5-2022

Math for Liberation: Designing a Math of Voting Systems Unit for Lewiston Middle School

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"A mathematics education that is ultimately worthy of Black children is one that prioritizes their liberation above all else." - Dr. Danny Bernard Martin

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A Senior Thesis

Presented to the Department of Mathematics

Bates College

in partial fulfillment of the requirements for the

Degree of Bachelor of Arts

by

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Lewiston, Maine

April 13, 2022

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Acknowledgments

I would like to extend my gratitude to Nicole Pendexter, Alicia Ricker, and Ty Thurlow of Lewiston Middle School, all of whom took time out of their busy days to help me develop this project. Thanks to Patti Buck for connecting me with Ms. Burke and helping me develop initial ideas for this project. Thank you to Andee Alford for her encouragement to present this work at the National Numeracy Network and for her persistent belief in me. I also appreciate Andee Alford and Grace Coulombe for always chatting with me while I was doing my thesis break laps in the library, and for their mentorship throughout my Bates experience. I must also thank Adriana Salerno and Katy Ott both for introducing me to the topic of the mathematics of democracy in their class Math for Social Justice, and for their unwavering support since my freshman year. I wouldn't be here without you two.

I also had the pleasure of working with Darby Ray and Sam Boss of the Harward Center and the rest of the Community Engaged Research Fellows this winter, all of whom contributed valuable insights into the community-engaged nature of this research.

I am also exceptionally grateful to the Gene A. Clough Fund for Precollege Science and Math Education, which allowed me to properly compensate Ms. Burke for her time and efforts.

To my friends, of whom there are too many to name, who kept me sane with karaoke, movie nights, Italian Bakery, craft time in the common room, and their endless support. I want to thank especially Thomas Costin, for all of our thesis study halls and your endless patience. You listened to every sentence of this thesis before it was written, from breakfast in Commons, to between sit-ups at the gym, to the library basement at 9pm, and I could not have done it without you.

I'd like to thank my mom, dad and grandma for the endless hours they spent listening to me talk out ideas on the phone until I made sense of them. I'm also grateful to my sister for at least pretending to listen, and for always making me laugh.

I cannot begin to express my thanks to my advisor, Carrie Diaz Eaton. You were the best air traffic controller I could ask for. You pushed me beyond what I knew I was capable of, and I'm deeply grateful to have worked under your guidance.

Finally, I would like to express my deepest gratitude to the incomparable Angela Burke and the fabulous students of room 203. Angela, I've learned not only a great deal about how to be a good teacher from working with you these past seven months, but also a great deal about how to be a good person. Your eagerness to learn, compassion for your students, and ability to find laughter in every moment (and I mean *every* moment) is something I deeply admire and will carry forward into the rest of my life. To the students of 203, without you there is no thesis. Thank you for accepting me into your classroom, and for your deep thinking about math and voting (and superpowers!). I am glad to have crossed paths with all of you. Keep being the confident, brilliant mathematicians that you are!

Introduction

Math equity reform has longtime been a question of access and achievement. Scholars have asked questions like, who gets recommended for gifted programs? What types of schools offer calculus? And, most prominently, who "does well" in mathematics by traditional measures like standardized test scores, and how can we close this "achievement gap?" Decades of research in this field have suggested only incremental changes in math education, none of which change the fundamental structures that keep historically underrepresented students such as Black, Indigenous, Latinx, poor and female students from thriving in the math classroom.

The recent shift in math equity research locates the deficit not in oppressed students but in the field of mathematics itself. Scholars have begun to ask, instead of making learners "play the game" of mathematics by the dominant culture's rules, how can we instead "change the game" of mathematics so all learners can thrive?

Axiomatic proof is an apt metaphor here. The field of mathematics has been built on certain axioms – statements which we have long-taken to be self-evident without proof. An incomplete list of these assumptions includes meritocracy, that the best will succeed, inherent ability, that some people are good at math and others aren't, and objectivity, that math is neutral and culture-free. Beginning with these axioms in the context of white supremacy, heteropatriarchy, settler colonialism, and other oppressive structures, it is no surprise that what followed logically was a system of mathematics that efficiently ranks white men above everyone else.

Professor of Education and Mathematics Dr. Danny Bernard Martin writes that any reform "in service to acknowledging and valuing Black humanity can start with the axiom of Black learners' brilliance" [**57**]. What would it mean to begin mathematics anew with new axioms? What if we began with the axioms that math is a site of connection and community, that all students bring valuable knowledge to the math classroom, that the creation of math knowledge is a continual, cultural and communal pursuit, and that math both shapes social realities and can be used to transform them? Then what follows logically may be a liberatory mathematics, which all students can use to understand and to change their world.

CHAPTER 1

Lewiston Middle School Context

Lewiston is unlike many other places in Maine. It is a former mill town that began to decline in the 1970s, shrinking in population and jobs [23]. At that time, its main cultural influence was French-Canadian [50]. Change came in 2001, when Somali refugees began settling in Lewiston, drawn to its cheap housing prices and small city setting [23, 50]. Word of mouth spread, and between 2001-2009 alone over 4,000 new migrants made Lewiston their home [23]. These migrants are primarily Somali, within which just under half are ethnically Bantu, but there are also migrants from Sudan, the Congo, and other African countries [23, 50]. This influx of people has revitalized Lewiston, increasing per-capita income and driving down crime rates [23].

Due to this unique community in which it is situated, Lewiston Middle School is far more racially diverse than the general population of Maine public schools (Figure 1.1). It currently houses 781 students and 61 teachers with 29 support staff [53]. The student population is a little over half White, with Black students making up the second-largest group, and all other racial and ethnic identities composing 5% or less of the school population. Compared to the general population of public school students in Maine, the most significant gap is LMS's significantly higher relative proportion of Black students. At LMS, 36% of students identify as Black or African American, compared to 4.08% of all public school students in Maine [53, 56]. 53.7% of LMS students are white, compared to 87.85% of students statewide. 3.3% of LMS students are Hispanic or Latino compared to 2.66% statewide, Asian students are equal at 1.4% both at LMS and statewide, and 5% of LMS students identify as being two or more races compared to 3.07% statewide. 0.4% of LMS students are Native Hawaiian or Pacific Islander compared to 0.1% statewide. Finally, the only racial category LMS has less of than the state average is American Indian or Alaska Native at 0.3% of LMS students compared to 0.83% statewide.

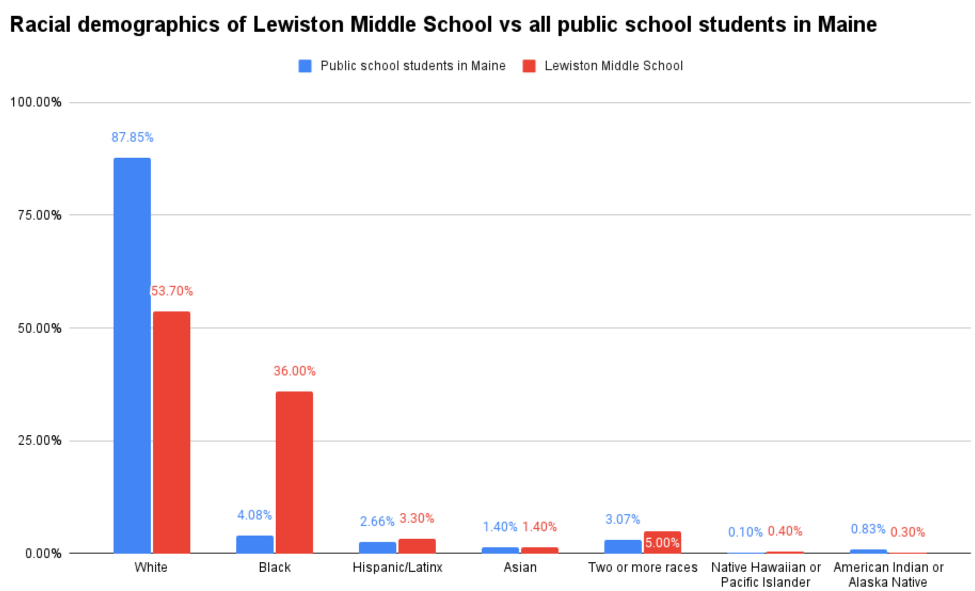

FIGURE 1.1. Racial demographics of Lewiston Middle School compared to all public school students in Maine as of 2021 Maine Department of Education data.

Lewiston Middle School also has a higher concentration of economically disadvantaged students and English Language Learners than the general population of public school students in Maine, and has a slightly higher rate of chronic absenteeism (Figure 1.2). At LMS, 65%

of students are economically disadvantaged, meaning they qualify for free or reduced lunch [53]. Statewide, 39.85% of public school students are economically disadvantaged [56]. The Maine DOE defines English Language Learners as "students with a primary or home language other than English who are in the process of acquiring English" [52]. At LMS, 23.1% of students are English Language Learners [53]. Statewide, only 3.10% of public school students are English Language Learners [56]. The most common home language for English Language Learners statewide in 2021 was Somali [55]. About 22.7% of LMS students are chronically absent, meaning they are absent 10% of the time or more [53]. Statewide, 18.3% of students are chronically absent [53].

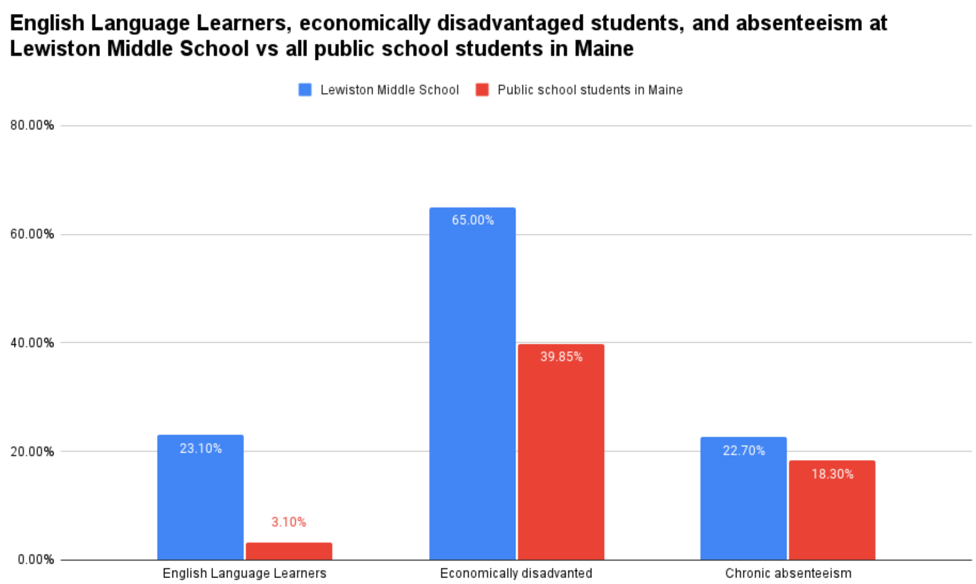

FIGURE 1.2. English Language Learners, economically disadvantaged students and absenteeism rates at Lewiston Middle School compared to all public school students in Maine, 2021 data from Maine Department of Education.

The differences in state assessment scores between the LMS average and Black students or ELL students at LMS is striking (Figure 1.3). In the four school years from 2015-2019, the average percentage of students performing at, or above state expectations was 20.6% in math, 40.7% in science, and 26.4% in English Language Arts (ELA) [53]. The average percentage of Black students performing at, or above state expectations was 5.7% in math, 23.1% in science, and 12.3% in ELA. The average percentage of ELL students performing at, or above state expectations was 3.5% in math, 11.1% in science, and 3.95% in ELA.

The score gap for Black students is most striking in mathematics. Whereas the percentage of students meeting or exceeding standards drops by about half for Black students in literacy and science, it drops by almost three-quarters in mathematics. It is important to remember that these gaps in scores are not indicative of student deficits but rather of the failure of the system to serve these students.

In 2017, the Maine ACLU, Disability Rights Maine, and Kids Legal concluded a two-year-long investigation into race and disability accommodations in Lewiston Public Schools (LPS) [35]. One of the key issues they identified as a civil rights violation was disproportionate discipline based on race and disability. They found that Black students were three times as likely to be suspended as White students, and two times as likely to receive an out-of-school suspension. Students with disabilities were three times as likely to receive an out-of-school suspension. Black students with disabilities had the highest rate of suspension, 8.7 times higher than White students without disabilities (the group with the lowest suspension rate). The organizations requested that LPS "develop a concrete set of strategies, objectives,

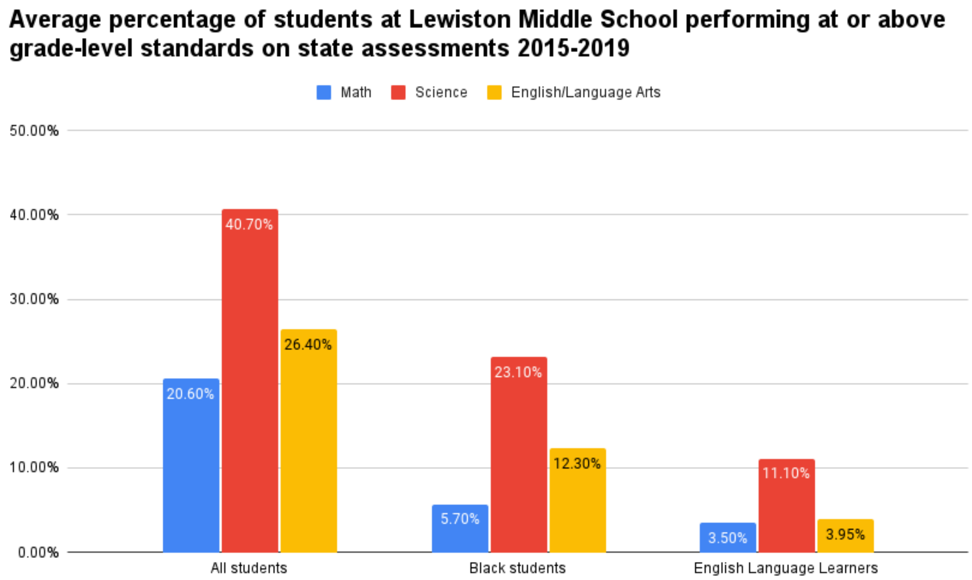

FIGURE 1.3. Average percentage of students at Lewiston Middle School performing at or above grade-level standards on state assessments between 2015 and 2019 by race and English Language Learner status. Data from Maine Department of Education.

and timelines to eliminate race and disability disparities in school discipline" and continue to monitor these disparities in collaboration with community members. The report also found that the teaching force in LPS is overwhelmingly White. Students interviewed for the report said they'd never even heard of a Black teacher in LPS. The organizations requested that LPS begin an aggressive recruitment campaign for more teachers and staff of color, especially Somali teachers.

More recently in 2021, The U.S. Department of Justice investigated ELL and disabled students' rights in LPS. They found the LPS ELL program to be inadequate and "fail[ing] to... provide appropriate EL services" [82]. Students could be in the program for years and not become fluent in English. The most impactful requirement from this report is a mandatory period of English instruction for ELL students every day.

The school climate of LMS is currently in flux, as the school goes through several transitions. It gained a new principal this year in Ty Thurlow. Students, teachers, and staff are also trying to transition back to in-person instruction after over a year of distance and hybrid learning due to the COVID-19 pandemic. Anecdotally, many teachers at LMS cite the pandemic as the cause of a perceived rise in disruptive student behavior. A recent article in the Sun Journal featured a former LMS ed-tech who reported fights almost every day and a climate of fear as a result [65]. In the same article, Superintendent Jake Langlais speculated that the unpredictability of COVID-19 and pandemic restrictions have stressed out students and staff. He also cited concerns about high levels of disrespect and talking back from students.

The daily schedule of LMS is slightly different this year than previous because of the Department of Justice mandate that all ELL students receive a period of English instruction every day. Students start the day with homeroom. Then they have a period called What I Need, or WIN. During this time, ELL students receive their English instruction, and all other students go to the class they need the most support in for remedial lessons. After WIN, students have three hour-long academic classes that follow a rotating block schedule based on the day of the week. After their third class, they attend CREW, a 25-minute advisory period. On Mondays and Tuesdays during CREW, students have RULER training, a mandated social-emotional learning program. On Wednesdays, CREW is an academic check-in, during which students can complete missing assignments. On Thursdays and Fridays, CREW is team building, with the activity determined by the teacher. Then they attend their last academic class of the day as determined by the block schedule. Students have math and literacy every day but have social studies and science every other day. The fourth rotating block is either art, PE, or music on a trimester system. Lunch time also rotates on the trimester system.

Lewiston Middle School uses a team approach with six teams total: Voyageur, Yosemite, Haleakala, Glacier, Denali and Olympic. Students only attend classes with other students on their team. There are six teachers on each team: two math teachers, two literacy teachers, one social studies teacher, and one science teacher. Teams are also divided by grade with half of the teams being 7th grade and the other half being 8th. Teachers loop with the same students for two years. The building is three floors tall with each team getting one wing of one floor.

Finally, as of 2022, the first student-voice committee was founded at LMS, entitled the CREW Leadership Team. It is composed of the school's Expeditionary Learning Coordinator and two students from each team (twelve students total) and convenes every Friday. It was first proposed by school staff in January, and elections took place in February. Students self-nominated for the team. Students on each team then voted for their representatives via Google Form. The approach each team used for voting varied. Haleakala chose a checklist approach, asking students to choose two of the candidates listed. There were a few issues with students choosing more than two, but not many compared to the amount who voted.

CHAPTER 2

Review of Relevant Pedagogical Theory

Overview of Theories Studied

Thirteen well-known pedagogies for teaching learners on the margins were included in this review. The theory of *Universal Design* began in architecture in the 1980s, which aimed to create environments that are usable by as many people as possible with a critical eye towards how the environment determines who is "disabled" [**12**]. An example of universal design in architecture is curb cuts, which make getting around more accessible for wheelchair users as well as kids on scooters or parents with strollers. Educators began to apply this principle to curricula in the 1990s.

The fundamental principle of Universal Design for Learning is to create lessons that are inclusive and flexible from the start in order to meet needs across learner variation. The goal is to produce "expert learners," individuals who want to learn and know how to learn. It recognizes the curriculum as disabled in who, what and how it can teach, rather than the learners. It addresses these curricular disabilities through providing an adaptable curricula based on three principles: providing students multiple means of representation, multiple means of action and expression, and multiple means of engagement.

Diaz Eaton *et al.* applied Universal Design principles to modeling, creating what the authors call a *Rule of Five* framework [**19**]. The five types of modeling they identify are symbolic, verbal, experiential, numerical and visual. To fully understand a model, students need to be able to move fluently through these five representations. This method of teaching modeling is inclusive of many types of knowledge and learning styles and allows for cross-disciplinary and cross-cultural communication based on this agreed-upon and flexible understanding of modeling.

Culturally Relevant Pedagogy (CRP) has been a popular pedagogy for teaching students of color since Ladson-Billings theorized it in 1995. It aims to both help students succeed academically and affirm their cultural identity while learning to recognize and critique oppressive structures [**43**]. The three dimensions of Cultural Relevance are conceptions of self and others, social relations and conceptions of knowledge.

According to Yosso, *Critical Race Theory* (CRT) is "a framework that can be used to theorize, examine and challenge the ways race and racism implicitly and explicitly impact on social structures, practices and discourses" [**88**]. CRT developed out of the Critical Legal Studies (CLS) movement in the late 1980s. Scholars of color critiqued the CLS movement for its failure to consider race as a central element in the legal system. These scholars moved to form their own field analyzing race and racism in the U.S, drawing from law, sociology, history, ethnic studies and gender studies. Though initially focused on the unfulfilled promises of the Civil Rights era, CRT is today a broad field with applications in many disciplines. In this case, we are talking about how to recognize and challenge the ways racism impacts the classroom. Solórzano (1998) provides a useful five-part framework for understanding Critical Race Theory in an educational setting.

Solórzano's theory marks a transition in the field towards using CRT to interpret educational contexts [**74**]. He specifically focused on the educational experience of Chicanx scholars. He defines CRT in an educational setting as "challeng[ing] the dominant discourse on race and racism as they relate to education by examining how educational theory, policy, and practice are used to subordinate certain racial and ethnic groups." The five principles of CRT in education are the centrality and intersectionality of race and racism, the challenge to dominant ideology, the commitment to social justice, the centrality of experiential knowledge, and the interdisciplinary perspective.

The idea of *Community Cultural Wealth* was first proposed by Yosso (2005). Yosso argues that traditional conceptions of cultural capital, such as the theory proposed by Bordieu and Passeron in 1977, wrongfully use a deficit perspective [**88**]. This older theory frames schooling as a passive activity, where teachers are meant to give students knowledge that is deemed valuable by the dominant society. It argues that white and wealthy students succeed because they already have this valuable knowledge, and poor and non-white students fail because they do not. Yosso asks what would happen if we instead choose to explicitly value the knowledge of poor and non-white students and their communities.

Yosso defines culture as "behaviors and values that are learned, shared, and exhibited by a group of people" [**88**]. We will take this to be our definition of culture as well. She then defines Community Cultural Wealth as "an array of knowledge, skills, abilities and contacts possessed and utilized by Communities of Color to survive and resist macro and micro-forms of oppression." Within this definition, she identifies six dimensions of community cultural wealth: aspirational capital, linguistic capital, familial capital, social capital, navigational capital and resistant capital. Community Cultural Wealth theory aims to empower students of color to recognize and utilize assets in their communities.

The term *Open Education Resources* (OERs) was coined in 2002 during a UNESCO forum on on Open Courseware in Higher Education [80]. OERs do not have an agreed upon definition. UNESCO defines them as teaching and learning materials that can be used and altered freely. However, Lambert argues that "open" is not enough in and of itself, and we must define OERs as specifically in the service of social justice [44]. Thus, Lambert defines OERs as: "free digitally enabled learning materials and experiences primarily by and for the benefit and empowerment of non-privileged learners who may be under-represented in education systems or marginalised in their global context. Success of social justice aligned programs can be measured not by any particular technical feature or format but instead by the extent to which they enact redistributive justice, recognitive justice and/or representational justice."

Open Education Pedagogy (OEP) begins with these goals and applies them to broader interactions in the classroom [18]. It asks how the ideals of collaboration and sharing knowledge contained in Open Education Resources can be applied to classroom practices rather than simply materials. Using OEP, students are to understand knowledge as something continually constructed and revised, and see themselves as active and valuable participants in that process. The content should be inextricable from the community in which it is learned.

Inclusive Pedagogy was coined by Tuitt (2003) as a recommendation for undergraduate professors teaching in diverse classrooms [78]. After reviewing inclusive literature, Tuitt identified four key principles for teachers to create a more inclusive classroom environment: social interaction, sharing power, dialogical teacher-student interaction, and utilization of personal narratives.

Gender-Complex Education “takes into consideration the existence and experiences of transgender and gendernonconforming people” [**67**]. Much of previous math education research has assumed that students fall into the categories “boy” or “girl” based on their bodies. It has also relied on gender essentialism, assuming that all girls behave one way and all boys behave another [**67, 24**]. Gender-Complex Education draws on theories of gender as a performance rather than an inherent trait [**67**]. The field of mathematics is traditionally associated with masculinity. Rubel argues that because of this, girls have to choose between their femininity and “being good at math” [**69**]. Esmonde expands this argument to say that all students have to think about their performance of gender in the math classroom in order to align with the gender performance that social culture says does well in math [**24**]. A shift from such a gendered mathematics – that affects all students – to a gender-complex math education would involve questioning mechanisms of gender oppression in the math classroom, using math to analyze gender oppression, and including representations of gender diversity in the math curriculum [**67**].

Living Mathematx, (pronounced mathematesh), proposed by Gutiérrez in 2017, asks us to consider all living beings, including the natural world, as doers and teachers of mathematics [**30**]. It acknowledges that the knowledge of each living thing is “legitimate, partial and interdependent” and that we must consider all this knowledge together in order to create the math we need to solve the problems that face us collectively, such as climate change. There are three principles within Mathematx. First is “In Lak’ech,” based on a Mayan philosophy, that asks us to recognizes ourselves in others and others in ourself. Second is reciprocity, and third is “Nepantla,” an Aztec word, which in this theory means to seek multiple knowledges and knowers.

A year later, Gutiérrez expanded these ideas into *Rehumanizing Mathematics* [**31**]. She argues that all humans naturally do mathematics in a way that builds off their culture, existing knowledge and interests, but school dehumanizes math by removing this holistic view of math and instead focusing on a “culture-free” procedure-based mathematics. She argues for restoring math to a whole-person endeavor that draws on culture, identity, emotion and the body, for “decoupl[ing] mathematics from wealth, domination, and compliance [and] recoupl[ing] it with connection, joy, and belonging.” Rehumanizing has eight dimensions: participation/positioning, cultures/histories, windows/mirrors, living practice, creation, broadening mathematics, body/ emotions and ownership.

A *Decolonial* mathematics framework analyzes how mathematics is complicit in indigenous oppression, and how we can begin to disrupt settler-colonialism in math. In his talk “Justice and Equity in Mathematics Comes with Land Back,” Belin Tsinnajinnie discusses that inclusion in math is often framed in terms of the benefit of the field, but should be framed in terms of the needs and goals of marginalized communities [**77**]. Settler-colonialism destroyed indigenous ways of knowing and being in math, and math continues to be used as a tool of indigenous oppression, such as the 30-meter telescope being built on Native land in Hawai’i, which wouldn’t be possible without the cooperation of mathematicians. In order to disrupt this, we must ask how progress can be made in mathematics without exploiting people, land or resources, and how we can do math in such a way that we are accountable to the community and places we come from.

The final framework we will discuss is *Abolitionism.* Love defines abolitionist teaching as "the practice of working in solidarity with communities of color while drawing on the imagination, creativity, refusal, (re)membering, visionary thinking, healing, rebellious spirit, boldness, determination, and subversiveness of abolitionists to eradicate injustice in and outside of schools" [**49**]. Abolitionist teachers explicitly teach their students about oppressive social structures and how to resist, yet they equally draw upon the strengths of their students to imagine a better future. They make Black students matter in and out of the classroom.

Themes Identified in the Literature

After conducting a review of inclusive pedagogy, four themes were identified. The first is community and relationships. Many of the theories emphasize the importance of establishing caring relationships in the classroom where students feel a responsibility for each other. The second theme is student expertise, which involves valuing the cultural knowledge and styles students bring to the classroom and intentionally drawing on these strengths in instruction. The third is communal knowledge production, which involves modes of instruction that move away from the traditional structure of teacher and text experts and students as knowledge receivers towards conceptions of knowledge that center students. The fourth is social critique and action, which centers on giving students tools to fight oppression in their lives and communities.

