Workshop on

Tractable special cases of hard combinatorial optimization problems

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Vector connectivity in graphs

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Motivated by challenges related to domination, connectivity, and information propagation in social and other networks, we study the Vector Connectivity problem. This problem, introduced by Boros et al. in [1], takes as input a graph G and an integer $r(v)$ for every vertex v of G, and the objective is to find a vertex subset S of minimum cardinality such that every vertex v either belongs to S, or is connected to at least $r(v)$ vertices of S by disjoint paths. If we require each path to be of length exactly 1, we get the well-known vector domination problem, which is a generalization of the dominating set and vertex cover problems. Consequently, the vector connectivity problem becomes NP-hard if an upper bound on the length of the disjoint paths is also supplied as input. Due to the hardness of these domination variants even on restricted graph classes, like split graphs, Vector Connectivity seems to be a natural problem to study for drawing the boundaries of tractability for this type of problems.

In the talk, I will give an overview of known complexity results for the Vector Connectivity problem. In particular, the problem can be solved in polynomial tim e on split graphs, in addition to cographs and trees. On the other hand, the problem is NP-hard for planar line graphs and for planar bipartite graphs, APX-hard on general graphs, and can be approximated in polynomial time within a factor oflog $n + 2$ on all n-vertex graphs. Vertex covers and dominating sets in a graph G can be easily characterized as hitting sets of derived hypergraphs (of G itself, and of the closed neighborhood hyper graph of G, respectively). Using Menger's Theorem, a similar characterization of vector connectivity sets can be derived.

Based on joint works with Endre Boros, Ferdinando Cicalese, Pinar Heggernes, Pim van 't Hof, and Romeo Rizzi (see [1]-[3]).

Literatur

[1] E. Boros, P. Heggernes, P. van 't Hof, and M. Milanič. Vector connectivity in graphs. In *Theory and applications of models of computation*, volume **7876** of *Lecture Notes in Comput. Sci.*, pages 331–342. Springer, Heidelberg, 2013.

[2] E. Boros, P. Heggernes, P. van 't Hof, and M. Milanič. Vector connectivity in graphs. *Networks*, **63(4)**, 277–285, 2014.

[3] F. Cicalese, M. Milanič, and R. Rizzi. On the complexity of the vector connectivity problem. Submitted, 2014.