buildings

Article

Supplier Selection through Multicriteria Decision-Making Algorithmic Approach Based on Rough Approximation of Fuzzy Hypersoft Sets for Construction Project

Atiqe Ur Rahman [1], Muhammad Saeed [1], Mazin Abed Mohammed [2], Arnab Majumdar [3] and Orawit Thinnukool [4,*]

[1] Department of Mathematics, University of Management and Technology, Lahore 54000, Pakistan; aurkhb@gmail.com (A.U.R.); muhammad.saeed@umt.edu.pk (M.S.)
[2] College of Computer Science and Information Technology, University of Anbar, Anbar 31001, Iraq; mazinalshujeary@uoanbar.edu.iq
[3] Department of Civil Engineering, Imperial College London, London SW7 2BX, UK; a.majumdar@imperial.ac.uk
[4] College of Arts, Media, and Technology, Chiang Mai University, Chiang Mai 50200, Thailand
* Correspondence: orawit.t@cmu.ac.th

Abstract: The suppliers play a significant role in supply chain management. In supplier selection, factors like market-based exposure, community-based reputation, trust-based status, etc., must be considered, along with the opinions of hired experts. These factors are usually termed as rough information. Most of the literature has disregarded such factors, which may lead to a biased selection. In this study, linguistic variables in terms of triangular fuzzy numbers (TrFn) are used to manage such kind of rough information, then the rough approximations of the fuzzy hypersoft set (FHS-set) are characterized which are capable of handling such informational uncertainties. The FHS-set is more flexible as well as consistent as it tackles the limitation of fuzzy soft sets regarding categorizing parameters into their related sub-classes having their sub-parametric values. Based on these rough approximations, an algorithm is proposed for the optimal selection of suppliers by managing experts' opinions and rough information collectively in the form of TrFn-based linguistic variables. To have a discrete decision, a signed distance method is employed to transform the TrFn-based opinions of experts into fuzzy grades. The proposed algorithm is corroborated with the help of a multi-criteria decision-making application to choose the best supplier for real estate builders. The beneficial facets of the put forward study are appraised through its structural comparison with few existing related approaches. The presented approach is consistent as it is capable to manage rough information and expert's opinions about suppliers collectively by using rough approximations of FHS-set.

Keywords: linguistic variable; rough approximation; triangular fuzzy number; signed distance method; fuzzy hypersoft set

Citation: Rahman, A.U.; Saeed, M.; Mohammed, M.A.; Majumdar, A.; Thinnukool, O. Supplier Selection through Multicriteria Decision-Making Algorithmic Approach Based on Rough Approximation of Fuzzy Hypersoft Sets for Construction Project. *Buildings* **2022**, *12*, 940. https://doi.org/10.3390/buildings12070940

Academic Editors: S. A. Edalatpanah and Jurgita Antucheviciene

Received: 13 May 2022
Accepted: 30 June 2022
Published: 2 July 2022

1. Introduction

Multi-attribute decision-making ($MADM$) is a project for evaluating various alternatives (elements under consideration) in the initial universe by considering suitable attributes and their sub-attribute values. It is a special kind of multi-criteria decision-making ($MCDM$) that performs the same task by considering various criteria. In the classical literature, the $MADM$ has successfully been applied in Supply chain management (SCM) using different classical theoretical and analytical approaches [1–4]. The SCM is the managing of the course of commodities and services. It comprises all progressions that convert unprocessed substances into processed materials. It engages the dynamic reorganization of a business's delivery-side activities to optimize the client's worth and achieve a competitive benefit in the market. It characterizes suppliers' attempts to extend and put into practice supply chains that are as resourceful and reasonable as possible. Supply chains wrap all from production to product growth to the information systems required to direct these tasks. Usually, the SCM tries to centrally manage or connect the production, delivery, and supply of manufactured goods. By supervising the supply chain, the firms can reduce surplus costs and deliver products to the user more rapidly. This is accomplished by keeping firm control of domestic records, domestic production, supply, sales, and the inventories of firm wholesalers. With time, it is observed that various kinds of uncertainties are involved in the selection process of suitable suppliers to manage supply chains. Fuzzy set (F-set) [5] and rough set (R-set) [6] were initiated by Zadeh and Pawlak respectively to tackle uncertain data and information. In F-set, the condition "well defined" of classical set is characterized by a membership function μ_T defined by a membership grade $\mu_T(\hat{z})$ within $[0,1]$ for all members $\hat{z}$ of a non-empty initial universe $\hat{\mathcal{Z}}$ whereas in R-set, indiscernibility relation among the different entities in the information is tackled through their lower and upper approximations. In order to equip F-set and R-set with parameterization contrivance, Molodtsov conceptualized soft set (S-set) [7] which enhanced their applicability in daily-life uncertain scenarios. Many researchers played their role to characterize the elementary properties and operations of S-set like Maji et al. [8] investigated its basic algebraic and axiomatic properties, Ali et al. [9] introduced some of its new set-theoretic operations, Babitha & Sunil [10,11] discussed its relation, functions and ordering, Li [12] introduced some of its operational modifications, Pei & Miao [13] introduced its information system and Sezgin & Atagün [14] made discussion on its operations. Maji et al. [15] discussed the application of S-sets in DM process first time in literature. In order to study the characteristics of F-set, R-set and S-set collectively, Maji et al. [16], Feng et al. [17], and Ali [18] introduced fuzzy soft set (FS-set), soft rough set (SR-set) and fuzzy soft rough set (FSR-set) respectively which are hybridized structures of S-set with F-set and R-set. Çağman et al. [19] investigated various elementary properties of FS-sets along with their applications in daily-life situations. Also, the researchers [20,21] made rich contributions towards the applications of FS-sets.

In various real-life DM scenarios, parameters are required to be further partitioned into their respective parametric values in the form of non-overlapping sets. The theory of S-sets has limitations in tackling such scenarios; therefore, Smarandache [22] developed a hypersoft set (HS-set) which manages this situation by employing a multi-argument approximate function (MAA-function). In this function, the Cartesian product (C-product) of sub-parametric values classes is taken as its domain and the power set of a non-empty universal set as the co-domain. The DM process has become more flexible as well as consistent with the development of HS-sets. Saeed et al. [23] studied various axiomatic properties, set-theoretic cum aggregation operations, relations, functions and matrix representations of HS-sets with expounding numerical examples to make it applicable in various fields of study. Yolcu & Ozturk [24], Debnath [25] and Rahman et al. [26] developed fuzzy hypersoft set (FHS-set) and rough hypersoft set (RHS-set) respectively by combining HS-set with F-set and R-set. They also studied their daily-life applications through DM techniques. Although many researchers studied the extensions of HS-set the contributions of Rahman et al. [27,28] and Saeed et al. [29,30] are most prominent and relevant to this study.

With the digitalization and globalization of economic markets, the importance of well-deliberated supply chain management (SCM) has been increased. SCM is an assimilation of key business processes from the end client to the first supplier, and it provides item administration and data that add an incentive for clients. This way, all companies must have acquaintance with a few solid suppliers. The achievement of an organization is profoundly reliant upon the selection of legitimate suppliers. Thus, the supplier selection problem (SSP) is a significant piece of SCM. Choosing the right suppliers widely lessens the material buying cost and works on corporate seriousness. The SSP includes swapping between numerous criteria that are both qualitative and quantitative in nature. Consequently, SSP transforms into a $MCDM$ problem, and it is important to make a compromise between inconsistent material and immaterial factors to have optimal supplier selection. Construction supply chain management ($CSCM$) is an exceptional and challenging concern within the construction industry due to its expected peripheral hazards and deviations. Several investigators have accentuated the necessity for SCM in the construction-based procedures due to the enhanced complication and range in construction schemes [31]. The proficient $CSCM$ can enhance the working of a scheme and diminish dissipation due to incompetent substances supervision and organization [32]. Construction supply chains (CSC) are not straightforward sequences or procedures but are intricate systems that need supervision and organization of construction-based substances all through the construction-based procedures. This amplifies the hazard and intricacy attached to CSC. Construction-based schemes with complication and distinctiveness often produce various alterations and unexpected states of affairs during the supply progression, where interferences can occur on both domestic and peripheral resources. In various situations, suppliers are an unavoidable basis for peripheral hazards. The assortment of suppliers in CSC is considered an $MCDM$ issue that engages the deliberation of all quality-based aspects. Suppliers in the CSC must be capable of making available a proficient and efficient reaction to probable distractions. Conventionally, managers only concentrate on procuring from suppliers who can provide substances at a cheaper rate, a good quality, and a short due course [33]. Several researchers [34–44] made rich contributions for the optimal selection of suppliers with the discussion of $CSCM$-based case studies and daily life scenarios by using several techniques like grey relational analysis approach, multi-objective programming approach, grey combined compromise solution (CoCoSo-G) method, hybrid fuzzy-based approach and analytic hierarchy process approaches etc.

1.1. Research Gap and Motivation

In order to cope with expected uncertainties in SSP, the theory of fuzzy set and related modellings are utilized by several authors like Xu et al. [45], Huang & Jane [46] and Chang & Hung [47] discussed SSP based on the theory of R-sets. Xiao et al. [48], Patra & Mondal [49] and Chang [50] applied fuzzy and SS-like modelings to discuss SSP through DM. Chatterjee et al. [51] used hybrid model of R-set and FS-set to discuss SSP through DM method. Attributes and their respective sub-attributes play an important role in the $MCDM$ process. The consideration of these factors varies from scenario to scenario, i.e., some scenarios consider attributes merely, whereas partitioning attributes into their respective attribute-value disjoint sets ($AVDS$) is necessary for some other scenarios (also called HS-environment). Ignoring any of these two factors may affect the credibility and reliability of decisions made through the DM process. As discussed earlier, SSP is a $MCDM$ problem and involves many parameters that need to be further partitioned into their respective sub-parametric values in the form of non-overlapping sets. With the keen analysis of the above literature on SCM, it can easily be examined that it is incapable of presenting any model which may be proficient in undertaking the following issues collectively as a sole model:

1. **Ambiguousness of decision-makers:** When the decision-makers are unsure about the selection of suppliers, and they furnish their opinions in the form of linguistic terms which are auxiliary need to be converted to fuzzy membership grades (i.e., fuzzy numbers) for the approximation of suppliers based on chosen parameters to handle with approximation-based vagueness.
2. **Consideration of rough information:** The SSP as an $MCDM$ problem may have a variety of criteria for supplier selection. These criteria may be qualitative or quantitative and traditional or typical. Thus a firm is intended to conduct a general survey about the suppliers based on customary decisive factors like market-based exposure, community-based reputation, trust-based status etc., in addition to standard parameters set by employed decision-makers of the firm. With the help of this process, a dataset of rough information (information obtained through a local survey)

is developed. The concept of rough approximations is required to manage such informational roughness.

3. **The entitlement of MAA-function:** In various $MCDM$ scenarios like SSP, clinical diagnosis, human resource management etc., the consideration of parameters is insufficient to have reliable and unbiased decisions. Therefore, categorising the parameters into their associated disjoint sub-classes having their sub-parametric values is necessary. Such sub-classes are tackled with the entitlement of a novel approximate function called MAA-function, which provides multi-argument-based sub-parametric tuples by taking the C-product of these sub-classes as its domain. It further provides approximations for alternatives (suppliers) based on these tuples. Such kind of scenario is usually termed as HS-environment.

Since the above-described approaches are insufficient to collectively manage the above three problems, this limitation leads to demand for this study. The proposed method is more flexible as well as consistent as it can address the above issues collectively by modifying the concept of rough approximations for HS-environment with a fuzzy setting.

1.2. Main Contributions

Some of main contributions of this study are:

1. The FHS-set based rough approximations are characterized by taking attributes and their respective sub-attributes in the form of linguistic variables (L-variables).
2. As the rough information and opinions of hired experts are both pertinent to be emphasized, both aspects are considered as two separate FHS-sets, which are then integrated with rough approximations. These features are collected in the form of L-variables represented by TrFn.
3. A $MCDM$-based application is discussed for SSP with the proposal of an intelligent algorithm.
4. The signed method is employed to transform TrFn-based L-variables into fuzzy values for having a discrete decision.
5. The advantageous aspects of the proposed study are assessed through comparison with some existing relevant models.

The rest of the paper is structured as follows: In Section 2, some essential definitions like F-set, S-set, SR-set, FS-set, HS-set and HRS-set are reviewed from the literature to assist the readers for better understanding of proposed results. Section 3 characterizes rough approximations under the FHS-set environment and proposes an algorithm based on these modified approximations for SSP. An application in daily-life $MCDM$ is discussed to validate the proposed algorithm. Section 4 illustrates a comparison of the proposed study with some suitable existing structures, and lastly, the paper is summarized with the future directions.

2. Preliminaries

In order to support the main results, some essential definitions recaptured from literature are presented in this section.

The concept of F-set was introduced by Zadeh [5] as a generalization of a classical set to deal with uncertainties in the data under consideration. It employs a membership function to assign a real-valued membership grade to every element in the universal set $\hat{\mathcal{Z}}$ within $\mathbb{I}$ (unit closed interval).

