Three Essays in Experimental and Network Economics

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I am submitting herewith a dissertation written by John D. McMahan entitled "Three Essays in Experimental and Network Economics." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Economics.

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Three Essays in Experimental and Network Economics

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This dissertation is dedicated to my family, who work to instill a love of learning for everyone,

To my wife Susan, for her unconditional support,

and to my late grandmother Dorothy and father Doug.

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Abstract

This dissertation consists of the three essays in network and experimental economics. The first essay explores the importance of endogenous bilateral connections and punishment networks in public good settings. I conduct a laboratory experiment that varies the incentive to form links among participants in a traditional Voluntary Contribution Mechanism game. I find that when link benefits are zero very few connections are formed, and very little punishment takes place. When link benefits are positive many links are formed and cooperation levels are increased. In general, we find evidence that participants strategically use the bilateral linking process to avoid punishment and find significant differences in the impacts of the bilateral link formation process when compared with exogenous punishment institutions. The second essay studies heterogeneity in sequential Tullock contests in the form of increased prize valuations and probabilistic entry, in a theoretical and laboratory setting. Building upon a new modelling technique, I generate theoretical hypotheses about the impact of heterogeneity in sequential contests. Specifically, a change in prize valuation or effort cost has the largest impact when the individual with the heterogenous valuation moves earlier in the contest. We then design a laboratory test and find support for theoretical predictions. We also find evidence that overbidding tends to increase as players move later in the contest. Further, we find an interesting behavioral result that we call a *Winning Probability heuristic*. For final players in sequential contests, many subjects make decisions consistent with choosing a winning probability rather than expected payoff maximization predicted by Nash Equilibrium theory. The final essay adapts a theoretical model commonly used in pricing of goods on a network with consumption complementarities to a setting that deals with telecommuting and flexible work arrangements. I provide an example of how allowing an employee to work from home can impact connectivity among employees and firm profitability. I show that the network of employees, wage structure, and the position of the employee in the network are all important determinants on whether a working from home arrangement is profitable. I then explicitly model how a firm can invest to influence the connectivity of their employees through investments that facilitate connections among employees such as providing an office space, hosting get togethers, or setting up a team chat function for remote workers. I also find that optimal expenditure in facilitating connections has a nonlinear relationship to the cost.

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Chapter 1

Endogenous Punishment Networks in Public Goods

I Introduction

The ability of group members to impose costly penalties on others has been shown to increase cooperation in settings where private and public interests are at odds, such as common pool resource and public goods games (Ostrom et al. 1992, Fehr and Gächter, 2000). The effectiveness of a group in overcoming the social dilemma of free riding depends on their ability to establish norms that the group members follow. Further, research has shown that the design of the sanctioning (or reward) institution impacts the contribution expectations of group members and can take many forms (Andreoni and Gee 2012; Nicklisch et al. 2016; *among others*). These can range from exogenous punishment from a third party (such as a fine or law) to an endogenous choice from within the group (*e.g.* an elected leader sanctioning free riders or vigilante justice). Each aspect of the sanctioning institution interacts with the preferences of the members to determine whether the group is successful in overcoming the free riding incentives created by the public goods problem.

In this paper, we propose a new design for a sanctioning institution in a laboratory study of repeated public good games that we believe captures key features of an important set of real-world scenarios. The design employs a bilateral linking stage that requires pairwise agreement of group members to be connected such that each is exposed to sanction by the other. This linking choice allows for individual group members to "opt in" or "opt out" of sanctioning relationships depending on their preferences. This is a form of endogenous peer-to-peer sanctioning similar to Ramalingam et al. (2016) but differs critically in the ability for individual players to avoid being sanctioned. The bilateral linking choice is representative of situations that require both parties to engage in a relationship, one that potentially brings an additional benefit, before sanctioning can take place. Situations where this is applicable include international relations and group scenarios in personal or business relationships.

For example, consider how mutually beneficial relations between nations also expose them to potential sanctions. Such sanctions, which might occur in response to human rights violations or weapons proliferation, often come in form of a trade sanctions (*e.g.* tariffs), barriers to travel, or reduced financial aid. In these cases, there must be an established financial or trade relationship that is agreed on by both parties that makes the sanctioning actions possible. It is possible that these relationships are formed exclusively for the *potential* to punish in the future, but in many cases (such as free trade agreements) they bring benefits that separate from the discipline that may apply to, for example, weapons proliferation (which has a public good characteristics). Moreover, when choosing whether to sanction at all, the sanctioning party must weigh the potential deterioration of the relationship with the potential gains from a change in behavior of the sanctioned party. Thus, the value that is placed on the relationship outside of the sanctioning context may influence whether sanctioning takes place and the extent of the sanctions.

This paper seeks to answer the following questions. How does varying incentives to form connections in a decentralized bilateral punishment network affect the resulting network and subsequent public good contributions? Further, how strong do the incentives need to be for individuals with differing preferences to form connections and agree to monitor each other in a decentralized setting? If we consider where the most productive relationships in a punishment network would come from, it seems that individuals who are low contributors (and who are willing to increase their contributions) must believe that the punishment they will face will outweigh their utility loss from increasing their contributions and they must be matched with a willing punisher. Indeed, research has shown that increased cooperation from low contributors who face the threat of punishment is a primary driver of increased contributions from groups with sanctioning institutions. (Ramalingam et al, 2016; Nikiforakis, 2012).

However, in many of the studies that look at punishment in public goods games, members do not have the ability to completely elude the punishment institutions. In the seminal paper in this area by Fehr and Gätcher (2000) all players can punish one other. More recent literature that endogenizes the punishment institution implements group level choice such as a majority vote (Deangelo and Gee, 2020) or self-selection mechanism (*voting with their feet*) (Gürerk et al. 2014; Nicklisch et al., 2016), which allow for some individual control but do not allow for the presence of free riding without reproach. In our setting of bilateral linking, if group members understand that they face the threat of sanctions (or having to inflict costly sanction) by being connected with other group members who have mismatched preferences they may avoid forming those connections.

Many analyses of endogenous punishment institutions have focused on a voting mechanism (such as in DeAngelo and Gee, 2020) that determines whether the punishment institution forms. In these and other designs it is not the case that a single individual can unilaterally avoid exposure to punishment without impacting others punishment options. This separates the individual choice to be exposed to punishment from the social choice of the existence of a punishment network. In our setting punishment networks only exist to the extent individuals choose to expose themselves to sanction, and only by doing so can they sanction others. This is important because an individual who plans to free ride may see the value of increasing others' contributions from the existence of a punishment network, while not wanting to expose himself to punishment.

It could be the case that the requirement of mutual agreement to punish severely limits the ability for group members to self-organize and increase cooperation. In these cases, the most beneficial punishment relationships, likely high contributors punishing (or threatening to punish) free riders, can be avoided by a single party refusing to link together. Conversely, it could be the case that the bilateral linking process breaks the cycle of punishment and counter punishment, that can offset efficiency gains from increased contributions in a full punishment environment, while preserving the punishment opportunities that support higher levels of cooperation. Thus, the bilateral linking process has the potential to achieve the benefits of punishment environments without some of the drawbacks. The tension between these countervailing forces makes it non-obvious how behavior in a bilateral linking punishment network game will compare to other punishment institution settings.

To study the above setting, we design a laboratory experiment that consists of a three-stage game. In the first stage, group members simultaneously propose links to other members of the group. If links are proposed by both players a link is formed and both players gain the ability to punish each other in the third stage. Before the punishment stage and after the link stage, the group makes public goods contributions in a standard Voluntary Contribution Mechanism

(VCM) game. To capture the (net) benefits that result from bilateral relationships we assign a fixed payment for each link that is formed. The treatments vary the fixed payment of established links from 0 to strongly positive. We include baseline comparisons of no punishment (a standard VCM game) and an exogenously imposed complete punishment network (replicating the setting of Fehr and Gächter, 2000).

To preview the results, we find that when link benefits are zero, very few links are formed, little punishment is administered, and public good contributions are similar to the baseline comparison of no punishment. That is, in the absence of an external benefit of the link relationship, subjects generally do not choose to expose themselves to future punishment, despite the benefit that might be anticipated from disciplining free riding. As the link benefit is increased, we see increased connections between players and improved contributions, but in many cases, groups fail to reach a complete punishment network even when the value of a link is as high as 50% of the possible punishment that could be inflicted. With positive link benefits we observe public good contributions that fall between the levels exhibited in the standard (no punishment) VCM game and the levels in the exogenous complete punishment network setting. Additionally, we find that a strong predictor of broken links is punishment in the previous period, indicating that subjects actively break links to avoid punishment. This provides a mechanism that reduces the presence of anti-social punishments that often offset the gains from increased contributions which is frequently observed in research on punishment networks (Nikiforakis, 2008; Nikiforakis et al. 2012). On balance, we find that the bilateral linking process, when the link benefit is positive, achieves similar overall efficiency as a full punishment network but with differing underlying behavior. That is, bilateral linking does not elicit a complete punishment network, and resulting contributions are lower in the VCM game than when the punishment network is complete, but bilateral linking results in significantly less socially costly realized punishment or sanctioning.

II Related Literature

Several papers have shown that sanctioning institutions can be effective in improving contributions by reducing free riding (Fehr and Gächter 2000; DeAngelo, Gee 2020; De Geest et al. 2017). The pioneering work by Fehr and Gächter (2000) had players making contribution choices in a VCM game with the option to inflict costly penalties on any group member after the contribution decisions had been made. The authors found that this punishment option resulted in significantly higher contributions than situations without punishment. Variations of this design on many dimensions soon followed (see Chaudhuri, 2011 for a survey). For example, Van Leeuwen et al. (2019) show that the ability for participants to vote to exclude (exile) members from participating in a public good production results in higher contributions. Multiple studies have shown that there are differences between third-party (monitor outside the group) (Andreoni and Gee, 2012 & 2015) and second party (within-group) punishment (Carpenter & Matthews, 2009; Carpenter et al., 2012; Leibbrandt et al., 2015). In field settings, the availability of punishment may be limited by assignment of property rights or institutional design. For example, territorial user fishing rights (TURFS) and other types of fishing cooperatives (Deacon 2012; De Geest et al., 2017) assign rights to a group of individuals who protect the resource from outside use, as well as overexploitation by members.

Other research has restricted the ability of players to punish other group members to a limited, fixed, subset of the group. This is referred to as an incomplete punishment network (Boosey and Isaac, 2016; Leibbrandt et al., 2015; Carpenter et al., 2012). In these settings, punishment is restricted to be between players who are connected to each other in the punishment network, or who are a part of a particular group. These punishment networks can be either exogenously or endogenously determined. It should be noted that punishment in this literature is designed to be socially wasteful. The player who is punishing imposes costs on themselves to inflict a larger punishment on their group member. Thus, while punishment generally increases contributions, these gains can be offset by increased punishment, which yields an ambiguous result on efficiency.

Research dealing with exogenous punishment networks has been both theoretical as well as experimental. Bramoullé and Kranton (2007) were the first to present theoretical equilibrium results with players producing public goods on a network. In this research and much of what has followed in the theoretical literature, the benefits of the public good generated by a player's production are restricted only to the players they are linked with. This is referred to as a *local* public good (Allouch, 2015; Bramoullé Kranton D'Amour, 2014). This network structure is quite different than the punishment networks that we are concerned with in this paper. Related research deals with global public goods but the networks serve to a way to exchange information (Elliott and Golub, 2019).

More closely related to this paper is the study of fixed exogenous networks in the laboratory. Leibbrandt et al. (2015) study exogenously imposed incomplete networks and conclude that the primary determinant of the level of (and effectiveness) of punishment was the density of the punishment network rather than the scope of punishment. In other words, more connections, which are opportunities for punishment, resulted in higher contributions to the public good. Whereas the impact of who was connected to whom showed little to no effect on contribution levels. On the other hand, Boosey and Isaac (2016) study fixed punishment networks and find that the structure of the network, holding the number of punishment possibilities constant, results in different mean contribution levels. Fatás et al. (2020) finds a similar result. They study four different exogenous punishment networks and find that central players have an important impact on punishment behaviors. [1]

Another literature explores endogenously formed punishment institutions in public good games.[2] Theoretically, Brekke et al. (2011) showed that players who are more pro-social opt into groups together and contribute more in a VCM game.[3] In the laboratory, and closely related to this study, Ramalingam et al. (2016) conduct a laboratory experiment where participants choose whether to participate in the sanctioning process in a public goods game. If participants opt into the sanctioning stage, they were allowed to freely punish others. Treatments in which participants had to pay for the option to punish and costless a costless option were compared. They find that, as the cost of joining the punishment network increases, the participation in the punishment network decreases. Further, that the contributions in the costless treatment are higher than the costly treatment, and contributions decline as costs increase. Another study by Kosfeld, Okada, and Reidl (2009) show that when institutions are formed endogenously, contributions and group welfare are increased. They also find that individuals who vote to implement punishment institutions contribute more, on average, than those that do not. Similarly, Gächter and Thoni (2005) find that cooperation among individuals who know they are grouped with other like-minded individuals is higher. This suggests that the composition of preferences

[1] (Shreedhar, Gange & Marchiori 2020) also study a similar idea.
[2] Sutter et al. (2010) study endogenous choice between punishment and reward institutions.
[3] See (Takács et al., 2008; Zschache, 2012; Kinateder Merline, 2017) for additional studies.

for those who agrees to interact in a punishment network likely differ from those who do not. This is not accurately reflected in an exogenously imposed punishment environment described by Leibbrandt et al. (2015). We hypothesize that the bilateral linking mechanism in our study is more likely to link players who have similar contribution preferences than those without.

This idea of preferential linking is very similar to the well-documented concept of homophily in the social network literature (*see* Jackson, 2010)[4]. Currarini and Mengel (2016) show that individuals show preference to people with similar characteristics (homophily) and preferences for others within their group (in group bias) and this bias disappears when groups are endogenously formed rather than exogenously imposed. Further, much of the research on cooperative action, and why it prevails and some cases and not others, has been tied to presences of formal and informal institutions created by preferential linking (Jackson et al. 2017).

Next, we discuss the literature on the determinants of punishment rather than the design of the punishment institution. Perceived norms have been shown to be an important factor in decision making as well as the punishment levels administered by players in the lab (Carpenter and Matthews, 2009; Michaeli and Spiro 2015; Boosey and Isaac, 2016). In another study Eckel et al. (2010) theoretically study how central players in a network (who have higher social status) ultimately impact the coordination problem. They show that if players have strong preference for matching contributions, then the central player works as a coordinating device. This could be viewed as establishing a social norm for others to follow. Other studies have used more explicit designs to elicit social norm behavior and treat the internal preference and the preference for conformity as two terms in the utility function (Andreoni et al. 2020; Granovetter 1978). The enforcement of norms (through sanctions) by third parties have also been studied with reference to public goods (Fehr and Fischbacher 2004; Bendor and Swistak 2004). A robust conclusion in this literature is that deviation from norms is a motivation for punishment for the deviators.

Some of the first studies in economics dealing with other-regarding preferences have deep ties to social norms. Whether individuals express preference for inequality aversion, efficiency (Fehr & Schmidt, 1999; Bolton & Ockenfels 2000), or maximin preferences (Charness & Rabin, 2002), these preferences could be unique to the individual or could be enforced on a

[4] Over 100 different studies across social science research have detected the presence of homophily (Block & Grund 2018).

group level. When other regarding preferences form the expectation of behavior in the group, they become a norm. Relatedly, norms can be either *absolute* or *relative.* Carpenter and Matthews (2009) showed that, as an explanatory variable, absolute norms often outperformed relative norms (generally assumed to be the group average) in a laboratory setting. Further, the enforcement of norms by a third party has been shown to punish based on efficiency. Whereas second party (in-group) punishment is centered on the sanctioning of free riders. Nikiforiakis et al. (2012) showed that feuds (punishment in retaliation to previous punishment) most often developed in normative conflict when individuals had differing values of the public good.

Lastly, we look at the literature that studies exactly who in the group is punishing and other types of preference heterogeneity. In the standard VCM punishment game, the most common type of punishment is pro-social, *i.e.* punishment by higher contributors inflicted on free riders. However, anti-social punishment (from low to high) has also been observed and has been suggested as a form of retaliation or seeking to enforce a low-contribution norm (Nikiforakis, 2012). Additionally, Nikiforakis (2008), finds that punishment is lower when a second counter-punishment stage is available which could suggest a fear of retaliation. Heterogeneity among players has also been found to be an important determinant in outcomes. Conditional cooperators, or players who will cooperate so long as others are not observed to be free riding, make up a substantial portion of players in the laboratory (Fischbacher et al., 2001). Moreover, a portion of players will always free ride and yet another will always contribute high amounts (Fehr and Gächter, 2001). Ones and Putterman (2007) test types of heterogeneity test the stability of these types in a repeated environment. They find that sorting individuals based on willingness to cooperate and avoiding anti-social punishment result in higher outcomes. Albrecht et al. (2018) finds that many conditional cooperators do not punish and that many free riders punish pro-socially. Lastly, they find that information on punishment explains much of the variation in cooperation. Relating to the current paper, the importance of the interaction between types of players is likely to be intimately tied to the bilateral linking choice and should have important impacts on cooperation levels.

The literature in this area is clearly expansive. Specifically, this study contributes to the literature by being the first to experimentally study the effects of an endogenous decentralized punishment network on cooperation in a public goods setting. The remaining sections will highlight the basic payoffs and hypothesis generated for testing in the laboratory. The following experimental section will cover the design and procedures. We then cover the results and conclude with a summary of results and potential extensions.

III Experimental Design and Procedures

III.A Experimental Design

The complete game consisted of three-stages: 1) subjects proposed links with other members of a group; 2) subjects chose contributions in a standard VCM game, and 3) subjects had an opportunity to punish group members to whom they were linked in the first stage. This experiment consisted of three treatments plus two baseline comparison treatments. The treatments varied the benefit of forming bilateral links in stage 1 from 0 (*LB-0*), 2 (*LB-2*), and 4 (*LB-4*). The baseline comparison treatments had no link formation stage. The comparison treatments had either a complete punishment network, (everyone could punish everyone) or an empty punishment network (no punishment stage).

III.B Experimental Procedures

The experiment was conducted virtually using z-Tree Unleashed (Fischbacher, 2007; Duch et al., 2020) through the Experimental Economics Laboratory at the University of Tennessee-Knoxville. A total of 192 subjects participated over the course of 15 sessions, with 3 sessions per treatment. Subjects were recruited from a database of undergraduate students who had previously agreed to receive recruiting e-mails for paid economics studies.

At the beginning of the session students were given a copy of the experimental instructions and the instructions were read by a moderator[5]. The participants remained in a Zoom chat were encouraged to follow along with the instructions as they were read. Each experimental session was divided into 3 parts: A series of three preference elicitation tasks, the main experiment with treatments described above and a brief questionnaire.In the first of the three preference elicitation tasks students chose between 10 possible lotteries of $4 and $0 or a certain payoff of $2, similar to Holt and Laury (2002). The second task was a variation of the

[5] Instructions are available for review in the Appendix.

Ultimatum Game that utilized the strategy method (see Brandts and Charness, 2011) to elicit preferences over the range of possible outcomes. Participants were asked to split 10 tokens (exchanged at a rate of 15 cents per token) with another participant who could either accept the decision paying out the proposed split or reject the decision and both players receive zero. Each participant was asked (simultaneously) whether they would accept or reject each of the 11 possible (integer) splits offered by a Player A. Each participant's decision as Player A was distributed to one other player and the corresponding accept or reject decision as Player B was applied. Thus, they received a single outcome as Player A and Player B. The third task also used the strategy method but this method was applied to a VCM setting. Following the task developed by Fischbacher et al. 2012, participants were sorted in random and anonymous groups of four and made an unconditional contribution choice in standard VCM setup. Before revealing any choices, they then made conditional contribution choices based on the 10 (integer) random hypothetical contribution averages from the remaining group members. Three random unconditional choices were chosen to form the group average which the fourth group members contribution choice based on the average was automatically applied. The results for the three preference elicitation tasks were revealed at the end of the session. Monetarily incentivized practice questions to check for understanding were given for the second two elicitation tasks and the main experiment.

The second part of the experiment was the administration of one of the treatments described above. Participants were randomly sorted into groups of four participants and were only viewed by their other group members through an identifier on the screen (N1, N2, or N3). These groups remained fixed for the 20 rounds of this session. Tokens were used and the exchange rate was adjusted to 25 tokens to $1 USD. Participants were told there would be between 15 and 25 rounds to limit last round effects[6]. A round in the LB treatments of the main experiment consisted of three stages. In the first stage, participants proposed links to their three other group members. For a link to be formed *both* players must have proposed a link to each other. If one or both did not propose a link, no link was formed. The link is therefore pairwise and undirected. Links were used in the third stage to allow subjects to deduction earnings from any group members to whom they are linked. Depending on the treatment, participants earned a

[6] The ending point could potentially be inferred from the range, but the goal was simply to avoid a hard stopping point. This step was intended to limit the presence of end of round effects rather than eliminate it.

number of tokens for each linked formed (either 0, 2, or 4). While decisions in each stage are made simultaneously, the results from each of the stages were presented visually to the participants in such a way that they can review all possible information in the round up to the current decision stage[7]. Participants could also review results from all previous rounds.

In the second stage, players played a linear VCM game where they could contribute up to their endowment (which does not depend on the link formation stage) tokens to the public good. All contributions to the public good were multiplied by 0.4 and distributed to all players. Any of the ten tokens not contributed by a player, were kept. The payoff for a player in stage 2 is:

$$\pi_i^2 = 10 - c_i + 0.4\sum_{j=1}^{4} c_j$$

where c_i is the contribution of player i.

In the third stage, subjects earn an additional endowment of 6 tokens and could spend to sanction their other group members. If they wished to punish a group member with whom they are linked with they could spend up to 2 tokens[8] out of their endowment, per player, that they were linked. All spent tokens were lost and for each token spent, 3 tokens are deducted from the linked group member's total.[9] We chose parameter values to be identical to the closely related study by Boosey and Isaac (2016). The payoff in stage 3 for player i is then:

$$\pi_i^3 = 6 - \sum_{j\in p_{ij}=1} (d_{ij} + 3d_{ji})$$

Where d_{ij} is the amount of tokens player i used to punish player j and p_{ij} are the positive elements of the symmetric $4x4$ punishment adjacency matrix P determined by the linking decisions in the linking stage, $p_{ii} = 0$. The elements of P, p_{ij}, represent whether a link is formed between players i and j and is equal to 1 in the presence of a link. Since links must be proposed bilaterally, $p_{ij} = p_{ji}$. The total number of positive links is known as the *degree* of a

[7] Example screens are given in the instructions, in the Appendix.
[8] By tenths.
[9] Note that it is possible for players to have negative payoffs in this stage, as the group members that they are linked with have to the ability to sanction them as well.

network and is defined by $\frac{\sum_i \sum_j p_{ij}}{2}$. Both the number of punishment opportunities that face any given player and the number they can give out is equal to $\sum_j p_{ij}$.

In the comparison treatments, there was no linking stage. In the *Complete* treatment an exogenous complete network of punishment opportunities was applied, and all group members could deduct from one another. In the empty network, there was no deduction stage and no links. These represent the classic VCM game and the standard representation of a complete punishment network first studied by Fehr and Gächter (2000).

After each round a summary of the stages was given to players that displayed their earnings for the round. After completion of 20 rounds, participants filled out a brief questionnaire that elicited qualitative responses on decision making for the different decision settings and collected basic demographic information. For payment, the sum from the tasks in the elicitation tasks were added to the 20 rounds of play in the main experiment. Average earnings were $23.78 and sessions lasted between 70-90 minutes.

IV Predictions

IV.A Hypotheses

Using backward induction in a *single-period* three-stage game, without including regarding preferences, sub-game perfect Nash-equilibrium predictions are zero allocation and zero punishments in a linear VCM game. In the third stage, no solely self-interested player would undertake costly punishment. Thus, anticipating no punishment in the next period, each player contributes zero to the public good. Then, given zero contributions and zero punishment, players only connect with other players if they receive positive benefits from connections. Therefore, a complete network is the equilibrium choice in the positive link benefit treatments, and when the connection benefit is zero, players should be indifferent about the formation of links. The efficient outcome for any VCM is full contribution and zero punishment and a complete (empty) network when the punishment benefit is positive. This tension between the efficient outcome and the self-interested Nash is the source of interest in the VCM game design.

Much empirical evidence has shown us that the Nash equilibrium strategy is rarely played by the group. A player’s decision in each game and round could depend on a myriad of factors, such as conditional cooperation, other regarding preferences, or strategic behavior in a repeated game. In the link formation phase, participants receive a payoff for formed links and gain the ability to punish others but also expose themselves to risk of punishment. Intuitively, if the link cost is negative (benefit is positive) linking with players that they are not expected to be punished by or compelled to punish is the optimal choice. This involves either players that they expect will not punish them, potentially due to similar contribution choices or who have displayed willingness to permit deviating behavior. Moreover, if they know they are willing to punish others for contributing low amounts, then connecting to a low contributor may not be in their best interest. However, if they themselves are someone who punishes but believe the threat of punishment increases contributions of a low contributor then then they may still wish to link. As the rounds progress, the participants receive additional information about the history of punishment from those they are linked, which will further inform link formation and severance choices.

Taking the links as given in the VCM phase, a higher number of links results in higher possibility of punishment. Prior work has also found that punishment is linked to deviations from the average (Boosey and Isaac, 2016). On the margin, If the individual believes the threat of punishment will be larger than the payoff gained from withholding a token from the public pot, contributions would be expected to increase. Additionally, prosocial preferences and other factors determine levels of contributions in a standard VCM game (Ostrom, 1992; *others*). With this information we assert the following hypotheses:

Hypothesis 1a: *The total degree of the punishment network will increase when the link benefit is larger.*

Hypothesis 1b: *Total (average) amounts of punishment and punishment levels will decrease as the benefits of linking are increased and in later periods.*

The first part of this hypothesis is straightforward. Increasing the incentives for link formation is expected to increase the number of links that are formed. Regarding part (b), previous research on fixed networks has shown that increasing the amount of punishment opportunities works to increases total contributions. As contributions rise, we expect punishment to fall. This would suggest that punishments should decrease over time. However, in our study given the player's endogenous choice of punishment network, players who wish to make a low contribution could opt out of the punishment network. This creates the possibility that, as the link benefits are increased, the likelihood of linking with a punisher (and the likelihood of being punished) may *increase*. Additionally, as players gain more information about other player's types and likelihood of punishing only links will remain that do not result in punishing behavior.

***Hypothesis 2a**: Contributions (and efficiency) will be larger in the treatments where the benefit from linking is higher.*

***Hypothesis 2b**: When the benefit from linking is higher, deviations from the group contribution mean will decrease.*

When the incentive to link is higher, more links are expected to form. When more links are formed the potential punishment levels are higher, and total contributions should rise. This should increase the connectivity between otherwise higher and lower contributors. Higher connectedness then, potentially, results in higher punishment possibilities for deviators from the mean and more convergent contribution levels. With less punishment and higher contributions, efficiency would increase.[10]

***Hypothesis 3** The likelihood of proposing a link to another player is higher when previous round contributions from the other player—the receiver--is closer to the previous round contributions the proposer. This gap will be lower when the difference between proposer and receiver is positive.*

Previous research has suggested that when contributions are similar, punishment is less likely to occur. Therefore, in the cases where links are incentivized players would prefer to be linked. Additionally, players who wish to punish low contributors may seek out the opportunity and propose links to historically low contributors. It is unknown whether low contributors will attempt to link with high contributors with the intention to punish. Punishment of high contributors has been documented, but traditionally this has been a result of anti-social punishment which can be potentially avoided in the bilateral linking environment. Further as shown, in theory, by Takács et al. (2009), when the rewards for conformity are low (high risk of

[10] Efficiency is defined as the total group contributions less the punishment inflicted.

punishment) then contributors will only link with contributors and defectors with defectors. This is potentially an expression of the sorting mechanism behind homophily.

IV.B Power Calculations

A pilot session was conducted to better inform experimental design in order to test the hypotheses listed above. The pilot session conducted used the *LB-2* treatment. Participants were drawn from the same population and experimental procedures closely mirrored the processes described in the design. The results from the session along with guidance from previously related research were used to motivate the selection of sample sizes for the treatments. Ultimately, based on the results from the pilot sessions, budget and time considerations, and participant availability, a treatment size of 40 individuals per treatment (10 groups per treatment) was selected. Due to the random nature of participant attendance, the totals for each of the treatments were 36, 40, 40, 40 and 36 (*Empty, LB-0, LB-2, LB-4, Complete)*.

We use these sample sizes and the pilot data to predict the minimum detectable effect sizes. For the aggregate treatment level comparisons, our power calculations (80% level) suggested that we could detect differences of 5.5 for Contributions and 3.75 for Efficiency across treatments (Hypothesis 3). We are also powered to detect differences of 1.05 in Punishment totals and .68 in Network Size across treatments (Hypothesis 1). For Contributions and Efficiency, these are reasonably large effect sizes and with comparison to previous research, our design may be underpowered to detect more subtle differences between treatments, if they exist. For the variance calculations we are powered to detect differences of 6 (Contribution), .14 (Connections), .9 (Link Proposals), 2.9 (Efficiency), and .4 (Punishment). For the individual comparisons, we are powered to detect the response of Contributions to Neighborhood and Punishment Size at effect sizes of .7 and 3.33. To detect responses the presence of anti-social punishment we are powered to detect minimum effect sizes of .357.

It should be noted that these power calculations are approximations of the true underlying distributions that are guided by informed choices and a small sample of pilot data. The key feature of our design, changing the link benefit, is expected to have significant impacts on the resulting punishment network enabling participants to sanction their fellow group members. Further, with partner matching and the repeated nature of the game, the outcomes of the groups could vary significantly, and in ways that are not adequately represented by the small sample of the pilot data. Nevertheless, for the variables where we observer similar levels of deviation in the data, the power calculations can provide confidence that the results we report represent the true characteristics of the underlying distributions.

V Analysis

Overall, the average age of participants was 21.1 years of age. 63% of participants were female and 46% had participated in a previous economics experiment. Self-reporting on a 1-5 Likert Scale where 1 is “strongly disagree” and 5 “strongly agree” 91.5% of the participants stated they agreed with the statement “I understood the instructions for experiment 1” and 82.3% stated they agree with the statement “I understood the instructions for experiment 2”. This provides some evidence to support the claim that most participants had a strong grasp of the instructions. In a similar fashion, 91% reported that they were well compensated.[11] Table 1.1 provides descriptive statistics for the variables used in the remainder of the analysis section.

In the rest of the section, we will present results starting with aggregate results on total contributions, link formation, punishment, and efficiency. We then look more closely at the differences in distribution of group choices between treatments. Lastly, we compare individual behavior with respect to contribution choices, link offers, link establishment, breaking and formation of new links, punishment, and player types that were determined from the two pre-game preference elicitations.

To summarize the main findings from the experiment, we find many similarities in behavior between the two positive link benefit treatments, *LB-2* and *LB-4*. In these treatments we see evidence of many links being formed, modest increases in contributions over the no punishment treatment, a portion of groups achieving very high levels of cooperation, and some punishment behavior. In most cases, these levels do not reach the levels found in *Complete*. We also find that when the outside incentive to link is zero (*LB-0*), very few links form and we see behavior that closely resembles *Empty*. Subjects do not appear to take advantage of the opportunity to link for the sole purpose of disciplining free riding. The activity in the

[11] Some questionnaire data was lost through the use of virtual method. However, all participants were recruited from the same database and there is a no systematic reason to expect participant characteristics to differ in the missing data.

Table 1.1. Data Description: Endogenous Networks

Variable	Description	Mean	Std. Dev.
Contribution	0 to 10 integer input of public goods contribution	3.50	2.79
LinkProposed	=1 if a link was proposed from player *i* to player *j*.	.561	.496
LinkEstablished	=1 if a link was established between player *i* and player *j*.	.476	.499
NeighborhoodSize	Takes values 0 to 3. Number of links established for each player	1.43	1.36
PunishedBy	Takes values 0 to 2. Amount of tokens spent by player *j* to punish player *i*.	.091	.378
UGPref	Number of offers rejected out of 11 in the Ultimatum Game preference elicitation.	2.70	1.56
Free Ride	=1 if participant is classified as “Free Rider’ from the Public Goods preference elicitation	.179	.384
CondCoop	=1 if participant is classified as “Conditional Cooperator” from the Public Goods preference elicitation	.613	.487
Age	Participant’s age, in years	45.55	45.59
Female	=1 if participant was a female	.633	.482
TotalEarnings	Total earnings for the experiment	23.99	4.29

Note: Other variables are used in the analysis but are derived from the variables defined here. Any new variable that is introduced is explained at that time.

punishment network in *LB-0* falls below *LB-2, LB-4,* and *Complete.* Further, we find that there is very limited evidence of groups achieving high levels of cooperation without the use of the punishment network. This indicates that some level of punishment (or threat of punishment) is necessary to sustain high cooperation.

In terms of the novel bilateral link design, we find strong evidence that links are broken in response to punishment. This indicates that participants are strategically utilizing the linking network to either 1) avoid repeated punishment interactions or 2) provide a secondary source of "punishment" via the reduction in benefits associated with severing a link. Moreover, we find clear evidence that as the link benefit is increased across treatments networks become denser and more players become connected to the network. This suggests that, on average, subjects view the potential benefits/costs of a link (in absence of the benefit imposed by the treatment) differentially and each subsequent link requires increased compensation to form.[12] We also find that there is a strong correlation within treatments between the contribution choices of Free Riders and Conditional Cooperators. We see a high proportion of 0 contribution choices by Free Riders in the *LB* treatments, indicating potential behavioral differences (*e.g.* link proposals acting as a signal) between treatments with link formation, and those without. Lastly, we find muted evidence of homophily and no evidence of the convergence of contributions within groups across treatments.

V.A Aggregate Results

Starting with aggregate comparisons across treatments, Table 1.2 presents the group level mean Contributions, Network Size, Punishment, and Efficiency. Figure 2.1 shows the average results, by period, between treatments for Contributions, Links Proposed, Network Size, Punishment, and Efficiency. Pairwise tests for significance across treatments are presented in the Appendix. For public good contributions we find significant differences between *Complete* and *Empty* and *LB-0*. While the difference between *LB-4, LB-2* and *Empty* and *LB-0* are positive, we do need see statistical differences. Comparisons of contributions are discussed further later in the section but generally we find that contributions are highest in *Complete* with *LB-2* and *LB-4* falling

[12] This is likely a result of the increased compensation required to get uncooperative individuals into the punishment network, either free riders, or subjects with a high willingness to punish.

Table 1.2. Aggregate Group Outcomes by Treatment

	Complete	*LB-4*	*LB-2*	*LB-0*	*Empty*
Contributions	17.51	14.99	14.87	10.73	12.08
	(1.86)	(3.20)	(3.09)	(2.20)	(1.84)
Links Proposed		10.21	7.994	3.562	
		(.736)	(.954)	(.717)	
Network Size		4.617	3.102	0.663	
		(.535)	(.610)	(.330)	
Punishment	8.375	3.852	3.376	0.846	
	(2.30)	(1.10)	(1.26)	(.275)	
Efficiency	42.13	45.15	45.55	45.59	47.25
	(2.77)	(2.64)	(2.50)	(1.38)	(1.10)
Observations	180	180	197	190	180
Groups	10	9	10	9	10

Note: Observations are at the period-group level. Standard errors in parentheses are clustered at the group level.

between *Complete* and *LB-0/Empty*. One clear result from the aggregate data is the difference in connections in the *LB* treatments. All differences are significant at the 10% level ($p_{LB4,LB2} =$ .058; $p_{LB4,LB2} =$ .000; $p_{LB2,LB0} = 0$). This leads to Result 1:

Result 1 *Bilateral punishment links (and link proposals) follow the law of demand.*

There is a clear positive relationship between the number of connections and the benefit received for connections. This result provides strong evidence in support of Hypothesis 1b. The result indicated that subjects consider the expected value of each connection in the group and compare this directly to the link benefit. Given the relatively low level of connections in *LB-0*, we can infer most links are viewed as having a negative net value by at least one player in each pair of players and are not formed. This is somewhat in contrast with the results of Ramalingam et al. (2016) who find that participants are willing to pay for the option to punish everyone in their group. The bilateral nature of the links allows for a single person to opt out of the institution and provides a clear distinction between this study and Ramalingam et al. This difference appears to negatively impact the value participants place on forming a single link (or more simply, that the proposed link is not reciprocated).

Looking at link proposal rates in Table 1.2 we see that the point estimate difference in links proposed between *LB-2* and *LB-4* is 2.22, and the difference in connections formed is 1.51. Given that a single unconnected individual can reduce the connections of an otherwise fully connected group from 6 to 3 this suggests the possibility that the difference in links between the treatments are coming from a single member of the group, namely a free rider who views links as net-negatives only and must be compensated to form links. We explore the individual level determinants of links a bit later in the section.

Figure 1.1 shows the treatment mean, by period, for each variable of interest. We observe consistent behavior across treatments. Network Size and Proposals are quite stable over the periods and follow the patterns described above. In general, *LB-2* and *LB-4* display similar behavior in terms of Contributions, Efficiency, and Punishment, which indicates subjects perceive a limited value to the increased connectivity from increased linking benefits. Efficiency

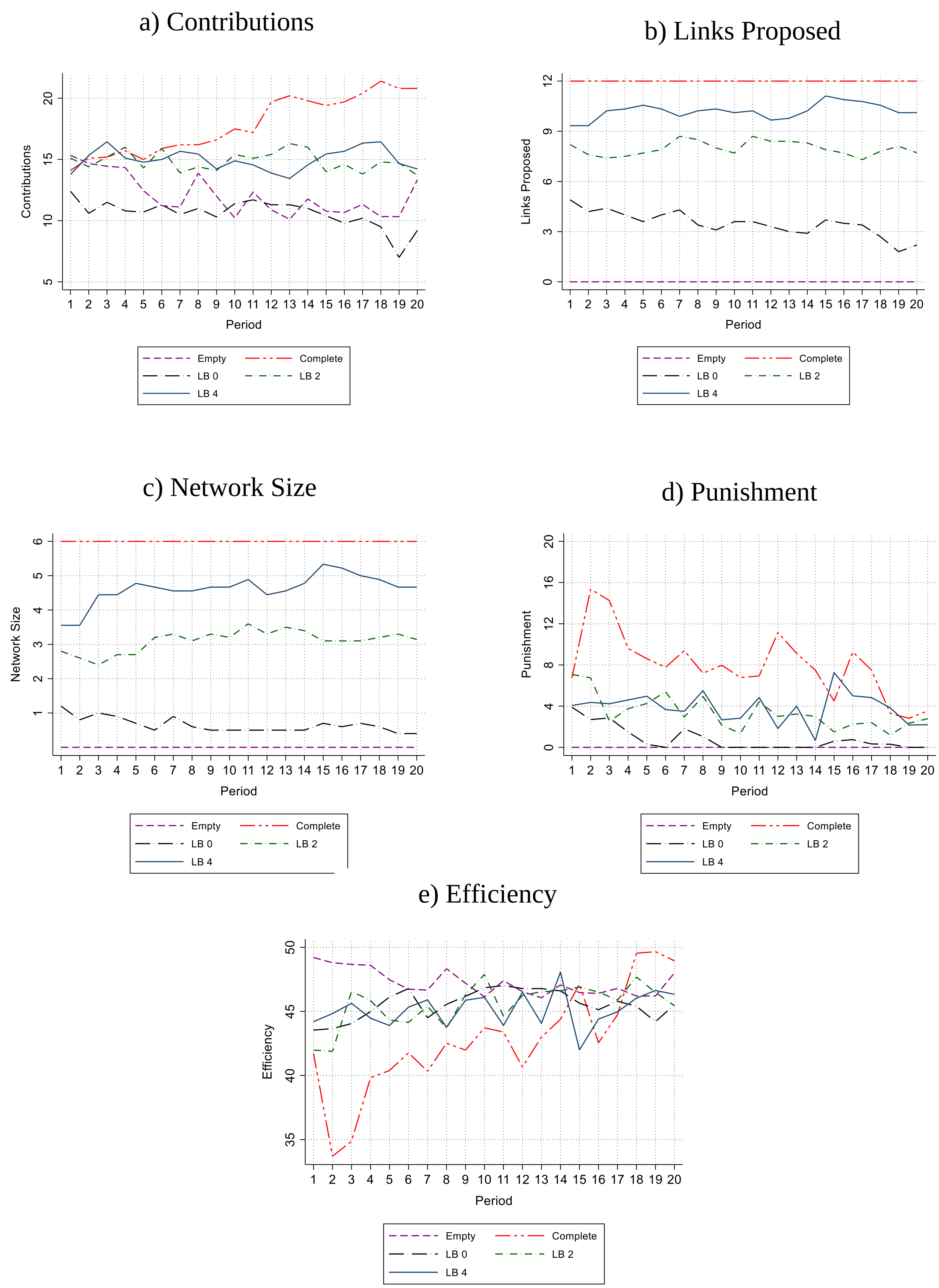


Figure 1.1. Treatment Averages

is defined on the group level as the total tokens created by the public and private goods minus the tokens expended punishment and the total deductions.

We see a clear pattern of increased contributions over all treatments in *Complete*. Despite the treatment average only being significantly different that *LB-0* and *Empty*, *Complete* has higher contributions than *LB-2* and *LB-4* in the final 13 periods. Similarly, despite no statistical difference in average contributions, *LB-2* and *LB-4* display higher contributions than *LB-0* and *Empty* in the final 17 periods. The gains in contributions are not without cost, however. As we observe higher punishment in *LB-2, LB-4*, and *Complete* relative to *LB-0* and *Empty*, with the largest average punishment in *Complete*.[13] In general, the elevated levels of punishment result in reduced efficiency. This is especially the case in *Complete* where the gains in contributions are not seen until earlier periods of high punishment translate into higher contributions in the later periods with less punishment. The persistent difference in contributions and punishment across periods yields support for Hypothesis 2a and the following result:

Result 2 *Contribution and Punishment in positive link benefit treatments fall between the Complete and empty network. Contributions and Punishment in the no link benefit treatment behave similarly to the empty network.*

An important caveat is need for the interpretation of relatively small differences in efficiency in the Link Benefit treatments. Depending on the lens that the link benefit itself is viewed from, we can reach different conclusions. If we connect this to the real-world context of the link benefit arising endogenously through interactions from participants in the group then the efficiency of the mechanism (when defined as contributions minus punishment), we understate the attractiveness of the bilateral linking environment. In this case the additional benefits of the links would reasonably be included in efficiency calculations and serve to increase the overall efficiency. However, from a social planner's perspective, if the links need to be subsidized then the benefit of the links to the subjects represents a social transfer, and there is no gain in efficiency resulting from connections. So far, we have looked at aggregate treatment average data. We will now compare the within-treatment variation at the group level, across treatments. Figure 1.2 shows group by period contribution levels for *Empty LB-2* and *LB-4* for each group

[13] Statistically significant differences are found at the 10% level for each treatment pair except *LB-2, LB-4*, and *LB-0* and *Empty*

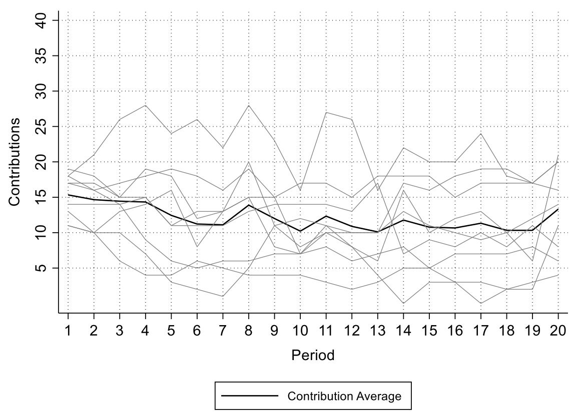
within the treatments. Clearly Figure 1.2 shows a divergence in contributions in later rounds for *LB-2* and *LB-4,* despite reasonably similar overall treatment averages by period. In *LB-2* and *LB-4,* a few groups tend toward full or nearly full contributions while other groups fail to coordinate and contribution below average. Contributions in *Empty* remain relatively centered around the group average for all periods and only a single group reaches contributions levels greater than 20 (half) as opposed to 3 groups in both *LB-2* and *LB-4* in the final period.

Table 1.3 examines the variance for each of the 5 variables of interest presented in Table 1.1. Deviations from the treatment mean by period are used to calculate the variance. Pairwise tests for significant differences between treatments are given in the Appendix. We see significant differences in the variance of contributions in *LB-2* and *LB-4* and *Empty*. The differences between *Complete* and *LB-0* are large in point estimates but statistically insignificant. The point estimates for the variance in connections and links proposed is highest in *LB-2* and is statistically different from all other treatments at the 1% level, in connections. *LB-2* serves as the midpoint between *LB-0* which has very few connections on average and *LB-4* where connections trend toward the complete network. In these extreme cases, the networks in some group change very little. This heightened activity around *LB-2* suggests possible sweet spot for interpreting how participants view the value of connections.

