South African Statistical Journal
2019, Vol. 53, No. 1, 15–30

THE ABSENCE OF DIFFUSION IN THE SOUTH AFRICAN SHORT RATE

G. L. Grobler
Unit for Business Mathematics and Informatics, North-West University,
Potchefstroom, South Africa
e-mail: *gerrit.grobler@nwu.ac.za*

In the field of Financial Mathematics, stochastic differential equations are used to describe the dynamics of interest rates. An example is a model for the short rate, which is a mathematically defined rate not directly observable in any market. However, observable rates such as short dated Treasury rates or the Johannesburg Interbank Agreement Rate (JIBAR) can be used as proxies for the short rate.

The short rate dynamics are traditionally modelled by one-factor diffusion processes. These type of models remain popular due to the analytical tractability of the pricing formulae of interest rate derivatives under these models. To capture the leptokurtic nature of interest rate returns in the South African market, two types of models can be used: a pure jump model or a jump diffusion model. In this paper we investigate whether jumps are present and whether a diffusion component is evident. Our initial investigation showed that jumps were present in the South African market, and that no diffusion component was evident at low interest rate levels. This result was found using a Monte Carlo method to test for jumps. We therefore conclude that a pure jump process is an appropriate model for the South African short rate.

Key words: JIBAR, Jump diffusion models, Pure jump models, Short rate models, T-bill, Testing for jumps.

1. Introduction

A classical approach in interest rate modelling is to model the short rate r_t. This rate can be defined in terms of the price of a zero-coupon bond, given by

$$p(t,T) = E^* \left[e^{-\int_t^T r_s \mathrm{d}s} \right],$$

where expectation is taken under a risk-neutral martingale measure $\mathbb{P}^*$ (Björk, 2004). Various models for the short rate exist, with basic models initially developed from the family of one-factor pure diffusion models; see, for example, Vasicek (1977), Cox, Ingersoll and Ross (1985) and Hull and White (1990). The dynamics of these models are described by

$$\mathrm{d}r_t = \mu(r_t)\mathrm{d}t + \sigma(r_t)\mathrm{d}W_t, \tag{1}$$

where W_t is a Brownian motion under $\mathbb{P}^*$ and the functions μ and σ are the drift and diffusion coefficients respectively.

MSC2010 subject classifications. 91G30, 62P05, 60J75.

Globally, economic events in markets lead to infrequent jumps in interest rates, which can be modelled by adding a jump component to the diffusion process (Johannes, 2004). The dynamics of the resulting jump diffusion process can, in general, be described by

$$\mathrm{d}r_t = \mu(r_t)\mathrm{d}t + \sigma(r_t)\mathrm{d}W_t + \int_{-\infty}^{\infty} J(t, y)N(\mathrm{d}t, \mathrm{d}y), \tag{2}$$

where $N(\mathrm{d}t, \mathrm{d}y)$ is a marked point process and J a real valued function on $\mathbb{R}$ (Protter, 2005, p. 26). Initially, models of this type had N as a Poisson process with constant intensity λ and J representing i.i.d. random variables. An advantage of this model is addressing the issue of leptokurtic interest rate returns observed in markets.

In this paper, evidence will be presented that, at times, no diffusion component is evident in the South African short dated interest rate market. This is especially evident at low interest rate levels. We will show that the volatility in the market is fully described by the jump component of a jump diffusion model, making the diffusion component redundant. Therefore, the model most appropriate for the South African short rate is a pure jump model i.e. a model with no diffusion component.

As the short rate is non-observable, historical data on the level of the short rate do not exist. We will analyse proxies of the short rate, in the form of the 3-month Johannesburg Interbank Agreement Rate (JIBAR) as well as the 91-day Treasury bill (T-bill) rate. Both these rates are short dated, risk-free rates, however these rates have differing characteristics. The 91-day T-bill is traded on a weekly auction open to the public, and may therefore be an appropriate market rate to use. On the other hand, using JIBAR data may be appropriate, for this study, due to the fact that JIBAR data is available on a daily basis. The procedures used to test for the presence of jumps rely on the availability of high-frequency data. To be transparent, we will present results for both sets of data.

The time period under investigation is 1 March 2000 until 25 July 2017 during which the monetary policy in South Africa adopted an inflation targeting framework. Aling and Hassan (2012) found market characteristics to differ in the period before and after the change in monetary policy, which changed the diffusion model they found most appropriate for the South African short rate during this time.