Definition 1 ([5])**.** *A F-set $\mathcal{F}$ over $\hat{\mathcal{Z}}$ is defined as $\mathcal{F} = \{(\hat{z}, \mu_{\mathcal{F}}(\hat{z}))|\hat{z} \in \hat{\mathcal{Z}}\}$ where $\mu_{\mathcal{F}}$ is a membership function stated by $\mu_{\mathcal{F}} : \hat{\mathcal{Z}} \to \mathbb{I}$ and $\mu_{\mathcal{F}}(\hat{z}))$ is a membership grade corresponding to every $\hat{z} \in \hat{\mathcal{Z}}$.*

Definition 2 ([6])**.** *Let $\hat{\mathcal{Z}} \neq \emptyset$ be a finite universe and $\mathcal{R}$ be an equivalence relation (an indiscernibility relation) over $\hat{\mathcal{Z}}$, then the pair $(\hat{\mathcal{Z}}, \mathcal{R})$ is known as a Pawlak approximation space and for a set of parameters $\mathfrak{E}$, the pair $(\hat{\mathcal{Z}}, \mathfrak{E})$ is known as an information system such that each parameter $\hat{e}$ repre-*

sents a function $\hat{e} : \hat{\mathcal{Z}} \rightarrow \mathbb{V}^{\hat{e}}$ where $\mathbb{V}^{\hat{e}}$ is a collection of values of $\hat{e}$. For $\mathcal{R}$ and $\hat{\mathcal{Z}}_1 \subseteq \hat{\mathcal{Z}}$, the following operations $\overleftarrow{\mathcal{R}}_{\hat{\mathcal{Z}}_1} = \{\hat{z} \in \hat{\mathcal{Z}} : [\hat{z}]_{\mathcal{R}} \subseteq \hat{\mathcal{Z}}_1\}$ & $\overrightarrow{\mathcal{R}}_{\hat{\mathcal{Z}}_1} = \{\hat{z} \in \hat{\mathcal{Z}} : [\hat{z}]_{\mathcal{R}} \cap \hat{\mathcal{Z}}_1 \neq \emptyset\}$ are called $\mathcal{R}_{lower}$ and $\mathcal{R}_{upper}$ approximations of $\hat{\mathcal{Z}}_1$ respectively. The relations $\mathcal{R}_{(+)} = \overleftarrow{\mathcal{R}}_{\hat{\mathcal{Z}}_1}$, $\mathcal{R}_{(-)} = \hat{\mathcal{Z}} - \overrightarrow{\mathcal{R}}_{\hat{\mathcal{Z}}_1}$ and $\mathcal{R}_{(B)} = \overrightarrow{\mathcal{R}}_{\hat{\mathcal{Z}}_1} - \overleftarrow{\mathcal{R}}_{\hat{\mathcal{Z}}_1}$ are known as $\mathcal{R}$-positive, $\mathcal{R}$-negative and $\mathcal{R}$-boundary regions of $\hat{\mathcal{Z}}_1$ respectively. If $\mathcal{R}_{(B)} \neq \emptyset$ then the pair $(\overleftarrow{\mathcal{R}}_{\hat{\mathcal{Z}}_1}, \overrightarrow{\mathcal{R}}_{\hat{\mathcal{Z}}_1})$ is called a R-set.

Definition 3 ([7])**.** *A S-set $\mathbb{S}$ over $\hat{\mathcal{Z}}$ is a pair $(\psi_{\mathbb{S}}, \mathfrak{A})$, where $\psi_{\mathbb{S}} : \mathfrak{A} \rightarrow \mathbb{P}(\hat{\mathcal{Z}})$ is an approximate function of $\mathbb{S}$ and $\mathfrak{A} \subseteq \mathfrak{E}$ (a set of parameters). For any $\hat{a} \in \mathfrak{A}$, $\psi_{\mathbb{S}}(\hat{a})$ is called an approximate element of $\mathbb{S}$.*

Example 1. *Let $\hat{\mathcal{Z}} = \{\hat{z}_1, \hat{z}_2, \hat{z}_3, \hat{z}_4, \hat{z}_5, \hat{z}_6\}$, $\mathfrak{E} = \{\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_9\}$ and $\mathfrak{A} = \{\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4\}$ then approximate elements of approximate function $\psi_{\mathbb{S}}$ for s-set $\mathbb{S} = (\psi_{\mathbb{S}}, \mathfrak{A})$ are given as $\psi_{\mathbb{S}}(\hat{p}_1) = \{\hat{z}_1, \hat{z}_3, \hat{z}_6\}$, $\psi_{\mathbb{S}}(\hat{p}_2) = \{\hat{z}_2, \hat{z}_3, \hat{z}_5\}$, $\psi_{\mathbb{S}}(\hat{p}_3) = \{\hat{z}_4, \hat{z}_5, \hat{z}_6\}$, $\psi_{\mathbb{S}}(\hat{p}_4) = \{\hat{z}_1, \hat{z}_2, \hat{z}_5\}$ and s-set is stated as $\mathbb{S} = \{\psi_{\mathbb{S}}(\hat{p}_1), \psi_{\mathbb{S}}(\hat{p}_3), \psi_{\mathbb{S}}(\hat{p}_3), \psi_{\mathbb{S}}(\hat{p}_4)\}$ or*

$$\mathbb{S} = \{ \ (\ \hat{p}_1, \{\hat{z}_1, \hat{z}_3, \hat{z}_6\} \ ), (\ \hat{p}_2, \{\hat{z}_2, \hat{z}_3, \hat{z}_5\} \ ), (\ \hat{p}_3, \{\hat{z}_4, \hat{z}_5, \hat{z}_6\} \ ), (\ \hat{p}_4, \{\hat{z}_1, \hat{z}_2, \hat{z}_5\} \ ) \ \}.$$

While stating the following definition, Feng et al. [17] argued that every S-set can be regarded as an information system, and in this way, every Pawlak's R-set model may be regarded as a particular case of Molodtsov's S-set. They justified their claim in detail with the help of theoretical illustration in their research work ([17], p. 1127).

Definition 4 ([17])**.** *Let $\mathbb{S} = (\psi_{\mathbb{S}}, \mathfrak{A})$ be a S-set over $\hat{\mathcal{Z}}$ then $\mathfrak{B} = (\hat{\mathcal{Z}}, \mathbb{S})$ is regarded as soft-approximation space. For $\mathfrak{B}$ and $\hat{\mathcal{Z}}_1 \subseteq \hat{\mathcal{Z}}$, the following operations $\underbrace{appr_{\mathfrak{B}}}(\hat{\mathcal{Z}}_1) = \{\hat{z} \in \hat{\mathcal{Z}} : \exists \hat{a} \in \mathfrak{A}, [\hat{z} \in \psi_{\mathbb{S}}(\hat{a}) \subseteq \hat{\mathcal{Z}}_1]\}$ and $\overbrace{appr_{\mathfrak{B}}}(\hat{\mathcal{Z}}_1) = \{\hat{z} \in \hat{\mathcal{Z}} : \exists \hat{a} \in \mathfrak{A}, [\hat{z} \in \psi_{\mathbb{S}}(\hat{a}), \psi_{\mathbb{S}}(\hat{a}) \cap \hat{\mathcal{Z}}_1 \neq \emptyset]\}$ are called soft-$\mathfrak{B}_{lower}$ and soft-$\mathfrak{B}_{upper}$ approximations of $\hat{\mathcal{Z}}_1$ respectively. These are also known as soft rough approximations of $\hat{\mathcal{Z}}_1$ w.r.t $\mathfrak{B}$. The relations $\mathfrak{B}_{(+)} = \underbrace{appr_{\mathfrak{B}}}(\hat{\mathcal{Z}}_1)$, $\mathfrak{B}_{(-)} = \sim \overbrace{appr_{\mathfrak{B}}}(\hat{\mathcal{Z}}_1)$ and $\mathfrak{B}_{(B)} = \overbrace{appr_{\mathfrak{B}}}(\hat{\mathcal{Z}}_1) - \underbrace{appr_{\mathfrak{B}}}(\hat{\mathcal{Z}}_1)$ are known as soft-$\mathfrak{B}$-positive, soft-$\mathfrak{B}$-negative and soft-$\mathfrak{B}$-boundary regions of $\hat{\mathcal{Z}}_1$ respectively. If $\mathfrak{B}_{(B)} \neq \emptyset$ then the pair $(\underbrace{appr_{\mathfrak{B}}}(\hat{\mathcal{Z}}_1), \overbrace{appr_{\mathfrak{B}}}(\hat{\mathcal{Z}}_1))$ is called a soft $\mathfrak{B}$-rough set or simply SR-set. Note: $\sim$ denotes set complement.*

Definition 5 ([16])**.** *A FS-set $\mathbb{P}$ over $\hat{\mathcal{Z}}$ is a pair $(\psi_{\mathbb{P}}, \mathbb{Z})$, where $\psi_{\mathbb{P}} : \mathbb{Z} \rightarrow \mathbb{P}_{\mathcal{F}}(\hat{\mathcal{Z}})$ and $\mathbb{Z} \subseteq \mathfrak{E}$. For any $\hat{z} \in \mathbb{Z}$, $\psi_{\mathbb{P}}(\hat{z})$ is known as approximate element of FS-set $\mathbb{P}$ and can be represented as $\psi_{\mathbb{P}}(\hat{z}) = \{(\hat{z}, \mu_T^{\hat{z}}(\hat{z})) | \hat{z} \in \hat{\mathcal{Z}}\}$ where $\mu_T^{\hat{z}}(\hat{z})$ represents membership grade of $\hat{z} \in \hat{\mathcal{Z}}$ subject to the condition that $0 \leq \mu_T^{\hat{z}}(\hat{z}) \leq 1$.*

Definition 6 ([22])**.** *A HS-set $\mathcal{H}$ over $\hat{\mathcal{Z}}$ is a pair $(\mathcal{W}, \mathcal{D})$, where $\mathcal{D}$ is the C-product of $\mathcal{D}^i$, $i = 1, 2, 3, \ldots, n$, $\mathcal{D}^i \cap \mathcal{D}^j = \emptyset \ \forall \ i \neq j$ having attribute values of attributes $\hat{d}^i$, $i = 1, 2, 3, \ldots, n$, $\hat{d}^i \neq \hat{d}^j$, $i \neq j$ respectively and $\mathcal{W} : \mathcal{D} \rightarrow \mathbb{P}(\hat{\mathcal{Z}})$ is called approximate function (so-called MAA-function) of $\mathcal{H}$ and for all $\hat{d} \in \mathcal{D}$, $\mathcal{W}(\hat{d})$ is called approximate element of $\mathcal{H}$.*

As HS-set is the extension of S-set therefore Rahman et al. [27] modified the definition of SR-set and developed the following definition in HS-set environment.

Definition 7 ([27])**.** *Let $\mathcal{H} = (\mathcal{W}, \mathcal{D})$ be a HS-set over $\hat{\mathcal{Z}}$ then $\mathfrak{C} = (\hat{\mathcal{Z}}, \mathcal{D})$ is regarded as HS-approximation space. For $\mathfrak{C}$ and $\hat{\mathcal{Z}}_2 \subseteq \hat{\mathcal{Z}}$, the following operations $\underbrace{appr_{\mathfrak{C}}}(\hat{\mathcal{Z}}_2) = \{\hat{z} \in \hat{\mathcal{Z}} : \exists \hat{d} \in \mathcal{D}, [\hat{z} \in \psi_{\mathbb{S}}(\hat{d}) \subseteq \hat{\mathcal{Z}}_2]\}$ and $\overbrace{appr_{\mathfrak{C}}}(\hat{\mathcal{Z}}_2) = \{\hat{z} \in \hat{\mathcal{Z}} : \exists \hat{d} \in \mathcal{D}, [\hat{z} \in \psi_{\mathbb{S}}(\hat{d}), \psi_{\mathbb{S}}(\hat{d}) \cap \hat{\mathcal{Z}}_2 \neq \emptyset]\}$ are called HS-$\mathfrak{C}_{lower}$ and HS-$\mathfrak{C}_{upper}$ approximations of $\hat{\mathcal{Z}}_2$ respectively. These are also known as hypersoft rough approximations of $\hat{\mathcal{Z}}_2$ w.r.t $\mathfrak{C}$. The relations $\mathfrak{C}_{(+)} = \underbrace{appr_{\mathfrak{C}}}(\hat{\mathcal{Z}}_2)$, $\mathfrak{C}_{(-)} = \sim \overbrace{appr_{\mathfrak{C}}}(\hat{\mathcal{Z}}_2)$*

and $\mathfrak{C}_{(B)} = \overbrace{appr_{\mathfrak{C}}}(\hat{\mathcal{Z}}_2) - \underbrace{appr_{\mathfrak{C}}}(\hat{\mathcal{Z}}_2)$ *are known as HS-*$\mathfrak{C}$*-positive, HS-*$\mathfrak{C}$*-negative and HS-*$\mathfrak{C}$*-boundary regions of* $\hat{\mathcal{Z}}_2$ *respectively. If* $\mathfrak{C}_{(B)} \neq \emptyset$ *then the pair* $(\underbrace{appr_{\mathfrak{C}}}(\hat{\mathcal{Z}}_2), \overbrace{appr_{\mathfrak{C}}}(\hat{\mathcal{Z}}_2))$ *is called a hypersoft* $\mathfrak{C}$*-rough set or simply HSR-set.*