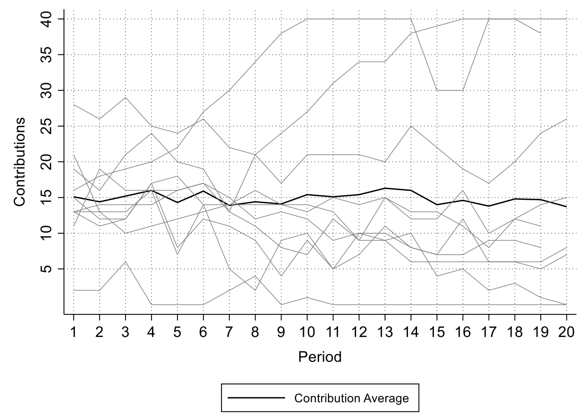
Result 3 *Between group variance is higher in the Complete, LB-2, and LB-4 treatments relative to the Empty and LB-0 treatments.*

The observed variance in Punishment is highest in *Complete* but is marginally insignificant when compared with *LB-2* and *LB-4*. A potential mechanism that reduces the variance that is observed *LB-2* and *LB-4* is the ability for any player to sever a link in the *Link Benefit* treatments. This avoids repeated anti-social punishment that can escalate punishment totals. We will provide evidence that this is indeed the case later in the section. Lastly, variance in Efficiency is significantly higher in *LB-2 LB-4* and *Complete* relative to *LB-0* and *Empty.* This result is a derived directly from the variance in contributions for *LB-2* and *LB-4* and the variance in punishment in the *Complete* treatment. These variances produce the differences in variances observed in Efficiency.

a) *Empty*



b) *LB-2*



c) *LB-4*

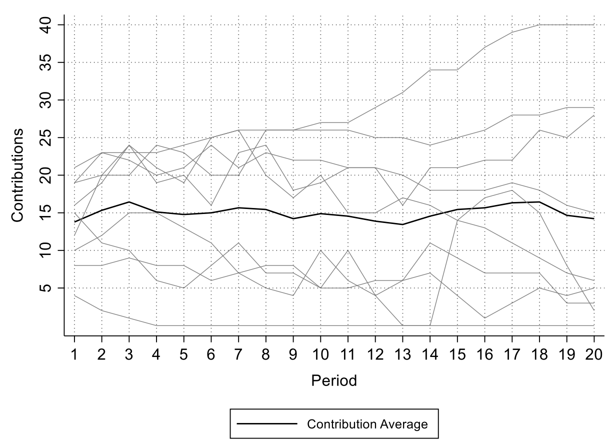


Figure 1.2. Group Level Contributions by Treatment Type

Table 1.3. Group Outcomes by Treatment: Variance

	Dependent Variable: Variance				
	Contributions	Connections	Links Proposed	Punishment	Efficiency
Complete	1.152			9.394**	10.26**
	(2.494)			(3.943)	(4.146)
LB-0	1.608	0.131**	0.788***	0.661***	1.363*
	(1.972)	(0.0607)	(0.184)	(0.179)	(0.762)
LB-2	7.821*	0.463***	1.206***	3.283**	7.159***
	(4.714)	(0.100)	(0.349)	(1.383)	(2.456)
LB-4	7.896**	0.363**	0.770*	4.317***	8.668***
	(3.653)	(0.168)	(0.417)	(1.091)	(2.033)
Constant	4.427***				1.594***
	(1.406)				(0.506)
Observations	927	927	927	927	927
R-squared	0.074	0.240	0.184	0.106	0.108

Note: Reported results are point estimates from a pooled OLS regression. The comparison group is the Empty treatment. Standard errors in parentheses are clustered at the group level. *** p<0.01, ** p<0.05, * p<0.1

To further explore differences in group choices among treatments, we compare decisions in each of the treatments conditional on the punishment network being full, and also conditional on the punishment network being empty. In these cases, the punishment networks are identical and the decision environment of the groups (in the absence of dynamic effects) are otherwise identical, but we expect that behavior may differ when the extant punishment network has arisen endogenously. Table 1.4 shows significant differences in contributions conditional on an empty network when such a network has arisen endogenously, *i.e.* when there is a positive link benefit but no players chose to link. Contributions are much lower in endogenous empty networks. When the punishment networks are complete, we see higher contributions in the *LB-2* treatment relative to an exogenously complete case, but the difference is not significant. Particularly in the empty network observations, these results suggest that the dynamic interaction of the heterogeneity of individuals and the bilateral linking incentive influences how the punishment opportunities (or absence of) affect contribution choices. We also present figures in the Appendix that show the network size and contributions for each group in the final period, as well as the maximum network size and contribution.

IV.B Individual Decisions

So far, we have compared treatments and seem some similarities at the aggregate level across treatments. The differences arise when we look more closely at the group decisions. We now restrict our attention to the individual decisions within the groups and connect this to the differences we see in the groups between treatments. We first present results which use the conditional cooperator and free rider classifications from the pre-stage preference elicitation. We then look at behavior at the individual linking and punishment choices focusing on the *LB* treatment. Finally, we will focus on pairwise linking and punishment decisions.

Previous research has shown that much of the dynamic response that we observe in repeated public goods games is a result of "optimistic" conditional cooperators lowering their contributions in response to low contributions from other members of the group/session. We use the results of the pre-experiment public goods preference elicitation to "type" players as Conditional Cooperator, Free Rider, or Other (Fischbacher et al. 2012). We find similar

Table 1.4. Group Contributions with Full or Empty Networks

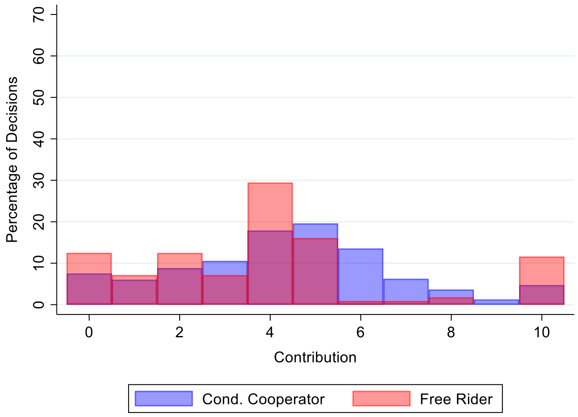
	Dependent Variable: Group Contribution	
	Full Network	Empty Network
LB-0		-3.041
		(2.965)
LB-2	5.435	-7.539**
	(7.388)	(3.210)
LB-4	-1.758	-6.685***
	(5.350)	(1.938)
Constant	15.36***	14.65***
	(1.491)	(1.306)
Observations	315	343

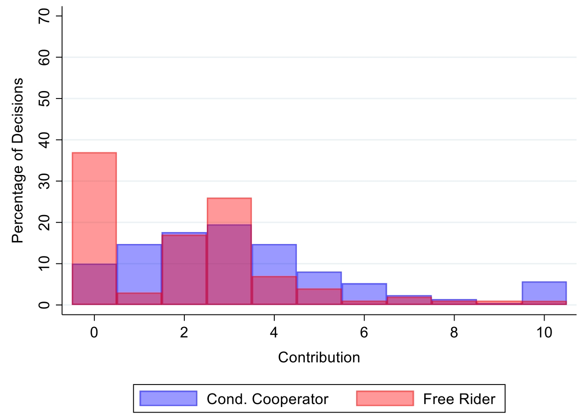
*Note: The table reports pooled OLS regression results where the dependent variable is group contributions. Samples are restricted to observations in each treatment where the punishment network is either full or empty. The constant for each column is the corresponding baseline treatment, Complete for the full network and Empty for the empty network. Period fixed effects are included but omitted. Standard errors clustered at the group level are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1* proportions of players as Fischbacher et al. (2012), with 61.3% of players classified as Conditional Cooperators, 17.9% as Free Rider, and 20.8% as Other.[14] We then compare the individual frequency of contribution choices, by treatment and player type (Conditional cooperators and Free Rider), in Figure 1.3. This provides us with some clarity in understanding how the player types interacted within groups and how the composition of types within groups may have affected contributions. From Figure 1.3 we can see a few clear differences between the treatments. First, in all treatments there are a substantial portion of Free Riders who contribute positive amounts despite responding with zero to all possible group averages in the single shot preference elicitation. A likely explanation for this behavior is the presence of a repeated game in which players recognize the importance of reputation formation and positive group contributions.

Secondly, we see quite a bit of overlap in the density of contributions between Conditional Cooperators and Free Riders, where the center of the mass varies between treatments (and which generally trends up as the link benefit is increased). While Conditional Cooperators contribute higher for almost all positive contribution levels, in all treatments, there looks to be interdependence between Free Riders and Conditional Cooperators. This also suggests the possibility of strong path dependence for groups. Moreover, we see positive values at full contributions for both Free Riders and Conditional Cooperators in *LB-2, LB-4*, and *Complete.*

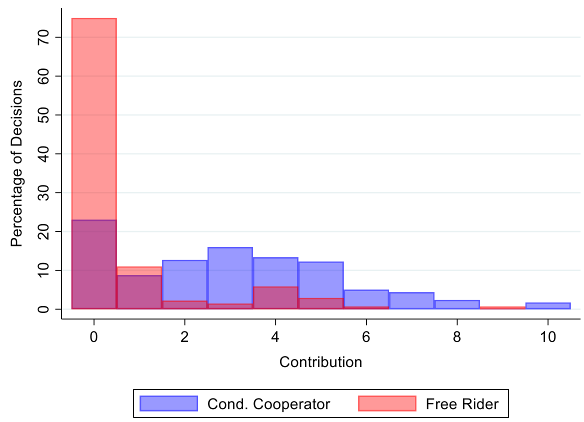
The last striking difference is the noticeably larger portion of free riding, by Free Riders, in each of the *LB* treatments relative to both of the baseline treatments. A potential explanation for such behavior is that the link formation process serves as a signaling mechanism for contributions in the following stage. It could be the case that the link formation stage induces players to cognitively think through their contributions and their group members response to their contributions (*i.e.* their expectations about punishment conditional on a link) and thus avoid linking. And by avoiding linking they then feel justified in staying true to their “Type” and ultimately decide to free ride. Alternatively, this could be a result of the punishment mechanism, in which Free Riders respond to punishment by removing links and reverting to free riding behavior in subsequent periods, similar to a Grim Trigger strategy studied in repeated games.

[14] Classifications are defined exactly as in Fischbacher et al. 2012. We exclude “Triangle Cooperators” and classify them in “Other”. Free Riders are classified by “0” conditional responses to all possible group contribution averages. Conditional cooperators have correlation coefficient of at least 1 with response to changes in the group average.

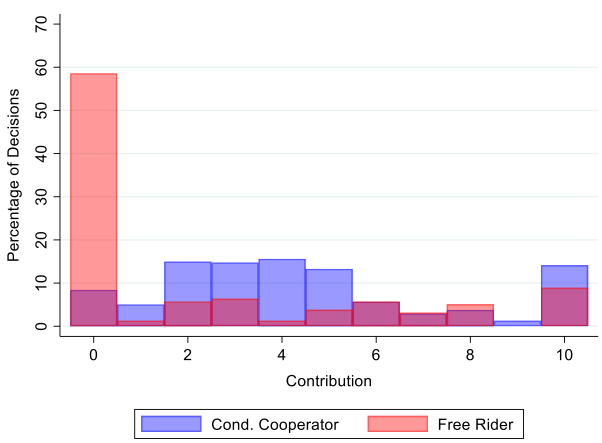




c) *LB-0*



d) *LB-2*



d) *LB-4*

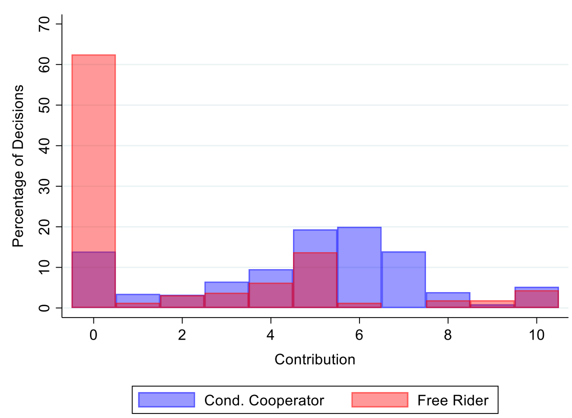


Figure 1.3. Contributions by “Type”

On the other hand, particularly in *Complete*, when they consider the potential for punishment from low contributions, they are forced to accept the punishment or alter their behavior if they wish to avoid punishment in future periods. In *Empty*[15]¸ low contributing individuals do not have to worry about punishment or linking but may still recognize the importance of positive contributions and settle on a small but positive contribution. We will explore the individual determinants of contributions next.

Table 1.5 reports the results from a random effects panel regression for each treatment with public account contributions as the dependent variable. The regressors include lagged contributions, lagged punishment, number of connections, lagged group average, and a period variable. We see across treatments that the strongest predictors of contributions are both previous round contributions and the previous round group average. An interesting result is that the point estimates on punishment received in the previous round are negative or near zero for all treatments. The estimates are negatively significant at the 5% level in *LB-0* and *LB-4 (p=0.000 and p=0.047, respectively)*.

It should be noted that the direction of the causality is not clear. It could be the case that players who have punished in the previous period exerted lower effort levels, but it could also be the case that higher levels of punishment result in "counter-punishment" through the lowering of contribution levels. Shifting attention to the effect of individual network size on contribution levels we see significant positive estimates in both *LB-0* and *LB-4* at the 1% level (p=.000, p=0.010, respectively). The positive coefficient on *NeighborhoodSize* in *LB-2* is marginally insignificant (p=.135). Results in the Appendix use dummy variables for each network size and find significant positive coefficients in all treatments for each level of network connections. This shows that higher network sizes are correlated with higher levels of contributions (again the causality could run in both directions). Together these results suggest support for the idea that the actual mechanism that increases contributions is the threat of punishment as opposed to the actual act. This leads us to Result 4:

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[15] The low levels of free riding could also be a small sample size issue with only 9 groups comprising the treatment. This persistence in positive contributions, particularly among free riders, is a likely source for the limited amount of decay over time we observe in the *Empty* treatment.

Table 1.5. Individual Contributions by Treatment

	Dependent Variable: Contributions				
	Complete	*LB-0*	*LB-2*	*LB-4*	*Empty*
ContLag	0.589***	0.336***	0.571***	0.577***	0.470***
	(0.062)	(0.114)	(0.040	(0.041)	(0.144)
PunishInLag	0.008	-0.283***	-0.042	-0.0608**	
	(0.030)	(0.068)	(0.072)	(0.034)	
NeighborhoodSize		0.863***	0.249	0.167***	
		(0.108)	(0.166)	(0.0646)	
GroupAveLag	0.370***	0.492***	0.352***	0.405***	0.393***
	(0.085)	(0.160)	(0.072)	(0.044)	(0.122)
Period	0.001	0.002	-0.005	-0.013	0.012
	(0.007)	(0.008)	(0.007)	(0.008)	(0.008)
Constant	0.239	0.210	-0.0125	-0.116	0.257
	(0.195)	(0.170)	(0.134)	(0.165)	(0.240)
Observations	680	720	748	684	684
Number of Subjects	40	40	40	36	36

*Note: Regression is a random effects panel data model with standard errors clustered at the group level. NeighborhoodSize describes the total number of links for each subject (0 to 3). The number of clusters is equal to the number of subjects divided by 4 and the reported standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Result 4a *Higher levels of punishment received in the previous period are not correlated with higher levels of contributions in the next period.*

Result 4b *Large punishment network degrees are correlated with higher levels of public goods contributions.*

Table 1.6 displays the results for a pooled OLS decision which regresses a set of covariates on the individual total punishment decisions conditional on a link existing. The data is decomposed so that each subject has 3 potential punishment decisions, conditional on the number of links formed, in each period. Previous research (Nikiforakis, 2012) has shown that there is substantial retaliatory punishment observed in public goods games with punishment. We do see evidence of a similar trend in *Complete* and *LB-4.* We do not observe coefficients that are significantly different from zero in *LB-0* and *LB-2,* (however point estimates are positive). This is likely due, in part, to the ability for subjects to break links in which punishment has occurred. When the link benefit is higher, such as in *LB-4*, the opportunity cost of weighing the potential punishment in the next period is higher, thus a link is more likely to be retained. In the *Complete* treatment it is not possible to opt out of links, so it is not surprising to see more robust evidence of anti-social punishment in this case. This is one strength of the pairwise linking environment. We show in Table 1.5 that a major predictor of destruction of a link is punishment in the previous period.

From Table 1.6 we can also observe that a major predictor of punishment is punishment of the same person in the prior period, suggesting that once punishment of another player has begun, the player will continue to punish (likely until contributions have been raised to acceptable level or the link is broken). Another notable finding is that we see very limited evidence for the other player contributions affecting the choice of punishment.[16] This could be the result of conditional cooperators responding to low contributions with low contributions of there own while still administering punishment. We also do not find a significant effect of *Period* in any of the treatments. Thus, we do not find support for Hypothesis 1b. We also find limited evidence that deviation from the group average is a

[16] Regressions with the difference in effort replacing the two contribution related covariates does not produce significant coefficients in any treatment.

Table 1.6. Individual Punishment Administered by Treatment

	Dependent Variable: PunishOut			
	Complete	*LB-0*	*LB-2*	*LB-4*
Contribution	0.024	-0.011	-0.009	-0.011
	(0.021)	(0.044)	(0.016)	(0.012)
PunishedByLag	0.142***	0.173	0.0910	0.188*
	(0.037)	(0.363)	(0.074)	(0.103)
PunishOutLag	0.421***	0.282**	0.743***	0.481***
	(0.049)	(0.290)	(0.155)	(0.068)
OtherPlayerCont	-0.0284	-0.020	-0.001	0.002
	(0.022)	(0.092)	(0.021)	(0.016)
OtherPlayerAveDev	-0.018	-0.003	-0.012	-0.032*
	(0.032)	(0.145)	(0.026)	(0.019)
UGPref	0.028	-0.021	0.0126	0.0329**
	(0.028)	(0.045)	(0.025)	(0.016)
LinkCreated		0.576	0.375**	0.528**
		(0.352)	(0.121)	(0.176)
Period	-0.006	-0.002	-0.002	0.003
	(0.004)	(0.013)	(0.003)	(0.002)
Constant	0.112	0.309	0.079	-0.047
	(0.087)	(0.354)	(0.114)	(0.076)
Observations	2,040	228	1,166	1,598
R-squared	0.277	0.303	0.479	0.313

*Note: Regression is a pooled OLS model with standard errors clustered at the group level. UGPref is the amount of rejected offers in the Ultimatum Game preference elicitation. Dependent variable is in tokens spent (0-2). The number of clusters is the number of groups per treatment (9-10). Bootstrapped standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1* significant predictor of punishment. Only in *LB-4* is the coefficient on *OtherPlayerAveDev* significantly different than zero (*p*=.084). *UGPreference* is a variable that captures the number of rejections in the Ultimatum Game preference elicitation that was conducted before the experiment began. Again, we see only limited evidence of this variable predicting punishment choices[17]. One quite striking result is the inclusion of the LinkCreated variable which is a binary variable that is equal to 1 when the link between the players was formed in the same period as the punishment. We see large positive estimates for each of the treatments[18] (*LB-0 p=.102; LB-2 p=.002; LB-4 p=.003*). This suggests that players are actively forming links with individuals they intend to punish.

Result 5 *Punishment is highly correlated with the formation of a link in the same period.*

Table 1.7 shows the marginal effects calculated at the variable means from a pooled probit regression where the dependent variable is the breaking of a link between two players from one period to the next. As mentioned above, a primary determinant of the breaking of link is the presence of punishment in a previous period.[19] Thus, players are responding to the negative payoffs associated with punishment by severing potentially profitable links. The breaking of a link may be a substitute for increasing contributions that would be required to avoid punishment in *Complete*. One other notable result from Table 1.7 is the negative coefficient on the round indicator. This suggests that networks become more stable as the rounds progress as fewer links are broken.

Result 6a *Punishment in the previous period is a strong predictor of broken links.*

Result 6b *The likelihood of a link being broken decreases over time; punishment networks become more stable over time.*

A more comprehensive regression with *Link Broken* as the dependent variable that includes player types is presented in the Appendix. This specification shows that Free Rider types are less likely to break a link when the Link Benefit is positive.

[17] Additional model with fixed effects is shown in the Appendix, these necessarily omit fixed variables.
[18] *LB-0* standard errors are likely influenced by the low number of connections/observations.
[19] Due to the bilateral nature of a link breakage, the PunishedByLag variable accounts for both ingoing and outgoing punishment.

Table 1.7. Determinants of Broken Links by Treatment

	Dependent Variable: Link Broken		
	LB-0	*LB-2*	*LB-4*
ContLag	0.002	-0.004	0.004
	(0.002)	(0.003)	(0.003)
PunishedByLag	0.050**	0.051***	0.063***
	(0.021)	(0.008)	(0.019)
OtherPlayerLagCont	-0.003	0.001	-0.002
	(0.003)	(0.003)	(0.005)
OtherPlayerLagAveDev	0.004	-0.004	0.001
	(0.003)	(0.004)	(0.007)
Period	-0.001**	-0.003***	-0.002*
	(0.001)	(0.001)	(0.001)
Observations	2,160	2,244	2,052

*Note: Regression results are from a pooled probit model clustered at the group level. The reported values are the marginal effects calculated at the variable means. The number of clusters is the number of groups per treatment (9-10). Bootstrapped standard errors are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

On the other hand, it also shows the interaction between a Conditional Cooperator and a Free Rider are likely to experience a severed links. This provides evidence that the likely mechanism of punishment is Conditional Cooperators punishing Free Riders (Nikiforakis et al., 2012), after which Free Riders respond by severing links.

Our last set of analysis looks the factors that influence whether links are established or proposed. We will focus on established links, but corresponding analysis for link proposals is available in the Appendix, results are largely the same. Table 1.8 presents a regression with an established link between two players as the dependent variable.[20] A presence of links between players of similar types would provide an example of the common finding of homophily in network science literature. We define types, as above, based on the pre-game preference elicitation. However, a type may simply be the contribution levels or some other observable characteristics. As we see however, contribution levels depend on group composition and previous contributions, so an exact measure is difficult to pin down. We see limited evidence of preferential linking between player types, most prominently in *LB-4*. In general, as the linking benefit is reduced, we see reduced point estimates and coefficient estimates that are not statistically different than zero.

Importantly these are dynamic estimates and will depend heavily on the interaction between players. We can easily observe that the strongest predictor of a link, is a link in the previous period. Thus, many links that are created, persist. Further, if punishment happened in a link in the previous period, the likelihood of a link in the next period is reduced (identical result to Table 1.7). The decreasing trend, as link benefit increases, of the coefficients on ContDiffLag and OtherPlayerLagAveDev indicates that the links formed (and maintained) when the link benefit is smaller are more likely to be between players that have higher contributions relative to the group or among players who have more disparate contribution differences. As link benefit increases and more subjects become connected, particularly free riders/low contributors, these relationships fall away. In general, when the link benefit is smaller, to maintain a link subjects must more be more tolerable to divergent behavior from their own. As the link benefit increases, subjects can exert more and more pressure in attempt to get divergent subjects to conform.

[20] Similar regressions were run as panel data with very little qualitative differences.

Table 1.8. Determinants of Established Links by Treatment

	Dependent Variable: Link Established			
	Combined	*LB-0*	*LB-2*	*LB-4*
ContLag	0.026*	-0.002	0.059*	0.009
	(0.016)	(0.006)	(0.033)	(0.010)
ContDiffLag	0.001	0.016**	-0.011	-0.006
	(0.012)	(0.007)	(0.030)	(0.011)
PunishedByLag	-0.170***	-0.052**	-0.164***	-0.136***
	(0.027)	(0.025)	(0.050)	(0.013)
LinkEstablishedLag	0.578***	0.167***	0.658***	0.367***
	(0.053)	(0.050)	(0.106)	(0.033)
FreeFree	0.104	-0.001	0.206*	0.064
	(0.104)	(0.020)	(0.107)	(0.057)
FreeCond	0.041	-0.003	-0.089	0.078***
	(0.069)	(0.019)	(0.074)	(0.021)
CondCond	0.071	0.017	0.202**	0.019
	(0.051)	(0.020)	(0.082)	(0.046)
FreeOther	0.102		0.039	0.130
	(0.125)		(0.106)	(0.086)
CondOther	-0.016	-0.003	0.146	-0.090***
	(0.068)	(0.035)	(0.139)	(0.028)
OtherPlayerLagAveDev	0.030**	0.025***	0.041	-0.004
	(0.013)	(0.009)	(0.031)	(0.014)
Observations	6,456	2,141	2,244	2,052

*Note: Reported results are from a dynamic probit regression with binary dependent variable =1 when a link was formed between players i and j, otherwise =0. Reported results are marginal effects calculated at the independent variable means. The comparison group is two players with types: Other. FreeOther was omitted due to a very small number of links <.1% in the second regression. Bootstrap standard errors are clustered at the group level and given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

VI Conclusion

This study builds on previous research into network effects, network formation, and outsiders in social dilemmas. Overall, the results paint a picture of how the bilateral linking environment differs from the traditional variations in the complete network punishment game. Subjects often utilize the breaking of links to avoid punishment and even when the link formation is subsidized to a total of 50% of the maximum possible negative payoff from a formed link, we still do not observe complete network formation in many groups. The ability for low contributing group members to avoid retaliation limits the contribution gains that we see in the more traditional punishment setting. However, to the extent that punishment interactions can be avoided we can observe reduced instances of anti-social punishment.

Ultimately many of the lessons that we learn from the more traditional framing of punishment carry over to the bilateral linking environment. When the linking benefit is positive, we see some ability for the network to enhance cooperative behavior and increase public good contributions but not to the level that a complete punishment setting enables. It is certain that the efficacy of a bilateral self-enforcement mechanism will depend on the strength of the link benefit between the parties, and the relative strength of punishment process itself.

Variations utilizing the bilateral approach provide potential extensions to this research. For example, the use of directed rather than undirected endogenous punishment networks may create an environment that would not allow low contributors to opt out of the punishment network. Ultimately, this could increase overall contributions as the total amount of punishable connections may increase. Potentially, this would then mirror a complete punishment network that has previously been studied, in the sense that any player that would like to punish would be available to do so. Also, in this study perfect information was available to all participants. Having link formation be related to information about contributions may potentially alter the link formation choice is interesting ways and provide an endogenous benefit for link formation. Lastly, varying link incentives, either through time or by individual, or utilizing a different punishment scheme (such as rewards) may create more a more externally valid environment. It could be the case that links and signaling in the early periods have disproportionate impact on the outcomes due to path dependence and more heavily subsidized links (or rewards) in early periods help groups achieve cooperative outcomes and reduced the need for strong link benefits in later periods. Hopefully, this study provides a first step to analyzing the impacts of bilateral interactions in public good settings.

Chapter 2

Probabilistic Entry and Heterogenous Valuations in Sequential Contests

I Introduction

The expansive literature on contests, which models a great variety of settings in which competitors incur costs in pursuit of winning a fixed prize or prizes, has long recognized the importance of sequential play. Many contests in the real world contain sequential elements. For example, in litigation arguments and evidence are brought forth sequentially. In industry, incumbent firms or contractual parties are often given the right to make a final offer in an agreement (Morgan, 2003). However, identifying subgame perfect Nash equilibria of sequential contests has posed a significant theoretical challenge.

This paper seeks to answer the following two questions. How does exogenous variation in the value of the contest prize alter the equilibrium contest expenditure when comparing simultaneous and sequential environments? Also, how does the threat of entry into a contest differ between a simultaneous and sequential environment? Recently, theoretical advancements in the study of contests by Hinnosaar (2018) have provided us with the tools to identify the SPNE of sequential contests with these characteristics. We then take the theoretically predicted solutions to the laboratory to test the theory and connect the findings to the robust experimental literature on simultaneous contests.

Previously, sequential modelling of contests had been primarily restricted to two-period and had found that, due to the shape of the best-response functions, and unlike the related Stackelberg quantity competition problem, a first mover advantage was only present when contest participants were asymmetric (Tullock, 1987; Linster, 1993). While some research had extended contest theory to a third sequential player, the results using the traditional backward induction technique are cumbersome and unwieldy (Kahana and Klunover, 2018).[21] Hinnosaar's innovation, using the *inverse best response function,* draws from techniques used in aggregative games (see Jensen 2018) and allows for the characterization the Sub-Game Perfect Nash Equilibrium (SPNE) for any homogenous (in valuations/costs) n-player T-period, and n_t player-per-period contest ($\sum n_t = n$). This novel technique provides a substantial step forward from the 3-person limitation of the backward induction approach.[22] Further, embedded within this framework, are a variety of strategically similar games, including quantity competition among firms, pari-mutuel betting, public goods games, and Tullock contests.

Hinnosaar's primary result generalizes the *first-mover advantage* (Dixit, 1987) result to an *earlier mover* advantage (in an otherwise symmetric contest) that says that any player moving earlier in a contest will choose higher effort choices, have higher chance to win, and will have ex ante higher expected utility, *ceteris paribus*. Further, he suggests that the total amount of effort expended in the contest is increasing when additional information about total expenditure is revealed (direct observations).

In this paper we extend Hinnosaar's model to incorporate heterogenous valuations of the contest prize as well as potential entry into a 3-person contest and test the predictions in a laboratory experiment. This paper provides one of the first theoretical and laboratory examinations of the effects of player heterogeneity in multi-period sequential contests[23]. While the experimental literature on simultaneous contests is robust and has generated many interesting behavioral phenomena, such as overbidding, relatively little experimental work has been done on sequential contests (see Dechaenaux et al. 2015 for a survey on contests). Fonseca (2009) test and confirm the prediction that sequential and simultaneous Tullock contests have equivalent outputs in the two-period game. Nelson (2019) and Nelson and Ryvkin (2019) recently studied whether subjects in the laboratory exhibit *first-mover* advantage in a variety of homogeneous 3-player sequential contests (as predicted by Hinnosaar). They find that, contrary to prediction, subjects do not exhibit *first-mover* advantage. Additionally, they find that subjects produce lower

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[21] Some other studies have looked at 3-periods models in different contexts (Glazer and Hazin, 2000 and Baik Lee, 2019 for contests; Daughety, 1990; Ino and Matsumura, 2012; Julien et al., 2012 for oligopolies with linear demand).

[22] Hinnosaar, in fact, proves that the most complicated situation that backward induction can provide an analytical solution for is the 3-player contest.

aggregate efforts in sequential contests relative to simultaneous contests. This is again contrary to theoretical predictions.

We build on this previous research but shift the focus to the impact of heterogeneity and the interaction between heterogeneity and sequential position in sequential contests. We explicitly model both heterogenous prize valuation (equivalently, costs) and potential entry (heterogenous entry probabilities) in each of the sequential positions in a 3-period, 3-player contest. While we operate in the framework of a Tullock contest, the strategic similarity to a quantity competition model generates implications for market competition settings. Incorporating heterogenous valuations in a sequential environment is similar to cost heterogeneity among firms in quantity competition or differences in market power. Potential entry and its impact on prices and quantity in markets is widely studied (MacDonald, 1986; Bresnahan and Reiss, 1990). Further, probabilistic entry has been studied in the closely related context of auctions (Bulow and Klemperer, 2009). Contests have also been closely linked with auctions in the game theory literature (see Dechenaux et al. 2015 for a survey).

Important results have come from the study of the interaction between heterogenous valuations and the sequential timing of the contest (primarily focused on two players and two periods). For example, Leininger (1993) reverses the result of Dixit (1987) who suggested that, even when players are given the option to move earlier or later in a contest, it is in their best interest to move first regardless of the choice by the other player. Leininger (1993) shows that if players have heterogenous valuation, which is theoretically identical to a cost advantage in a Tullock contest, the player with the larger valuation with always choose to move first, and the lower valuation player, second. Our paper extends this literature by exploring contests beyond two periods (or two players) who have heterogenous valuations of a prize. This allows for a more robust analysis of the interaction between the timing of players decisions and their valuations.

Lastly, the way in which we model heterogenous valuation is identical to the inclusion of a “joy of winning” (Sheremeta, 2010) into a subject’s utility function in a Tullock contest. Sheremeta suggests a “joy of winning” as one explanation for the persistent and well documented observation of overbidding in contests (see Sheremeta (2015) for a survey of overbidding in contests). Thus, our model can capture how joy of winning would affect players Nash Equilibrium bids in sequential contest, including the impact of heterogeneity in this value.

More generally, as the experimental investigation of sequential contests is relatively undeveloped, it is unclear how the mechanisms of overbidding in simultaneous experimental contests translates to the multi-period sequential setting. Nelson and Ryvkin (2019) find that in the homogenous 3-player sequential contest that aggregate output and overbidding is lower in sequential contests but find very little overbidding in general. As a preview of the experimental results in this paper, we also find that sequential contests tend to lower overbidding relative to simultaneous contests in a variety of treatments. However, unlike Nelson and Ryvkin (2019), we observe significant overbidding in both simultaneous and sequential contests. Further, we find that overbidding tends to *increase* as players make decisions later in the contest. We also explore an implied "joy of winning" and suggest a new mechanism to explain behavior in sequential contests which we call a "winning probability heuristic".

The theoretical results and corresponding experimental design are more easily understood within the context of key implications of Hinnosaar's model. In a sequential contest, the contest order is defined by the order of decisions and revelation of effort to other contestants. Hinnosaar's key result is that the more other players will observe a particular player's effort prior to making their effort, the more effort the observed player will exert and the higher *ex ante* probability he will win. This is the aforementioned *earlier-mover* advantage. Additionally, Hinnosaar shows that as a contest becomes "more sequential"[24], thus having more observations of earlier players' effort choices, the higher the aggregate output in the contest. This provides clear and important predictions for a contest designer. In situations where rent-dissipation (competition) and higher aggregate outputs is desired, such as an R&D race, a maximally sequential structure should be utilized. In settings where it is not, sequencing is undesirable.

Utilizing Hinnosaar's theory, we show that in a sequential contest of 3 players with heterogenous valuations *always* results in higher aggregate output than the corresponding simultaneous contest, regardless of ordering and valuation types. Moreover, similar to Leininger (1993) and Morgan (2003), placing players with higher (lower) valuations later (earlier) in the contest results in lower aggregate totals. However, in contrast to Leininger and Morgan, the totals in any sequential contest structure are greater than the simultaneous case. Lastly, we find

[24] You can think of more sequential as "flatter". Imagine a 3-player contest where 3 players made decisions at the same time (simultaneous). Then moving to a situation with 2 leaders and 1 follower. And finally, a fully sequential setting where each decision is made one after another. These contests are subsequently flatter.

that the total aggregate output is irrespective of the final period players' valuation. This suggests that a contest designer can minimize the impact of an outlying participant (in terms of valuation) by placing him at the end of the contest. This result is generalizable to n-players when there is a single player who a heterogenous valuation (cost). Finally, the contest designer can maximize the impact of a value heterogeneity by placing it at the beginning of the contest.

The final theoretical extension fully characterizes the SPNE solution to a three-player fully sequential model in which an incumbent player (who moves first) responds to probabilistic potential entry from two following players. This captures the notion of an incumbent firm facing potential entry in an output game. The incumbent firm chooses an output level given the public knowledge of the probabilities of the entrants in the two subsequent periods. Once the output from the first firm is submitted, the firm in period two receives his random opportunity cost draw and enters if he receives the low draw. He then chooses his output. The third firm follows identically. The contest is then resolved, and the prize is paid out. Thus, there are four potential outcomes. Both firms two and three enter, only one of two enter, or both stay out. Expected aggregate output increases as the probability of entry for both firm increases. Moreover, the increase in output, as firm entry probability increases, is very similar rates regardless of which's firm chance of entry increases. These hypotheses are intuitive. Earlier firms respond to potential entry by increasing output to capture a larger market share to buffer themselves from the potential entrant.

Our experiment tests the implications of these theoretical extensions while also providing the first empirical work extensively testing the major theoretical advance by Hinnosaar. Only one other study, that we know of, has experimentally tested a sequential Tullock contest for contests of more than three players.[25] Further, their results provided contradictory evidence for support of the theory, thus more research is needed to determine exactly how the model translates to real world applications.

[25] Nelson and Ryvkin (2019).

II Theoretical Model

In the following section we will explain the model and derive the hypotheses that we will be tested in the proposed experiment. An n-player sequential model of a Tullock contest between risk-neutral players $i = 1,2,..,n$ choosing effort $x_i \in R^+$ is considered. All investments are made at the beginning of period t and are public information. The winner of the contest is determined after all players have made the investment decisions and the probability of winning for each player follows a Tullock contest payoff given by:

$$p_i = \begin{cases} \dfrac{x_i}{\sum_j^n x_j}, if \ \sum_j^n x_j > 0 \\ \dfrac{1}{n}, \ \ if \ \sum_j^n x_j = 0 \end{cases}$$

The winner of the contest receives a prize of value $v_i > 0$ and all players investments are lost. Individuals pay constant marginal effort costs c_i. Thus, the general payoff function for player i is given by:

$$\max_{x_i} EU_i(\boldsymbol{x}) = v_i \frac{x_i}{\sum_j^n x_j} - c_i x_i$$

Hinnosaar (2018) characterizes a sequential contest using a T-dimensional vector $\boldsymbol{n} = (n_1,..,n_t)$ where T is the number of stages, and $n_t \geq 1$ is the number of identical ($v_i = v = 1, c_i = c = 1$) players making investment decisions at period t. For a three-player contest this is represented by four different possibilities. $\boldsymbol{n} = (3); (2,1); (1,2); (1,1,1)$; with (3) representing the simultaneous contest; $(1,2)$ representing a Stackelberg game with one leader and two followers. Using an inverse best-response function, we rewrite the choice variable as a function of aggregate investment in the contest $X = \sum_j^n x_j, \ X \in [0,1]$. Defining X_t to be the aggregate investment up to period t, the objective function for a player i in period t looks like:

$$\max_X EU_i(\boldsymbol{x}) = \frac{X_t - X_{t-1}}{X} - (X_t - X_{t-1}) \tag{1}$$

Hinnosaar introduces the concept of an *inverse best response function* for participants in period t, such that $f_{t-1}(X)$ is the solution to first order condition of (1). $f_{t-1}(X)$ is defined as the choice in period $t-1$ that sets aggregate output to X given all other players behave optimally. Utilizing these functions Hinnosaar then characterizes the solution to the general Tullock contest to recursively define the relationship:

$$f_{t-1}(X) = f_t(X) - n_t f_t'(X)X(1-X)$$

Which yields the solution of total aggregate output X from the equation of the final player $f_0(X) = 0$, given by the highest root. Further, the subgame-perfect Nash equilibrium output of a player i is given by:

$$x_i^* = \frac{1}{n_t}[f_t(X) - f_{t-1}(X)]$$

As mentioned in the introduction, this novel solution concept overcomes the limitation of the traditional backward induction approach by incorporating the first-order conditions of all players into a single variable of interest, the aggregate output X. For games with identical players, players who move in earlier periods have increased effort choices, winning probability, expected utility. Additionally, for games with identical number of players, the more information that is revealed (the more sequential a game is) the larger the predicted aggregate output of the game. *I.e.* as we move from (3) to (1,2) to (2,1) to (1,1,1), SPNE predictions of X increase.

II.A Value Heterogeneity

In our first theoretical extension we extend the model to allow for heterogeneity in prize values or effort costs for player i. We use a fully sequential model with a single player in each time period, t. We normalized the cost parameter to 1, $c_i = 1$ which yields the expected payoff function:

$$\max_X EU_i(\boldsymbol{x}) = V_i \frac{X_t - X_{t-1}}{X} - (X_t - X_{t-1}) \qquad (2)$$

The normalization of c_i is innocuous, as v_i and c_i enter the optimal solutions in the same way (v_i/c_i). Thus, a percentage point increase in valuation is equivalent to a percentage point decrease in cost. V_i in our model can be then taken to represent a value to cost ratio v_i/c_i.

The major result we find from this model are given by Proposition 1 and Theorem 1 below. First as players with higher valuations are moved from the beginning to the end of contests, their impact on the contest is lessened, in the form of reduced aggregate output, reduced odds of winning, expected utility, and total investments.

Proposition 1 (*3-Player game) Moving players with higher (lower) valuations later in a purely sequential contest will reduce (increase) aggregate output of the contest.*

Proof of Proposition 1 is in the Appendix.

Proposition 1 states that the total impact on the aggregate output from a change in valuation is dependent on the sequential position of the player whose valuation has changed. With larger absolute impacts resulting from changes in valuations, the earlier the player is in the sequential contest. Additionally, we see that the aggregate output in unchanged with a change in valuation of the final player.

Theorem 1 (*n-player game) The aggregate output in a purely sequential contest is independent of the valuation of a player in the final period.*

Proof of Lemma 1 is given in the Appendix.

Theorem 1 is related to Proposition 1 but provides a more robust and general result. Theorem 1 states that the impact of a change in valuation on the aggregate output of final player(s) is exactly zero. This does not imply that the distribution remains unchanged, only that the aggregate output. Stated another way, any increases (decreases) of effort expenditure by the last player(s) in the group resulting from an increase in the valuation of the prize is *exactly* offset by reductions from earlier players. The equilibrium aggregate output from two sequential contests that differ only the valuation of the final player are identical. This generalizes the results in earlier research on two-player contests that finds that second movers gain no advantage from increased valuations. Further the aggregate outputs can be easily ordered. This has a powerful policy suggestion, depending on the maximization criteria the policy maker chooses. If he wishes to maximize (minimize) the impact (in terms of aggregate rent expenditure) of a contestant with a cost or value heterogeneity he should place the contestant at the beginning (end) of the contest. Further, if he places the contestant at the end, whether the contestant has advantage or disadvantage, the change in aggregate rent expenditure induced by the heterogeneity will be fully dissipated by the other participants in the contest.

II.B Probabilistic Entry – One Entrant

We now model probabilistic entry for players in a purely sequential game $(1,1,\ldots,n)$. Each player, noted by the period t that they enter, after the first enters the contest with exogenous probability q_t. If they join the contest, they play as a standard contestant in a sequential Tullock contest. Otherwise, they receive outside option $O_i \in \{L, H\}$. The higher q_t, the higher the chance to receive a low outside option value (low fixed cost). We assume that the gap is sufficient such that entry into the contest is optimal, that is the expected utility of entry into the contest exceeds L but is lower than H.

Thus, holding the valuations of all players equal, the player who has the outside option, realizes the option before he makes the decision, and chooses:

$$EU_t(x) = \begin{cases} H \text{ , } if\ O_i = H \\ \frac{x_t}{\sum_j^t x_j} - x_t \text{ , } if\ O_i = L \end{cases} \tag{3}$$

whereas the player who makes a move *before* an entrant who has a probabilistic outside option faces the problem:

$$EU_{t-1}(x) = q_t\left(\frac{x_{t-1}}{\sum_j^{t-1} x_j + x_t} - x_{t-1}\right) + (1-q_t)\left(\frac{x_{t-1}}{\sum_j^{t-1} x_j} - x_{t-1}\right)$$

Ultimately, the game yields the following solution.

$$Z'\left(q_T {X^H}^2 + (1-q_T)X^H - {X^H}^3\right) + Z\left(2q_T X^H + (1-q_T) - 3{X^H}^2\right) = 0 \tag{5}$$

Where $Z = \frac{2X^H - q_T}{(q_T X + 2(1-q_T))}$ and $Z' = \frac{dZ}{dX^H}$. The solution to equation (4) is a polynomial in X^H. Hinnosaar demonstrates that the highest root of this polynomial yields the desired solution. Additionally, numerical results show that this total is strictly increasing in q_T for $0 \le q_T \le 1$, *i.e.* $\frac{dX^H}{dq_T} > 0$. Similarly, *ex ante* equilibrium aggregate output is increasing in q_T which leads to the following theorem.

Proposition 2 (*3-player contest) Increasing (decreasing) the entry probability for the third player in a sequential game increases (decreases) the aggregate output of the contest.*

Details for solving the game characterized by equation (3) and the proof for Proposition 2 are presented in Appendix A.

Solving for a similar exercise[26]with a probabilistic entrant in a simultaneous game we can generate the following finding. Through numerical derivative, we find that a change in the probability of entry by the final player has a larger impact on ex ante expected aggregate output relative to entry in a simultaneous game when the entry probability is large, and a smaller impact when the probability of entry is low. *I.e.* $\frac{dE(X_{Seq})}{dq_T} > \frac{dE(Sim)}{dq_T}$ when q_T is closer to 1 and $\frac{dE(X_{Seq})}{dq_T} < \frac{dE(Sim)}{dq_T}$ when q_T is closer to zero.