Results from an initial investigation of our data showed that jumps occur frequently and that no diffusion component was evident at certain times. Figure 1 shows a time in history where both rates remained fairly flat for long periods. For example, the JIBAR stayed at a level of 5.58% for 125 consecutive working days from 27 January 2011 to 1 August 2011, while the T-bill rate did not change for 20 consecutive weeks from 20 July 2011 to 30 November 2011.

In Table 1 and Table 2 the summary statistics of both the 3-month JIBAR changes as well as the 91-day T-bill rate changes are shown. For comparison, the summary statistics of weekly 3-month JIBAR changes are shown in Table 1, while in Table 2 the summary statistics of the daily 3-month JIBAR changes are shown. We observe that the sample kurtosis for both rates are high, while the number of zero rate changes from the sample has a relatively high frequency.

Johannes (2004) showed that a pure diffusion model does not have sample paths with leptokurtic increments. In addition, a model with a diffusion component included will, almost certainly, have sample paths with a relatively low frequency of zero rate increments. In this paper, we will fit models of the forms in equations (1) and (2) to our data using conditional moment estimates. We will show that, in contrast to a jump diffusion model, a pure diffusion model cannot replicate interest rate

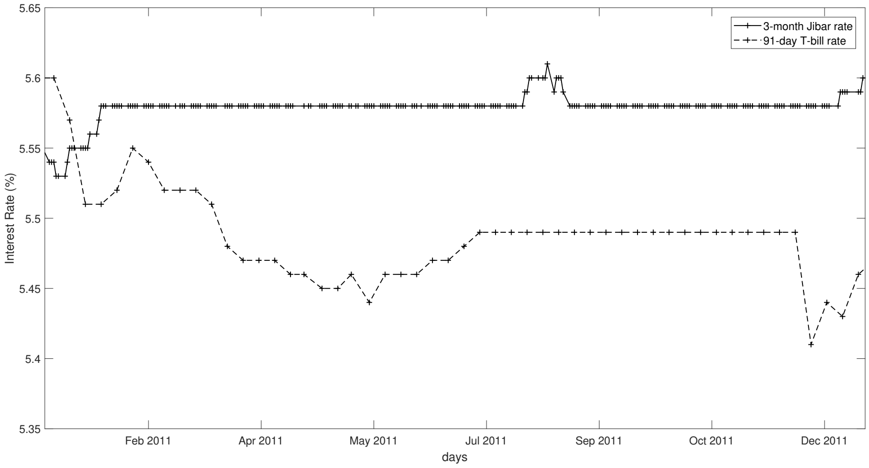


Figure 1. Historical values of the T-bill and JIBAR. A time period in which interest rates remained relatively unchanged.

Table 1. Descriptive statistics of the week-on-week JIBAR and T-bill rate changes (measured in basis points).

Description	**JIBAR**	**T-bill**
mean	−0.304 204	−0.259 669
standard deviation	11.082 663	12.032 036
skewness	−2.969 210	−1.195 373
kurtosis	21.787 060	16.643 180
number of zeroes	392	150
zeroes as % of total	43.36%	16.57%

paths with a similarly high sample kurtosis of observed rate changes. Numerical errors occur when fitting the jump diffusion model, which can be attributed to the volatility in the market mostly being generated by jumps. We also fit a pure jump model to the data using conditional moment estimates. We implement a method to test for jumps due to Lee and Mykland (2008) and discuss the dangers of interpreting the results when the test is applied to low-frequency data.

2. Testing for jumps

In this section, we first test the adequacy of various short rate models by simulating the null distributions of central sample moments. In Sections 2.1 and 2.2 the null distributions will be simulated from various short rate models. In Section 2.1 this will be done informally based on the following conditional moments: the sample mean, sample variance, normalised sample skewness and normalised sample kurtosis. In Section 2.2 a formal test will be performed for each hypothesis

Table 2. Summary statistics of the daily 3-month JIBAR rate changes (measured in basis points).

	JIBAR		
Description	**2000–2017**	**2000–2010**	**2010–2017**
mean	−0.07	−0.15	0.06
standard deviation	4.23	5.09	2.25
skewness	−6.59	−6.03	−1.45
kurtosis	141.22	100.99	302.20
number of zeros	3122	1610	1512
zeros as % of total	70.99%	59.72%	88.89%

based on the unconditional sample kurtosis.

A second test, which can be used to determine whether a jump occurred between each sample point, will be implemented in Section 2.3. We note that the method has a high misclassification error in identifying small jumps from daily data.

The results indicate that jumps should be included in a model for the short rate. We also find that estimates of the diffusion coefficients in a jump diffusion model are unreliable, which points to the absence of a diffusion component in the observed data. Hence, we suggest that the South African short rate should be modelled by a pure jump model.