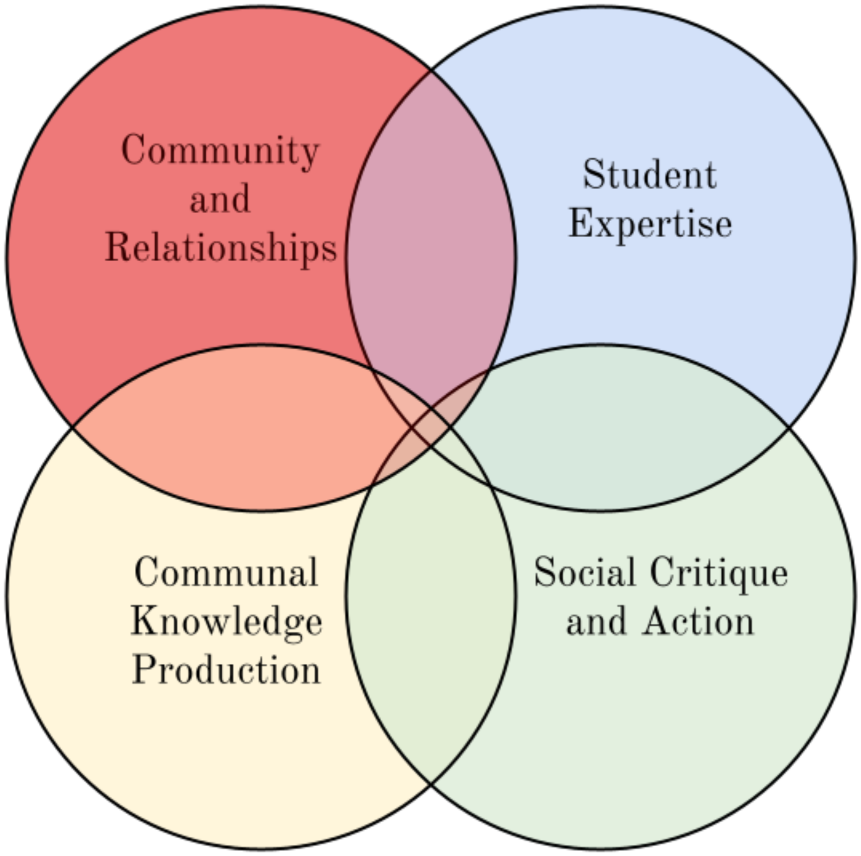

FIGURE 2.1. A visualization of the four themes. They are all interrelated, yet also important to distinguish.

Community and Relationships. We can see this theme clearly in the "social relations" principle of Culturally Relevant Pedagogy [43]. This principle states that teachers should connect with students, develop a classroom community, encourage collaboration and responsibility for others, and create equitable and reciprocal relationships with their students. We can see here that both student-teacher and student-student relationships are emphasized. The social relations principle can also mean connecting with students' families and communities: a classroom-community relationship. For example, in Abdulrahim and

Orosco's systematic review of Culturally Relevant practices in mathematics classrooms, they observed a teacher inviting family members into the classroom to do group work with students as a means of bridging the connection between home and school [**1**].

Joseph *et al.* has good examples of student-teacher relationships [**42**]. The Black girls in the study cited one-on-one teacher attention, approachability, patience and a perceived teacher desire to attend to their needs and help them understand as crucial in forming a good teacher-student relationship. The authors theorize that these attributes represent teachers understanding their Black female students' humanity. They also fit these responses into the "social interaction" principle of Inclusive Pedagogy [**78**]. This principle theorizes that positive social relationships are the basis of a classroom environment where students can understand each others' experiences and develop trust and care, which allows for collective knowledge construction. Social interaction once again covers both student-student and student-teacher relationships.

Leyva's conception of community and relationships focuses mostly on student-student [**48**]. The undergraduate students in this study cited lack of within-group (same-race, same-gender) peer group support as a major exclusionary force in mathematics. They thought that peer group support would lessen pressure on them as individuals to perform academically and help them process their emotional responses to marginalizing events in class. Further, they said that a feeling of within-group solidarity was critical to mitigating marginalizing and discouraging forces, and working with a group of individuals like you in class helps students feel like they belong in mathematics. Leyva recommends professors formalize study groups or otherwise formalize group work that helps students make connections and form networks of support. In the K-8 education context, this could look like assigned peer-group groups for group work or seating charts that group students by a certain identity.

In Living Mathematx, the community and relationships principle is a crucial underlying foundation [30]. Gutiérrez asks us to consider how we are responsible to and for each other and our world. She also acknowledges that knowledge transfer occurs only when a relationship has been formed between giver and receiver. We see this idea most clearly in the first two principles, In Lak'ech and reciprocity. In Lak'ech is fundamentally about empathy, about how we can appreciate both the uniqueness and similarity of others. Reciprocity asks how we can go beyond this appreciation to use our unique strengths to be mutually responsible for each other.

Along with relationships in the classroom, Mathematx also considers our relationship with the Earth, and both principles apply also to the student-nature relationship or student-universe relationship. In the classroom, we might think about asking students to appreciate the reasoning of others rather than critique it, or to appreciate patterns in the natural world. We might establish norms of gratitude for each other and our many knowledges, or analyze mathematical objects through similarity and difference lenses. In Gutiérrez's later theory Rehumanizing Math, we see a very similar theme in her third principle, "windows/mirrors," which asks how students can see themselves and others represented in curriculum and thus understand how they relate to others, and how we all relate to mathematics [31]. Windows/mirrors has a more content-based focus than In Lak'ech, but its end goal is much the same.

Student Expertise. We see the theme of student expertise perhaps most potently in Community Cultural Wealth which is explicitly about valuing the knowledge students of color bring to the classroom [88]. Yosso identified six types of capital we should value in students of color. First is aspirational capital, meaning the ability to dream, cultivate hope, and be resilient in the face of struggle. Second is linguistic capital, which encapsulates the many intellectual and social skills that come from experience with multiple languages or interaction styles. Third is familial capital, which is community connection that teachers community history, memory and cultural intuition. Fourth is social capital, which is networks of support; those who are able to access resources share with their community. Fifth is navigational capital, which is the ability to maneuver through structures of inequity. Finally, the sixth is resistant capital, which encompasses challenging the status quo and resisting oppression through self-love and the maintenance of cultural wealth. All students of color bring unique dimensions of these six types of cultural capital to the classroom, and lessons should be designed to celebrate and accentuate these strengths and funds of knowledge.

Solórzano's Critical Race Theory in education principle "centrality of experiential knowledge" recognizes the experiential knowledge of students of color as not only legitimate knowledge but as critical to understanding racism in education [74]. Inclusive Pedagogy's "personal narratives" also recognizes experiential knowledge as important and legitimate, but in this case for understanding material rather than analyzing racism [78]. It asks teachers to include opportunities for students to connect mathematics to their personal stories. When personal experiences as recognized as legitimate knowledge, student narratives can be used to recognize critique exclusionary structures in math and to connect math to their lives.

In Rehumanizing Mathematics, “cultures/histories” and “broadening mathematics” both draw on students’ expertise and experiences [31]. Cultures/histories respects all of the knowledge that students bring into the classroom, whether “standard” American mathematics or otherwise, and teaches students about the cultural and historical dimensions of math. Broadening mathematics encourages students to draw on the senses, the body and intuition to do mathematics. Through these principles, students are no longer asked to leave their cultural identity or intuition at the math classroom door, but rather assured that these things are assets to mathematics.

Often integration of cultural responsiveness into the classroom looks like connecting content to student context so students are able to apply their own knowledge to the subject at hand. Abdulrahim and Orosco provide plentiful examples [1]. One teacher connected the geography of students’ neighborhoods to the concepts of domain and range. Another linked factoring numbers to students’ kinship structures. A third linked word problems to everyday activities the students did with their families, like cooking and grocery shopping.

This valuing of students’ cultural context extends also to interactional styles, which was noted by Ladson-Billings, Leyva and Rubel. Ladson-Billings notes the importance of encouraging students to “to be themselves in dress, language style, and interaction styles while achieving in school” in order to model that healthy cultural identities do not clash with academic success [43]. Leyva noted that differential opportunities for participation across race and gender was one of the three major push-out forces from mathematics identified by students in the study [48]. Participation in the classes studied was often individualist and rewarded speed and competitiveness. Many of the students of color and women were uncomfortable participating using these styles. Rubel notes that efforts to involve women in math by making them more like men often emphasize this same interaction style: they try to make women more persistent, vocal or confident, or in other words, to communicate in traditionally masculine ways [69].

Instead of forcing students into interactional styles that are not their own, we should expand our idea of participation in math classes. Leyva suggests valuing thinking over speed and accuracy, and valuing questions as a way to increase collective understanding rather than viewing them a a representation of individual deficit [48]. Abdulrahim and Orosco give numerous examples on this theme [1]. An example that showed up often in their study was allowing students to use their home language when problem solving. Another example was teachers including oral storytelling in the same style as their students as an instructional technique.

Providing options for students in how they receive information, communicate information, and engage with the material is the foundation of Universal Design [12]. Not all learners in a classroom will understand all the same cultural references or use the same interaction styles, so the curriculum must be designed with options from the start. The first principle is multiple means of representation, which involves representing information in different ways. An example of this might be offering image and text definitions of a new vocabulary word. The second is to provide multiple means of action and expression, which means allowing learners to express what they know in different ways. This might look like allowing a variety of methods for solving a problem.

The third principle is to provide multiple means of engagement, which involves varying your approaches to ensure all learners are motivated to learn. This might look like allowing students to choose whether they work collaboratively or individually.

Diaz Eaton *et al.* offers a specific example of strength-based student choice in the context of constructing mathematical models [**19**]. The authors identify five ways of representing information: experiential, ex. experiments or simulations, numerical, ex. data, symbolic, ex. equations, verbal, ex. predictions, and visual, ex. graphs. They argue that successful modeling involves moving between these different representations. However, students can choose which representation to start with based on their strengths. The authors also argue that experiential representations of a model are often the most accessible "in" to a problem for a variety of learners, who can then choose to move from experiential to one of the other four representations based on their preference. For example, students could participate in the same simulation, and one student may choose to represent their observations verbally, while another chooses to make a chart.

When we value all students as experts and allow varied ways to participate, we should expect all students to succeed. High expectations and academic rigor are key themes in both Inclusive Pedagogy and Culturally Relevant Pedagogy [**78**, **43**].

Communal Knowledge Production. Critique of traditional classroom knowledge structures go hand-in-hand with recognizing student expertise. Once we establish a community and recognize that all students have valuable knowledge, we can begin to create knowledge collectively.

Open Education Practices empower students to become " active and visible participants in the construction of knowledge" [**18**]. First teachers must establish that knowledge is not set in stone, but continuously created and revised. OEP specifically recommends using Open Education Resources, which themselves are modifiable and collaboratively created, to get this point across. Then, move from a content-centered to a learner-centered approach by centering discussion, allowing critique and modifications of course materials and texts, and connecting the knowledge produced in the classroom to the world outside of its doors. Teachers may also ask students to set their own parameters for a class or lesson by asking questions like "what problems need to be solved?" or "what ideas need to be explored?" These approaches together allow learners to conceive of themselves as creators of valuable knowledge not only in class but in the world.

Culturally Relevant Pedagogy includes a very similar principle entitled "conceptions of knowledge" [**43**]. It recognizes that knowledge is changing and must be viewed critically, and that students should learn from one another. The three approaches Ladson-Billings suggests to change how knowledge is constructed in the classroom are recognizing student experts, open critique of the curriculum, and complex assessment. Recognizing student experts might mean students give presentations on an area of expertise, and other students are expected to respect their knowledge by listening and asking questions. Open critique of the curriculum involves teachers speaking openly about their critiques of required materials and encouraging student voice on the topic. Complex assessment means a move away from right answer approaches or allowing students to choose pieces of evidence to prove mastery of a skill.

Knowledge as an ever-changing entity also appears clearly in principle four of Gutiérrez's Rehumanizing Mathematics, "living practice" [**31**]. Gutiérrez argues that viewing math knowledge as static removes the human aspect of doing mathematics. Anecdotally, students often cite beliefs to me that math was "made up one day just because," and has always existed fully formed and indisputable. Within these creationist myth misconceptions, where is there any room for student participation in mathematics? Gutiérrez writes that when we intentionally teach math as the site of debates and divergent ideas we can reconceptualize math knowledge as a living, changing thing with room for our own ideas. Once students have room in mathematics to play, they will create their own form of mathematics, Gutiérrez's fifth principle, which leads to a sense of ownership, Gutiérrez's eighth principle, and belonging in mathematics. Mathematics becomes something students do, rather than something that is done to them.

To teach math as a living practice in the classroom, we might think about including more of the history and power dynamics in mathematics. This might look like teaching irrational numbers by telling the legend of Hippasus's revolutionary yet simple proof of their existence, and his subsequent drowning at sea for his discovery, or starting the school year with a larger-scale lesson on the history of math so all students begin with the assumption that math is created and revised over time by and for people.

Inclusive Pedagogy's principle "sharing power" explicitly addresses that this type of communal knowledge production asks the teacher to give up the usual power they hold over students [**78**]. Using this principle, everyone in the classroom in equally responsible for constructing knowledge. In practice, this might look like discussions, students being asked to build on each others' ideas, or collaborative inquiry based explorations. This idea feeds directly into the "dialogical teacher-student interaction" principle of IP, which asks the teacher to be a facilitator rather than a "teller."

Rehumanizing again has a nearly identical idea in its first principle, "participation/positioning" [31]. This principle advocates the shifting of authority from teacher and text to student. Gutiérrez imagines that in the classroom, students should respond to each others' ideas rather than seek validation from their teacher.

Mathematx also includes this idea that whoever gets to "know" holds power [30]. Gutiérrez quotes Appelbaum (2016) in calling Western mathematics a tool of "epistemicide," saying it effectively kills ways of knowing that are not traditional in Western math. Its third principle, Nepantla, asks how we can seek out multiple knowledges and knowers in order to build a more complete mathematics that benefits all involved, including the natural world. It asks how we can guide students to think critically about where their own knowledge comes from, and how to accept multiple knowledges at once and the tensions between them.

Tsinnajinnie also recognizes an epistemicide in mathematics [77]. He says that when colonizers arrived, there was a destruction and disruption of indigenous ways of knowing and being, including in mathematics. He notices in schools today, teachers either enforce one method or only demonstrate one method for problem solving in math. He believes this approach is assimilationist and does not allow room for many ways of knowing and being in math. According to him, one way to begin to decolonize math would be to not only accept multiple methods of problem solving, but to demonstrate and actively encourage various ways of knowing.

Social Critique and Action. One of Leyva *et al.*'s recommendations for practice is to explicitly call attention to whiteness and patriarchy in mathematics and its role in exclusion from the field [**48**]. This is a baseline level of social critique: recognizing oppressive structures.

Leyva's approach falls under one tenet of Solórzano's education theory of Critical Race Theory: challenging the dominant ideology [**74**]. Challenging the dominant ideology involves pointing out assumptions in the classroom – such as meritocracy, that the best succeed – and questioning who those assumptions benefit and who they oppress. In the math classroom, this could look like asking students to consider the assumption that math is separate from people, and who benefits from that assumption.

Another tenet of Solórzano's education theory of Critical Race Theory that is relevant in this section is the commitment to social justice. The commitment to social justice acknowledges the goal of Critical Race Theorists as eliminating racial subordination, and by extension all other forms of oppression. Here we extend recognition to action. This could look many ways in the math classroom, so long as you and your students are doing math with an explicit eye on eliminating racial oppression. Both of these tenets involve critiquing and working to change the status quo in the school system.

Martin uses a Critical Race Theory lens to argue that all previous math equity forms have simply tinkered around the edges in ways that are non-threatening to white supremacy and purposefully frame the deficit as within Black children rather than in the educational system that is built to fail them [**57**]. Math access, he writes, is often framed as allowing Black students to become "fuller citizens." But fuller citizens of what? If Black students are afforded such "citizenship" through math, they become complicit in American empire and its oppressive systems. Yet if they remain non-citizens, they are pathologized as threats to national progress. Instead of being caught in this double-edged sword of access, Black students should instead be taught a mathematics that prioritizes resistance, that puts the liberation of Black students above all else.

Culturally Relevant Pedagogy's "cultural critique" principle aims to "help students recognize, understand and critique current social inequities" [**43**]. Teachers should not shy away from socio-political realities Ladson-Billings provides the example of a middle school class who studied school location (in "wet" or "dry" areas of the city – whether it is legal to sell liquor) in relation to race and class of the students. They found that whiter, wealthier schools were in dry areas, while their school and other poor schools were in wet areas, and exposed this inequity using graphs, charts, maps and reports.

Abolitionist teaching begins with the same assumptions and takes them further. Activism and resistance should actively be taught in the classroom, and oppressive structures discussed explicitly [**49**]. Like Martin's theory, abolitionism believes that citizenship in the current America is not enough, and adds the idea of radical dreaming. Students should be empowered to not only recognize current oppressive structures and learn tools to resist and dismantle them, but also to imagine a better way.

Gender-Critical Education also advocates for critique and action. Math should be used as a tool to understand gendered oppression and take action on it [**67**]. Rands gives the example of teaching proportional reasoning and statistical analysis by analyzing GLSEN survey data on student interventions in gender-expression based harassment situations. Students develop their math skills, their understandings of gender, and learn how to challenge systems of gendered oppression when they see harassment take place.

Decolonizing takes on a similar lens. This framework argues that mathematics should center on the needs and goals of marginalized people, and that in practicing mathematics one should be responsible and accountable to their community [77]. Mathematics should be seen and taught as a means of social liberation, not just for oneself but for our communities.

CHAPTER 3

Math for Social Justice

Though "math for social justice" has become increasingly popular, multiple understandings exist, and differ based on ideological and educational agendas [9]. A grounding similarity is that students should see math as a tool to both understand the world and change it; math for social justice should prepare students to become an informed, critical and active citizenry through mathematics [46, 32, 36, 9, 15, 66]. Some approaches simply alter existing textbook problems by including real-world situations and data [73]. Others are completely open-ended explorations of an issue, where students generate questions that can be answered mathematically and conduct their own research and calculations [36]. Some focus only on understanding social issues, while others focus explicitly on action [67]. In most of the lessons that teach math for social justice, students complete one or more of the following steps, grounded in mathematical reasoning: analyze/understand an issue, critique existing systems, connect to their own life or other learning, imagine a better way, and act on their ideas to improve the world.

Though modern social justice math is an international movement across all student ages, it began in the United States in K-12 education [11]. Buell & Shulman (2019) point to Jonathan Osler as the founding father of modern social justice math. Osler founded Radical Math Teachers in 2006 while teaching at a public high school in Brooklyn, New York. Radical Math today holds national conferences and maintains a website of free resources for educators to bring social and racial justice into the math classroom [66]. One of the movement's other founding fathers can be found in Bob Moses: activist, founder of The Algebra Project, and author of *Radical Equations: Civil Rights from Mississippi to the Algebra Project* [11]. Moses maintains that math education is a civil right, and that math has been used to gatekeep Black people out of educational and personal success. Both Osler and Moses emphasize two goals: teaching students the rigorous math they need to succeed in the current system, and teaching students the mathematical tools they need to understand and change the system [64].

Math for social justice has become increasingly visible in recent years, from state standards to teacher education recommendations. Most recently California proposed new guidelines for math education in public schools that suggested using math to explore social issues and that teachers should be conscious of injustice in the classroom, such as being attentive to gender stereotypes in word problems [28]. Seattle Public Schools have also been locked in a long struggle to implement ethnic studies math standards that would include discussing power and privilege in the math classroom [29]. Social justice math also increasingly shows up in publications by prestigious math education organizations. The National Council of Teachers of Mathematics' 2020 publication "Catalyzing Change in Middle School Mathematics: Initiating Critical Conversations" advocates for helping students to understand and critique the world through math, a crucial foundation of social justice math [61]. Finally, the Association of Mathematics

Teacher Educators' 2017 Standards for Preparing Teachers of Mathematics included the standard "Understand Power and Privilege in the History of Mathematics Education" [6].

Though social justice math has been increasingly visible, it is also increasingly controversial. New math curricula have drawn criticism from a diverse opposition, from conservative media to professionals in STEM fields [28, 29]. This controversy is part of a larger tide of pressures on public institutions, especially public schools, in the United States. At the time of writing, 35 U.S. states have passed or considered legislation restricting discussion of race in the classroom or state agencies [4]. Out of those states, 16 signed these bills into law, 6 states failed to pass the bill, and 19 states are still considering. Further, in the wake of Florida passing the Parental Rights in Education bill (dubbed the "Don't Say Gay" bill by opponents) that prohibits education on gender or sexuality before 4th grade, 19 other states are considering similar bills [45].

Opponents of social justice math often pose a false dichotomy between math education and social literacy. It is true that a key challenge in social justice math is balancing social justice content with mathematical goals [9, 66]. Students are owed a rigorous mathematics education because of the gatekeeping role math plays in society [11]. Yet, if social issues are treated superficially, teachers risk their students misunderstanding a complex issue and reinforcing oppression [67, 9]. However, when done right, these goals are not mutually exclusive; the social justice context should enrich the mathematics, not dilute it [11]. One way to remedy this issue is cross-disciplinary support, such as collaborating with a social studies class to teach a topic through multiple disciplines and encourage connections and interdisciplinary reasoning [67].

Some opponents are also concerned that students of color and poor students will get taught social justice math, and wealthy white students will get taught traditional "rigorous" math, widening the gap in math resources and learning [64]. Osler argues that this gap will exist no matter what content is taught, and that teaching students of color and poor students social justice math will give them the tools to change some of the circumstances that make their education inequitable [63].

Finally, others argue that teaching math for social justice may be difficult for teachers given the constraints of public schooling. Opponents argue that math for social justice may not align with mandated standards or curricula, or adequately prepare students for standardized tests [66]. Finally, discussing social justice issues in the math classroom may require a skill set that math teachers rarely flex. They need to facilitate discussion, leave lessons open-ended and flexible, and handle high-emotion topics, which they may not be used to [46].

However, the benefits of teaching math for social justice, if done thoughtfully, far outweigh the pitfalls. Students gain a deeper understanding of math concepts and social issues [32, 66]. They learn to think critically about numbers, and to justify mathematical arguments [15, 9]. They are able to connect math to their own lives, the lives of others, and issues facing their communities [11, 32]. This makes math feel important and relevant, which encourages them to develop a healthy mathematical identity and motivates them to continue to engage with math and social justice issues. Teachers are also able to deepen their relationships with students and their understandings of students' lives and communities [66]. Taken together, these skills empower students as the mathematicians and activists of tomorrow.

Common Topics in Math for Social Justice

Teaching math for social justice most often looks like unit explorations of a particular social issue using math. Common categories for explorations include: criminal justice and incarceration, environmental justice and climate change, housing justice, economic exploitation, community health, and governance [66, 2, 46, 32, 40].

Exploring criminal justice and incarceration might look like Gutstein's "Driving While Black/Driving While Brown" lesson that explores racial profiling in traffic stops in Chicago through the lens of probability simulations [33]. Students start by randomly sampling colored cubes from a bag and estimating the total numbers of each color to conclude that random samples should represent the total population. Students then look at ACLU data about police stops and searches in Illinois to calculate the rate at which white drivers are searched, and then use that rate to calculate how many Black and Latinx drivers should have been searched if the search rate was the same as for white drivers. Finally, students compare that number to the actual number of searches of Black and Latinx drivers. Using this math exploration, they are able to conclude that driver searches in Chicago are not random.

Exploring environmental justice might look like Hendrickson's exploration of fracking, a local issue affecting her students' community [36]. In her lesson, Hendrickson has students brainstorm their own question about fracking that can be answered using math, such as "how many hydro plants does it take to generate the same amount of power as a fracking well produces in one day?" Students then conduct their own research, adapt their questions where necessary, and present their results to their class.

Teaching about housing justice might look like Hoke *et al.*'s exploration of eviction in San Francisco using calculus [**37**]. Students explore real data about rent and eviction rates in San Francisco three times throughout a semester of Calculus I. They use average rate of change, derivative, and area under the curve to better understand how eviction changed over time. They then compared their analysis to events in the news to hypothesize what effect certain taxes and regulations had on eviction in San Francisco.

Exploring economic exploitation could look like Wright's exploration entitled "What's a Fair Wage?," which uses data analysis methods to compare the wages of workers at different fictional fast-food chains [**86**]. Students compare the wages using mean, median, mode, range, and frequency, and think critically about which workers might cite which pieces of data when asked "what's a fair wage?" This lesson could easily be adapted to use real-world data, or explore gendered or racial wage gaps.

Using math to understand community health might look like Aguirre *et al.*'s modeling task of the Flint, Michigan water crisis [**2**]. In this task, students are presented with a media statement by several corporations who pledged to donate 6.5 million water bottles, enough to fulfill the needs of Flint's 10,000 schoolchildren for the rest of the calendar year. Students are then engaged in the open-ended mathematical modeling cycle of doing their own research, making assumptions, and ultimately assessing the validity of the corporations' statement.

Lastly, math can be used to assess the fairness of democracy. This topic will be explored in-depth later, but common topics include comparing voting systems (ex. plurality versus ranked choice), the apportionment of representatives to the House of Representatives in the

United States based on state population, and districting/gerrymandering [40].

However, teaching math for social justice also needs to involve smaller instructional changes that create an inclusive classroom environment. For example, Rands argues for eliminating gender roles and heteronormativity in math problems, such as moving away from problems about marriage, and explicitly including people of many gender identities and sexualities in word problems [67]. Many of the inclusive pedagogical practices reviewed earlier are important elements of teaching math for social justice, because they help all students feel included and valued in the classroom.

Choosing a Topic for Lewiston Middle School

The goals I want my lessons to meet are framed by my identity and relation to mathematics and mathematics education. I am a white, upper-class gay woman. I went to a middle school in Minneapolis, Minnesota that was similar to Lewiston Middle School in racial and national origin demographics, but with likely a wealthier population overall due to the high property values of its surrounding neighborhood. My school has since changed to a K-5 magnet, so current data is unable to capture the school population in 2014. I then went to wealthy, competitive Predominantly White Institutions (PWIs) for the rest of my education. I hated math in elementary and middle school, began to like it in high school, and in college came to truly appreciate its beauty and power. Because of this journey, my motivation for this thesis is to collaborate with LMS students (and specifically underserved students) to together discover the power and meaning of mathematics. In this section, I will be using “I” when speaking from a place of lived experience, and "we" when I am including the reader in my process. In proceeding chapters, "we" may also to refer to myself and the classroom teacher, Ms. Burke, as a team.

Goals for Intervention. As was discussed in Chapter 1, Lewiston Middle School has a shocking racial gap in math scores that indicates a potent failure to serve Black students in math. This unit was primarily designed to engage these students and other students on the margins. Given this context, my first goal is that students are able to see math as a powerful tool to both recognize injustice and to create more just systems. In order to do this, we must look at an issue where students are not only using math to understand and critique social issues, but are also able to propose solutions using mathematics. Ideally, students also are able to advocate for this social change. This goal is informed primarily by abolitionism and Critical Race Theory, which advocate teaching students tools for their own liberation in the classroom [**49**, **57**].

My second goal is to have students engage with genuine math in a genuine context. I want students to have the mathematical power to explore and understand an issue all the way to the bottom – not just skim the surface of an oversimplified issue. The math must be actually used in the real world to think about or solve the issue, and the students need to be able to access and understand all of the math involved. There should be no forcing the issue to fit the math or forcing the math to fit the issue. If the math and context is genuine, that is where students are able to do authentic, meaningful mathematics in the service of justice.

My third goal is for students to form and defend an opinion on an issue using math as a tool of reasoning in concert with other tools. This goal necessitates an open-ended exploration of an issue without clear answers. I do not want to simply create a math lesson for students to arrive at a predetermined discovery, even if it is a valuable and important one. Rather, I want students to practice real-world reasoning, where issues are not so clear-cut and they must evaluate for themselves, decide what they believe and what convinces them, yet also to listen to others and appreciate their point of view. I want there to be multiple paths so students can customize the ask to their needs and goals, and each student's work can be informed by their own perspectives and strengths.

