, photographs of the setup and the cavity with round ultra-high reflective mirrors are presented. At relatively high input power levels, the optical losses are markedly reduced (in the photograph, the input power is 8.5 W) when using such ultra-high reflective mirrors. The mirrors are suspended from each weighing balance with specially manufactured identical mounting mechanics and form an optical cavity. The mechanism provides an option for manual adjustments of the mirrors with respect to each other to achieve specular reflections.

The measurements were carried out for the three different multi-reflection configurations (21, 33, and 41 reflections), in each case with varying angle of incidence, angle of reflection, and patterns. The laser source used is a CW diode-pumped solid-state laser with a wavelength of 532.50 nm $\pm$ 0.01 nm and a maximum output power of 11 W. Its optical power stability is $\pm0.5\%$ over 2 h. The cross-section of the laser beam is $\sim4$ mm ($1/e^2$), and it is linearly polarized (vertical, $>100$:1).

### 3.3. Description of the reference detector

The detector used for the optical power measurement consists of an integrating sphere with a thermopile and a Si photodiode attached to it. Its responsivity was calibrated against PTB's reference standard (LM7) for optical power levels between 100 mW and 10 W, with both detectors attached to the integrating sphere, so that any detector configuration can be simultaneously used for the optical power measurements. This takes advantage of the optical characteristics of a thermal detector; specifically, for their nearly flat spectral response and high-power density capability, and the semiconductor-based detector, for their fast detection response. Moreover, because an integrating sphere is used, the detector can also be used to measure the mirror-cavity transmittance of the photon momentum apparatus.

In this experiment, the optical power $\Phi_{\text{in}}$ entering the mirror cavity was measured by the detector using the output signal from the Si photodiode, as shown in figure 6. Then, it was removed from the beam path to measure the optical power with the photon-momentum apparatus. Moreover, to determine the total transmittance of the photon-momentum mirror cavity, the output power $\Phi_{\text{out}}$ was also measured using the same detector. Here, a monitor detector is used to reduce the effects of laser power fluctuations that may occur during the cavity transmittance measurement. Thus, the total cavity transmittance is calculated by the ratio between the input and output power measurements as $T = (\Phi_{\text{out}}/\text{Mon2})/(\Phi_{\text{in}}/\text{Mon1})$, where Mon1 and Mon2 are the output signals of the monitor detector
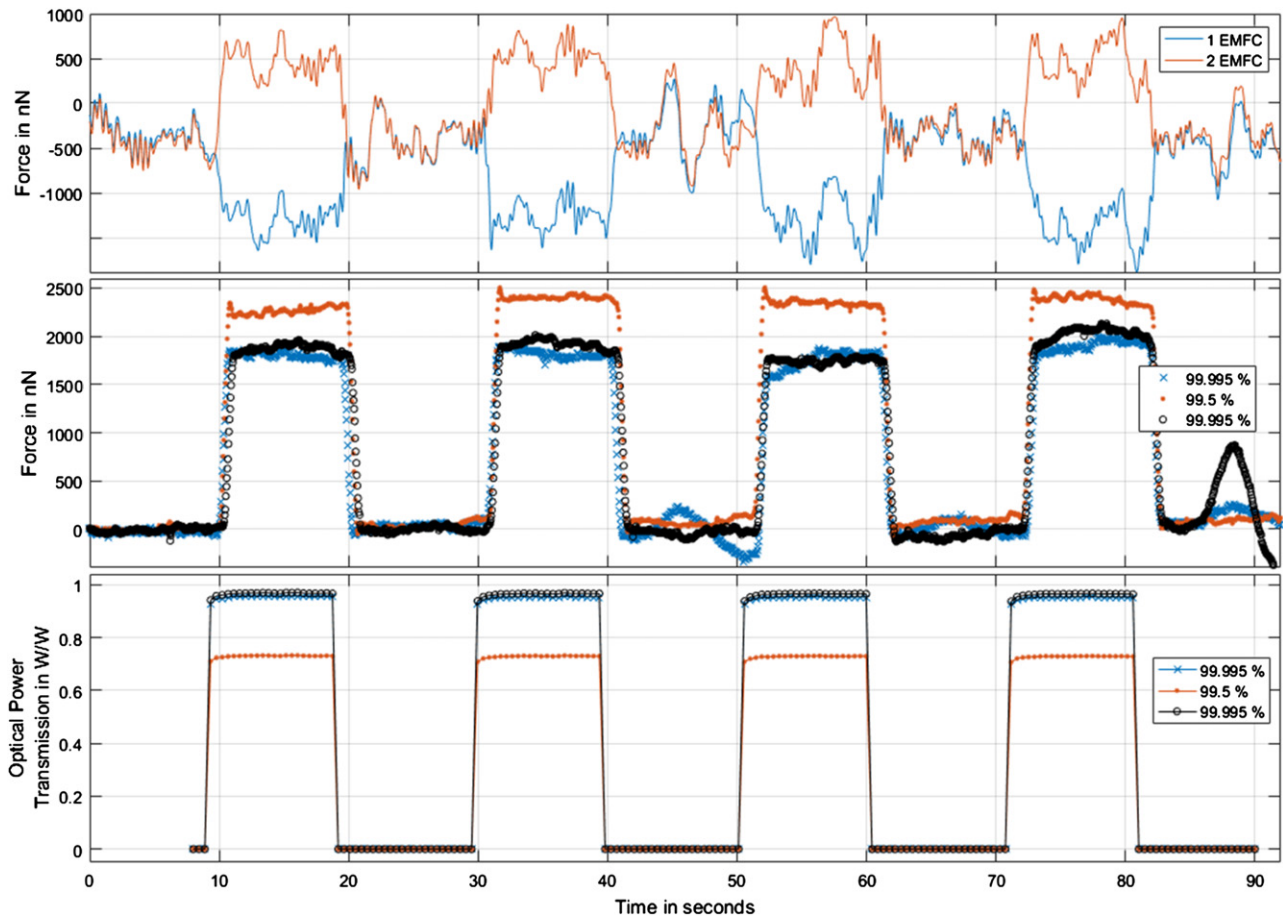
**Figure 7.** (*top*) Force signals from both balances separately during one measurement trial using a cavity with mirror reflectivity of 99.995%. (*middle*) Measured difference forces (no filtering is applied) using two different cavities with reflectivity of 99.5% for the red dotted line and 99.995% for the black circle and blue cross. (*bottom*) Optical power transmission measurements of cavities with an integrating sphere thermopile—Si photodiode detector during the photon momentum measurements; the nominal input powers are as follows: 8.57 W—blue cross, 8.77 W—black circle, and 8.69 W—red dot.