Definition 8 ([24,25])**.** *The HS-set* $\mathcal{H} = (\mathcal{W}, \mathcal{D})$ *over* $\hat{\mathcal{Z}}$ *is said to be FHS-set if* $\mathcal{W} : \mathcal{D} \to \mathbb{F}(\hat{\mathcal{Z}})$ *where* $\mathbb{F}(\hat{\mathcal{Z}})$ *denotes the family of all fuzzy subsets over* $\hat{\mathcal{Z}}$*. Let* $\mathcal{H}_1 = (\mathcal{W}_1, \mathcal{D}_1)$ *and* $\mathcal{H}_2 = (\mathcal{W}_2, \mathcal{D}_2)$ *be two FHS-sets then*

1. $\mathcal{H}_1 \subseteq \mathcal{H}_2$ *if* $\mathcal{D}_1 \subseteq \mathcal{D}_2$ *and for* $\hat{d} \in \mathcal{D}_1, \mathcal{W}_1(\hat{d}) \subseteq \mathcal{W}_2(\hat{d})$.
2. $\mathcal{H}^c = (\mathcal{W}, \mathcal{D})^c = (\mathcal{W}^c, \mathcal{D})$ *where* $\mathcal{W}^c(\hat{d})$ *is complement of set* $\mathcal{W}(\hat{d})$ *for all* $\hat{d} \in \mathcal{D}$.
3. $\mathcal{H}_1 \cup \mathcal{H}_2 = (\mathcal{W}_3, \mathcal{D}_3)$ *where* $\mathcal{D}_3 = \mathcal{D}_1 \cup \mathcal{D}_2$ *and*

$$\mathcal{W}_3(\hat{d}) = \begin{cases} \mathcal{W}_1(\hat{d}) & ;\hat{d} \in (\mathcal{D}_1 - \mathcal{D}_2) \\ \mathcal{W}_2(\hat{d}) & ;\hat{d} \in (\mathcal{D}_2 - \mathcal{D}_1) \\ max\{\mathcal{W}_1(\hat{d}), \mathcal{W}_2(\hat{d})\} & ;\hat{d} \in (\mathcal{D}_1 \cap \mathcal{D}_2) \end{cases}$$

4. $\mathcal{H}_1 \cap \mathcal{H}_2 = (\mathcal{W}_4, \mathcal{D}_4)$ *where* $\mathcal{D}_4 = \mathcal{D}_1 \cap \mathcal{D}_2$ *and for* $\hat{d} \in \mathcal{D}_4, \mathcal{W}_3(\hat{d}) = min\{\mathcal{W}_1(\hat{d}), \mathcal{W}_2(\hat{d})\}$.

Example 2. *Consider a multinational firm that intends to employ an assistant manager for its accounts department. Through advertisement, six candidates were inspected initially by a screening committee. These candidates constitute the set of alternatives i.e.,* $\hat{\mathcal{Z}} = \{\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \mathfrak{C}_4, \mathfrak{C}_5\}$*. With the mutual consensus of all recruitment committee members, the evaluation indicators are decided for the efficient evaluation and ranking of candidates. The parameters for this recruitment are qualification* $(\hat{p}_1)$*, relevant experience in years* $(\hat{p}_2)$ *and computer skill* $(\hat{p}_3)$*. In order to increase the reliability and flexibility of the evaluation, the sub-parametric values with respect to opted parameters are investigated which are arranged in the disjoint sets* $\mathcal{D}_1 = \left\{\hat{d}_{11} = MBA\right\}$*,* $\mathcal{D}_2 = \left\{\hat{d}_{21} = 5, \hat{d}_{22} = 7, \hat{d}_{23} = 10\right\}$ *and* $\mathcal{D}_3 = \left\{\hat{d}_{31} = MS.office\right\}$ *respectively such that* $\mathcal{D} = \mathcal{D}_1 \times \mathcal{D}_2 \times \mathcal{D}_3 = \left\{\hat{d}_1, \hat{d}_2, \hat{d}_3\right\}$*. Then a FHS-set* $(\mathcal{W}, \mathcal{D})$ *is structured as*

$$(\mathcal{W}, \mathcal{D}) = \left\{(\mathcal{W}(\hat{d}_1), \hat{d}_1), (\mathcal{W}(\hat{d}_2), \hat{d}_2), (\mathcal{W}(\hat{d}_3), \hat{d}_3)\right\}$$

where

$$\begin{array}{l} \mathcal{W}(\hat{d}_1) = \left\{ \frac{\mathfrak{C}_1}{<0.2>}, \frac{\mathfrak{C}_2}{<0.3>}, \frac{\mathfrak{C}_3}{<0.4>}, \frac{\mathfrak{C}_4}{<0.5>}, \frac{\mathfrak{C}_5}{<0.6>} \right\} \\ \mathcal{W}(\hat{d}_2) = \left\{ \frac{\mathfrak{C}_1}{<0.3>}, \frac{\mathfrak{C}_2}{<0.4>}, \frac{\mathfrak{C}_3}{<0.5>}, \frac{\mathfrak{C}_4}{<0.6>}, \frac{\mathfrak{C}_5}{<0.7>} \right\} \\ \mathcal{W}(\hat{d}_3) = \left\{ \frac{\mathfrak{C}_1}{<0.4>}, \frac{\mathfrak{C}_2}{<0.5>}, \frac{\mathfrak{C}_3}{<0.6>}, \frac{\mathfrak{C}_4}{<0.7>}, \frac{\mathfrak{C}_5}{<0.8>} \right\} \end{array}.$$

Hence

$$(\mathcal{W}, \mathcal{D}) = \left\{ \begin{array}{l} \left(\{\frac{\mathfrak{C}_1}{<0.2>}, \frac{\mathfrak{C}_2}{<0.3>}, \frac{\mathfrak{C}_3}{<0.4>}, \frac{\mathfrak{C}_4}{<0.5>}, \frac{\mathfrak{C}_5}{<0.6>}\}, \hat{d}_1\right), \\ \left(\{\frac{\mathfrak{C}_1}{<0.3>}, \frac{\mathfrak{C}_2}{<0.4>}, \frac{\mathfrak{C}_3}{<0.5>}, \frac{\mathfrak{C}_4}{<0.6>}, \frac{\mathfrak{C}_5}{<0.7>}\}, \hat{d}_2\right), \\ \left(\{\frac{\mathfrak{C}_1}{<0.4>}, \frac{\mathfrak{C}_2}{<0.5>}, \frac{\mathfrak{C}_3}{<0.6>}, \frac{\mathfrak{C}_4}{<0.7>}, \frac{\mathfrak{C}_5}{<0.8>}\}, \hat{d}_3\right) \end{array} \right\}.$$

3. Methodology

In this part of the paper, the adopted methodology of the proposed study is explained by characterizing some of its essential components. First, the following definitions are reviewed as they have a vital role in formulating the proposed methodology. These definitions and their various properties and features can be studied in detail from [52–58].

3.1. Essential Definitions

Definition 9. *A fuzzy number* $\tilde{A}$ *is a mapping* $\psi_{\tilde{A}} : \mathbb{R} \to \mathbb{I}$ *validating the following axioms:*

1. $\psi_{\tilde{A}}$ is upper semi-continuous.
2. For all $\hat{r}, \hat{s} \in \mathbb{R}, \alpha \in \mathbb{I}$, $\psi_{\tilde{A}}(\alpha\hat{r} + (1-\alpha)\hat{s}) \geq min\{\psi_{\tilde{A}}(\hat{r}), \psi_{\tilde{A}}(\hat{s})\}$.
3. $\exists\ \hat{u} \in \mathbb{R}$ such that $\psi_{\tilde{A}}(\hat{u}) = 1$
4. $\overline{\mathcal{S}(\tilde{A})}$ is compact, where bar denotes closure and $\mathcal{S}$ denotes support.

The graphical representation of fuzzy number is given in Figure 1.

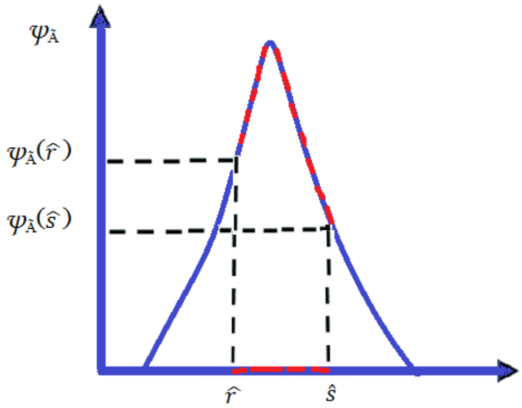

Figure 1. Graph of Fuzzy Number.

Definition 10. *A triangular fuzzy number (TrFn) $\tilde{T} = (\hat{t}_1, \hat{t}_2, \hat{t}_3)$ with its membership function $\psi_{\tilde{T}}$ is stated as*

$$\psi_{\tilde{T}}(\hat{z}) = \begin{cases} \frac{\hat{z}-\hat{t}_1}{\hat{t}_2-\hat{t}_1} & \hat{z} \in [\hat{t}_1, \hat{t}_2] \\ \frac{\hat{t}_3-\hat{z}}{\hat{t}_3-\hat{t}_2} & \hat{z} \in [\hat{t}_2, \hat{t}_3] \\ 0 & otherwise \end{cases}$$

If $\tilde{T} = (\hat{t}_1, \hat{t}_2, \hat{t}_3)$ and $\tilde{U} = (\hat{u}_1, \hat{u}_2, \hat{u}_3)$ are two TrFns then their arithmetic operations are given below:

1. *Addition:* $\tilde{T} \tilde{+} \tilde{U} = (\hat{t}_1 + \hat{u}_1, \hat{t}_2 + \hat{u}_2, \hat{t}_3 + \hat{u}_3)$.
2. *Multiplication:* $\tilde{T} \tilde{\times} \tilde{U} \approx (\hat{t}_1 \times \hat{u}_1, \hat{t}_2 \times \hat{u}_2, \hat{t}_3 \times \hat{u}_3)$.
3. *Scalar Multiplication:* $\hat{a} \tilde{\times} \tilde{U} = (\hat{a}\hat{u}_1, \hat{a}\hat{u}_2, \hat{a}\hat{u}_3)$.

The graph of TrFn is given in Figure 2.

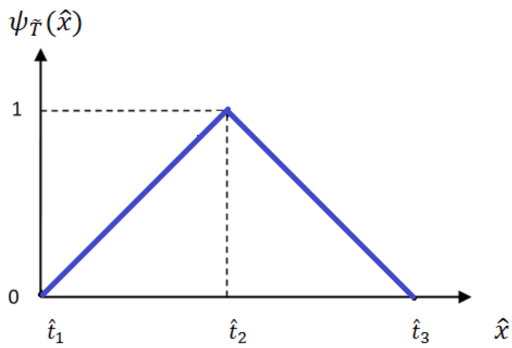

Figure 2. Graphical Representation of Triangular Fuzzy Number.

Definition 11. *Sometimes in DM while getting a specific point in the decision, it is necessary to have crisp values. The transformation of fuzzy numbers to crisp real numbers is known as defuzzification. Although several techniques exist in literature for such kind of transformation, the signed distance method [58] is preferred due to its simplicity. This method is explained below for the conversion of TrFn $\tilde{T} = (\hat{t}_1, \hat{t}_2, \hat{t}_3)$*

$$d(\tilde{T}) = \frac{1}{2}\int_{0}^{1} \{\tilde{T}_l(\eta) + \tilde{T}_r(\eta)\}d\eta \; \forall \; \eta \; \in \mathbb{I} \tag{1}$$

where $\tilde{T}_l(\eta) = \hat{t}_1 + (\hat{t}_2 - \hat{t}_1)\eta$ *and* $\tilde{T}_r(\eta) = \hat{t}_3 - (\hat{t}_3 - \hat{t}_2)\eta \Rightarrow d(\tilde{T}) = \frac{1}{4}(\hat{t}_1 + 2\hat{t}_2 + \hat{t}_3)$.

Definition 12. *In DM, it is often difficult to tackle qualitative entities (i.e., variables), e.g., large, very large, huge etc. Therefore, such entities are needed to regard numeric values. Such variables are tackled with the help of a linguistic variable, a kind of mapping from a set of linguistic entities to a specific range of real numbers. For example, Chatterjee et al. [51] considered the "quality of product" as a linguistic variable and presented it in the form of TrFns which is depicted in Table 1.*

Table 1. *L*-variables and their representation in TrFns.