Ms. Burke also has three main goals for the unit. First, she wants students to understand that math is an integrated part of their world. Second, she wants to engage her students in interdisciplinary learning. Finally, she wants to learn new teaching tactics and topics as a fourth-year teacher.

Structural Constraints. I was given permission to design one week's worth of instructional time, where each class runs exactly one hour. It happens that Wednesday of the intervention week is an early release day, so on Wednesday each class runs only 38 minutes, and Friday is a pep rally so each class runs 40 minutes. Since I am only teaching for one week, I am unable to lay a lot of groundwork for the subject covered or classroom routines and culture. Further, I must balance the needs of my host teacher, the students, the principal and administration, and my own needs. For example, I was asked to omit any references to political parties in my lessons. Finally, the lessons should fit naturally with Ms. Burke's existing classroom style and structure, so it is easier for her to take over the lessons in the future.

Theoretical Constraints. To do math for social justice, we must balance the twin goals to doing good justice and doing good math. To do good justice, we must be sure of giving proper nuanced context to the issue, or else risk reinforcing oppression [9, 67]. Drawing from abolitionism, teaching should balance critique of oppressive structures with celebrations of the wealth of marginalized communities [49]. If we spend a week in the classroom talking about how Black bodies are criminalized, that may be the only image white students take away of their Black classmates. In this way, social justice math done without contextual depth can have the opposite effect than intended.

Secondly, we want to be sure that we are doing good math. The math in my lessons should be rigorous, explorative, well-scaffolded and should fit within existing school curricula and structures. It should also align with state standards.

Voting Systems. Reckoning with all these goals and constraints, I landed on exploring different voting systems. At LMS, students start the year with a unit on civics in Social Studies, so they come to class in the spring only needing a refresher on the workings of the U.S. government rather than a full introduction to a social issue, leaving more time for rigorous math. Learning about how democracy works in the U.S. is also one of the overarching categories of grade-level standards in social studies, making this topic ideal for an interdisciplinary unit since it addresses grade-level standards in both disciplines simultaneously [51]. This topic is also specific to student place and context, as Maine adopted ranked-choice voting in 2016 and has been using it in presidential primary elections, general elections and eligible state and federal elections since 2018, one of only two states to do so [25, 59].

The mathematics involved in counting votes is fully accessible to students and fits with the grade-level math standards of frequency tables and data analysis and visualization. This project also addresses Maine state Guiding Principles for Mathematical Practice D and E. Guiding Principle D reads, "A responsible and involved citizen: Students make sense of the world around them through mathematics including economic literacy" [54]. In this unit, students use math to make sense of voting. Guiding Principle E reads, "An integrative and informed thinker: Students connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the role math plays in other disciplines and life." Students connect mathematical learning to social studies learning and the broader world.

Studying voting also has unique opportunities for modeling, forming and defending an opinion, and student advocacy. Voting is particularly well-suited to classroom simulations, which beyond engaging students, are a useful starting point for basic modeling. Further, it allows students to make their own mathematically informed decisions about which system is the most fair in their opinion, in a relevant national and local discussion with no right answers. Finally, voting systems allows for student activism in their school, via creating a proposal for which system should be used for student leadership team elections.

Creating Relationships

Importance of Relationships. The center of this project was the collaborative relationship between Ms. Burke and myself, and our shared knowledge. She was able to contribute deep knowledge of her students: how they engage, their interests, and their strengths. She also knows classroom and school routines and what it's like to be a full-time teaching professional. Her knowledge was crucial in both designing the unit and evaluating its efficacy. I was able to contribute mainly content-area knowledge about the mathematics of voting systems, but also a new framework for how to integrate interdisciplinary projects like this in the future. Through our partnership and shared knowledge, we were both able to fulfill individual goals (e.g. learn what it's like to be a teacher, learn more about voting systems) as well as create a far more effective unit than either of us would have been able to do alone. Finally, this collaborative relationship allowed co-ownership of the unit from the start, supporting long-term unit adoption.

Also important to the project was my relationship with students in order to establish trust and design the unit based around their strengths and interests. Finally, my relationship with administration was important for trust and transparency, since I was working with young people in a public school setting.

Teacher. I first met with Ms. Burke on October 1st, 2021 to discuss creating a week-long unit on mathematics and democracy for her class. The ideas of voting systems, apportionment and gerrymandering were presented. Ms. Burke was most interested in voting systems. We met again on October 15th to discuss scheduling the unit and class observations, grade-level standards, and how she structures her classes. A third meeting was scheduled for November but had to be postponed due to a Covid-19 outbreak at Lewiston Middle School.

During winter semester, Bates suspended in-person community engagement until February 14th, 2022 due to the Omicron variant. I met with Ms. Burke on January 20th via Zoom to discuss goal-setting for the unit and research updates from the fall semester. Once community engagement resumed, we met regularly on Fridays during Ms.

Burke's prep, for a total of five meetings. These meetings were devoted to planning the unit. We also met once more the Friday after the implementation week to discuss our reflections on the unit.

For Ms. Burke's time and efforts, she was compensated $2,000 using funds from the Gene A. Clough Fund for Precollege Science and Math Education.

Students. Ms. Burke teaches three sections of eighth-grade math. Period 3 is at-level and has 14 students. Period 7 is also at-level and has 17 students. Period 5 is algebra (advanced) and has 20 students. In total, 51 students were involved in the intervention. Students are 13 and 14 years old. In the fall semester of 2021, I observed each period once in order to build relationships with students. During observations, my activities varied. During lecture, I quizzed myself on student names and took notes on what students were doing to get a sense for how each student engages in class. During group work and transition times, I talked with students about their interests and helped them with the worksheets if needed.

Class observations continued in winter semester once community engagement resumed, with a goal of observing each class period once a week. This was not always possible due to the unpredictability of the school setting. Early releases, testing days, a field day, and school emergencies all disrupted the observation schedule. In the four weeks between community engagement resuming and the unit implementation, Period 5 was observed three times, Period 3 observed four times and Period 7 observed five times.

Administration. I met with principal Ty Thurlow on Wednesday, October 13th to secure administrative approval. He approved 5 school days of instructional time for the project with the only reported data to be mine and Ms. Burke's reflections on the week.

Assistant Principal Erik Anderson was also informed of the unit by Ms. Burke, and he visited class twice during unit implementation for very brief periods to observe.

CHAPTER 4

Voting Curricular Research

The Mathematics of Voting Systems

Voting is a means of determining a group preference from a set of individual preferences [8]. Debates regarding the countless methods of determining this group preference have continued unabated among mathematicians for many years, along with many meticulously defined "fairness" and "unfairness" criteria and impossibility theorems which draw on these criteria to state in different terms that a "fair" system is impossible [8]. For the purposes of this project we will focus on three vote-counting methods and two "fairness" qualifiers.

The three vote-counting systems, plurality, runoff, and ranked-choice, were chosen because they are the most common in the United States and most relevant to the local politics of the students' community in Maine, where many of them will grow up to vote in ranked-choice elections. The two fairness criterion chosen were the debate between plurality and majority and "spoilers." The plurality versus majority debate is the moral foundation of the differences between these three systems, is easily understood, and gives students room to form their own opinion on what is "fair" mathematically. Spoilers are particularly relevant in the American two-party system and affect voter strategy, so it is important for students to understand this concept as future voters. Spoilers is also the secondary, more practical, reason for the differences between these three different vote-counting methods. Though these two criteria are the main focus, students will also develop their own definition of a fair election throughout the unit (see Chapter 5: Implementation).

Traditional Plurality. Plurality voting is the most common method of selecting a winner in political elections in the United States [**10**]. Simply, the candidate that gets more votes than any other candidate wins. Plurality voting (and in fact all voting systems we will cover) makes use of a preference ballot, which asks voters to rank candidates, though in this case the voter is effectively only ranking one candidate above all of the others. Plurality is voting is widely understood, easy and fast to use, and results are quickly counted mechanically.

A common critique of plurality voting is that it may not represent the will of the majority. That is, if a winning candidate among 3 gets only 40% of the votes, that means 60% of voters oppose that candidate; a majority of voters would prefer someone else. This scenario begs the question of if plurality voting truly represents the will of the people [**84**]. This scenario is not uncommon; in the seven Maine gubernatorial elections between 1990-2014 only one candidate (Angus King in 1998) won with a majority of the votes [**7**].

A second common critique of traditional plurality voting is that it suffers from the "spoiler" problem [**10**]. Put simply, a spoiler is a candidate X whose presence changes the outcome of an election, other than X becoming a winner. This logic is easily understood through the following joke attributed to University of Columbia philosophy professor Sidney Morgenbesser:

> After finishing dinner, Professor Morgenbesser decides to order dessert. The waitress offers two choices: apple pie or blueberry pie. Morgenbesser orders the apple pie. After a few minutes the waitress comes back to the table and says, "I forgot to tell you, we also have cherry pie tonight." Morgenbesser replies, "Oh, in that case I'll have the blueberry pie" [**10**].

One can easily see that the only logical scenarios would be the newcomer, cherry pie, becoming the winner, or Morgenbesser's pie preference remaining unchanged. Any other change is illogical and would mean that cherry pie acts as a spoiler in Morgenbesser's pie preferences.

In the U.S., many consider third-party candidates to act as spoilers since they theoretically take votes away from the two main candidates, and these "lost" votes may change the outcome of an election. For example, in the 2000 presidential election, some claim that third-party candidate Ralph Nader pulled votes away from Democratic nominee Al Gore, clearing the way for a narrow George W. Bush victory [**68**]. If Nader voters whose second choice was Gore voted for Gore instead of Nader, they could have avoided their third-choice candidate being elected. In this way plurality can reward voters for voting dishonestly.

Runoff Elections. In a runoff election, a plurality vote is conducted as usual, and if the winner does not reach a required vote threshold, a second election is held only between the top two vote-getters. In the United States, Georgia and Louisiana use a runoff system in general elections and ten states use runoffs as part of their primary elections [**70**]. This system is also used in many countries internationally including France, Russia and Brazil [**10**]. Runoff rules differ, but each specify a threshold percentage of the votes a candidate must get to win, and some specify a percentage margin the winner must lead by in order to avoid a runoff [62]. Most U.S. states that use a runoff system require a candidate to get a majority (over 50%) of the votes in the first election to avoid a runoff. North Carolina is unique in that it only requires a runoff if the winner receives less than 40% of the votes. Ecuador uses the additional margin requirement, as it requires a candidate have more than 40% of the votes and at least a 10% lead on the runner-up in order to win in the first round [62].

The purpose of runoff elections is to elect a candidate with majority support. A runoff election increases the likelihood that the Condorcet winner will be elected [62]. [1] Put simply, a runoff increases the likelihood of electing the candidate most people prefer to all other candidates. It also helps with the spoiler problem, provided the spoiler is in third place or lower. In the 2000 presidential election, a runoff between Bush and Gore would have eliminated the spoiler effect of Ralph Nader and perhaps led to a Gore victory.

Though runoffs have some clear benefits, they also entail costs. Running elections twice can be expensive for local governments and campaigning politicians [62]. Voters have to turn out twice, and turnout almost universally declines for the runoff vote. A study by FairVote on voter turnout in the 248 primary runoff elections for the U.S. House and Senate between 1994 and 2020 found that turnout declined in 97% of runoffs, by an average of 38% and a median of 37% [5].

[1] The Condorcet winner is the candidate, if they exist, that beats all other candidates in a head-to-head comparison. [62]

It is also still possible for dishonest voting to be rewarded in the runoff system [62]. For instance, consider a voter who knows her preferred choice, strong candidate Katy, will make the second round. However, the vote between second and third place candidates Meredith and Peter respectively will be a close vote. The voter knows Katy would easily beat Peter in a runoff, but a win against Meredith would be more difficult. This voter then may choose to vote for their last choice, Peter, in the first round in order to push him into the runoff where he will be easily beaten by her preferred candidate. However, this is a risky choice, since she does not want to accidentally push Peter past the runoff threshold so he wins the first election outright and a runoff is not held. It's also possible Katy would have won the first election outright had the voter chose her in the first round, avoiding the possibility of the competitive runoff with Meredith. Though this strategy has its risks, it still creates a paradox where voting for one candidate *increases* the chances of a different candidate – voting for Peter makes Katy more likely to win.[2]

Ranked Choice Voting. Ranked choice voting is also known as instant runoff voting, since it applies the principle of a runoff to only one ballot. In ranked choice voting, voters rank n candidates as their 1st through nth choices [59]. If a candidate gets over half of the first-choice votes (a majority), that candidate wins. If no candidate gets over half of the first-choice votes, the candidate with the least number of first-choice votes is eliminated and those voters' second-choice votes are redistributed to the remaining candidates. This process continues until one candidate achieves a majority. Ranked choice is currently

[2]This is called violating the monotonicity criterion. [8]

used statewide in only Alaska and Maine, though about 20 other jurisdictions use it nationally. Most recently, New York City adopted ranked choice for its mayoral primaries in June of 2021. Internationally, Australia, Ireland, Malta, Scotland, and New Zealand use a version of ranked choice voting. A version of ranked choice is also used by the Academy of Motion Picture Arts and Sciences to choose Best Picture for the Oscars, and by the Olympic Committee to select host cities [**59**, **10**].

The main argument in favor of ranked choice is that it requires that a winner be favored by a majority of voters, rather than simply a plurality [**59**]. Similarly, many argue that ranked choice has a moderating effect, eliminating polarizing candidates in favor of those with broad support [**59**]. Ranked choice also makes spoiler candidates far less likely to occur in practice, though it is not impossible – all systems that use a preference ballot have the possibility for spoilers to occur [**10**]. The lower likelihood of spoilers may cause an increase in honest voting, where voters can feel good about ranking their favorite candidate first. Practically, ranked choice is more cost effective than traditional runoffs where candidates spend additional time campaigning and the jurisdiction must run polls and count votes multiple times [**59**].

However, ranked choice requires knowledgeable voters which can be difficult in elections with many candidates such as New York's mayoral primary, where the Democratic ballot featured 13 choices [**10**, **59**]. Only a quarter of voters in the New York race ranked their 1st through 5th choices [**59**]. Consider a voter who only ranked their top choice, but that candidate was eliminated in the vote-counting process. With no second-choice vote to be moved elsewhere, their vote essentially disappears. This case is called an "exhausted ballot." To remedy this situation, some places that hold ranked choice elections require voters to rank all candidates, like Australia. [59]. However, this would require all New York Democrats to have a strong ordered preference for all 13 individual candidates. Practically, the system is also slower to count and harder for voters to understand. One Maine policy analyst claims that about 10 to 11 percent of ballots are uncounted in ranked choice elections due to either marking errors or the ballot becoming exhausted, compared to about 2 to 3 percent in traditional plurality [72]. Some argue these discounted ballots are more likely to belong to less educated voters, voters who speak English as a second language, and the elderly, who are more likely to rank fewer choices or rank incorrectly [72].

Same Votes, Different Outcomes. It is possible for each of these three vote-counting systems to yield three different winners given the same votes. Consider the following table of voter preferences in an election between candidates A, B, C, and D. This example is from Borger's *Mathematics of Social Choice: Voting, Compensation and Division*.

Number of ballots	**6**	**5**	**4**	**3**
Voter's 1st choice	A	B	C	D
Voter's 2nd choice	B	C	B	C
Voter's 3rd choice	C	A	A	B
Voter's 4th choice	D	D	D	A

TABLE 1. Preference schedule of 18 voters in an election between candidates A, B C and D

Using the traditional plurality method, candidate A wins, since they have more first-choice votes than any other candidate. Using the runoff method, we hold a second election between A and B, since they were the top two first-choice vote-getters.

Number of ballots	**6**	**5**	**4**	**3**
Voter's 1st choice	A	B	B	B
Voter's 2nd choice	B	A	A	A

TABLE 2. Voter preferences after candidates C and D are eliminated

In this runoff election, B wins with 12 votes to A's 6. This tells us a vast majority (2/3) of voters would have preferred B to win over A, showcasing a weakness of plurality. Yet we are still not taking voter's preferences for candidates C and D into account. Using the ranked choice method, we eliminate candidate D since they got the fewest first-choice votes.

Number of ballots	**6**	**5**	**4**	**3**
Voter's 1st choice	A	B	C	C
Voter's 2nd choice	B	C	B	B
Voter's 3rd choice	C	A	A	A

TABLE 3. Voter preferences after candidate D is eliminated

Then we re-allocate those voter's second choice votes, adding 3 additional votes to C (Table 4).

No candidate has a majority yet, so we eliminate B next and reallocate those voter's second-choice votes (Table 5).

Candidate	**A**	**B**	**C**
Number of 1st choice votes	6	5	7

Table 4. Number of 1st choice votes each candidate has after the elimination of candidate D

Number of ballots	**6**	**5**	**4**	**3**
Voter's 1st choice	A	C	C	C
Voter's 2nd choice	C	A	A	A

Table 5. Voter preferences after candidate B is eliminated

Once we've done this, C gains an additional 5 votes and now has the necessary majority to win.

Candidate	**A**	**C**
Number of 1st choice votes	6	12

Table 6. Number of first-choice votes each candidate has after the elimination of candidate B. Candidate C now has the necessary majority to win.

We've seen that traditional plurality gave a result that was not preferred by most voters. The runoff election elected a more favorable candidate, yet the ranked choice election proved C to be even more favored by the majority. The fact that it is not the voters but the vote-counting method that chooses the winner makes understanding these systems imperative as a voter.

Review of Existing Materials

In a review of existing age-level lessons that address the mathematics of different voting systems, five lessons were found. The developed framework elucidated that none of them included a social critique and action component. Most failed to contextualize the issue of voting and its relevance to students, and all failed to incite student action. These results are summarized in Table 7.

Illustrative Math. The school's existing math curriculum, Illustrative Math by KendallHunt, actually already has an optional unit that compares plurality and ranked-choice voting. Illustrative Math defines itself as a "problem-based core curriculum" where students connect concepts to procedure [58]. According to Ms. Burke, this is the third year it's been used in the district, though implementation was uneven. Some students came to the middle school having done it in 6th grade, and others had never seen it. Ms. Burke also says the inquiry-based style of Illustrative Math has been difficult for some teachers, who have challenged district-level authorities about the curriculum's mandate. In general, she agrees that the curriculum does well in communal knowledge production, allowing students to discover mathematics for themselves, but is lacking in the other three areas: relationships and community, student expertise and relevancy, and social critique and action.

In its optional voting unit for 6th grade, More than Two Choices, the class votes on four different school lunch menus [47]. They then tabulate the results in groups, including a "satisfaction index" where they calculate what percent of students listed the winner as their first choice, second choice and so on. At the end of the lesson, they are

Lesson	**Age group**	**Length**	**Voting systems**	**C&R**	**SE**	**CKP**	**SC&A**
Illustrative Math	6th Grade	2-3 class periods	Plurality, runoff, ranked choice, Borda count, score voting	✓	×	✓	×
COMAP	10th-11th grade	About 3 weeks	Plurality, runoff, ranked choice, Borda count, Condorcet, approval voting	✓	✓	✓	×
Radical Math	7th-8th grade	One class	Plurality, Borda count, Condorcet	×	✓	×	×
AMLE	6th-8th grade	One class	Plurality, vote for two, Borda count, Condorcet, runoff	×	✓	×	×
MIT Blossoms	Unspecified	One class	Plurality, ranked-choice, Borda count	×	×	×	×

TABLE 7. Summary of existing lessons on the mathematics of voting systems. The farthest right columns represent whether each of the four aspects of the developed framework was present in the lesson. In order, they are community and relationships, student expertise, communal knowledge production, and social critique and action.

asked to consider which way was the most fair. In additional lessons, students practice tabulating ranked-choice results given data, and vote using variations on Borda count and score voting.

In terms of strengths, the satisfaction index is unique and a useful tool for understanding voter's opinions on different outcomes. This lesson also balances full class and small group work well, and secures class buy-in and a deeper understanding of voting systems by using real class data. The greatest weakness is that there is no justification given to students or teachers about why learning about the math of voting is important. There are also no reference to the fact that these voting systems are actually used in the real world. A smaller weakness is that the ranked-choice vote uses a confusing symbol system (a star, a smiley face, a neutral face, and a cross) that feels both overly complex and demeaning for students, and distances these votes from how they occur in real life. Finally, voting on school lunch menus could exclude some students. For example, at least two of the four menus include pork dishes, which Muslim and Jewish students could not eat.

Consortium for Mathematics and its Applications. The second set of lessons is a full unit developed by the Consortium for Mathematics and its Applications (COMAP), a non-profit that focuses on investigating real-world issues in math classrooms [**16**]. As part of this mission it develops multidisciplinary mathematics curricula. The consortium is composed of mathematicians, educators and corporate executives. This particular unit was designed by Joseph Malkevitch, an adjunct professor at the teachers college at Columbia University specializing in mathematics education. A main barrier to this lesson is that one must be a COMAP member to access it (though an unauthorized PDF of the first five sections and various teachers' personal materials related to this unit are available on the internet). This unit suggests Algebra I and II as prerequisites, which for most students would mean early high school, slightly older than the target of 8th grade.

This is a detailed and complete unit. It includes six sections, each of which has a class activity, student exercises and suggested extension projects. Most relevant to this project are sections 1 and 2, which explore various vote counting systems. The remaining sections discuss possible paradoxes, Arrow's Theorem, voting power, and proportional representation. In section 1, students rank five different soda brands and then design their own systems in small groups to create a class soda ranking given everyone's individual rankings. In section 2, students learn about plurality, Borda count, runoffs, and ranked-choice voting systems and apply these different methods to their class soda data.

This unit is strong overall. Students working together to design their own method of counting votes emphasizes communal knowledge creation, relationship building, and students' importance in mathematics. Students use their own class' data throughout the unit, making counting votes feel relevant to them. The unit also includes sidebars of news articles about voting, such as choosing the Oscar best picture or Olympic host city. It also uses real election data from diverse sources in the exercises, such as a Minnesota gubernial election, the Heisman trophy for outstanding college football players, and the Irish presidential election of 2011. Finally, the unit occasionally asks students to make arguments using data, a target skill for the developed lessons.

However, this unit lacks in critique and awareness of social issues. The only reference to why students should care about voting involves how TV shows are cancelled, and a short reference that all students will someday vote in American elections (which may not be true!). In its attempts to appeal universally through soda and TV shows, it loses actual relevance to students' communities and social realities. It also costs money for educators to access, limiting the students it can reach.

Radical Math. The next lesson is available for free with all associated materials on RadicalMath [87]. Peter Wright, an associate professor of Mathematics Education at University College London and founder of the Teaching Maths for Social Justice Network, developed this lesson in 2014 in response to students not understanding a recent UK referendum on "alternative voting." The target age is 12-14. In this lesson, students are given 11 filled-in ballots ranking 4 candidates, who are named after female world leaders. Students then tabulate the votes using relative majority, absolute majority, Borda points with arithmetic weighting, Borda points with geometric weighting, and Condorcet pair-wise counting. The ballots are set up so these different methods yield different winners. The lesson suggests conducting a class vote on a favorite film as an extension. Students discuss as a class which system to use, conduct the vote, and reflect on it. There is also a further suggested extension into tactical voting and vote-splitting where the class nominates another film they believe would split the winning film's votes.

This lesson was developed in direct response to a social issue, a new referendum on voting in the students' community. It also, like the COMAP lesson, lets students conduct their own vote on something interesting to them and use their class data. Its extension to tactical voting is also unique among the found materials. Finally, it asks students to evaluate the fairness of each system and make an argument for which is best.

Even though this lesson was developed in response to social context, it's unclear in the lesson materials themselves how this context will be discussed within the lesson. It's also far less detailed than the COMAP lessons, with the suggested extensions being as short as one sentence. Teachers would have to put extra time and planning in to fully implement this lesson effectively, though the ideas are good. Finally, it asks students to consider which system is most "fair" but offers no guidance as to how students should think about that weighty word.

Association for Middle Level Education. Next is a blog with a downloadable worksheet from the Association for Middle Level Education (AMLE) with no attributed author [79]. The AMLE is an international association of middle school educators. In this lesson, students are given a preference schedule of 3 female entertainers to elect as class president. Students then decide the winner using plurality, vote for two, Borda count, pairwise comparison (Condorcet) and runoff. The conclusion is that different vote-counting methods elect different winners, so students should be aware of the importance of vote-counting methods. The author also suggests "mentioning" Arrow's theorem.

In terms of strengths, like the other lessons this one features hands-on groupwork for students in a context they are familiar with, in this case electing student body president. However, this is a bare-bones resource. There isn't even an explanation of how each system the students are supposed to use works, or suggested further resources for the teacher to learn what each system is. It's a student handout and not a lesson plan. It also doesn't ask students to think critically about each systems' "fairness" or the social context of voting.

MIT Blossoms. The final math-specific lesson comes from MIT Blossoms, a site hosted by MIT that features free educational videos for math and science teachers to use in class [**26**]. This video and associated downloadable worksheets were made by Andy Felt, an associate professor of mathematics at the University of Wisconsin Stevens Point and his student Chris Natzke. This video is structured for one class period, with places for the teacher to pause for students to complete an activity. The video introduces candidates A, B, C and D, who are 3D animated people (who dip a toe in the uncanny valley) for the "best smiley face competition." Students then rank the 4 candidates. The remainder of the video follows a predictable pattern of the instructors evaluating their own data using a particular method, and then prompting the teacher to pause for students to tabulate their own classroom data. This video covers plurality, ranked-choice, and Borda count. There are also questions throughout that ask students to consider candidates with broad support versus polarizing candidates. In the end, they prompt students to discuss which method was their favorite.

This video's greatest strength is that it is a well-illustrated example of how each of the three systems discussed deal with candidates with broad support versus a minority candidate. There are some references to where these various systems are used globally, and the guiding questions after each system is introduced asks students to think deeply about each system and form their own opinions. However, like the others, this video lacks an explicit "so what" about voting and its connection to students. The students also aren't voting on anything relevant to them. The rivetingly named candidates A, B, C and D don't exactly inspire student buy-in to the election. As a lesson plan it's also partially incomplete, since it isn't clear to the instructor how students are to calculate votes or answer the questions posed to them. Is it meant to be group work? Writing independently? Discussed as a class? This lesson could look very different based on teacher decisions.

Citizen Advocacy Center. Finally, we have a lesson by the Citizen Advocacy Center entitled "What is Fair?" that is intended for social studies, but fills in some of the weaknesses of the math-specific lessons and lists four math standards among its outcomes [**13**]. The Citizen Advocacy Center is a non-profit legal organization dedicated to democracy. This lesson was made in 2003 for 9th-12th graders. In this lesson, students brainstorm goals of a "fair" voting system. Then, students nominate seven volunteers for four seats on a fictitious class council, who give short speeches. Students then vote on the candidates using plurality, cumulative voting, proportional representation and ranked-choice voting. The lesson suggests an extension where students are divided into interest groups. Finally, the class discusses the fairness and pros and cons of each system based on their earlier definition.

This lesson is the only one that asks students to be precise about evaluating "fairness." Dividing students into interest groups is also unique. Students co-creating definitions encourages communal knowledge production, and the whole-class discussion format encourages relationship-building. However, voting on actual students can be problematic because the election becomes a personal popularity contest, and like other lessons this one does not properly contextualize the issue or inspire student action.

In sum, these existing lessons all failed to situate voting in its proper social context or inspire student action. By not providing context for voting as a social issue, educators deny students the chance to recognize and critique injustice.