2.1 Tests based on conditional moments

One-factor diffusion model

For the first test, we assume a pure diffusion model under the null hypothesis of the form

$$\mathrm{d}r_t = \mu(r_t)\mathrm{d}t + \sigma(r_t)\mathrm{d}W_t. \tag{3}$$

We assume no specific paramatrisation of the drift and diffusion coefficients. Therefore, the model above can be seen as a nonparametric model. We now apply a method developed by Johannes (2004).

Denote the observed sample values by $R_1, R_2, ..., R_T$. The drift and diffusion coefficients can be approximated in terms of the conditional moments of rate increments, conditional on interest rate level, by

$$\mu(r_t) \approx \frac{1}{\triangle} E\left\{R_{t+\triangle} - R_t | R_t = r\right\}$$

and

$$\sigma^2(r_t) \approx \frac{1}{\triangle} E\left\{(R_{t+\triangle} - R_t)^2 | R_t = r\right\},$$

where $\triangle$ is the number of trading days between observations.

Therefore, to estimate μ and σ, the first two conditional moments of short rate increments need to be estimated. Johannes (2004) defines smoothed estimators for $\triangle^{-1}E[(R_{t+\triangle} - R_t)^j | R_t = r]$ by

$$m_j(r) = \frac{\frac{1}{\triangle}\sum_{t=1}^{T-1} (R_{t+1} - R_t)^j \, K\left(\frac{R_t - r}{h_j}\right)}{\sum_{t=1}^{T-1} K\left(\frac{R_t - r}{h_j}\right)}, \tag{4}$$

where the estimates are calculated for a discrete set of values of r. The function K is a normal kernel with bandwidth h_j.

Therefore, the drift and diffusion in equation (3) can be estimated by

$$\hat{\mu}(r) = m_1(r) \quad \text{and} \quad \hat{\sigma}(r) = \sqrt{m_2(r)},$$

which will enable us to simulate interest rate paths under the null hypothesis, which states that the South African short rate is driven by a one-factor diffusion model. From these simulated paths, the null distribution of the conditional sample mean and variance is estimated. A diffusion model will be able to replicate the first two sample moments.

To determine whether a fitted diffusion model can replicate leptokurtic historical interest rate returns, we estimate the null distributions of the conditional normalised skewness and kurtosis, given by

$$\hat{\Sigma}(r) = \frac{m_3(r)}{m_2(r)^{3/2}} \quad \text{and} \quad \hat{\kappa}(r) = \frac{m_4(r)}{m_2(r)}.$$

The starting point of the simulations is an interest rate level of $r = 0.1$, with 10 000 replications. The numbers of time steps are equal to the size of our samples, which are 4 399 and 906 for the JIBAR and T-bill samples respectively. From these simulated paths we calculate $\hat{\mu}(r)$, $\hat{\sigma}(r)$, $\hat{\Sigma}(r)$ and $\hat{\kappa}(r)$ for each path in order to obtain a distribution for each statistic via simulation. We then compare the 10% and 90% quantiles of each simulated distribution with the numerical value of the statistic calculated from the observed data. This method is called Monte Carlo hypothesis testing, as a null distribution of a test statistic is simulated from a fitted process defined under the null hypothesis.

The results are shown in Figure 2. In the top two windows we observe the sample drift and sample variance to be within the 10% and 90% bounds of the simulated null distributions at all interest rate levels. This indicates a good fit of the diffusion model to our sample. However, in the second row of windows we observe the sample skewness and kurtosis to be outside the 10% and 90% bounds at all interest rate levels. This shows an inability of a pure diffusion model to replicate rate changes which are skew and leptokurtic.

Jump diffusion model

We now test the statement that the South African short rate is driven by a jump diffusion model. Therefore, we assume a semiparametric jump diffusion short rate model, which, under the null hypothesis, has the form

$$\mathrm{d}r_t = \mu(r_t)\mathrm{d}t + \sigma(r_t)\mathrm{d}W_t + \mathrm{d}\left(\sum_{n=1}^{N_t} r_{\tau_n -}\left\{\exp(Z_n) - 1\right\}\right),$$

where N_t is a nonstationary Poisson process with rate dependent intensity $\lambda(r_{t-})$ and Z_n are independent and identically distributed random normal variables with mean zero and variance σ_z^2. The model is semiparametric since the distribution of the jumps are specified, while the intensity of the jumps, as well as the drift and diffusion coefficients, are functions of the underlying rate. The form of the model is similar to that in equation (2), with the function J equal to $r_{t-}(e^y - 1)$. However, a model for the logarithmic short rate can be obtained by applying Itô's lemma, resulting in

$$\mathrm{d}\log r_t = \mu(r_t)\mathrm{d}t + \sigma(r_t)\mathrm{d}W_t + \int_{-\infty}^{\infty} yN(\mathrm{d}t, \mathrm{d}y), \tag{5}$$