II.C Probabilistic Entry – Two Entrants

Finally, we consider is a fully sequential three-player game where one incumbent faces potential entry from two competitors. The game can be fully characterized by the following payoff functions:

$$EU_3(x) = \begin{cases} H\ , if\ O_3 = H \\ \frac{x_3}{x_1 + x_2 + x_3} - x_3\ , if\ O_3 = L \end{cases}$$

$$EU_2(x) = \begin{cases} H\ , if\ O_2 = H \\ q_3\left(\frac{x_2}{x_1 + x_2 + x_3} - x_2\right) + (1 - q_3)(\frac{x_2}{x_1 + x_2} - x_2)\ , if\ O_2 = L \end{cases}$$

$$EU_1(x) = q_2\left(q_3\left(\frac{x_1}{x_1 + x_2 + x_3} - x_1\right) + (1 - q_3)\left(\frac{x_1}{x_1 + x_2} - x_1\right)\right) +$$

$$(1 - q_2)(q_3\left(\frac{x_1}{x_1 + x_3} - x_1\right) + (1 - q_3)(1 - x_1))$$

Using the inverted best response correspondence, we can solve by backward induction.

Optimizing and setting equal to zero solves for the total aggregate output in terms of X^{HH}. The equation is a polynomial in X^{HH} and is readily solvable. Numerical analysis finds that X^{HH} is

[26] Omitted.

decreasing in both q_2 and q_3. Also, given equivalent changes in q_2 and q_3, the change in aggregate output from a change in q_3 is very similar to that in q_2. Further, along either dimension, as the threat of entry increases, the expected aggregate output also increases. [27] In the following section we build a laboratory experiment to test the theoretical extensions of the sequential contest model with both heterogeneous treatment values and probabilistic entry for a single player.

III Experimental Design and Procedures

III.A Experimental Design and Hypotheses

The design of the experiment consists of 6 separate treatments, in which participants compete in 10 rounds of simultaneous and 10 rounds of sequential contests (with an additional 2 practice rounds for each)[28]. In each round, in all treatments, each of the 3 players in the group received an endowment of 120 and competed to win a prize in a traditional Tullock contest. In the simultaneous contests, all players made decisions simultaneously. In the sequential contests players made decisions one after another (1,1,1) and the total effort choices were revealed to the following player. Players were allowed to bid up to their endowment. The first treatment was the *Control*, in which the prize is valued at 100 lab dollars. The second set of treatments were the Heterogenous Valuations (HV or *HetP*). In this case the value of the prize for a single player is set to 120. Three treatments were derived using the HV, by varying the position of the player in the sequential game who had the HV (Player 1, 2 and 3). [29] The simultaneous decisions in these treatments are functionally equivalent. The last set, the Random Entry (RE; *Random*) treatments, consisted of two treatments. The RE contests used the baseline prize values but introduced a 50% entry probability for either player 2 or 3. This decision probability was known to all players, and whether the player had entered was not revealed until the end of the contest. Again, the simultaneous rounds of these treatments are functionally identical. Nash equilibrium predictions for each of the treatments are provided in Table 2.1.

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[27] An example showing the responses based on varying entrant probabilities is given in the Appendix.

[28] The ten rounds of each type were played concurrently, which type of contest was played first was randomly determined before the Session.

[29] This was the aggregate effort only, so Player 3's did not see individual effort totals. Player 2's only had input from one previous player, so the aggregate total displayed was equivalent to the individual total.

Hypotheses

The experiment was designed to test the following hypotheses.

Hypothesis 1 *Aggregate contest expenditures are higher in sequential contests than simultaneous contests.*

Nash predictions for sequential contests are higher than simultaneous contests which is a general proposition proposed by Hinnosaar (2018) for symmetric contests and shown in a heterogenous contest by this paper.

Hypothesis 2 *The aggregate output in sequential contest is independent of the valuation of a player in the final period.*

Hypothesis 2 was generated from Theorem 1. By comparing the aggregate output of the *Control* treatment with the aggregate output from the *HetP3* we provide support for the behavioral validity of Theorem 1.

Hypothesis 3 *Moving players with higher (lower) valuations later in a sequential contest will reduce (increase) aggregate output of the contest.*

Hypotheses 3 is similar to Hypothesis 2 but explores the relationship between value/cost heterogeneity, aggregate rent expenditures, and sequential position more generally. From Proposition 1 we predict that the aggregate totals will be highest when a higher prize value is given to Player 1, second highest for Player 2, and lowest (and equivalent to a symmetric sequential contest) for Player 3.

Hypothesis 4 *Introducing an entry probability for the final player in a sequential game decreases the aggregate output of the contest.*

Our design allows us to compare a situation where a final player has a random entry probability and its ultimate impact on aggregate outputs. By comparing a 50% entry probability in the *RandomP3* treatment to a guaranteed entrant in the *Control* treatment we can analyze how the threat of entry affects rent dissipation.

Table 2.1. Nash Equilibrium Aggregate Effort Predictions: Control

Treatment – Game Type	Nash Equilibrium Aggregate Effort
Control	
Sequential	78.87
Simultaneous	66.67
Heterogenous Prize Value	
Player 3 – Sequential	78.87
Player 2 – Sequential	82.39
Player 1 – Sequential	91.47
Simultaneous	70.59
Random Entry	
Player 2 – Sequential	64.28
Player 3 – Sequential	64.31
Simultaneous	58.66

Note: Predictions are generated with prize value 100. Players valuation of the prize are identical. A fully sequential game has (1,1,1) structure and a simultaneous game has structure (3).

Hypothesis 5 *Increasing the entry probability for the third player in a three-period contest has a* ***nearly*** *identical impact on the expected aggregate output than increasing the entry probability for the second player.*

This hypothesis is related to the position of potential entry in a sequential game. Numerical analysis generated from the model shows that the position of potential entry by participants has very little impact on total expenditures in the contest.

Hypothesis 6 *Overbidding will be lower in sequential contests relative to simultaneous contests in multi-period contests in aggregate.*

This is the exact finding by Nelson and Ryvkin (2019) that was discussed earlier. Our introduction of heterogeneity could induce behavioral change, but we would expect to find similar results as the previous study of multi-period contests.

Hypothesis 7 *Overbidding will not differ between player types in Sequential Contests.*

There is very limited evidence to anticipate the behavioral response to the revelation of efforts in contests affects subject's choices. Results from Fonseca (2009) and Nelson (2019) find little evidence of overbidding by first players. However, introducing heterogeneity into the sequential decision setting may cause changes in behavior.

Hypothesis 8 *Players moving first in sequential contests will exhibit higher effort and receive higher expected payoffs by leveraging their earlier-mover advantage.*

Hypothesis 8 is one of the primary results of Hinnosaar (2018) from extending contests into *n*-player sequential setting. This expands Dixit's (1987) first mover advantage to a more a general earlier mover advantage. We can observe effort totals and calculated expected values for each treatment by player position.

III.B Experimental Procedures

The experiment was conducted virtually using z-Tree Unleashed (Fischbacher, 2007; Duch et al., 2020) through the Experimental Economics Laboratory at the University of Tennessee-Knoxville. A total of 318 subjects participated over the course of 21 sessions.

Sessions varied in size from 9 to 21 participants. Subjects were recruited from a database of predominately undergraduate students who had previously agreed to receive recruiting e-mails for paid economics studies.

At the beginning of the session students were given a copy of the experimental instructions and the instructions were read by a moderator[30]. The participants remained in an online video chat were encouraged to follow along with the instructions as they were read. Each experimental session was divided into 3 parts. The first part was a risk preference elicitation task where students chose between 10 possible lotteries of $4 and $0 or a certain payoff of $2, similar to Holt and Laury (2002). The results for the risk elicitation were revealed at the end of the session. The second part of the experiment was the administration of one of the treatments described above. The final part consisted of a brief questionnaire that elicited qualitative responses on decision making for the different player types and basic demographic information.

In the main section of the experiment, participant played 24 rounds total of 3-person contests. Depending on the randomly determined treatment, participants either faced 12 sequential contests followed by 12 simultaneous or vice versa.[31] The first two rounds of each decision type were used as non-paid practice rounds. Participants were unaware of the number of rounds in the experiment and were also not told of the different game type until the corresponding round was reached. In each round, participants were sorted in an anonymous, random group of 3 participants and given a random player number: 1,2, or 3 (both group and player number changing each round). These player numbers determined the order of play in the sequential contest and the prize value or probabilistic entry chance in the HV and RE treatments, respectively.

Subjects contributed tokens out of their endowment for a chance to win the prize specified by their prize value. They were allowed to contribute up to their endowment of 120 and every token not bid was kept. The chance to win the prize was the ratio of their bids to the total bids from their group including their own. In the RE treatments, for the players that had the random entry chance and selected not to enter, they received a message saying they did not enter the contest and will receive 80 tokens in lieu of participation to simulate a high draw of the

[30] Instructions are available for review in the Appendix.
[31] Practice rounds were conducted at the start of each set of sequential or simultaneous contests.

outside option. The contest proceeded as a two-player contest in this case and the other players were not notified of the result.

After each round, the winner was randomly chosen and each player received a summary of the total group contributions for their round, their chance to win, the result of the contest, and their earnings for the round based on remaining endowment and prize value, if applicable.

Each of the 20 non-practice rounds were paid out. Lab Dollars were exchanged at a rate of 170 to 1 US Dollar. One of the ten scenarios from the risk-preference elicitation was chosen randomly for each participant and paid out according to the decision for that scenario. After the total for the experiment was determined, participants were paid via Amazon gift card. On average participants earned $15.88 and sessions lasted about 70 minutes.

Overall, the average age of participants was 20.96 years of age. 53% of participants were female and 46% had participated in a previous economics experiment. Self-reporting on a 1-5 Likert Scale where 1 is "strongly disagree" and 5 "strongly agree" 88% of the participants stated they agreed with the statement "I understood the instructions for experiment 2". This provides some evidence to support that most participants had a strong grasp of the instructions. In a similar fashion, 87% reported that they were well compensated.[32] Table 2.2 shows a description of the data used in the subsequent results section.[33]

IV Results

In this section, we provide three main aggregate results from the data. The first is that the position of the heterogeneity, both in valuation and entry probability, had a significant impact on the aggregate totals of effort in sequential contests. The second is that we observe that more overbidding relative to Nash Equilibrium in simultaneous contests relative to sequential Contests. The third result is that overbidding tends to increase with player position in sequential contests, *i.e.* later players tend to overbid more than earlier players. After exploring the three main results in more detail, we then explore heterogeneity in individual behavior in attempt to

[32] Some questionnaire data was lost through the use of virtual method. However, all participants were recruited from the same database and there is a no systematic reason to expect the characteristics to differ in the missing data.
[33] Balance tests are not reported but show no evidence of non-random assignment of observables across treatments.

Table 2.2. Data Description: Contests

Variable	Description	Mean	Std. Dev.
Effort	0 to 120 main input for effort choice in the contest	48.46	38.08
ExUtilPreDraw	calculated expected payoff conditional on effort choices by all players in the group before the winner of the contest was drawn	106.39	26.30
LastWin	=1 if the contest was won in the previous period	.333	.4714
Female	=1 if participant was a female	.633	.482
GPA	self-reported GPA of subject (out of 4)	3.41	.454
RiskPref	Number of certain payoffs selected in pre experiment risk preference elicitation	5.39	1.56
ComprehensionAgree	=1 if participants self reported a score of 4 or higher out of 5 when agreeing to the statement "I understood the instructions for experiment 1"	.179	.384
Treatment	=1 binary indicators with the associated names: *RandP2 RandP3 HetP1 HetP2 HetP3* and *Control*	-	-
Player #	=1 binary indicator for the player position in sequential contests: Player 1, Player 2, and Player 3	-	-

Note: Other variables are used in the analysis but are derived from the variables defined here. Any new variable that is introduced is explained at that time.

understand the source of overbidding, primarily in sequential contests. We show that individuals who make decisions with complete information about the aggregate effort choices of the members of their group (Player 3s in Sequential contests) still exhibit overbidding, but a significant portion do display behavior consistent with expected utility maximization. We also study the implied "Joy of Winning" and relate this to the observed behavior of Player 3's choosing a "winning percentage" in a sequential contest. Lastly, we explore the difference in outcomes in the RE treatments.

IV.A Aggregate Results

Tables 2.3 and 2.4 summarize the effort totals by group for each treatment type relative to the Nash equilibrium predictions along with the corresponding overbidding amount. The standard errors presented in parentheses are derived from simple regressions that control for session-by-period and individual correlation. The first striking feature is the direction of the totals in the Sequential HV treatments.

Result 1 *The introduction of heterogeneity into sequential contests significantly impacts the aggregate totals. Further, the change in totals depend on which player in the sequential contest has the heterogeneity.*

In the HV treatments, we see strong support for the theoretical predictions of the change in aggregate outputs as we move the high value player later in the contest. Supporting Hypothesis 2, the observed point estimates decrease as the HV player is moved from the first position to the last. Comparing with the control, as predicted by the theoretical model, when the HV is in the third position the point estimates are nearly equivalent. Pairwise t-tests comparing the aggregate totals between treatments are reported in Tables B.1 and B.2 (Appendix). We see statistically significant differences between the *HetP1* and *HetP2* treatments when compared with *HetP3* and *Control.* Thus, we can reject the null hypothesis of Hypothesis 3 of no change in aggregate output between *Control* and the *HetP1* and *HetP2* treatments. Further, we fail to reject the null hypothesis of no difference between *HetP3* and *Control* in Hypothesis 2. Theorem 1 asserted that the aggregate output in a sequential contest is independent of the valuation of the final player. We see convincing evidence that this is the case when comparing *HetP3* and

Table 2.3. Group Level Outcomes: Sequential

Treatment Type	N	Nash Eq.	Observed	Overbidding Amount
Control	220	78.87	140.35 (4.57)	78.0% (5.80)
HetP1	200	91.47	160.36 (5.57)	75.3% (6.09)
HetP2	140	82.39	156.74 (6.97)	90.2% (8.46)
HetP3	190	78.87	140.22 (4.05)	77.8% (5.13)
RandomP2	140	64.28	140.88 (5.45)	119.2% (8.66)
RandomP3	150	64.31	121.02 (4.46)	88.2% (7.09)

Note: Totals shown are total group contributions by treatment type, standard errors are in parentheses. Overbidding is calculated as the percentage increase of the observed values relative to the Nash Equilibrium predictions.

Table 2.4. Group Level Outcomes: Simultaneous

Treatment Type	N	Nash Eq.	Observed	Overbidding Amount
Control	220	66.67	149.34 (5.96)	124.0% (8.94)
Het. Value	530	70.59	145.59 (3.19)	106.5% (4.52)
Random Entry	290	58.66	131.12 (3.17)	123.5% (5.40)

Note: Totals shown are total group contributions by treatment type, standard errors are in parentheses. Observations are pooled across relevant treatment types where decisions are functionally identical. Overbidding is calculated as the percentage increase of the observed values relative to the Nash Equilibrium predictions.

Control. Moreover, we see that point estimates of the aggregate totals decrease as the valuation moves to later players in the contest (*HetP1* to *HetP2* to *HetP3*). These results persist in spite of significantly elevated bids in all treatments, lending support for our extension of Hinnosaar's model with regard to heterogenous prize valuations.

We find less support for the theory in the *Random* treatments as we observe deviations in aggregate output between *RandomP2* and *RandomP3* when we expected to find none. We see that the point predictions for *RandomP2* and *Control* are quite similar with *RandomP3* deviating downward substantially. Indeed, these differences with *RandomP3* are statistically significant. These results suggest that the introduction of entry probability for the final player has a stronger impact relative to the intermediate player on reducing aggregate output. Thus, we do not find support for Hypothesis 5. We will explore some potential causes of this deviation later in the section.

Result 2 *Overbidding is higher in simultaneous contests relative to sequential contests.*

Tables 2.2 and 2.3 clearly show significant overbidding for all treatments. While the overbidding is high (~121% in the simultaneous case), these findings are well within observed overbidding rates in contests (Sheremeta, 2013). Our study is the second to observe lower amounts of overbidding in sequential contests relative to simultaneous contests in multi-player environments. (Nelson and Ryvkin, 2019). This provides support for Hypothesis 6. Examining the effects of different types of heterogeneity, we see that observed values of overbidding vary depending on treatment more widely in the sequential treatments than in the simultaneous environments. Overbidding in the sequential treatments is the largest in RE and close to the *Control* and the HV treatments.[34] Further, we do not find ample evidence to support Hypothesis 1, as the sequential contest outputs are not statistically different than the simultaneous outputs in the *Control* and *Random* treatments. In HV we observe differences in the *HetP1* and *HetP2* only.[35]

Result 3 *Overbidding rates differ by player position in sequential contests.*

[34] Pairwise t-tests and the regressions used to generate standard errors are presented in the Appendix.
[35] Reports are once again provided in the Appendix, sample comparisons are restricted to across treatment comparison only in all but the *Control*.

Figure 2.1 shows the outputs by player position (on aggregate) for each of the treatments compared to the Nash Equilibrium prediction and the *Control* treatment (and it's Nash Eq. prediction). Visual inspection shows that individuals respond similarly to theoretical prediction, albeit at elevated effort levels. Table B.6 in the Appendix shows the fully specified regressions for each player type relative to the control treatments. Further, we see that overbidding behavior tends to increase as players move later in contests. Table 2.5 confirms this by regressing overbidding relative to the Nash by player type pooling across all treatments. We observe that overbidding increases by roughly 15 tokens for each increase in player position and the difference between type is significant at the 1% level ($p<.001$ for Type 2 against Type 3), rejecting Hypothesis 7.

A full specification of Table 2.5 is shown in Appendix and provides evidence for other observed patterns in contests, such as "Hot Hand", decreasing effort choices in risk aversion, higher contributions from Females, and decreased contribution from previous participation. While at the aggregate level, we see evidence that later players in the sequential contests exhibit a peculiar response and increase bids when they observe earlier players effort choices. This is at odds with the theoretical predictions, especially considering the overbidding that is still taking place by earlier players. Generally, at these elevated levels of effort choices, effort contributions should be strategic substitutes so we would expect lower effort levels when higher levels of effort are being expended by earlier players. We explore this further in the next section.

IV.B Additional Analyses

Player 3 Decisions

One unique feature of sequential contests (not limited to multi-player contests) is the ability to observe a direct response to output total of the rest of the participants. For example, Player 3 in our Sequential contests observes all total bids from the players before her and gets to precisely choose her winning percentage. Thus, based on expected value, we can observe behavior that directly measures their (perceived) value of the prize that is not conditional on other player's choices, such as in simultaneous contests or earlier players in a sequential contest.

Table 2.5. Deviation from the Nash Equilibrium by Player Type: Sequential

	Dependent Variable: Deviation from Nash
2nd Player	19.76***
	(1.643)
3rd Player	33.62***
	(1.818)
Constant	9.160***
	(1.322)
Observations	2,632
R-squared	0.109

*Note: Deviations from predicted Nash Eq. values based on total inputs from previous players and assuming values of prizes are equal to the assigned prize value. Sample does not include RandomP2, the 2nd player Nash is given by an approximation. The comparison group is the overbidding amount of the first player. Cluster robust standard errors at the group-period-session level are in parentheses *** p<0.01, ** p <0.05, * p<0.1.*

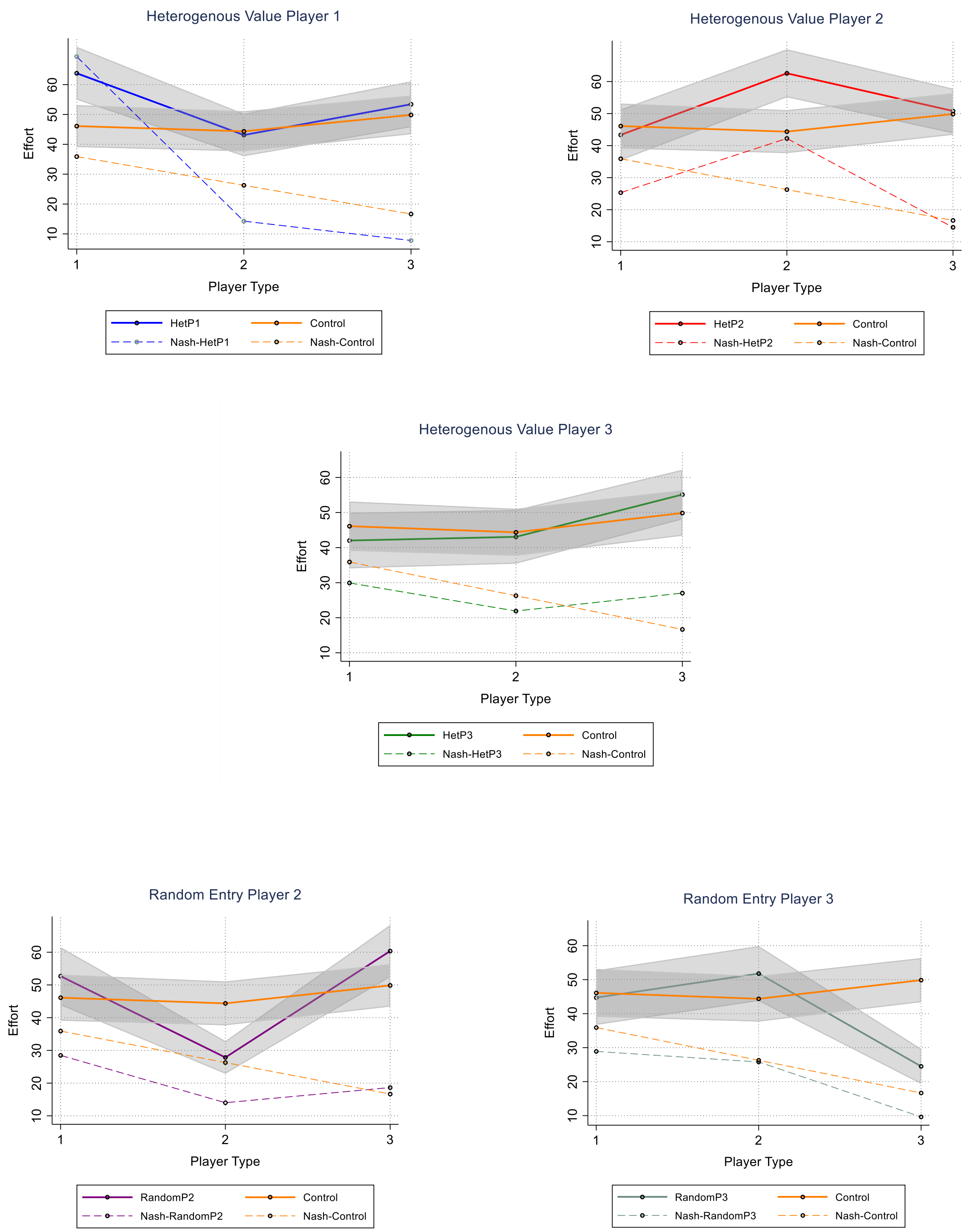

Figure 2.1. Average Effort by Player Type and Treatment: Sequential

We also consider the possibility that players make decisions that are not based on maximizing the expected value (or their failure to correctly compute the value).

Figure 2.2 shows a scatter plot of the individual responses as Player 3s relative to the Nash Equilibrium predictions. We observe that approximately 11.2% of decisions are within 5 tokens of the Nash predicted values. However, this number drops to 4.7% if we condition on positive values of Effort only, suggesting that many of the decisions that follow Nash predictions are "correct" zeros. Together these numbers paint a picture of at least a portion of individuals who respond according to Nash predictions.

Table 2.6 explores this relationship between effort choices and Nash predictions further by regressing effort choices of Players 3 on the Nash with and without observable covariates. The Nash prediction in our case, conditional on a total input of 100 or less, is $x_3 = 100 * \sqrt{\frac{X_{-i}}{100}} - X_{-i}$.[36] Thus, we would expect to see a coefficient of 1 on the square root term and -1 on the linear term. We observe significant point estimates of .685 and -.399, respectively. For a portion of subjects, the Nash Equilibrium predict behavior, but the coefficients are not as high as theory would predict[37].

Player 3 Decisions-Winning Percentage Heuristic:

Additionally, from Figure 2.2 we can see that despite a noisy decision environment there are clusters along positively sloped lines (slopes of 1 and .5 are shown). This indicates the possibility that subjects are making decisions based on choosing a fixed "winning percentage" rather than the Nash equilibrium. Figure 2.3 shows histograms based on the resulting winning chance based on the final decision of player 3s for all input totals and for positive predicted Nash Values (Input totals less than 100), respectively. The "Nash" bins represent the winning percentage if the expected payoff maximizing Nash equilibrium choice was played. Bins are set to a single percent. Visually we can see clear spikes at 33%, 50%, and 66% (0% and 100% correspond to 0 effort by a third player and 0 input total by the two players before the third). When the input total from the first two players yields a positive predicted Nash effort choice,

[36] Conditional on entry for Players in *RandomP3* and multiplied by 120 in the *HetP3* treatment. X_{-i} is the total contributions from the previous players.

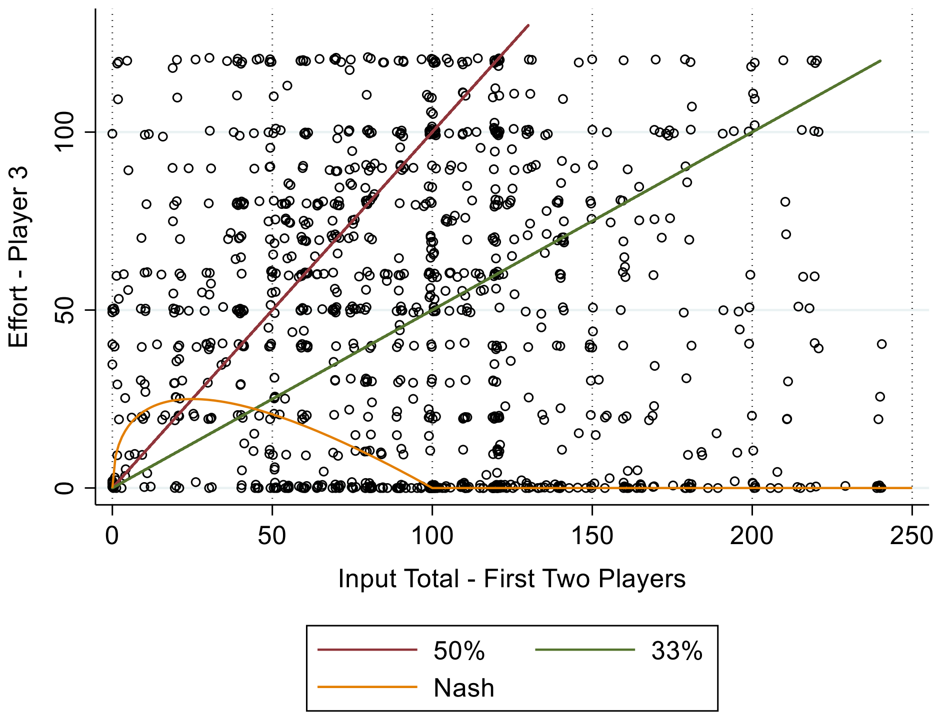

Figure 2.2. Effort by Player 3 Against Input Total: All Treatments

Table 2.6. 3rd Player Efforts Against Nash: Sequential

	Dependent Variable: Effort	
	Sparse Specification	Full Specification
PreviousInputTotSqrt	0.639**	0.685**
	(0.286)	(0.282)
PreviousInputTotLinear	-0.350	-0.399*
	(0.230)	(0.224)
Age		-0.278
		(0.595)
Experience		-12.45***
		(3.910)
Female		-2.901
		(3.681)
GPA		2.910
		(3.254)
Economics		2.037***
		(0.533)
RiskPref		-5.291***
		(1.033)
LastWin		-1.201
		(2.209)
ComprehensionAgree		0.982
		(6.407)
Constant	27.11***	50.45***
	(8.971)	(10.59)
Observations	456	456
R-squared	0.025	0.126

*Note: Coefficients are the point estimates from a pooled OLS regression. Robust standard errors are in parentheses and clustered at the session-period level (173). Sample is restricted to 3rd players in Sequential contest only when the Input Total from the first two players was less than 100 and the prize value was 100. *** $p<0.01$, ** $p<0.05$, * $p<0.1$*

8.6% of Player 3 choices give a win probability that is between 50-51% whereas only .33% and 1.0% of player decisions result in a winning probability of 49-50% and 48-49%, respectively. The exact totals for a winning probability of 33-34% is 3.4% and 66-67% is 3.9%.[38]

Figure 2.3 provides compelling visual evidence that subjects make decisions based on the winning percentage which we refer to as a *Winning Percentage Heuristic*. [39] Whether the percentage serves as a cognitive anchor or a calculated outcome cannot be identified but both are plausible explanations. Even as Player 3, when all inputs from other players are known, the reasonably complex form of $x_3 = \sqrt{V_3 X_{-i}} - X_{-i}$ (when $X_{-i} < V_3$) is likely a source of difficulty for participants to intuit. Particularly, the response to the effort of other players transitioning from complements to substitutes is a quite complicated relationship. Further, subjects are experiencing large overbidding from other participants which can potentially muddle the ideal strategy. Subjects were not given a calculator in z-Tree and were not instructed to use one but were free to use resources that were available to them to do so.

A winning percentage of 50% is likely the simplest to calculation to perform mentally and may serve as a cognitive anchor for the decision. In Figure 2.4 we observe much higher proportions of decisions that are slightly greater than 50% relative to slightly below 50% which suggests that participants may start from they want a slightly larger than 50% chance to win.[40] While this strategy is likely to reduce the cognitive complexity of the task, it is unlikely to be in the best interest of the player in terms of expected payoffs. We can see visually from the scatter plot that many of the decisions that fall near the positive sloped lines are well outside of the predicted Nash values. However, given the relatively flat payoff functions around the Nash, perhaps this an improvement in utility for participants.

Player 3 Decisions-Joy of Winning:

Using Player 3 data, we also construct an implied value of the prize for each participant as Player 3. This number is the average of the implied Nash values of an additive joy of winning

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[38]There are not the only noticeable trends. There looks to be a spike at 40% along with 60% and potentially 25%, but these are less pronounced. In general, *some* percentage choice seems to be driving behavior for a large portion of participants.
[39] Additional figures are shown in the appendix for all decisions, with and without Nash predicted comparisons.
[40] This shares a similarity to a bunching response observed in the taxation response literature (Le Maire et al., 2016).

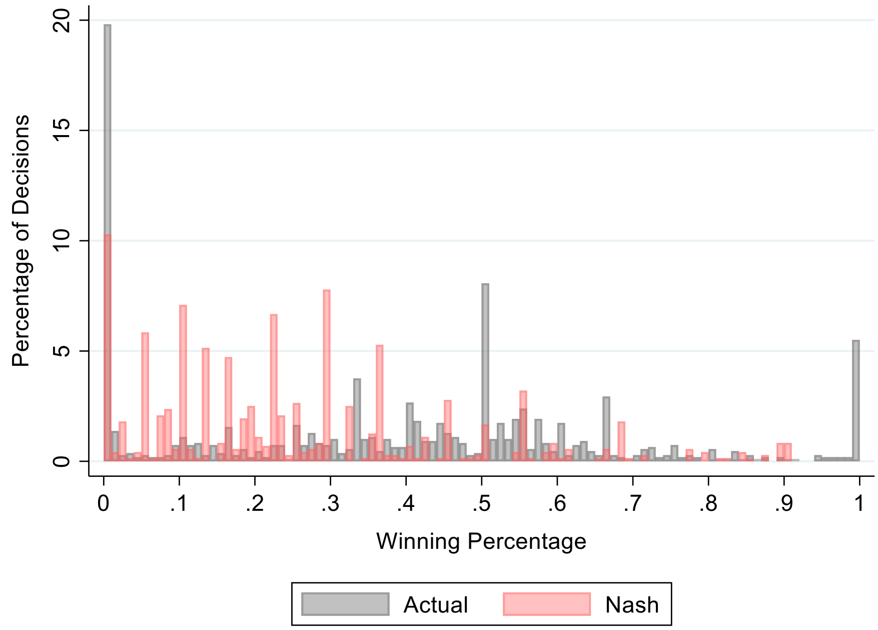


Figure 2.3. Induced Winning Percentage for Player 3: Observed Values Against Nash Predictions - Input Totals Less Than 100

$Value = (Nominal\, Value + Joy\ of\ Winning)$ such as presented in (Sheremeta, 2013). This characterization conveniently fits the heterogenous valuation Table 2.6 presents the descriptive statistics for such a variable. This variable indicates that individuals are behaving as if they had a prize value of 292.72 on average with median 243.90. This average is high relative to the median due to a set of very large observations (which result from large inputs as Player 3 when the input total they face is low). The median corresponds to about 140% overbidding which is about 15% higher than what we observe in the simultaneous cases. Combining this with Result 3 which indicates that overbidding is higher for later players in the contest gives a sense of how participants are viewing the contest.

We can view these high values of implied "joy of winning" and the indication of a *Winning Percentage Heuristic* together. Relating back to the theoretical model of heterogenous valuations, when the Joy of Winning enters additively in the success function it behaves inversely proportional to a decrease in the marginal cost of the effort. For illustration, the optimal response to the other player's effort in a two person contests are given by $x_i = \sqrt{\frac{V}{c} x_j} - x_j$, where V is the value of the prize, and c is the marginal cost of effort. Thus, if players are making a decision according to a *Winning Percentage Heuristic* that can easily be represented a reduced response to the cost of effort. Effectively they optimize based on the first term only and "face" values of c that are lower than 1 (or even 0). Further, the response to c could be affected by the saliency of c either through experimental design or in terms of cognitive mechanism such as the Winning Percentage Heuristic. Thus, the Joy of Winning calculations provided by Table 2.7 are more correctly interpreted as the ratio between Joy of Winning and marginal cost.

Earlier-Mover Advantage

Here we discuss the evidence regarding first mover advantage. Tables 2.8 and 2.9 present the data for effort choices and expected utility before the prize drawing by player position in sequential contests for each treatment type. From these tables we see little evidence of first mover advantage across all treatments. Only in *HetP1* do we see significantly larger effort levels by Player 1, but this translates to an expected utility increase only over Player 3 and Player 1 end with less expected utilities than Player 2s, on average, despite a 20-token prize advantage. Also, the 1st player efforts in the *HetP1* treatment are closest to the Nash equilibrium prediction,

Table 2.7. Implied "Joy of Winning": 3rd Player

	Implied Joy of Winning
Mean	292.72
	(556.15)
Median	243.90
Observations	770

Note: Sample is restricted to Player 3s in sequential contests whose prize value is equal to 100. The standard deviation is in parentheses.

Table 2.8. Effort Choices by Player Position: Sequential Contest

	Dependent Variable: Effort					
	HetP1	HetP2	HetP3	RandomP2	RandomP3	Control
Player 2	-23.08***	16.23***	-0.607	0.0599	4.986	1.141
	(4.025)	(4.213)	(4.333)	(5.863)	(4.371)	(4.081)
Player 3	-10.56**	5.294	12.23***	6.149	-1.829	3.518
	(4.131)	(4.535)	(3.974)	(4.407)	(5.851)	(4.028)
Constant	65.94***	46.28***	43.56***	53.43***	45.84***	46.78***
	(3.015)	(3.107)	(3.029)	(3.292)	(3.089)	(2.615)
Observations	576	408	549	337	371	638
R-squared	0.053	0.036	0.022	0.006	0.006	0.002

*Note: Standard errors in parentheses clustered at the group-session-period level (134-218). Sample is restricted to 3rd players in sequential contests only when the input total from the first two players was less than 100 and the prize value was 100. *** p<0.01, ** p<0.05, * p<0.1*

Table 2.9. Pre-Draw Expected Utilities by Player Position: Sequential Contest

	Dependent Variable: Expected Utility – Pre-Draw					
	HetP1	HetP2	HetP3	RandomP2	RandomP3	Control
Player 2	3.827**	2.826	3.287*	-5.806*	0.559	-2.211
	(1.704)	(1.901)	(1.789)	(3.213)	(2.302)	(1.716)
Player 3	-2.994*	-1.361	9.518***	2.418	-6.528**	-1.938
	(1.695)	(2.082)	(1.855)	(2.442)	(3.141)	(1.742)
Constant	100.9***	102.0***	104.4***	104.1***	112.90***	107.70***
	(1.953)	(1.933)	(1.783)	(2.176)	(2.210)	(1.580)
Observations	576	408	549	337	371	638
R-squared	0.011	0.005	0.022	0.010	0.010	0.002

*Note: Standard errors in parentheses clustered at the group-session-period level (134-218). Participants in Random treatments who did not enter the contest are omitted. *** p<0.01, ** p<0.05, * p<0.1* indicating that even when players are utilizing strategies closer to the Nash, they are not translating to higher payoffs in the experiment.[41]

As we saw from Result 3, the significant overbidding by the later periods reduces the incentive of exercising the earlier mover advantage in the contest. If players are anticipating overbidding from players later in the contest, they may reduce their bids to compensate. Looking at the rest of the Table 2.8 we see only significant increases in effort in the positions that correspond to higher valuations in HV treatments and only in the case of Player 3 in *HetP3* does this translate into higher expected utilities. Lastly, we see significant negative deviations for the players who randomly entered contest in the corresponding RE treatments (Player 2 in *RandomP2*, and Player 3 in *RandomP3*). For the simplest comparison, in the *Control* treatment we find point estimates of increasing effort as players move later in the contest but negative returns to expected value. The coefficients for the *Control* treatment are not statistically significant and support previous research that does not find evidence of first mover advantage in contests (Fonseca, 2009). Lastly, we find that in all treatments and player positions, average expected payoffs are negative for each round.

The lack of first mover advantage is tied closely to the *Winning Probability Heuristic* and the Joy of Winning discussion above. If later players are using behavioral heuristics to make decisions then the deterrence effect of an earlier player increasing the bid is drastically reduced. Further, effort choices by earlier players may be seen as a more cognitively complex task as a player must correctly anticipate later players movement when making your effort choice.

Another potential explanation that has been put forth in the literature is effect of the contest structure itself. Generally, payoffs are "flat" around the Nash Equilibrium choices, in the sense that the reduction in expected value from deviations from the Nash are not large. For example, if a Player 3 in a symmetric sequential contest with prize value of 100 and endowment of 100, faces an output total of 100 (thus, has Nash prediction of 0), loses only 16.7 tokens (16.7%) in expectation from a contribution of half of his endowment. Further, if endowments are increased relative to the prize value, then the percentage loss lowers. Lastly, feedback in a Tullock contest is probabilistic (random draws for the prize) could make learning the Nash

[41] This result would be reversed, as predicted by theory, in a case where later players also responded according to Nash predictions.

strategy difficult. Lastly, we use random player roles in each period which may result in difficulty learning to exercise the positional advantage. Taken together, these factors could explain the inability of subjects to respond to the incentives provided by the theory.

Random Entry vs. Heterogenous Entry:

Our final analysis examines the unusually high difference in overbidding rates between the *Random* treatments and relative to the *Heterogenous* treatments. We look to the individual decisions by player type for clues on the divergent behavior. Figure 2.4 shows the average effort totals by type for each of the RE treatments, conditional on both entry and no entry from the randomly participating subject. We find evidence that there is significantly higher bidding for the Player 3s in the *RandomP2* sequential contest relative to the *RandomP3* contest conditional on entry by all players. Table 2.10 summarizes the difference means among the third players for *RandomP2* and *RandomP3*. Additionally, Table 2.10 shows that the totals that Player 3s receive conditional on full entry are not significantly different (although the point estimate is negative for *RandomP3*). We can also observe that the simultaneous decisions look very similar across treatments. Taking these results in sum, it is difficult to discern the source of the significant deviations in Player 3 behaviors across treatments.

Figure 2.5 displays a scatterplot of the inputs by Player 3s against the input totals they face.[42] From the figures you can see there is a significant difference in the number of inputs of 5 or less by the third players in the *RandomP3* treatment relative to *RandomP2* (31.7% to 13.7%, respectively). probabilistic entry from another player. Similarly, there is a much larger portion of bids that are 100 or above in the *RandomP2* treatment relative to the *RandomP3* treatment (32.9% to 17.1%, respectively).

While bids over 100 points to a potential lack of understanding of the incentives of the game and are clearly influencing the average, there is a substantial fraction of 0 bids from the *RandomP3* treatment which potentially indicate a behavioral response conditional on entry into the contest. We observe simultaneous sections of the same treatment that do not show statistically different Effort choices, with a point estimate of .046 (2.28 *sd*, *p=.984*). This is in

[42] Figure B.2 and B.3 in the Appendix displays bar charts of Effort and Input Totals separately.

Table 2.10. Comparisons of Contributions in Random Entry Treatments

	Dependent Variable		
	Input Total-Sequential	Effort – Sequential Type 3	Effort-Simultaneous
RandomP3	-6.355	-23.18***	0.0460
	(7.518)	(9.106)	(2.283)
Constant	103.8***	68.01***	52.86***
	(4.958)	(6.662)	(1.635)
Observations	155	155	719
R-squared	0.004	0.078	0.000

*Note: Treatment RandomP2 serves as the baseline comparison. Input Total refers to the total inputs that the third player faces. Effort in the middle column refers to the efforts by the Player 3 conditional on entry by the corresponding random player in each treatment. Effort in the third column is the effort reported by players who enter the contest only. Standard errors in parentheses clustered at the session-period level (67,70) first the first and third columns and the subject level for the second (75). *** p<0.01, ** p<0.05, * p<0.1*

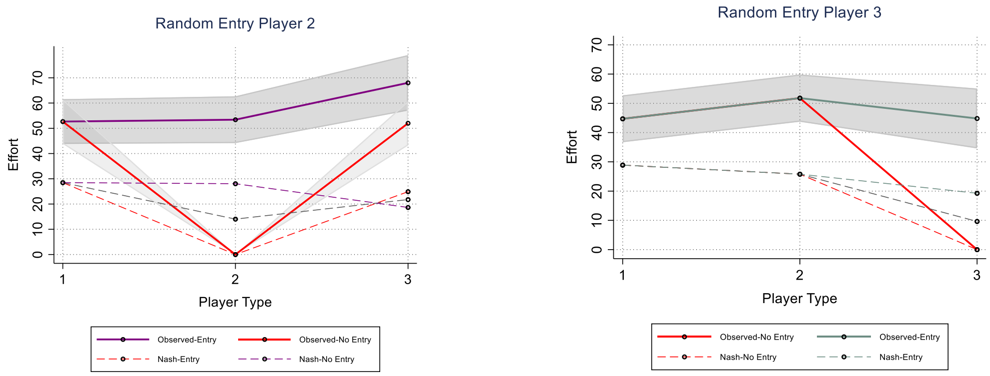


Figure 2.4. Average Effort Levels by Player Type: Random Treatments

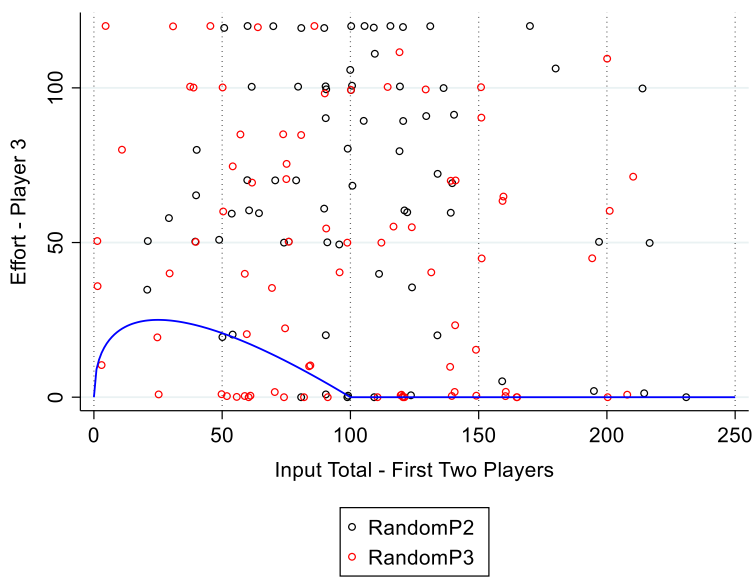


Figure 2.5. Effort by Player 3 Against Input Total: Random Treatments

conjunction point estimates for the difference in input totals that are being seen by Player 3s, as reported in Table 2.8. The results are robust to the inclusion of observable characteristics, which suggests that the driver of the results is unobservable.

It is difficult to speculate why we see such differing results between these treatments. A likely explanation for these differences seems to be a spurious correlation between player characteristics or session level effects. In particular, the *RandomP2* treatment is the driver for most of the differences between the *Random* and Heterogenous treatments as the *RandomP3* choices as conditional on entry and are fairly similar to *Control.* Further, in the case of *RandomP3,* we observe the expected effects predicted by theory as the 50% entry on Player 3s (who are doing the most overbidding) results in significantly lower total contest outputs relative to *Control* when you incorporate the 0 efforts in the case of non-entry by Player 3s. The use of an online environment may introduce noise in the decision process, as well.