obtained when measuring the input power $\Phi_{\text{in}}$ and the output power $\Phi_{\text{out}}$, respectively. In addition, in this experiment, the monitor detector was used to be able to compare '*online*' the optical power $\Phi_{\text{in}}$ with that measured by the photon momentum apparatus. For this purpose, a conversion factor including the reflectivity of mirrors 1 and 2 was determined.

### 3.4. Description of the experimental method

The measurements were performed at two different metrology laboratories for mass/force, at TU Ilmenau (January–February 2019) and the radiometry clean room at PTB, Braunschweig (December 2019). In both cases, the portable force measurement setup, the laser unit, and the pneumatic optical beam shutter were used in the general measurement schematics without any alterations (see figure 6). For the measurements at TU Ilmenau, before each trial, the laser beam was initially set to a defined optical power level and directly entered the optical cavity after passing the shutter, without other intermediate steps. Only the cavity with the mirrors of 99.5% reflectivity was tested because of the not-clean-room conditions of the operation. After the specular type of multiple reflections in the cavity, the laser beam entered the reference power meter head.

The power meter was a PTB-calibrated commercial (Ophire model: 10A-PPS) thermopile sensor with a 0.4% ($k = 2$ coverage factor) expanded measurement uncertainty. The optical transmission losses obtained during the measurements were later confirmed during the second measurement campaign at PTB, Braunschweig. The measurement results presented in this paper are data collected from the trials made at PTB, Braunschweig. In this case, during the course of all measurements, the optical power of both in- and outgoing laser beams (which enter the cavity and then leave) were simultaneously monitored with different power meters having expanded measurement uncertainties of 0.2% ($k = 2$ coverage factor); the full description and schematics are presented in figure 6.