Linguistic Terms	Relevant TrFns
Very Poor (η_1)	(1, 1, 2)
Poor (η_2)	(2, 3, 4)
Mild Poor (η_3)	(3, 4, 5)
Fair (η_4)	(4, 5, 6)
Mild Good (η_5)	(5, 6, 7)
Good (η_6)	(6, 7, 8)
Very Good (η_7)	(8, 9, 10)

3.2. Procedure for Criteria Selection

In *SSP*, the criteria (parameters) play a significant role in approximating alternatives (suppliers). Usually, they perform the role of input variables for any algorithm that is proposed for the appropriate selection of suppliers. Therefore decision-makers should select the decisive criteria with great care and investigation. Several criteria (parameters) have already been discussed by numerous researchers for supplier selection in *CSCM* but the criteria discussed in researches [34–44] are found more significant and relevant to the proposed study. As the proposed study is concerned majorly with the consideration of *HS*-environment, therefore with the keen analysis of criteria given by these researchers, only those criteria are likely to opt which may further be categorized into their sub-parametric valued disjoint classes to fulfil the requirements of *HS*-environment. As the *MAA*-function has a major role in *HS*-environment, the *C*-product of sub-parametric valued disjoint classes is determined to obtain sub-parametric tuples for the domain of the *MAA*-function. In other words, the opted criteria are transformed to multi-arguments based sub-parametric tuples in *HS*-based *CSCM*. The sub-parametric tuples are further filtered on the preferential criterion set by the mutual consensus of decision-makers.

3.3. Decision Makers and Their Role

A decision-maker is a person or group responsible for making strategically important decisions based on a number of variables, including time constraints, resources available, the amount and type of information available and the number of stakeholders involved. They are important because their main goals are typically to keep the company functioning efficiently and make decisions that help it continue growing. Decision-makers determine larger company decisions and work to keep it efficiently running so other employees can focus primarily on their day-to-day projects. In *CSCM*, the procurement department usually performs tasks like employment of experts for *DM*, advertisement of bids for bids, scrutiny of suppliers, etc. This department is headed by a procurement manager who leads a team of procurement agents and specialists. In the case of the proposed study, there are the following three committees of decision-makers to whom the procurement manager assigns some specific tasks:

Committee A: This committee consists of some firm employees who have expertise in scrutinizing the procurement proposals. A procurement officer heads it. The members of this committee are responsible for assessing products, services and suppliers and negotiating contracts. They are also responsible for ensuring that approved purchases are of sufficient quality and are cost-efficient. However, in the case of this study, their major role is to scrutinize the proposals of suppliers received through the adopted procedure (call for bids) by the procurement department.

Committee B: This committee consists of those local employees of the firm who have expertise in surveying. The surveying officer of the firm heads it. The committee members are responsible for collecting rough information about the suppliers who applied their proposals by conducting various surveys in the markets and other relevant localities. They are bound to collect such information in the form of linguistic terms.

Committee C: This committee consists of experts employed by the procurement department through the standard recruitment process. This committee plays a significant role throughout the supplier selection process. It is headed by an operational manager responsible for regulating appropriate policies to make intelligent decisions. Their main role is to accomplish the DM process by analyzing the collected rough information and approximating their opinions about the suppliers.

4. Rough Approximation of Fuzzy Hypersoft Set

In this part of the paper, rough approximations of FHS-sets are characterized which have membership grades as linguistic entities represented by TrFns.

Definition 13. *(Rough Approximations of FHS-sets) Assume that* $\hat{\mathcal{Z}} = \left\{\hat{\mathbb{C}}_1, \hat{\mathbb{C}}_2, \hat{\mathbb{C}}_3, \ldots, \hat{\mathbb{C}}_n\right\}$ *be a set of alternatives,* $\mathcal{P} = \{\hat{e}_1, \hat{e}_2, \hat{e}_3, \ldots, \hat{e}_p\}$ *be a set of attributes and their corresponding* $AVDS$ *are* $\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \ldots, \mathcal{D}_p$ *such that* $\mathcal{D} = \mathcal{D}_1 \times \mathcal{D}_2 \times \mathcal{D}_3 \times \cdots \times \mathcal{D}_p = \{\hat{d}_1, \hat{d}_2, \hat{d}_3, \ldots, \hat{d}_q\}$ *where each* $\hat{d}_i$ $(i = 1, 2, \ldots, q)$ *is a p-tuple element of* $\mathcal{D}$ *and* $q = \prod_{i=1}^{p} |\mathcal{D}_i|$, $|\bullet|$ *denotes set cardinality. Take* $\mathcal{J} = \{\hat{d}_1, \hat{d}_2, \hat{d}_3, \ldots, \hat{d}_r\} \subseteq \mathcal{D}$ *with* $r \leq q$. *Let* $\mathbb{E} = \{\mathbb{E}_1, \mathbb{E}_2, \mathbb{E}_3, \ldots, \mathbb{E}_k\}$ *be a set of experts hired by relevant firm to evaluate alternatives through parameters. Two FHS-sets* $\mathcal{H}_1 = (\mathcal{U}, \mathcal{D})$ *and* $\mathcal{H}_2 = (\mathcal{V}, \mathbb{E})$ *are constructed with their membership functions* $\mathcal{U} : \mathcal{D} \to \mathbb{P}(\hat{\mathcal{Z}})$ *and* $\mathcal{V} : \mathbb{E} \to \mathbb{P}(\hat{\mathcal{Z}})$ *respectively. For* $\mathcal{H}_1 = (\mathcal{U}, \mathcal{D})$, *approximate elements are* $\mathcal{W}(\hat{d}_1) = \left\{ \frac{\hat{\mathbb{C}}_1}{\vartheta_{11}}, \frac{\hat{\mathbb{C}}_2}{\vartheta_{12}}, \frac{\hat{\mathbb{C}}_3}{\vartheta_{13}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\vartheta_{1n}} \right\}$, $\mathcal{W}(\hat{d}_2) = \left\{ \frac{\hat{\mathbb{C}}_1}{\vartheta_{21}}, \frac{\hat{\mathbb{C}}_2}{\vartheta_{22}}, \frac{\hat{\mathbb{C}}_3}{\vartheta_{23}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\vartheta_{2n}} \right\}, \ldots, \mathcal{W}(\hat{d}_r) = \left\{ \frac{\hat{\mathbb{C}}_1}{\vartheta_{r1}}, \frac{\hat{\mathbb{C}}_2}{\vartheta_{r2}}, \frac{\hat{\mathbb{C}}_3}{\vartheta_{r3}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\vartheta_{rn}} \right\}$.

Hence

$$(\mathcal{W}, \mathcal{D}) = \left\{ \left(\{\frac{\hat{\mathbb{C}}_1}{\vartheta_{11}}, \frac{\hat{\mathbb{C}}_2}{\vartheta_{12}}, \frac{\hat{\mathbb{C}}_3}{\vartheta_{13}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\vartheta_{1n}}\}, \hat{d}_1\right), \left(\{\frac{\hat{\mathbb{C}}_1}{\vartheta_{21}}, \frac{\hat{\mathbb{C}}_2}{\vartheta_{22}}, \frac{\hat{\mathbb{C}}_3}{\vartheta_{23}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\vartheta_{2n}}\}, \hat{d}_2\right), \ldots, \left(\{\frac{\hat{\mathbb{C}}_1}{\vartheta_{r1}}, \frac{\hat{\mathbb{C}}_2}{\vartheta_{r2}}, \frac{\hat{\mathbb{C}}_3}{\vartheta_{r3}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\vartheta_{rn}}\}, \hat{d}_r\right) \right\}$$

where ϑ_{ij}, ${}_1i^r$ & ${}_1j^n$, *is the linguistic variable denoted by TrFn* $(\vartheta^1_{ij}, \vartheta^2_{ij}, \vartheta^3_{ij})$. *Similarly for* $\mathcal{H}_2 = (\mathcal{V}, \mathbb{E})$, *approximate elements are* $\mathcal{V}(\mathbb{E}_1) = \left\{ \frac{\hat{\mathbb{C}}_1}{\theta_{11}}, \frac{\hat{\mathbb{C}}_2}{\theta_{12}}, \frac{\hat{\mathbb{C}}_3}{\theta_{13}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\theta_{1n}} \right\}$, $\mathcal{V}(\mathbb{E}_2) = \left\{ \frac{\hat{\mathbb{C}}_1}{\theta_{21}}, \frac{\hat{\mathbb{C}}_2}{\theta_{22}}, \frac{\hat{\mathbb{C}}_3}{\theta_{23}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\theta_{2n}} \right\}, \ldots, \mathcal{V}(\mathbb{E}_r) = \left\{ \frac{\hat{\mathbb{C}}_1}{\theta_{k1}}, \frac{\hat{\mathbb{C}}_2}{\theta_{k2}}, \frac{\hat{\mathbb{C}}_3}{\theta_{k3}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\theta_{kn}} \right\}$.

Hence

$$(\mathcal{V}, \mathbb{E}) = \left\{ \left(\{\frac{\hat{\mathbb{C}}_1}{\theta_{11}}, \frac{\hat{\mathbb{C}}_2}{\theta_{12}}, \frac{\hat{\mathbb{C}}_3}{\theta_{13}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\theta_{1n}}\}, \mathbb{E}_1\right), \left(\{\frac{\hat{\mathbb{C}}_1}{\theta_{21}}, \frac{\hat{\mathbb{C}}_2}{\theta_{22}}, \frac{\hat{\mathbb{C}}_3}{\theta_{23}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\theta_{2n}}\}, \mathbb{E}_2\right), \ldots, \left(\{\frac{\hat{\mathbb{C}}_1}{\theta_{k1}}, \frac{\hat{\mathbb{C}}_2}{\theta_{k2}}, \frac{\hat{\mathbb{C}}_3}{\theta_{k3}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\theta_{kn}}\}, \mathbb{E}_k\right) \right\}$$

where θ_{ij}, ${}_1i^k$ & ${}_1j^n$, *is the linguistic variable denoted by TrFn* $(\theta^1_{ij}, \theta^2_{ij}, \theta^3_{ij})$. *Now rough approximation of FHS-set* $\mathcal{H}_2 = (\mathcal{V}, \mathbb{E})$ *is characterized by using FHS-set* $\mathcal{H}_1 = (\mathcal{U}, \mathcal{D})$ *which leads to construction of two new FHS-sets* $(\mathcal{V}^L, \mathbb{E})$ *and* $(\mathcal{V}^U, \mathbb{E})$ *called lower and upper approximation respectively. Lower approximation* $(\mathcal{V}^L, \mathbb{E})$ *is defined as*

$$(\mathcal{V}^L, \mathbb{E}) = \left\{ \begin{array}{l} \left(\mathcal{V}^L(\mathbb{E}_1) = \{\frac{\hat{\mathbb{C}}_1}{\omega_{11}}, \frac{\hat{\mathbb{C}}_2}{\omega_{12}}, \frac{\hat{\mathbb{C}}_3}{\omega_{13}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\omega_{1n}}\}, \mathbb{E}_1\right), \left(\mathcal{V}^L(\mathbb{E}_2) = \{\frac{\hat{\mathbb{C}}_1}{\omega_{21}}, \frac{\hat{\mathbb{C}}_2}{\omega_{22}}, \frac{\hat{\mathbb{C}}_3}{\omega_{23}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\omega_{2n}}\}, \mathbb{E}_2\right), \\ \ldots, \left(\mathcal{V}^L(\mathbb{E}_k) = \{\frac{\hat{\mathbb{C}}_1}{\omega_{k1}}, \frac{\hat{\mathbb{C}}_2}{\omega_{k2}}, \frac{\hat{\mathbb{C}}_3}{\omega_{k3}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\omega_{kn}}\}, \mathbb{E}_k\right) \end{array} \right\}$$

where ω_{ij}, ${}_1i^k$ & ${}_1j^n$, *is the linguistic variable denoted by TrFn* $(\omega^1_{ij}, \omega^2_{ij}, \omega^3_{ij})$ *and* $\omega_{ij} \in \mathcal{V}^L(\mathbb{E}_i)$ *can be computed as*

$$\omega_{ij} = \bigcup_{\alpha} \left\{ \ \zeta_{\alpha j} = (\zeta^1_{\alpha j}, \zeta^2_{\alpha j}, \zeta^3_{\alpha j}) \in \mathcal{W}(\hat{d}_\alpha) : \zeta_{\alpha j} < \theta_{ij} \in \mathcal{V}(\mathbb{E}_i), \ _1i^k \,\&\, _1j^n \ \right\}. \tag{2}$$