V Conclusion

In this study we utilize a recent advancement by Hinnosaar (2018) in the modelling to sequential contests to explore common forms of heterogeneity in sequential contests. We generate Nash Equilibrium predictions for sequential contests that involve heterogenous valuations and probabilistic entry. From a contest design perspective, in a 3-player contest, we find that the effects of a heterogenous valuation for a single player, on aggregate expenditure, are maximized when the player with the heterogenous valuation moves earlier in the contest. This compounds the first mover advantage of the sequential position. We derive a general result that in the *n*-player contest the aggregate expenditures in a contest with a single heterogenous value player that moves last has identical output to an aggregate contest in which all valuations are equal. Suggesting that if a designer sought to minimize the impact of a cost advantage or maximize the output when a firm is at a cost disadvantage, they would seek to place that player or firm at the end of the contest.

We then test these theoretical implications using an experiment. Linking the theory with the extensive literature in experimental economics on contests. We find some support for the theoretical predictions in terms of directions and the invariance of aggregate totals to a heterogenous value final player. However, we find a deviation from theoretical predictions in the probabilistic entry treatment which may be linked to behavioral aspects of the decision-making process in a probabilistic entry environment. Like many other studies we find significant overbidding in our contests, and we confirm a recent result that suggests that overbidding in sequential contests tends to be lower than simultaneous environments. Additionally, we provide evidence that overbidding tends to increase with player position relative to Nash predictions, but also with increased bidding by earlier players. We also find support that players in a contest make decisions in accordance with a *Winning Percentage Heuristic*, in which a subset of players choose effort to target a winning percentage as opposed to deciding based on a more complicated expected value. The ability to observe behavior for the final player in the group in terms of a direct observation for the conditional response of a player's effort relative to the group effort provides another interesting tool for experimental scientists who study contests.

This study is limited to the examination of 3-player contests in terms of theoretical predictions (except for the invariance proposition) and experimental design. This limitation is primarily driven by the inclusion of heterogeneity in player types which can make the theoretical analysis unwieldy. Further, the study exhibited relatively high levels of overbidding, which could indicate some procedural anomalies, although this overbidding was well within the overbidding amounts that have been observed in related studies. The high levels of overbidding and the *Winning Percentage Heuristic* behavior we observe is likely a function of the random nature of the contest itself.[43] It is possible that the combination of overbidding and the heuristic drown out the earlier mover advantage that is predicted by the model. Deterrence of later effort is the source of earlier mover advantage but if players respond to high effort with even higher effort of their own the benefit of deterrence is lost. What we, and other studies, could be observing is learning by subjects that deterrence is ineffective (and reduces payoffs). Some other possible extensions could examine different contest structures by adding players, either in the same sequential position or creating contests with more sequential positions. Introducing uncertainty or imperfect information into the output revelations may reflect more realistic settings where contestants do not have full information over the outputs of their competitors but receive signals of prior contributions could be interesting.

[43] Overbidding is observed in more deterministic contests such as auctions, but there is a gambling element involved in a standard Tullock contest.

Chapter 3

A Network Model of Flexibility in the Workplace

I Introduction

Collaborative work in firms and other organizations has been studied from many perspectives dating at least to Coase's (1937) seminal work. Modern network theory provides a new tool for understanding particular aspects of the structure of workplaces. In particular, this theory provides a tool for understanding how one worker's effort may impact other "connected" workers, and from those workers on to others in the web of a worker network. With this tool we can understand how a firm or other organization may optimal differentially motivate otherwise similar workers depending on their position in a network. Further, we can model how the importance of connections in the network may impact organizational decisions that impact the formation of connections, such as how workplaces of physically structured, and whether workers are permitted to work from home.

With societal progress into the digital age, the work, and the workplace itself has evolved. Online collaboration tools, such as Zoom, have drastically heightened the level of integration possible between separated employees and enable expansion of remote work, while also raising questions about what might be lost when workers don't share a space. While telecommuting and flexible work arrangements have been on the rise for a few decades, we still see prominent examples of companies who maintain large office spaces/campuses (such as Google). Further, in the COVID-19 pandemic we have seen many examples of companies rolling back working from home policies, even amid employee objections. [44]

In this paper we develop a theoretical model that captures the value to an employer of investing in facilitating connections among her employees. We model this with a formalized network of complementarities between employees that improve worker productivity ala Ballester et al. (2006). Additionally, we allow for the employer to sets wages differentially based on

[44] Web article by O'Connor (2021) details Apple's push to return to the office.

observed network connections and make costly investments to improve the likelihood of generating connections and complementarities between employees. By modelling the workplace in this way we can capture many of the important elements of flexible work arrangements that have been suggested by previous research. Further, the model serves to connect literatures that distinctly focus on employee outcomes and employer outcomes but rarely in conjunction.[45]

Flexible work arrangements have been shown to have positive impacts on employees as well as reduced costs for employers. Research in management, social psychology and more recently economics, have explored flexible work arrangements. Much of the work is centered on the impact of remote work on employees (see Allen et al., 2015 for a summary). This research has suggested mostly benefits with few costs. Benefits include but are not limited to: increased productivity, increased satisfaction, lower stress, increased organization commitment, higher wages, reduced commute times, and reduced turnover (Gajendran and Harrison, 2007; Golden 2007; Kurland & Egan, 1999; among others). Drawbacks include decreased satisfaction, in the form of isolation, decreased work-life separation (being contacted at odd hours), an inability to disconnect from work. Other research suggests lower productivity (distraction) could also result.

Less studied are the costs and benefits accrued by the firms. Research has suggested that having employees work remotely allows firms to recruit in a wider area (resulting in cheaper, or more productive labor) and increased profits through increased productivity and lowered costs (*e.g.* rent/electricity) (Martinez Sanchez et al., 2007; Meyer et al., 2001). However, this result is likely situation-dependent, as some firms have opted to discourage telecommuting and create workplace environments that maximize interaction (see Schmidt & Rosenberg, 2014).

One drawback with the current literature on remote working is that much of the existing data related to working remotely is observational, which makes it challenging to draw causal inferences. Explicitly, Gao and Hitt (2003) note that the telecommuting literature, while helpful for understanding telecommuting in practice, is largely empirical and has lacked sufficient theoretical foundation to build hypotheses. This statement is echoed by other researchers (Bailey and Kurland, 2002: Bloom et al, 2015). Bloom et al. (2015) state that the lack of experimental data in telecommuting research makes studies “hard to evaluate due to their non-random nature”.

[45] To our knowledge, no studies focus on firm and employee outcomes simultaneously.

This fault does not necessarily fall on the researchers however, as the Council of Economic Advisors (2010) cite a lack of data as a "factor hindering deeper understanding about the costs and benefits of flexibility."

Recent research in economics has worked to fill this void. In a field experiment, Bloom et al. (2015) randomly sent home a subset of call-center workers in a Chinese firm and observed increased output and increased employee satisfaction for those employees who worked from. Additionally, a recent paper by Chen et al. (2019) examines the surplus gained (relative to their reservation wage) by Uber drivers in a flexible work arrangement and finds that flexible work contracts gain twice the surplus they otherwise would. A similar element in these two studies is the type of work that is done by the firms they are studying benefit very little from employee-employee interactions (*e.g.* Uber drivers and call center workers). We believe this is a crucial aspect that must be considered to accurately generalize conclusions based on flexible work arrangements.

A related paper by Calvó-Armengol et al. (2015) studies the investment in information transfers among agents in a network. They describe a setting in which agents can undertake costly actions to either speak or listen in order to coordinate actions. Calvó-Armengol et al. connect network structure into a broader literature about the role of communication in the theory of organizational design (Arrow, 1974). Their model highlights the natural use of social networks when thinking about interactions among employees in a firm. Networks are a good representation of a few important phenomenon in organizations, such as communication flows, hierarchies (or other firm organization), and influence. In addition the model by Calvó-Armengol et al. has many similarities to ours in the importance of centrality and network structure in a linear-quadratic payoff structure and the connection to organizational design. In contrast to their focus on communication, our model focuses on production complementarities between members and the impact of the altering the network structure by a principal.

With widely cited benefits and few costs, the proliferation of flexible work arrangements should not be surprising. But this begs the question, why is the office still the primary way to conduct business? Quotes from managers such as CEO Jamie Diamond of Goldman Sachs may help elucidate some of the drawbacks of remote work. He listed four challenges associated with working from home regarding the COVID-19 pandemic: 1) Performing jobs remotely is more successful when people know one another and already have a large body of existing work to do. It does not work as well when people do not know one another. 2) Most professionals learn their job through an apprenticeship model, which is almost impossible to replicate in the Zoom world. Over time, this drawback could dramatically undermine the character and culture you want to promote in your company. 3) A heavy reliance on Zoom meetings slows down decision making because there is little immediate follow-up. 4) And remote work virtually eliminates spontaneous learning and creativity because you do not run into people at the coffee machine, talk with clients in unplanned scenarios, or travel to meet with customers and employees for feedback on your products and services. While this example provides only anecdotal evidence, a central theme is clear. There is a social component that is created when employees work near each other that is difficult to replicate through remote work. We characterize this social component in our model as the primary drawback of offering a flexible work arrangement. This is the key assumption for the tradeoff a firm faces when deciding to offer remote work. Specifically, it takes the form of complementarities in a social network.

This model presented below builds directly on the seminal paper by Ballester et al. (2006) (BAZ) who examine games with strategic network complementarities among players using a linear-quadratic utility function. They show that a player's Nash equilibrium output is proportional to their Katz-Bonacich centrality in the network. The BAZ framework has been applied to a wide range of topics such as crime (Ballester and Zenou, 2010; Pattachini and Zenou, 2012). Additional papers have incorporated the framework provided by BAZ and developed second-stage principal optimizations to analyze different questions such as pricing on a network of consumers (Candogan et al., 2012; Chen et al., 2018; and others). Candogan et al. (2012) (CBO) provided the seminal example of this class of models. These models generally frame a monopolist (or multiple firms) who price a good that is purchased by a network of consumers with consumption externalities. Specifically, CBO finds that the optimal pricing strategy is such that prices should be set inversely proportional to a consumers Katz-Bonacich centrality.

A key contribution of this paper is to adapt the framework pioneered by CBO to a workplace setting to show how the profit maximizing piece-rate wage to individual employees in a network is set *proportional* to their centrality. We then employ this framework to explore how potential network outcomes may impact employer decisions that influence the formation of connections. For example, and employer may risk connections being broken by allowing a worker to work remotely. Conversely, an employer may make investments in facilitating connections between employees that create more cohesive networks, such as through “team building” events.

Another contribution to the literature by adapting the network pricing model (ala Candogan et al. 2012) to a corresponding wage setting environment in a principal-agent problem. In this new setting we add a theoretical framework to a burgeoning literature on working from home environments. We highlight the importance of thinking of the firm’s decision in conjunction with the employees when considering remote work. Lastly, we add a third stage to the model that allows for a firm to influence the shape of the network before making price/wage decisions. This reflects a more realistic setting when thinking about the models in a wage-setting environment.

II The Model

In this model the principal (employer) first invests in influencing the complementarity network of the agents (employees), $\boldsymbol{G}$. The network is then realized (randomly) and the employer chooses a wage vector $\boldsymbol{\omega}$ from allowable wage strategies $\boldsymbol{\Omega}$ to pay for production of a divisible good that maximizes profit. Employees take the wage as given and choose production effort (output) accordingly. The problem is solved using backward induction. The effort and wage setting stages of the model are a re-characterization of Candogan et al (2012) but the incentives are otherwise identical. In the standard characterization of this type of network model with complementarities (Ballester et al., 2006) the network of connections between the agents generates complementarities in output, which serve to increase the effort level (output) of the employee holding wage rate fixed. We describe the stages in more detail below:

Stage 1 (Wage setting) Conditional on a network of connections, the employer sets the optimal wage schedule. The employer chooses wage vector $\boldsymbol{\omega}$ to maximize profits associated with network $\boldsymbol{g}$ ($\pi_{\boldsymbol{g}}$): $\max \pi_{\boldsymbol{g}} = \max_{\boldsymbol{\omega} \epsilon \boldsymbol{\Omega}} \sum_i \phi q_i - \omega_i q_i$ where ϕ is the price received on the market by the employer from selling output q_i from the employees. For ease, we us a linear production function $q_i = e_i$ allowing for substitution of q_i and e_i (e_i henceforth). Also note that this profit function implies that effort from the employees is treated as perfect substitutes in production. Spillovers resulting from employee effort are captured through the disutility of work described below.

Stage 2 (Effort) Employee i chooses effort e_i to maximize utility given the wages chosen by the employer, $\boldsymbol{\omega}$ and the effort levels of the other employees, $\boldsymbol{e}_{-i}$, i.e.

$$\max_{e_i \epsilon [0,\infty)} u_i(e_i, \boldsymbol{e_{-i}}, \boldsymbol{\omega})$$

Each employee is represented in the set $\mathcal{F} = \{1, \dots, n\}$ selects a divisible effort $e_i \epsilon [0, \infty)$ and receives the following utility:

$$u_i(e_1, \dots, e_n) = \omega_i e_i - \alpha_i e_i - \frac{1}{2}\delta e_i^2 + \sum_{j \neq i}^{n} \gamma_{ij} e_i e_j g_{ij} \tag{1}$$

Note that utility decreases with own effort at an increasing rate, $\frac{d^2 u_i}{de_i^2} = \delta > 0$. An individual's (strictly positive) wage, ω_i, is set by the employer. The parameter α_i is a "skill" factor where lower α_i suggests less disutility from production. The model accommodates varying quadratic own-effort costs, however we set them equal for all players, $\delta_i = \delta$. Interactions between players are captured by γ_{ij}. The influence of network-connected employees' efforts on the disutility of production by worker i is captured by the parameter γ. For any two workers this term captures the interaction effect with $U_{e_i e_j} = \gamma_{ij}$. In general, for $\gamma_{ij} < 0$ efforts are substitutes, while $\gamma_{ij} > 0$ represents complements. In this paper we assume $\gamma_{ij} > 0 \; \forall i, j$, thus, all interactions are complementary. This captures in a simple manner the team aspect of production. For any worker i connected in the network to worker j the disutility i experiences to produce a given output e_i decreases with e_j. The adjacency matrix captures the full network of complementarities $\boldsymbol{g}$ and is given by the $n \times n$ matrix $\boldsymbol{G} = [\gamma_{ij} g_{ij}]$. The ijth entry of $\boldsymbol{G}$ represents the presence of a link from player i on player j with positive indicator variable which can take value $g_{ij} \epsilon \{0,1\}$ multiplied by weight γ_{ij}. By convention $g_{ii} = 0$, thus $\boldsymbol{G}$ is a zero-diagonal non-negative square matrix. When $g_{ij} = g_{ji}$, the matrix is symmetric and the links between players are undirected (if i is connected to j then j is connected to i), in this case $\boldsymbol{g}$ is an undirected network. Additionally, the convention $g_{ii} = 0$ rules out loops (which is a path [sequence of links from i to j through links $c < g_{ic_1}, g_{c_1c_2}, \ldots g_{c_f j} >$] that travels through node i more than once.)

III Effort Equilibrium

We solve the model through backward induction starting with the effort choice of workers. Assumption 1 is needed to ensure that effort choices remain finite.

$$Assumption\ 1: 2\delta > \mu(\boldsymbol{G})$$

Where $\mu(\boldsymbol{G})$ is the spectral radius of adjacency matrix $\boldsymbol{G}$. This assumption states that the disutility of own effort parameter is smaller than the utility gained from increases in effort by other employees in the network. Assumption 1 (with Assumption 2 below) guarantees an interior solution to the vector of efforts. Assumption 1 states that the utility returns on increase in fellow employee's effort (or consumption in CBO) are likely to outweigh the cost of own effort when effort becomes large. This is a standard assumption (parallel to Assumption 1 in BO; and $2\delta > \lambda\mu(\boldsymbol{G})$ in BAZ) for this class of models. The assumption restricts employees from experiencing a feedback loop where complementary increases in effort from connected employees increasing utility more than the disutility of increasing the own effort resulting incentives to increase own effort infinitely.

An additional assumption is needed to ensure positive wages (effort) choices for each employee and is the following:

$$Assumption\ 2: \phi > \alpha_i$$

This ensures that the return to the employer is positive at small levels of effort and guarantees that the offered wage is positive and large enough so that there is positive effort from all employees and ultimately an interior solution in terms of optimal offered price and effort vectors. From CBO, under Assumptions 1 & 2, optimizing (1) we know that this problem solves to give *unique* optimal efforts:

$$\boldsymbol{e}^* = (\boldsymbol{\Lambda} - \boldsymbol{G})^{-1}(\boldsymbol{\omega} - \boldsymbol{\alpha}) \tag{2}$$

where $\boldsymbol{\Lambda}$ is a $n \times n$ diagonal matrix with elements $2\delta_i$. This expression is linked to the concept of Bonacich Centrality where $(\boldsymbol{\Lambda} - \boldsymbol{G})^{-1}$ is the component matrix of Bonacich centrality measures. Relating this with the centrality concept to our employee's optimization we can return to the solution of the employee's optimization problem (2) and set $\gamma_{ii} = \gamma$. Equation 2 becomes:

$$\boldsymbol{e}^* = \boldsymbol{\beta}(\boldsymbol{g}, \lambda)(\boldsymbol{\omega} - \boldsymbol{\alpha}) \tag{3}$$

Thus, the optimal effort for each employee is a linear function of her wage and her Bonacich centrality. More central players receive larger effects from increases in effort on the network, thus have ultimately higher effort levels.[46]

IV Optimal Wages

The principal's problem when he optimizes individual wages for a given network is[47]:

$$\max_{\boldsymbol{\omega} \epsilon \boldsymbol{\Omega}} \sum_i \phi e_i - \omega_i e_i = (\phi \mathbf{1} - \boldsymbol{\omega})\boldsymbol{e}^{\boldsymbol{T}}$$

where he sells output from production function $f(\boldsymbol{e})$ on the market for price ϕ and pays total wages $\omega_i e_i$ to each employee i. In our model $f(\boldsymbol{e}) = \sum e_i$ which suggests the employer production function is linear. To solve the two-stage problem we can substitute the solution to the employee's problem and optimize. The important consideration here is the set of available wage choices Ω. For the first setting we allow the employer to freely set wages for each employee. Again, following CBO, we can substitute equation (2) into the employer's optimization problem and find that the optimal wage vector is:

$$\boldsymbol{\omega} = \boldsymbol{\alpha} + (\Lambda - \boldsymbol{G})\left(\Lambda - \frac{G + G^T}{2}\right)^{-1} \frac{\boldsymbol{\phi 1} - \boldsymbol{\alpha}}{2} \tag{4}$$

If the network is symmetric, *i.e.* the influence of employee i on employee j is identical to the influence of j on i the above equation reduces to:

$$\boldsymbol{\omega} = \frac{\boldsymbol{\phi 1} + \boldsymbol{\alpha}}{2} \tag{5}$$

[46] More information on Bonacich Centralities is given in the Appendix.
[47] A reminder that we assume effort from the employees is directly converted, at a 1:1 ration, to inputs for the employers $e_i = q_i$

This shows that when the network is symmetric the optimal wage vector does not depend on the structure of the network, and that the employer simply sets the wage proportional to the skill parameter of the worker where higher skill employees receive higher wages. However, many networks are not symmetric – some workers have increased influence over other workers in the network, in which case the network influences the optimal wage vector. We can use equations (4) and (5) to solve for the indirect profit function yielding:

$$\pi^* = \left(\frac{\phi\mathbf{1} -}{2}\right)^T \left(\Lambda - \frac{G + G^T}{2}\right)^{-1} \left(\frac{\phi\mathbf{1} - \boldsymbol{\alpha}}{2}\right) \tag{6a}$$

Or equivalently:

$$\pi^* = \left(\frac{\phi\mathbf{1} - \boldsymbol{\alpha}}{2}\right)^T \beta(\boldsymbol{g}, \lambda) \left(\frac{\phi\mathbf{1} - \boldsymbol{\alpha}}{2}\right) \tag{6b}$$

Thus, we see that more connected networks (with higher centralities) yield higher profits, *ceteris paribus*.

An important difference from prior network theory models arises when the adjacency matrix is asymmetric. In our workplace context, the optimal wages are given by the equation:

$$\boldsymbol{\omega} = \frac{\phi\mathbf{1} + \boldsymbol{\alpha}}{2} - G\Lambda^{-1}\tilde{\beta}\left(\tilde{G}, \Lambda^{-1}, \tilde{\boldsymbol{v}}\right) + G^T\Lambda^{-1}\tilde{\beta}\left(\tilde{G}, \Lambda^{-1}, \tilde{\boldsymbol{v}}\right) \tag{7}$$

where $\tilde{\beta}\left(\tilde{G}, \Lambda^{-1}, \tilde{\boldsymbol{v}}\right)$ is the weighted Bonacich Centrality matrix, $\tilde{G} = \frac{G+G^T}{2}$ and $\tilde{\boldsymbol{v}} = \frac{\boldsymbol{\phi 1 - \alpha}}{2}$. The first term in this equation is a common term regardless of network position and based on the vector of own costs of effort. The second term indicates that the optimal wage for player i *decreases* in the complementarities that player j exerts over player i, and the third term states the optimal wage increases in the influence that employee i exerts over player j (relevant elements of G^T) which is scaled by how central each employee j is (relevant elements of $\tilde{\beta}\left(\tilde{G}, \Lambda^{-1}, \tilde{\boldsymbol{v}}\right)$). Thus, the employer provides increased differential compensation based the ability of an employee to exert asymmetric influence over central employees in the network and takes advantage of the decreased effort costs (through complementarities) that the influential employee generates for other employees by reducing their piece-rate. These terms cancel out when the influence matrix is symmetric and reduces to equation (5).

IV.A Application—The Cost of Lost Connections

We will now present an example that demonstrates how the model informs that choice a firm would face when consider allowing one individual in a pre-existing workplace to enter a flexible working or telecommuting arrangement that could result in broken network connections. This will allow us to visualize how the model presented so far can be used in this setting to increase understanding of relationship between employer and employee that exist in flexible work settings.

Consider an 9-person network with the structure provided in Figure 3.1. The network represents a potential workplace structure with a "boss" (Player 1), two "managers" (Players 2 and 3) who each oversee 3 employees (Players 4-9). The "types" of employees are given a corresponding letter. We will examine both a symmetric and asymmetric network. The arrows on the right represent the influence that each type of employee exert on one another in the asymmetric case. We can see that the "boss" asymmetrically influences the "managers" but does not (directly) influence the "employees". [48] In a similar fashion, the managers asymmetrically influence the employees. This example is very different from a symmetric setting where all influences are identical and which can be thought of as representing a non-hierarchical, or "flat", organizational structure.[49]

Throughout this application we set the parameter values to $\phi = 2, a_i = a = 1,$ and $\delta_i = \delta = 3$ unless otherwise noted. Thus, the price the monopolist receives is 2, the own-effort cost and quadratic own-effort cost parameters are set equal for each employee and set to 1 and 3, respectively. We will assume that, when an employee accepts a flexible work arrangement, they potentially lose links within the network due to decreased interactions with their fellow employees.

In this application we want to capture the potential benefits of flexible work arrangements arising to the worker, as well as the potential cost of broken connection. Our primary way of representing the benefit to the worker as a reduction in quadratic own-effort cost from $\delta = 3$ to $\delta_i = 1.25$. This reflects the idea that production in the flexible work environment

[48] Adjacency matrices are provided in the Appendix.
[49] We will refer to the players according to their "type".

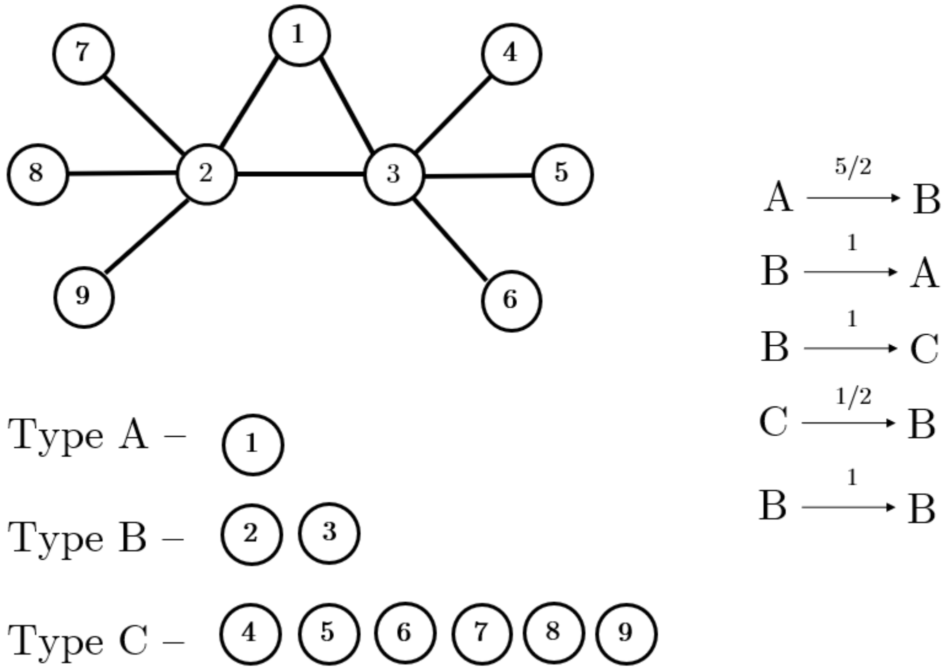


Figure 3.1. Expected Profit by Link Probability

is easier than in the office environment through channels such as flexible working hours, reduced distractions, etc. Alternatively, we also present results keeping $\delta_i = 3$ which represents environments where the benefits may be fixed, such as reduced commuting times. However much of the example will focus on the case where $\delta_i = 1.25$, and we will highlight the important details from the $\delta_i = 3$ example when they arise. Lastly, we assume that the employer does not re-optimize the wage-incentive structure based on the resulting network (*i.e.* she does not observe the broken connections).[50]

We first identify the effects that occur if an individual work of the various types were to lose *all* their connections in the workplace. Table 3.1 summarizes the change in efforts, wages, utilities, and profits in both the identical and differential incentive schemes, and in both the symmetric and asymmetric network cases. Each row of the table represents a different network structure that is associated with the removal of the listed player type ("Complete" has no removals) from the network with the associated wage structure and change in δ, if applicable.

For example, the 6th row of table 3.2 starting with "Type B – Asymmetric" corresponds to the network associated with the removal of a "manager" (Player 2 or 3) from the network. In the second column we see a 5-element vector of efforts when the wage is set equal for all players (and set based on the "Complete" network before any changes in the network are induced by the working from home agreement). This is vector of elements corresponds to the efforts for each player type: $\{e_A, e_{B_0}, e_{B_1}, e_{C_0}, e_{C_1}\}$. The differing efforts between player types result from the asymmetric changes in connections that the player types face. Assume Player 2 is the employee whose connections are lost, the "managers" (Type B) now have different centralities (number of connections) and thus, different effort levels; e_{B_0} for Player 2 and e_{B_1} for Player 3. the "employees" (Type C) connected to Player 2 lose their only connection to network (but are otherwise identical) and are represented by e_{C_0}. The employees connected to Player 3 retain their connections and efforts are given by e_{C_1}. This exercise is repeated for the differentially set wages in column 3 and then for utilities in columns 4 and 5. Then, given wages and efforts, profits are calculated in columns 6 and 7. Thus, any entry for effort or utility that has more than

[50] Results when the employer reoptimizes based on the new network structure are presented in the Appendix. Generally, the profit loss will be less dramatic when the employer can reoptimize, but the results are often in the same direction.

3 entries represents the different centralities for players of the same type that are induced by the loss of connections. We can then compare each row with the “Complete” rows to understand the impact that the working from home arrangement has for each employee type.

The wage vector for each player in the symmetric case in both the identical and differential incentive structure is $\omega = \omega_{Iden} = \omega_{Diff} = 1.5$. In the asymmetric case when the wage when is set identically is equal to $\omega_{Iden} = 1.5$. When set differentially, $\omega_{Diff} = [\omega_A, \omega_B, \omega_C] = [1.84, 1.26, 1.56]$. Since we assume that the wage is set before the employer knows the impact of the changes to the network structure that are induced by the flexible working arrangement, the wages are identical for each of the possible resulting networks.

Focusing on the “Complete” networks in Tables 3.1 and 3.2 we can see that the employer only profits from the use of differential incentive schemes when the network is asymmetric (conditional on equivalent own-cost parameter values). We can also observe that in nearly all cases (except Asymmetric-Type C when $\delta = 1.25$) the employer is set to lose profits from a working from home arrangement. It is important to understand that this represents the worst-case scenario for the employer, in that an employee who takes the flexible working arrangement then maintains no links. When there is no change in δ for the employee (no reduction in effort costs from working at home) then the profit losses will reach their maximum. In this case, the profit difference represents the minimum that the employer would need to receive via fixed reduction in the employee’s compensation to guarantee a positive return.

Another noticeable result is that often the profit loss when using differential incentive schemes is higher relative to the identical incentive scheme in the asymmetric environment. In general, if the employer is optimally setting differential incentive schemes, they must do at least as well as setting a fixed wage scheme; however, the loss in these cases is a result of the larger misappropriation of wages that comes from the change in the underlying network. If the employer could re-optimize wages this difference would be reversed. Nevertheless, it does suggest that using an identical wage structure can lead to less loss when you have a change in the underlying network structure, and the network structure is asymmetric.

Another potentially interesting result in the “Complete” networks comes from examining the utilities of the “boss” and the “managers”. When incentive structures are identical, and in both the symmetric and asymmetric settings, the “boss” receives lower utility (and produces less output) relative to the “managers”. This is because the “managers” have higher centralities within the network (without the asymmetric influence the “boss” description is less apt). However, when the employer can set differential incentive schemes in the asymmetric setting, the “boss” is compensated above the other employees and realizes significantly higher utility. This is an example of how the employer optimally leverages the influence asymmetries, essentially taking advantage of the decreased utility costs that are generated by the output increases of the “boss”. This could also represent a situation where the employer makes incorrect assumptions about the network structure and sets wages incorrectly leading to a loss in utility for her most influential worker.

The last notable result is that firm profits increase when Type C players enter a work from home arrangement, regardless of a change in δ. It should be noted that, mechanically, this result is based on the parameter choices and influence network that we have defined in the example. This is easiest to see in the symmetric, $\delta = 3$ case where profit decreases when a Type C player exits the network. When a Type C is disconnected this lowers the centrality for all players in the network and lowers outputs/profits. In the asymmetric, differential wage case, the profit increased. You can see that the efforts from the remaining Type C connected players are increased relative to the complete network case. This is due to the increased relative influence of the “managers” and “bosses” on the Type Cs. The last important consideration is the choice of the change in δ. For demonstration, our parameter choice was a reasonably large change in δ, suggesting large reductions in own-effort costs associated with working in a flexible environment. This change depends critically on the context, but it is intuitive to understand that the larger the reduction in δ, the smaller the profit losses will be (or the larger the gains) when flexible work is introduced. Similarly, the more employees who have the arrangements the more own-cost advantages can be leveraged.

So far, this example has examined the case where a flexible working arrangement resulted in an unanticipated complete loss of connections. An alternative assumption may be that each link that has formed has a chance to fail. Mechanically the employer could attach some probability of failure for each link and calculate the expected profit loss from a failure of that link. In other words, they could calculate the profits associated with or without the link. Then, given the probabilities of failure for each link, calculate the likelihood of the formation of any given network and the associated probabilities and determine whether the expected value of the profit is greater with the introduction of the flexible work environment compared to a setting without.

This is potentially a complicated calculation and there are a significant number of possible networks and parameter combinations if the employer made the choice for each employee. We will continue the application restricting our attention to the results from a *single* offer of a telecommuting arrangement to Player 2. Figure 3.2 shows the relevant connection types for Player 2. Due to the parameter values chosen in the example, a link between Player 2 and her "employees" are functionally identical (Type III). She also has a link between her "boss", Player 1, and the other "manager", Player 3 which have different influencing weights.

Let us assume that the probability of failure of any link is equivalent and given by $(1 - p)$ then the chance that a link persists is given by p. We can then calculate the resulting profits for each network. We will use $\delta = 1.25$. Table 3.3 displays the profits for each possible network. In Table 3.3, each row of the table lists the profits associated with the removal a link (or multiple links) of the listed type corresponding to Figure 3.2. For example, the fifth row (I, II) is the profit associated with the removal of two links, the link between the two managers (Player 2 and Player 3 - II) and a link between Player 2 and one employee (I). The last row of the table is the profit in the case where there is no working from home arrangement (and no reduction in δ for manager). Examination of Table 3.3 reveals a pattern that suggests that the most valuable connections in order are I, II, and III. Thus, the employer has strong incentive to keep the "boss" connected to each of the managers and the managers to each other. In this example, we treat the probability of each connection being maintained as identical, but this the table suggests that targeted intervention is the ideal choice if possible.Comparing the profit without the flexible work arrangement at the bottom of Table 3.3 with the values above in the corresponding columns, we can see that if three or more links are formed profits will always be higher in the telecommuting environment. This level is highly dependent on the choice of δ and the required links for profitability will increase as δ increases. Moving along with our example,

Table 3.1. Optimal Values with Complete Link Loss, $\delta = 3$

Network Type	Effort - Iden $[e_A, e_B, e_C]$	Effort - Diff $[e_A, e_B, e_C]$	Utility - Iden $[U_A, U_B, U_C]$	Utility - Diff $[U_A, U_B, U_C]$	Profit π_{Iden}	Profit π_{Diff}
"Complete" - Symmetric	[.150, .200, .117]	[.150, .200, .117]	[.458, .580, .368]	[.458, .580, .368]	0.625	0.625
"Complete" - Asymmetric	[.215, .225, .112]	[.165, .245, .104]	[.522, .716, .347]	[.755, .653, .395]	0.638	0.667
Type A – Symmetric	[.083, .167, .111]	[.083, .167, .111]	[.188, .333, .241]	[.188, .333, .241]	0.542	0.542
Type A – Asymmetric	[.083, .158, .096]	[.140, .112, .102]	[.135, .220, .152	[.223, .123, .145]	0.489	0.462
Type B - Symmetric	[.109, .083, .156 .083, .109]	[.109, .083, .156, .083, .109]	[.135, .109, .171, .109, .135]	[.135, .109, .171, .109, .135]	0.464	0.464
Type B - Asymmetric	[.113, .083, .180, .083, .098]	[.167, .042, .165, .093, .107]	[.135, .106, .177, .107, .121]	[.249, .054, .151, .133, .148]	0.461	0.447
Type C - Symmetric	[.145, .178, .195, .083, .113, .116]	[.145, .178, .195, .083, .113, .116]	[.380, .448, .481, .233, .306, .313]	[.380, .448, .481, .233, .306, .313]	0.588	0.588
Type C - Asymmetric	[.161, .224, .239, .083, .102, .103]	[.210, .203, .219, .093, .110, .111]	[.457, .595, .624, .256, .308, .311]	[.670, .534, .566, .302, .352, .357]	0.611	0.635

Note: Under the Network Type header, Type A, B, and C refer to the player Type who works from home and subsequently has zero connections. Symmetric and Asymmetric labels refer to the use of the symmetric or asymmetric adjacency matrixes described in the text. Values in the Effort and Utility columns for "Type B" correspond with $\{e_A, e_{B_0}, e_{B_1}, e_{C_0}, e_{C_1}\}$ *and* $\{U_A, U_{B_0}, U_{B_1}, U_{C_0}, U_{C_1}\}$ *where* B_0 *is the Type B player who has zero links and* B_1 *the traditional link. Similarly,* C_0 *represents the Type C players who are connected to the* B_0 *player and* C_1 *with* B_1*. In the Type C column, the vector shown is* $\{e_A, e_{B_0}, e_{B_2}, e_{C_0}, e_{C_1}, e_{C_2}\}$ *where* C_0 *represents the zero-link type C player and* B_0*, the connected "manager". The* C_1 *represents the other two Type Cs connected to* B_0 *with* C_2 *and* B_2 *representing the remaining.*

Table 3.2. Optimal Values with Complete Link Loss, $\delta = 1.25$

Network Type δ	Effort - Iden $[e_A, e_B, e_C]$	Effort - Diff $[e_A, e_B, e_C]$	Utility - Iden $[U_A, U_B, U_C]$	Utility - Diff $[U_A, U_B, U_C]$	Profit π_{Iden}	Profit π_{Diff}
"Complete" - Symmetric	[.150, .200, .117]	[.150, .200, .117]	[.458, .580, .368]	[.458, .580, .368]	0.625	0.625
"Complete" - Asymmetric	[.215, .225, .112]	[.165, .245, .104]	[.522, .716, .347]	[.755, .653, .395]	0.638	0.667
Type A – Symmetric	[.222, .167, .111]	[.222, .167, .111]	[.188, .333, .241]	[.188, .333, .241]	0.542	0.542
Type A – Asymmetric	[.222, .158, .096]	[.372, .112, .102]	[.135, .220, .152]	[.223, .123, .145]	0.489	0.462
Type B - Symmetric	[.109, .222, .156 .083, .109]	[.109, .222, .156, .083, .109]	[.135, .292, .171, .109, .135]	[.135, .292, .171, .109, .135]	0.533	0.533
Type B - Asymmetric	[.113, .222, .180, .083, .098]	[.167, .113, .165, .093, .107]	[.135, .284, .177, .107, .121]	[.249, .145, .151, .133, .148]	0.530	0.500
Type C - Symmetric	[.145, .178, .195, .222, .113, .116]	[.145, .178, .195, .222, .113, .116]	[.380, .448, .481, .622, .306, .313]	[.380, .448, .481, .622, .306, .313]	0.657	0.657
Type C - Asymmetric	[.161, .224, .239, .222, .102, .103]	[.210, .203, .219, .247, .110, .111]	[.457, .595, .624, .684, .308, .311]	[.670, .534, .566, .806, .352, .357]	0.680	0.704

Note: Under the Network Type header, Type A, B, and C refer to the player Type who works from home and subsequently has zero connections. Symmetric and Asymmetric labels refer to the use of the symmetric or asymmetric adjacency matrixes described in the text. Values in the Effort and Utility columns for "Type B" correspond with $\{e_A, e_{B_0}, e_{B_1}, e_{C_0}, e_{C_1}\}$ *and* $\{U_A, U_{B_0}, U_{B_1}, U_{C_0}, U_{C_1}\}$ *where* B_0 *is the Type B player who has zero links and* B_1 *the traditional link. Similarly,* C_0 *represents the Type C players who are connected to the* B_0 *player and* C_1 *with* B_1*. In the Type C column, the vector shown is* $\{e_A, e_{B_0}, e_{B_2}, e_{C_0}, e_{C_1}, e_{C_2}\}$ *where* C_0 *represents the zero-link type C player and* B_0*, the connected "manager". The* C_1 *represents the other two Type Cs connected to* B_0 *with* C_2 *and* B_2 *representing the remaining.*

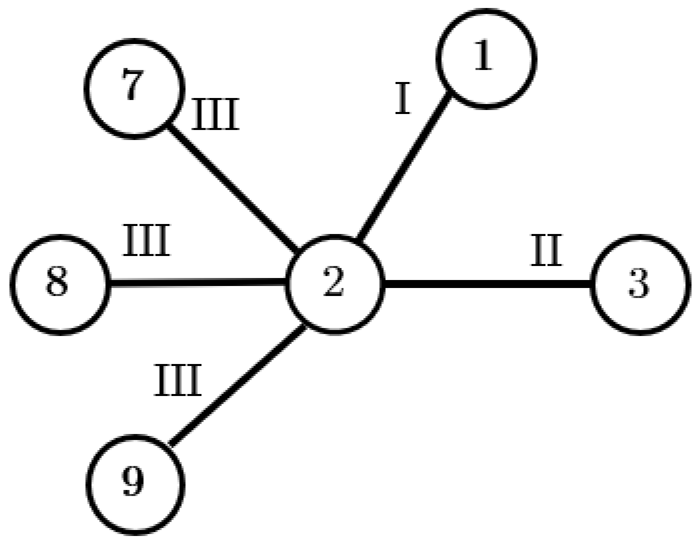


Figure 3.2. Link Types for Player 2

Table 3.3. Profit Values: Incomplete Networks

Missing Links	Adjacency Matrix Type		
	Symmetric		Asymmetric
	$\pi_{Iden} = \pi_{Diff}$	π_{Iden}	π_{Diff}
Complete	1.760	1.711	1.887
I	.9475	.808	.809
II	.890	.928	.999
III	1.003	1.167	1.273
I, II	.743	.653	.644
I, III	.767	.839	.896
II, III	.836	.908	.965
III, III	.856	1.038	1.123
I, II, III	.654	.606	.589
I, III, III	.710	.679	.661
II, III, III	.673	.762	.807
III, III, III	.743	.928	.995
I, II, III, III	.585	.565	.541
I, III, III, III	.631	.627	.601
II, III, III, III	.601	.697	.730
All Missing	.533	.530	.500
Profit with no arrangement	.625	.638	.667

Note: The Roman Numerals correspond to the links shown in Figure 3.2 If the number is listed that link is missing. There are 3 possible III links, and 1 of I and II. Profits correspond to a δ of 1.25.

each network above has a probability of forming that is equal to $P = p^{5-k}(1-p)^k$ where k is the number of missing links. Thus, the expected profit is a function of p. Summing across all possible networks we have:

$$E[\pi] = \sum_{k}^{5} \sum_{z} \pi_z p^{5-k}(1-p)^k - \pi_{Baseline}$$

where z is each possible network associated with k missing connections. If the expected total from the working from home environment exceeds the profit associated with the baseline network, then it is the employer's best interest to offer the arrangement.[51] Figure 3.3 plots profits in each network environment less the baseline scenario against the probability of maintaining each connection.   One can observe that the expected profit for each of the three scenarios differs based on the values of p. This results from the benefits derived from higher levels of connectivity. Unsurprisingly, when the network is asymmetric, and the employer can exploit the structure with differential incentives, the value of highly connected networks is increased. We can also observe that the benefits to highly connected networks are nonlinear and see increasing returns at higher levels of connectedness. Ultimately the value of p that is required for a flexible work arrangement to be profitable, in expectation, is $p = \{.239, .208, .263\}$. These values correspond to the symmetric, asymmetric identical wage, and asymmetric differential wage cases, respectively. Thus, the choice of whether to offer flexible work arrangement does depend on the compensation scheme. Explicitly, the resulting values of p above state that if the probability of maintaining connections is above 23.9% (20.8%, or 26.3%) then the working home arrangement for the manager will be profitable, in expectation.

As mentioned before, these values will depend on the parameter choices so the results should be interpreted with caution. However, in many cases the directional relationships will persist regardless of the magnitude of the change in underlying parameters. Further, this stylized example only deals with a single employee working from home. The example should be thought of as a demonstration of how the re-characterization of the popular network pricing model can be

[51] Provided she is risk neutral.

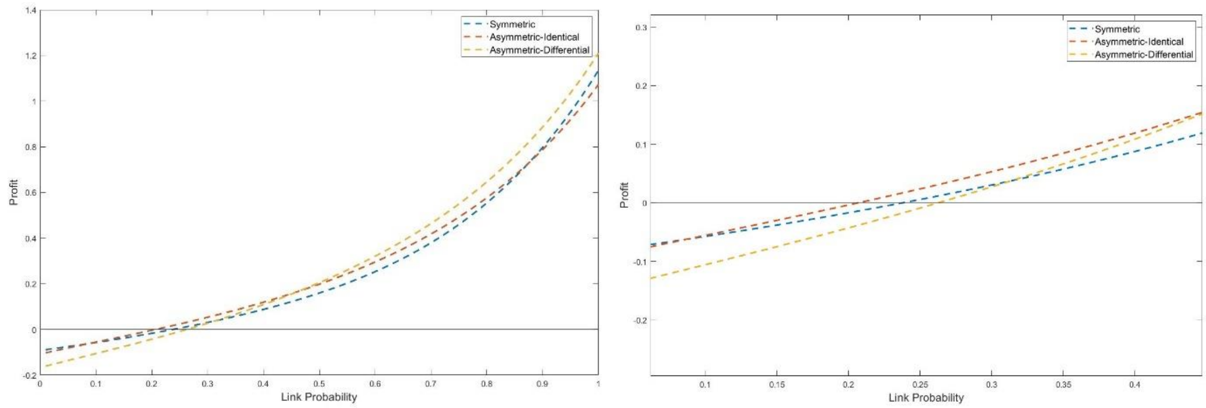


Figure 3.3. Expected Profit by Link Probability

used in the context of flexible work arrangements to provide insights in this novel setting. One of the biggest differences in between the pricing scenario and a flexible work setting is the control that an employer has over the composition of the network. The above example showed one such way that an employer may make an impact on the underlying network structure. Moreover, the application started with an established baseline network and then examined how an induced change in network structure (and parameter values) alters profits and creates a tradeoff that depends on the likelihood of a network remaining intact.