The measurements presented in this section can also be seen as a key measurement case when using specially chosen and tailored opto-electro-mechanical components, among a variety of possible parameters and operational/measurement ranges. Each individual measurement was made by periodically switching the shutter on and off (with a laser irradiation exposure time of 10 s) several times to create stepwise applied optical power (ABA type of measurements) for different measurement settings, as given in table 1.
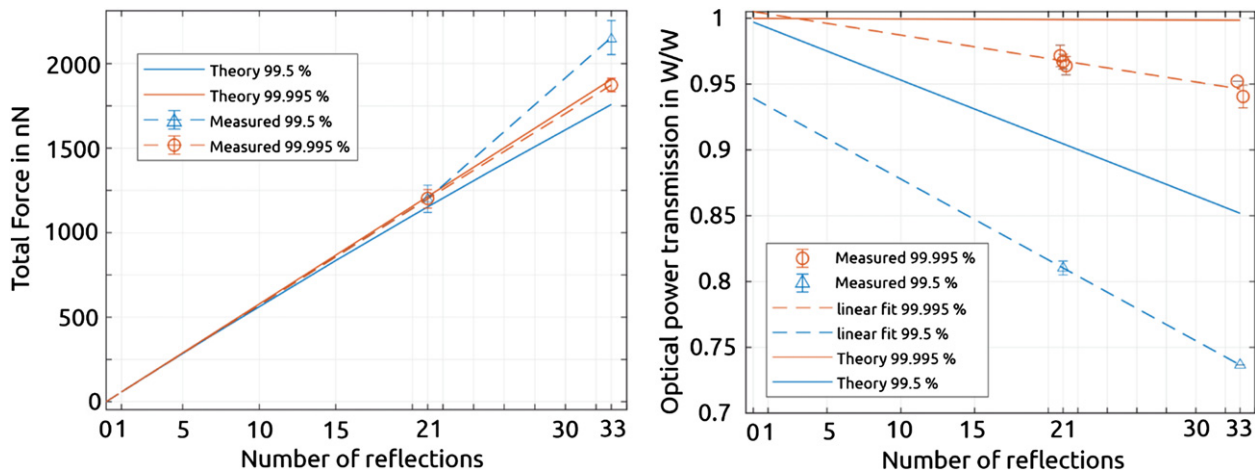
**Figure 8.** (*left*) Total measured photon-momentum force in configuration with 21 and 33 reflections with an input optical power of 8.66 W (*symbol-dashed line*) and the theoretically calculated values (*solid line*). (*right*) Optical power transmission measurement with integrating sphere thermopile—Si photodiode detector for all numbers of reflection configurations (*symbols*), their corresponding linear fits (*dashed line*), and the theoretically calculated values (*solid line*).

In both metrology laboratories, the vibration isolation is yet incomplete; therefore, we have carried out multiple trials of extensive measurements for statistical evaluation of the data, despite clear evidence of different types of interferences considering both magnitude and frequency (see figure 7). The temperature measurements of the air surrounding the force measurement system (several cm away from the mirrors) show no short-term correlations/interference with respect to the final results. The temperature change during the measurements at TU Ilmenau was detected to be less than 10 mK min$^{-1}$ from the typical average 293.45 K, and at PTB, Braunschweig, in the clean room laboratory under temperature-stabilized conditions, the maximum change was detected to be less than 2 mK min$^{-1}$ from the averages of 292.395 K and 291.432 K. These temperature variations led to linear thermal drift associated with deformations of the metrology frame caused by existing thermo-mechanical stresses.

## 4. Results and discussion

The force measurements were performed in accordance with the optical schematics presented in figure 6. The input optical power values were used to theoretically calculate the expected photon-momentum-generated forces for comparison with the data from the measurements. In this configuration, we are able to directly compare the reference for the force measurements and the reference for the optical power measurements, towards the SI-based traceable comparison of the force/mass (actually to the Planck constant) and laser power references.

In figure 7, we show one of the typical sets of force and corresponding laser power measurements. At first glance, from the raw data of the force measurements for the 33-reflection configuration (mirror reflectivity: 99.995%, input power: approximately 8.66 W, output power: 8.3 W), the mean value of 1873 nN is obtained with a relative combined standard deviation of 2.3% (43 nN). If the calculations are made assuming the sample standard deviation, then the value
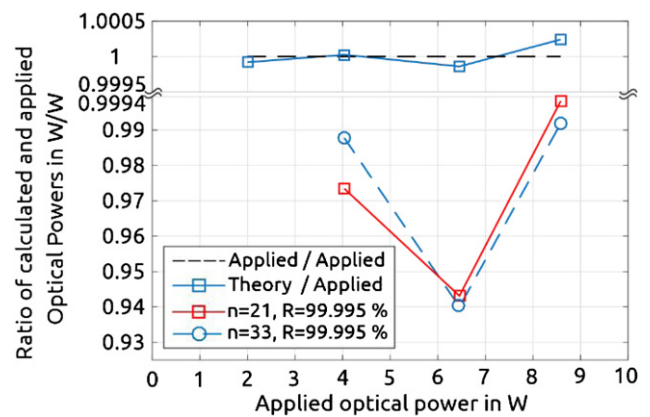


