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RAINBOW CONNECTION NUMBER AND TOTAL RAINBOW CONNECTION NUMBER OF AMALGAMATION RESULTS DIAMOND GRAPH(Br_4) AND FAN GRAPH(F_3)

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Abstract. *If $G = V(G), E(G)$be a graph and edge coloring of G is a function $f: E(G) \to C$, rainbow connection number is the minimum-k coloration of the rainbow on the edge of graph G and denoted by rc(G). Rainbow connection numbers can be applied to the result of operations on some special graphs, such as diamond graphs and fan graphs. Graph operation is a method used to obtain a new graph by combining two graphs. This study performed amalgamation operations to obtain rainbow connection numbers and rainbow-total-connection numbers in diamond graphs (Br_4) and fan graphs (F_3) or $Amal(3Br_4 * F_3, v)$. Based on the research, it is obtained that the rainbow-connection number theorem on the amalgamation result of the diamond graph (Br_4) and fan graph (F_3) is $rc(G) = diam$ with $t = 4$. Furthermore, the theorem related to the total rainbow-connection number on the amalgamation result of the diamond graph(Br_4) and the fan graph (F_3), is obtained, namely $trc(G) = 2diam = 8$ with $t = 4$.*

Keywords: *diamond graph, fan graph, amalgamation operation, rainbow connection numbers*

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1. INTRODUCTION

One branch of mathematics that is specifically a study in discrete mathematics and is used to represent discrete objects and the relationships between objects is graph theory. The representation usually has a dot to represent the object, while the object's relationship is expressed with an edge. Previously, graphs were first used to solve the Konigsberg bridge problem in 1736 [1]. One of the theories developed in graph theory is Chartrand et al. 2005, which first introduced rainbow connection number, the concept of rainbow connection number. A rainbow connection number is coloring on a graph with the condition that there is a rainbow path on the graph side and two adjacent sides can have the same color. However, the side included in the rainbow path must have a different color, whereas the rainbow path is a path where each side does not have the same color. The rainbow connection number is the k-minimum coloring of the rainbow in graph G. It is denoted by *rc(G)* [2].

Rainbow connection numbers can be applied to the operating results of some special graphs, such as diamond graphs and fan graphs [3], [4]. Graph operation is a method used to obtain a new graph by combining two graphs. There are several forms of operation, including joint, corona, comb, shackle, and amalgamation [5]. Several studies have been carried out, namely examining rainbow-connection numbers on the results of amalgamation operations of several special graphs, such as what was done by Fitriani and Salman in 2016 [6]. Then, in 2019, several studies were carried out on the amalgamation of diamond graphs, including those carried out by Afifah et al. [7] and Cindy et al. [8]. Therefore, from several studies that have been done previously, the author tried to develop rainbow connection numbers by adding amalgamation operations from two special graphs.

In this study, amalgamation operations of two special graphs were performed, including the diamond graph (Br_4) and the fan graph (F_3). The amalgamation operations were performed by agglutinating all terminal points v_{0i} of several graphs G_i into one point [9]. Next, a rainbow coloring will be determined from the amalgamation of a diamond graph(Br_4) and fan graph (F_3). This study aimed to determine the rainbow-connection number and the rainbow-total-connection number from the amalgamation of the diamond graph (Br_4) and the fan graph (F_3), so that two theorems related to rainbow connection numbers, and rainbow-total-connection numbers will be obtained. This study hopefully can contribute to the development of science in the field of graph theory related to rainbow connection numbers and rainbow total connection numbers.

2. RESEARCH METHOD

In this study, the method used was a literature study research method. The steps used are as follows:

1. Formulate the research problem to be discussed.
2. Study and understand literature sources related to rainbow connection number coloring.
3. Analyze the problems that had been obtained. Then, patterns and theorems to be proven would be obtained. The steps were as follows:
   (a) Draw the graph to be studied, namely the diamond and fan graphs.
   (b) Apply amalgamation operations to diamond and fan graphs.
   (c) Determine the pattern of rainbow connection numbers and rainbow-total-connection numbers result of amalgamation of diamond and fan graph operations.
   (d) Proving the rainbow-connection number theorem and the rainbow-total-connection number resulting from the amalgamation of diamond graphs and fan graphs.
4. Formulate the conclusions that had been obtained.