Another natural way to think of an employer interacting with the employee network is forming the network is from the ground up. If we continue to think of the network as representing complementarities in effort through productive interactions, an employer can choose the level of the investment in things such as an office space, holiday parties, communication technologies, etc. to enhance the formation of productive relationships among her workforce and create a more cohesive network. These investments come with nominal costs and conceivably there should be some level of investment that is optimal for her to undertake. We explore this scenario below by building on an additional stage to the presented model and provide an example of how this can be used.

V Investing in Network Formation

The analysis in the model in Section IV corresponds to a *single* materialized network. The employer may be able to exert control or invest in the network to influence the shape/connectivity of the network and yield potentially higher profits (depending on the cost to change the network). In Section IV, we characterized the firm and the employees' response for any given **fixed** network $\boldsymbol{g}$. We now model a prior stage in which the firm makes costly investments to influence the formation of $\boldsymbol{g}$ which potentially yields many different fixed networks (such as the example presented in section V). After this stage, the game proceeds as previously modeled, with the firm setting wages after observing the network, and workers then choosing effort. More explicitly we classify the stage in the following way:

Investment Stage The principal chooses investment level vector $\boldsymbol{x}$, with elements x_{ij}, in the feasible set of investment strategies $\boldsymbol{X}$, at marginal cost Q to maximize expected profits. The investment influences the probability of link formation in network $\boldsymbol{g}$ through function $p(\boldsymbol{x})$. The function $p(\boldsymbol{x})$ maps investment levels, x_{ij} to probabilities of the formation of link g_{ij}, i.e. p_{ij}. Given a resulting profit from stage 2, π, The employer solves the following problem:

$$\max_{\boldsymbol{x} \epsilon \boldsymbol{X}} E[\pi] - I\boldsymbol{x}$$

Clearly, if the employer could choose the network directly, they would choose the network that provides him the highest profit. This is easily verified as the complete network in which a link exists between each pair of workers. However, suppose connection formation to be probabilistic, rather than the deterministic. In other words, an employer may force a set of employees to work together, but whether their work becomes complementarity requires an overlapping of skills, personality traits, developing friendships/better working relationships, type of work etc. Relating this to flexible work, we assume that an employer can invest in technologies that enhance the likelihood that co-workers can discover and benefit from these complementarities. For example, providing work laptops with meeting software such as skype, company cell phones, paying for an office space (rent, electricity, etc.) where individuals can come to work, providing a gym at the office so employees can spend leisure hours together, financing company parties/get togethers etc. are taken to represent a gradient of spending that increases the likelihood of forming connections that lead to complementarities. Research has shown that a critical drawback from utilizing flexible work arrangements is that of decreased employee interactions and potentially isolation. This suggests a mechanism where the firm can foster connections between employees, which require costly investments.

Consider a setting with 3 players: A, B, C. There are several ways in which these players could interact. A with B only. A with B and C but no connection between B and C. All three players interact, etc. Given each of the network we can solve for an associated total effort level, and total profit that is given by equation (2) for each possible configuration (Figure 3.4 in the Example section below provides a visual representation). If these players share common parameter values or the connections strength between them are identical then some of the networks are functionally identical. For instance, if players A, B, and C all have identical parameters and connection strength then there is no difference in (total) profit or efforts if only A and B share a link as opposed to only B and C. The distributions amongst the players may differ, but from the principal's perspective, the differences are relevant. More generally, players can be classified according to their centrality measures, and any network with an identical centrality matrix will have equivalent profits.

It is important to note that this does not imply that networks with an equivalent *number* of links have equivalent profits. In fact, along the lines of other research, the shape of the network critically impacts on the associated profits. This is not, however, the focus of this paper. Taking the relationship between centrality and profit a step further, adding links to a network strictly increases profits when wages can be set differentially. This conclusion is drawn from Lemma 2:

Lemma 2: The weighted sum of the profits, π_m*, is strictly increasing in m;* $\pi_a > \pi_b\ for\ all\ a > b\ \ a, b \in \{1, M\}$

where *M* is the total number of possible connections[52] between *n* employees and *m* is the number of connections on a network $\boldsymbol{g}$. $m \in \{0, M\}$. Lemma 2 follows from the fact that for any network initial network by adding a connection to the network you weakly increase the centrality of all members of the network. When the centrality is increased, the employer can keep wages the same and receive higher output as increased centralities lowers effort cost through complementarities.[53] Thus, the employer can be no worse off from an increase in links. Further, any network with *m+1* connections can be built by adding a link to a network with m connections, therefore the weighted average must be larger.

We will use the idea of weighted profit to characterize the investment stage of the model. Define the function, π_m is defined by a weighted averaged of the profits of different networks $\boldsymbol{g}$ that have m connections, $\boldsymbol{g}_m$. Let there be s unique network configurations $\boldsymbol{g_m}$ with m connections that have associated Bonacich centrality vector $\boldsymbol{\beta}(\boldsymbol{g}, \lambda)$. Let t be the number of networks that have each configuration s. Let *T* be the total number of networks with connections m. Further π_{m_s} is the profit associated with the s configuration with m connections. Thus, the weighted profit associated with m connections is:

$$\pi_m = \frac{1}{T}\sum_{i=1}^{s} t * (\pi_{m_i})$$

[52] M is given by $= \binom{n}{2} = \frac{n(n-1)}{2}$

[53] It is important to note that this is not necessarily true in the presence of substitutability.

In terms of the model, we assume that the principal can invest in technology x at price I to influence the probability links forming. Any individual link g_{ij} has p probability of forming. Thus, a binomial distribution appropriately represents the probability of a network configuration forming with m links from n players.

$$P = p^m(1-p)^{\frac{(n)(n-1)}{2}-m}$$

Or,

$$P = p^m(1-p)^{M-m} \tag{7}$$

For example, in a 3-person network, the maximum number of links that can be formed is $\binom{n}{2} = 3$. The probability that a link forms is p, thus the probability that *each* link forms is $p * p * p = p^3$. The probability that no links form is $(1-p)^3$. The probability that a certain 1-link network configuration forms is $p^1(1-p)^2$ and so on. For our consideration, the probability of *any* 1-link network forming is:

$$\binom{3}{1} p^1(1-p)^2 = \frac{3!}{3-1!\,1!} p^1(1-p)^2 = 3p(1-p)^2$$

or more generally,

$$\binom{n}{m} p^m(1-p)^{\frac{(n)(n-1)}{2}-m}$$

For the future examples we use the logistic function, $P(x) = 1 - e^{-\beta x}$. This has S-shape characteristics such that the first investment in x provides small increases in probability of formation, then beings to increase convexly. After sufficient investment, the ability to translate investment into substantial increases in connectivity decrease. *E.g.* moving from no office to an office compared with already having an office space and undertaking team-building activities.

Putting these aspects together an employer can maximize expected profit, π_e. The firm pays cost Ix to influence the formation of networks. The profit resulting from the investment contains all possible networks G which form with probability, p_G and have given profit π_G. Thus, the equation can be expressed as:

$$\max_x \pi = \sum_{m=0}^{M} P_m(x)\pi_m - Ix$$

which can be rewritten as

$$\max_x \pi = \left(\sum_{m=0}^{M} \pi_m \binom{M}{m} p(x)^m (1 - p(x))^{M-m} \right) - Ix + \mu I \tag{8}$$

where π_m is the weighted profit function from above.

Solving yields,

$$0 = \sum_{m=1}^{M} (\pi_m - \pi_{(m-1)}) m \binom{M}{m} (p(-x)^{M-(m-1)} p(x)^m) - I \tag{9}$$

The summation term is the marginal benefit of increasing investment in x. The benefit is derived from the probabilistic increase of adding connections and increasing centrality. Thus, the change in probability from the term $(\pi_m - \pi_{(m-1)})$ is strictly positive. The rest of the terms $m \binom{M}{m} (p(-x)^{M-(m-1)} p(x)^m$ are strictly positive and, marginal cost is constant and equal to I. Thus, the *unique* optima of the objection function are implicitly defined by equation (5). Uniqueness and existence criteria are provided in the Appendix.

V.A Application of Investment in Network Formation

We will now put the third stage to work and provide a numerical example of the three-stage model to generate qualitative insights on how an employer sets wages and invests in connections among her work force. For simplicity, we restrict the analysis to a network of three employees. We again compare both a symmetric and asymmetric network. Further, we will show how the optimal total investment in network complementarities changes in response to changes in model parameters. In each case, we present both the results when the employer sets a homogenous wage and is free to set the wage individually.

For this application, the parameters of the model are set to $\phi = 2, a_i = a = 1, \delta_i = \delta = 3, I = .01$. This sets the marginal investment cost is set to .01. We first start with a symmetric network such that $\gamma_{ij} = \gamma_{ji} = 1$. The number of undirected networks that can be formed from any n nodes are equal to $2^{\frac{n(n-1)}{2}}$. In this example, the total number of possible networks is 8. Due to the symmetry assumptions and number of players in the example, the profit associated with a network with y number of connections is identical. Therefore, we have four representative networks shown in Figure 3.1.

The solutions to the optimal wage and effort choices given by Equations (2) and (4) represent the solution for *each* of the above networks. For demonstration, we show the results for Network 2 (N2) listed in Figure 3.4. In this case there are two connections in the network and Employee 2 functions as the center of the line. Due to the symmetry conditions, any other two-player network would yield identical results with different employees serving the central role.

When the employer is restricted to offering a homogenous wage, the optimal solutions is setting wage $\omega_{N2} = 1.5$. The effort vector is $e_{N2} = [e_1, e_2, e_3] \approx [.1029, 0.1176, 0.1029]$. With the linear production function, effort is directly converted to output and the optimal profit is the sum over the workers of the difference between the selling price, 2, and the wage multiplied by the amount of effort giving $\pi_{N2} \approx .1618$. From (5) we know that the optimal wages do not depend on the network structure when the adjacency networks are symmetric (*i.e.* $g_{ij} = g_{ji}$). This means that the optimal values are identical when wages can be set freely. Repeating this exercise for each of the possible networks results in a vector of profits across the possible network formations: $\pi = [\pi_{empty}, \pi_{N1}, \pi_{N2}, \pi_{N3}] = [.125, .142, .162, .188]$.

Incorporating the employer's investment choice in the probability of link(network) formation results in the explicit expected profit equation below:

$$E[\pi] = \pi_{empty}\big(1 - p(x)\big)^3 + 3\pi_{N1}\big(1 - p(x)\big)^2 p(x) + 3\pi_{N2}\big(1 - p(x)\big)p(x)^2 + \pi_{N3}p(x)^3 - Ix$$

The employer optimizes this function with respect to investment choice x. While increasing x increases the chance of more complete networks, and higher profits, it requires payment of marginal investment cost I. From this equation it is apparent that the larger the gap in profit between less connected and more connected networks critically defines the level of investment. This gap will depend on the underlying parameters in the model and the ability of the employer

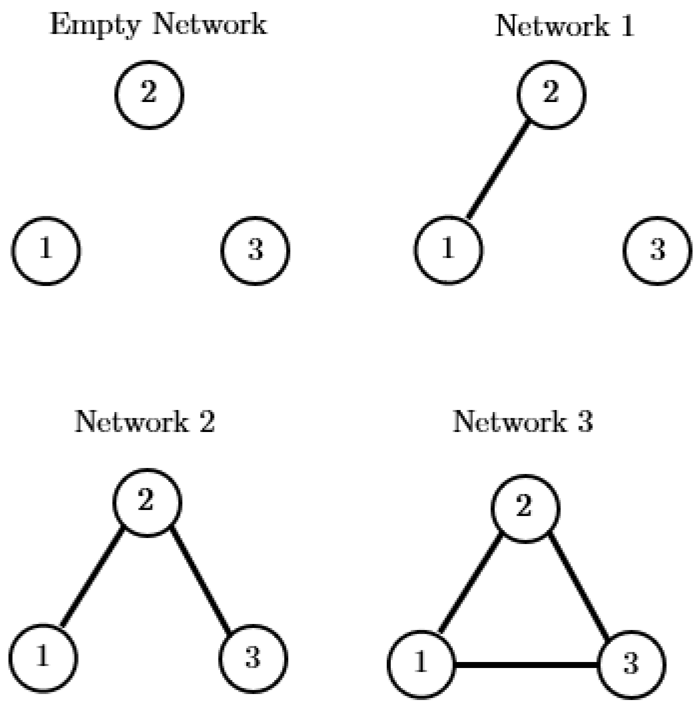


Figure 3.4. Possible Network Structures with 3 players

to exploit the network structure to increase profits. The above equation yields optimum investment choice, $x = 2.44$ corresponding to a probability of link formation $p(x) =$.705. The expected profit is .142 and total investment in network formation Ix=.024.

The total expenditure in this constitutes between 3.3-4.9% of the total revenue for the employer, depending on the resultant network structure and 14.16% of expected profits. While the expenditures can represent more than just office space rental these percentages fall within common suggested rent-to-revenue ratios of 2-20% depending on industry (*citation*). Clearly, the percentage of expenditures relative to revenue/profit depends heavily on the parameter values and network structure chosen in the model, but this type of analysis demonstrates one potential use of the model.

We now alter the example to examine how an asymmetric network affects the solution. Keeping the model parameters identical, we alter the adjacency matrix to be asymmetric such that the optimal freely set wage will vary between individuals based on network position. Let the adjacency matrix be the following:

$$G = \begin{bmatrix} 0 & 2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 2 & 0 \end{bmatrix}$$

In the case of a complete network. This G matrix describes a situation where Employee 2 has four times the impact on Employees 1 and 3, holding the sum of total influences $\sum_i \sum_j g_{ij}$ the same provides a reasonable comparison.

Optimal wages and efforts will now vary based on network position, in addition the profits associated with a network with y connections will now vary. Figure 3.5 displays the different possible networks with 2 connections and their associated profits. Note that Network 2B from Figure 3.5 is functionally identical to a network with a link between Employee 2 and 3 in lieu of Employee 2 and 1. Tables 1 and 2 present the solutions the model for each of the possible network types along with the optimal investment in link formation and the expected profits. From Tables 3.4 and 3.5 we can see optimal investment in the network is lower in the asymmetric network environments relative to the symmetric network environments. While this is sensitive to the choice of network connection strength, we can see that the average profit across the types of networks with the same number of connections are:

$$\pi_{sym} = [\pi_{empty}, \pi_{N1}, \pi_{N2}, \pi_{N3}] = [.125, .1417, .1618, .1875]$$

$$\pi_{asym_{single}} = [\pi_{empty}, \pi_{N1}, \pi_{N2}, \pi_{N3}] = [.125, .1410, .1595, .1815]$$

$$\pi_{asym_{free}} = [\pi_{empty}, \pi_{N1}, \pi_{N2}, \pi_{N3}] = [.125, .1421, .1627, .1883]$$

The profits confirm that the comparisons are quite similar. Moreover, it shows that when the network is asymmetric there is a larger penalty to using a single wage for the employer. While the differences are small in this example, due to modest changes in the model parameters, when the employer can freely set wages, there are gains to the employer from an asymmetric network. These gains translate into increased optimal investment in the formation of connections between employees. This is due to the ability of the employer to motivate more strongly the (relatively) influential employee and take advantage of the impact that the influential employee creates on others. In each of the asymmetric cases, there is a possibility for a lower realized profit than the symmetric case when the influential employee is not connected to the network. This leads us to important conclusion in the difference between being able to freely set wage and when the employer is restricted to a single wage, which can be seen by comparing Tables 3.4 and 3.5. For any given network, in the single wage case, the employer is unable to differentially motivate the influential employee, and this results in lower output from this employee and lower realized utility for this employee (displayed in Appendix). However, when the employer can freely set wages, this is reversed. The employer increases the wage of the influential employee relative to the others, and this results higher output and more utility.

Next, we examine how the optimal investment in network formation changes when model parameters vary. Unless otherwise noted, we retain the asymmetric network and parameter values used in the previous example. One natural question that can be answered withthe model, is how does the optimal total investment in network generating technologies change when the marginal investment cost changes?

Table 3.4. Optimal Values -Single Wage

Network	Wage ω	Effort Vector $e_i = [e_1, e_2, e_3]$	Profit π
$N0_{empty}$	1.5	[0.083,0.083,0.083]	0.125
$N1_{(1,2)}$	1.5	[0.114,0.093,0.083]	0.145
$N1_{(1,3)}$	1.5	[0.091,0.083,0.091]	0.133
$N1_{(2,3)}$	1.5	[0.083,0.093,0.114]	0.145
$N2_{(1,2),(1,3)}$	1.5	[0.122,0.094,0.094]	0.155
$N2_{(1,2),(2,3)}$	1.5	[0.118,0.103,0.118]	0.169
$N2_{(1,3),(2,3)}$	1.5	[0.094,0.094,0.122]	0.155
$N3_{Complete}$	1.5	[0.129,0.105,0.129]	0.181
Investment - x			2.169
Probability - $p(x)$			0.662
Expenditure - Ix			0.022
Expected Profit - $E[\pi]$			0.138
Expenditure % of Revenue			2.9-4.3%

Note: Listed numbers in the parentheses in the network column are the links present between each player type listed in Figure 3.4. Expenditure percentage revenue is calculated based on the expenditure divided by the range of realized revenues across each network type.

Table 3.5. Optimal Values-Freely Set Wages

Network	Wage $\omega_i = [\omega_1, \omega_2, \omega_3]$	Effort Vector $e_i = [e_1, e_2, e_3]$	Profit π
$N0_{empty}$	[1.500,1.500,1.500]	[0.083,0.083,0.083]	0.125
$N1_{(1,2)}$	[1.421,1.579,1.500]	[0.105,0.105,0.083]	0.147
$N1_{(1,3)}$	[1.500,1.500,1.500]	[0.091,0.083,0.091]	0.133
$N1_{(2,3)}$	[1.500,1.579,1.421]	[0.083,0.105,0.105]	0.147
$N2_{(1,2),(1,3)}$	[1.420,1.585,1.500]	[0.113,0.107,0.093]	0.157
$N2_{(1,2),(2,3)}$	[1.403,1.665,1.403]	[0.11,0.129,0.11]	0.175
$N2_{(1,3),(2,3)}$	[1.500,1.585,1.420]	[0.093,0.107,0.113]	0.157
$N3_{Complete}$	[1.400,1.682,1.400]	[0.121,0.134,0.121]	0.188
Investment - x			2.458
Probability - $p(x)$			0.707
Expenditure - Ix			0.025
Expected Profit - $E[\pi]$			0.141
Expenditure % of Revenue			3.3-4.9%

Note: Listed numbers in the parentheses in the network column are the links present between each player type listed in Figure 3.4. Expenditure percentage revenue is calculated based on the expenditure divided by the range of realized revenues across each network type.

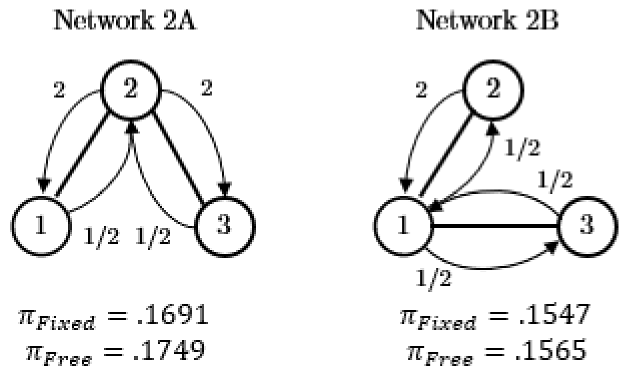


Figure 3.5. Possible Types of 2-Link Networks-Asymmetric Networks

This scenario represents a potential response to a change in network building and communication technologies such as e-mail or Zoom. Technologies that foster connections between employees at lower costs should generate higher complementarities among those employees at lower costs and result in increased connectivity, but the effect on total investment in unclear. It may be the case the decreased cost of investment is completely offset by the increase in investment. This is similar to the relationship between a monopolist's pricing decision and elasticity of demand. Figure 3.6 shows how the total expenditure in network forming activities change.

The figure shows a distinctly nonlinear relationship. Shown in the Appendix is that the probability of link formation decreases, at an increasing rate, as marginal investment cost rises. Thus, we can see that when marginal investment costs it is relatively inexpensive for the employer to achieve high levels of connectivity. As the investment cost rises, the employer seeks to increase expenditure to maintain high levels of connectedness. As the cost continues to rise, expenditure eventually drops. This is caused by the conversion of investment, x, into probability of link formation $p(x)$, which can be recalled as $1 - e^{-\beta x}, \beta = .5$. However the decreasing rate of change in investment expenditure remains. More figures are displayed in the Appendix with varied β.

Naturally, the choice of investment in network building depends on a multitude of factors. The example presented in this section and Section V hopefully provides an illustration of the value of applying a network model to thinking about employer and employee relationships when it comes to telecommuting arrangements.

VI Conclusion

The model presented above highlights the intricate tie between the firm and its' employees when it comes to deciding the structure of the work arrangements, in terms of flexibility and telecommuting, that has been relatively under researched. While there has been ample research showing the benefits (sometimes costs) of telecommuting on the employees, the importance of studying how those arrangements affect the firm has not been studied. Drawing on framework of network models of interactions we provide a new tool to study the explicit

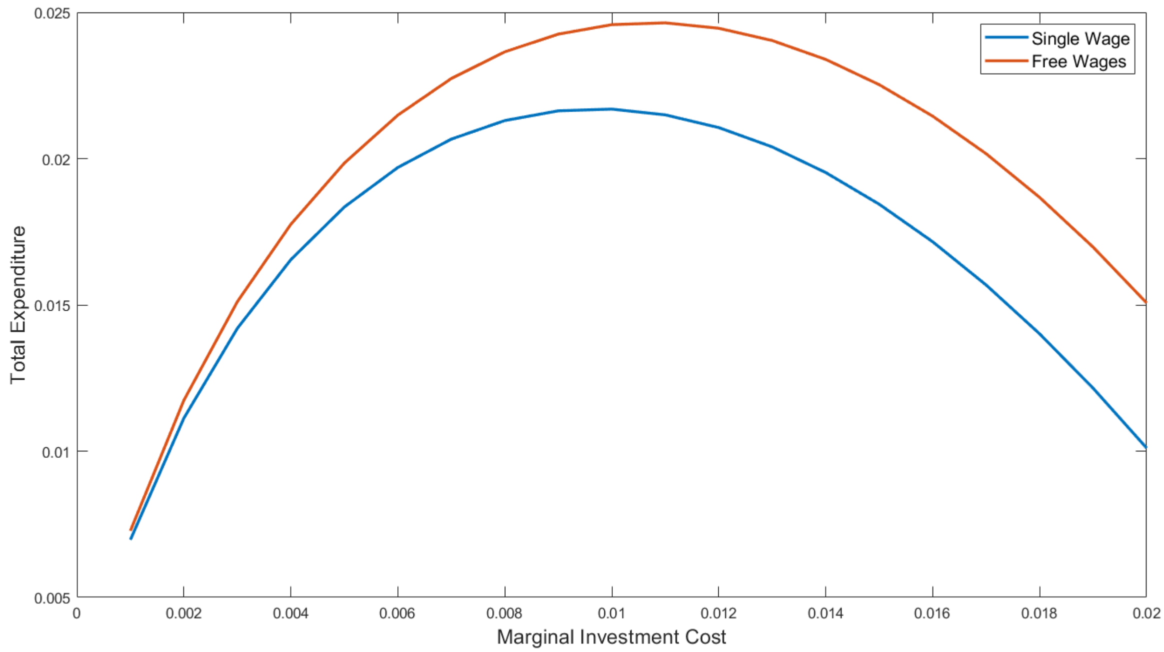


Figure 3.6. Optimal Investment Expenditure

effects of networks of complementarities among employees in a firm and how a firm may invest to influence that network of complementarities.

By highlighting the ability of the firm to influence the network of their employees we can show how increased investment expenditure in network formation technologies can improve profitability for firms. We show that it is more advantageous for a firm to invest in network building technologies when the network is asymmetric, and they can freely set wages to differentially motivate individual workers. Further, we showed that the change in optimal investment in these technologies as advances are made is nonlinear and depend on the underlying characteristics of the model. Finally, we showed that profitability is gained by allowing some workers to telecommute if their effort costs are significantly reduced but only when there remains a sizable core of connected employees.

One drawback of this study is the inability to generate analytical results for the investment stage of the model, thus limiting the generalizability of the results. It may be the case that there are alternative objective functions for this stage that generate more generalizable results and is a potential avenue for future research. The investment stage also relates to the pricing problem framed by the original model as a firm can potentially alter the connectivity of the consumption network through practices such as advertising.

The above insights are primarily drawn from considering how the firm makes decisions regarding offering alternative working arrangements. This appears to be of critical importance when studying telecommuting and flexible work arrangements. The connection between this the presented theoretical model and telecommuting behaviors can provide a useful tool when considering problems in this area of research. One may also consider applying the model empirically. The most difficult challenge is finding a suitable representation of the network of complementarities. Recent research on the use of Enterprise Social Networks may yield such a dataset. Careful consideration should be given in regard the network of interactions, information/knowledge transfer, institutional structure, and other important organizational details and how they translate to complementarities in production among employees. The network data could then be married with more easily observed wage or productivity data to test the implications of the model. Hopefully, this paper will provide a first step toward the integrating of principal-agent modelling in telecommuting and flexible work research.

Bibliography

Albrecht, Felix, Sebastian Kube, and Christian Traxler. "Cooperation and norm enforcement-The individual-level perspective." *Journal of Public Economics* 165 (2018): 1-16.

Allen, Tammy D., Timothy D. Golden, and Kristen M. Shockley. "How effective is telecommuting? Assessing the status of our scientific findings." *Psychological Science in the Public Interest* 16.2 (2015): 40-68.

Allouch, Nizar. "On the private provision of public goods on networks." *Journal of Economic Theory* 157 (2015): 527-552.

Andreoni, James, and Laura K. Gee. "Gun for hire: Delegated enforcement and peer punishment in public goods provision." *Journal of Public Economics* 96.11-12 (2012): 1036-1046.

Andreoni, James, and Laura K. Gee. "Gunning for efficiency with third party enforcement in threshold public goods." *Experimental Economics* 18.1 (2015): 154-171.

Arifovic, Jasmina, and John Ledyard. "Individual evolutionary learning, other-regarding preferences, and the voluntary contributions mechanism." *Journal of Public Economics* 96.9-10 (2012): 808-823.

Arrow, Kenneth J. (1974), The Limits of Organization. Norton, New York. [650, 654]

Baik, Kyung Hwan, Jong Hwa Lee, and Seokho Lee. "Endogenous Timing in Three-Player Tullock Contests." *Working Paper*

Bailey, D. E., & Kurland, N. B. (2002). A review of telework research: Findings, new directions, and lessons for the study of modern work. *Journal of organizational behavior*, *23*(4), 383-400.

Ballester, Coralio, Antoni Calvó-Armengol, and Yves Zenou. "Who's who in networks. Wanted: The key player." *Econometrica* 74.5 (2006): 1403-1417.

Ballester, Coralio, Yves Zenou, and Antoni Calvó-Armengol. "Delinquent networks." *Journal of the European Economic Association* 8.1 (2010): 34-61.

Bendor, Jonathan, and Piotr Swistak. "The evolution of norms." *American Journal of Sociology* 106.6 (2001): 1493-1545.

Block, Per, and Thomas Grund. "Multidimensional homophily in friendship networks." *Network science (Cambridge University Press)* 2.2 (2014): 189.

Bloom, N., Liang, J., Roberts, J., & Ying, Z. J. (2014). Does working from home work? Evidence from a Chinese experiment. *The Quarterly Journal of Economics*, *130*(1), 165-218.

Bolton, Gary E., and Axel Ockenfels. "ERC: A theory of equity, reciprocity, and competition." *American Economic Review* 90.1 (2000): 166-193.

Boosey, Luke, and R. Mark Isaac. "Asymmetric network monitoring and punishment in public goods experiments." *Journal of Economic Behavior & Organization* 132 (2016): 26-41.

Bramoullé, Yann, and Rachel Kranton. "Public goods in networks." *Journal of Economic Theory* 135.1 (2007): 478-494.

Bramoullé, Yann, Rachel Kranton, and Martin D'amours. "Strategic interaction and networks." *American Economic Review* 104.3 (2014): 898-930.

Brandts, Jordi, and Gary Charness. "The strategy versus the direct-response method: a first survey of experimental comparisons." *Experimental Economics* 14.3 (2011): 375-398.

Brekke, Kjell Arne, et al. "Playing with the good guys. A public good game with endogenous group formation." *Journal of Public Economics* 95.9-10 (2011): 1111-1118.

Bresnahan, Timothy F., and Peter C. Reiss. "Entry in monopoly market." *The Review of Economic Studies* 57.4 (1990): 531-553.

Bulow, Jeremy, and Paul Klemperer. "Why do sellers (usually) prefer auctions?." *American Economic Review* 99.4 (2009): 1544-75.

Calvó-Armengol, Antoni, Joan De Martí, and Andrea Prat. "Communication and influence." *Theoretical Economics* 10.2 (2015): 649-690.

Candogan, Ozan, Kostas Bimpikis, and Asuman Ozdaglar. "Optimal pricing in networks with externalities." *Operations Research* 60.4 (2012): 883-905.

Carpenter, Jeffrey, and Peter Hans Matthews. "What norms trigger punishment?." *Experimental Economics* 12.3 (2009): 272-288.

Carpenter, Jeffrey, Shachar Kariv, and Andrew Schotter. "Network architecture, cooperation and punishment in public good experiments." *Review of Economic Design* 16.2-3 (2012): 93-118.

Charness, Gary, and Matthew Rabin. "Understanding social preferences with simple tests." *The Quarterly Journal of Economics* 117.3 (2002): 817-869.

Chaudhuri, Ananish. "Sustaining cooperation in laboratory public goods experiments: a selective survey of the literature." *Experimental economics* 14.1 (2011): 47-83.

Chen, M. Keith, et al. "The value of flexible work: Evidence from uber drivers." *Journal of political economy* 127.6 (2019): 2735-2794.

Chen, Ying-Ju, Yves Zenou, and Junjie Zhou. "Competitive pricing strategies in social networks." *The RAND Journal of Economics* 49.3 (2018): 672-705.

Chowdhury, Subhasish M., and Roman M. Sheremeta. "A generalized Tullock contest." *Public Choice* 147.3 (2011): 413-420.

Cinyabuguma, Matthias, Talbot Page, and Louis Putterman. "Cooperation under the threat of expulsion in a public goods experiment." *Journal of Public Economics* 89.8 (2005): 1421-1435.

Coase, Ronald Harry. "The nature of the firm." *economica* 4.16 (1937): 386-405.

Coenen, M., & Kok, R. A. (2014). Workplace flexibility and new product development performance: The Council of Economic Advisors (2010), 'Work-life Balance and the Economics of Workplace Flexibility'

Currarini, Sergio, and Friederike Mengel. "Identity, homophily and in-group bias." *European Economic Review* 90 (2016): 40-55.

De Geest, Lawrence R., John K. Stranlund, and John M. Spraggon. "Deterring poaching of a common pool resource." *Journal of Economic Behavior & Organization* 141 (2017): 254-276.

Deacon, Robert T. Fishery Management by Harvester Cooperatives, Review of Environmental Economics and Policy, Volume 6, Issue 2, Summer 2012, Pages 258-277 https://doi.org/10.1093/reep/res008

DeAngelo, Gregory, and Laura K. Gee. "Peers or police?: The effect of choice and type of monitoring in the provision of public goods." *Games and Economic Behavior* 123 (2020): 210-227.

Dechenaux, Emmanuel, Dan Kovenock, and Roman M. Sheremeta. 2015. "A survey of experimental research on contests, all-pay auctions and tournaments." *Experimental Economics*, 18(4): 609-669.

Dixit, A. (1987). "Strategic behavior in contests." *The American Economic Review* 77 (5), 891–898.

Duch, M. L., Grossmann, M. R. P. and Lauer, T. (2020). "z-Tree unleashed: A novel client-integrating architecture for conducting z-Tree experiments over the Internet". Journal of Behavioral and Experimental Finance, Volume 28, December 2020, Article 100400.

Eckel, Catherine C., Enrique Fatás, and Rick Wilson. "Cooperation and status in organizations." *Journal of Public Economic Theory* 12.4 (2010): 737-762.

Elliott, Matthew, and Benjamin Golub. "A network approach to public goods." *Journal of Political Economy* 127.2 (2019): 730-776

Fatás, Enrique, et al. "Public goods and decay in networks." *SERIEs* 6.1 (2015): 73-90.

Fatás, Enrique, Miguel A. Meléndez-Jiménez, and Hector Solaz. "Social hierarchies: a laboratory study on punishment patterns across Networks." *Economic Inquiry* 58.1 (2020): 104-119.

Fehr, Ernst, and Klaus M. Schmidt. "A theory of fairness, competition, and cooperation." *The Quarterly Journal of Economics* 114.3 (1999): 817-868.

Fehr, Ernst, and Simon Gächter. "Cooperation and punishment in public goods experiments." *American Economic Review* 90.4 (2000): 980-994.

Fehr, Ernst, and Urs Fischbacher. "Third-party punishment and social norms." *Evolution and Human Behavior* 25.2 (2004): 63-87.

Fischbacher, U., Gächter, S., & Quercia, S. (2012). The behavioral validity of the strategy method in public good experiments. *Journal of Economic Psychology*, *33*(4), 897-913.

Fischbacher, Urs, Simon Gächter, and Ernst Fehr. "Are people conditionally cooperative? Evidence from a public goods experiment." *Economics Letters* 71.3 (2001): 397- 404.

Fischbacher, Urs. "z-Tree: Zurich toolbox for ready-made economic experiments." *Experimental Economics* 10.2 (2007): 171-178.

Fischbacher, Urs. "z-Tree: Zurich toolbox for ready-made economic experiments." *Experimental Economics* 10.2 (2007): 171-178.

Gächter, Simon, and Christian Thöni. "Social learning and voluntary cooperation among like-minded people." *Journal of the European Economic Association* 3.2-3 (2005): 303-314.

Gajendran, R. S., & Harrison, D. A. (2007). The good, the bad, and the unknown about telecommuting: meta-analysis of psychological mediators and individual consequences.

Gajendran, R. S., Harrison, D. A., & Delaney-Klinger, K. (2015). Are telecommuters remotely good citizens? Unpacking telecommuting's effects on performance via i-deals and job resources. *Personnel Psychology*, *68*(2), 353-393.

Gao, G., & Hitt, L. (2003). The economics of telecommuting: Theory and evidence. *ICIS 2003 Proceedings*, 54.

Glass, J. (2004). Blessing or curse? Work-family policies and mother's wage growth over time. *Work and Occupations*, *31*(3), 367-394.

Glazer, A. and R. Hassin (2000). "Sequential rent seeking." *Public Choice* 102 (3-4), 219– 228.

Golden, T. (2007). Co-workers who telework and the impact on those in the office: Understanding the implications of virtual work for co-worker satisfaction and turnover intentions. *Human Relations*, *60*(11), 1641-1667.

Golden, T. D. (2006). The role of relationships in understanding telecommuter satisfaction. *Journal of Organizational Behavior*, *27*(3), 319-340.

Golden, T. D., & Veiga, J. F. (2005). The impact of extent of telecommuting on job satisfaction: Resolving inconsistent findings. *Journal of Management*, *31*(2), 301-318.

Granovetter, Mark. "Threshold models of collective behavior." *American journal of sociology* 83.6 (1978): 1420-1443.

Gürerk, Özgür, Bernd Irlenbusch, and Bettina Rockenbach. "On cooperation in open communities." *Journal of Public Economics* 120 (2014): 220-230.

Hanges, P. J., & Wang, M. (2012). Seeking the Holy Grail in organizational science: Uncovering causality through research design.

Hinnosaar, Toomas. "Optimal sequential contests." *Available at SSRN 3119014* (2018).

Holmstrom, Bengt, and Paul Milgrom. "Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design." *Journal of Law, Economics, & Organization* 7 (1991): 24-52.

Holt, Charles A., and Susan K. Laury. "Risk aversion and incentive effects." *American Economic Review* 92.5 (2002): 1644-1655.

Holt, Charles A., and Susan K. Laury. "Risk aversion and incentive effects." *American economic Review* 92.5 (2002): 1644-1655.

Hunton, J. E. (2005). Behavioral Self-Regulation of Telework Locations: Interrupting Interruptions!(Retracted). *Journal of Information Systems*, *19*(2), 111-140.

Ino, H. and T. Matsumura (2012, November). “How many firms should be leaders? Beneficial concentration revisited.” *International Economic Review* 53 (4), 1323–1340.

Jackson, Matthew O. *Social and economic networks*. Princeton university press, 2010.

Jackson, Matthew O., and Asher Wolinsky. "A strategic model of social and economic networks." *Journal of Economic Theory* 71.1 (1996): 44-74.

Jackson, Matthew O., Brian W. Rogers, and Yves Zenou. "The economic consequences of social-network structure." *Journal of Economic Literature* 55.1 (2017): 49-95.

Jensen, M. K. (2010). “Aggregative games and best-reply potentials.” *Economic Theory* 43 (1), 45–66.

Jensen, Martin Kaae. "Aggregative games." *Handbook of Game Theory and Industrial Organization, Volume I*. Edward Elgar Publishing, 2018.

Julien, L. A., O. Musy, and A. W. Saïdi (2012). “On hierarchical competition in oligopoly.” *Journal of Economics* 107 (3), 217–237.

Kahana, Nava, and Doron Klunover. "Sequential lottery contests with multiple participants." *Economics Letters* 163 (2018): 126-129.

Kinateder, Markus, and Luca Paolo Merlino. "Public goods in endogenous networks." *American Economic Journal: Microeconomics* 9.3 (2017): 187-212.

Kopányi-Peuker, Anita, Theo Offerman, and Randolph Sloof. "Fostering cooperation through the enhancement of own vulnerability." *Games and Economic Behavior* (2015).

Kosfeld, Michael, Akira Okada, and Arno Riedl. "Institution formation in public goods games." *American Economic Review* 99.4 (2009): 1335-55.

Kurland, N. B., & Egan, T. D. (1999). Telecommuting: Justice and control in the virtual organization. *Organization Science*, *10*(4), 500-513.

Linster, Bruce G. "Stackelberg rent-seeking." *Public Choice* 77.2 (1993): 307-321.

Le Maire, Daniel, and Bertel Schjerning. "Tax bunching, income shifting and self-employment." *Journal of Public Economics* 107 (2013): 1-18.

Leibbrandt, A., Ramalingam, A., Sääksvuori, L., & Walker, J. M. (2015). Incomplete punishment networks in public goods games: experimental evidence. *Experimental Economics*, *18*(1), 15-37.

Leininger, W. (1993). More efficient rent-seeking – A Münchhausen solution. *Public Choice*

MacDonald, James M. "Entry and exit on the competitive fringe." *Southern Economic Journal* (1986): 640-652.

Mago, Shakun D., and Roman M. Sheremeta. "New Hampshire Effect: behavior in sequential and simultaneous multi-battle contests." *Experimental Economics* 22.2 (2019): 325-349.

Markussen, Thomas, Louis Putterman, and Jean-Robert Tyran. "Self-organization for collective action: An experimental study of voting on sanction regimes." *Review of Economic Studies* 81.1 (2014): 301-324.

Martínez Sánchez, A., Pérez, M., de Luis Carnicer, P., & José Vela Jiménez, M. (2007). Teleworking and workplace flexibility: a study of impact on firm performance. *Personnel Review*, *36*(1), 42-64.

Meyer, C. S., Mukerjee, S., & Sestero, A. (2001). Work-family benefits: which ones maximize profits?. *Journal of managerial Issues*, 28-44.

Michaeli, Moti, and Daniel Spiro. "Norm conformity across societies." *Journal of public economics* 132 (2015): 51-65.

Morgan, John. "Sequential contests." *Public choice* 116.1-2 (2003): 1-18.

Morgan, S. J., & Symon, G. (2002). Computer-mediated communication and remote management: integration or isolation?. *Social Science Computer Review*, *20*(3), 302-311.

Nelson, Arthur B. 2019. “Deterrence in sequential contests: An experimental study.” *Working paper*, Florida State University.

Nelson, Arthur, and Dmitry Ryvkin. *Contests with sequential moves: An experimental study*. No. wp2019_11_01. 2019.

Nicklisch, Andreas, Kristoffel Grechenig, and Christian Thöni. "Information-sensitive leviathans." *Journal of Public Economics* 144 (2016): 1-13.

Nikiforakis, Nikos. "Punishment and counter-punishment in public good games: Can we really govern ourselves?." *Journal of Public Economics* 92.1-2 (2008): 91-112.

Nikiforakis, Nikos, Charles N. Noussair, and Tom Wilkening. "Normative conflict and feuds: The limits of self-enforcement." *Journal of Public Economics* 96.9-10 (2012): 797-807.

O'Connor, B. (2021, June 22). *The workers pushing back on the return to the office*. BBCWorklife. https://www.bbc.com/worklife/article/20210618-the-workers-pushing-back-on-the-return-to-the-office.

Ones, Umut, and Louis Putterman. "The ecology of collective action: A public goods and sanctions experiment with controlled group formation." *Journal of Economic Behavior & Organization* 62.4 (2007): 495-521.

Ostrom, Elinor, James Walker, and Roy Gardner. "Covenants with and without a sword: Self-governance is possible." *The American Political Science Review* (1992): 404-417.

Patacchini, Eleonora, and Yves Zenou. "Juvenile delinquency and conformism." *The Journal of Law, Economics, & Organization* 28.1 (2012): 1-31.

Price, Curtis R., and Roman M. Sheremeta. "Endowment effects in contests." *Economics Letters* 111.3 (2011): 217-219.

Putterman, Louis, Jean-Robert Tyran, and Kenju Kamei. "Public goods and voting on formal sanction schemes." *Journal of Public Economics* 95.9-10 (2011): 1213-1222.

Ramalingam, A., Godoy, S., Morales, A. J., & Walker, J. M. (2016). An individualistic approach to institution formation in public good games. *Journal of Economic Behavior & Organization*, *129*, 18-36.

Schlager, Edella, and Elinor Ostrom. "Property-rights regimes and natural resources: a conceptual analysis." *Land Economics* (1992): 249-262.

Schmidt, E., & Rosenberg, J. (2014). *How google works*. Hachette UK.

Sheremeta, Roman M. "Experimental comparison of multi-stage and one-stage contests." *Games and Economic Behavior* 68.2 (2010): 731-747.

Sheremeta, Roman M. "Overbidding and heterogeneous behavior in contest experiments." *Journal of Economic Surveys* 27.3 (2013): 491-514.

Shreedhar, Ganga, Alessandro Tavoni, and Carmen Marchiori. "Monitoring and punishment networks in an experimental common pool resource dilemma." *Environment and Development Economics* 25.1 (2020): 66-94.

Sutter, Matthias, Stefan Haigner, and Martin G. Kocher. "Choosing the carrot or the stick? Endogenous institutional choice in social dilemma situations." *The Review of Economic Studies* 77.4 (2010): 1540-1566.

Takács, Károly, Béla Janky, and Andreas Flache. "Collective action and network change." *Social Networks* 30.3 (2008): 177-189.

Taskin, L., & Bridoux, F. (2010). Telework: a challenge to knowledge transfer in organizations. *The International Journal of Human Resource Management*, *21*(13), 2503-2520.

Tullock, G. (1967, June). The welfare costs of tariffs, monopolies, and theft. *Economic Inquiry* 5 (3), 224–232.

Tullock, G. (1980). Efficient rent-seeking. In J.M. Buchanan et al. (Eds.), *Toward a Theory of the Rent-Seeking Society*, College Station: Texas A&M Press.75: 433–462.

van Leeuwen, Boris, et al. "Centrality and cooperation in networks." *Experimental Economics* 22.1 (2019): 178-196.

Appendices

A Appendix-Chapter 1

Tables A.1.1-5 report pairwise Wald test of proportions between treatments. Distributions are generated by a pooled OLS regression of each variable on treatment indicators. Observations are at the group level in each period. Standard errors are clustered the group level. Tables A1.1.6-10 report pairwise χ^2 test of proportions between treatment. Distributions are generated by a pooled OLS regression with boostrapped standard errors. Observations are again at the group level by period. Standard errors are clustered at the group level.

Table A.1.12 provides the corresponding random effects regression to Table 1.4. There are some econometric concerns with using lagged dependent variables in the random effects model. Specifically, there are potential violations of the assumption of no cross-temporaneous correlations between the covariates and the error terms. Further, due to a small number of observations in certain network sizes and treatments which does not permit adequate sample sizes for bootstrapping, thus due to the small number of clusters, there is potential misspecification with the random effects model.