The symbol $\bigcup$ *is operated by* $(min, mean, max)$ *rule. Similarly upper approximation* $(\mathcal{V}^U, \mathbb{E})$ *is defined as*

$$(\mathcal{V}^U, \mathbb{E}) = \left\{ \begin{array}{l} \left(\mathcal{V}^U(\mathbb{E}_1) = \{\frac{\hat{\mathbb{C}}_1}{\sigma_{11}}, \frac{\hat{\mathbb{C}}_2}{\sigma_{12}}, \frac{\hat{\mathbb{C}}_3}{\sigma_{13}}, ..., \frac{\hat{\mathbb{C}}_n}{\sigma_{1n}}\}, \mathbb{E}_1\right), \left(\mathcal{V}^U(\mathbb{E}_2) = \{\frac{\hat{\mathbb{C}}_1}{\sigma_{21}}, \frac{\hat{\mathbb{C}}_2}{\sigma_{22}}, \frac{\hat{\mathbb{C}}_3}{\sigma_{23}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\sigma_{2n}}\}, \mathbb{E}_2\right), \\ \ldots, \left(\mathcal{V}^U(\mathbb{E}_k) = \{\frac{\hat{\mathbb{C}}_1}{\sigma_{k1}}, \frac{\hat{\mathbb{C}}_2}{\sigma_{k2}}, \frac{\hat{\mathbb{C}}_3}{\sigma_{k3}}, \ldots, \frac{\hat{\mathbb{C}}_n}{\sigma_{kn}}\}, \mathbb{E}_k\right) \end{array} \right\}$$

where σ_{ij}, $_1i^k \,\&\, _1j^n$, *is the linguistic variable denoted by TrFn* $(\sigma^1_{ij}, \sigma^2_{ij}, \sigma^3_{ij})$ *and* $\sigma_{ij} \in \mathcal{V}^U(\mathbb{E}_i)$ *can be computed as*

$$\sigma_{ij} = \bigcup_{\beta} \left\{ \ \xi_{\beta j} = (\xi^1_{\beta j}, \xi^2_{\beta j}, \xi^3_{\beta j}) \in \mathcal{W}(\hat{d}_\beta) : \xi_{\beta j} \cap \theta_{ij} \neq \emptyset, \ _1i^k \,\&\, _1j^n \ \right\}. \tag{3}$$

The symbol $\bigcup$ *is operated by* $(min, mean, max)$ *rule. If there does not exist any* $\xi_{\beta j}$ *then* $\sigma_{ij} = \theta_{ij}$*. The pictorial representation of these approximations is presented in Figure 3.*

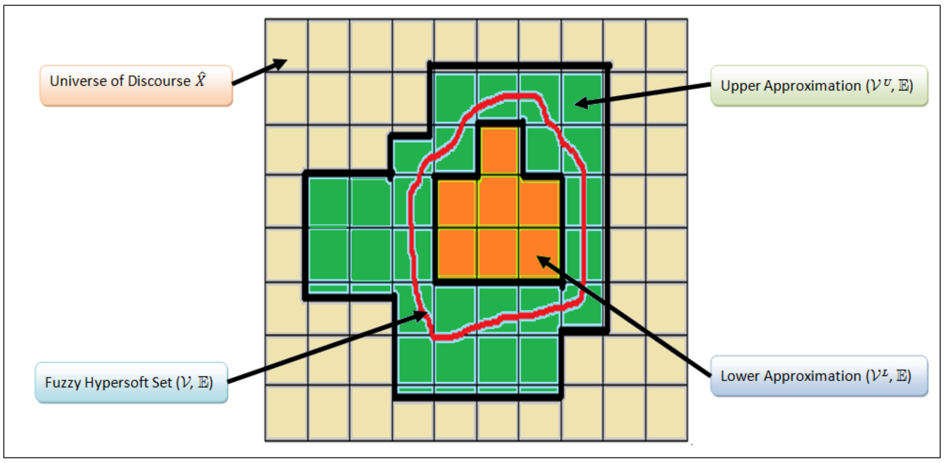

Figure 3. Graphical Representation of Rough Approximations of *FHS*-set $(\mathcal{V}, \mathbb{E})$.

These approximations are explained in the following example.

Example 3. *Taking the data given in Example 2, the given scenario is transformed to judge the overall "job suitability" of candidates and all the sub-parametric tuples are regarded as linguistic terms with their representation as TrFns given in Table 1 therefore a FHS-set* $(\mathcal{U}, \mathcal{D})$ *for fairly poor "job suitability" of candidates is given as*

$$(\mathcal{U}, \mathcal{D}) = \left\{ \ \left(\{\frac{\hat{\mathbb{C}}_1}{\eta_2}, \frac{\hat{\mathbb{C}}_2}{\eta_3}, \frac{\hat{\mathbb{C}}_3}{\eta_4}, \frac{\hat{\mathbb{C}}_4}{\eta_4}, \frac{\hat{\mathbb{C}}_5}{\eta_1}\}, \hat{d}_1\right), \left(\{\frac{\hat{\mathbb{C}}_1}{\eta_4}, \frac{\hat{\mathbb{C}}_2}{\eta_4}, \frac{\hat{\mathbb{C}}_3}{\eta_2}, \frac{\hat{\mathbb{C}}_4}{\eta_3}, \frac{\hat{\mathbb{C}}_5}{\eta_4}\}, \hat{d}_2\right), \left(\{\frac{\hat{\mathbb{C}}_1}{\eta_2}, \frac{\hat{\mathbb{C}}_2}{\eta_1}, \frac{\hat{\mathbb{C}}_3}{\eta_4}, \frac{\hat{\mathbb{C}}_4}{\eta_2}, \frac{\hat{\mathbb{C}}_5}{\eta_7}\}, \hat{d}_3\right) \ \right\}$$

and a FHS-set $(\mathcal{V}, \mathbb{E}_2)$ *based on the evaluation of expert 2 is given as*

$$(\mathcal{V}, \mathbb{E}_2) = \left\{ \ \left(\{\frac{\hat{\mathbb{C}}_1}{\eta_4}, \frac{\hat{\mathbb{C}}_2}{\eta_3}, \frac{\hat{\mathbb{C}}_3}{\eta_3}, \frac{\hat{\mathbb{C}}_4}{\eta_6}, \frac{\hat{\mathbb{C}}_5}{\eta_4}\}, \mathbb{E}_2\right) \ \right\}.$$

On the basis of $(\mathcal{V}, \mathbb{E}_2)$ *and* $(\mathcal{U}, \mathcal{D})$*, two FHS-sets* $(\mathcal{V}^L, \mathbb{E}_2)$ *and* $(\mathcal{V}^U, \mathbb{E}_2)$ *are computed as* $(\mathcal{V}^L, \mathbb{E}_2) = \left\{ \ \left(\{\frac{\hat{\mathbb{C}}_1}{\omega_{21}}, \frac{\hat{\mathbb{C}}_2}{\omega_{22}}, \frac{\hat{\mathbb{C}}_3}{\omega_{23}}, \frac{\hat{\mathbb{C}}_4}{\omega_{24}}, \frac{\hat{\mathbb{C}}_5}{\omega_{25}}\}, \mathbb{E}_2\right) \ \right\}$ *where*
$\omega_{21} = \bigcup\{(2,3,4),(4,5,6)\} = (2,4,6)$,
$\omega_{22} = \bigcup\{(1,1,2),(3,4,5)\} = (1,1/2,5)$,
$\omega_{23} = \bigcup\{(2,3,4)\} = (2,3,4)$,
$\omega_{24} = \bigcup\{(4,5,6),(5,6,7),(2,3,4)\} = (2,14/3,7)$,
$\omega_{22} = \bigcup\{(1,1,2),(4,5,6)\} = (1,3,6)$ *and*

$$(\mathcal{V}^U, \mathbb{E}_2) = \left\{ \left(\{ \frac{\hat{\mathbb{C}}_1}{\xi_{21}}, \frac{\hat{\mathbb{C}}_2}{\xi_{22}}, \frac{\hat{\mathbb{C}}_3}{\xi_{23}}, \frac{\hat{\mathbb{C}}_4}{\xi_{24}}, \frac{\hat{\mathbb{C}}_5}{\xi_{25}} \}, \mathbb{E}_2 \right) \right\} \textit{ where}$$

$\xi_{21} = \bigcup\{(2,3,4),(4,5,6)\} = (2,4,6)$,
$\xi_{22} = \bigcup\{(3,4,5),(5,6,7)\} = (3,5,7)$,
$\xi_{23} = \bigcup\{(4,5,6)\} = (4,5,6)$,
$\xi_{24} = \bigcup\{(4,5,6),(5,6,7)\} = (4,11/2,7)$,
$\xi_{22} = \bigcup\{(4,5,6)\} = (4,5,6)$.

Now some axiomatic properties of TrFn-based linguistic terms, $(\mathcal{V}^L, \mathbb{E})$ and $(\mathcal{V}^U, \mathbb{E})$ are studied. Let $\tau^{\hat{a}}$ and $\tau^{\hat{b}}$ are two linguistic terms with their respective representations in terms of TrFns are $(\hat{a}_1, \hat{a}_2, \hat{a}_3)$ and $(\hat{b}_1, \hat{b}_2, \hat{b}_3)$ then we have the following properties:

1. If $\hat{a}_3 > \hat{b}_3$ then $\tau^{\hat{a}} > \tau^{\hat{b}}$.
2. If $\hat{a}_1 > \hat{b}_1$ and $\hat{a}_3 = \hat{b}_3$ then $\tau^{\hat{a}} > \tau^{\hat{b}}$.
3. If $\hat{a}_2 > \hat{b}_2$, $\hat{a}_1 = \hat{b}_1$ and $\hat{a}_3 = \hat{b}_3$ then $\tau^{\hat{a}} > \tau^{\hat{b}}$.

The rough approximations $(\mathcal{V}^L, \mathbb{E})$ and $(\mathcal{V}^U, \mathbb{E})$ validate the following axiomatic properties:

1. If $\theta_{kj} \leq \min\{\vartheta 1j, \vartheta 2j, \ldots, \vartheta rj\}$ then for any $\hat{\mathbb{C}}_j$, $(\mathcal{V}^L, \mathbb{E}_k)$ is remained unchanged from the respective decision of $\mathbb{E}_k$. In Example 3, the expert $\mathbb{E}_2$ has recommended η_2 as decision for candidate $\hat{\mathbb{C}}_3$ therefore organization rated $\hat{\mathbb{C}}_3$ by η_4, η_2, η_4 w.r.t all attribute-valued tuples i.e., $\eta_2 = \min\{\eta_4, \eta_2, \eta_4\}$ therefore its $(\mathcal{V}^L, \mathbb{E}_2)$ in TrFn is $(2,3,4)$.
2. For any $\hat{\mathbb{C}}_j$, $(\mathcal{V}^U, \mathbb{E}_k)$ is remained unchanged from the respective decision of $\mathbb{E}_k$ if
   (i) $\vartheta^1_{ij} < \theta^1_{kj}$ then it necessary implies that $\vartheta^3_{ij} < \theta^1_{kj}$.
   (ii) $\vartheta^1_{ij} < \theta^1_{kj}$ then it necessary implies that $\vartheta^1_{ij} < \theta^3_{kj}$.
   (iii) $\vartheta_{ij} = \theta_{kj}$ where ${}_1i^r$ keeping j fixed.

   In Example 3, the expert $\mathbb{E}_2$ has recommended η_4 as decision for candidate $\hat{\mathbb{C}}_5$ therefore organization rated $\hat{\mathbb{C}}_5$ by η_1, η_4, η_7 w.r.t all attribute-valued tuples and conditions (i), (ii) and (iii) are validated by η_1, η_4 and η_7 respectively therefore its $(\mathcal{V}^U, \mathbb{E}_2)$ is η_4 that is expressed in TrFn as $(4,5,6)$.
3. For any expert $\mathbb{E}_k$, $\omega_{ij} \leq \sigma_{ij}$ for all ${}_1i^k$ & ${}_1j^n$.
4. If $M_1 = [\theta_{ij}]$ and $M_2 = [\omega_{ij}]$ are two matrices containing opinions of experts and lower approximation respectively then identical valued entries in jth column of M_1 will have similar result in M_2. This result is also valid for upper approximation as well.

4.1. Procedure for Optimal Selection of Supplier for Construction Project

In this section, a procedure is followed from [51] with partial modifications for the best selection of supplier. Suppose that $\hat{\hat{\mathbb{C}}} = \left\{\hat{\mathbb{C}}_1, \hat{\mathbb{C}}_2, \hat{\mathbb{C}}_3, \ldots, \hat{\mathbb{C}}_n\right\}$ be a set of suppliers (alternatives), $\mathfrak{E} = \{\hat{e}_1, \hat{e}_2, \hat{e}_3, \ldots, \hat{e}_p\}$ be a set of attributes and their corresponding $AVDS$ are $\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3, \ldots, \mathcal{K}_p$ such that $\mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2 \times \mathcal{K}_3 \times \cdots \times \mathcal{K}_p = \{\hat{k}_1, \hat{k}_2, \hat{k}_3, \ldots, \hat{k}_q\}$ where each $\hat{k}_i$ $(i = 1, 2, \ldots, q)$ is a p-tuple element of $\mathcal{K}$ and $q = \prod_{i=1}^{p} |\mathcal{K}_i|$, $|\bullet|$ denotes set cardinality. Take $\mathcal{J} = \{\hat{k}_1, \hat{k}_2, \hat{k}_3, \ldots, \hat{k}_r\} \subseteq \mathcal{K}$ with $r \leq q$. Let $\mathbb{E} = \{\mathbb{E}_1, \mathbb{E}_2, \mathbb{E}_3, \ldots, \mathbb{E}_m\}$ be a set of experts hired by relevant firm to evaluate alternatives through parameters. The following Algorithm 1 is proposed to rank the suppliers:

Algorithm 1: Optimal Selection of Supplier by using rough approximation of FHS-set.