Furthermore, several definitions and theorems that were used in this study were presented.

Definition 1 If *n* and *t* be positive integers with size of $diam = 4$ is an amalgamation of diamond graph and fan graph. A diamond graph with *n* = 4 denoted by (Br_4) is a graph with [3]:

$$\begin{aligned} V(Br_4) &= \{v\} \cup \{v_i | i \in \{1,2, \dots\dots, n\}\} \cup \{u_i | i \in \{1,2, \dots n-1\}\} \\ E(Br_4) &= \{vv_i | i \in \{1,2, \dots n\}\} \cup \{v_i v_{i+1} | i \in \{1,2, \dots . n-1\}\} \\ &\quad \cup \{u_i u_{i+1} | i \in \{1,2, .. n-2\}\} \cup \{u_i v_i | i \in \{1,2, \dots . n-1\}\} \\ &\quad \cup \{u_i v_{i+1} | i \in \{1,2, \dots n-1\}\} \end{aligned}$$

Fan graph with $n = 3$ denoted by F_3 is a graph with:

$$V(F_3) = \{v\} \cup \{v_i | i \in \{1,2, \dots n\}\}$$
$$E(F_3) = \{vv_i | i \in \{1,2, \dots n\}\} \cup \{v_i v_{i+1} | i \in \{1,2\}\}$$

For more details, the amalgamation of diamond and fan graphs can be seen in Figure 1 below.

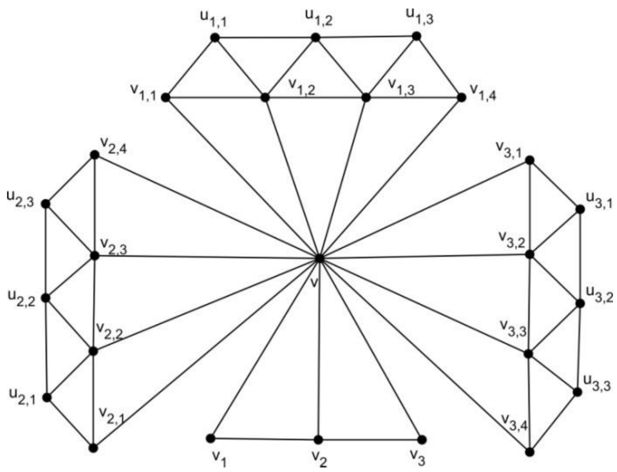

Figure 1. ***Amal*(3*Br*4*F_3, *v*)**

It can be seen in Figure 1 which is the result of the amalgamation operation of a diamond graph with n = 4 and a fan graph with n=3 for t=4. The amalgamation operation above is carried out by agglutinating all v_{0i} terminal points on three diamond graphs and one fan graph, so that they become one point..

Theorem1 [2] If G is a nontrivially connected graph of size m and $diam(G) = \max\{d(u,v)|u,v \in V(G)\}$, so:

$$diam(G) \leq rc(G) \leq src(G) \leq m$$

3. RESULTS AND DISCUSSION

3.1. Rainbow connection numbers Result of Amalgamation of Diamond Graph(〖Br〗 _4) and Fan Graph(F_3)

Figure 1 is an amalgamation of three diamond graphs and one fan graph which will then be searched for rainbow connection numbers from each of these graphs so that the following theorem is obtained.:

Theorem 2. *If* $n = 4$ *and* $t = 4$ *for* $i \in [1,t]$. *If* $G \cong Amal(3Br_4 * F_3, v)$ *and* (Br_4) *is a diamond graph and* (F_3) *is a fan graph, so,*

$$rc(G) = diam(G)$$

Proof:

Based on theorem 1, so that we get $rc\{Amal(3Br_4 * F_3, v)\} \geq diam\{Amal(3Br_4 * F_3, v)\}$. Next, it will be shown that $rc\{Amal(3Br_4 * F_3, v)\} \leq diam\{Amal(3Br_4 * F_3, v)\}$. It is defined by a coloring $c: E(G) \rightarrow \{1,2,3,4\}$ as follows:

$c(u_{i,j}u_{i,j+1}) = j$ for $i \in \{1,2, \dots . t\}$ and $j \in \{1,2, \dots . . n-1\}$

$c(v_{i,k}v_{i,k+1}) = ((k+1)mod2) + 1$ for $i \in \{1,2, \dots . t\}$ and $k \in \{1,2, \dots . n\}$

$c(u_{i,j}v_{i,k}) = ((k+1)mod2) + 3$ for $i \in \{1,2, \dots . t\}, j \in \{1,2, \dots n-1\}$ and $k \in \{1,2, \dots . n\}$

$c(v_{i,k}, v) = ((k+1)mod2) + 1$ for $i \in \{1,2, \dots . t\}$ and $k \in \{1,2, \dots . n\}$

$c(v_{i,}v) = i$ for $i \in \{1,2,3\}$ with $i \neq i \in \{1,2, \dots . t\}$

Furthermore, for the coloring of rainbow connection numbers, the results of the amalgamation of diamond graphs and fan graphs can be seen in Figure 2 below.

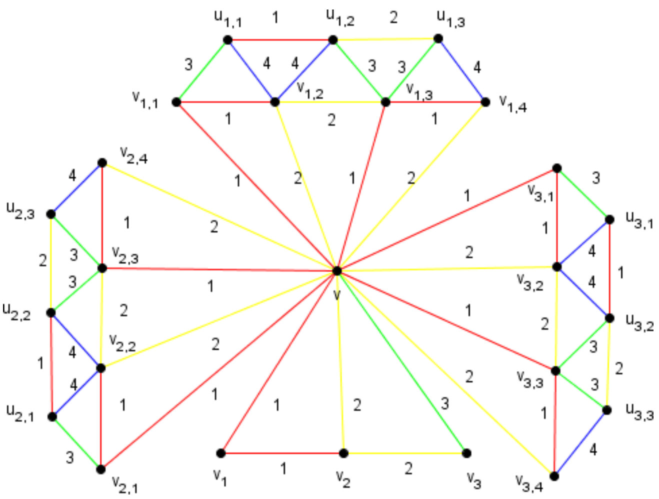

Figure 2: Coloring the sides of the rainbow $Amal(3Br_4*F_3, v)$

It can be seen in Figure 2, which is the result of coloring the rainbow connection numbers on $Amal(3Br_4*F_3, v)$, From the coloring, it can be concluded that the result of the amalgamation operation of the diamond graph and the fan graph is that there is a rainbow-connection number coloring. A rainbow coloring is determined by calculating the graph's diameter, then labeling the graph is done on $Amal(3Br_4*F_3, v)$. Furthermore, it will be shown in table 1 that for each *x* and *y* with $d(x, y) \geq 2$, there is a path x to y, which is a rainbow path.

Table 1 Rainbow Path of $Amal(3Br_4 * F_3, v)$

Cases	*x*	*y*	Conditions	Rainbow Paths
1	$u_{i,j}$	$u_{i,k}$	$j,k \in [1,2,3,4], i \in [1,2,3], t \in [1,2,3]$	$u_{i,j}, v_{i,j}, v, v_{i,k}, u_{i,k}$
2	$u_{i,j}$	$v_{i,m}$	$j \in [1,2,3] \wedge m \in [1,2..n], i \in [1,2,3]$	$u_{i,j}, v_{i,k+1}, v_{i,k+2}, v_{i,m}$
3	$u_{i,j}$	v_i	$j \in [1,2,3,4], i \in [1,2,3]$	$u_{i,j}, v_{i,j}, v, v_i$
4	$v_{i,k}$	$u_{i,j}$	$k,j \in [1,2,3,4], i \in [1,2,3], t \in [1,2,3]$	$v_{i,k}, v, v_{i,j}, u_{i,j}$
5	$u_{i,j}$	$v_{i,k}$	$j,k \in [1,2,3,4], i \in [1,2,3], t \in [1,2,3]$	$u_{i,j}, v_{i,j}, v, v_{i,k}$
6	$u_{i,j}$	$u_{i,m}$	$j < m, d(x,y) = 2 \wedge i \in [1,2,3]$	$u_{i,j}, u_{i,j+1}, u_{i,m}$
7	$v_{i,m}$	$u_{i,n}$	$m \in [1,2,3,4] \wedge n \in [1,2,3], i \in [1,2,3]$	$v_{i,m}, u_{i,n}, u_{i,n+1}$
8	$v_{i,k}$	$v_{i,m}$	$k < m, d(x,y) = 2 \wedge i \in [1,2,3]$	$v_{i,k}, v, v_{i,m}$
9	v_i	v_j	$i < j \wedge i,j \in [1,2,3]$	v_i, v_{i+1}, v_j