**Figure 9.** Ratio of the optical powers calculated from photon-momentum-based force measurements (equation (3)) using the cavity with 99.995% reflectivity against the nominally applied optical power measured by the integrating sphere thermopile—Si photodiode detector. The black dashed line presents a unity level, the solid blue square is the calculated optical power from theoretically obtained force values, the blue dashed circle is obtained for the 33-reflection configuration, and the solid red square is obtained for the 21-reflection configuration.

is 0.3%. However, the relative standard error was 3.1% as a result of unforeseen environmental noise (temperature and mechanical vibrations). Furthermore, in this particular case, the mean value of the measured forces in comparison with the value calculated theoretically differs by an average of 1.08%; for all measurements with the 99.995% reflective mirror, this difference is within 1.7% (see figure 8 *left*). These results show a major improvement regarding the reliability of the comparison of actual photon-momentum-generated forces obtained from the real measurement data against those obtained by the idealized theoretical computations presented in [1–3]. The authors in [3] carried out a very diligent metrological study and reported this difference to be approximately 4%. Agreeably, this could be considered an error because of the assumptions, leading to a systematic underestimation of theoretically

**Table 2.** Measured and retrieved quantities of the parameters with their associated uncertainties. The uncertainties given in parentheses are calculated as the root sum of the squares based on the standard deviations of different measurements and from the datasheets. All data provided for the case of 8.660(73) W input power measured with an integrating sphere thermopile—Si photodiode detector. Relative uncertainty of the optical power is calculated using equation (9).

| Parameter | | Multi-reflection configuration cases | | | |
|---|---|---|---|---|---|
| Number of reflections | | 21 | 33 | 21 | 33 |
| Reflectivity, % | | 99.5(2) | | 99.9950(20) | |
| Optical transmission, % | Measured | 81.03(50) | 73.67(60) | 96.70(120) | 94.68(118) |
| | Theory | 90.46 | 85.18 | 99.89 | 99.85 |
| | Difference[a] | 9.43 | 11.51 | 3.19 | 5.17 |
| Force, nN | Measured | 1202(80) | 2154.3(100) | 1200(55) | 1873(43) |
| | Theory | 1152(30) | 1758(30) | 1213(30) | 1906(30) |
| | Difference | 4.34% | 22.54% | 1.08% | 1.7% |
| Rel. uncer. of opt. power, % | Measured | 6.6586 | 4.6462 | 4.5833 | 2.2958 |
| | Theory | 2.5039 | —[b] | 2.5000 | 1.6017 |

[a]Such a difference in the optical transmission values between those theoretically calculated (equations (3b) and (10)) and those measured with an integrating sphere thermopile—Si photodiode detector (equation (10)) shows that the idealized theoretical model given by equation (3a) should be elaborated in the future to obtain a better estimate of the residual optical power.

[b]The relative error between the theoretically calculated and measured absolute forces surpasses the relative uncertainty calculated by equation (9), so other effects should be taken into account.

**Table 3.** Uncertainty components in terms of relative deviations in percentage.

| Parameter | | Type | Multi-reflection configuration cases | | | |
|---|---|---|---|---|---|---|
| Number of reflections | | | 21 | 33 | 21 | 33 |
| Reflectivity (datasheet) | | B | 0.2 | | 0.002 | |
| Force | Measured photon momentum | B | 6.6 | 4.6 | 4.5 | 2.3 |
| [a] | Balance reproducibility (datasheet, 29.44 nN) | | 2.5 | 1.4 | 2.5 | 1.6 |
| | Balance readability (datasheet, 9.81 nN) | | 0.53 | 0.83 | 0.46 | 0.83 |
| | Floor noise of measured diff. signal (std, 5 nN) | | 0.45 | 0.25 | 0.45 | 0.29 |
| Optical power | | A | 0.6 | 0.6 | 0.2 | 0.6 |

[a]If considering only the instrumentation (balance)-based portion of the uncertainty within the total measured force value for idealized conditions, namely a noise-free environment.

calculated force values. In contrast, however, in accordance with values obtained from our measurements, this difference is reduced, leading us to consider it to be some kind of an overdeveloped value of the measured forces, not only resulting from the portions of the absorbed or diffusely scattered optical power of the stray light, but also other undescribed physical effects.