Establishing the Context of Voting Systems

In the developed lessons, we want to build on the strengths of the existing lessons while providing proper social context for voting systems. Returning to the cultural critique principal of Culturally Relevant Pedagogy, students should “recognize, understand and critique current social inequities” [**43**] in order to learn the tools to change their world. In order to recognize and critique these inequities, students need to first understand the landscape of the issue. This context will ground fairness in a local student context – their school elections – while also showing that voting systems are a current global debate with no right answer. First, we consider voting systems in student home countries around the world, then in the United States, and finally in Lewiston Middle School.

First, students will consider whether every country votes in the same way. The following seven countries were provided by Ms. Burke as locations where students are from or have family. A brief overview of the voting systems of these home countries is listed in Table 8. These descriptions are vastly oversimplified and sometimes only consider one office because the full scope of election systems is beyond this unit’s ability. The main takeaways for students is to demonstrate that elections are not the same everywhere and to make connections to the three systems we will study in the unit where possible.

The runoff systems in Kenya, Sudan and Nigeria are similar yet different. In Sudan, the candidate must achieve a simple majority – over 50% of the popular vote [**39**]. Kenya and Nigeria additionally require

Country	**Voting System**
Angola	Cast ballots for list of candidates, get seats in proportion to votes [76]
Canada	Plurality vote for members of Parliament [22]
Kenya	Runoff for president [3]
The Philippines	Plurality for president [14]
Somalia	Moving towards plurality from indirect voting based on clans [81]
Sudan	Runoff for president [39]
Nigeria	Runoff for president [38]

TABLE 8. Table of student home countries and their associated voting systems.

the candidate to have broad support across districts. In addition to the majority of popular vote, Kenya requires a candidate to get at least 25% of the votes in half of the country's counties [3]. Nigeria requires this same 25% of votes, but in two-thirds of of the country's states [38].

The Philippines uses simple plurality, and Somalia is moving towards that system. Previously, Somalia used a complex voting system where a body of clan elders was primarily responsible for appointing officials to government offices [14, 81]. Canada uses a constitutional monarchy system, where the people directly elect only members of the House of Commons, and do not directly elect the Prime Minister or royalty [22]. Angola's system is especially unique. Voters cast a single vote for a list of candidates, where the top name is the president, and the other names correspond to other governmental offices, mainly the legislature [76]. These lists of candidates then get seats in the government in proportion to how many votes they received.

Next, we move from learning about global voting systems to discussing what students know about how we vote in the United States. First, we consider who can vote in the United States. Generally, voters must be 18 years old, a U.S. citizen, and be a registered voter [85]. However, some areas allow non-citizens to vote in local elections, and some states ban convicted felons from voting [85]. Maine is one of only two states where felons never lose their right to vote [60]. One pertinent issue for students is refugee voting rights, since many students have connections to the local immigrant and refugee community. After living in the U.S. for five years, refugees can apply for citizenship [41]. If granted citizenship, they are allowed to vote in all elections.

When thinking about U.S. democracy, we must pay credit to the oldest living participatory democracy in the world, the Iroquois Confederacy [34]. Founded in 1142, the Iroquois model was influential in designing the U.S. system. The Iroquois are a confederacy of six Native American Nations that covered much of New York State in the 17th and 18th centuries [21]. Today, the six nations live mainly on reservations in New York State and Canada, and continue to practice participatory democracy today, as well as fight for rights to their land and resources [21, 71, 75].

Next, we move from the federal level to the state level. A main reason this topic was chosen is because the students' home state of Maine uses ranked-choice voting in state and federal primaries and in federal general elections [**25**]. The Ranked Choice Voting Act was approved by voters in November 2016, and initially only included state elections and congressional elections. The act was controversial, even being challenged before the State Supreme Court, and has faced continued challenges by opponents. However, in 2018 voters chose to expand ranked-choice to included presidential primaries and general elections. In 2020 Maine voters voted in the state's first ranked-choice presidential election, and there are no current legal challenges to the system.

Finally, once we have touched upon global and local context for vote counting systems, we reach the level where students are able to affect change: CREW leadership team elections. As discussed in Chapter 1, these elections are currently conducted using a vote-for-two system to elect two students per team as representatives. On team Haleakala, students filled out a checklist Google Form where they selected their top-two candidates.

This context helps students to locate this unit within a global landscape of different systems and debates, with room for them to make connections to their own world.

CHAPTER 5

Implementation

In a literature review of existing grade-level lessons on the mathematics of voting, it was found that none of the lessons included a social critique and action aspect. The lessons failed to situate voting systems in their proper social context, and failed to inspire student action. The developed unit draws on the strengths of the existing work, while incorporating social critique and action. In the developed unit, the first full class period is spent contextualizing voting for students, and the last class period is spent entirely on a student action project. In mid-week, classroom simulations along with a balance of mathematical reasoning and contextual reasoning ensure that the math does not become procedural, and retains its connections to the real world. The four facets of the developed framework – community and relationships, student expertise, communal knowledge production, and social critique and action – are integrated throughout.

Unit implementation took place March 21st through March 25th, 2022. Each day of the unit I came in about 15 minutes before the first class. Ms. Burke and I then proceeded to co-teach the lesson for the day to the three different sections. We were sometimes able to discuss reflections on the day's lesson during prep periods, but often meetings were scheduled or other priorities arose. Even with scheduling challenges, we found small amounts of time to reflect on the lesson of the day, which I wrote up in a shared Google Doc. We also met once the week after implementation to reflect on the unit as a whole, during which I also took notes on our reflections. This is the data source from which I draw for instructor reflections.

Since this unit takes place across five class days (Monday through Friday), each lesson will be referred to by the day of the week on which it occurred. In this section, "we" refers to myself and Ms. Burke. All materials are included in the appendix. The daily lesson plan, worksheet(s), and worksheet key(s) are organized by day (Monday through Friday being Appendices A through E), followed by the slidedecks for each day (Appendix F). Materials can also be found online through BatesConnect, a platform for sharing resources with teachers in the Lewiston Public Schools, and on QUBEShub, an open-source repository for quantitative classroom resources. [1]

Monday. Monday's lesson is one of the main departures from the existing lessons on voting systems. Monday was spent entirely on socially contextualizing voting globally and locally, from other countries, to the United States, to Maine, to the students' school elections. See Appendix A.1 for the day's detailed lesson plan.

The class began with introducing the project for the week: comparing and contrasting voting systems using math, and then creating a proposal for which system to use for school leadership team elections. Then, students engaged kinesthetically in the "two sides game" where they chose sides on different prompts (such as hot pizza or cold pizza) by moving to different sides of the room. At the end of this game, students were asked to take a side on whether they think every country votes in the same way, and then whether every state votes in the same

[1] Materials can be found on QUBEShub at the following link: `https://qubeshub.org/publications/2916/1`

way. In all three sections, students were very engaged in this activity. There were only two sit-outs all day out of 51 students. For every prompt, multiple students on each side contributed a reason to support their opinion. In one of the sections, the country prompt caused a high-energy debate, with students enthusiastically yelling across the divide and gesturing wildly. Ms. Burke said this was one of her favorite moments of the week, and that she wants to integrate movement like this more in the future.

Next, we discussed and defined "voting." Every class had one or two students contribute a definition, which was usually similar to "a way for a group of people to make a decision." After showing students the mathematical definition of voting, "a method of determining a group preference from a set of individual preferences," we asked students if they have ever tried to decide on a movie with a group of people. Then we prompted them to think about what other decisions they make in groups, with whom, and how. This discussion also saw exceptionally high engagement across classes, with students often talking over each other trying to share their ideas. Most of the contributions were about making decisions in their family.

Next, we discussed what students know about voting in the United States. Students mainly drew on knowledge from social studies in this discussion. The discussion followed different paths in each class, but in every class we discussed who can vote currently in the United States and who could vote historically. With prompting, there was always a student ready to talk across every section. One section was particularly engrossed in mandatory voting, and we had another high-energy debate, with even the quietest students offering an opinion. One Black student was particularly engaged in discussing racial exclusion from voting, volunteering lots of ideas and walking closer to the front of the room from where he had been working at a standing desk in the back. It was sometimes challenging to keep students on-topic in this section, as questions occasionally strayed to other social studies topics unrelated to voting (e.g. "has a president ever been removed from office or just impeached?"), but even these somewhat off-topic questions demonstrate student thinking and curiosity.

The next section was guided notes on the voting systems of three countries and students' home state of Maine. Angola and Nigeria were selected along with the United States because they are student home countries with voting systems other than traditional plurality. For more details, see Chapter 4, "Establishing the Context of Voting," Appendix A.2 for the student worksheet, and Appendix F.1. for the slidedeck. Students were quietest in this section of the lesson.

The next major activity was a discussion of when students have voted before. Students gave widely varied answers: other classes, mock elections, family dinner, and more. Eventually a student in every section mentioned the leadership team elections, and we asked students to reconstruct what that process was like. Students also asked good questions in this section that showed they were really considering what voting is, such as one student asking if a lottery system is voting, and if so, what kind.

Finally, we discussed what should be true in a fair election. While sometimes needing prompting, there was always a student ready to respond. Students also came up with ideas I did not expect, such as private ballots and no intimidation or bribery. Truthful campaigning also came up in every class. Finally, every section said something about a majority unprompted, which allowed for a nice foreshadowing of Tuesday's class where we find out majority does not always rule.

Overall, students were highly engaged in this lesson. Even the quietest students contributed. Worksheets from the day were complete and correct with very few exceptions. Students asked questions throughout the lesson that showed they were engaged, thinking deeply, and curious about the topic. The most difficult part of the lesson was that students were confused about why they were doing it in a math class. Several students voiced that this should have been a social studies lesson instead, or "this doesn't seem like math." In the future, teachers should better roadmap the day or week for students so they know what to expect.

Tuesday. Tuesday's lesson focused on plurality voting. Tuesday established the structure of class for the rest of the week: learn about a voting system, enact an election, analyze the election data, and discuss the pros and cons of the system. For the full lesson plan, see Appendix B.1.

First, we introduced the mock election on superpowers. Students were captivated by the superpower topic in the first section of the day and eagerly asked if they could make speeches in favor of their preferred candidate. This activity saw exceptionally high buy-in throughout the day. Most students advocated from their seats, but one of the most previously disengaged students came to the front of the room to advocate for her superpower. Other students throughout the day were so moved by their enthusiasm they had to stand up.

After hearing speeches, we conducted the election. Students lined up four or five at a time to vote at the front of the room, where a tri-fold poster had been set up to act as a voting booth. Behind the tri-fold was a shoebox with a slot cut in it (the ballot box), and four colors of post-it notes with labels indicating which superpower they correspond to. Students selected one post-it and put it in the ballot box. After a "voter fraud" incident in first period (a student putting more than one post-it in the box), an adult supervised the voting process more closely. The election process also saw high engagement, with very few students abstaining all day.

After voting, students worked on the post-voting worksheet to keep them occupied while their classmates voted. The worksheet consisted of a face scale where students colored in how they would feel if each superpower won, to get them thinking about the complex nature of their preferences. See Appendix B.4 for the worksheet.

After students voted, we quickly created a bar chart on the board using the post-it notes. One section was quite split among the candidates (Figure 5.1). In another section, teleportation had an easy majority, with invisibility in second and almost no votes for the other two. In the third section, invisibility had an easy majority with teleportation second and almost no votes for the other two. There was enough competition between invisibility and teleportation to keep things interesting across classes, but other instructors may wish to use their own candidates.

Then, after a brief review of how to calculate percentages, students were released to individual or group work on their worksheets. They calculated the percentage of votes each candidate received, and then considered the questions "what percent of voters preferred the winning superpower? What percent of voters preferred a different superpower?" Ms. Burke observed a "great deal of effort" on these calculations. It

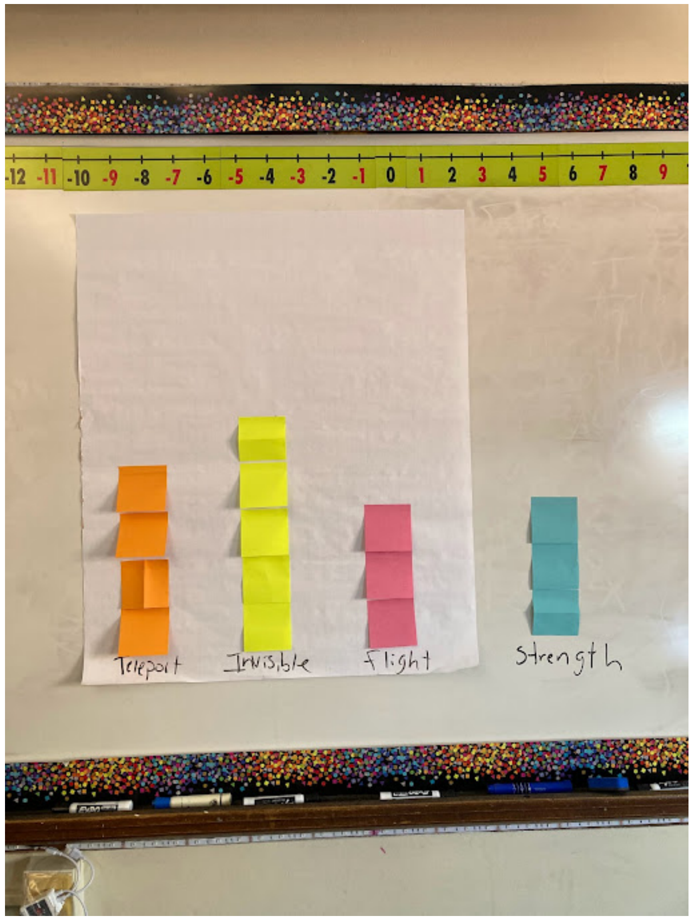

FIGURE 5.1. Bar chart of votes from one section.

seemed like a good topic to practice for students, since many needed some help getting started.

Next, we discussed plurality and majority. We used mock election data using Star Wars characters to present the definitions of plurality and majority. The new trilogy Star Wars characters were chosen due to their pop-culture status and racial diversity (a Black man, an Asian woman, a Latino man and a white woman). Most students were actually unfamiliar with the Star Wars characters. Students expressed varied opinions on the fairness of winning by plurality and not majority. Some thought it was fair because that's just how elections work, or because when you have a lot of candidates it would be nearly impossible to get a majority. It may have been hard for students to imagine a system other than plurality where a majority was achievable with many candidates. Many thought it was unfair because most people wanted someone else. Many students also expressed that this was a new concept to them.

Next, we discussed vote splitting. Students had been observed talking about Harry Potter and wearing Harry Potter merchandise, so Harry Potter characters were chosen for the vote-splitting example to align with student interests. Students were audibly excited about the Harry Potter example when the slide was revealed. [2] Students got a feel for how vote-splitting happens fairly quickly, but some were confused about why it was necessarily a bad thing. For instance, it could also happen that two "bad guys" run and split votes and allow Harry to win. To bridge this conceptual leap, I told them the joke about ordering pie that is attributed to Dr. Sidney Morgenbesser, which demonstrates why spoilers are illogical. The joke can be found in its entirety in Chapter 4, "The Mathematics of Voting Systems," or in Tuesday's lesson plan in Appendix B.1. After telling the joke, there was always a moment where students were shocked into silence by the punchline, and then many burst out at once saying things like "that doesn't make any sense!" We then broke down *why* Professor Morgenbesser's choice does not make sense: a losing candidate changed the winner. Students

[2] Teachers should note that Harry Potter author J. K. Rowling has repeatedly made public transphobic statements, and should carefully weigh student interest in the series against this reality when choosing to use this example [27]

had “ah-ha moments” here about why the results of vote-splitting, in their words, “don’t make any sense.”

Finally, we reviewed the pros and cons of plurality elections. This was difficult since students did not have anything to compare it to yet. However, with enough prompting from Ms. Burke and myself, we were still able to generate a decent list in each class.

This lesson also saw high student engagement and effort. Students particularly enjoyed debating the superpowers, participating in the mock election, the post-voting worksheet, and the Harry Potter example. Several voiced that the ideas of plurality versus majority and vote-splitting were something they had never thought of before. Students also got to review calculating percentages, which most of them had forgotten how to do.

Wednesday. Wednesday’s lesson focused on the runoff system. Wednesday was an early release day, so each class was 38 minutes instead of an hour. Wednesday followed the same structure as Tuesday. See Appendix C.1 for the detailed lesson plan.

After all the classes voted on Tuesday, the vote totals from all three sections were added. Teleportation and Invisibility were very close with 18 and 19 votes respectively, and Flight and Super Strength trailed with 5 and 6 votes each (Figure 5.2). Since there was a clear top-two in our election data, we decided to begin Wednesday’s lesson with presenting the previous day’s data and asking students what we should do now. In every section at least one student said we should re-vote on just Teleportation and Invisibility. It was hard to tell, though, if all of their classmates followed this logic.

Next, we introduced the concept of a runoff and where it is used in the world. Students were a bit confused by how a runoff works as

Superpower	Votes
Flight	5
Teleportation	18
Invisibility	19
Super-strength	6
	Total votes: 48

FIGURE 5.2. Aggregate election data from Tuesday as shown to students at the beginning of class on Wednesday.

evidenced by a lot of blank post-voting worksheets and some misconceptions that they voiced during the pros and cons discussion. Many students had trouble understanding that a runoff election included both Tuesday's election and Wednesday's. Some of these misconceptions persisted all the way through Friday. Future teachers should spend more time on introducing the idea of a runoff and make sure that all students understand that voters must vote twice. Teachers should also include a review of plurality in contrast (and in relationship) with a runoff at this point.

Next, we carried out the steps of a runoff election. Students followed along with the steps on the flowchart on their worksheets (Appendix C.2.). First, we discussed as a class if any candidate had a majority, and then identified the candidates in first and second place. Students responded confidently to these questions.

Then, we voted on the two runoff candidates. Students were less enthusiastic on Wednesday than on Tuesday. Two of the sections already had a majority the previous day, so the results of the runoff were predictable. To engage students more, I reminded them that although they could easily predict their own section's outcome, when we look at all the sections the race is a nail-biter, so every vote counts. This seemed to help with energy. The method we chose for voting (using the same color but writing your new vote on the post-it if you voted for Flight or Super Strength, see Appendix F.3. for the student voting instructions slide) was a bit confusing for students. The numbers didn't quite work out; a few students must have just chosen their new candidate's color. However, in closer elections, this method is worth trying just in case it's possible to identify vote-splitting.

For the post-voting worksheet, students were to write down one way that a runoff is similar to plurality and one way that it is different. This activity saw the least engagement all week, with most of the worksheets across all sections left blank. When asked, students responded that they were confused and couldn't think of ways the systems were similar or different. As discussed before, teachers should spend longer in the beginning of the lesson comparing plurality and runoff.

Like the previous day, students recorded election data and calculated the percentages. Students did quite well on calculating percentages without help on Wednesday after practicing on Tuesday, though some needed to be redirected from neighbors or screens.

Finally, we discussed the pros and cons of a runoff system, including going back to vote-splitting. Students were much quieter on Wednesday, with often only one student responding to each prompt. Students also continued to be confused about how a runoff works. The most common confusion was that voters only vote once on two of the candidates. However, at least a few students in every class participated in the discussion, and a few were able to connect things they had heard at home about voting to a runoff, such as their mom saying she can't vote for her first-choice candidate because she knows he won't win.

For their exit ticket, students filled out a ranked choice ballot. This went better than expected; only two ballots were mismarked across all three sections. Students seemed to understand how to use it quickly. In accordance with the laws in Maine, it was explained to students that they could rank the same candidate in multiple positions, and leave columns blank if they wanted. Two students chose to only rank one or two candidates, but everyone else ranked all four and did not repeat a choice.

Overall, Wednesday was the most low-energy day. Ms. Burke hypothesized that this was because students found the idea of a runoff difficult, as we have discussed. It is also possible some students found it too easy, as several students in algebra brainstormed some ways other than a runoff to determine a winner among two close candidates, such as introducing a surprise third candidate. The election results being predictable in two sections as well as the novelty of voting wearing off may have also contributed. Finally, both adults became sick on Wednesday so could not stoke the energy as much as we would have liked.

Thursday. Thursday's lesson covered ranked choice voting. No election was held since students submitted ballots at the end of class on Wednesday, but otherwise the class was similarly structured to Tuesday and Wednesday. See Appendix D.1. for the full lesson plan.

After Wednesday's class, I had to transform student ballots into data they could work with on Thursday. Since two sections had a majority winner, I picked the data from the section that was more divided for students to use. I transformed each ballot into a sequence of letters indicating the voter's choices. For example, if a student's first choice was Teleportation, second choice was Invisibility, third choice was Super Strength and fourth choice was Flight, their sequence was TISF. See Figure 5.3 for all the letter sequences students worked with, and Appendix D.2. for the full worksheet.

On Thursday, we first revealed the data from the previous day's runoff election, where Invisibility narrowly beat out Teleportation 23 to 22. Then, we told students where ranked choice is used in the real world and introduced the idea of ranked choice as many runoff elections in a row, but instead of keeping the top-two you eliminate from the bottom up, and instead of re-voting, all your opinions go on the same ballot.

Next, we worked through another Harry Potter example as a class using five voter's ballots. We translated the ballots into letter combinations and then carried out the steps of ranked choice. The most difficult conceptual leap for students in the whole lesson was abstracting voter's preferences – "my first choice is Harry, my second choice is Hermione and my third choice is Voldemort" – to sequences of letters. We slowed this process down when we saw students looking lost at this point in the presentation, and methodically stepped them through going from ballots to letters. Once they were oriented in this system, they understood it well, and were able to keep the context in mind ("the first column is people's first choices") even when working through repetitive procedure with the abstracted voter preferences.

Day 4: Ranked Choice

Our Ranked Choice Election
Directions

1. Count the farthest left letter of each vote
2. Decide if one superpower has a majority
3. If not, cross out all of the votes for the last-place candidate
4. Count the farthest left letters again, skipping any that are crossed-out
5. Repeat until one superpower has a majority

Ballots

T S I F
F T I S
S T I F
T I F S
T I S F
I
S T F I
T I F S
I T F S
T I
T I F S
I T S F
I T F S
I T S F
I T F S
I S T F

FIGURE 5.3. Student worksheet showing the letter-sequence data for one section

Many of them latched onto the algorithm quickly, and were expeditiously completing the worksheet before we were even finished with the example. When released to tabulate the ranked choice votes on their worksheet, almost every student completed the process far faster than we had anticipated, without help. Students were overall quite focused during this activity, and the worksheets show a great deal of effort. I had previously told them that ranked choice was challenging to understand, and that some mathematicians had told me that college students sometimes had trouble understanding it (these were math professors who attended a talk I gave about the project at the 2022 National Numeracy Network Meeting). Students expressed disbelief that college students could not understand ranked choice.

In two of the sections, we led a thorough review of plurality and runoff before comparing them with ranked choice, since it seemed some students were still confused about runoffs. Then, we discussed the pros and cons of ranked choice. Students had a lot of "what if" questions in all sections, sometimes even writing down their own sequences of letters to try out scenarios. They were curious and thinking deeply about how ranked choice works. We had a good discussion on the pros and cons of ranked choice with high participation across sections. One student kept saying "what's the catch?" because he felt ranked choice was so obviously a good system. He later expressed he felt the week was "an advertisement for ranked choice."

One topic we had not planned but ended up having time to cover was the way each system treats polarizing candidates versus candidates with broad support (this is perhaps best covered in the MIT lesson on voting systems discussed in Chapter 4). We improvised some letter sequences where one candidate was polarizing (appeared either at the start or end) and one had broad support (appeared most often in the middle positions). To demonstrate what these candidates might be like in real life, we talked about polarizing versus broad support foods. Black olives are polarizing, because people either love them or hate them. Mashed potatoes are a broad support candidate because they're not anyone's favorite or least favorite food, and most people would feel pretty okay about eating them. This activity saw high engagement, with students talking over each other about food preferences, and led to critical thinking and debate about whether it's best to elect polarizing candidates or broad support candidates ("but they're no one's favorite!"). One student who currently serves on the student leadership team said even though ranked choice seems fairer, he'd rather use plurality because he knows he's a polarizing candidate.

Thursday was another high-effort, high-engagement day. Students were curious and thought deeply about the ranked choice system. Once they understood the letter combination notation, they were quick to understand the algorithm. Remarkably, they were able to stay grounded in the context of an election and voter preferences even when working with something quite abstract (an algorithm and letter sequences). They were able to fluidly transition between the letters and speaking about voter's opinions. This lesson was also easily differentiated, with students who were ready able to take it to high levels of thinking like voter strategy or complicated "what if" scenarios with their own letter sequences. In the future, educators should plan to step slowly through turning opinions into letter sequences and review the two previous voting systems thoroughly. The food preference activity should also be further developed and included.

Friday. The main goal of Friday's lesson was to decide which of the three voting systems should be used for student leadership team elections the following year. Then, students created a slidedeck advocating for their chosen system to be shown to the school administrator who leads the leadership team. For the full lesson plan, see Appendix E.1.

Friday was a unique day for students because it was the end of spirit week, so the school was preparing for the pep rally later in the day. Classes were 40 minutes instead of an hour, and student distraction was high. Due to Covid-19, the current eighth graders had never experienced a pep rally in their middle school careers. Several other teachers on the team gave students permission to paint banners in the hallway, dress up/apply face paint, and otherwise decorate for the pep rally during class. Students from other classes also came in and out of the classroom during instruction. As a result of this excitement, students were highly distracted and reluctant to complete the day's lesson.

In all periods, we began with a review of each of the systems so far, where students volunteered what they remembered about each election method. Students were handed back their worksheets from Tuesday, Wednesday and Thursday to help them recall. Then, we asked for arguments for a particular system. All three sections settled on ranked choice quickly. In one section, confusion about a runoff requiring two separate elections persisted, but once this was cleared up, this section also agreed unanimously on ranked choice.

The slidedeck activity was different in every class. In the first period of the day, we assigned small groups of students to each slide and released them to individual or group work. About half of the class worked on their slides, and the other half took this worktime as free time to talk to friends and work on pep rally posters. Since it was a unique day for students, we agreed to allow this.

In the second section of the day, we instead tried creating the slidedeck as a whole class rather than in small groups. Students dictated ideas for each slide, and I typed the ideas onto the template. Only about three students contributed consistently to this discussion. During this activity, increasing numbers of students began standing on tables to draw on the windows. We released students about 15 minutes early for pep rally prep, once it became clear that the number of distracted students had hit critical mass.

The final section was the advanced section. They entered the room less distracted than the previous two periods, so we agreed to try again on allowing group work. All students worked diligently to complete their assigned slide. After completing their slide, some students wanted to do more. One conducted research online about ranked choice to add civil campaigning as a reason for ranked choice, which we had not discussed during the lessons. Another chose to make an outline of the key reasons for the viewer. When other students were done, they chose to play online games, socialize, or work on pep rally posters.

Friday was the most disorderly day because of the pep rally. However, the structure of the slidedeck seemed to work well overall. It was a medium students were familiar with and a versatile way to present information. Some students particularly enjoyed decorating the slides, with one identifying herself as a slide decoration expert. One issue was the universal edit ability, which caused one student's work to get accidentally deleted by another student in the first section. It may be useful to review Google Drive edit etiquette before releasing students to work. Students were also meant to work on the reflection (see Appendix E.2.) once they completed their slide, but in order to get higher quality work, we agreed to move the reflection to Monday. This activity would have mitigated student distraction once they finished their slide.