3.2. Rainbow-Total Connection Numbers Result of Amalgamation Operation of Diamond Graph (Br_4) and Fan Graph (F_3)

Theorem 3. *If $t \in N, t = 4$ and $\{G_i | i \in [1, t]\}$ is a nontrivial connected graph. If*

$$G \cong Amal(3Br_4 * F_3, v)$$

with (Br_4) is a diamond graph and (F_3) is a fan graph, then

$$trc(G) = 8$$

Proof:

First shown $trc(G) \geq 2diam(G) - 1$. because $diam(G) = 4$. Then, $trc(G) = 2diam(G) = 8$ is obtained for $t = 4$.. If $trc(G) \leq 2diam(G) - 1$ with $t = 4$. Without reducing the generality there will be *c* a total 7 coloring of the rainbow on G. Then look at two different points $u_{i,j}, u_{im} \in V(G)$ with $i \in [1,t]$ and $j, m \in [1,2..n-1]$. Path $u_{i,j} - u_{im}$ with length = *diam(G)*. If given a total of 7 colors of the rainbow on G, there will be the same color. Assign a color to the point v, so there will be 6 colors left to color the point and edges on $u_{i,j}, v_{i,k}, v, v_{i,m}, u_{i,m}$ with any point and side coloring. In such a way that the points and edges on the path acquire the same color. Therefore, there is no total rainbow path $u_{i,j} - u_{im}$. This is against the supposition $trc(G) \leq 2diam(G) - 1 = 7$. Then, it must be $trc(G) = 2diam(G) = 8$ for $t = 4$.

Next shown $trc(G) = 2diam(G) = 8$. If $u, v \in V(G)$ and $e \in E(G)$. What defines a coloring $c: V(G) \cup E(G) \rightarrow [1,2 \dots .8]$ as follows:

$$c(u,v) = \begin{cases} u_{i,j} = u_{i,j+2} = 1 & ; i \in [1,t], j \in [1,2,\dots.n-1] with\ j \in odd \\ u_{i,j+1} = 3 & ; i \in [1,t], j \in [1,2,\dots.n-1] with\ j \in even \\ v_{i,k} = v_{i,k+2} = 4 & ; i = 1, k \in [1,2,\dots n] with\ k \in odd \\ v_{i,k+1} = v_{i,k+3} = 5 & ; i \in [1,2], k \in [1,2 \dots n] with\ k \in even \\ v_{i,k} = v_{i,k+2} = 6 & ; i \in [2,3], k \in [1,2 \dots n] with\ k \in odd \\ v_{i,k+1} = v_{i,k+3} = 7 & ; i = 2, k \in [1,2 \dots n]\ with\ k \in even \\ v = 8 & ; where\ v = center\ point \end{cases}$$

$$c(e) = \begin{cases} u_{i,j}u_{i,j+1} = j\ with\ ; i \in [1,t], j \in [1,2 \dots n-1] \\ v_{i,k}v_{i,k+1} = ((k+1) mod\ 2) + 1\ with\ ; i \in [1,t], k \in [1,2,\dots.n] \\ u_{i,j}v_{i,k} = i + 2\ \ dengan\ \ ; i \in [1,t], j \in [1,2 \dots n-1] and\ k \in [1,2,\dots.n] \\ v_{i,k}v = ((k+1) mod\ 2) + 1\ with\ \ ; i \in [1,t], k \in [1,2,\dots.n] \\ v_i v = 1\ with\ ; i \in [1,2], and\ i \neq i \in [1,t] \\ v_i v = 2\ with\ ; i = 3, and\ i \neq i \in [1,t], i \neq i \in [1,2] \end{cases}$$

Coloring of the rainbow-total-connection numbers on the amalgamation of the diamond graph and the fan graph in Figure 3 below.