Additionally, the experimental setup allows us to track the residual power of the outgoing laser beam synchronously during the full period of measurements, instead of using a beam dump. In addition to the expected losses in the optical power after the laser beam undergoes multiple specular reflections in the optical cavity, we have encountered a substantial amount of other unidentified losses when measuring the residual optical power. In figure 8 (*right*), the power transmissions of the optical cavity calculated using equation (10) for the data obtained from the theoretical calculations using equation (3a) and from the measurements via reference power detectors as a ratio of outgoing and ingoing optical powers are shown. We note that

the theoretical calculations of the force values require only the notion of the input power, and its comparison with the measured values shows that the difference is less than 1.7%; however, we admit that the idealized theoretical model and the experimental procedures should be further elaborated to fully describe the aforementioned results.

$$\text{Power} = \frac{c}{1 + R_L} \sum_{i=1}^{N} \frac{F_i}{R_L^{i-1}} \tag{8}$$

$$\frac{u(\text{Power})}{\text{Power}} = 100\% \times \sqrt{\left(\frac{u(F)}{F}\right)^2 + \left(\frac{u(R_L)}{R_L}\right)^2} \tag{9}$$

$$\text{Optical transmission} = \frac{\text{Power}_{N+1}}{\text{Power}} = \frac{\text{Power}_{\text{output}}}{\text{Power}_{\text{input}}} \tag{10}$$

Using the values of the force measurements plugging in equation (8), which is the equations (1) and (5) solved for optical power, the magnitude of the input optical power can be calculated. In figure 9, we show the results of this calculation

normalized against the actual input optical power, which was measured using the reference optical monitor detector (refer schematics in figure 6). Similarly, the resulting relative standard uncertainties of the input optical power can be calculated from the photon-momentum-based force measurement data and can readily be compared with the results of actual input power measurement using the reference detector. More details on data collected for the measured and retrieved quantities are given in table 2.

We performed a general uncertainty analysis of the measurements considering only the main contributing components. The simplified uncertainty analysis is presented in table 3. Because of the 1.7% systematic error between the measured and theoretically calculated force values, which further are adding up on systematic error in the calculated absolute value of the input optical power (refer figure 9), we have avoided giving a single value for the relative combined expanded uncertainty.

Thus, in this work, we have rather attempted to evaluate the main uncertainties associated with the optical power measurements through force measurements using the photon-momentum-based method and demonstrated that upon increasing the input power entering the optical cavity and the number of reflections (figure 10, top four lines), the resulting uncertainties of the generated small force values or, vice versa, the measurements of the input optical power, can be markedly reduced. However, it is necessary to obtain more empirical data at the most critical parameters and measurement ranges to improve the accuracy of the measurements and to further eliminate the systematic errors associated with the measurements, such as for higher-power lasers, for other wavelengths, and for other highly reflective mirrors and incidence angles. The forces generated by the photon-momentum-based method can evidently serve as a means for obtaining direct and accurate measurements of small (calibration) forces below 10 μN through traceable optical power measurements. Thereafter, by virtue of the same relation, the photon-momentum-based force measurements can be used to develop a viable, accurate, and absolute optical power meter (which interchangeably is considered to be a detector or sensor in other applications) at higher ranges with a direct traceability to SI units. In accordance with the existing technical limits and instrumentation capabilities, as well as the theoretical predictions (refer to comparison of figure 3 with figure 10), there is certainly room to improve the accuracy and precision of the optical power and small force measurements by at least several orders of magnitude.