CHAPTER 6

Conclusion and Recommendations

Successes. One pattern of success in the unit was that utilizing student expertise led to high engagement. Students were almost universally engaged in activities that asked them to draw on their existing knowledge and interests, such as discussing what they know about voting, how they vote in their family, or using Harry Potter characters for election examples. This outcome is corroborated by multiple well-known pedagogies that advocate centering student experiences, including Community Cultural Wealth and Critical Race Theory [**88**, **74**].

A second pattern of success was the adaptable difficulty of this unit. Students who were ready to extend concepts were able to dive deep, think critically, make connections, and raise new questions about voting. Students who weren't yet ready were still able to grasp the basics of the unit, and hopefully still take away something meaningful. This outcome is corroborated by Universal Design, which is about creating adaptable lessons that are able to meet student needs across learner variation [**12**].

A third major success was that students were able to make change in their school using mathematics and advocacy skills. All three sections completed a slidedeck that advocated for ranked choice voting for CREW Leadership Team elections. These presentations have been passed along to the administrator who heads the team, who has committed to change next year's voting process in accordance with student proposals.

Returning to Ms. Burke's goals for the unit, she had stated she wanted to show students that math is an integrated part of their world and to get them out of routine and engaging in interdisciplinary learning. She also hoped to gain new knowledge about voting herself, and new skills in the classroom. After the unit, her reflections indicate that these goals were met. She particularly enjoyed teaching her students a concrete example of how math is used in the real world and what careers exist in math. She reflected that she learned a great deal about voting and its connections to math. She also thought getting students out of their seats during the two sides game and the voting process was beneficial, and wants to incorporate movement more in her class going forward.

Ms. Burke reported that she intends to continue teaching the unit, saying she "can't wait to do this unit every year." She is hoping to develop the unit further with the social studies teacher on the team. This was a main success for this project, as many Bates projects with community partners last only until graduation. It also indicates that the co-ownership of the unit was successful, as Ms. Burke felt confident about teaching it herself in coming years.

Challenges. One major challenge was that students had met me a maximum of six times, which was not enough to fully build trusting relationships. This manifested mainly as inappropriate comments from some students about disliking me or the unit. When asked, very few students could provide a reason for their opinion or a way to improve the unit. Ms. Burke reported that a certain level of complaining is normal in her class, which I had also noticed during observations. However, this complaining was never personal to Ms. Burke as far as I had observed, meaning my situation was slightly different and a product of mistrust or weak relationships. This challenge will no longer be an issue in the future once Ms. Burke takes over the unit on her own.

When students were able to provide a reason for their reported dislike of the unit, it was always that there was not enough math. These student reports should be taken with a grain of salt, because my recollection from observations is that these same students complained about too much math during lessons prior to this unit. I asked one of these students about this apparent paradox, and she reported, "Oh yeah, we just like to complain." However, even if some of these reports may be unreliable, some may also be genuine. In planning the unit, the main tension was balancing social context with mathematics, and ultimately the lessons ended up with less mathematics than I would have liked. This tension is well-documented in literature about teaching Mathematics for Social Justice [**9, 66, 11, 67**].

Another similar challenge was that students struggled to understand why fair elections should matter to them. On Monday, after discussing that mathematicians are working on finding the fairest way to vote, we asked students if that seems like a worthwhile goal. The answers we got were all "no." One student said there are bigger issues in the world, like climate change. Another said that maybe it's important for other people to work on, but it's not important for her. These responses demonstrate that students struggled to recognize the ways that certain elections were unjust. This may be because this question was asked too early in the unit, before students truly understood any voting system other than plurality. However, it would be worthwhile to better establish why voting is a justice issue on day one.

Ms. Burke reflected that overall she saw high levels of student participation, engagement, and effort this week. However, we both reflected that it was difficult to tell if the unit engaged Black students in particular. Even though the main motivation for this project was to engage Black students in math, the project was not set up to tangibly measure outcomes for Black students. This was because of the nature of the Bates – Lewiston Public Schools partnership. This project was approved as a curricular development project which collected only instructor personal reflections as data. Approval for collecting student outcomes data is much more involved because students as minors need parental permission, and the school has not often approved these types of studies. A student who is interested in collecting this type of data should be in touch with the Harward Center very early in the project.

Finally, related to engaging Black students, there was less direct discussion of race and racism in the unit than I would have liked. We only discussed it on Monday in talking about how Black people were historically excluded from voting, both *de jure* and *de facto*. We also were able to talk briefly in one class about the racial composition of the U.S. government, and whether it reflects the racial composition of the United States. During these discussions, one Black boy was especially engaged. Part of this challenge is the nature of the topic: voting systems does not lend itself as well to the discussion of race as other math and democracy topics do. However, more could be done to connect voting systems to other systems of racial injustice, like gerrymandering, polling locations, and other forms of Black disenfranchisement.

Recommendations. The unit should convince students that voting systems is a justice issue on day one. Ms. Burke and I discussed doing a data analysis lab on Monday, where students analyze real-world election data to practice recognizing injustice. For example, students might explore plurality and majority during this lab on Monday using real data instead of waiting until Tuesday, since many of them found the idea of a plurality winner to be new and compelling. This change would also help with student complaints about too little math, since it introduces slightly more challenging math (still calculating percentages, but with bigger numbers) on the first day.

There was also extra time on Friday, which could be used to bookend Monday's introduction by connecting back to the real-world work going on in the world of math and voting systems. Students could be shown actual mathematicians in the field and a little of what their work looks like. There may also be an opportunity here for Arrow's Impossibility or something similar. One challenge is that the field of social choice mathematics is overwhelmingly white and male. To remedy this, mathematicians of color in related fields such as gerrymandering or voting site mapping could also be brought into the conversation. Alternatively, each class could start with a spotlight feature on an underrepresented mathematician doing work in the democracy area.

This unit should continue to be developed alongside a parallel social studies curriculum. The parallel curriculum could do more heavy lifting on social context, so students have more time to do math in math class. It could also more fully explore race and racism in U.S. democracy, which would further enrich the unit. Ms. Burke's ultimate goal for the unit is for it to be a week-long "walls-down" (meaning fully integrated, interdisciplinary) math and social studies unit taught around election season. Hopefully, future eighth-grade classes will continue to have the opportunity to learn a liberatory mathematics, one they can use to build a better world.

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Appendix A.1. Monday Lesson Plan

Grade: 8th **Subject:** Math **Day:** 1

GETTING YOURSELF READY		
Materials: – Slidedeck – Guided notes handouts – pencils	**Objective(s) and Proving Behavior:** **Objectives:** **Content** - Students understand that there are many voting systems in use in both the U.S. and around the world - Students understand how they vote in their school **Language** - Students can define "voting" - Students can explain aspects of a fair election **Proving behavior:** - Students complete worksheet definition activity - Students complete exit ticket showing a main idea - Students participate in discussion on what makes an election fair	**Agenda (w/times)**: 10 minutes: students come in, get through introduction slides 15 minutes: two sides activity 20 minutes: guided notes 15 minutes: what is a fair vote discussion

STUDENT ACTIVE LEARNING & CONNECTION TO CONTENT

Active Learning Plan:

- Intro slides
    - Introduction slide of myself
    - Introduce project/purpose for the week
    - Content and language objectives for the day
        - Have a student read each
- "Two Sides" activity
    - Build relationships, ease us into both the content and activity of voting (voting on two candidates)
    - Read the brief instructions
    - For each question have one or more students from each side defend their stance
    - Don't yet reveal the answers to the voting questions
- Guided notes
    - What is voting?
        - Let one or more students attempt their own definition, ask other students to add on
        - Reveal definition on slide; mention it is the definition of voting mathematicians use
        - Give deciding on a movie example: you and your friends all have opinions (individual preferences) on what movie to watch, and you need to figure out which movie to pick so the most people enjoy it – the group preference
        - Ask students how they decide on things with their family or friend group, and what they need to decide on
    - What do you know about voting in the U.S.?
        - Group discussion, can prompt with questions like "can everyone vote?"
        - Then, share that our form of government was inspired by the Iroquois (pronounced ee-rah-kwa) Confederacy, a powerful group of Native American nations
    - Go through country and state slides and students fill out worksheet
        - Scripted sentence for each slide, should go fairly quick
            - Every country does not vote in the same way
                - Comment on how the class reasoned earlier during the two-sides activity

- In Angola, instead of voting for just one person, voters choose a list of candidates. Then, if the first list got ⅔ of the votes, the first ⅔ of people on that list are elected. If the second list got ⅓ of the votes, then the first ⅓ of people on that list are elected.
    - Can prompt if this system looks different from how they usually think of voting
- In most of the United States, we vote for one candidate. Then, whichever candidate gets the most votes wins.
- In Nigeria, they vote in two rounds. The first round is like the United States. Voters vote for one candidate. Then, the candidates that are in first and second place enter a second round where voters pick between only those two.
- Every state does not vote in the same way (again, comment on how the students reasoned earlier)
    - Maine uses ranked-choice voting. We'll talk about this more on Thursday, but for now notice how the voter does not pick just one candidate, but is able to say what their first, second, third and fourth choices are.

    - Is there one best way to vote?
        - Ask for student input
        - Emphasize that mathematicians are working on trying to find the fairest way to vote
- When have you voted?
    - Lead discussion, hopefully students mention student council
    - Ask them to recap the process of voting for student council
- What should be true of a fair vote?
    - Whole-group discussion
    - Prompt with scenarios if necessary: would it be fair if…?
        - I just chose? (no dictatorship)
        - What about only some of you? (universal suffrage, everyone's vote counts the same)
        - If we were voting on our favorite color and all of the options were shades of orange? (diverse candidates)
        - There were lots of ties or it was hard to tell who won? (decisive outcomes)
        - Should most people like the winner? (majority rule)
        - Student additions:
            - Private ballots
            - No bribery or threats
            - Truthful campaigning

Release to Asynchronous Work: Practicing or Application Skills/Content

Exit Ticket:
Write or draw a main idea from today

Anticipated Challenges
Students may be confused about why we are talking about this in a math classroom. There are many possible approaches to remedy this, but it may be most helpful to start the day by saying today's class will have more of a social studies focus, and tomorrow we will do more math. It may also be helpful to discuss that real-world issues are never limited to a single school subject, so we are practicing using tools of reasoning from several subjects.

Appendix A.2. Monday Guided Notes Worksheet

Name:_______________

Day 1: Voting Around the World

1. Fill in the definition of voting in the box below:

Voting is a way of determining a _________ preference from a set of _______________ preferences

2. What do you know about voting in the United States? Write one thing you know in the circle below

3. Match the country to their voting system

Angola	Vote in two rounds
The United States	Vote for lists
Nigeria	Vote for one

4. What is different about the voting system in Maine? Circle one answer
    a. You vote for lists of people instead of individual candidates
    b. You rank more than just your first choice
    c. You vote with your feet

Exit Ticket
Write or draw a main idea from today's class in the shape below.

Appendix A.3. Monday Guided Notes Worksheet Key

Name:____KEY__________

Day 1: Voting Around the World

1. Fill in the definition of voting in the box below:

Voting is a way of determining a __group__ preference from a set of ___individual___ preferences

2. What do you know about voting in the United States? Write one thing you know in the circle below

Many possible answers: To vote you must be 18 and a U.S. citizen (though in very few places non-citizens can vote in local elections), in some states if you've committed a felony you can't vote, voting is not mandatory, most places voters pick one candidate and the candidate with the most votes wins, our government is a representative democracy, voting happens on several different levels of government… students will have many ideas

3. Match the country to their voting system

Angola	Vote in two rounds
The United States	Vote for lists
Nigeria	Vote for one

4. What is different about the voting system in Maine? Circle one answer

   a. You vote for lists of people instead of individual candidates

   b. You rank more than just your first choice

   c. You vote with your feet

Exit Ticket
Write or draw a main idea from today's class in the shape below.

Student answers vary

Appendix B.1. Tuesday Lesson Plan

Grade: **8th** **Subject:** **Math** **Day: 2**

GETTING YOURSELF READY		
Materials: -Post-it notes (4 colors) - pencils - calculators - daily worksheet - post-voting worksheet - ballot box - "voting booth" (trifold poster)	**Objective(s) and Proving Behavior:** **Objectives:** **Content** - Students can carry out the steps of a plurality election **Language** - Students can define plurality and majority - Students can describe some pros and cons of a plurality election using math to support their argument **Proving behavior:** - Students successfully tabulate the votes in groups using the plurality process - Students participate in discussion about pros and cons of plurality and fill in pros and cons on their worksheet	**Agenda (w/times)**: Intro slides: 10 minutes Mock election 10 minutes Group work: 15 minutes Plurality/Majority: 10 minutes Pros/Cons, vote-split, wrap up: 15 minutes

STUDENT ACTIVE LEARNING & CONNECTION TO CONTENT

Active Learning Plan:

- Brief intro slides
  - Read content and language objectives
  - Get excited about mock election!
    - I had a whole story about the people from Marvel calling me up and saying we can all have a superpower, but they're super busy so we all have to get the same one
  - Slide explaining "vote for one"
    - Reference yesterday's lesson and where they've used this system before
- Conduct mock election
  - Students have a chance to defend their chosen candidate (5 minutes or so)
  - Students come up to the front of class, go behind the voting booth, choose one post it and place it into the ballot box
    - One teacher may want to supervise in case of voter fraud (picking multiple post-its)
  - Students work on post-voting worksheet at their tables
  - After all students have voted, quickly create bar graph on board out of post-its
- Group work/Individual work
  - Students transform bar graph data into table data
  - Then, calculate percentages
  - Then, consider questions
  - Review worksheet as a class
- Discuss worksheet questions
  - Takeaway: it is possible for a candidate to win when over half of voters preferred a different candidate
- Plurality vs Majority
  - Explanation and guided notes
  - Discussion
- Vote splitting
  - Talk about how similar candidates can split a majority
    - Student council election example: don't run with someone in your friend group and split your mutual friends' votes!

- Pros and Cons
    - Discussion, students write down one pro and one con they come up with
    - Pros: Easy to understand, fast to use, makes the most people the most happy, only have to rank one candidate (don't need to know how you feel about the rest of them)
    - Cons: Winning candidate may not be liked by most of the voters, similar candidates can split votes (spoiler problem), rewards dishonest/strategic voting, does not capture full opinion of voters (just their first choice – can reference post-voting here, they have detailed opinions!)
- Exit Ticket

RELEASE TO ASYNCHRONOUS WORK: PRACTICING OR APPLICATION SKILLS/CONTENT

Exit Ticket:
Circle the argument that is most convincing to you in your table

Anticipated Challenges:

1. It is hard to come up with pros and cons without a different system to compare with. This will get easier as the week goes on, but feel free to prompt students heavily if needed (How easy was it to vote this way? How many candidates do you really need to know about? Etc.).

2. Students understood how and why vote splitting happens easily, but many of them struggled to understand why it's arguably a bad thing. To remedy this, I adapted a famous joke attributed to Sidney Morgenbesser to illustrate why a losing candidate changing the winner of an election is a logical fallacy. The original joke is as follows:

   After finishing dinner, Professor Morgenbesser decides to order dessert. The waitress offers two choices: apple pie or blueberry pie. Morgenbesser orders the apple pie. After a few minutes the waitress comes back to the table and says, "I forgot to tell you, we also have cherry pie tonight." Morgenbesser replies, "Oh, in that case I'll have the blueberry pie"

   Students were very engaged in this example, and had visible "aha!" moments about why this type of winner-change does not make sense.

3. Finally, some students got confused in the spoiler discussion and thought candidates were actually entering elections last-minute and changing the outcome. It's important to be clear that in real life surprise candidates do not enter elections last-minute, but that this is just a way to think about one candidate's effect on the election outcome. If it is easier, you can reframe this as disqualifying a candidate after the fact rather than them joining the field.

Appendix B.2. Tuesday Worksheet

Name:_______________

Day 2: Plurality

Steps in a Plurality Election

1. Vote for one candidate
2. The candidate that gets the most votes wins

Our Plurality Election

Superpower	Votes	Percentage of total votes
Flight		
Invisibility		
Teleportation		
Super-strength		
	Total votes:	Total percent: 100%

1. What percent of voters preferred the winning superpower?

2. What percent of voters preferred a different superpower?

Guided Notes

A ______________ is when a candidate gets more votes than any other candidate.

A ______________ is when a candidate gets over half of the votes.

Plurality Pros and Cons

Pros	Cons

Exit Ticket

Which reason, either before or against plurality voting, is most convincing to you? Circle it in your table above.

Appendix B.3. Tuesday Worksheet Key

Name:____KEY__________

Day 2: Plurality

Steps in a Plurality Election

1. Vote for one candidate
2. The candidate that gets the most votes wins

Our Plurality Election

Superpower	Votes	Percentage of total votes
Flight		
Invisibility		
Teleportation		
Super-strength		
	Total votes:	Total percent: 100%

1. What percent of voters preferred the winning superpower?

(depends on class data)

2. What percent of voters preferred a different superpower?

(depends on class data)

Guided Notes

A ______plurality_____ is when a candidate gets more votes than any other candidate.

A _______majority____ is when a candidate gets over half of the votes.

Plurality Pros and Cons

Pros	Cons
Easy to understand Fast to use Many people are familiar with it Less knowledgeable voters – only need to know your first choice	Winner only needs a plurality to win and not a majority → winner may not be liked by over half of voters Similar candidates split votes The presence of a losing candidate can change the winner (spoiler effect) Rewards dishonest/strategic voting Does not capture much information about voters' opinons

Exit Ticket

Which reason, either before or against plurality voting, is most convincing to you? Circle it in your table above.

Appendix B.4. Tuesday Post-Voting Worksheet

Name:_______________

<u>Day 2: Post-Voting Worksheet</u>

Color in a face for how you would feel if each superpower won:

Flight:

Teleportation:

Super-strength:

Invisibility:

Appendix C.1. Wednesday Lesson Plan

Grade: 8th **Subject:** Math **Day:** 3

GETTING YOURSELF READY		
Materials: -Post-it notes in 4 colors -Printed worksheets -pencils -calculators -"voting booth" (trifold poster) -"ballot box" shoebox -ranked-choice ballots	**Objective(s) and Proving Behavior:** **Objectives:** - Students can carry out the steps of a runoff election given ballot data -Students can explain some pros and cons of a runoff using math to support their argument **Proving Behavior:** -Students complete the worksheet on tabulating the results of an election using the runoff method -Students fill out the pros and cons table on the worksheet, students participate in discussion of pros and cons	**Agenda (w/times)**: Intro slides: 5 minutes Conduct the runoff: 10 minutes Group work: 10 minutes Discussion: 13 minutes

STUDENT ACTIVE LEARNING & CONNECTION TO CONTENT

Prep
- Compile the previous day's voting data in such a way that there is no majority
  - Can use only one section's data, all the data in aggregate, or create a "what if?" example

Active Learning Plan:
- Students come in, will need a pencil and calculator
- Intro slides
  - Read content and language objectives
  - Briefly explain a few places the style we are studying today is used
  - Briefly explain algorithm referring to flowchart on student worksheets
- Conducting a runoff
  - Step through the flowchart on worksheet
  - Look at yesterday's data to figure out if a runoff is needed and who is in the runoff
    - Our data had two far-and-above top contenders, so I actually had students try to develop the algorithm on their own based on a table of the previous day's votes before telling it to them – every class succeeded in doing so
  - Vote!
    - Students use same COLOR as previous day
    - If they voted for a candidate NOT in the runoff on Tuesday, they write their runoff choice on the post-it
    - Students work on post-voting worksheet
    - Create bar graph once all students have voted
- Student group work
  - Students fill out table calculating the percent of votes each of the two runoff candidates got
  - Goals: in a two-candidate vote the winner always wins by majority
    - Takeaway: we at least know that a majority of people like this candidate better than the other one
- Discussion
  - Discuss group worksheet questions, aimed at takeaway above
  - Spoiler discussion continues, consider sticky note colors
    - If most of the same color sticky notes are together, that's evidence of vote splitting

            - For us, so few students voted for anything other than the top two candidates that the results weren't interesting
    - Walkthrough the Harry Potter example again, with students directing the runoff steps
        - Did the runoff help with the vote-splitting between similar candidates?
    - Pros and Cons discussion
        - Pros: Easy enough to understand, vote-splitting less likely, know that the winner is not the *last* choice of more than half of the voters – slightly more likely to elect a winner more people favor, voters can vote more honestly
        - Cons: Eliminated the third-place candidate (was that fair? What if they're really close to the second place candidate?), fewer people re-vote in real life, have to run polls twice and campaign longer, vote splitting is still possible
- Ranked choice ballot
    - In Maine, you are allowed to choose the same candidate for multiple rankings
    - You are also allowed to leave choices blank, but you may not skip two in a row
    - For more information, see the voting booth poster here: https://www.maine.gov/sos/cec/elec/upcoming/pdf/HORZ.boothposterFINALDRAFT.061218.pdf

RELEASE TO ASYNCHRONOUS WORK: PRACTICING OR APPLICATION SKILLS/CONTENT

Exit Ticket:
Fill out your ranked-choice ballot

Anticipated Challenges

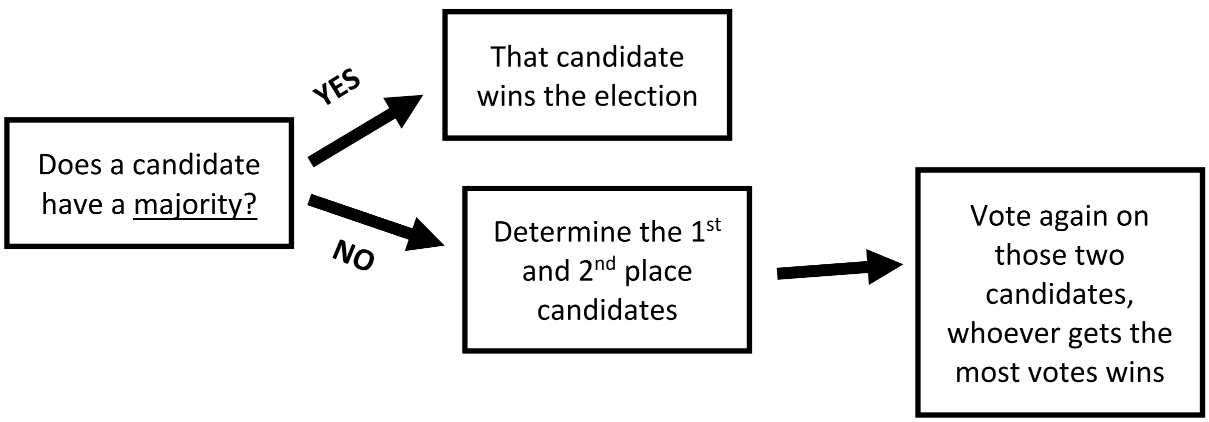
1. Students struggled initially to understand that the runoff included both Tuesday's election and today's. Emphasize what a runoff means multiple times throughout the day, and ask students to explain it.

2. Some of the in-class votes were boring because that period already had a clear majority on Tuesday. Hype up the idea that we're adding up all the classes' data at the end of the day to see which superpower won the runoff in all the classes.

Name:_______________

Day 3: Runoff

Steps in a Runoff Election



Our Runoff Election

1. Looking at the votes from all of the classes added up yesterday, does one superpower have a majority? Yes or no?

2. Who are the 1st and 2nd place candidates? Fill them in in the first column of the table on the other side of this page.

3. Conduct the runoff election!

4. Fill in the table below with the votes from the runoff and calculate the percentages.

Superpower	Votes	Percentage of total votes
	Total votes:	Total percent:

Did the winner win with a plurality or a majority?

Runoff Pros and Cons

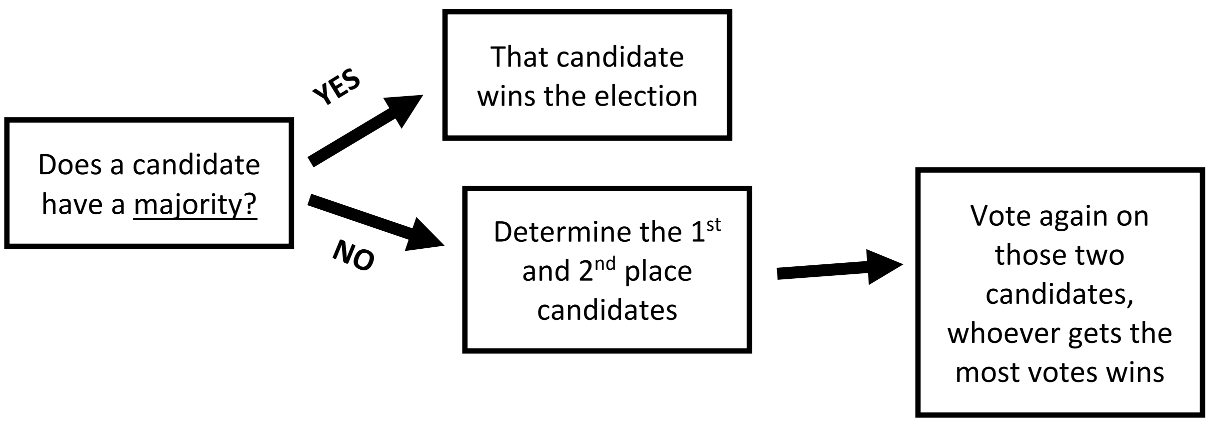
Pros of this system	Cons of this system

Exit Ticket
Fill out your ranked-choice ballot!

Name:_____KEY_________

Day 3: Runoff

Steps in a Runoff Election



Our Runoff Election

1. Looking at the votes from all of the classes added up yesterday, does one superpower have a majority? Yes or no?

(depends on data)

2. Who are the 1st and 2nd place candidates? Fill them in in the first column of the table on the other side of this page.

(depends on data)

3. Conduct the runoff election!

4. Fill in the table below with the votes from the runoff and calculate the percentages.

Superpower	Votes	Percentage of total votes
	Total votes:	Total percent:

Did the winner win with a plurality or a majority?

Majority

Runoff Pros and Cons

Pros of this system	Cons of this system
Somewhat easy to understand Helps with vote splitting and spoilers Winner must have a majority Voters can vote more honestly	Eliminates all candidates but the top two, even if many are close Have to vote twice Voter turnout declines

Exit Ticket
Fill out your ranked-choice ballot!