Table A.11 provides the corresponding fixed effects regression to Table 1.4. There are some econometric concerns with using lagged dependent variables in the random effects model. Specifically, there are potential violations of the assumption of no cross-temporaneous correlations among the covariates in the error terms. However, the fixed effects model does not allow use of the type variables defined by the pre-experiment preference elicitation. Table A.12 provides an alternative specification with network size indicators.

Table A.13 provides an alternative specification to Table 1.5 with the removal of the variable derived from the pre-experiment Ultimatum Game preference elicitation and using a fixed effect panel regression. Tables A.14-A.16 provides additional alternative specifications. Table A.1.14 provides an alternative specification to Table 1.6 with the inclusion of indicators for the relationship between the proposer and proposee. For example, *FreeCond* indicates that the proposer is classified as a Free Rider and the proposee is the Conditional Cooperator. Some additional specifications for Table 1.7 are also provided along with regressions for link proposals as opposed to established links. Reported results are the coefficients in a probit model, marginal effects are not shown. Figure A.1 shows the two missing treatments from Figure 1.2. Figures A.2

and A.3 correspond with Table 1.4. Figure A.2 shows the max connections and max contributions for each group. Figure A.3 shows the group contributions and connections in the final period. Figures A.4 and A.5 show the proportions for each treatment type corresponding to Figure 1.3.

Table A.1. Group Contributions by Treatment: Wald Test

	Complete	*LB-0*	*LB-2*	*LB-4*
Empty	4.69**	0.24	0.66	0.68
	(.035)	(.625)	(.422)	(.413)
Complete		6.01**	0.58	0.50
		(.018)	(.451)	(.481)
LB-0			1.30	1.32
			(.261)	(.257)
LB-2				0.00
				(.977)

*Note: Reported statistics are F-test values for Wald Tests comparison of group averages between treatments. Test statistics are generated from a pooled OLS regression clustered with standard errors at the group level where each group is observed over 20 periods. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.2. Group Punishment by Treatment: Wald Test

	Complete	*LB-0*	*LB-2*	*LB-4*
Empty	14.40***	10.27***	7.79***	13.43***
	(.000)	(.002)	(.008)	(.000)
Complete		11.47***	3.94*	3.42*
		(.001)	(.053)	(.071)
LB-0			4.18**	7.69***
			(.047)	(.008)
LB-2				0.09
				(.768)

*Note: Reported statistics are F-test values for Wald Tests comparison of group averages between treatments. Test statistics are generated from a pooled OLS regression with standard errors clustered at the group level where each group is observed over 20 periods. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.3. Links Formed in Each Group by Treatment: Wald Test

	Complete	*LB-0*	*LB-2*	*LB-4*
Empty	2.20e32***	4.30**	27.94***	81.59***
	(.000)	(.044)	(.000)	(.000)
Complete		278.58***	24.40***	7.33***
		(.000)	(.000)	(.009)
LB-0			13.31***	43.00***
			(.001)	(.000)
LB-2				3.79**
				(.058)

*Note: Reported statistics are F-test values for Wald Tests comparison of group averages between treatments. Test statistics are generated from a pooled OLS regression with standard errors clustered at the group level where each group is observed over 20 periods. Empty and Complete treatments have fixed values of 0 and 6 connections, respectively. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.4. Group Efficiency by Treatment: Wald Test

	Complete	*LB-0*	*LB-2*	*LB-4*
Empty	3.19*	0.95	0.42	0.59
	(.080)	(.333)	(.520)	(.446)
Complete		1.35	0.91	0.68*
		(.251)	(.345)	(.415)
LB-0			0.00	0.02
			(.988)	(.877)
LB-2				0.01
				(.908)

*Note: Reported statistics are F-test values for Wald Tests comparison of group averages between treatments. Test statistics are generated from a pooled OLS regression with standard errors clustered at the group level where each group is observed over 20 periods. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.5. Link Proposals in Each Group by Treatment: Wald Test

	Complete	*LB-0*	*LB-2*	*LB-4*
Empty	1.82e30***	24.62***	69.92***	192.07***
	(.000)	(.000)	(.000)	(.000)
Complete		138.64***	17.75***	5.94**
		(.000)	(.000)	(.019)
LB-0			13.72***	41.84***
			(.000)	(.000)
LB-2				3.41*
				(.071)

*Note: Reported statistics are F-test values for Wald Tests comparison of group averages between treatments. Test statistics are generated from a pooled OLS regression with standard errors clustered at the group level where each group is observed over 20 periods. Empty and Complete treatments have fixed values of 0 and 12 link proposals, respectively p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.6. Group Contribution Variance by Treatment: χ^2 Test

	Complete	*LB-0*	*LB-2*	*LB-4*
Empty	0.21	0.67	2.75*	4.67**
	(.644)	(.415)	(.097)	(.031)
Complete		0.03	1.85	2.86*
		(.859)	(.174)	(.091)
LB-0			1.77	2.90*
			(.183)	(.089)
LB-2				0.00
				(.989)

*Note: Reported statistics are pairwise test statistic values for chi-squared tests of equivalent proportions of group averages between treatments. Test statistics are generated from a pooled OLS regression with standard errors clustered at the group level using bootstrapped standard errors where each group is observed over 20 periods. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.7. Group Punishment Variance by Treatment: χ^2Test

	Complete	*LB-0*	*LB-2*	*LB-4*
Empty	5.68**	13.69***	5.63**	15.67***
	(.017)	(.000)	(.018)	(.000)
Complete		4.91**	2.14*	1.53*
		(.027)	(.143)	(.216)
LB-0			3.54*	10.93***
			(.060)	(.000)
LB-2				0.34
				(.559)

*Note: Reported statistics are pairwise test statistic values for chi-squared tests of equivalent proportions of group averages between treatments. Test statistics are generated from a pooled OLS regression with standard errors clustered at the group level using bootstrapped standard errors where each group is observed over 20 periods. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.8. Links Formed Variance by Treatment: χ^2Test

	LB-0	*LB-2*	*LB-4*
Empty	4.68**	21.37***	4.65**
	(.031)	(.000)	(.031)
LB-0		7.39***	1.69
		(.005)	(.194)
LB-2			0.26
			(.611)

*Note: Reported statistics are pairwise test statistic values for chi-squared tests of equivalent proportions of group averages between treatments. Test statistics are generated from a pooled OLS regression with standard errors clustered at the group level using bootstrapped standard errors where each group is observed over 20 periods. Empty treatment is set to 0 variance. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.9. Group Efficiency Variance by Treatment: Wald Test

	Complete	*LB-0*	*LB-2*	*LB-4*
Empty	6.13**	3.20*	8.50***	18.18
	(.013)	(.074)	(.004)	(.000)
Complete		4.63**	0.42	0.12
		(.031)	(.516)	(.728)
LB-0			5.56**	12.50***
			(.018)	(.000)
LB-2				0.24
				(.626)

*Note: Reported statistics are pairwise test statistic values for chi-squared tests of equivalent proportions of group averages between treatments. Test statistics are generated from a pooled OLS regression with standard errors clustered at the group level using bootstrapped standard errors where each group is observed over 20 periods. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.10. Link Proposals in Each Group by Treatment: Wald Test

	LB-0	*LB-2*	*LB-4*
Empty	18.37***	11.95***	3.42*
	(.000)	(.000)	(.065)
LB-0		1.09	0.00
		(.297)	(.969)
LB-2			0.61
			(.434)

*Note: Reported statistics are pairwise test statistic values for chi-squared tests of equivalent proportions of group averages between treatments. Test statistics are generated from a pooled OLS regression clustered at the group level using bootstrapped with standard errors standard errors where each group is observed over 20 periods. Empty treatment is set to 0 variance. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.11. Individual Contributions by Treatment:Random Effects-No Lag

	Dependent Variable: Group Contributions		
	LB-0	*LB-2*	*LB-4*
GroupConnections	0.582	2.704**	1.846
	(0.977)	(0.865)	(1.082)
Constant	10.34***	6.486**	6.472
	(0.648)	(2.683)	(4.995)
Observations	190	197	180
R-squared	0.007	0.274	0.156

*Note: Regression is fixed effects panel data model with standard errors clustered at the group level. GroupConnections describes the total number of links for each group (0 to 6). The number of clusters is equal to the number of groups in each treatment (9 to 10) the reported standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.12. Individual Contributions by Treatment: Random Effects-No Lag

	Dependent Variable: Contributions				
	Complete	*LB-0*	*LB-2*	*LB-4*	*Empty*
PGChoice	0.214*	0.252***	0.171*	0.536***	0.751***
	(0.116)	(0.082)	(0.094)	(0.121)	(0.181)
TotalPunishInLag	-0.00997	-0.255***	-0.045	-0.008	
	(0.038)	(0.063)	(0.038)	(0.030)	
NeighborhoodSize		0.753***	0.598***	0.592***	
		(0.165)	(0.186)	(0.226)	
GroupAveLag	0.629***	0.537***	0.566***	0.605***	0.400***
	(0.191)	(0.090)	(0.145)	(0.106)	(0.078)
Period	0.036	-0.001	-0.013	-0.018	-0.017
	(0.023)	(0.019)	(0.025)	(0.020)	(0.015)
Constant	0.595	0.0602	0.241	-1.71***	-0.941*
	(0.751)	(0.534)	(0.704)	(0.467)	(0.545)
Observations	680	720	748	684	684
Number of SubjectID	40	40	40	36	36

*Note: Regression is a random effects panel data model with standard errors clustered at the group level. NeighborhoodSize describes the total number of links for each subject (0 to 3). The number of clusters is equal to the number of subjects divided by 4 and the reported bootstrapped standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.13. Contributions by Treatment: Network Size Indicators

	Dependent Variable: Contribution		
	LB-0	*LB-2*	*LB-4*
PunishInLag	-0.255***	-0.037	-0.027
	(0.061)	(0.037)	(0.022)
1.NeighborhoodSize	1.004**	0.837**	2.051***
	(0.398)	(0.387)	(0.667)
2.NeighborhoodSize	1.508***	1.590***	2.525***
	(0.282)	(0.408)	(0.634)
3.NeighborhoodSize	2.431***	1.864***	2.453***
	(0.184)	(0.473)	(0.593)
GroupAveLag	0.529***	0.596***	0.655***
	(0.083)	(0.097)	(0.089)
Period	-0.000	-0.010	-0.007
	(0.018)	(0.017)	(0.015)
Constant	1.063***	0.551*	-0.803
	(0.403)	(0.331)	(0.663)
Observations	720	748	684
Number of SubjectID	40	40	36

*Note: Regression is a random effects panel data model with standard errors clustered at the group level. NeighborhoodSize describes the total number of links for each subject (0 to 3). The number of clusters is equal to the number of subjects divided by 4 and the reported standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.14. Individual Contributions by Treatment: Fixed Effects

	Dependent Variable: Contributions				
	Complete	*LB-0*	*LB-2*	*LB-4*	*Empty*
ContLag	0.349***	0.091	0.383***	0.272***	0.005
	(0.047)	(0.063)	(0.066)	(0.089)	(0.058)
TotalPunishInLag	0.0419*	-0.250***	-0.054	-0.004	
	(0.024)	(0.067)	(0.050)	(0.029)	
NeighborhoodSize		0.429*	0.193	0.516***	
		(0.252)	(0.172)	(0.138)	
GroupAveLag	0.430***	0.455***	0.364***	0.448***	0.348***
	(0.071)	(0.144)	(0.105)	(0.065)	(0.087)
Period	0.022**	-0.003	-0.005	-0.018	-0.020
	(0.010)	(0.014)	(0.018)	(0.015)	(0.018)
Constant	0.739***	1.152**	0.748	0.051	2.13***
	(0.282)	(0.510)	(0.483)	(0.399)	(0.779)
Observations	680	720	748	684	684
R-squared	0.374	0.125	0.358	0.313	0.066

*Note: Regression is a fixed effects panel data model with standard errors clustered at the group level. NeighborhoodSize describes the total number of links for each subject (0 to 3). The number of clusters is equal to the number of subjects divided by 4 and the reported bootstrapped standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.15. Punishment by Treatment: Fixed Effects – Established Links Only

	Dependent Variable: PunishOut			
	Complete	*LB-0*	*LB-2*	*LB-4*
Contribution	0.042**	-0.009	-0.016	-0.030***
	(0.020)	(0.138)	(0.017)	(0.011)
PunishedByLag	0.109***	0.102	0.081	0.154***
	(0.036)	(0.381)	(0.051)	(0.059)
PunishOutLag	0.228***	0.131	0.053	0.176**
	(0.039)	(0.356)	(0.041)	(0.089)
OtherPlayerCont	-0.024	-0.035	0.015	0.010
	(0.021)	(0.321)	(0.018)	(0.022)
OtherPlayerAveDev	-0.023	-0.006	-0.026***	-0.033**
	(0.015)	(0.259)	(0.010)	(0.015)
LinkCreated		0.184	0.022	0.272**
		(0.387)	(0.030)	(0.119)
Period	-0.007	0.001	-0.005*	0.002
	(0.005)	(0.015)	(0.003)	(0.003)
Constant	0.150	0.353	0.220**	0.150**
	(0.139)	(0.626)	(0.100)	(0.067)
Observations	2,040	228	1,166	1,598
R-squared	0.140	0.077	0.035	0.096

*Note: Regression is a fixed effect panel regression with standard errors clustered at the group level. Dependent variable is in tokens spent (0-2). The number of clusters is the number of groups per treatment (9-10). Bootstrapped standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.16. Punishment by Treatment: Fixed Effects – All Observations

	Dependent Variable: PunishOut			
	Complete	*LB-0*	*LB-2*	*LB-4*
Effort	0.039*	-0.001	-0.004	-0.004
	(0.023)	(0.002)	(0.012)	(0.017)
PunishedByLag	0.131***	0.087**	0.182***	0.158**
	(0.040)	(0.044)	(0.070)	(0.064)
OtherPlayerCont	-0.017	-0.003	0.009	0.029
	(0.024)	(0.003)	(0.015)	(0.018)
OtherPlayerAveDev	-0.034*	0.002	-0.000	-0.045***
	(0.019)	(0.003)	(0.017)	(0.017)
LinkCreated		0.682***	0.357**	0.466***
		(0.223)	(0.142)	(0.165)
Period	-0.009	-0.002**	-0.002	0.001
	(0.005)	(0.001)	(0.003)	(0.003)
Constant	0.194	0.031*	0.055	-0.045
	(0.159)	(0.019)	(0.047)	(0.087)
Observations	2,040	2,160	2,244	2,052
R-squared	0.091	0.264	0.092	0.115

*Note: Regression is a fixed effect panel regression with standard errors clustered at the group level. Dependent variable is in tokens spent (0-2). The number of clusters is the number of groups per treatment (9-10). Bootstrapped standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.17. Punishment by Treatment: Fixed Effects – Established Links Only No Lag

	Dependent Variable: PunishOut			
	Complete	*LB-0*	*LB-2*	*LB-4*
Effort	0.039*	-0.014	-0.015	-0.032**
	(0.023)	(0.091)	(0.017)	(0.014)
PunishedByLag	0.131***	0.096	0.084*	0.178***
	(0.040)	(0.270)	(0.050)	(0.064)
OtherPlayerCont	-0.017	-0.040	0.014	0.013
	(0.024)	(0.120)	(0.018)	(0.022)
OtherPlayerAveDev	-0.034*	-0.004	-0.026***	-0.038***
	(0.019)	(0.087)	(0.010)	(0.014)
LinkCreated		0.122	0.007	0.224**
		(0.139)	(0.036)	(0.108)
Period	-0.009	0.001	-0.005*	0.002
	(0.005)	(0.015)	(0.003)	(0.003)
Constant	0.194	0.426	0.232**	0.172***
	(0.159)	(0.493)	(0.103)	(0.066)
Observations	2,040	228	1,166	1,598
R-squared	0.091	0.071	0.033	0.074

*Note: Regression is a fixed effect panel regression with standard errors clustered at the group level. Dependent variable is in tokens spent (0-2). The number of clusters is the number of groups per treatment (9-10). Bootstrapped standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.18. Punishment by Treatment: Fixed Effects – All Observations

	Dependent Variable: PunishOut			
	Complete	*LB-0*	*LB-2*	*LB-4*
Effort	0.042**	-0.001	-0.003	-0.004
	(0.020)	(0.002)	(0.011)	(0.014)
PunishedByLag	0.109***	0.057	0.101***	0.106**
	(0.036)	(0.058)	(0.038)	(0.051)
PunishOutLag	0.228***	0.083	0.347***	0.187***
	(0.039)	(0.078)	(0.093)	(0.055)
OtherPlayerCont	-0.024	-0.002	0.009	0.024
	(0.021)	(0.003)	(0.014)	(0.016)
OtherPlayerAveDev	-0.023	0.002	-0.003	-0.037**
	(0.015)	(0.003)	(0.014)	(0.015)
LinkCreated		0.686***	0.396***	0.492***
		(0.223)	(0.145)	(0.172)
Period	-0.007	-0.001**	-0.000	0.002
	(0.005)	(0.001)	(0.002)	(0.003)
Constant	0.150	0.024*	0.005	-0.047
	(0.139)	(0.014)	(0.026)	(0.075)
Observations	2,040	2,160	2,244	2,052
R-squared	0.140	0.272	0.207	0.146

*Note: Regression is a fixed effect panel regression with standard errors clustered at the group level. Dependent variable is in tokens spent (0-2). The number of clusters is the number of groups per treatment (9-10). Bootstrapped standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.19. Links Broken by Treatment

	Dependent Variable: Link Broken			
	Combined	*LB-0*	*LB-2*	*LB-4*
EffortLag	-0.020	0.063	-0.097**	0.044
	(0.034)	(0.044)	(0.041)	(0.052)
PunishedByLag	0.842***	1.361***	0.729***	0.941***
	(0.096)	(0.186)	(0.187)	(0.139)
PunishWantedWithLag	0.821***	1.371***	0.733***	0.878***
	(0.099)	(0.189)	(0.189)	(0.150)
OtherPlayerLagCont	0.027	-0.082	0.054	0.002
	(0.034)	(0.107)	(0.051)	(0.080)
FreeFree	0.436	0.310	0.355	
	(0.293)	(1.483)	(0.263)	
FreeCond	0.685**	0.061	0.908***	0.977
	(0.295)	(1.589)	(0.224)	(0.998)
CondCond	-0.107	-0.196	-0.162	0.083
	(0.181)	(0.519)	(0.245)	(0.310)
CondFree	0.237	-0.137	0.531**	0.248
	(0.170)	(0.594)	(0.260)	(0.297)
OtherPlayerLagAveDev	-0.041	0.149	-0.142	0.051
	(0.065)	(0.128)	(0.091)	(0.127)
FreeRide	-0.444*	-0.135	-0.404**	-0.728
	(0.258)	(1.546)	(0.200)	(0.970)
Period	-0.046***	-0.026	-0.044*	-0.062***
	(0.012)	(0.023)	(0.023)	(0.024)
Constant	-1.807***	-2.094***	-1.539***	-1.948***
	(0.146)	(0.525)	(0.223)	(0.417)
Observations	6,456	2,160	2,244	1,976

*Note: Regression is a random effects panel data model with standard errors clustered at the group level. NeighborhoodSize describes the total number of links for each subject (0 to 3). FreeFree was omitted in the LB-4 regression due to multicollinearity. The number of clusters is equal to the number of subjects divided by 4 and the reported standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.20. Links Proposals by Treatment

	Dependent Variable: Link Proposed			
	Combined	*LB-0*	*LB-2*	*LB-4*
EffortLag	0.036	-0.073**	0.156***	0.006
	(0.049)	(0.034)	(0.031)	(0.026)
EffortDiffLag	-0.014	0.109**	-0.057	-0.023***
	(0.065)	(0.050)	(0.063)	(0.009)
PunishedByLag	-0.787***	-0.870***	-0.752**	-0.842***
	(0.188)	(0.117)	(0.333)	(0.206)
LinkEstablishedLag	2.482***	2.110***	2.153***	2.140***
	(0.239)	(0.421)	(0.298)	(0.406)
FreeFree	-0.387	-0.637***	-0.375	0.639***
	(0.511)	(0.209)	(0.287)	(0.053)
FreeCond	-0.482***	-1.167***	-0.976*	0.194***
	(0.160)	(0.328)	(0.551)	(0.010)
CondCond	0.113	0.067	0.001	0.368***
	(0.103)	(0.115)	(0.309)	(0.006)
FreeOther	-0.418**		-0.696	0.190
	(0.205)		(0.444)	(0.165)
CondOther	0.011	0.238	0.115	-0.170
	(0.261)	(0.204)	(0.208)	(0.219)
OtherPlayerLagAveDev	-0.007	0.075	-0.003	-0.004
	(0.043)	(0.047)	(0.061)	(0.006)
Constant	-0.577***	-0.520*	-0.576*	-0.150
	(0.144)	(0.276)	(0.301)	(0.261)
Observations	6,456	2,141	2,244	2,052

*Note: Regression is a pooled probit model with standard errors clustered at the group level. FreeOther was dropped due to insufficient observations in LB-0. The number of clusters is the number of groups per treatment (9-10). Bootstrapped standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.21. Links Proposed by Treatment-No Types

	Dependent Variable: LinkProposed			
	Combined	*LB-0*	*LB-2*	*LB-4*
ContributionLag	0.092**	-0.037	0.195***	0.039
	(0.043)	(0.040)	(0.062)	(0.056)
ContDiffLag	-0.053	0.076	-0.144**	-0.014
	(0.046)	(0.101)	(0.070)	(0.091)
PunishedByLag	-0.532***	-0.541*	-0.460***	-0.656***
	(0.101)	(0.306)	(0.116)	(0.174)
LinkProposedLag	1.720***	1.671***	1.648***	1.938***
	(0.128)	(0.308)	(0.084)	(0.258)
OtherPlayerLagAveDev	0.053	0.123	0.028	0.048
	(0.045)	(0.124)	(0.061)	(0.083)
Constant	-0.793***	-1.380***	-0.950***	0.074
	(0.204)	(0.170)	(0.305)	(0.326)
Observations	6,456	2,160	2,244	2,052
Number of SubjectID	116	40	40	36

*Note: The table is created from the results of a dynamic panel probit regression with binary dependent variable equal to 1 when a link was formed between players i and j. The comparison group is two players with types: Other. FreeOther was omitted due to a very small number of links <.1% in the second regression. Bootstrap standard errors are clustered at the group level and given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.22. Links Established by Treatment

	Dependent Variable: LinkEstablished			
	Combined	*LB-0*	*LB-2*	*LB-4*
ContributionLag	0.096	-0.025	0.184***	0.041
	(0.059)	(0.089)	(0.070)	(0.063)
ContDiffLag	0.009	0.228*	-0.036	-0.025
	(0.047)	(0.119)	(0.071)	(0.070)
PunishedByLag	-0.622***	-0.733***	-0.482***	-0.758***
	(0.096)	(0.272)	(0.152)	(0.117)
LinkEstablishedLag	2.156***	2.388***	2.093***	2.080***
	(0.118)	(0.509)	(0.124)	(0.270)
OtherPlayerLagAveDev	0.121**	0.363***	0.126	0.004
	(0.053)	(0.134)	(0.084)	(0.096)
Constant	-1.493***	-2.276***	-1.580***	-0.364
	(0.212)	(0.269)	(0.280)	(0.289)
Observations	6,456	2,160	2,244	2,052
Number of SubjectID	116	40	40	36

*Note: The table is created from the results of a dynamic panel probit regression with binary dependent variable equal to 1 when a link was formed between players i and j. The comparison group is two players with types: Other. FreeOther was omitted due to a very small number of links <.1% in the second regression. Bootstrap standard errors are clustered at the group level and given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table A.23. Determinants of Established Links by Treatment-Panel

	Dependent Variable: Link Established			
	Combined	*LB-0*	*LB-2*	*LB-4*
ContLag	0.098*	-0.026	0.184**	0.051
	(0.056)	(0.085)	(0.090)	(0.059)
ContDiffLag	0.005	0.225**	-0.034	-0.032
	(0.044)	(0.112)	(0.092)	(0.061)
PunishedByLag	-0.632***	-0.744***	-0.514***	-0.749***
	(0.096)	(0.161)	(0.155)	(0.115)
LinkEstablishedLag	2.152***	2.389***	2.061***	2.017***
	(0.128)	(0.426)	(0.121)	(0.260)
FreeFree	0.388	-0.016	0.645**	0.349
	(0.391)	(0.280)	(0.312)	(0.295)
FreeCond	0.152	-0.040	-0.279	0.427***
	(0.258)	(0.270)	(0.237)	(0.105)
CondCond	0.265	0.245	0.634**	0.107
	(0.192)	(0.303)	(0.269)	(0.248)
FreeOther	0.381		0.122	0.717*
	(0.471)		(0.330)	(0.424)
CondOther	-0.061	-0.045	0.457	-0.493***
	(0.253)	(0.501)	(0.429)	(0.134)
OtherPlayerLagAveDev	0.113**	0.355***	0.128	-0.023
	(0.049)	(0.123)	(0.104)	(0.076)
Constant	-1.637***	-2.342***	-1.869***	-0.436**
	(0.257)	(0.337)	(0.340)	(0.200)
Observations	6,456	2,141	2,244	2,052

*Note: The table is created from the results of a dynamic panel probit regression with binary dependent variable equal to 1 when a link was formed between players i and j. The comparison group is two players with types: Other. FreeOther was omitted due to a very small number of links <.1% in the second regression. Bootstrap standard errors are clustered at the group level and given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

b) *Complete* a) *LB-0*

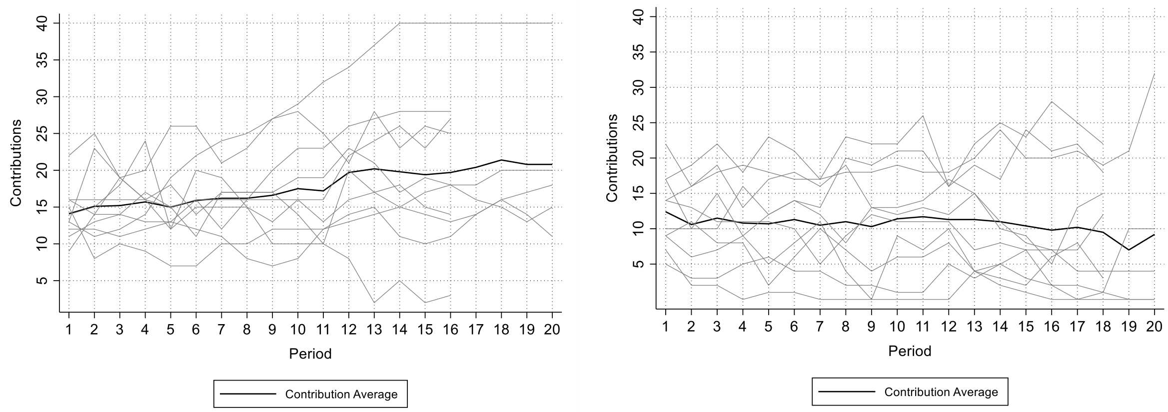


Figure A.1. Group Contributions by Treatment: Remaining

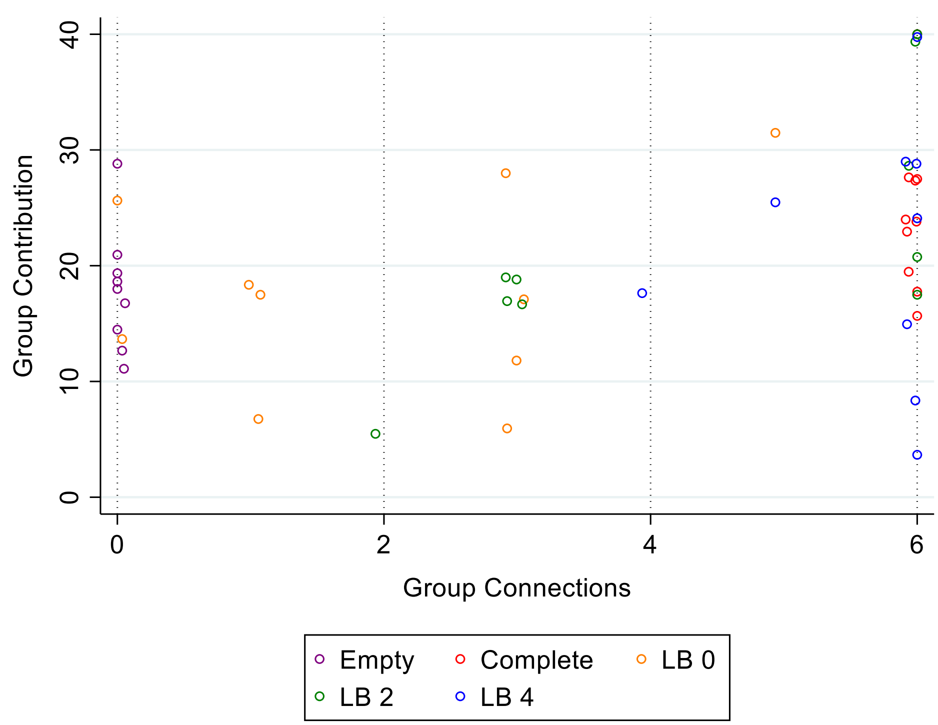


Figure A.2. Maximum Group Contributions and Network Size

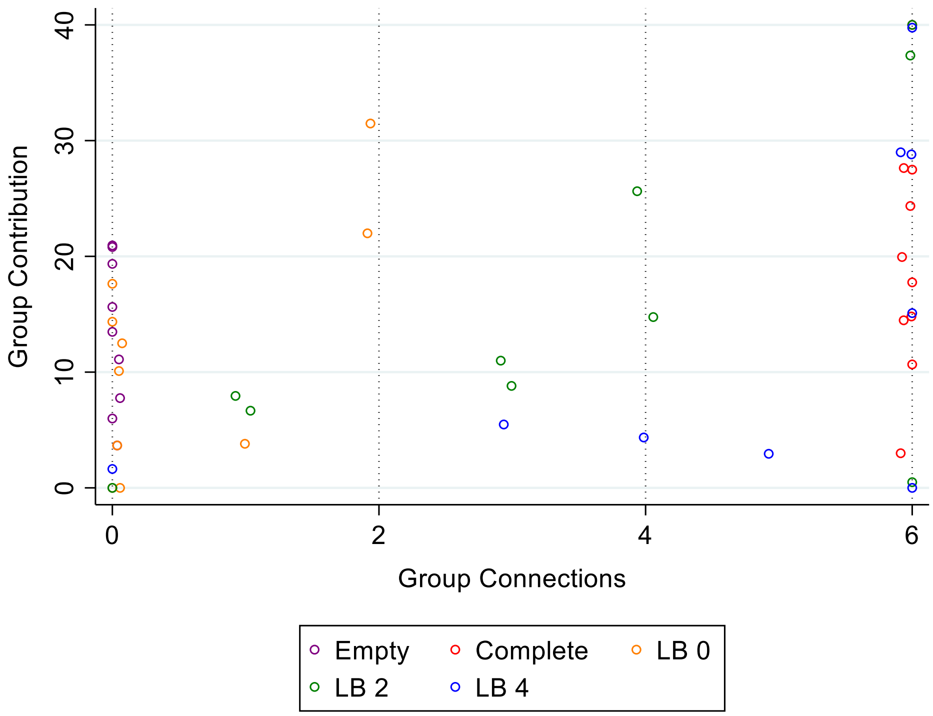


Figure A.3. Group Contributions and Network Size in Final Period

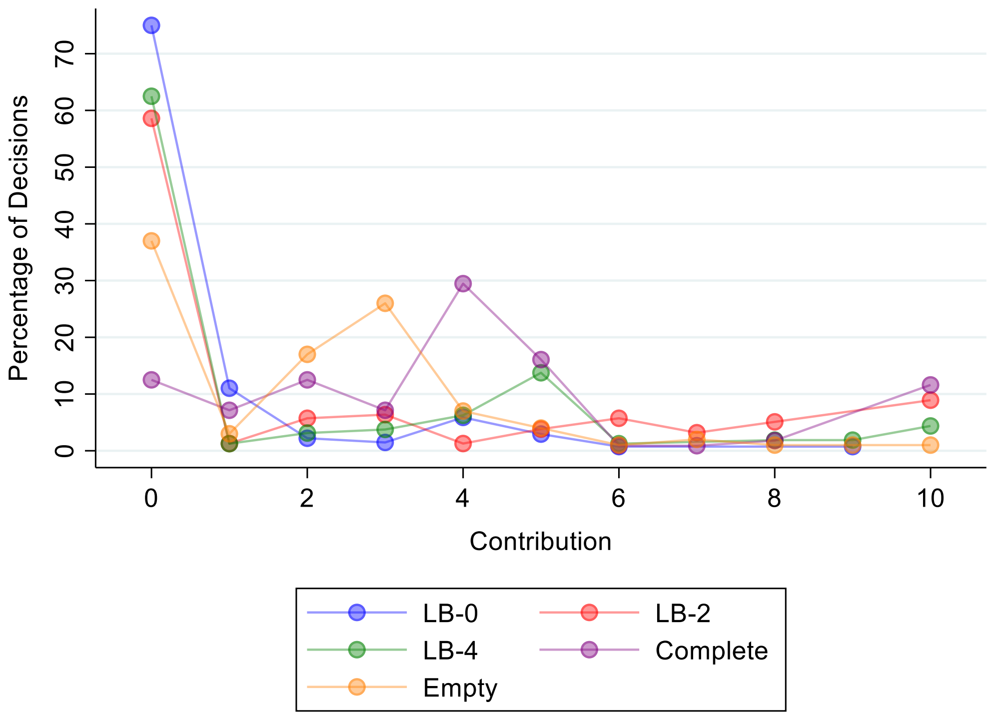


Figure A.4. Percentage of Contribution Choices by Treatment: Free Riders

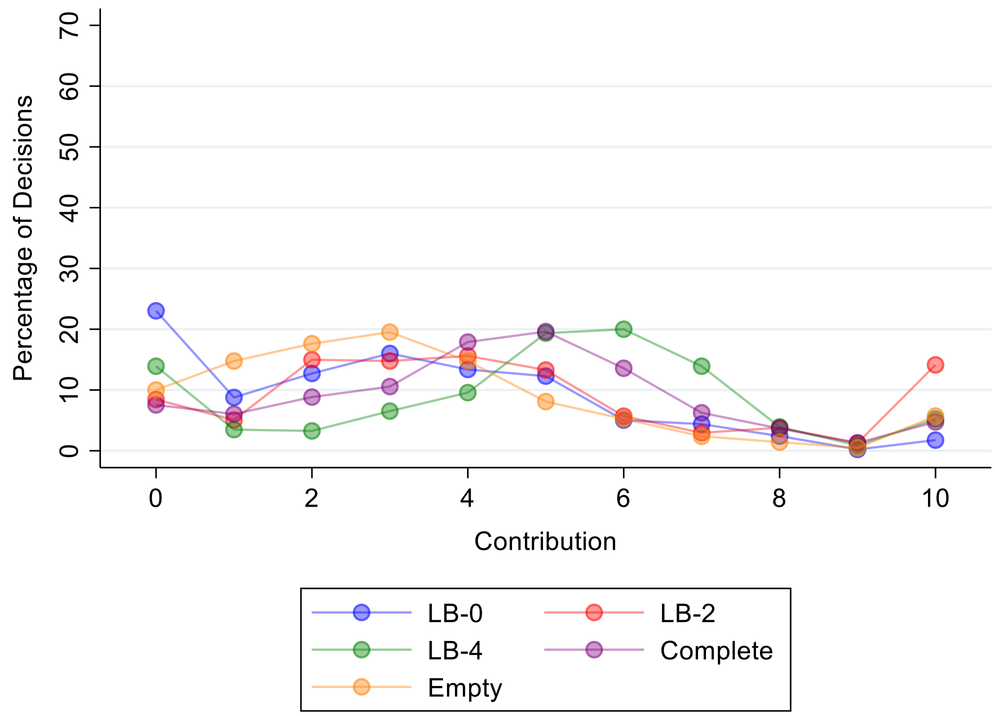


Figure A.5. Percentage of Contribution Choices by Treatment: Conditional Cooperators

Below are sample instructions for the *LB-2* treatment and the end of experiment questionnaire. Instructions for the *LB-0* and *LB-4* treatments are otherwise identical with small updates or removal of the link benefit line. The *Complete* and *Empty* treatment used 2 and 1 stage, respectively.

Experiment Instructions

Introduction

Thank you for participating in today's study. Please follow the instructions carefully. At any time, please feel free to raise your hand if you have a question. If you follow the instructions carefully, you can earn more money depending both on your own decisions and on the decisions of others.

You will make decisions using a computer. In order to keep your decisions private, please do not reveal your choices or otherwise communicate with any other participant.

Today's session has three parts: Experiment 1, Experiment 2, and a short questionnaire. You will have the opportunity to earn money in both experiments based on your decisions. You will be paid your earnings privately, and in Amazon gift card, at the end of the experiment session. We will proceed through the written materials together. Please do not enter any decisions on the computer until instructed to do so.

At certain stages in the experiment, you will see a timer in the top right corner of your screen. In many cases, if you do not make a decision in the allotted time the program will automatically move along to the next decision and will input a 'default' decision. These defaults are likely not in your best interest, so it's important to stay focused on the experiment screen. These timers are not designed to rush you and you should have plenty of time to make decisions. However, in many cases we can not move on until everyone has hit "Continue" "Submit" etc. So, again, please stay focused on your screen so that we may complete the experiment in a timely manner.

Experiment 1

Please refer to your computer screen while we read the instructions.

Experiment 1 will consist of three different decision tasks. We will proceed through the instructions for each one at a time and make the decision(s) before proceeding onto the next. We will reveal the results to each of the tasks at the end of the session.

1 - Lottery Task

In the first decision setting, we would like you to make a decision for each of 10 scenarios. Each scenario involves a choice between receiving $2 for sure (Option B) or playing a lottery that pays $4 or $0 with the stated chances (Option A).

You will notice that the only differences across scenarios are the chances of receiving the high or low prize for the lottery. At the end of the session, **ONE** of the 10 scenarios will be selected at random and you will be paid according to your decision for this selected scenario **ONLY**. Each scenario has an equal chance of being selected.

Please consider your choice for each scenario carefully. Since you do not know which scenario will be played out, it is in your best interest to treat each scenario as if it will be the one used to determine your earnings.

Before making decisions, are there any questions?

Please proceed to entering decisions on your computer. Once you are ready to submit your decisions, please click the "Submit" button.

2- Offer Task:

Tokens will be used as the currency in the remaining tasks in Experiment 1. Tokens are exchanged at a rate 6.66 of to $1 USD (15 cents per token).

In this task, there are two types of decisions: Player As and Player Bs.

Player As will be given 10 tokens and will decide how to split that amount with Player B.

Player Bs have the option to accept or decline the split. If Player B *accepts* the offer, then the proposed split proposed is paid out. If Player B *rejects,* then both players receive zero.

Illustrative Examples:

Example 1: Player A offers 2 tokens to Player B (which is a split of 8 (A) / 2 (B)). If Player B accepts, Player A receives 8 and Player B receives 2. If Player B declines, both players receive zero.

Example 2: If Player A offers 5 to Player B (a split of 5 (A) and 5 (B)). If Player B accepts, Player A receives 5 and Player B receives 5. If Player B declines, both players receive zero.

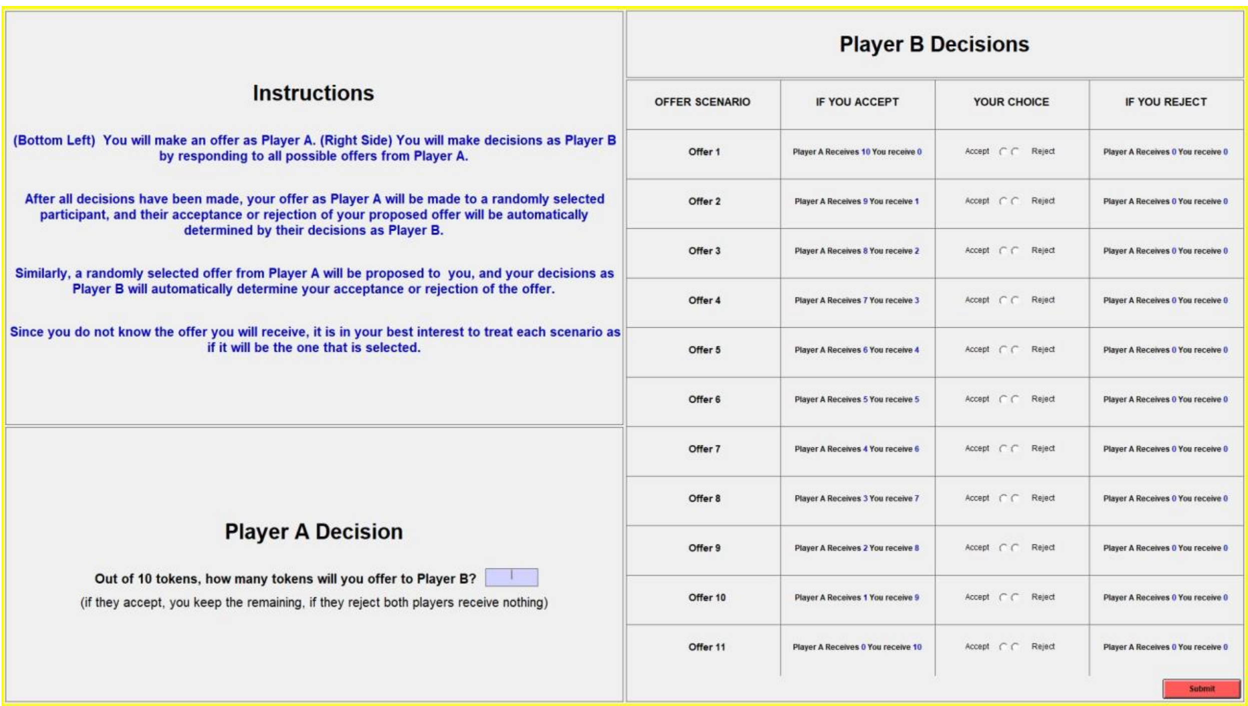
We will now have some practice questions to help with understanding. For each practice question you get correct you receive 2 tokens (30 cents). Once you are ready, please hit continue.

<u>Decision Task:</u>

Your first part of the task is, as Player A, to decide the split to offer to Player B.

Then, in the second part of the task, as Player B, you will make an *accept* or *reject* decision based on each of the possible splits received from another participant who is Player A.

Your decision screen will look like this:



Your split offer as Player A will be given to one other randomly selected participant and their accept/reject decision as Player B will be automatically applied. Then as Player B, you will receive one offer from a random Player A, in which your previously made accept/reject decisions will be applied.

As Player B, since you do not know which offers you will receive, it is in your best interest to carefully decide your choices as if you will receive that offer.

Before we continue, are there any questions?

Please proceed to entering decisions on your computer. Once you are ready to submit your decisions, please click the "Submit" button.

3- Account Contribution Task

In our last decision task, you will be randomly matched into groups of four participants.

At start of the task each member is endowed with 10 tokens. Your task is to allocate them into a Public or Private account. Each token that is not put into the Public account is automatically put into your Private account. Every other player in your group faces the same situation.

For each token in your Private Account you will receive one token.

For each token put into the Public Account ***all players*** in the group receive .4 tokens. Thus, 1 token turns into a total 1.6 tokens that gets distributed evenly to the group. The total contributions to the Public Account will come from the sum of contributions of all group members.

Your earnings from the decision task will be the following:

Your earnings = Total tokens in the Public Account * (.4) + Tokens from your Private Account

Illustrative examples:

Example 1: *Everyone in your group contributes 0 to the Public Account*

Your Public Account contribution=0

Your Private Account contribution (automatically determined)=10 – 0 =10

Total Public Account contributions from your other group members = 0

Total in Public Account (you + your other group members) =0

Payoff to you=Public Account*.4 + Private Account= 0 +10 = 10

Example 2: *Everyone contributes 6 to the Public Account*

Your Public Account contribution=6

Your Private Account contribution (automatically determined)=10 – 6 =4

Total Public Account contributions from your other group members = 18

Total in Public Account (you + your other group members) =24

Payoff to you=Public Account*.4 + Private Account= 9.6 +4 = 13.6

Example 3: *You contribute 2 your group members contribute 8*

Your Public Account contribution=2

Your Private Account contribution (automatically determined)=10 – 2 =8

Total Public Account contributions from your other group members = 24

Total in Public Account (you + your other group members) =26

Payoff to you=Public Account*.4 + Private Account= 10.4 +8 = 18.4

Payoff to your group members = 10.4 + 2=12.4

As you can see from these examples, increasing your contributions to the Public Account increases the value of your tokens and increases everyone payoffs. On the other hand, increasing your Private Account contributions results in a higher payoff for you.