▷ **Start**

▷ **Input:**

1. Consider $\hat{\hat{\mathbb{C}}}, \mathbb{E}, \mathfrak{E}$ and $\mathcal{K}$ as initial universe (set of suppliers), set of experts, set of parameters and C-product of corresponding $AVDS$ respectively and $\mathcal{J} \subseteq \mathcal{K}$.

▷ **Construction:**

2. Construct a FHS-set $\mathcal{H}_1 = (\mathcal{U}, \mathcal{J})$ based on predefined real linguistic terms of the firm with FHS- variables (e.g., Effectual Suppliers) over $\hat{\hat{\mathbb{C}}}$.

3. Construct module FHS-sets $\mathcal{U}(\hat{k}_i)$ for all $\hat{k}_i \in \mathcal{J}$ over $\hat{\hat{\mathbb{C}}}$ characterized by firm's TrFn-based L-variables $\vartheta_{ij} = (\vartheta^1_{ij}, \vartheta^2_{ij}, \vartheta^3_{ij})$ for ${}_1i^r$ & ${}_1j^n$ and tabulate them with (i,j)th entries such that ith row and jth column for attribute-valued tuples $\hat{k}_i \in \mathcal{J}$ and suppliers $\hat{\mathbb{C}}_j \in \hat{\hat{\mathbb{C}}}$ (see Table 2).

4. Construct a FHS-set $\mathcal{H}_2 = (\mathcal{V}, \mathbb{E})$ based on opinions of experts hired by the firm with FHS- variables (e.g., Effectual Suppliers) over $\hat{\hat{\mathbb{C}}}$.

5. Construct module FHS-sets $\mathcal{V}(\mathbb{E}_i)$ for all $\mathbb{E}_i \in \mathbb{E}$ over $\hat{\hat{\mathbb{C}}}$ characterized by TrFn-based L-variables $\theta_{ij} = (\theta^1_{ij}, \theta^2_{ij}, \theta^3_{ij})$ for ${}_1i^m$ & ${}_1j^n$ assigned by experts and tabulate them with (i,j)th entries such that ith row and jth column for experts $\mathbb{E}_i \in \mathbb{E}$ and suppliers $\hat{\mathbb{C}}_j \in \hat{\hat{\mathbb{C}}}$ (see Table 3).

▷ **Computation:**

6. Compute rough approximations $\mathcal{V}_1 = (\mathcal{V}^L, \mathbb{E})$ and $\mathcal{V}_2 = (\mathcal{V}^U, \mathbb{E})$ of FHS-set $\mathcal{H}_2$ w.r.t $\mathcal{H}_1$ and tabulate the values of module FHS-sets $(\mathcal{V}^L(\mathbb{E}_i)$ and $(\mathcal{V}^U(\mathbb{E}_i)$ for i^{th} expert with ω_{ij} and σ_{ij} respectively as (i,j)th-entries and can be computed in accordance with Equations (2) and (3). Their tabular representations are provided in Tables 4 and 5 respectively.

7. Compute three FHS-sets $\mathfrak{W}_{\mathcal{V}_1}, \mathfrak{W}_{\mathcal{H}_2}$ and $\mathfrak{W}_{\mathcal{V}_2}$ based on $\mathcal{V}_1, \mathcal{H}_2$ and $\mathcal{V}_2$ respectively and compute their respective membership functions $\psi_{\mathfrak{W}_{\mathcal{V}_1}}(\hat{\mathbb{C}}_j) = \mathcal{Q}^L(\hat{\mathbb{C}}_j)$, $\psi_{\mathfrak{W}_{\mathcal{H}_2}}(\hat{\mathbb{C}}_j) = \mathcal{Q}(\hat{\mathbb{C}}_j)$ and $\psi_{\mathfrak{W}_{\mathcal{V}_2}}(\hat{\mathbb{C}}_j) = \mathcal{Q}^U(\hat{\mathbb{C}}_j)$ in the following way:

$$\mathcal{Q}^L(\hat{\mathbb{C}}_j) = (\min_x\{\omega^1_{ij}\}, \frac{1}{m}\sum_{y=1}^{m}\omega^2_{yj}, \max_z\{\omega^3_{zj}\}) \tag{4}$$

$$\mathcal{Q}(\hat{\mathbb{C}}_j) = (\min_x\{\theta^1_{ij}\}, \frac{1}{m}\sum_{y=1}^{m}\theta^2_{yj}, \max_z\{\theta^3_{zj}\}) \tag{5}$$

$$\mathcal{Q}^U(\hat{\mathbb{C}}_j) = (\min_x\{\sigma^1_{ij}\}, \frac{1}{m}\sum_{y=1}^{m}\sigma^2_{yj}, \max_z\{\sigma^3_{zj}\}) \tag{6}$$

for ${}_1x^m$ & ${}_1z^n$.

8. Compute a FHS-set $\mathcal{H} = (\Psi, \mathcal{Y})$ over $\hat{\hat{\mathbb{C}}}$ based on computations done in previous step where $\mathcal{Y}$ is the C-product of $AVDS$ with respect to confidence-level based attributes (i.e. Low (SLC), medium (SMC), high (SHC)) such that $\Psi_{SLC} = \mathcal{Q}^L(\hat{\mathbb{C}}_j)$, $\Psi_{SMC} = \mathcal{Q}(\hat{\mathbb{C}}_j)$ and $\Psi_{SHC} = \mathcal{Q}^U(\hat{\mathbb{C}}_j)$.

9. Compute score $\Delta_j = \Delta(\hat{\mathbb{C}}_j) = (\varpi_1 \times \Psi_{SLC}) + (\varpi_2 \times \Psi_{SMC}) + (\varpi_3 \times \Psi_{SHC})$ for each supplier where ϖ_1, ϖ_2 and ϖ_3 are weights estimated for SLC, SMC and SHC respectively such that $\sum_{f=1}^{3} \varpi_f = 1$.

10. Compute crisp score Θ_j by applying Equation (1).

▷ **Output:**

11. Select the Supplier with maximum crisp score.

▷ **End**

The brief description of this algorithm is presented in Figure 4.

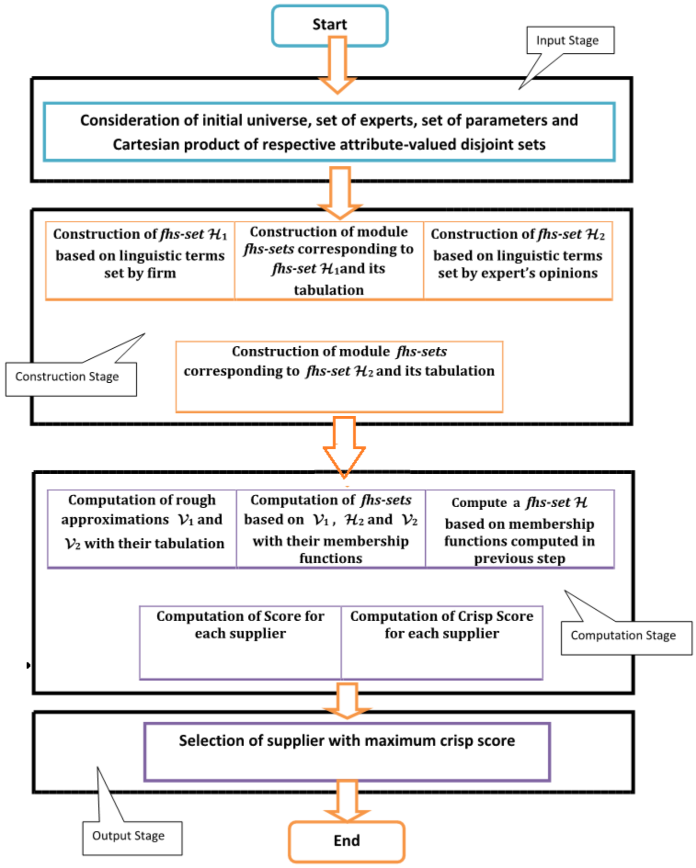

Figure 4. Flowchart of Proposed Algorithm.

Table 2. Tabular Representation of FHS-set $\mathcal{H}_1$.

	$\hat{\mathbb{C}}_1$	$\hat{\mathbb{C}}_2$	$\hat{\mathbb{C}}_3$	.....	$\hat{\mathbb{C}}_n$
$\hat{k}_1$	ϑ_{11}	ϑ_{12}	ϑ_{13}	.....	ϑ_{1n}
$\hat{k}_2$	ϑ_{21}	ϑ_{22}	ϑ_{23}	.....	ϑ_{2n}
$\hat{k}_3$	ϑ_{31}	ϑ_{32}	ϑ_{33}	.....	ϑ_{3n}
.....	.....	.....	.....	.....	.....
$\hat{k}_r$	ϑ_{r1}	ϑ_{r2}	ϑ_{r3}	.....	ϑ_{rn}

Table 3. Tabular Representation of FHS-set $\mathcal{H}_2$.

	$\hat{\mathbb{C}}_1$	$\hat{\mathbb{C}}_2$	$\hat{\mathbb{C}}_3$	.....	$\hat{\mathbb{C}}_n$
$\mathbb{E}_1$	θ_{11}	θ_{12}	θ_{13}	.....	θ_{1n}
$\mathbb{E}_2$	θ_{21}	θ_{22}	θ_{23}	.....	θ_{2n}
$\mathbb{E}_3$	θ_{31}	θ_{32}	θ_{33}	.....	θ_{3n}
.....	.....	.....	.....	.....	.....
$\mathbb{E}_r$	θ_{r1}	θ_{r2}	θ_{r3}	.....	θ_{rn}

Table 4. Tabular Representation of Lower Approximation $\mathcal{V}_1 = (\mathcal{V}^L, \mathbb{E})$.

	$\hat{\mathbb{C}}_1$	$\hat{\mathbb{C}}_2$	$\hat{\mathbb{C}}_3$	.....	$\hat{\mathbb{C}}_n$
$\mathbb{E}_1$	ω_{11}	ω_{12}	ω_{13}	.....	ω_{1n}
$\mathbb{E}_2$	ω_{21}	ω_{22}	ω_{23}	.....	ω_{2n}
$\mathbb{E}_3$	ω_{31}	ω_{32}	ω_{33}	.....	ω_{3n}
.....	.....	.....	.....	.....	.....
$\mathbb{E}_m$	ω_{r1}	ω_{r2}	ω_{r3}	.....	ω_{mn}

Table 5. Tabular Representation of Upper Approximation $\mathcal{V}_2 = (\mathcal{V}^{\mathcal{U}}, \mathbb{E})$.

	$\hat{\mathbb{C}}_1$	$\hat{\mathbb{C}}_2$	$\hat{\mathbb{C}}_3$	.....	$\hat{\mathbb{C}}_n$
$\mathbb{E}_1$	σ_{11}	σ_{12}	σ_{13}	.....	σ_{1n}
$\mathbb{E}_2$	σ_{21}	σ_{22}	σ_{23}	.....	σ_{2n}
$\mathbb{E}_3$	σ_{31}	σ_{32}	σ_{33}	.....	σ_{3n}
.....	.....	.....	.....	.....	.....
$\mathbb{E}_m$	σ_{r1}	σ_{r2}	σ_{r3}	.....	σ_{mn}

4.2. Application of Proposed Rough Approximations for Supplier Selection

In this section, the proposed algorithm is validated with the help of real-life perception.

Problem Statement:

The administration of a well-established real estate builders firm BUILDCOM (a hypothetical name), is intended to search for the best supplier to provide all kinds of construction materials (bricks, cement, sand, etc.) for their building projects in different areas of the city. The BUILDCOM advertises a call for bids for this purpose, and some companies submit their proposals. After advertising a call for bids, the firm has constituted three committees: Committee A consists of two experts $\mathbb{E}_1$ and $\mathbb{E}_2$ who have relevant experience in the scrutinizing the proposals received in response to an advertisement. The members of this committee's major role are to shortlist the proposals based on their experience and in accordance with the firm's scrutiny policy collectively and implicitly. The short-listed proposals are then forwarded to committee B, and committee C. Committee B consists of two experts: $\mathbb{E}_3$ and $\mathbb{E}_4$ who are domestic employees of the firm. The major role of this committee is to collect rough data in the form of linguistic terms by considering some effective parameters. The committee C consists of three experts: $\mathbb{E}_5, \mathbb{E}_6$ and $\mathbb{E}_7$, who have been hired to analyze the findings of committee B and provide their opinions in the form of linguistic terms in accordance with their acceptance level. The administration considers this process of evaluation due to biasing market situation therefore, a comprehensive cum reliable technique may help them to have proper selection. The proposed algorithm finds its place to help them in this regard.