Appendix C.4. Wednesday Post-Voting Worksheet

Name:_______________

Day 3: Post-Voting Worksheet

In the boxes below, write one way a runoff is SIMILAR to plurality, and one way it is DIFFERENT

Appendix C.5. Wednesday Post-Voting Worksheet Key

Name:____KEY__________

Day 3: Post-Voting Worksheet

In the boxes below, write one way a runoff is SIMILAR to plurality, and one way it is DIFFERENT

Similar: voters pick one candidate

Different: runoff is two rounds and plurality is only one round, in a runoff you only vote on two candidates

Appendix C.6. Student Ranked Choice Ballots

Best Superpower	**1st Choice**	**2nd Choice**	**3rd Choice**	**4th Choice**
Flight				
Teleportation				
Super-strength				
Invisibility				

Best Superpower	1st Choice	2nd Choice	3rd Choice	4th Choice
Flight				
Teleportation				
Super-strength				
Invisibility				

Best Superpower	1st Choice	2nd Choice	3rd Choice	4th Choice
Flight				
Teleportation				
Super-strength				
Invisibility				

Appendix D.1. Thursday Lesson Plan

Grade: 8th	Subject: Math Day: 4	
Getting Yourself Ready		
Materials: - Slidedeck - Pencils - Worksheet	**Objective(s) and Proving Behavior:** **Objectives:** **Content** - Students can carry out the steps of a ranked choice election given ballot data **Language** - Students can explain some pros and cons of a ranked choice election using math to support their argument **Proving behavior:** - Students accurately tabulate the results of their class's ranked choice election on the worksheet - Students fill out the pros and cons table on the worksheet	**Agenda (w/times)**: Review, runoff data: 5 minutes Intro to RC: 3 minutes HP example: 7 minutes Student worksheet: 15 minutes Discussion: 20 minutes Last example: 10 minutes

Student Active Learning & Connection to Content

Prep

- Go through the ranked choice ballots students cast the previous day
  - Actually enact each election separately following the steps of ranked choice
- Choose the section with the most interesting data (most rounds of elimination, or a surprise winner)
- Write all the ballot data from that section on the worksheet in the form of letter sequences
  - First letter is first choice, second is second choice, etc.
  - See our worksheet for an example

Active Learning Plan:

- Students enter, will need pencil and worksheet
- Show results of the runoff election, get student reactions
- Briefly review plurality and runoff and their strengths and weaknesses
- Intro to ranked choice
  - Where it's used
  - It's easiest to think of ranked choice as many runoffs in a row
    - Instead of eliminating everyone but the top two, we eliminate the person in last place over and over
    - The other difference is voters only vote once – students have experienced this as they've voted in both systems
    - This is why it's also called "instant runoff"
- Harry Potter example
  - Show ballot data with five voters
  - Slowly step students through converting this data to the letter sequences
  - Walk through the example
    - How many votes are needed to win?
    - Count first-choice votes (the first column)
    - Determine if one candidate has a majority (no one will)
    - Eliminate the candidate in last place
      - Cross out G everywhere it appears
    - Recount first-choice votes

            - Might help to circle the first-choice votes
        - Harry should win 3 to 2
- Group/Individual work
    - Students attempt this process on their own/with tablemates using the superpower ballot data on the worksheet
- Discussion
    - Ranked-choice is most useful when you have many competitive candidates – it doesn't change much if there's a clear winner or even a clear top-two
        - Give the example of runoff again: was it fair to eliminate the third-place candidate if they were super close to the second-place candidate? Ranked choice only eliminates the last-place candidate, who surely can't win
    - Polarizing candidates versus candidates with broad support
        - This discussion was added in the moment during implementation because of extra time, but I highly recommend getting to it if possible
        - Discuss what "polarizing" versus "broad support" means
            - Food examples! We used black olives as polarizing – you either love them or you hate them – and mashed potatoes as broad support – not anyone's favorite food, but most people would be okay with it
        - Ranked choice tends to elect these broad support candidates
            - Not anyone's first or last choice, but most people are okay with them
            - In contrast, plurality is more likely to elect polarizing candidates
        - In terms of those letter sequences, polarizing candidates are often the first or last letter and rarely in the middle, and broad support candidates are often in the middle and rarely first or last
- Pros and Cons discussion
    - Pros: guarantees majority winner, only vote once (as opposed to a runoff), encourages honest voting, elects candidates with broad support, captures a lot of information on voters' opinions, may give a more "fair shot" to candidates (ex. Third place elimination in runoff), vote splitting/spoilers likelihood significantly reduced
    - Cons: requires knowledgeable voters (to be able to rank many candidates), complicated and lengthy process to count votes, ballots may be confusing and are more likely to be mismarked and thus uncounted
- Electing different winners with the same votes
    - Walk through calculating this last example as a class using
        - Plurality: only count first-choice votes (T wins)
        - Runoff: eliminate everyone but top-two (T and I stay), recount (I wins)
        - Ranked choice: follow algorithm as you've been doing all class (F wins)

Release to Asynchronous Work: Practicing or Application Skills/Content

Exit Ticket:
Circle the reason in your table that is most convincing to you

Anticipated Challenges

Students' biggest conceptual leap was actually going from voter preferences (my first choice is Harry, my second choice is Hermione, my third choice is Voldemort) to those sequences of letters. If you take your time with this, they'll also have a better conceptual understanding of what each column means when tabulating votes.

Name:_______________

Day 4: Ranked Choice

Our Ranked Choice Election
Directions

1. Count the farthest left letter of each vote
2. Decide if one superpower has a majority
3. If not, cross out all of the votes for the last-place candidate
4. Count the farthest left letters again, skipping any that are crossed-out
5. Repeat until one superpower has a majority

Ballots

T S I F
F T I S
S T I F
T I F S
T I S F
I
S T F I
T I F S
I T F S
T I
T I F S
I T S F
I T F S
I T S F
I T F S
I S T F

Total number of votes:

Number of votes needed to win:

Round 1

Superpower	Number of Votes
T	
F	
I	
S	

a. Does any superpower have the number of votes needed to win?

b. If no, which candidate has the least number of votes? Cross that letter out everywhere it appears.

c. Now, fill in the three remaining superpowers in the table below, and count the left-most letters in each vote again.

Round 2

Superpower	Number of Votes

a. Does any superpower have the number of votes needed to win?

b. If no, which candidate has the least number of votes? Cross that letter out everywhere it appears.

c. Now, fill in the two remaining superpowers in the table below, and count the left-most letters in each vote again.

Round 3

Superpower	Number of Votes

Does one superpower have a majority?

Ranked Choice Pros and Cons

Pros of this system	Cons of this system

Exit Ticket

Circle the reason in the table above that is most convincing to you.

Appendix D.3. Thursday Worksheet Key

Name:____KEY________

Day 4: Ranked Choice

Our Ranked Choice Election

Directions

1. Count the farthest left letter of each vote
2. Decide if one superpower has a majority
3. If not, cross out all of the votes for the last-place candidate
4. Count the farthest left letters again, skipping any that are crossed-out
5. Repeat until one superpower has a majority

Ballots

T S I F
F T I S
S T I F
T I F S
T I S F
I
S T F I
T I F S
I T F S
T I
T I F S
I T S F
I T F S
I T S F
I T F S
I S T F

Total number of votes:
16

Number of votes needed to win:
9

Round 1

Superpower	Number of Votes
T	6
F	1
I	7
S	2

Does any superpower have the number of votes needed to win?

No

If no, which candidate has the least number of votes? Cross that letter out everywhere it appears.

F

Now, fill in the three remaining superpowers in the table below, and count the left-most letters in each vote again.

Round 2

Superpower	Number of Votes
T	7
I	7
S	2

Does any superpower have the number of votes needed to win?

No

If no, which candidate has the least number of votes? Cross that letter out everywhere it appears.

S

Now, fill in the two remaining superpowers in the table below, and count the left-most letters in each vote again.

Round 3

Superpower	Number of Votes
T	9
I	7

Does one superpower have a majority?

Yes, T has a majority.

Ranked Choice Pros and Cons

Pros of this system	Cons of this system
Winner has a majority Only vote once Voters can vote honestly/feel good about their choices Elects candidates with broad support, not polarizing candidates Lots of information about each voter's preferences Gives all candidates a "fair shot" Vote splitting/spoilers likelihood greatly reduced	Lengthy and complicated process to count votes Requires knowledgeable voters Ballots may be confusing, are more likely to be mismarked and then uncounted Hard to understand

Exit Ticket

Circle the reason in the table above that is most convincing to you.

Appendix E.1. Friday Lesson Plan

Grade: 8th **Subject:** Math **Day:** 5

<table>
<tr><th colspan="3">GETTING YOURSELF READY</th></tr>
<tr>
<td>Materials:
- Slidedeck
- Template for student slidedeck (shared with students so they can all edit)
- Reflection document (individual to each student)
- Computers
- Completed worksheets from Tuesday, Wednesday, Thursday</td>
<td>Objective(s) and Proving Behavior:

Objectives:
Content
- Students understand math as a tool to understand and change our world

Language
- I can use math to support an argument for social change

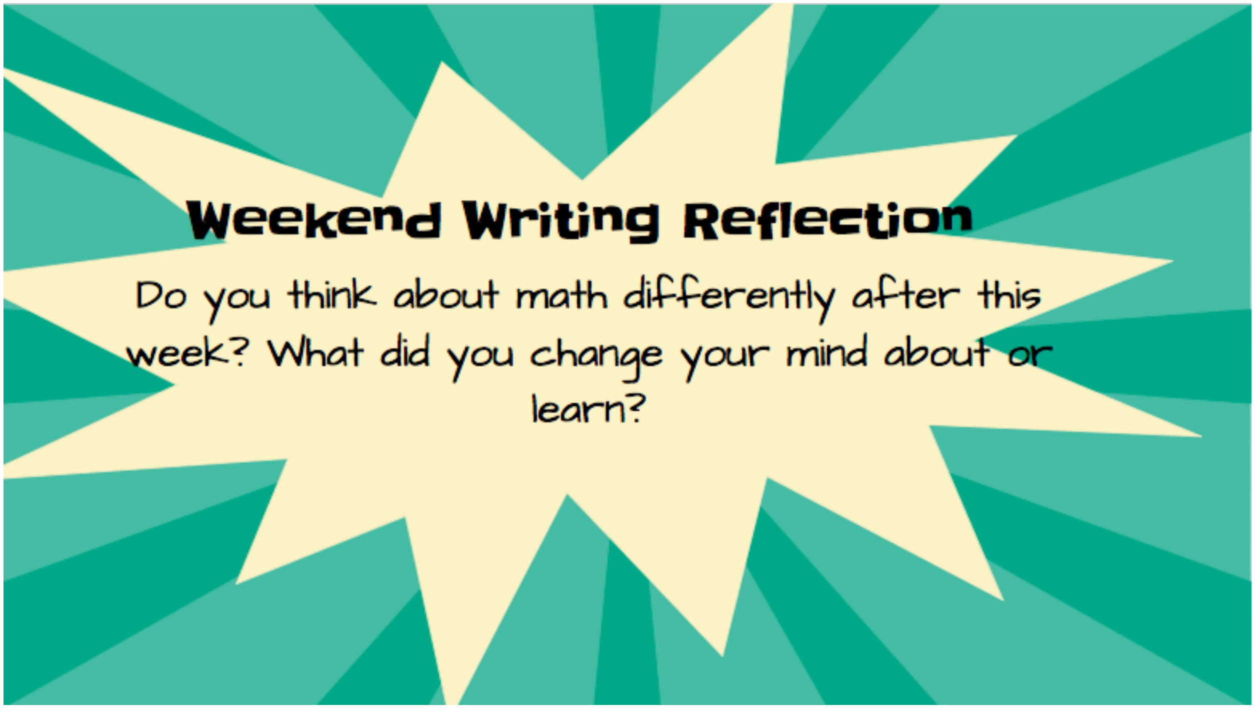
Proving behavior:
- Students complete reflection about if they think about math differently
- Students complete slidedeck arguing for a particular system and use math in their reasoning</td>
<td>Agenda (w/times):
Intro, review and debate: 15 minutes
Outline/what to include: 5 minutes
Student work: 40 minutes</td>
</tr>
<tr><th colspan="3">STUDENT ACTIVE LEARNING & CONNECTION TO CONTENT</th></tr>
<tr><td colspan="3">Active Learning Plan:

Go over today's goal with students: create slidedeck that makes an argument for which of the three systems to use next year for student council elections
Hand back completed worksheets for students to reference throughout class
Review the three systems we talked about this week
Student debate for which system to use
(Brief) discussion: what should we include?

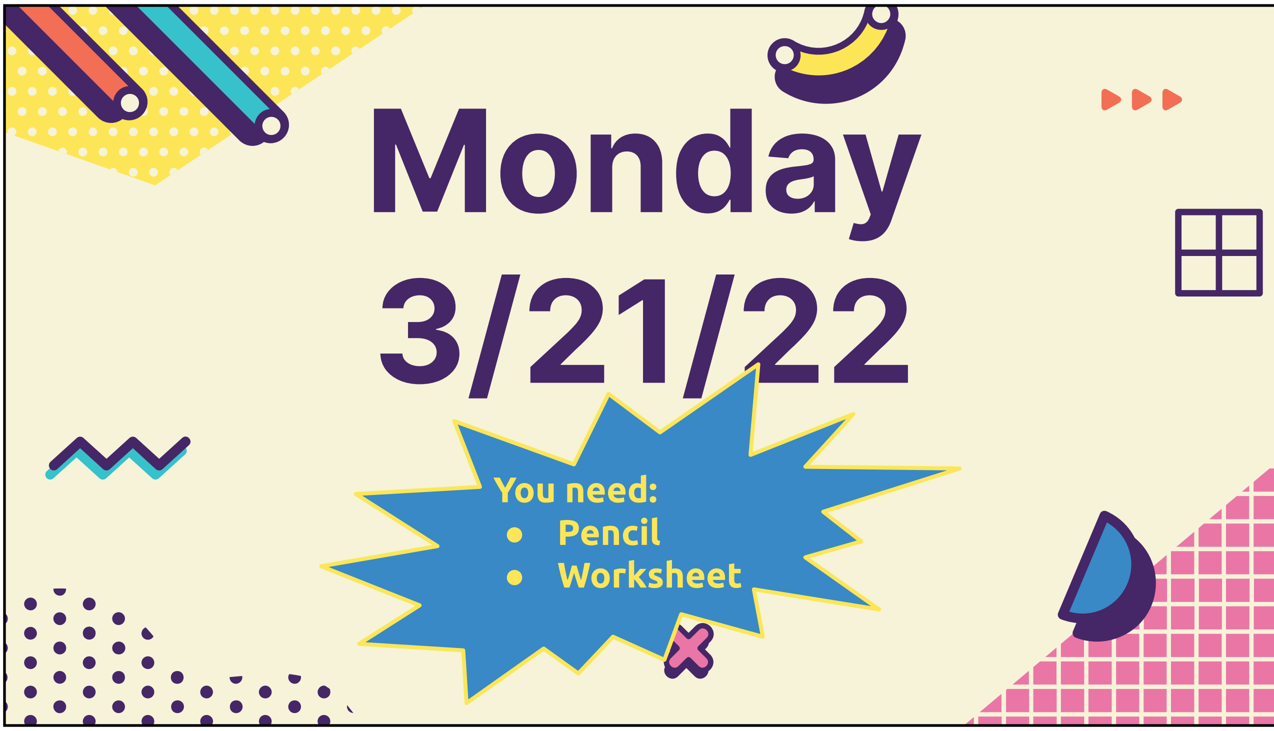
Consider our audience: what might they not know about voting that we should share?
How the system works
Evidence
A thesis at the beginning

Students work on slidedeck individually or in small groups
Once their slide has been approved, they can work on the student reflection
</td></tr>
<tr><th colspan="3">RELEASE TO ASYNCHRONOUS WORK: PRACTICING OR APPLICATION SKILLS/CONTENT</th></tr>
<tr><td colspan="3">Exit Ticket/Weekend:
Written reflection (3-4 sentences) on the prompt: Do you think about math differently after this week? What did you change your mind about or learn?</td></tr>
<tr><td colspan="3">Anticipated Challenges
For some classes working on a collective slidedeck was difficult. It might be helpful to review some Google Slides etiquette before starting, such as don't edit a slide that isn't yours and don't delete the work of others. Assigning small groups to work on a particular concept based on student strengths and interests (reason 1, algorithm, beautification/design) worked best across the sections. Bigger classes can add more reasons.</td></tr>
</table>

Appendix E.2. Friday Student Reflection Template



Appendix F.1. Monday Slidedeck



This week

We're going to take a close look at 3 different voting systems and compare/contrast them using math!

And on Friday create a proposal for which system to use for CREW Leadership Team elections to share with Mrs. Ricker!

Content and language objectives

Content Objectives

- I understand that there are many different types of voting systems in use around the world and within the U.S.
- I understand how I vote in my school

Language Objectives

- I can define "voting"
- I can explain aspects of a "fair election"

Two Sides game

If you agree with the text in PURPLE, go to the LEFT side of the room

If you agree with the text in GREEN, go to the RIGHT side of the room

Summer or winter?

Hot pizza or cold pizza?

Does every country vote in the same way?

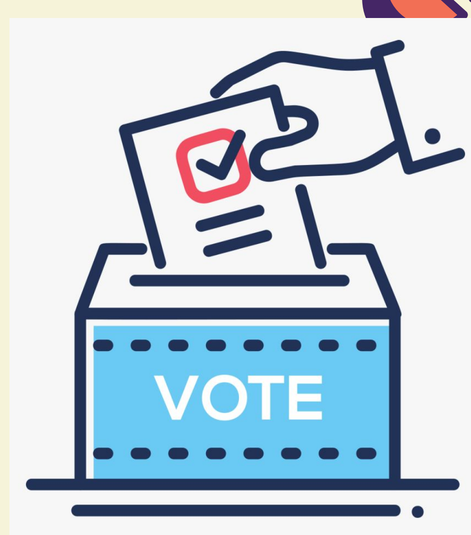
YES or NO

Does every U.S. state vote in the same way?

YES or NO

What is voting?

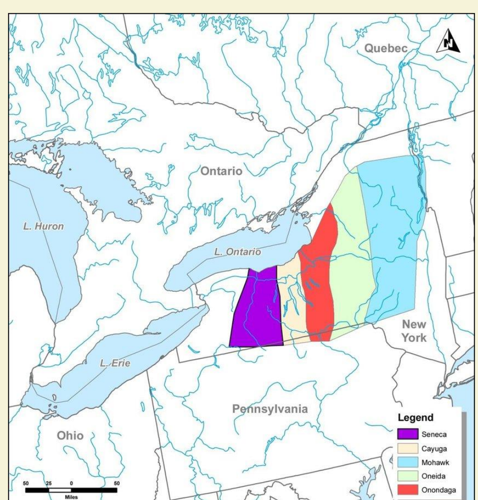
One definition: Voting is a way of determining a <u>group preference</u> from a set of individual preferences



What do you know about voting in the United States?

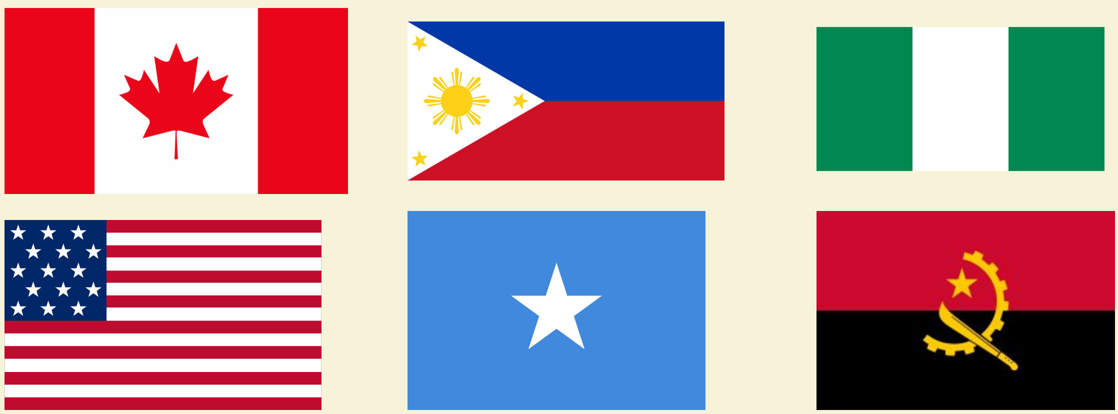
What do you know about voting in the United States?

- Representative democracy
- Inspired by the Iroquois Confederacy
    - Group of 6 Native American nations
    - Oldest living participatory democracy



Does every country in the world vote in the same way?

No!



Angola

Vote for lists

Vote for one group

Hadhira
Zeke
Abdi

2/3

Sophia
Ash
Anika

1/3

The United States

Vote for one

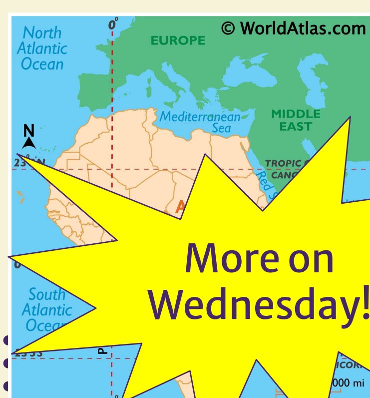
More on Tuesday!

Vote for one

- Hadhira
- Zeke
- Abdi
- Sophia
- Ash

Nigeria

Vote in two rounds



More on Wednesday!

Vote for one

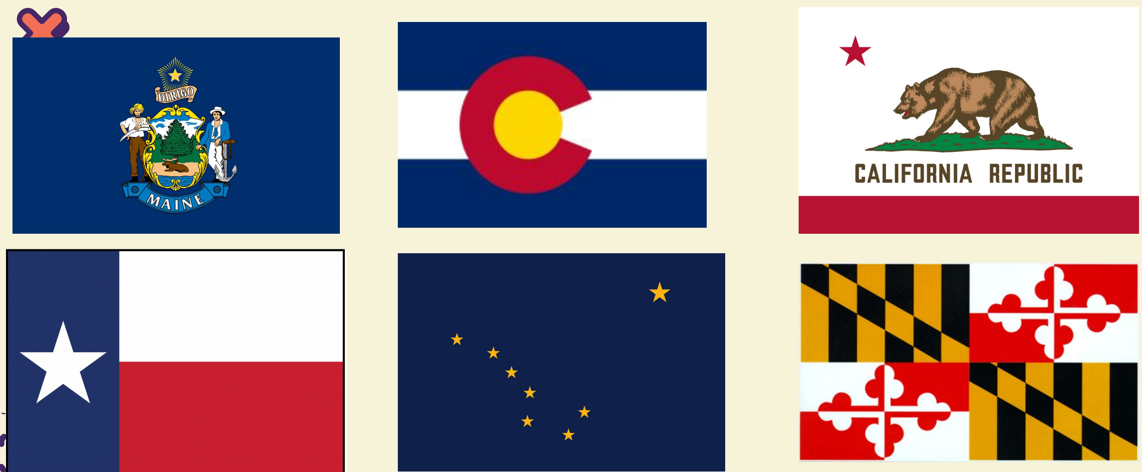
- Hadhira
- Zeke
- Abdi
- Sophia
- Ash

Vote for one

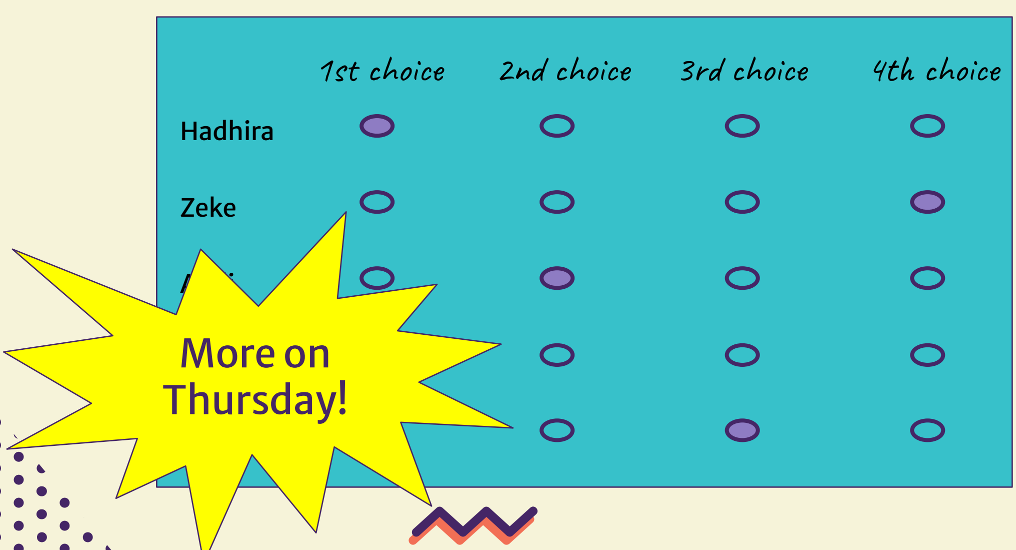
- Abdi
- Sophia

Does every state in the U.S. vote in the same way?

No!



Maine: Ranked Choice Voting



Do you think there is one best way to vote?

- No! Everyone would use that way then!
- There are many, many different systems that all have their strengths and weaknesses
- This is a global issue and debate
- Mathematicians are some of the key people involved in trying to find the fairest way to vote

When have you voted?

What should be true in a fair vote?