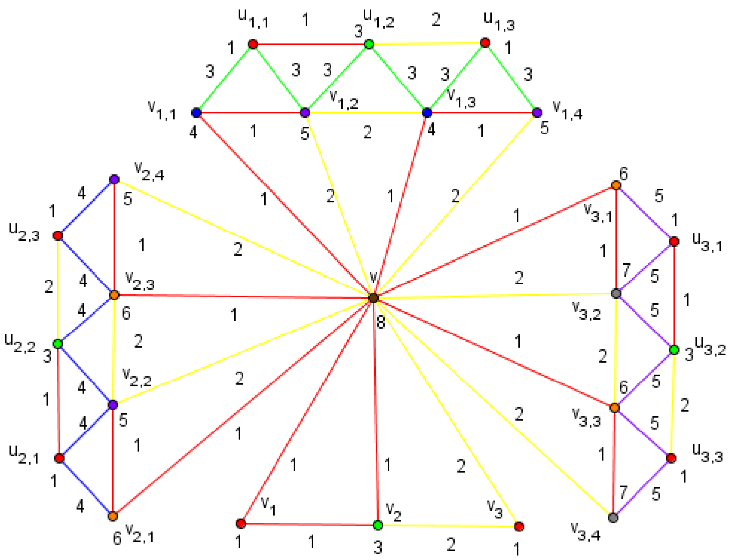

Figure 3: Rainbow-total coloring of $\boldsymbol{Amal(3Br_4 {*} F_3, v)}$

It is clear that two points are neighbors $x, y \in V(G)$. There is a rainbow path-total *x* to ithat is *xy*. Then, it will be shown in table 2 that for every *x* and *y*, there is a rainbow total path.

Table 2 Rainbow Total Path of $Amal(3Br_4 * F_3, v)$

Cases	*x*	*y*	Conditions	Rainbow Paths
1	$u_{i,j}$	$u_{i,k}$	$j,k \in [1,2,3,4], i \in [1,2,3], t \in [1,2,3]$	$u_{i,j}, v_{i,j}, v, v_{i,k}, u_{i,k}$
2	$v_{i,k}$	$u_{i,j}$	$k \in [1,2,3,4] \wedge j \in [1,2,3] i \in [1,2,3]$	$v_{i,k}, v_{i,k+1}, v_{i,k+2}, u_{i,j}$
3	$v_{i,k}$	$u_{i,j}$	$k,j \in [1,2,3,4], i \in [1,2,3], t \in [1,2,3]$	$v_{i,k}, v, v_{i,j}, u_{i,j}$
4	$u_{i,j}$	$v_{i,k}$	$j,k \in [1,2,3,4], i \in [1,2,3], t \in [1,2,3]$	$u_{i,j}, v_{i,j}, v, v_{i,k}$
5	$u_{i,j}$	v_i	$j \in [1,2,3,4], i \in [1,2,3], t \in [1,2,3,4]$	$u_{i,j}, v_{i,j}, v, v_i$
6	$u_{i,j}$	$u_{i,m}$	$j < m, d(x,y) = 2 \wedge i \in [1,2,3]$	$u_{i,j}, u_{i,j+1}, u_{i,m}$
7	$v_{i,m}$	$u_{i,n}$	$m \in [1,2,3,4] \wedge n \ \in [1,2,3], i \in [1,2,3]$	$v_{i,m}, v_{i,m+1}, u_{i,n}$
8	$v_{i,k}$	$v_{i,m}$	$k < m, \wedge \ k,m \in [1,2..n] i \in [1,2,3]$	$v_{i,k}, v, v_{i,m}$
9	v_i	v_j	$i < j \wedge i,j \in [1,2,3]$	v_i, v_{i+1}, v_j