In particular, the variability of possible improvements in the relative uncertainty of the optical power measurements, in comparison with conventional methods, can be seen with reference to the solid black line with 0.05% notation in figures 3 and 10. It shows approximately the typical limit of the relative standard uncertainties of conventional power meters [11] in measuring the laser power at 10 W and above. While our current measurements were made for the 10 W power level, upon its further increase, optical power measurements with improved relative measurement uncertainty can be expected because of the measurements of higher nominal force values with the
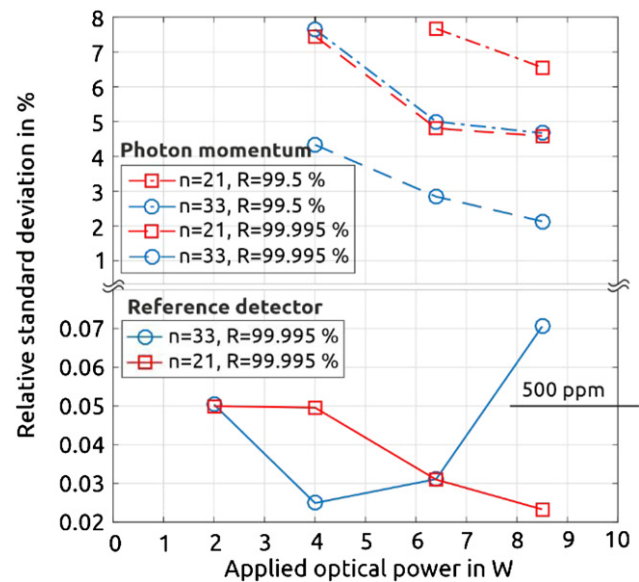


**Figure 10.** Relative standard deviation of the optical power measurement calculated from the photon-momentum-based force measurements (top four lines) and the reference detector (integrating sphere thermopile/Si photodiode) detector (lower two lines).

same force measurement uncertainties (refer figure 10 and table 2). In table 3, we show the nominally existing force measurement, the so-called instrumentation-based (balance) portions, of contributing relative uncertainties. These can be further improved by implementing a more adequate vibration isolation scheme or by using another class of custom-developed force measurement system that would resolve forces below the 10 nN level.

Similarly, one may consider a measurement configuration utilizing an increased or decreased number of multi-reflections to generate higher nominal force values or to reduce the optical losses during the optical transmission. In general, the figure 3 (right) demonstrates a possible roadmap for improvements based on evaluating the main physical quantities underlying the photon momentum method. For example, if one considers using a laser with a power of 100 kW (see *x*-axis), then, independently from the number of reflections, the relative standard uncertainty can be expected to be approximately 70 ppm, which is actually conditioned by the limitations of the reflectivity coefficient (power losses) of the mirrors. Similarly, if one considers a laser with a power of 500–600 W, then for the measurements with 33 reflections (green dashed line), it could be theoretically expected to obtain a relative standard uncertainty of approximately 100 ppm. One should also consider, however, that it is possible to obtain even better relative force measurement uncertainties using a custom-developed force measurement set-up with a fully characterized system and well-established SI traceable chain for the measurement quantities. Our study, however, make use of the more widely accepted approach of calibrating force and mass values in relation to the state-of-the-art EMFC balances and the standard weight pieces, which should have specified expanded relative uncertainties (maximum permissible errors) recommended by 'uncertainties of the weights of the classes E1, E2, F1, and F2 according to OIML R111'.

## 5. Conclusion

We presented a comparison of optical power measurements performed by a photon-momentum-based force measurement system and those performed using a traceable reference standard detector (integrating sphere with attached thermopile and Si photodiode) within an optical power range between 1 W and 10 W at a wavelength of 532 nm. The photon-momentum-based force measurement system developed consisted of two differential EMFC balances, which uses the multi-reflection principle to amplify the generated effective forces created by two quasi-parallel ultra-high reflective mirrors ($>$99.5% and $>$99.995%). Using different measurement configurations (reflectivity, number of reflections), the resulting forces (less than 2000 nN) and the optical powers ingoing and outgoing from the optical cavity were simultaneously monitored, and a preliminary evaluation of the relative standard deviations were demonstrated. The systematic error between the theoretically calculated forces and the forces obtained from the measurements, which was in the range of 10%–20% in our earlier measurements [1, 2], was reduced to less than 1.7%. We consider it an important step toward accurate measurement of the optical power using a non-absorbing apparatus for optical power levels between 1 W and 10 W, which correspond to a force of approximately 2000 nN at the upper limit. Moreover, from the computation principle, the absolute value of the applied laser power, the total number of specular type of laser beam reflections, and the environmental conditions, further improvements to the measurement accuracies and associated uncertainties were discussed.

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