Content and language objectives

Content Objectives

- I understand that there are many different types of voting systems in use around the world and within the U.S.
- I understand how I vote in my school

Language Objectives

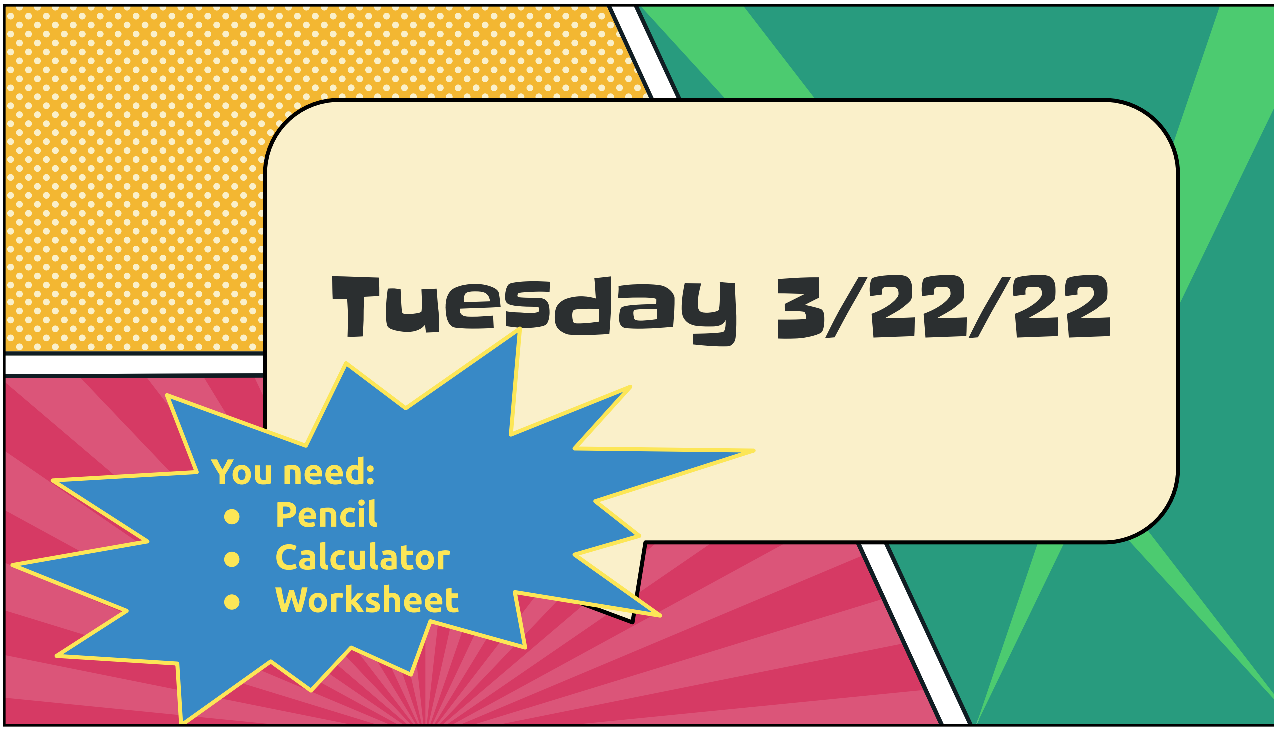
- I can define "voting"
- I can explain aspects of a "fair election"

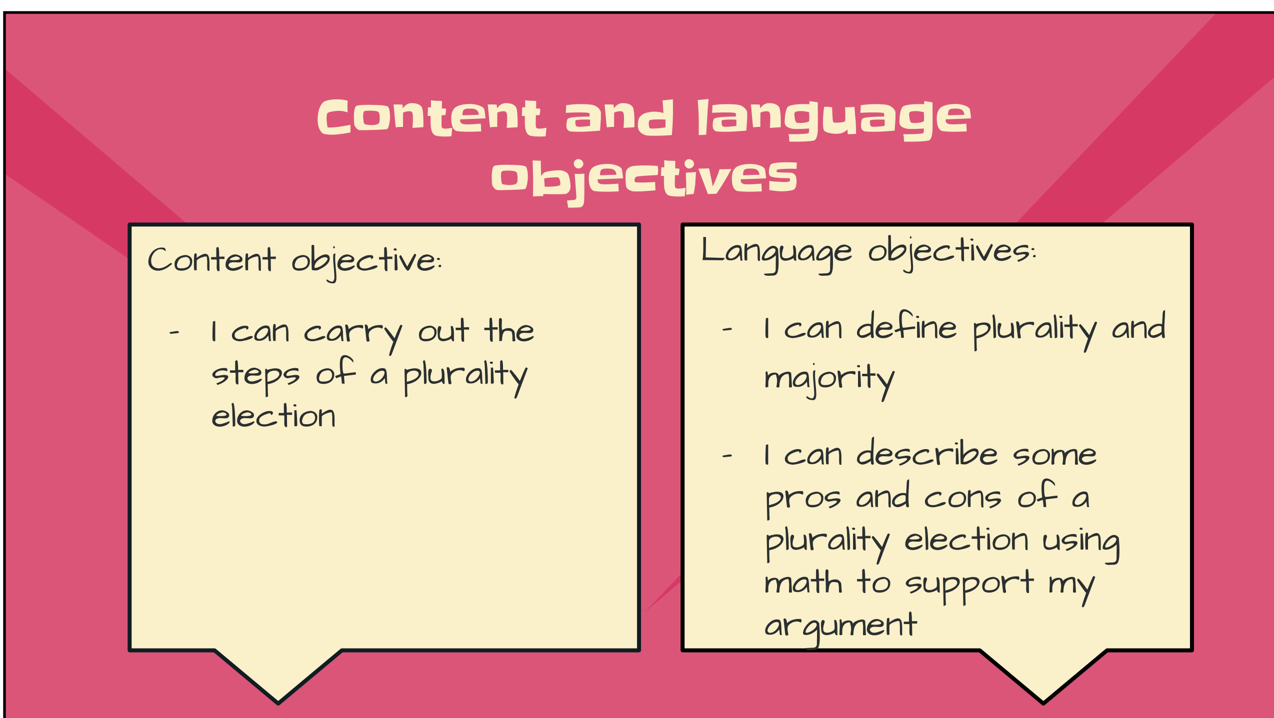
Exit Ticket and Looking Ahead

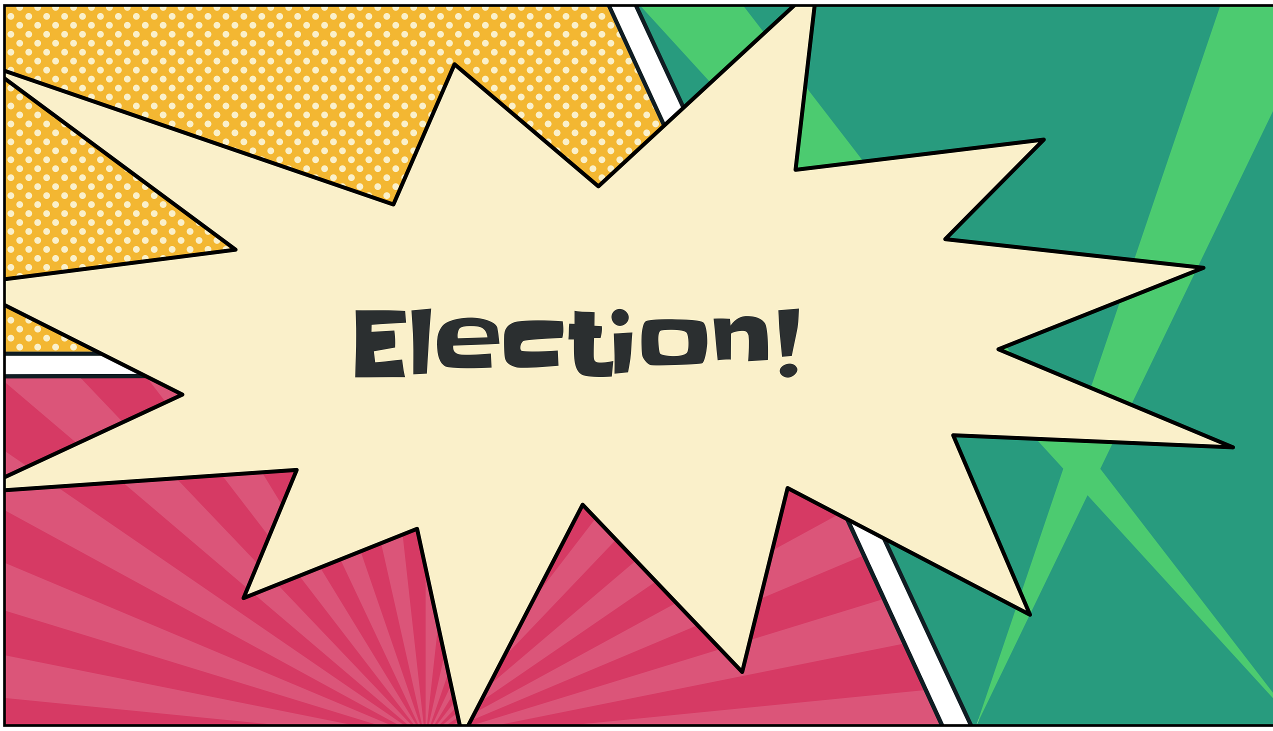
Exit Ticket: Write or draw a main idea from today

Looking ahead: Tomorrow, we'll do an election simulation using the "vote for one" method

Appendix F.2. Tuesday Slidedeck






Election!

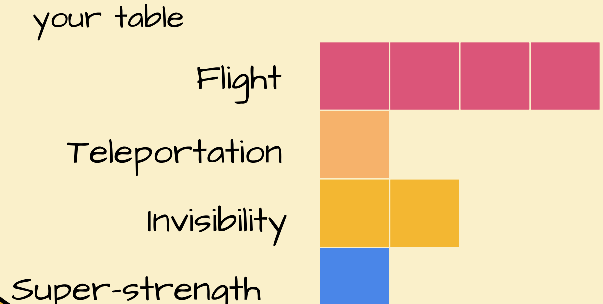

What is the best superpower?

Today: “Vote for one”

- From yesterday:
  - Most common voting system in the U.S.
  - Very similar to what you did for CREW leadership team
    - You voted for two, but it works almost the same
- Vote for one candidate
- Whichever candidate gets the most votes wins

Election Steps

1. Line up for the voting booth!
2. Select ONE post-it note and place it in the ballot box
3. Work on the post voting worksheet at your table



PINK is flight

ORANGE is teleportation

YELLOW is invisibility

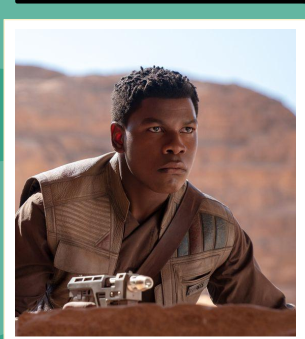
BLUE is super-strength

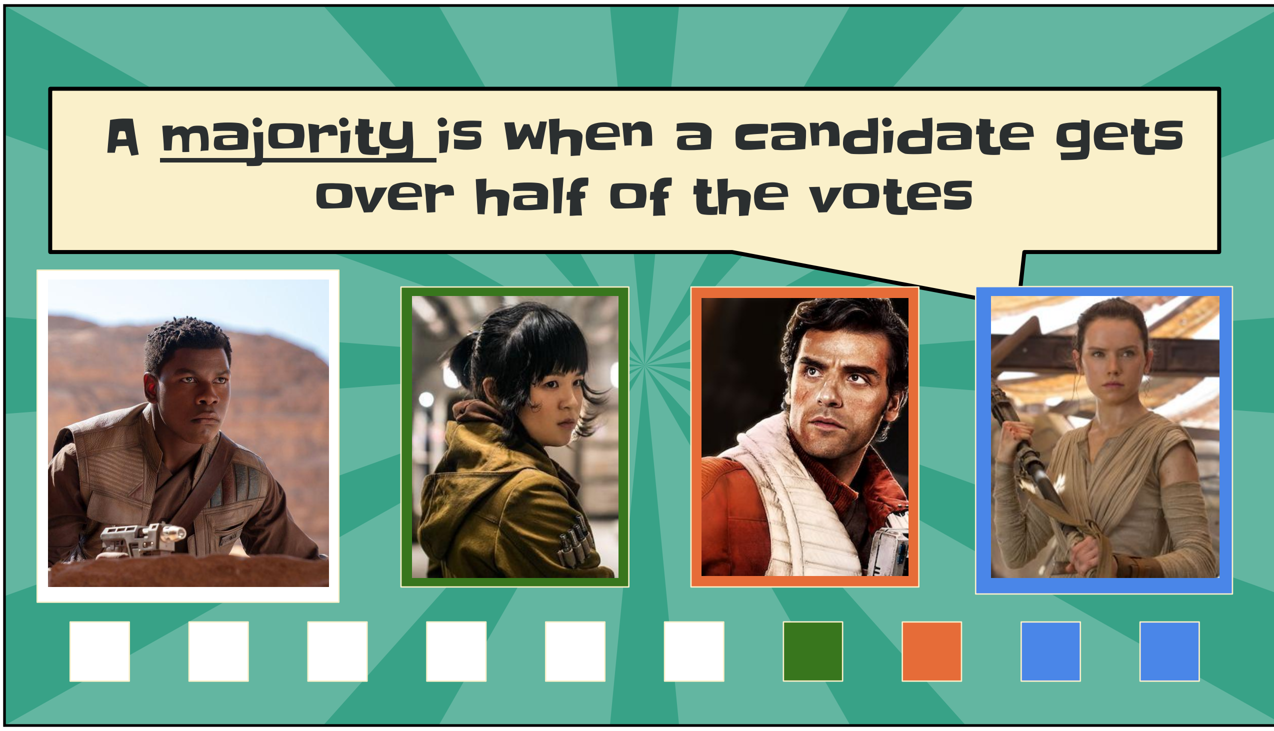
Superpower	**Votes**	**Percentage of total votes**
Flight		
Invisibility		
Teleportation		
Super-strength		
	Total votes:	**Total percent: 100%**

What did you notice?

1. What percent of voters preferred the winning superpower?
2. What percent of voters preferred a different superpower?

A <u>plurality</u> is when a candidate gets more votes than any other candidate

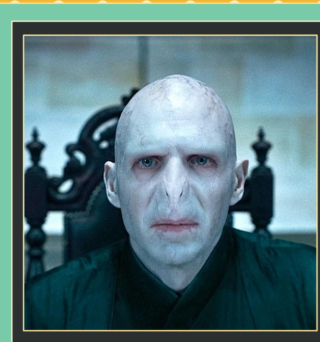




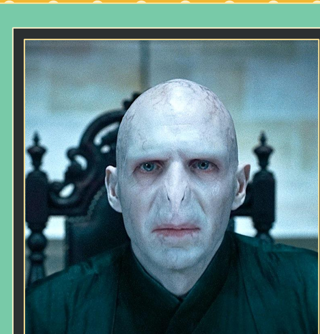
Discussion

1. Did the winning superpower win by plurality or majority?
2. In your opinion, did the winner win fairly? Why or why not?
3. Should a candidate need a majority to win? Why or why not?

Split the Vote



Split the Vote



Summing Up

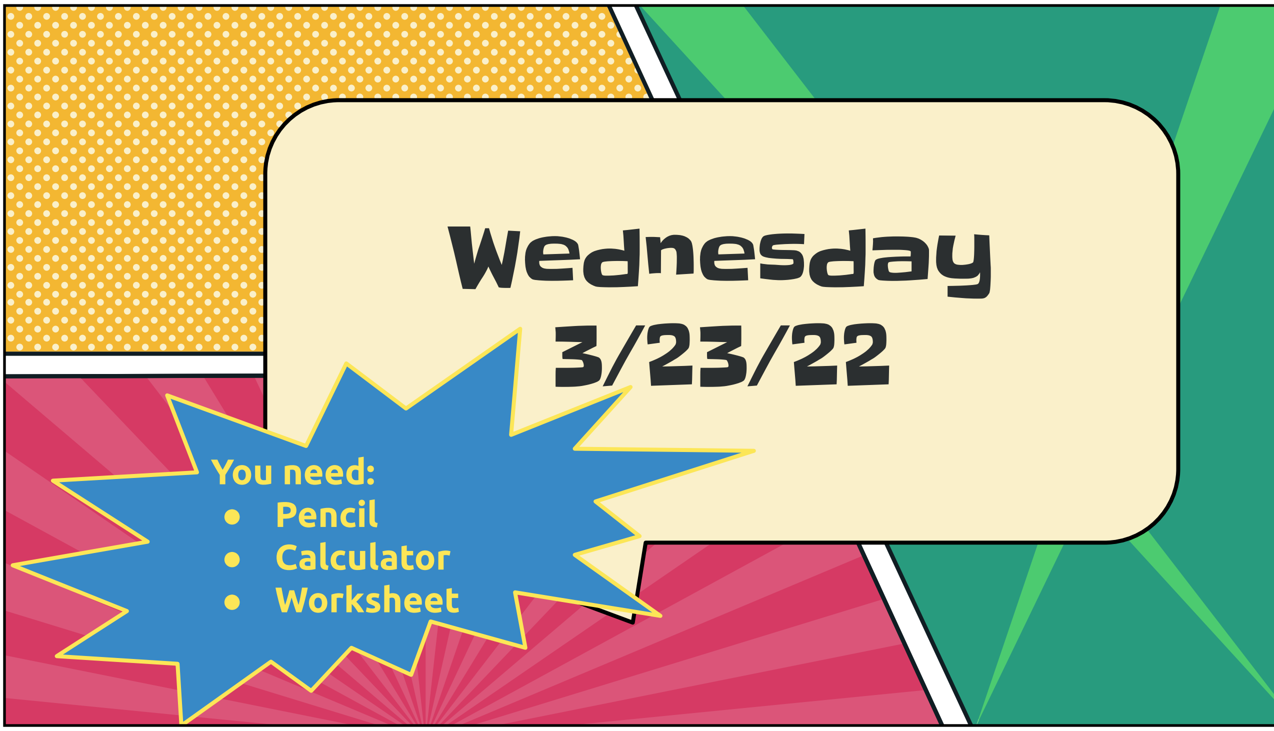
Pros	Cons

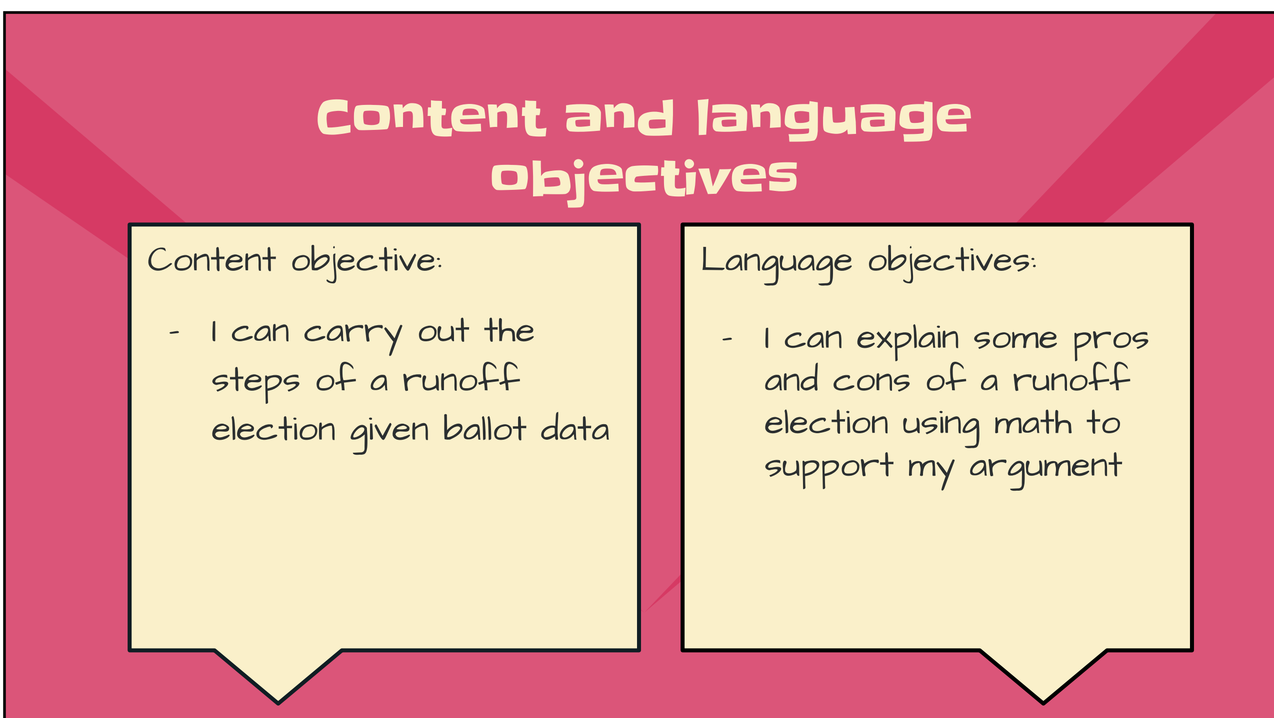
Exit Ticket

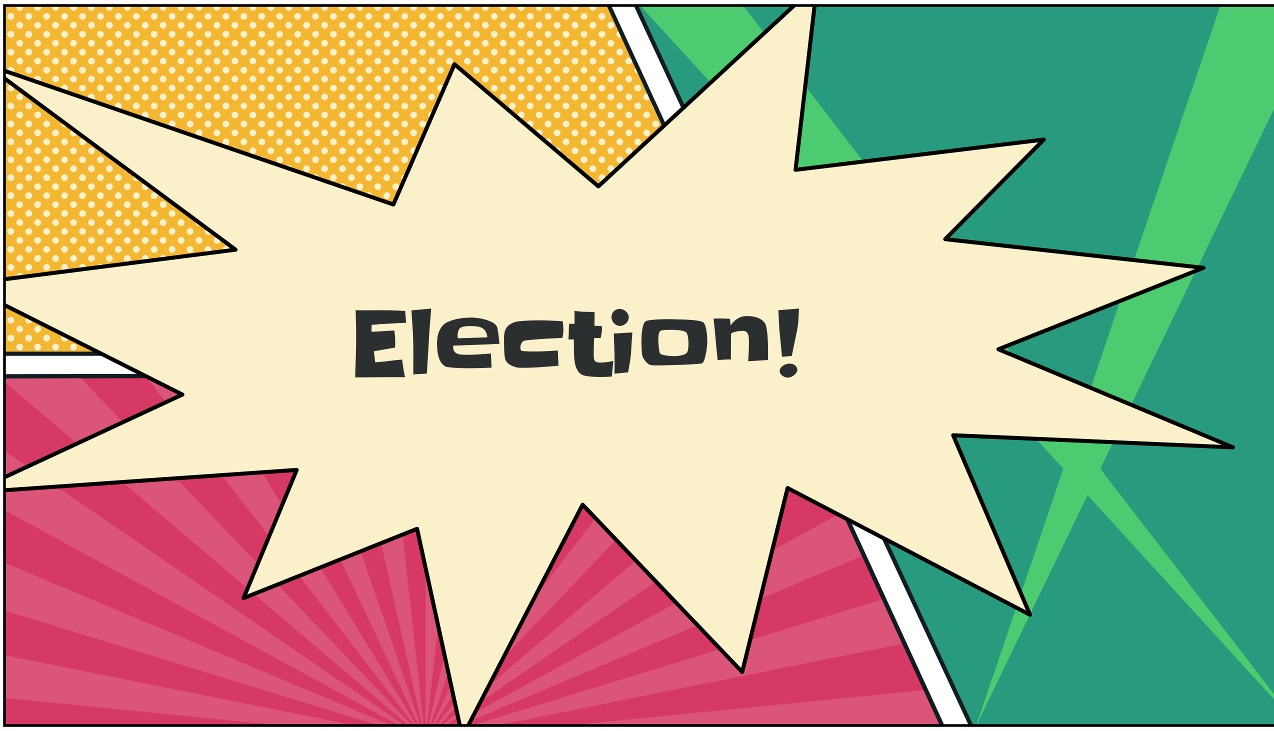
Which reason either for or against plurality voting is most convincing to you? Circle it in your table

Tomorrow we'll look at a voting system called a runoff!

Appendix F.3. Wednesday Slidedeck



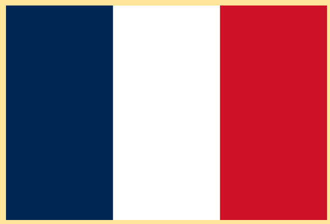
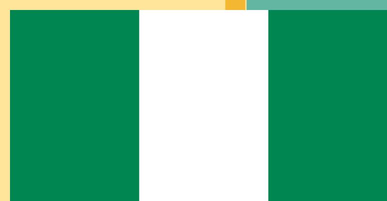




Superpower	Votes
Flight	5
Teleportation	18
Invisibility	19
Super-strength	6
	Total votes: 48

Today: Runoff

- Nigeria, France, and Brazil
- Ten U.S. states

Today: Runoff

1. Vote for one candidate like plurality (we did this yesterday)
2. Determine if a candidate has a majority
   a. If yes, that candidate wins
   b. If no, we hold a runoff
3. Determine the 1st and 2nd place candidates
4. Hold a second vote where you vote for one of those two candidates

Superpower	Votes	Percentage of total votes
Flight	5	[redacted]
Teleportation	18	[redacted]
Invisibility	19	[redacted]
Super-strength	6	[redacted]
	Total votes: 48	[redacted]

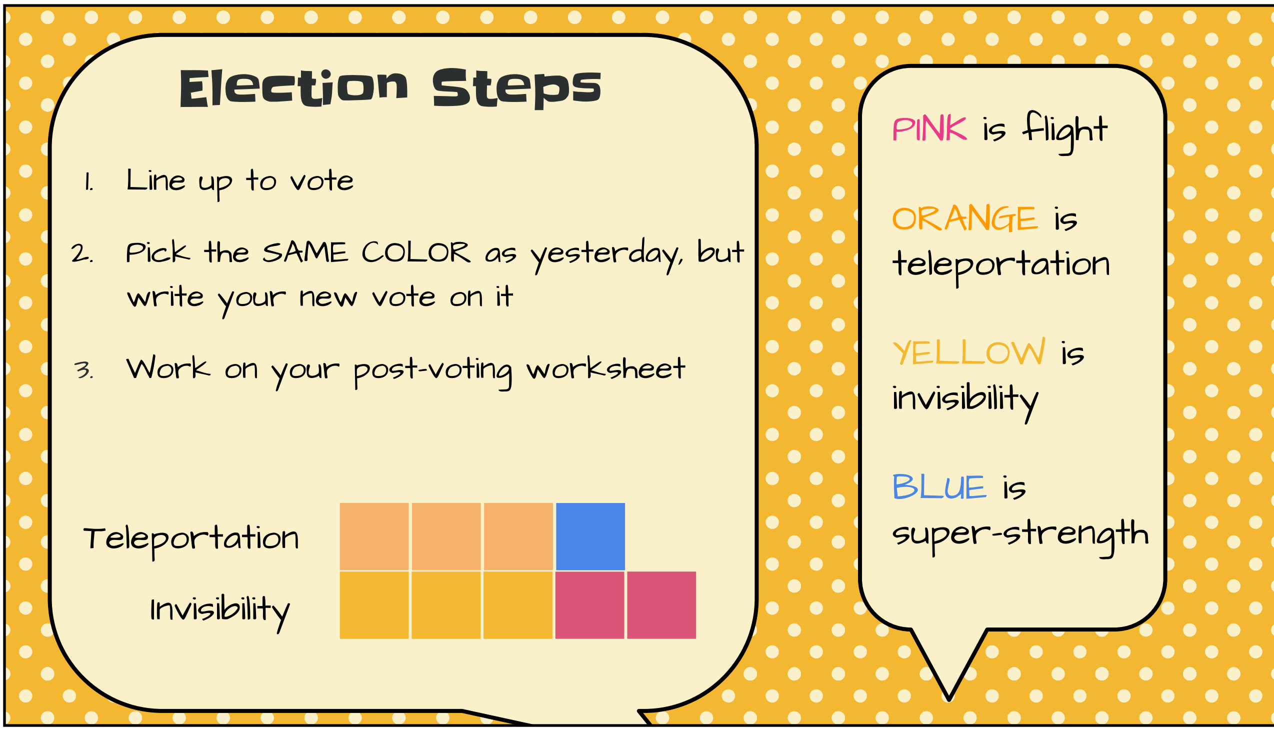
Does one candidate have a majority?

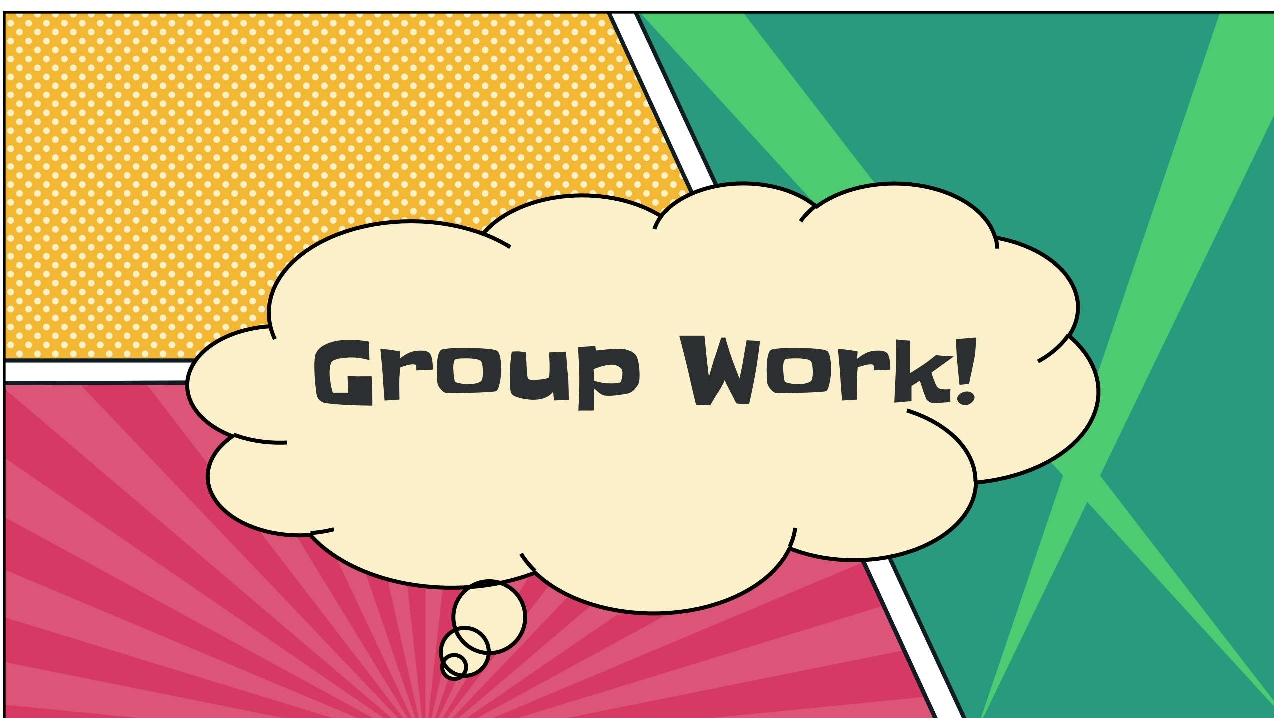
Which two candidates are in 1st and 2nd place?