Input Stage:

(Step 1)—Consider four suppliers are initially scrutinized by committee A for further evaluation who form an initial universe $\hat{\mathbb{C}} = \{\hat{\mathbb{C}}_1, \hat{\mathbb{C}}_2, \hat{\mathbb{C}}_3, \hat{\mathbb{C}}_4\}$ and the members of committee B are agreed upon a set of effective parameters $\mathfrak{E} = \{\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4\}$ describing product worth degree, service worth degree, delivery speed and cost respectively. These parameters are further classified into their respective sub-parametric valued disjoint sets $\mathcal{K}_1 = \{\hat{e}_{11} = mid, \hat{e}_{11} = high\}$, $\mathcal{K}_2 = \{\hat{e}_{21} = well, \hat{e}_{22} = superior\}$, $\mathcal{K}_3 = \{\hat{e}_{31} = before\, due - date, \hat{e}_{32} = ondue - date\}$ and $\mathcal{K}_4 = \{\hat{e}_{41} = silghty\, costly\}$ respectively such that $\mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2 \times \mathcal{K}_3 \times \mathcal{K}_4 = \{\hat{k}_1, \hat{k}_2, \hat{k}_3, \ldots, \hat{k}_8\}$. Both committees has made mutual consensus on $\mathcal{J} = \{\hat{k}_5, \hat{k}_6, \hat{k}_7, \hat{k}_8\} \subseteq \mathcal{K}$.

Construction Stage:

(Steps 2–5)—Suppose committee C consists of three experts who form a set $\mathbb{E} = \{\mathbb{E}_5, \mathbb{E}_6, \mathbb{E}_7\}$ and committee B has finalized linguistic terms in the form TrFns as given in Table 1. Also it has provided collected linguistic terms-based information about the suppliers w.r.t set $\mathcal{J}$ as given in Table 6. The committee C has provided its linguistic terms-based decision regarding acceptance-level of set $\hat{\hat{\mathbb{C}}}$ as given in Table 7.

Table 6. Linguistic Terms-Based Collected Information by Committee B.

	$\hat{k}_5$	$\hat{k}_6$	$\hat{k}_7$	$\hat{k}_8$
$\hat{\mathbb{C}}_1$	η_6	η_4	η_5	η_6
$\hat{\mathbb{C}}_2$	η_4	η_6	η_5	η_4
$\hat{\mathbb{C}}_3$	η_5	η_6	η_4	η_7
$\hat{\mathbb{C}}_4$	η_3	η_5	η_6	η_4

Table 7. Linguistic Terms-Based acceptance-level by Committee C.

	$\mathbb{E}_5$	$\mathbb{E}_6$	$\mathbb{E}_7$
$\hat{\mathbb{C}}_1$	η_4	η_5	η_4
$\hat{\mathbb{C}}_2$	η_4	η_3	η_6
$\hat{\mathbb{C}}_3$	η_6	η_5	η_6
$\hat{\mathbb{C}}_4$	η_3	η_4	η_5

Computation Stage:

(Steps 6–10)—With the help of formulations given in Equations (2) and (3), rough approximations $\mathcal{V}_1 = (\mathcal{V}^L, \mathbb{E})$ and $\mathcal{V}_2 = (\mathcal{V}^U, \mathbb{E})$ are computed and tabulated in Tables 8 and 9.

Table 8. Tabulation of Lower Approximation $\mathcal{V}_1 = (\mathcal{V}^L, \mathbb{E})$.

	$\hat{\mathbb{C}}_1$	$\hat{\mathbb{C}}_2$	$\hat{\mathbb{C}}_3$	$\hat{\mathbb{C}}_4$
$\mathbb{E}_5$	$(4, 5, 6)$	$(4, 5, 6)$	$(4, 6, 8)$	$(3, 4, 5)$
$\mathbb{E}_6$	$(4, 5.5, 6)$	$(3, 4, 5)$	$(4, 5.5, 7)$	$(3, 4.5, 6)$
$\mathbb{E}_7$	$(4, 5, 6)$	$(4, 5.75, 8)$	$(4, 6, 8)$	$(3, 5, 7)$

Table 9. Tabulation of Upper Approximation $\mathcal{V}_2 = (\mathcal{V}^U, \mathbb{E})$.

	$\hat{\mathbb{C}}_1$	$\hat{\mathbb{C}}_2$	$\hat{\mathbb{C}}_3$	$\hat{\mathbb{C}}_4$
$\mathbb{E}_5$	$(4, 6.25, 8)$	$(4, 5.75, 8)$	$(4, 7, 10)$	$(3, 5, 7)$
$\mathbb{E}_6$	$(4, 6.25, 8)$	$(3, 5.33, 7)$	$(4, 6, 8)$	$(3, 5.5, 8)$
$\mathbb{E}_7$	$(4, 6.255, 8)$	$(4, 5.75, 8)$	$(4, 7, 10)$	$(3, 5.5, 8)$

Similarly from Equations (4) to (6), the membership grades are computed as

$$\psi_{\mathfrak{W}_{\mathcal{V}_1}} = \left\{ \frac{\hat{\mathbb{C}}_1}{(4,5.167,7)}, \frac{\hat{\mathbb{C}}_2}{(3,4.916,8)}, \frac{\hat{\mathbb{C}}_3}{(4,5.833,8)}, \frac{\hat{\mathbb{C}}_4}{(3,4.5,7)} \right\} \tag{7}$$

$$\psi_{\mathfrak{W}_{\mathcal{H}_2}} = \left\{ \frac{\hat{\mathbb{C}}_1}{(4,5.333,7)}, \frac{\hat{\mathbb{C}}_2}{(3,5.333,8)}, \frac{\hat{\mathbb{C}}_3}{(5,6.667,8)}, \frac{\hat{\mathbb{C}}_4}{(3,5,7)} \right\} \tag{8}$$

$$\psi_{\mathfrak{W}_{\mathcal{V}_2}} = \left\{ \frac{\hat{\mathbb{C}}_1}{(4,6.25,8)}, \frac{\hat{\mathbb{C}}_2}{(4,5.611,8)}, \frac{\hat{\mathbb{C}}_3}{(4,6.667,10)}, \frac{\hat{\mathbb{C}}_4}{(3,5.333,8)} \right\} \tag{9}$$

In the same manner, by applying weights $\varpi_1 = 0.25$, $\varpi_2 = 0.5$ and $\varpi_3 = 0.25$, score values are computed as $\Delta(\hat{\mathbb{C}}_1) = 5.625$, $\Delta(\hat{\mathbb{C}}_2) = 5.461$, $\Delta(\hat{\mathbb{C}}_3) = 6.479$, and $\Delta(\hat{\mathbb{C}}_4) = 5.042$.

Output Stage:

(Step 11)—Hence Suppliers are ranked as $\hat{\mathbb{C}}_3 > \hat{\mathbb{C}}_1 > \hat{\mathbb{C}}_2 > \hat{\mathbb{C}}_4$ with preference to $\hat{\mathbb{C}}_3$ (see Figure 5).

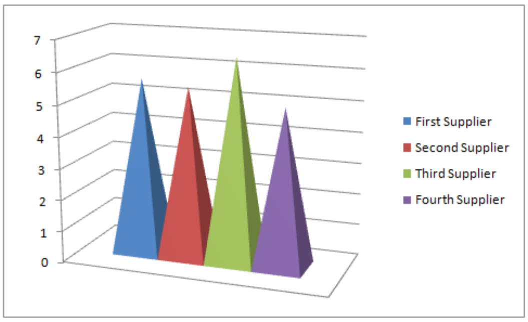

Figure 5. Ranking of Suppliers.

5. Discussion and Comparison Analysis

Before summarizing the paper, an illustrative discussion is provided for the comparison of the presented approach with some pre-developed approaches. In this regard, a suitable criterion is followed for the accomplishment of this evaluation process, i.e., whether the existing approaches realize the adopted decisive factors or not. The presented approach is assumed to be superior with respect to computational straightforwardness and logical reflection.

1. As procurement has turned out to be imperative in shaping the effectiveness and endurance of production groups, it has been getting significant interest. As Sarkis & Talluri [59] specified, purchaser-dealer correlations based merely on cost are not adequate to any further extent. The growing significance of supplier selection decisions is compelling companies to reconsider their procuring and assessment approaches as a thriving procuring assessment directly depends on selecting the "right" supplier.
2. As discussed earlier in the literature review, the SSP is a $MCDM$ problem, and it can easily be examined that the key aspect of each $MCDM$ is the partiality shown by experts for the objects under observation regarding each decisive factor. It can also be scrutinized that the views of experts are the major source of study in numerous researches. However, if the views of experts depict some sort of inaccuracies, then the computational process may likely be influenced. In this context, computational and informational roughness is observed to be involved.
3. In SSP, several features like market-based experience, community-based character, trust-based status etc., are necessitated to be regarded along with the views of employed experts. These features are generally named rough information. Such information can be collected by carrying out various surveys in the locality or by interviewing the firm's local employees.
4. As parameters and their respective sub-parametric values have an imperative part in $MCDM$. The meditation of such aspects may vary from situation to situation basis, i.e., in some states of affairs, only parameters are considered, whereas others prefer to necessitate categorising parameters into their related sub-classes consisting of their relevant parametric values. The former is used in the S-set environment, and the latter is utilized in HS-environment. The disregarding of such decisive factors may influence the integrity and trustworthiness of decisions. The SSP, being the $MCDM$ problem, may involve several decisive parameters which are required to be classified into their respective sub-classes having their sub-parametric values to have reliable results. In other words, the SSP demands for HS-environment.
5. Keeping in view the above discussion, the contributions of researchers Chang & Hung [47], Xiao et al. [48], Chatterjee et al. [51], Liu et al. [60] and Mukherjee et al. [61] are observed as the most significant and relevant to proposed approach for SSP. These approaches have ignored the HS-setting, the consideration of rough approximations to tackle with rough information and impreciseness in the views of experts. Whereas the proposed approach is capable of managing the above aspects collectively. The decisive features like rough information and impreciseness in the opinions of experts are tackled by introducing the concept of rough approximations with the fuzzy setting.

The MAA-function is employed to equip the approach with an HS-environment. This function is meant to tackle sub-classes of parameters by taking their C-product as its domain. Consequently sub-parametric tuples are then used to approximate the alternatives (suppliers).

6. It is now vivid that the proposed approach is thoroughly distinct from existing approaches therefore its computational results are not comparable with above existing models. However, for the sake of advantageous assessment, its structural comparison is elaborated with the above mentioned approaches in Table 10. In this regards, the following evaluating features are considered:
   (i). Consideration of fuzzy membership (FM),
   (ii). Soft approximate function (SAF),
   (iii). Hypersoft approximate function (HAF) and
   (iv). The entitlement of company's collected information rather than opinions of experts (ECCI).

   The first feature is meant to judge whether the impreciseness relating to the opinions of experts is tackled or not, the second feature is used to assess whether the S-set environment is employed or not, the third feature is used to check whether the HS-environment is observed or not and similarly the last feature is meant to examine whether the concept of rough approximation is used to tackle rough information or not. With the help of this comparison, it is vivid that the proposed approach is superior to the existing ones as it addresses all above features collectively as single model. In Table 10, the symbols ✓ and × stand for YES and NO respectively.

Table 10. Structural assessment of presented model with accessible appropriate models.

References	FM	SAF	HAF	ECCI
Chang & Hung [47]	×	×	×	×
Xiao et al. [48]	✓	✓	×	✓
Chatterjee et al. [51]	✓	✓	×	✓
Liu et al. [60]	×	×	×	×
Mukherjee et al. [61]	✓	×	×	×
Proposed Approach	✓	✓	✓	✓

6. Conclusions

This research mainly aims to address existing literature's limitations on SCM and $CSCM$ for supplier selection. Usually, the rough information about suppliers' reputations is ignored, making the selection process biased and risky. Therefore this study has utilized the concept of rough approximations of FHS-set, which is a more generalized and flexible structure. It can manage the hypersoft setting along with the entitlement of the MAA-function. The consideration of these features provides a strong foundation for having a reliable decision regarding the selection of suppliers. The rough information and expert opinions are observed and emphasized for evaluating suppliers. These kinds of information are considered in the form of linguistic terms and then represented by TrFn. The upper and lower approximations of the FHS-set are then characterized, which are further used in the proposal of an intelligent algorithm for the supplier selection in $CSCM$. In order to assess the applicability of the proposed algorithm, a real-world $MADM$ scenario is discussed to help the administration of construction companies in the selection of a suitable supplier. The main advantage of the proposed study is that TrFn-based L-variables are utilized to characterize the uncertain attitude of attributes and their respective sub-attributes with the entitlement of experts and rough information for evaluation of suppliers. As this study is based on the characterization of the FHS-set, which is a flexible model, this study is not meant for those scenarios in which experts provide their opinions in the form of an intuitionistic fuzzy setting or neutrosophic setting. Therefore this study may be extended in future for the characterization of rough approximations under such environments to tackle experts' two and three-dimensional opinions. Similarly, as the concept of TrFn is used due to its easy computation in this research, other kinds of fuzzy numbers like trapezoidal numbers etc., can also be used to represent experts' opinions. This approach may also be employed to discuss a case study based on real data set to discuss any $MCDM$ problem like product selection, medical diagnosis, energy source management, etc.

Author Contributions: Conceptualization, A.U.R., M.S., M.A.M. and O.T.; Methodology, A.U.R., A.M. and O.T.; Software, A.U.R., M.A.M. and O.T.; Validation, A.U.R., M.A.M. and A.M.; Formal analysis, A.U.R., M.S. and O.T.; Investigation, A.U.R., M.S. and M.A.M.; Data curation, M.A.M., A.M. and O.T.; Writing of the original draft, A.U.R., M.S., M.A.M. and O.T.; Writing of the review and editing, M.S., M.A.M. and A.M.; Visualization, A.U.R., M.S. and M.A.M.; Supervision, M.S., M.A.M. and A.M.; Project administration, M.A.M. and A.M. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: This study has no associated data.