We will now have some practice questions to help with understanding. For each practice question you get correct you receive 2 tokens (30 cents). Once you are ready, please hit continue.

Your Decision:

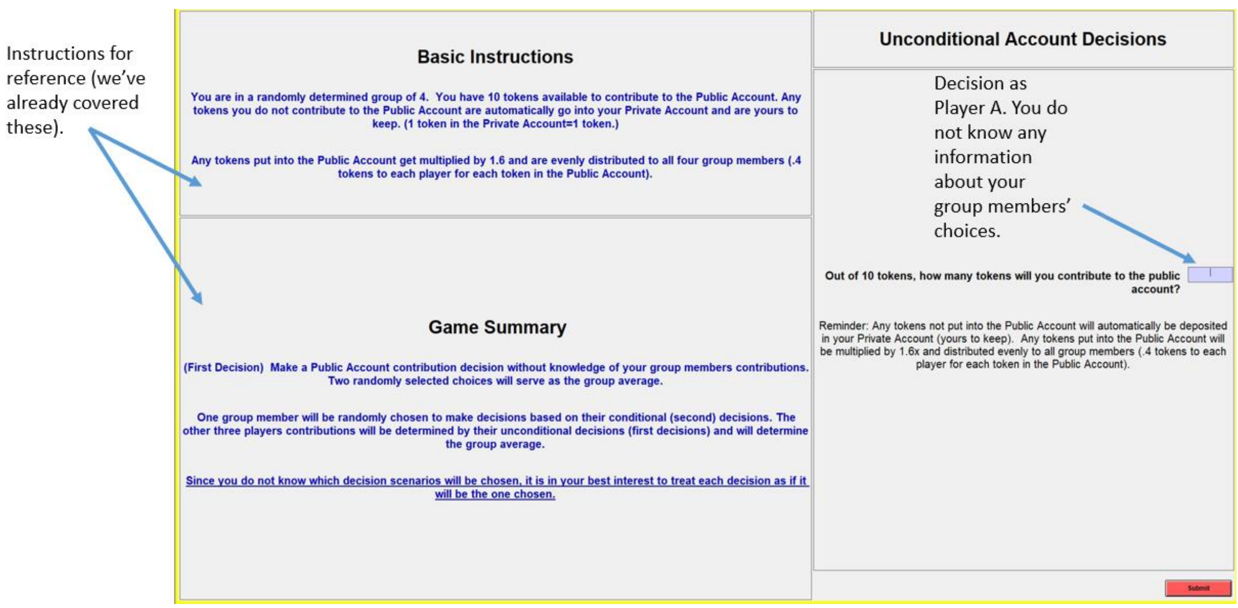
(First Decision – Unconditional Choice) Your task will be first to decide how much to contribute to the Public Account without information about your group members' choices.

(Second Decision- Conditional Choice) Then, you will make the same contribution decisions based on what you would do if you knew the average of what your other group members contributed. You will be asked this for a range of different group averages. For example, what would your contribution be if the average contribution from your group was 5 (15 total)?

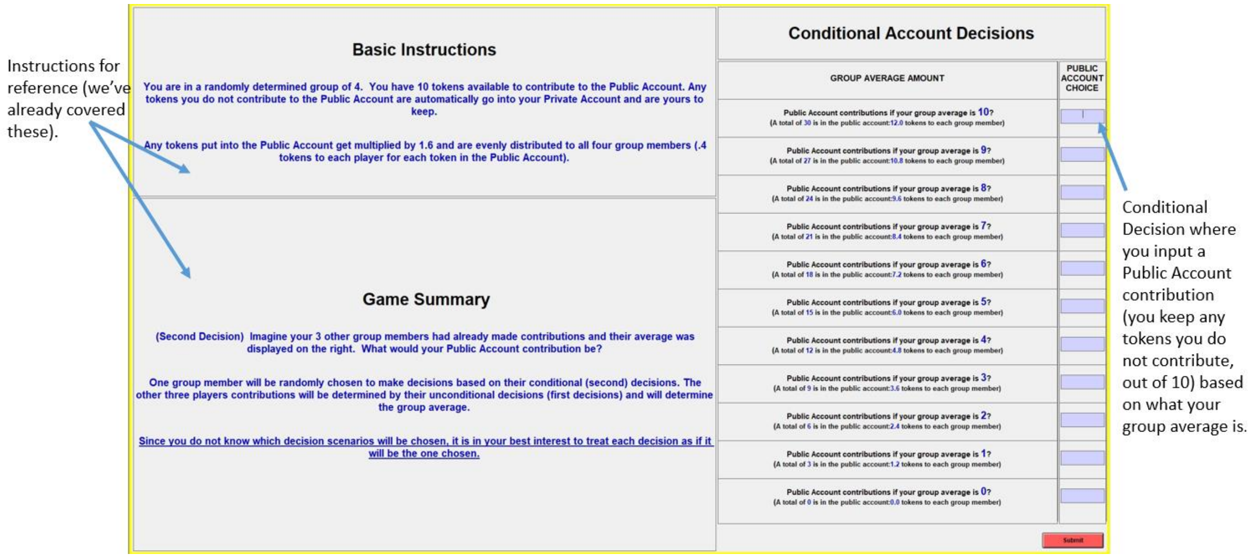
For payment, 3 random group members' unconditional contributions will form the group average (rounded) and 1 player will have their conditional choice selected based on the group average. Thus, the total contributions will be made up 3 unconditional contributions and 1 conditional contribution.

The earnings are determined based on the total of Public and Private contributions for each group member.

<u>Player A Decision (Unconditional):</u>



Player B Decisions (Conditional):



Illustrative Example:

Example: You are randomly determined to make the conditional choice. Your 3 group members contribute a total of 15 from their unconditional contributions (for an average of 5). Your conditional choice when the average was 5, was 8.

The payoff would be resolved in the following:

The computer takes the 3 first decisions (unconditional) from your other group members and adds them up. The total is 15, giving an average of 5.

The computer then takes your second decision (conditional) corresponding to the question that asked about an average of 5. Your response was 8.

8 is then added to the Public Account and this is the total in the Public Account for the group.

Payoff Calculation:

Your Public Account contribution=8

Your Private Account contribution (automatically determined)=10 – 2 =2

Total Public Account contributions from your other group members =15

Total in Public account (you + your other group members) =23

Payoff to you=Public Account*.4 + Private Account= 9.2 +2 = 11.4

<u>Since you do not know which scenario will be played out it is in your best interest to treat each scenario as if it is the one that will be chosen.</u>

Do you have any questions?

Once you are ready, please hit continue.

Experiment 2

This experiment is made up a series of similar decisions over the course of several rounds. At the start of the experiment, you will be randomly put into a group of 4 players. This group is different than the group from the previous experiment and **your group members will remain the same throughout the remainder of the experiment.** There will be between 15-25 rounds and each round will be comprised of 3 separate stages. We will adjust the token exchange rate to 25 tokens to $1 US. (1 token=$.04)

We will describe each stage in the following instructions. We will start with the second stage, as this is the easiest way to understand.

Second Stage

This task is identical to the unconditional (first) decision in the account task. You are one of four members of a group. At the beginning of each round each member is endowed with 10 tokens. Your task is to allocate them into a Private or Public account. Each token that is not put into the Public Account is automatically put into your Private Account. Every other player in your group will make the same decision.

For each token in your Private Account, you will receive one token.

For each token put into the Public Account ***all players*** in the group receive .4 tokens. Thus, 1 token turns into a total 1.6 tokens that gets distributed evenly to the group. The total contributions to the group account will come from the contributions of all four players.

Your earnings from Stage 2 will be the following:

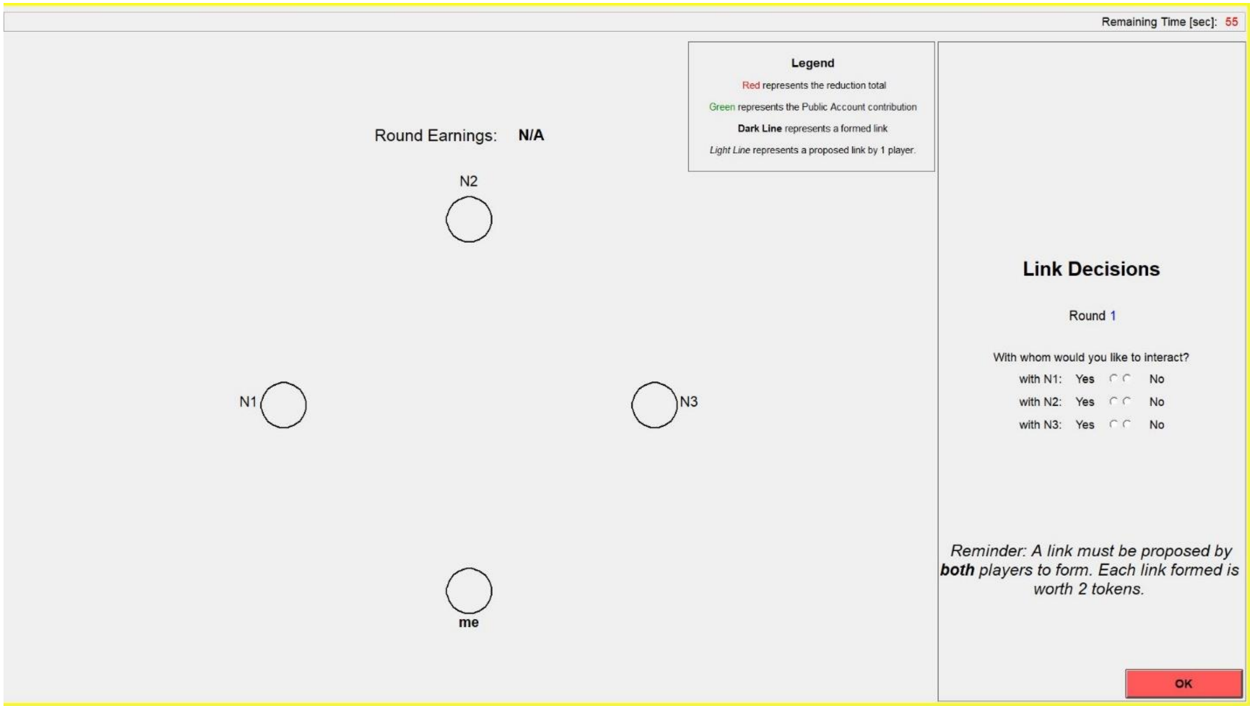
Your earnings = Total tokens in the Public Account * (.4) + Tokens from your Private Account

Stages 1 and 3 Overview

Stages 1 and 3 are related and are most easily explained together. In Stage 1 you will propose "links" to the members in your group. For a link to be formed both players must propose a link to each other. The primary use of a link is for Stage 3. In Stage 3, whichever players are linked have the ability to deduct from the player they are linked with. In this case, players can pay *up to 2 tokens* to reduce the player they are linked with by 3 times the amount spent. (.5 tokens reduces by 1.5; 1 token reduces by 3; etc.)

Stage 1- Link Formation

Your choice in this stage is to whether or not to propose a link to each of your group members. For a link to form, both players must propose a link to each other. If one player proposes a link and the other does not, no link is formed (or if neither player proposes a link then no link is formed). **For each link formed you receive 2 tokens.**



Your payoff for stage 1 is made up from the benefits/costs of linking:

Stage 1 payoff= Number of links formed*2.

Stage 3- Reduction

You will receive 6 tokens of endowment in Stage 3 (each round). If you are linked with someone you may spend *up to 2* of your tokens (by .1) to reduce the payoff of the group member you are linked with by 3 times the amount spent. Therefore, depending on how many group members you are linked with, you have up to 3 choices, and you can spend up to 2 tokens on *each* group member that you are linked with.

Note that each player you are linked with can reduce your payoff as well.

Your payoff for stage 3 is made up of the following:

Stage 3 payoff= Endowment of 6 tokens – Tokens spent on reductions for others – 3*amount of tokens spent reducing your payoff *from* others.

You can potentially have negative payoffs for this stage/round.

<u>Your total payoff for each round will be the sum of the payoffs from stages 1, 2, and 3</u>.

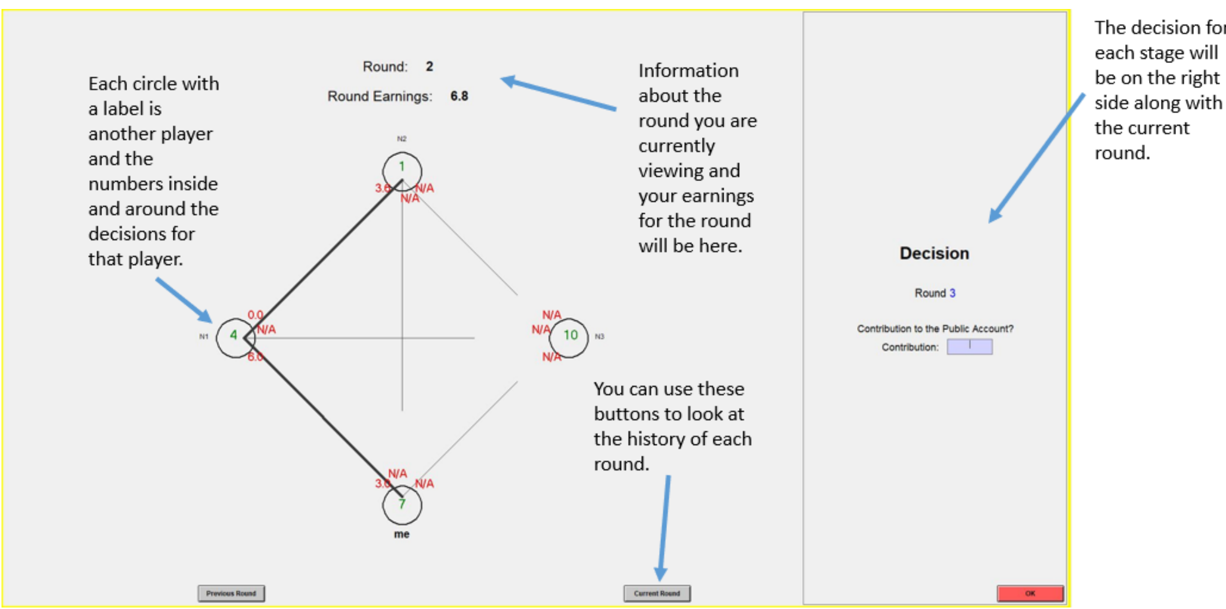
Round payoff= Stage 1 payoff + Stage 2 payoff + Stage 3 payoff

Are there any questions on the basic structure of the game?

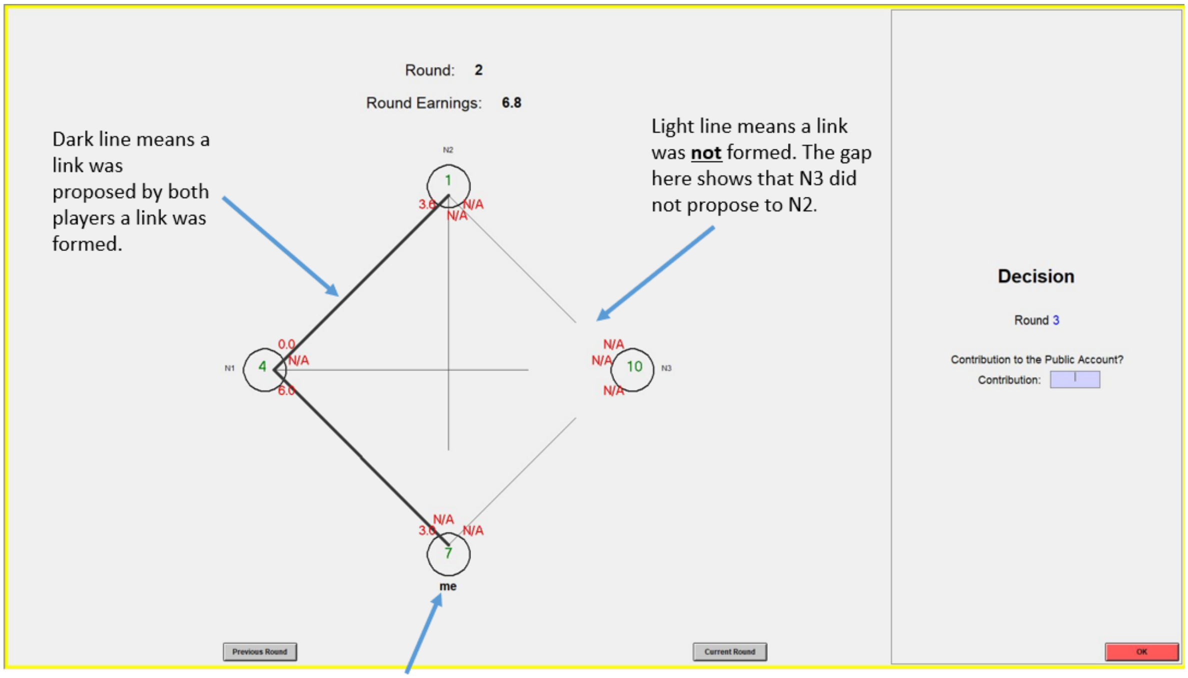
After the decisions are made in each stage, the relevant information will be displayed on your screen. In each stage, all of the information from the previous stages will be available for your review as well by clicking the "previous round" buttons on the left hand of the screen as seen below.

You'll also be given a summary of your total payoff for the round. It is possible to receive negative payoffs for a round. In the unlikely case that your payoffs for experiment 2 become negative the total will be counted as 0. The screen information looks like the following:

<u>General Information</u>

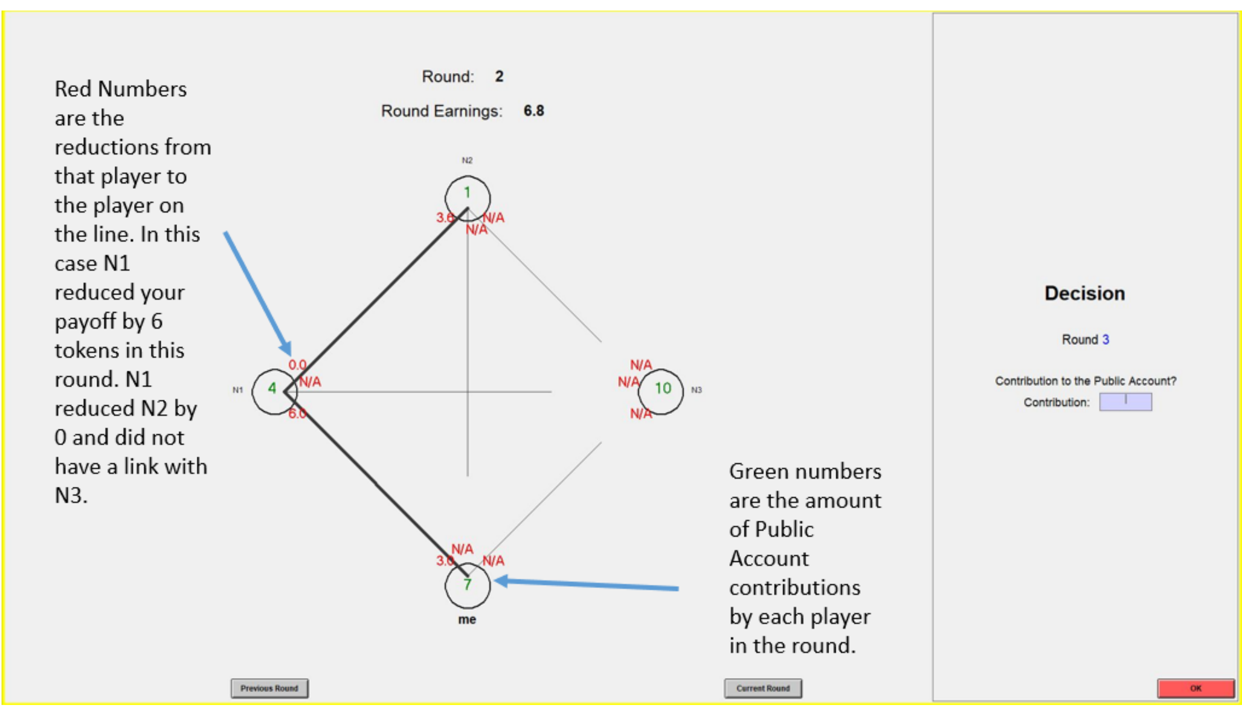


Link Formation:

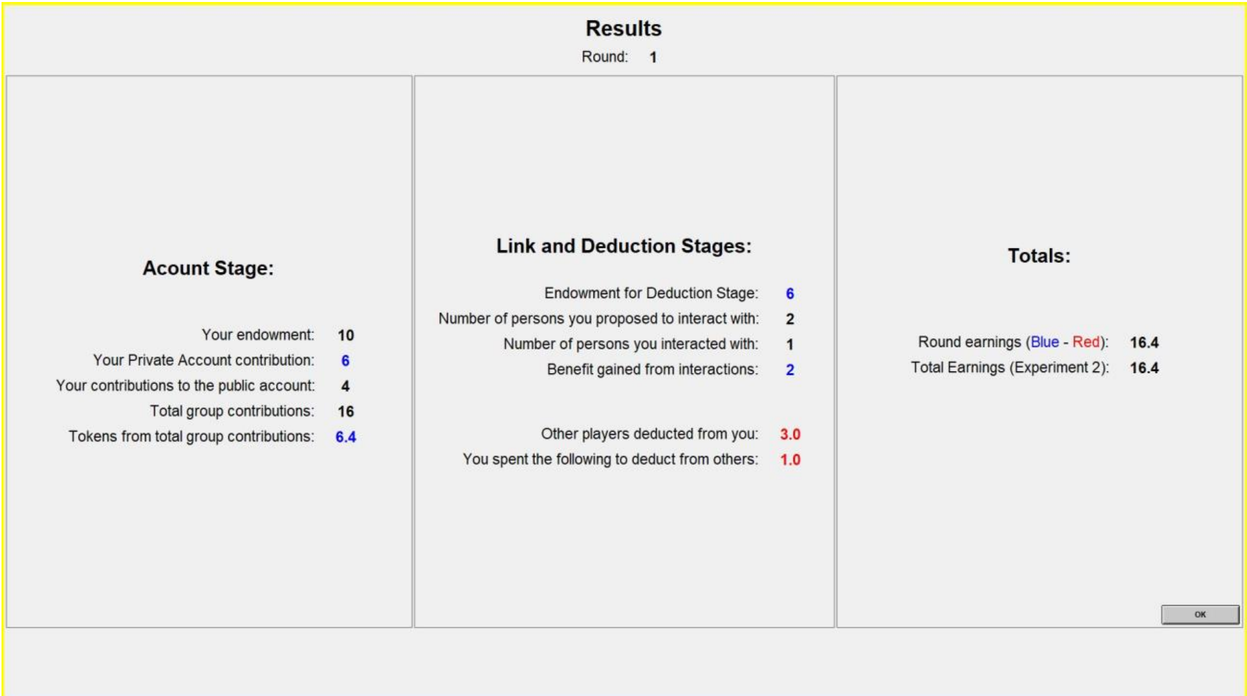


You are here. You proposed a link to N1 and N3 but not N2. N1 proposed a link back so a link was formed.

Contributions and Deductions:



Results:



We will have a final set of practice questions to help with understanding before we begin. (7.5 tokens per correct answer; 30 cents)

Reminder: Please stay focused on your screen and submit your decisions when you have made them. The first two rounds will have extended timers before the screen automatically moves forward. After the second round the timers will be reduced.

Do you have any questions?

Once you are ready, please hit continue.

B Appendix-Chapter 2

Proof for Proposition 1

We solve for the model of three players who have heterogenous valuation. Let the game be fully sequential (1,1,1) $t \in 1,2,3$. For one player i, set valuation to $V_i \neq 1 > 0$ and the others $V_{\sim i} = 1$. Let V_t be the valuation of the player in period t.

For Case 1 (C1) set,

$$V_1 = V \;\; V_2 = 1 \; V_3 = 1$$

The resulting optimizations give:

$$f_2(X) = X^2 \quad f_1(X) = X^2(2X - 1) \quad f_0(X) = 6X^2 - X[4V + 2] + V$$

Solving for the highest root of the equation $f_0(X) = 0$ yields the total aggregate output:

$$X = \frac{2V + 1 \pm \sqrt{4V^2 - 2V + 1}}{6}$$

For Case 2 (C2),

$$V_1 = 1 \quad V_2 = V \quad V_3 = 1$$

$$f_2(X) = X^2 \quad f_1(X) = X^2\left(\frac{2X}{V} - 1\right) \quad f_0(X) = \frac{6}{V}X^2 - X\left[\frac{4}{V} + 2\right] + 1$$

$$X = \frac{2 + V \pm \sqrt{4 - 2V + V^2}}{6}$$

For Case 3 (C3),

$$V_1 = 1 \quad V_2 = 1 \quad V_3 = V$$

$$X = \frac{1}{2} \pm \frac{\sqrt{12}}{12} \approx .7887$$

$$f_2(X) = \frac{X^2}{V} \quad f_1(X) = \frac{X^2}{V}(2X - 1) \quad f_0(X) = \frac{1}{V}(6X^2 - X[4 + 2] + 1)$$

Note: This is identical to a setting where $V_i = 1$ in a fully sequential game.

Now solving for the simultaneous game (CS).

Let V_i represent te value to player $i \in 1,2,3$:

$$V_i = V \; V_2 = 1 \; V_3 = 1$$

Taking the FOCs and imposing symmetry for players 2 and 3, the FOCs reduce to:

$$x_2(2V - 1) = x_1$$

Solving for equilibrium outputs:

$$x_2 = \frac{2V}{(2V + 1)^2} \qquad x_1 = \frac{2V(V - 1)}{(2V + 1)^2}$$

Aggregate output is given by:

$$X = \frac{2V}{2V + 1}$$

The expected utilities for each player type:

$$EU_1(\boldsymbol{x}) = \frac{1}{(2V + 1)^2} \qquad EU_2(\boldsymbol{x}) = \frac{V(4V^2 - 4V + 1)}{(2V + 1)^2}$$

Given any value $V_{max} > V > 1$ the aggregate outputs can be easily ordered.$CS < C3 < C2 < C1$. For $V_{min} < V < 1$, the sequential case order is reversed; $CS < C1 < C2 < C3$.

Setting $f_n(X) = X_{-n}$ for any previous contribution by the players before them X_{-n} gives a relationship between the previous contributions and the Nash equilibrium choice of the player such that $X > X_{-n}$.

Proof of Theorem 1

Let the game consist of a general t period game with n_t players per period who have value V_t. The maximization choice by the last player, m, is given by

$$EU_m = \frac{V_m(X - X_{m-1})}{X} - (X - X_{m-1})$$

Taking FOC yields,

$$V_m \frac{X_{m-1}}{X^2} - 1 = 0 \quad \rightarrow \quad f_m(X) = X_{m-1} = \frac{X^2}{V_m}$$

Which can be written as,

$$\frac{1}{V_m} z_m(X) \quad where\ z_m(X) = f_m(X)\ if\ V_m = 1$$

Thus, for player $m-1$

$$U_{m-1} = \frac{V_{m-1}(X_{m-1} - X_{m-2})}{X} - (X_{m-1} - X_{m-2}) = \frac{V_{m-1}\left(\frac{X^2}{V_m} - X_{m-2}\right)}{X} - \left(\frac{X^2}{V_m} - X_{m-2}\right)$$

$$FOC\colon\ V_{m-1}\left(\frac{1}{V_m} + \frac{X_{m-2}}{X^2}\right) - \frac{2X}{V_m} = 0$$

$$X_{m-2} = \frac{1}{V_m} X^2(2X - V_{m-1})$$

Which is equal to,

$$\frac{1}{V_m} z_{m-1}(X)$$

$$where\ z_{m-1}(X) = f_m(X) = X^2(2X - V_{m-1})$$

is the function if $V_m = 1\ and\ V_{m-1} = V_{m-2}$.

Doing this recursively for any n

$\frac{1}{V_m} z_{m-n}(X)$

The final equation that determines the aggregate output is then:

$$\frac{1}{V_m} z_0(X) = 0$$

Which clearly does not depend on V_m.

Details for the solving of equation (3) and the proof of Proposition 2*:*

A special case of this setup is a fully sequential setting in which only the final entrant in period T has probabilisitic entry. Utilizing the model by Hinnosaar, we rewrite the final players problem as

$$\max_X \frac{X - X_{T-1}}{X} - (X - X_{T-1})$$

Solving yields

$$X_{T-1} = X^2$$

Thus, the player in period before the final player faces an entrant who will make the final aggregate outcome X^2 or he will not play at all, $X_{T-1} = X$. We label the final value X generated in the case where the final entrant does enter the market, X^H, and X^L corresponds to the setting in which he does not enter.

The $T-1$ period player then maximizes his expected value given by:

$$EU_{T-1}(x) = (q_T)(\frac{({X^H}^2 - X_{T-2})}{X^H} - ({X^H}^2 - X_{T-2})) + (1 - q_T)(\frac{X^L - X_{T-2}}{X^L} - (X^L - X_{T-2}))$$

However, from the above equation we know the relationship between X^H and X^L;

$${X^H}^2 = X_{T-1} = X^L$$

Therefore, we can rewrite the expected maximization problem in only one choice variable

$$\max_{X_L} EU_{T-1}(x) = (q_T)\,(\frac{({X^H}^2 - X_{T-2})}{X^H} - ({X^H}^2 - X_{T-2})) + (1 - q_T)(\frac{{X^H}^2 - X_{T-2}}{{X^H}^2} - ({X^H}^2 - X_{T-2})$$

Taking the derivative and setting equal to zero,

$$q_T\left(1 + \frac{X_{T-2}}{{X^H}^2}\right) + (1 - q_T)\left(\frac{2X_{T-2}}{{X^H}^3}\right) - 2X^H = 0$$

$$X_{T-2} = X^{H^3} \frac{2X^H - q_T}{(q_T X + 2(1 - q_T))}$$

Restricting ourselves to a three-person model, the final player maximizes expected value:

$$\max_{X^H} q_T \left(\frac{(X_{T-2} - 0)}{X^H} - (X_{T-2} - 0 \right) + (1 - q_T)\left(\frac{X_{T-2} - q_T}{X^L} - (X_{T-2} - 0)\right)$$

Substituting for X_{T-2} and X_L and optimizing

$$Let\ Z = \frac{2X^H - q_T}{(q_T X + 2(1 - q_T))}\ and\ derative\ Z' = 4(1 - p) + \frac{p^2}{q_T X + 2(1 - q_T)},$$

$$Z' \left(q_T X^{H^2} + (1 - q_T) X^H - X^{H^3} \right) + Z \left(2 q_T X^H + (1 - q_T) - 3X^{H^2} \right) = 0$$

Probabilistic Entry Setting B.

Given the third player enters the contest he optimizes to produce:

$$X_2 = X^2 = X^{HH^2}$$

Otherwise 0. X^{HH} represents the total output given both players entered.

Thus, if the second player joins the contest, he faces the problem:

$$EU_2(x) = q_3 \left(\frac{X_2 - X_1}{X^{HH}} - X_2 - X_1 \right) + (1 - q_3) \left(\frac{X_2 - X_1}{X^{HL}} - (X_2 - X_1) \right)$$

Substituting, and using the fact that $X_2 = X^{HL} = X^{HH^2}$ when the third player does not join the contest; we can rewrite the expected value as a function of only one variable

$$\max_{X^{HH}} EU_2(x) = q_3 \left(\frac{X^{HH^2} - X_1}{X^{HH}} - (X^{HH^2} - X_1) \right) + (1 - q_3) \left(\frac{X^{HH^2} - X_1}{X^{HH^2}} - \left(X^{HH^2} - X_1 \right) \right)$$

Yielding,

$$X_1 = X^{HH^3} \frac{2X^{HH} - q_3}{(q_3 X^{HH} + 2(1 - q_3))}$$

The first player, who always enters, then optimizes:

$$EU_1(x) = q_2\left(q_3\left(\frac{X_1 - 0}{X^{HH}} - (X_1 - 0)\right) + (1 - q_3)\left(\frac{X_1 - 0}{X^{HL}} - (X_1 - 0)\right)\right) +$$

$$(1 - q_2)(q_3\left(\frac{X_1 - 0}{X^{LH}} - (X_1 - 0)\right) + (1 - q_3)\left(\frac{X_1 - 0}{X^{LL}} - (X_1 - 0)\right)$$

In the case where the second player does not join, $X_2 = X_1$ thus, $\sqrt{X_1} = X^{LH}$ and $X_1 = X_2 = X_{LL}$ in the case where both players do not join. Substituting, and simplifying:

$$EU_1(x) = q_2\left(q_3\left({X^{HH}}^2 \frac{2X^{HH} - q_3}{(q_3 X + 2(1 - q_3))}\right) + (1 - q_3)\left(X^{HH} \frac{2X^{HH} - q_3}{(q_3 X + 2(1 - q_3))}\right)\right) +$$

$$(1 - q_2)(q_3\left(\sqrt{{X^{HH}}^3 \frac{2X^{HH} - q_3}{\left(q_3 X^{HH} + 2(1 - q_3)\right)}}\right) + (1 - q_3)) - \frac{2X^{HH} - q_3}{2 - q_3} {X^{HH}}^2$$

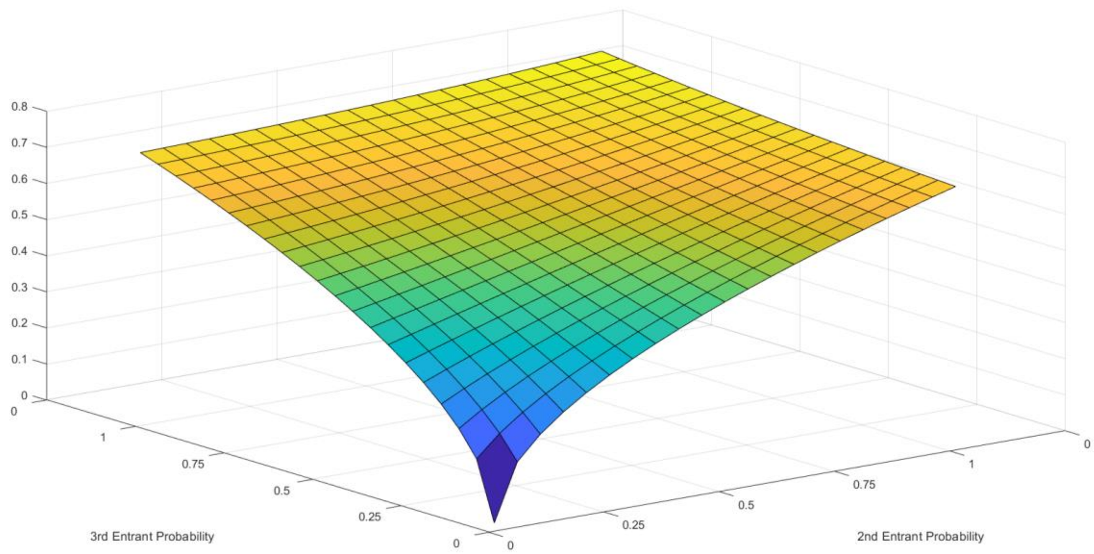


Figure B.1. Aggregate N.E. Predictions: Multiple Probability Combinations

B.I Additional Results

Tables B.1-B.5 Below are results for the pairwise comparisons for each of the listed values in Tables 2.3 and 2.4. Reported values are t-statistics. Table B.6 shows the full specification corresponding to Table 2.5.

Table B.1. Group Total Comparisons: Sequential

	HetP1	*HetP2*	*HetP3*	*RandomP1*	*RandomP2*
Control	3.211***	2.52**	-.021	.081	-3.06***
	(.001)	(.012)	(.984)	(.936)	(.002)
HetP1		.493	2.985***	2.62***	5.53***
		(.622)	(.003)	(.009)	(.000)
HetP2			2.342**	2.08**	4.91***
			(.020)	(.039)	(.000)
HetP3				-.092	2.80***
				(.927)	(.005)
RandomP1					2.67***
					(.008)

*Note: Reported statistics are t-statistic values for two-sided t-test comparisons of total group effort between treatments. Each group observation in a period is treated as an independent observation. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table B.2. Group Total Comparisons: Simultaneous

	HetP1	*HetP2*	*HetP3*	*RandomP1*	*RandomP2*
Control	.172	.308	-2.13**	-2.68***	-2.52**
	(.864)	(.758)	(.033)	(.008)	(.012)
HetP1		-.135	2.19**	2.68***	2.54**
		(.893)	(.029)	(.008)	(.011)
HetP2			2.30**	2.74***	2.66***
			(.022)	(.006)	(.008)
HetP3				-.862	.579
				(.389)	(.563)
RandomP1					-.298***
					(.766)

*Note: Reported statistics are t-statistic values for two-sided t-test comparisons of total group effort between treatments. Each group observation in a period is treated as an independent observation. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.*

Table B.3. Group Overbidding Comparisons: Sequential

	HetP1	*HetP2*	*HetP3*	*RandomP1*	*RandomP2*
Control	-.361	1.52	-.021	4.93***	-1.63
	(.718)	(.130)	(.984)	(.000)	(.104)
HetP1		-1.79*	-.313	-5.03***	-1.86*
		(.074)	(.755)	(.000)	(.064)
HetP2			1.42	-3.12***	4.91***
			(.157)	(.002)	(.000)
HetP3				-4.56***	-1.52
				(.000)	(.129)
RandomP1					2.67***
					(.008)

*Note: Reported statistics are t-statistic values for two-sided t-test comparisons of total group effort between treatments. Each group observation in a period is treated as an independent observation. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table B.4. Group Overbidding Comparisons: Simultaneous

	HetP1	*HetP2*	*HetP3*	*RandomP1*	*RandomP2*
Control	-1.16	-.969	-3.44***	-.222	-.139
	(.247)	(.333)	(.000)	(.825)	(.890)
HetP1		-.135	2.19**	-.670	-1.12
		(.893)	(.029)	(.485)	(.261)
HetP2			2.30**	-.556	-.967
			(.022)	(.578)	(.335)
HetP3				-2.49**	-3.09***
				(.013)	(.002)
RandomP1					-.298***
					(.766)

*Note: Reported statistics are t-statistic values for two-sided t-test comparisons of total group effort between treatments. Each group observation in a period is treated as an independent observation. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table B.5. Group Overbidding Comparisons: Sequential

	HetP1	*HetP2*	*HetP3*	*RandomP1*	*RandomP2*	*Control*
	-3.67***	-2.38**	2.97***	-1.80*	1.54	2.37**
	(.000)	(.018)	(.003)	(.074)	(.126)	(.019)
Observations	250	240	250	200	200	220

*Note: Reported statistics are t-statistic values for two-sided t-test comparisons of total group effort between treatments. Each group observation in a period is treated as an independent observation. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

Table B.6. Individual Overbidding by Player Type: All Treatments

	Dependent Variable: Overbid
	All Treatment
Player 2	14.98***
	(1.533)
Player 3	29.23***
	(1.622)
RiskPref	-4.126***
	(0.450)
GPA	-3.071**
	(1.218)
Economics	0.636**
	(0.296)
Age	0.380
	(0.234)
Female	7.589***
	(1.561)
Comprehension	-10.32***
	(2.691)
LastWin	8.127***
	(1.465)
Constant	36.82***
	(3.327)
Observations	3,120
R-squared	0.138

*Note: The above table reports the results of a regression with dependent variable that is equal to the difference between individual effort choice and predicted ex-ante individual Nash choice. Standard errors are clustered at the group-period-session level. p-values are given in parentheses. *** p<0.01, ** p<0.05, * p<0.1*

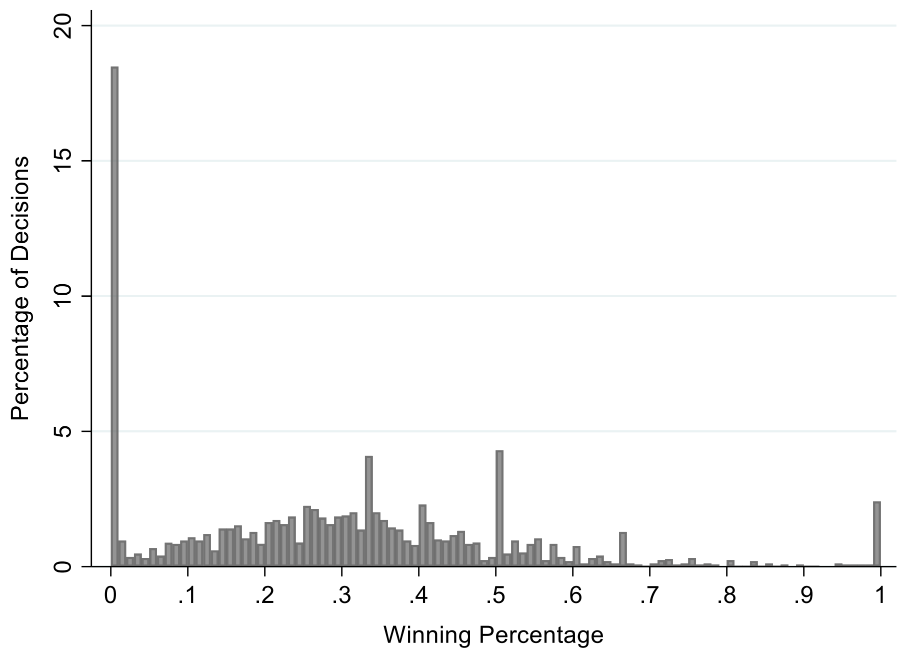


Figure B.2. Induced Winning Percentage for Player 3: All Decisions

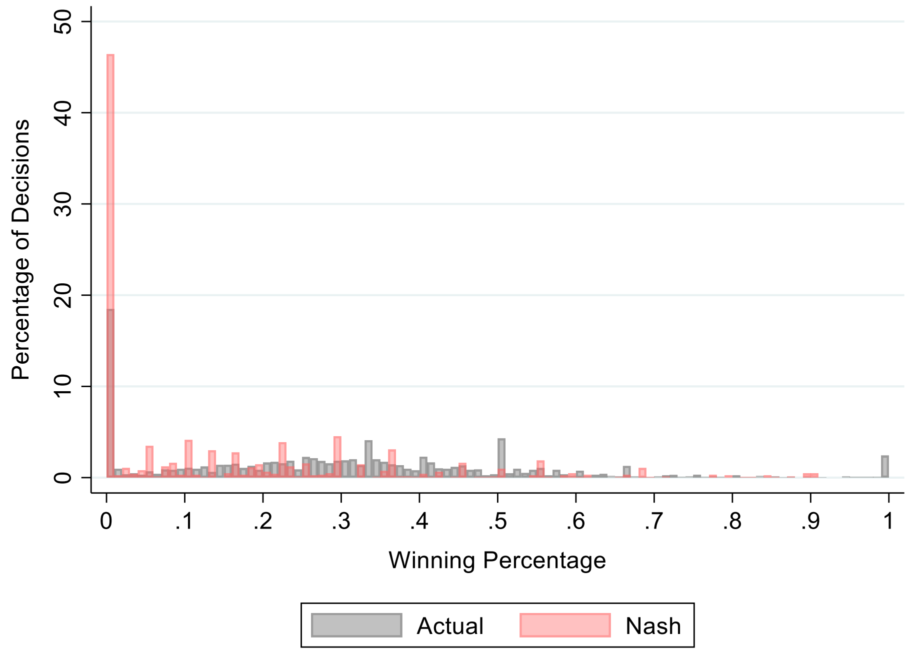


Figure B.3. Induced Winning Percentage for Player 3: Nash Against Actual All Decisions

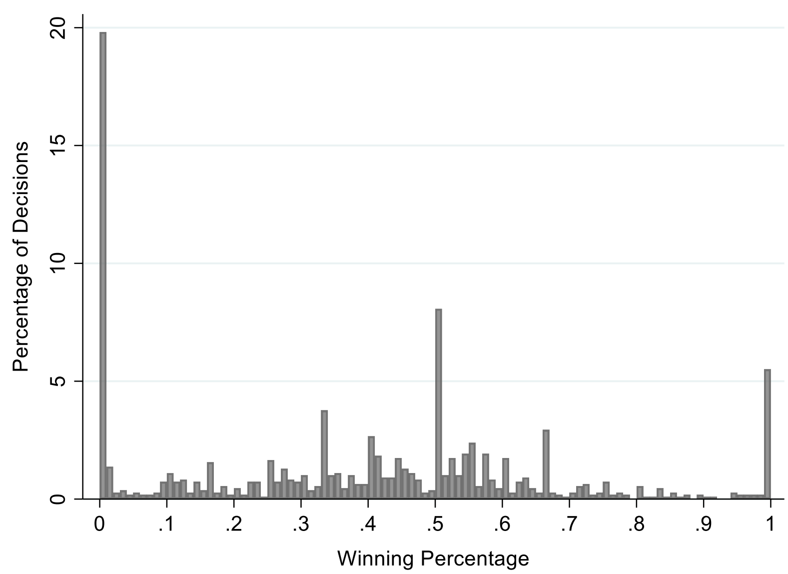


Figure B.4. Induced Winning Percentage for Player 3: Input Totals Less than 100

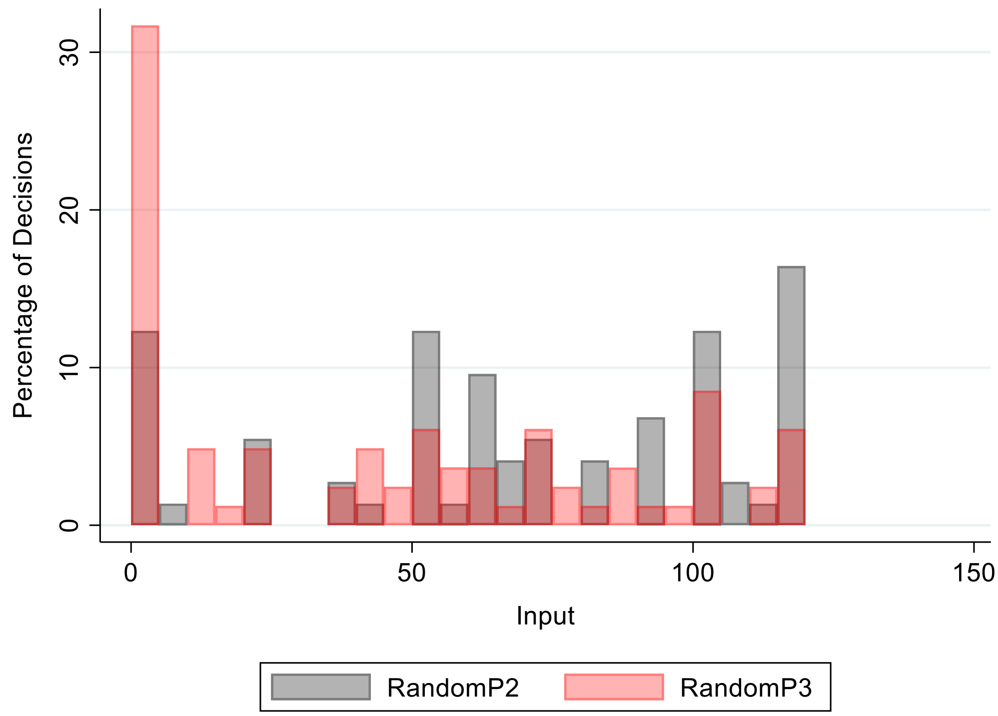


Figure B.5. Player 3 Effort Choices: Random Treatments

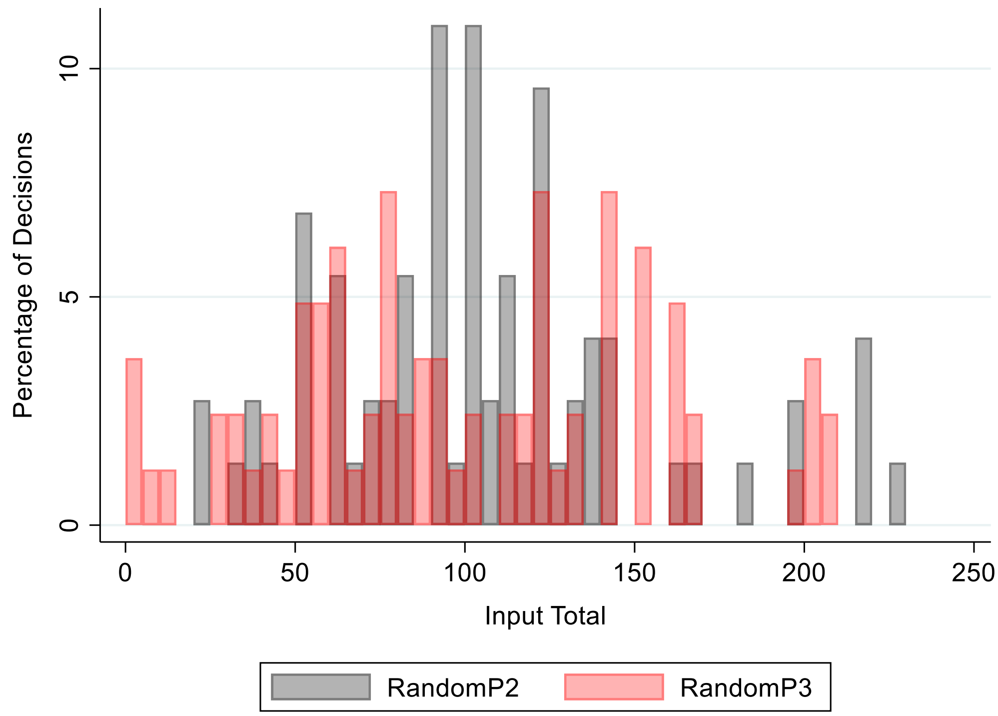


Figure B.6. Player 1 and Player 2 Effort: Random Treatments

Experiment Instructions

Introduction

Thank you for participating in today's study. Please follow the instructions carefully. At any time, please feel free to raise your hand if you have a question.