4. CONCLUSIONS

Based on the results and discussion, it can be concluded that:

1. Rainbow connection numbers Result of Diamond Graph Amalgamation Operation(Br_4) and fan graph (F_3)

   Theorem 1. *If n and t is a natural number with $n = 4$ and $t = 4$ for $i \in [1, t]$. If $G \cong Amal(3Br_4 * F_3, v)$ with (Br_4) is diamond graph and (F_3) is fan graph, so*

$$rc(G) = diam(G)$$

2. Rainbow Total Connection Numbers Result of Diamond Graph Amalgamation Operation(Br_4) and Fan Graph (F_3)

   Theorem 2. *If $t \in N, t = 4$ and $\{G_i | i \in [1, t]\}$ is a nontrivial connected graph.. If $G \cong Amal(3Br_4 * F_3, v)$ with (Br_4) is a diamond graph and(F_3) is a fan graph, So*

$$trc(G) = 8$$

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BIBLIOGRAPHY/REFERENCES

[1] M. E. Supiyandi, "Penerapan Teknik Pewarnaan Graph Pada Penjadwalan Ujian Dengan Algoritma Welch-Powell," ["Application of Graph Coloring Techniques in Scheduling Exams With the Welch-Powell Algorithm,"] *J. Ilmu Komput. dan Inform.*, vol. 03, no. 01, 2018.
[2] G. Chartrand, G. L. Johns, K. A. McKeon, and P. Zhang, "Rainbow connection in graphs," *Math. Bohem.*, vol. 133, no. 1, pp. 85–98, 2008, doi: 10.21136/MB.2008.133947.
[3] M. . Shulhany and A. N. M. Salman, "Bilangan Terhubung Pelangi Graf Berlian," ["Rainbow Connection Numbers of Diamond Graphs,"] in *Prosiding Seminar Nasional Matematika dan Pendidikan Matematika UMS 2015*, 2015, pp. 916–924.
[4] S. Sy, G. H. Medika, and L. Yulianti, "The rainbow connection of fan and sun," *Appl. Math. Sci.*, vol. 7, pp. 3155–3159, 2013, doi: 10.12988/ams.2013.13275.
[5] J. L. Gross, J. Yellen, and M. Anderson, *Graph Theory and Its Applications*. USA: CRC Press, 2018.
[6] D. Fitriani and A. N. M. Salman, "Rainbow connection number of amalgamation of some graphs," *AKCE Int. J. Graphs Comb.*, vol. 13, no. 1, pp. 90–99, Apr. 2016, doi: 10.1016/j.akcej.2016.03.004.

[7] A. F. Akadji, D. Taha, N. Lakisa, and N. I. Yahya, “BILANGAN TERHUBUNG TITIK PELANGI PADA AMALGAMASI GRAF BERLIAN,” ["NUMBER CONNECTION RAINBOW POINTS IN DIAMOND GRAPH AMALGAMATION,"] *Euler J. Ilm. Mat. Sains dan Teknol.*, vol. 7, no. 2, 2019, doi: 10.34312/euler.v7i2.10345.

[8] C. A. P. Noor, K. A. Mamonto, and W. E. Pranata, “BILANGAN TERHUBUNG PELANGI PADA AMALGAMASI GRAF BERLIAN,” ["RABBIN CONNECTION NUMBER IN DIAMOND GRAPH AMALGAMATION,"] *Euler J. Ilm. Mat. Sains dan Teknol.*, vol. 7, no. 1, 2019, doi: 10.34312/euler.v7i1.10327.

[9] R. Ardiyansah and D. Darmaji., “Bilangan kromatik graf hasil amalgamasi dua buah graf,” ["The chromatic number of a graph resulting from the amalgamation of two graphs,"] *J. Sains dan Seni POMITS*, vol. 2, no. 1, 2013.