Acknowledgments: This research work was partially supported by University of Anbar, Imperial College London and Chiang Mai University.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Lambert, D.M.; Cooper, M.C. Issues in supply chain management. *Ind. Mark. Manag.* **2000**, *29*, 65–83. [CrossRef]
2. Mentzer, J.T.; DeWitt, W.; Keebler, J.S.; Min, S.; Nix, N.W.; Smith, C.D.; Zacharia, Z.G. Defining supply chain management. *J. Bus. Logist.* **2001**, *22*, 1–25. [CrossRef]
3. Tan, K.C. A framework of supply chain management literature. *Eur. J. Purch. Supply Manag.* **2001**, *7*, 39–48. [CrossRef]
4. Power, D. Supply chain management integration and implementation: a literature review. *Supply Chain. Manag.* **2005**, *10*, 252–263. [CrossRef]
5. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
6. Pawlak, Z. Rough sets. *Int. J. Comput. Inf. Sci.* **1982**, *11*, 341–356. [CrossRef]
7. Molodtsov, D. Soft set theory-first results. *Comput. Math. Appl.* **1999**, *37*, 19–31. [CrossRef]
8. Maji, P.K.; Biswas, R.; Roy, A.R. Soft set theory. *Comput. Math. Appl.* **2003**, *45*, 555–562. [CrossRef]
9. Ali, M.I.; Feng, F.; Liu, X.; Min, W.K.; Sabir, M. On some new operations in soft set theory. *Comput. Math. Appl.* **2009**, *57*, 1547–1553. [CrossRef]
10. Babitha, K.V.; Sunil, J.J. Soft set relations and functions. *Comput. Math. Appl.* **2010**, *60*, 1840–1849. [CrossRef]
11. Babitha, K.V.; Sunil, J.J. Transitive closure and ordering in soft set. *Comput. Math. Appl.* **2011**, *61*, 2235–2239. [CrossRef]
12. Li, F. Notes on soft set operations. *ARPN J. Syst. Softw.* **2011**, *1*, 205–208.
13. Pei, D.; Miao, D. From soft set to information system. In Proceedings of the IEEE International Conference of Granular Computing, Beijing, China, 25–27 July 2005; Volume 2, pp. 617–621. [CrossRef]
14. Sezgin, A.; Atagün, A.O. On operations of soft sets. *Comput. Math. Appl.* **2011**, *61*, 1457–1467. [CrossRef]
15. Maji, P.K.; Biswas, R.; Roy, A.R. An application of soft sets in a decision making problem. *Comput. Math. Appl.* **2002**, *44*, 1077–1083. [CrossRef]
16. Maji, P.K.; Biswas, R.; Roy, A.R. Fuzzy soft sets. *J. Fuzzy Math.* **2001**, *9*, 589–602.
17. Feng, F.; Liu, X.; Leoreanu-Fotea, V.; Jun, Y.B. Soft sets and soft rough sets. *Inf. Sci.* **2011**, *181*, 1125–1137. [CrossRef]
18. Ali, M.I. A note on soft sets, rough soft sets and fuzzy soft sets. *Appl. Soft Comput.* **2011**, *11*, 3329–3332. [CrossRef]
19. Cağman, N.; Enginoğlu, S.; Citak, F. Fuzzy soft set theory and its applications. *Iran. J. Fuzzy Syst.* **2011**, *8*, 137–147. [CrossRef]
20. Arora, R.; Garg, H. A robust correlation coefficient measure of dual hesitant fuzzy soft sets and their application in decision making. *Eng. Appl. Artif. Intell.* **2018**, *72*, 80–92. [CrossRef]
21. Athira, T.M.; John, S.J.; Garg, H. A novel entropy measure of Pythagorean fuzzy soft sets. *AIMS Math.* **2020**, *5*, 1050–1061. [CrossRef]
22. Smarandache, F. Extension of soft set of hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic Sets Syst.* **2018**, *22*, 168–170. [CrossRef]
23. Saeed, M.; Rahman, A.U.; Ahsan, M.; Smarandache, F. An inclusive study on fundamentals of hypersoft set. In *Theory and Application of Hypersoft Set*; Pons Publication House: Brussels, Belgium, 2021; pp. 1–23.

24. Yolcu, A.; Ozturk, T.Y. Fuzzy hypersoft sets and it's application to decision-making. In *Theory and Application of Hypersoft Set*; Pons Publication House: Brussels, Belgium, 2021; pp. 50–64.
25. Debnath, S. Fuzzy hypersoft sets and its weightage operator for decision making. *J. Fuzzy Ext. Appl.* **2021**, *2*, 163–170. [CrossRef]
26. Rahman, A.U.; Hafeez, A.; Saeed, M.; Ahmad, M.R.; Farwa, U. Development of rough hypersoft set with application in decision making for the best choice of chemical material. In *Theory and Application of Hypersoft Set*; Pons Publication House: Brussels, Belgium, 2021; pp. 192–202. [CrossRef]
27. Rahman, A.U.; Saeed, M.; Smarandache, F. Convex and concave hypersoft sets with some properties. *Neutrosophic Sets Syst.* **2020**, *38*, 497–508. [CrossRef]
28. Rahman, A.U.; Saeed, M.; Hafeez, A. Theory of bijective hypersoft set with application in decision making. *Punjab Univ. J. Math.* **2021**, *53*, 511–526. [CrossRef]
29. Saeed, M.; Ahsan, M.; Abdeljawad, T. A development of complex multi-fuzzy hypersoft set with application in mcdm based on entropy and similarity measure. *IEEE Access* **2021**, *9*, 60026–60042. [CrossRef]
30. Saeed, M.; Ahsan, M.; Rahman, A.U. A novel approach to mappings on hypersoft classes with application. In *Theory and Application of Hypersoft Set*; Pons Publication House: Brussels, Belgium, 2021; pp. 175–191. [CrossRef]
31. Rajesh, R.; Ravi, V. Supplier selection in resilient supply chains: A grey relational analysis approach. *J. Clean. Prod.* **2015**, *86*, 343–359. [CrossRef]
32. ÓBrien, W.J.; Formoso, C.T.; Vrijhoef, R.; London, K.A. Construction Supply Chain Management Handbook. *Constr. Manag. Econ.* **2009**, *27*, 1265–1266.
33. Kannan, D.; Khodaverdi, R.; Olfat, L.; Jafarian, A.; Diabat, A. Integrated fuzzy multi criteria decision making method and multi-objective programming approach for supplier selection and order allocation in a green supply chain. *J. Clean. Prod.* **2013**, *47*, 355–367. [CrossRef]
34. Aretoulis, G.N.; Kalfakakou, G.P.; Striagka, F.Z. Construction material supplier selection under multiple criteria. *Oper. Res.* **2010**, *10*, 209–230. [CrossRef]
35. Safa, M.; Shahi, A.; Haas, C.T.; Hipel, K.W. Supplier selection process in an integrated construction materials management model. *Autom. Constr.* **2014**, *48*, 64–73. [CrossRef]
36. Stević, Ž.; Pamučar, D.; Vasiljeviéć, M.; Stojić, G.; Korica, S. Novel integrated multi-criteria model for supplier selection: Case study construction company. *Symmetry* **2017**, *9*, 279. [CrossRef]
37. Yin, S.; Li, B.; Dong, H.; Xing, Z. A New Dynamic Multicriteria Decision-Making Approach for Green Supplier Selection in Construction Projects under Time Sequence. *Math. Probl. Eng.* **2017**, *2017*, 7954784. [CrossRef]
38. Cengiz, A.E.; Aytekin, O.; Ozdemir, I.; Kusan, H.; Çabuk, A. A multi-criteria decision model for construction material supplier selection. *Procedia Eng.* **2017**, *196*, 294–301. [CrossRef]
39. Yazdani, M.; Wen, Z.; Liao, H.; Banaitis, A.; Turskis, Z. A grey combined compromise solution (CoCoSo-G) method for supplier selection in construction management. *J. Civ. Eng. Manag.* **2019**, *25*, 858–874. [CrossRef]
40. Hoseini, S.A.; Fallahpour, A.; Wong, K.Y.; Mahdiyar, A.; Saberi, M.; Durdyev, S. Sustainable supplier selection in construction industry through hybrid fuzzy-based approaches. *Sustainability* **2021**, *13*, 1413. [CrossRef]
41. Matić, B.; Jovanović, S.; Das, D.K.; Zavadskas, E.K.; Stević, Ž.; Sremac, S.; Marinković, M. A new hybrid MCDM model: Sustainable supplier selection in a construction company. *Symmetry* **2019**, *11*, 353. [CrossRef]
42. Eshtehardian, E.; Ghodousi, P.; Bejanpour, A. Using ANP and AHP for the supplier selection in the construction and civil engineering companies; Case study of Iranian company. *KSCE J. Civ. Eng.* **2013**, *17*, 262–270. [CrossRef]
43. Yazdani, M.; Chatterjee, P.; Pamucar, D.; Abad, M.D. A risk-based integrated decision-making model for green supplier selection: A case study of a construction company in Spain. *Kybernetes* **2020**, *49*, 1229–1252. [CrossRef]
44. Wang, T.K.; Zhang, Q.; Chong, H.Y.; Wang, X. Integrated supplier selection framework in a resilient construction supply chain: An approach via analytic hierarchy process (AHP) and grey relational analysis (GRA). *Sustainability* **2017**, *9*, 289. [CrossRef]
45. Xu, J.; Li, B.; Wu, D. Rough data envelopment analysis and its application to supply chain performance evaluation. *Int. J. Prod. Econ.* **2009**, *122*, 628–638. [CrossRef]
46. Huang, K.Y.; Jane, C.J. A hybrid model for stock market forecasting and portfolio selection based on ARX, grey system and RS theories. *Expert Syst. Appl.* **2009**, *36*, 5387–5392. [CrossRef]
47. Chang, B.; Hung, H.F. A study of using RST to create the supplier selection model and decision-making rules. *Expert Syst. Appl.* **2010**, *37*, 8284–8295. [CrossRef]
48. Xiao, Z.; Chen, W.; Li, L. An integrated FCM and fuzzy soft set for supplier selection problem based on risk evaluation. *Appl. Math. Model.* **2012**, *36*, 1444–1454. [CrossRef]
49. Patra, K.; Mondal, S.K. A supplier selection model with fuzzy risk analysis using the balanced solution technique with a soft set. *Pac. Sci. Rev. Nat. Sci. Eng.* **2016**, *18*, 162–168. [CrossRef]
50. Chang, K.H. Enhanced assessment of a supplier selection problem by integration of soft sets and hesitant fuzzy linguistic term set. *Proc. Inst. Mech. Eng. Part B J. Eng. Manuf.* **2015**, *229*, 1635–1644. [CrossRef]
51. Chatterjee, A.; Mukherjee, S.; Kar, S. A rough approximation of fuzzy soft set-based decision-making approach in supplier selection problem. *Fuzzy Inf. Eng.* **2018**, *10*, 178–195. [CrossRef]
52. Dubois, D.; Prade, H. Additions of interactive fuzzy numbers. *IEEE Trans. Autom. Control* **1981**, *26*, 926–936. [CrossRef]
53. Dijkman, J.G.; Van Haeringen, H.; De Lange, S.J. Fuzzy numbers. *J. Math. Anal. Appl.* **1983**, *92*, 301–341. [CrossRef]

54. Zadeh, L.A. Linguistic variables, approximate reasoning and dispositions. *Med. Inf.* **1983**, *8*, 173–186. [CrossRef]
55. Buckley, J.J. Possibilistic linear programming with triangular fuzzy numbers. *Fuzzy Sets Syst.* **1988**, *26*, 135–138. [CrossRef]
56. Fullér, R. On product-sum of triangular fuzzy numbers. *Fuzzy Sets Syst.* **1991**, *41*, 83–87. [CrossRef]
57. Dubois, D.; Prade, H. Fuzzy numbers: An overview. *Readings Fuzzy Sets Intell. Syst.* **1993**, *1*, 112–148. [CrossRef]
58. Chang, H.C. An application of fuzzy sets theory to the EOQ model with imperfect quality items. *Comput. Oper. Res.* **2014**, *31*, 2079–2092. [CrossRef]
59. Sarkis, J.; Talluri, S. A model for strategic supplier selection. *J. Supply Chain. Manag.* **2002**, *38*, 18–29. [CrossRef]
60. Liu, F.H.F.; Hai, H.L. The voting analytic hierarchy process method for selecting supplier. *Int. J. Prod. Econ.* **2005**, *97*, 308–317. [CrossRef]
61. Mukherjee, S.; Kar, S. Multi attribute decision making based on fuzzy logic and its application in supplier selection problem. *Oper. Supply Chain. Manag. Int. J.* **2014**, *5*, 76–83. [CrossRef]