You have been randomly assigned an ID number for this session. You will make decisions using a computer. You will never be asked to reveal your identity to anyone. Your name will never be associated with any of your decisions. In order to keep your decisions private, please do not reveal your choices or otherwise communicate with any other participant. Importantly, please refrain from verbally reacting to events that occur.

Today's session has three parts: Experiment 1, Experiment 2, and a short questionnaire. You will have the opportunity to earn money in both experiments based on your decisions. You will be paid your earnings privately, and in Amazon gift card, at the end of the experiment session. We will proceed through the written materials together. Please do not enter any decisions on the computer until instructed to do so.

Experiment 1

Please refer to your computer screen while we read the instructions.

We would like you to make a decision for each of 10 scenarios. Each scenario involves a choice between receiving $2 for sure (Option B) or playing a lottery that pays $4 or $0 with the stated chances (Option A).

You will notice that the only differences across scenarios are the chances of receiving the high or low prize for the lottery. At the end of the session, **ONE** of the 10 scenarios will be selected at random and you will be paid according to your decision for this selected scenario **ONLY**. Each scenario has an equal chance of being selected.

Please consider your choice for each scenario carefully. Since you do not know which scenario will be played out, it is in your best interest to treat each scenario as if it will be the one used to determine your earnings.

Before making decisions, are there any questions?

Please proceed to entering decisions on your computer. Once you are ready to submit your decisions, please click the "Submit" button

Experiment 2

In this experiment, all money amounts are denominated in lab dollars, and will be exchanged at a rate of 170 lab dollars to 1 US dollar. You will make a series of similar decisions over the course of several decision rounds. Your final payment will depend on the decisions you make in each round. Aside from decisions in "training" rounds, each decision you make impacts your earnings, which means that it is very important to consider each decision prior to making it. Each decision round is separate from the other rounds, in the sense that the decisions you make in one round will not affect the outcome or earnings of any other round.

In each round you will be randomly placed into three-person groups. You will compete with your group members in order to win a prize of 100 lab dollars. Only one member in each group will win the prize. Your task in each decision round is to decide how many lab dollars to bid toward winning that prize. Who wins the prize depends upon the total bids from you and the other members in your group. The chance to win the prize depends on the following formula:

$$Chance\ of\ winning = \frac{Your\ bid}{Total\ group\ bids(including\ your\ own)}$$

For example, if you and each of your two other group members bid 10 lab dollars then a total of 30 (10+10+10=30) lab dollars were bid. Thus, you will have a 33.33% chance of winning (10 out of 30, or 10/30, .3333). Additionally, each other player in your group would also have a 33.33% chance of winning due to their bid of 10 out of 30.

For another example, say you were to bid 10 lab dollars and both of your group members bid 20, then you have a 10 out of 50 (10+20+20) chance of winning, or 20%. While your group members each have a 20 out of 50 chance, or 40% chance. Note that you do not necessarily know how many lab dollars will be bid by the other players before the round, this is up to each player to decide.

In each round, you will receive 120 lab dollars in fixed income. This amount does not depend on your decision or whether you win the prize. You can bid anywhere from 0 to 120 lab dollars into the pot but *your bid is subtracted from your fixed income.* While increasing your bid will increase the chance you win the prize, any bid lab dollars are lost.

IF you win :

$$Your\ earnings = Value\ of\ the\ Prize + Fixed\ Income - Bid$$

IF you do not win the prize…

$$Your\ earnings = Fixed\ Income - Bid$$

<u>Proceeding through the experiment</u>

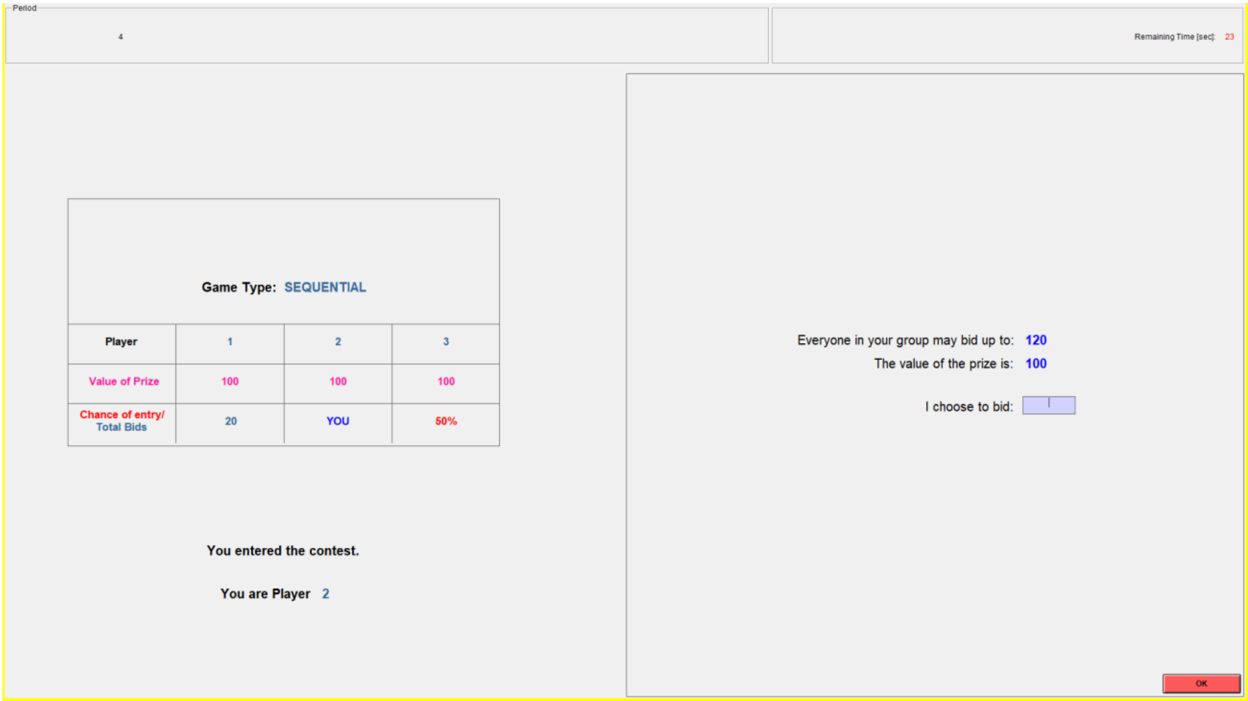
As mentioned before, at the start of each round, you will be randomly matched into a group of three players. This group will change each round. This means that the members of your group will vary from one round to the next. At the start of each round you will be randomly given an identifier, 1, 2, or 3. Each group member has an identical probability of being either member 1, 2, or 3.

In this experiment, players will make their decisions in order. We will call this a *Sequential* game. Player 1 will start, followed by Player 2, then Player 3. In the case of Player 2 and Player 3, these players will know the total of bids that have been made before them.

Additionally, in all decision rounds, Player 3 will have a 50% chance of not entering the contest (or, equivalently, an 50% chance of entering). In either case (whether Player 3 joins or not), you will not know if your fellow participant entered or not. If you are one of the players who is randomly selected to not enter, you will receive 80 lab dollars (plus your 120 in fixed income) for the round, but will make no contribution decision and no contribution. In this case, a screen is automatically displayed that prevents your decision from being made, and nothing needs to be done on your end.

As for the group members that are participating in the contest, it proceeds as normal. However, now the total contributions are made up of only those who have entered the contest. For example, if Player 3 does not enter, the total contributions are made up of only those from Player 1 and 2. Say that Player 1 contributes 10 and Player 2 contributes 20, then there are 30 total bids in the pot (Player 1 has a 10/30=33.33% chance of winning and Player 2 has a 20/30=66.67% chance of winning). The prize and cost remain identical. (Note: There will *always* be at least 2 people in the contest).

This information will be displayed on your screen whenever you make a decision, as shown below



On this screen, you can that Player 2 is the decision maker (when they enter the contest). He/she can observe that a bid total of 20 has been made so far. He/she can also see that Player 3 is still to play but has a 50% chance of not entering. He/she can also see that everyone in the group may bid up to 120 and the value of the prize for all players is 100.

One last possibility is that there are zero contributions, in this case each player who *entered* will have an equal probability of winning the prize. A player who does not enter, can never win the prize.

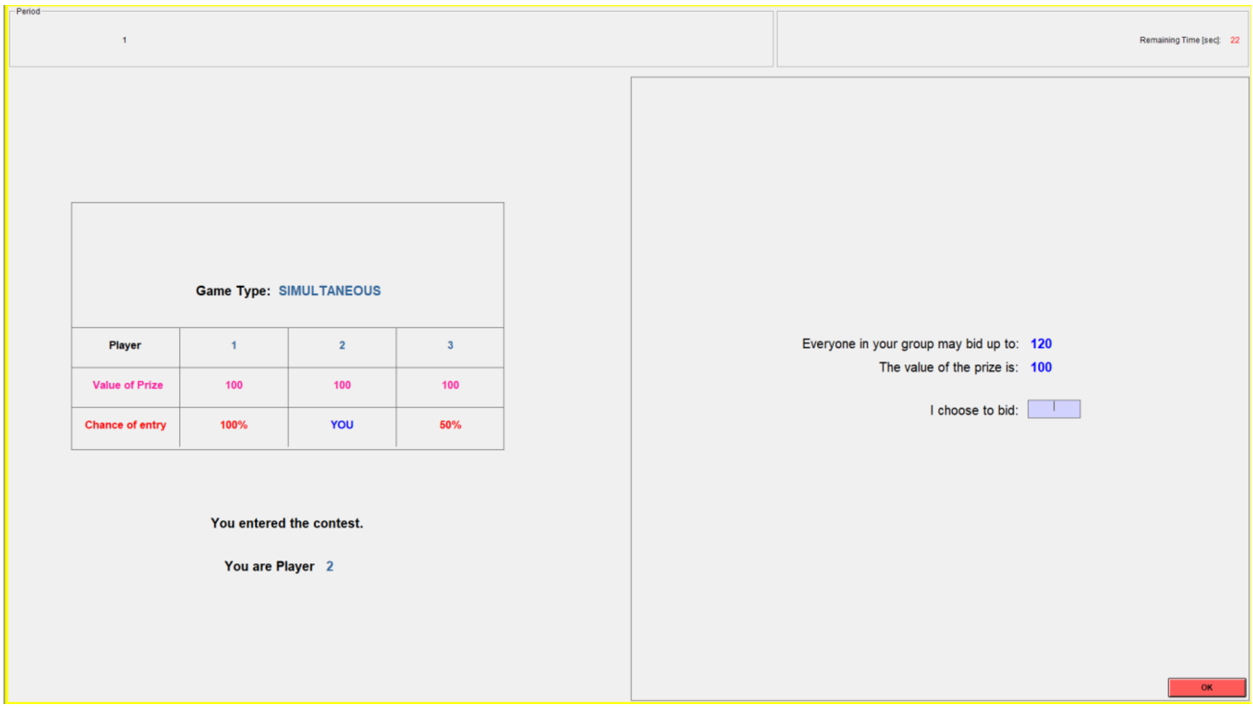
Lastly, you will be given a summary of the total contributions from your group, your chance of winning (if applicable), and your earnings for the round.

We will begin with two training rounds to help you understand the procedures. In the first practice round all players will enter (to practice making contributions). In the second practice round (and for all paid rounds) players will enter randomly as explained above.

Before we continue, do you have any questions?

Ok, thank you for your participation so far. At this time, we will make a slight change to the experiment. All rules for the contest will remain the same but instead of players making decisions sequentially, they will be made *Simultaneously*. Nothing in the rules of the game changes otherwise. Player 3 will still each have a 50% chance of entering.

The information about prize values and total bids will be displayed on your screen.



On this screen, you can that Player 2 is the decision maker (when they enter the contest). He/she can see that Player 1 will is simultaneously taking part in the contest with 100% chance while there is only a 50% chance that Player 3 is taking part in the contest. He/She also knows that any player that has entered the contest is making a decision at the current time. He/she can also see that everyone in the group may bid up to 120 and the value of the prize for all players is 100.

We will again have 2 practice rounds. In the first practice round, each player will join. In the second (and all subsequent paid rounds), the random entry chances described above will be followed. We will then resume the paid decision rounds.

Do you have any questions?

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Experiment 2

In this experiment, all money amounts are denominated in lab dollars, and will be exchanged at a rate of 170 lab dollars to 1 US dollar. You will make a series of similar decisions over the course of several decision rounds. Your final payment will depend on the decisions you make in each round. Aside from decisions in "training" rounds, each decision you make impacts your earnings, which means that it is very important to consider each decision prior to making it. Each decision round is separate from the other rounds, in the sense that the decisions you make in one round will not affect the outcome or earnings of any other round.

In each round you will be randomly placed into three-person groups. You will compete with your group members in order to win a prize of 100 lab dollars. Only one member in each group will win the prize. Your task in each decision round is to decide how many lab dollars to bid toward winning a prize. Who wins the prize depends upon the total bids from you and the other members in your group. The chance to win the prize depends on the following formula:

$$Chance\ of\ winning = \frac{Your\ bid}{Total\ group\ bids(including\ your\ own)}$$

For example, if you and each of your two other group members bid 10 lab dollars then a total of 30 (10+10+10=30) lab dollars were bid. Thus, you will have a 33.33% chance of winning (10 out of 30, or 10/30, .3333). Additionally, each other player in your group would also have a 33.33% chance of winning due to their bid of 10 out of 30.

For another example, say you were to bid 10 lab dollars and both of your group members bid 20, then you have a 10 out of 50 (10+20+20) chance of winning, or 20%. While your group members each have a 20 out of 50 chance, or 40% chance. Note that you do not necessarily know how many lab dollars will be bid by the other players before the round, this is up to each player to decide.

You can bid anywhere from 0 to 120 lab dollars into the pot. While increasing your bid will increase the chance you win the prize, any bid lab dollars are *lost*.

In each round, you will receive 120 lab dollars in fixed income. This amount does not depend on your decision or whether you win the prize. Your earnings for the decision round will be calculated as follows:

IF you win the prize…

$$Your\ earnings = Value\ of\ the\ Prize + Fixed\ Income - Bid$$

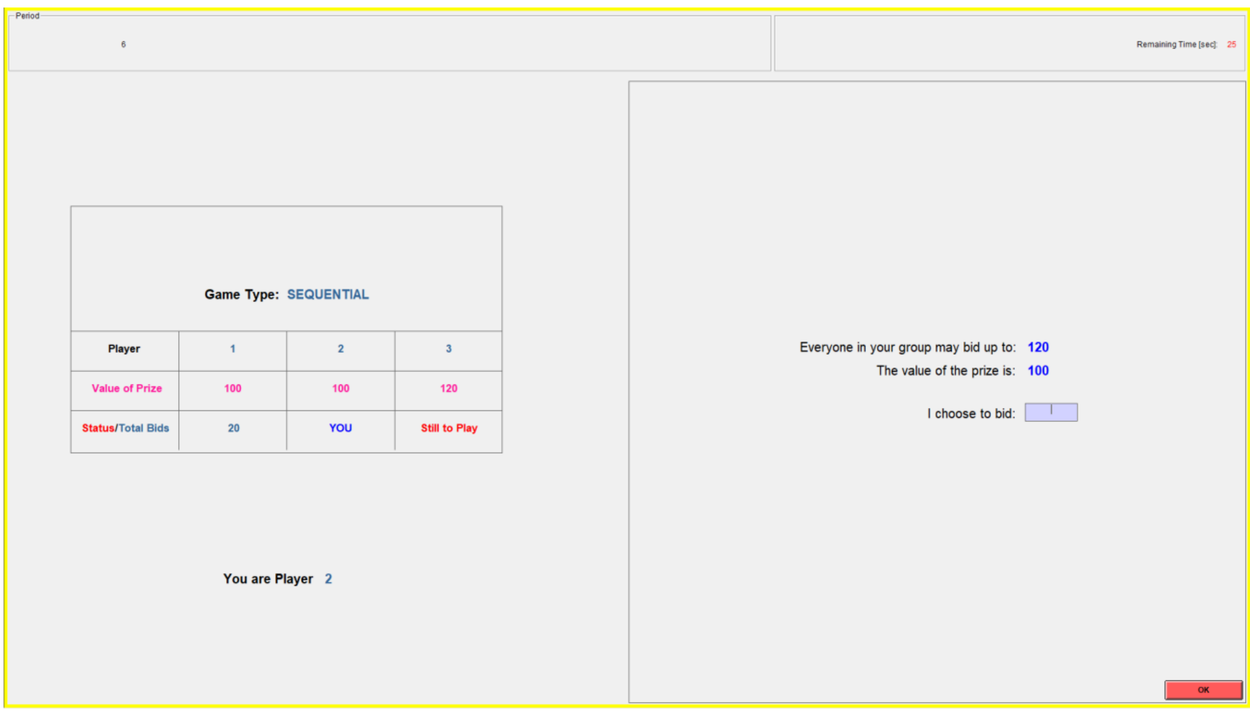
IF you do not win the prize…

$$Your\ earnings = Fixed\ Income - Bid$$

<u>Proceeding through the experiment</u>

As mentioned before, at the start of each round, you will be randomly matched into a group of three players. This group will change each round. This means that the members of your group will vary from one round to the next. At the start of each round you will be randomly given an identifier, 1, 2, or 3. Each group member has an identical probability of being either member 1, 2, or 3.

In this experiment, players will make their decisions in order. This is known as a *Sequential* game. Player 1 will start, followed by Player 2, then Player 3. In the case of Player 2 and Player 3, these players will know the total of bids that have been made before them. Additionally, in all decision rounds, one player (Player 3) will receive 120 from the prize rather than 100, if they win. This information will be displayed on your screen whenever you make a decision, as shown below.



In this screen, the decision maker is Player 2. He/She can see that Player 1 and Player 2 receive 100 from the prize and Player 3 receives 120. He/She can also see that 20 tokens have already been bid and that Player 3 will play after him/her.

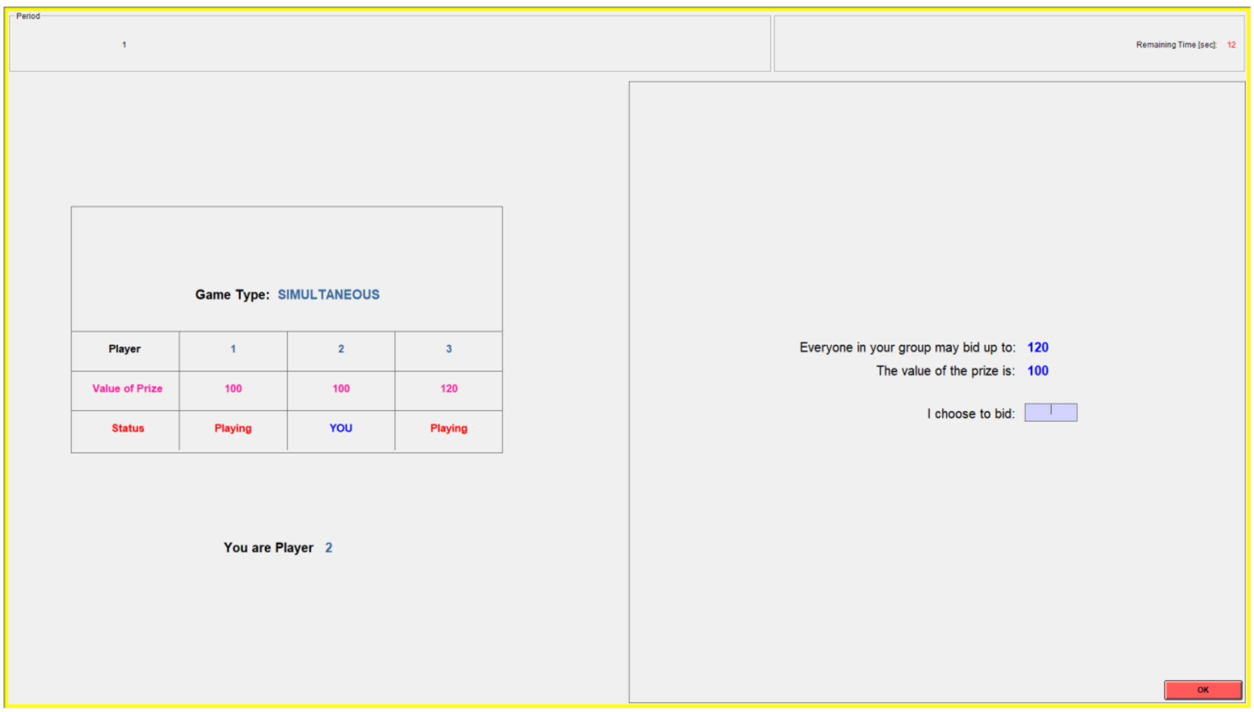
One last possibility is that there are zero bids, in this case each player will have an equal probability of winning, Aside from decisions in “training” rounds, each decision you make impacts your earnings, which means that it is very important to consider each decision prior to making it. As mentioned before, each decision round is separate from the other rounds, in the sense that the decisions you make in one round will not affect the outcome or earnings of any other round. Lastly, you will be given a summary of the total contributions from your group, your chance of winning (if applicable), and your earnings for the round.

We will begin with two training rounds to help you understand the procedures.

Before we continue, do you have any questions?

Ok, thank you for your participation so far. At this time, we will make a slight change to the experiment. All rules for the contest will remain the same but instead of players making decisions at the sequentially, they will be made *simultaneously*. Thus, each player will make their decision at the same time and will have no knowledge of other players decisions. Player 3 will still have a value of the prize of 120, while Players 1 and 2 have valuations of 100. Nothing in the rules of the game changes otherwise. The information about prize values and total bids will be displayed on your screen.

Please click continue and you will see an example screen.



In this screen, the decision maker is Player 2. He/She can see that Player 1 and Player 2 receive 100 from the prize and Player 3 receives 120. He/She can also see that Player 1 and Player 3 are making the decisions at the same time

We will again have two practice rounds. We will then resume the paid decision rounds.

Do you have any questions?

C Appendix-Chapter 3

Bonacich Centrality

Centrality is a concept that is closely related to degree (number of connections a node has) that captures the influence of a node on the network. In the example of this paper, let the network be unweighted (the influence among linked agents is equivalent) and equal to λ, thus increasing effort of agent A generates a complementarity λ, that increases the output of other agents B linked to the agent A. The resulting increase to agents B then increases the output of agents C (also including A), at a rate of λ^2, and so on. In the limit the benefit becomes $\lim_{k\to\infty} \lambda^k$, requiring $\lambda \leq 1$ for convergence. Additionally, the effort values are complementarity and strictly increasing the output of connected agents, thus the cost of own effort, b_i, must be sufficiently high to outpace the increase gained from complementarities. With this set up, Bonacich (1987) proposed the following measure of centrality:

Using the n-square adjacency matrix $\boldsymbol{G}$, the matrix that keeps track of all of the connections in network $\boldsymbol{g}$, let $\boldsymbol{G}^k$ be the kth power of $\boldsymbol{G}$. $\boldsymbol{G}^k$ has coefficients $[\boldsymbol{g}_{ij}^k]$, where k is an integer. The elements of $\boldsymbol{G}^k$ then represent the number of paths of length k from $\boldsymbol{g_i}$ to $\boldsymbol{g_j}$**.** Given a scalar $\lambda \geq 0$ and a network $\boldsymbol{g}$**,** define the matrix

$$\boldsymbol{M}(\boldsymbol{g}, \lambda) = \sum_{\boldsymbol{k=0}}^{\infty} \lambda^{\boldsymbol{k}} \boldsymbol{G}^{\boldsymbol{k}}$$

Using the equivalent matrix property of a geometric series,

$$\boldsymbol{M}(\boldsymbol{g}, \lambda) = [\boldsymbol{I} - \lambda \boldsymbol{G}]^{-\boldsymbol{1}}$$

Thus, under the condition $\lambda \leq 1$, λ^k is a decay factor that scales down the influence of each path, $\boldsymbol{g}_{ij}^k$, more for longer path lengths, k. The components of matrix $\boldsymbol{C}(\boldsymbol{g}, \lambda)$, $c_{ij}(\boldsymbol{g}, \lambda) = \sum_{\boldsymbol{k=0}}^{\infty} \lambda^{\boldsymbol{k}} \boldsymbol{g}_{\boldsymbol{ij}}^{\boldsymbol{k}}$ counts the number of paths in $\boldsymbol{g}$ that start in i and end at j, where paths of length k, are weighted by λ^k.

The Bonacich centrality of a node i is $\beta(\boldsymbol{g},\lambda) = \sum_{j=i}^{n} c_{ij}(\boldsymbol{g},\lambda)$ and it counts the *total* number of paths in $\boldsymbol{g}$ that start at *i*. It is the sum of the loops $c_{ii}(\boldsymbol{g},\lambda)$ from i to i itself and of all the outer paths $\sum_{j\neq i}^{n} c_{ij}(\boldsymbol{g},\lambda)$ from i to every other player $j \neq i$. That is,

$$\beta_i(\boldsymbol{g},\lambda) = c_{ii}(\boldsymbol{g},\lambda) + \sum_{j\neq i}^{n} c_{ij}(\boldsymbol{g},\lambda)$$

By definition, $c_{ii}(\boldsymbol{g},\lambda) \geq 1$ and thus, $\beta_i(\boldsymbol{g},\lambda)$.

Proof:

For any given network $\mathbf{g}_{\mathbf{m_s}}$, and corresponding adjaceny matrix $\mathbf{G}_{\mathbf{m_s}}$, the corresponding Bonacich centrality vector is given by $\mathbf{b}(\mathbf{g},\lambda)$. With individual components

$$b_{im}(\boldsymbol{g},\lambda) = c_{iim}(\boldsymbol{g},\lambda) + \sum_{j\neq i}^{n} c_{ijm}(\boldsymbol{g},\lambda)$$

Now for m+1, results in the adding of a connection to the network. Any configuration, $\mathrm{s}_{\mathrm{m+1}}$, can be built by adding a single connection so some configuration s_{m}. Thus, for any

$$b_{i(m+1)}(\boldsymbol{g},\lambda) = c_{ii(m+1)}(\boldsymbol{g},\lambda) + \sum_{j\neq i}^{n} c_{ij(m+1)}(\boldsymbol{g},\lambda) \geq b_{im}(\boldsymbol{g},\lambda) = c_{iim}(\boldsymbol{g},\lambda) + \sum_{j\neq i}^{n} c_{ijm}(\boldsymbol{g},\lambda)$$

With strict inequality for at least 1 i. ∴ $\mathbf{b}_{(\mathbf{m+1})}(\mathbf{g},\lambda) > \mathbf{b}_{\mathbf{m}}(\mathbf{g},\lambda)$ in turn $\pi_{\mathrm{m+1}} > \pi_{\mathrm{m}}$, applying this logic iteratively for all $\mathrm{i} \in \{1, \mathrm{M} - \mathrm{m}\}$ we get the result, $\pi_{\mathrm{m+i}} > \pi_{\mathrm{m}}$.

(Technical Assumption) Where μ is a fixed parameter that allows us to restrict the support of $p(x)$ to (μ,∞), allowing us to optimize x at marginal cost A. *i.e.* The principal pays for any x above μ:

$$(x-(\mu))A$$

This does suggest that with no investment x, there is some small probability that connections form. We maximize the expected profit with respect to investment to get FOC: (Utilizing the even property of $P(x); p'(x) = -p'(-x)$

$$\frac{d\pi}{dx} = 0 = \sum_{m=0}^{M} p'(x)\pi_m \binom{M}{m} [-(M-m)\big(1-p(x)\big)^{M-m-1} p(x)^m + (m)\big(1-p(x)\big)^{M-m} p(x)^{m-1}] - I$$

Rewriting,

$$\frac{d\pi}{dx} = \sum_{m=1}^{M+1} p'(x)\pi_{m-1} \binom{M}{m-1} [-(M-(m-1))\big(1-p(x)\big)^{M-(m-1)-1} p(x)^{(m-1)} + (m-1)\big(1-p(x)\big)^{M-(m-1)} p(x)^{(m-1)-1}] - I$$

Utilizing the relationship,

$$\big(M-(m-1)\big)\binom{M}{m-1} = m\binom{M}{m}$$

$$\big(M-(m-1)\big)\frac{M!}{(M-(m-1)!\,(m-1)!} = m\frac{M!}{(M-m)!\,m!}$$

And comparing the second term in the first equation and first term in the second equation the FOC can be reorganized as follows:

$$0 = \sum_{m=1}^{M} (\pi_m - \pi_{(m-1)}) m \binom{M}{m} (p(-x)^{M-(m-1)} p(x)^m) - I \quad (5)$$

The summation term is the marginal benefit of increasing investment in x. The benefit is derived from the changing of the probabilities by adding a connection. From Lemma 1 the term $(\pi_m - \pi_{(m-1)})$ is strictly positive. The rest of the terms $m\binom{M}{m}(p(-x)^{M-(m-1)} p(x)^m$ are strictly positive. The Marginal cost is constant and equal to I. Thus, the optima of the objection function are implicitly defined by equation (5). The next section shows that the investment choice x defined by equation (5) exists and is a unique maximum and minimum of the objective function.

Existence

Lemma 2 The Marginal Benefit of the objective function approaches zero as $x \to \pm\infty$.

Proof: $\underset{x\to\infty}{lim} f(x) = \lim_{x\to\infty} \sum_{m=1}^{M} (\pi_m - \pi_{(m-1)}) m \binom{M}{m} (p(-x)^{M-(m-1)} p(x)^m$

$$\underset{x\to\infty}{lim} f(x) = \underset{x\to\infty}{lim} \sum_{m=1}^{M} (\pi_m - \pi_{(m-1)}) m \binom{M}{m} (p(-x)^{M+1} \left(\frac{p(x)}{p(-x)}\right)^m)$$

$$\underset{x\to\infty}{lim} f(x) = \sum_{m=1}^{M} (\pi_m - \pi_{(m-1)}) m \binom{M}{m} \underset{x\to\infty}{lim} (p(-x)^{M+1} \underset{x\to\infty}{lim} \left(\frac{p(x)}{p(-x)}\right)^m)$$

$$\underset{x\to\infty}{lim} f(x) = \sum_{m=1}^{M} \left(\pi_m - \pi_{(m-1)}\right) m \binom{M}{m} (1)(0)$$

$$\underset{x\to\infty}{lim} f(x) = -I$$

$$\underset{x\to\infty}{lim} f(x) = \lim_{x\to-\infty} \sum_{m=1}^{M} (\pi_m - \pi_{(m-1)}) m \binom{M}{m} (p(-x)^{M-(m-1)} p(x)^m$$

$$\underset{x\to-\infty}{lim} f(x) = \underset{x\to-\infty}{lim} \sum_{m=1}^{M} (\pi_m - \pi_{(m-1)}) m \binom{M}{m} (p(-x)^{M+1} \left(\frac{p(x)}{p(-x)}\right)^m$$

$$\underset{x\to-\infty}{lim} f(x) = \sum_{m=1}^{M} (\pi_m - \pi_{(m-1)}) m \binom{M}{m} \underset{x\to-\infty}{lim} (p(-x)^{M+1} \underset{x\to-\infty}{lim} \left(\frac{p(x)}{p(-x)}\right)^m$$

$$\underset{x\to-\infty}{lim} f(x) = \sum_{m=1}^{M} \left(\pi_m - \pi_{(m-1)}\right) m \binom{M}{m} (1)(0)$$

$$\underset{x\to-\infty}{lim} f(x) = -I$$

Given sufficiently low investment cost I such that,

$$\sum_{m=1}^{M}\left(\pi_m - \pi_{(m-1)}\right)m\binom{M}{m}\left(p(-x)^{M-(m-1)}p(x)^m\right) > I$$

for range $x \in \{x^*_{min}, x^*_{max}\}$ $x^*_{max} > x^*_{min}$ and that $MB \rightarrow 0$ for $x \rightarrow \pm\infty$ the function must cross I at least twice which include $\in \{\mathrm{x}^*_{\mathrm{min}}, \mathrm{x}^*_{\mathrm{max}}\}$

C.I Additional Results

Figure C.1 shows the adjancency matricies used in section IV.A. Figures C.2 shows the marginal change in Invesment with changes in cost of I. Figures C.3 and C.4 show additional results corresponding to Figure 3.6 with differing values of β.

$$G_{Symm} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G_{Asymm} = \begin{bmatrix} 0 & 5/2 & 5/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1/2 & 1/2 & 1/2 \\ 1 & 1 & 0 & 1/2 & 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure C.1. Adjacency Matrices- Working from Home Example

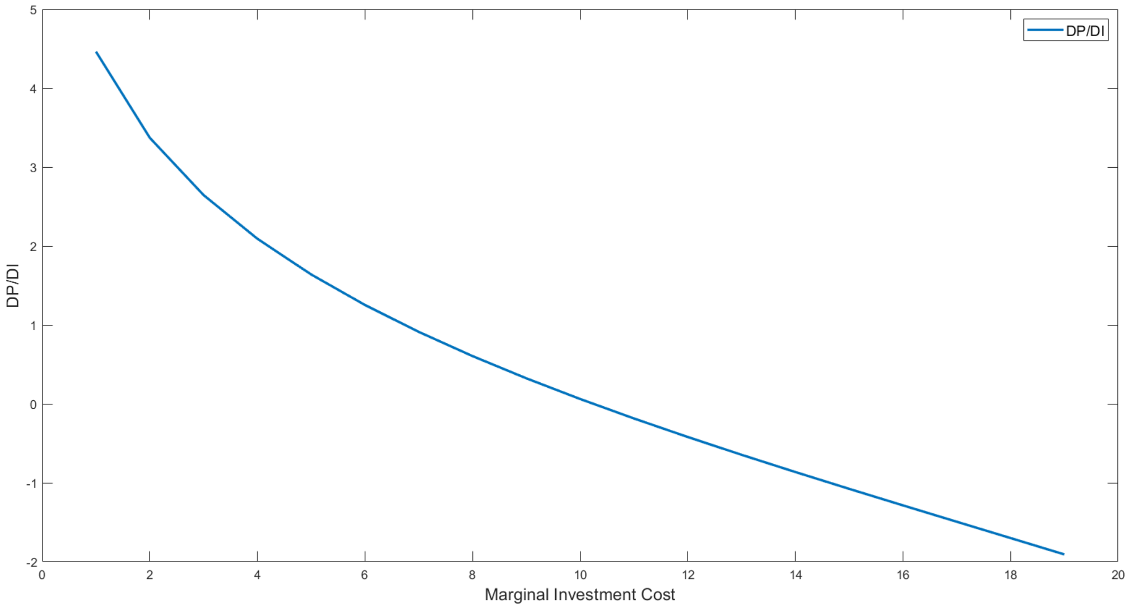


Figure C.2. Change in Link Probability with Change in *I*

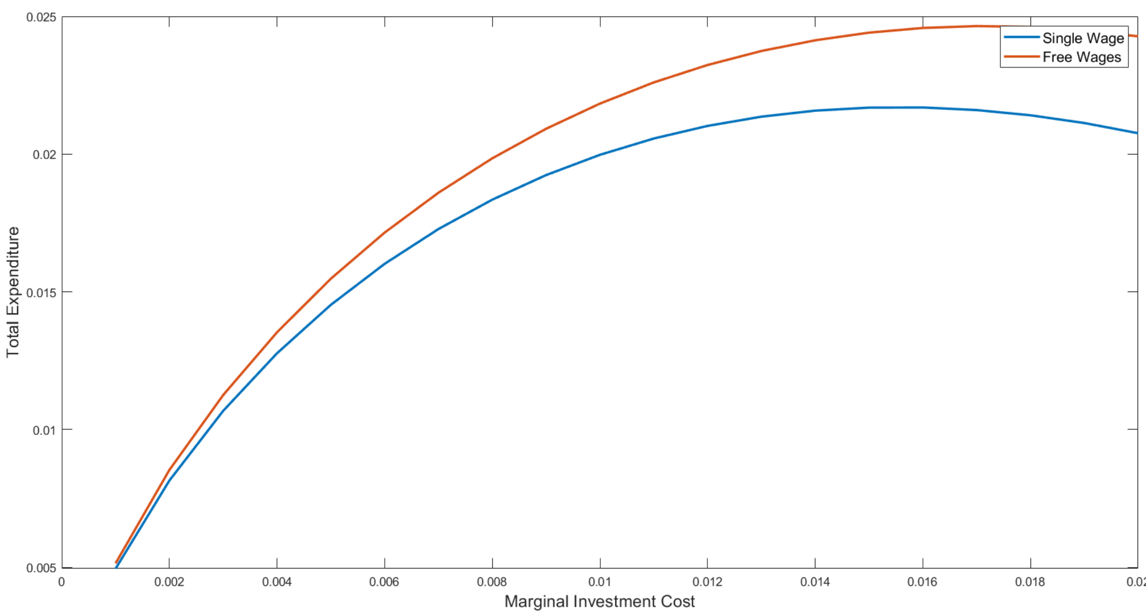


Figure C.3. Total Investment Expenditure: $\beta = .8$

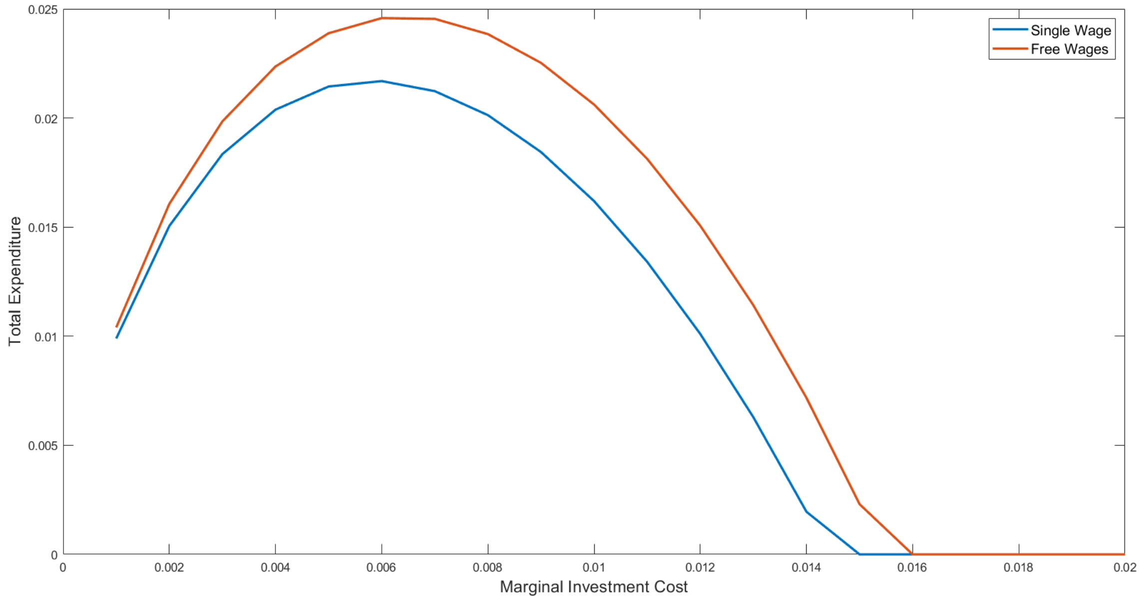


Figure C.4. Total Investment Expenditure: $\beta = .3$

Vita

John McMahan was born Lincoln, Nebraska on May 21st, 1991. He received a Bachelor of Science degree in Mathematics and Psychology from the University of Nebraska-Lincoln in May of 2015. He received a Masters of Arts degree in Economics from the University of Tennessee-Knoxville in January of 2017 and his Doctor of Philosophy degree in Economics from the University of Tennessee-Knoxville in December of 2021.