Superpower	Votes	Percentage of total votes
Flight	5	10.41%
Teleportation	18	37.5%
Invisibility	19	39.58%
Super-strength	6	12.5%
	Total votes: 48	Total percent: 100%

Does one candidate have a majority?

Which two candidates are in 1st and 2nd place?


Election Steps
1. Line up to vote
2. Pick the SAME COLOR as yesterday, but write your new vote on it
3. Work on your post-voting worksheet
Teleportation
Invisibility
PINK is flight
ORANGE is teleportation
YELLOW is invisibility
BLUE is super-strength


Group Work!

Superpower	Votes	Percentage of total votes
Teleportation		
Invisibility		
	Total votes:	Total percent: 100%

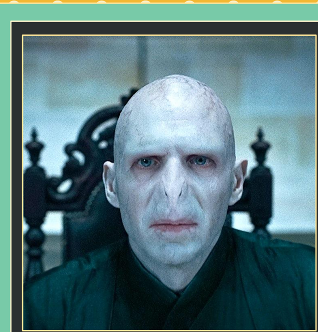
What did you notice?

1. Did the winner win with a plurality or a majority?
2. Will this always be the case in a runoff? Why or why not?

Back to Harry Potter...



Back to Harry Potter...

Summing Up

Pros

Cons

Starting Ranked Choice

Best Superpower	1st Choice	2nd Choice	3rd Choice	4th Choice
Flight	⬭	⬭	⬭	⬭
Teleportation	⬭	⬭	⬭	⬭
Super-strength	⬭	⬭	⬭	⬭
Invisibility	⬭	⬭	⬭	⬭

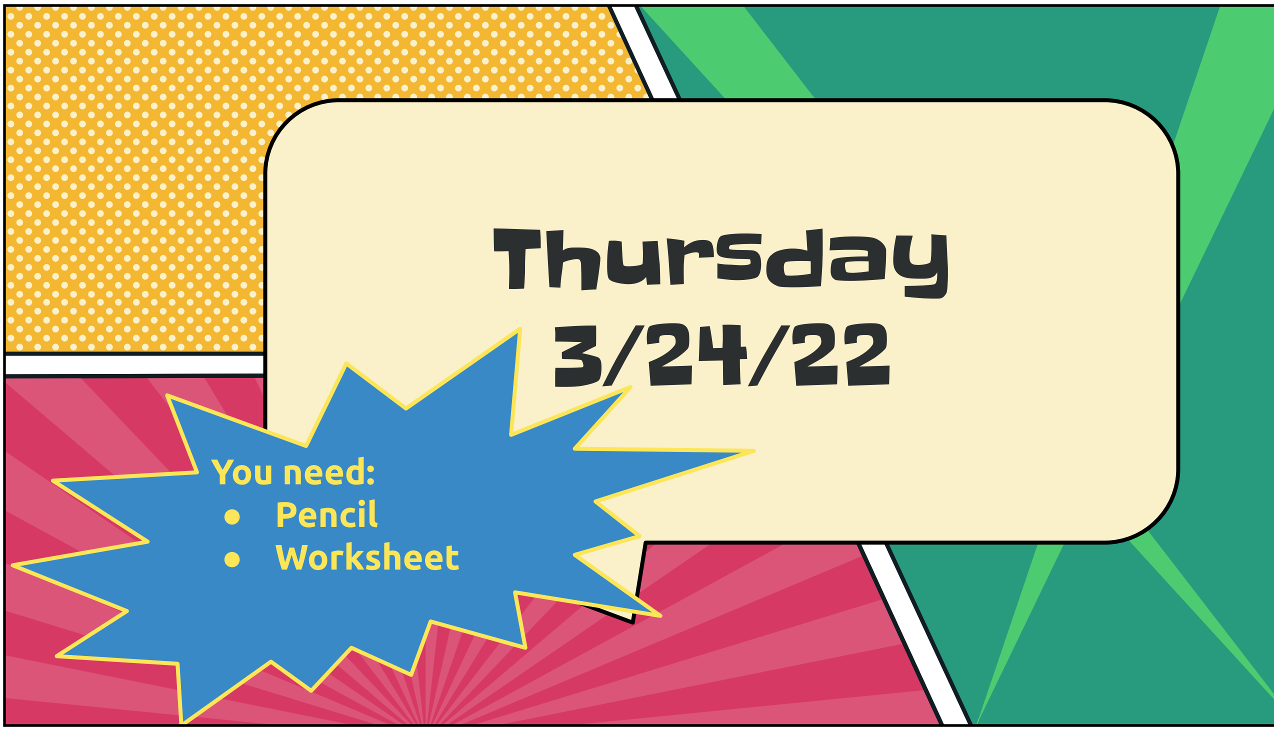
Maine Voting Rules

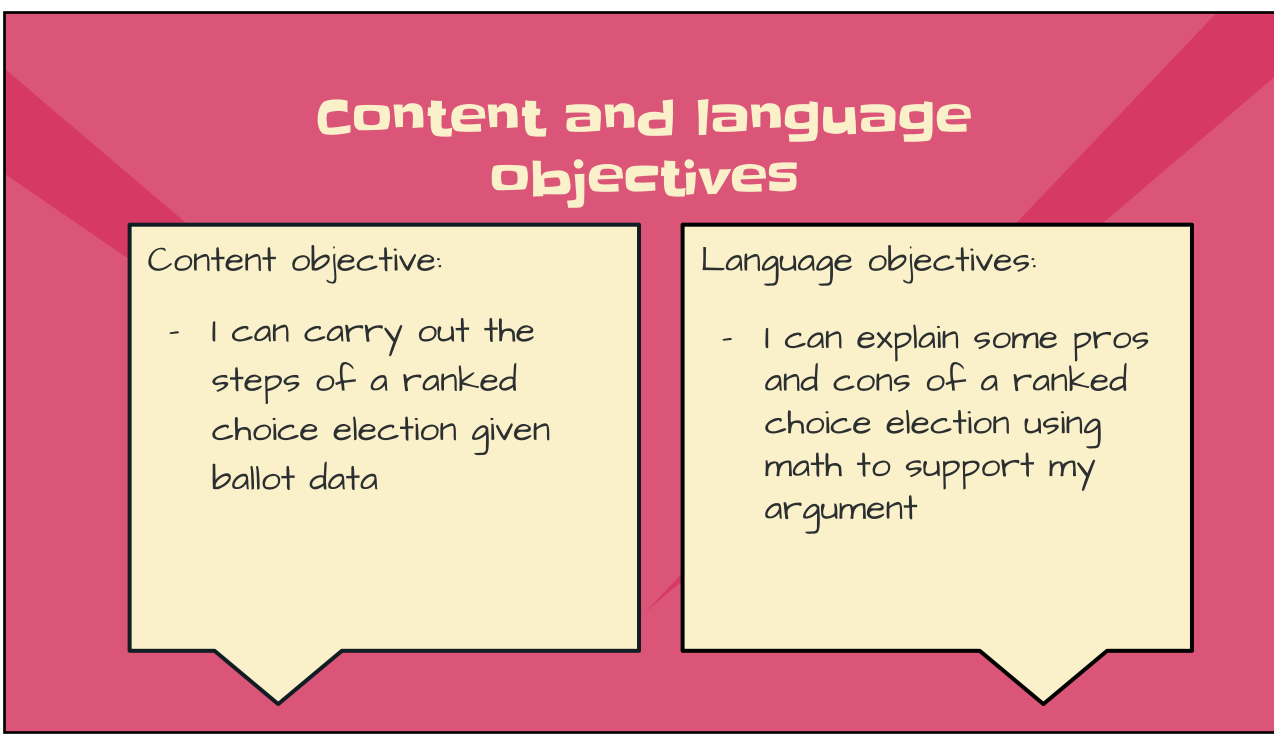
Exit Ticket

Hand in your ranked choice ballot!

Tomorrow we'll look at our final voting system called ranked choice!

Appendix F.4. Thursday Slidedeck

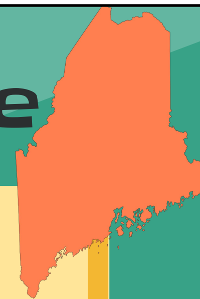




Superpower	Votes
Flight	5
Teleportation	18
Invisibility	19
Super-strength	6
	Total votes: 48

Superpower	Votes
Teleportation	22
Invisibility	23
	Total votes: 45

Today: Ranked Choice



- Maine (since 2018)! Also, Alaska
- 20 other U.S. towns and cities (ex. New York City)
- Australia, Ireland, Scotland, New Zealand, Malta
- Olympic Host Cities
- Best Picture at the Oscars

Today: Ranked Choice

Also called instant runoff,

<u>Ranked Choice is many runoffs in a row</u>

Remember: what is a runoff?

Today: Ranked Choice

Instead of eliminating everyone but the top-two candidates, ranked choice eliminates the loser of each round

Today: Ranked Choice

Instead of re-voting, all the voters' opinions go on one ballot

Back to Harry Potter...



	1st choice	2nd choice	3rd choice
1	Potter	Granger	Voldemort
2	Potter	Granger	Voldemort
3	Granger	Potter	Voldemort
4	Voldemort	Potter	Granger
5	Voldemort	Granger	Potter

P G V
P G V
G P V
V P G
V G P

P G V
P G V
G P V
V P G
V G P

Total votes:

Number of votes needed to win:

P G V
P G V
G P V
V P G
V G P

Number of 1st Choice votes:
P:
G:
V:

Has anyone won?

P G V
P G V
G P V
V P G
V G P

Number of 1st Choice votes:
P:
G:
V:

Eliminate the candidate with the LEAST first-choice votes and count again!

P G V
P G V
G P V
V P G
V G P

Number of 1st Choice votes

P:

V:

Has anyone won?

Your Turn!

1. Count the farthest left letter of each vote
2. Decide if one superpower has a majority
3. If not, cross out all of the votes for the last-place candidate
4. Count the farthest left letters again, skipping any that are crossed-out
5. Repeat until one superpower has a majority

Discussion

How is a ranked choice similar to a runoff? How is it different?

When would it be useful to use ranked choice?

Summing Up

Pros

Cons

One last note...

It is possible for each system to give a different outcome

One last note...

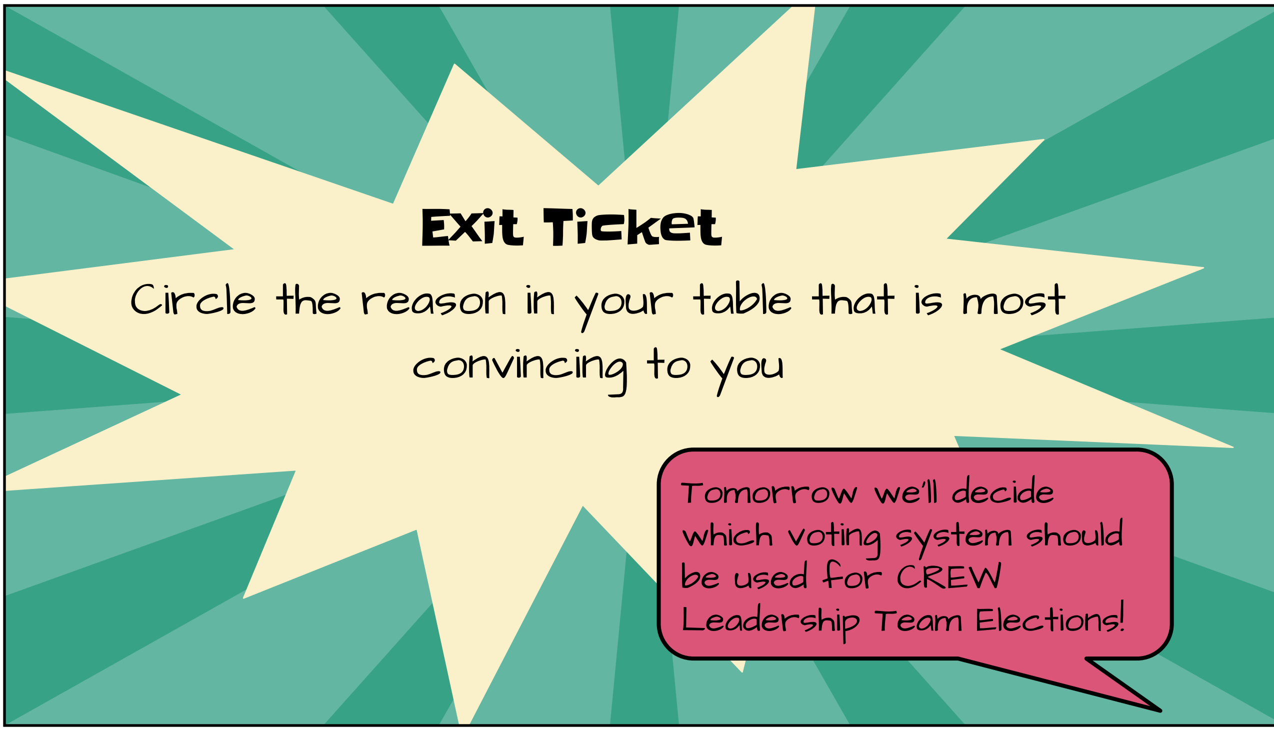
T I F S	I F T S	F I T S	S F I T
T I F S	I F T S	F I T S	S F I T
T I F S	I F T S	F I T S	S F I T
T I F S	I F T S	F I T S	
T I F S	I F T S		
T I F S			

One last note...

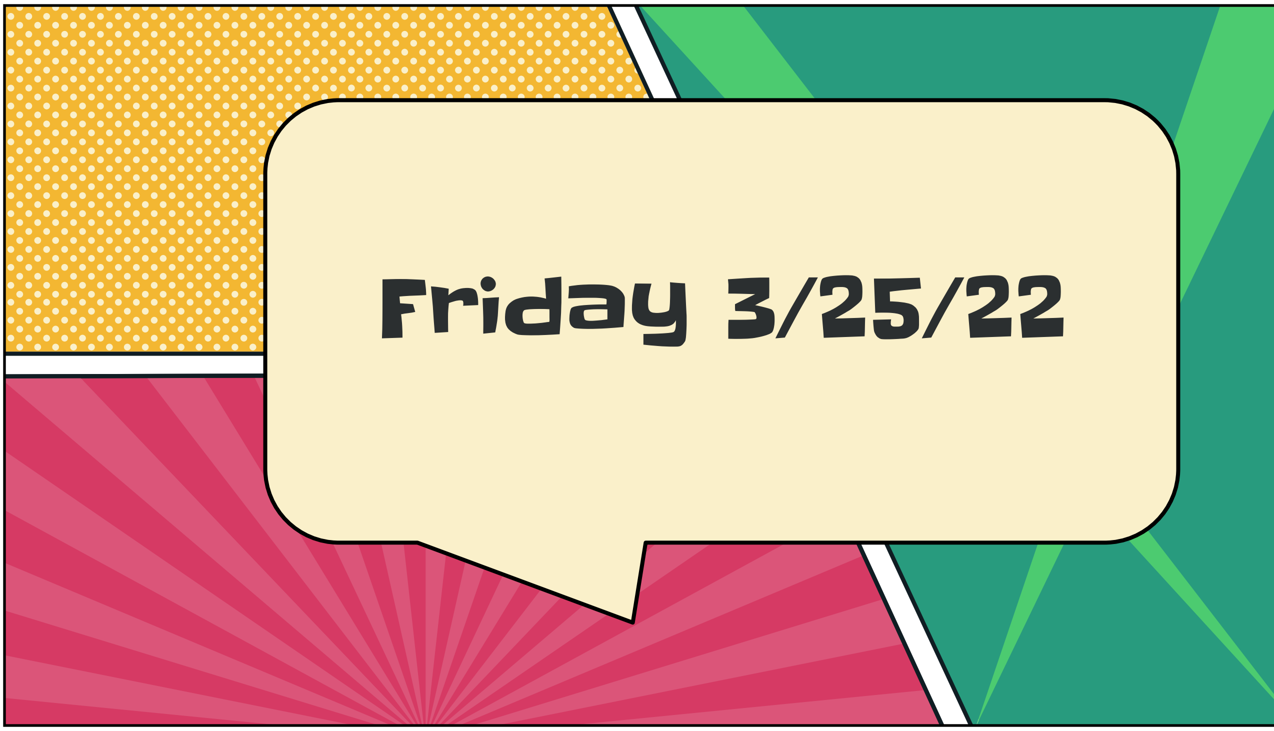
Plurality winner:

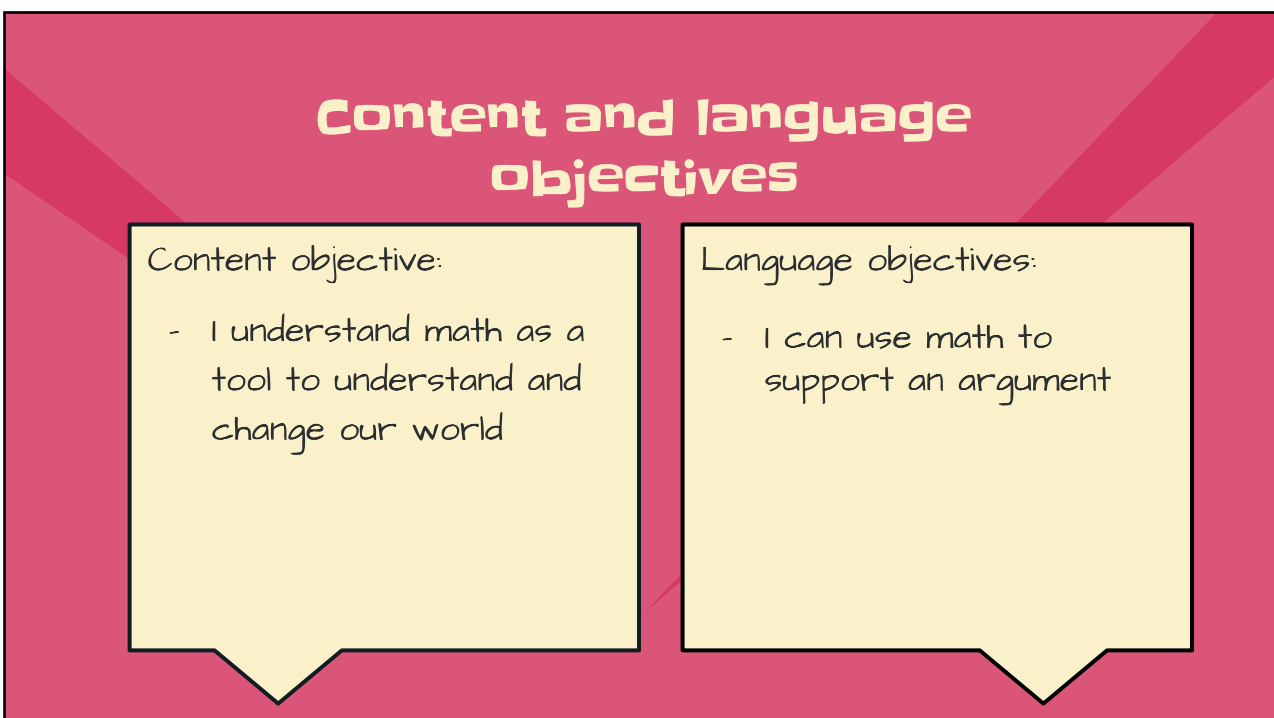
Runoff winner:

Ranked choice winner:


Exit Ticket
Circle the reason in your table that is most convincing to you
Tomorrow we'll decide which voting system should be used for CREW Leadership Team Elections!

Appendix F.5. Friday Slidedeck





Today: Action

Goal: Create a set of slides to show Mrs. Ricker that argues which voting system should be used for CREW Leadership Team elections next year

Tuesday: Plurality

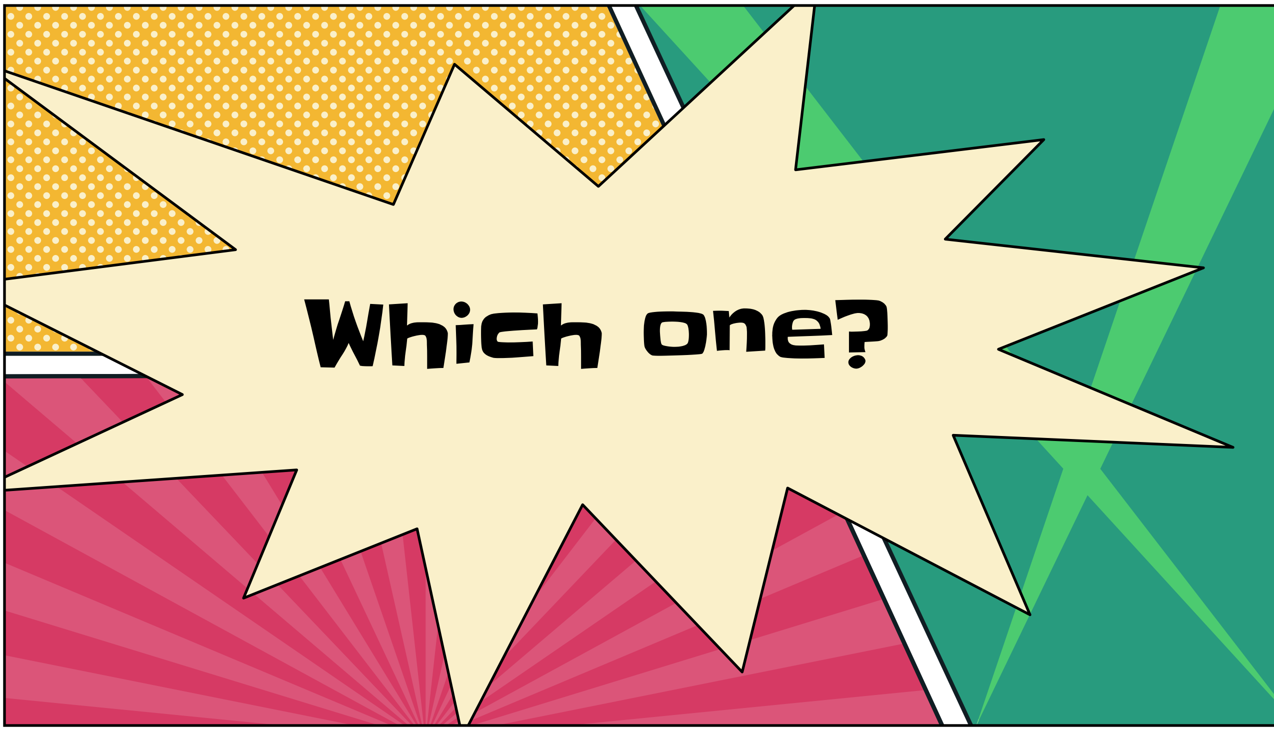
1. Vote for one
2. Whoever gets the most votes wins

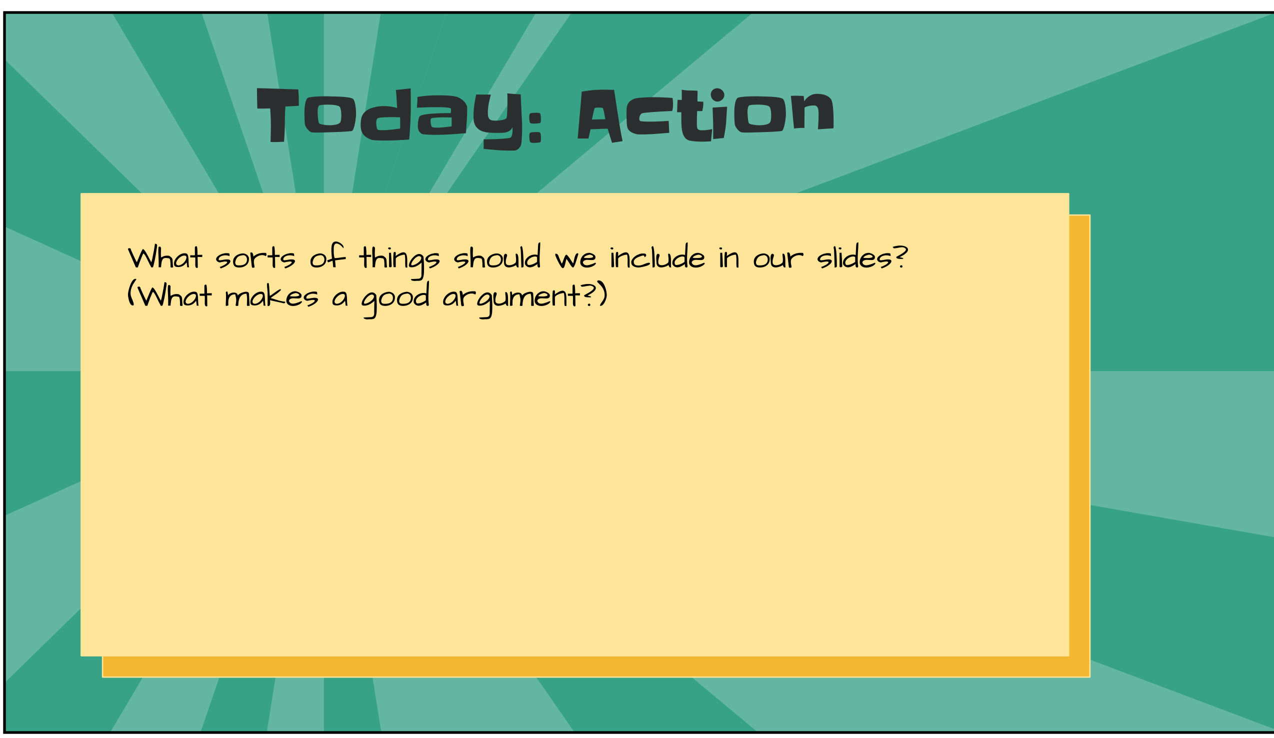
Wednesday: Runoff

1. Vote for one
2. If no candidate gets a majority, eliminate everyone but the top two
3. Vote again on those two candidates

Thursday: Ranked Choice

1. Rank multiple choices on one ballot
2. If no candidate gets a majority, eliminate the loser
3. Recalculate the votes based on second choices
4. Repeat until one candidate gets a majority


Which one?


Today: Action
What sorts of things should we include in our slides?
(What makes a good argument?)

Today: Action

Work in a group of three or four on one slide of the presentation

Weekend Writing Reflection

Do you think about math differently after this week? What did you change your mind about or learn?