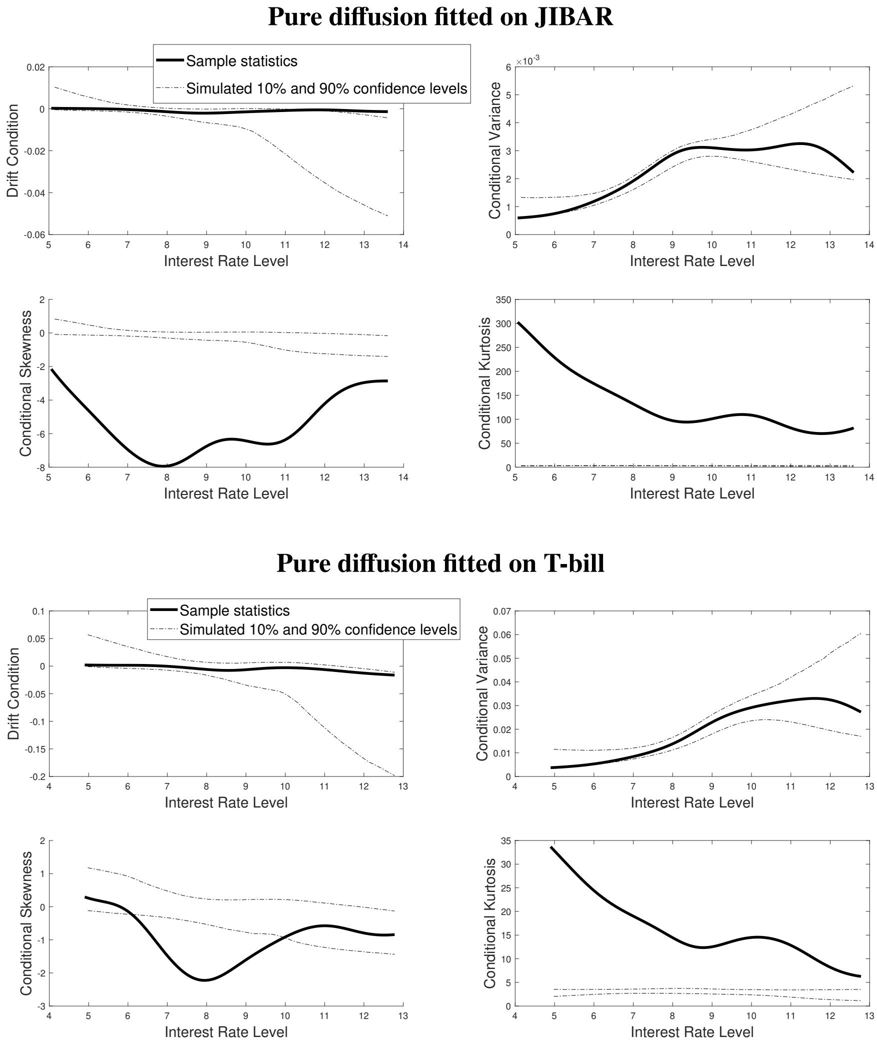


Figure 2. The simulated null distributions of the conditional drift and variance in the two top windows as well as the conditional normalised skewness and kurtosis in the two windows below them, compared with the matching test statistics. The null distributions were generated by nonparametric pure diffusion models fitted on the historical 3-month JIBAR returns as well as 91-day T-bill returns.

with redefined drift and diffusion coefficients μ and σ. From this formula the conditional moments can be calculated, and are given by

$$\frac{1}{\triangle}E\left[\log(R_{t+\triangle}/R_t)|R_t = r\right] \approx \mu(r_t),$$
$$\frac{1}{\triangle}E\left[\log(R_{t+\triangle}/R_t)^2|R_t = r\right] \approx \sigma^2(r_t) + \lambda(r_{t-})\sigma_z^2,$$
$$\frac{1}{\triangle}E\left[\log(R_{t+\triangle}/R_t)^4|R_t = r\right] \approx 3\lambda(r_{t-})\sigma_z^4,$$

and

$$\frac{1}{\triangle}E\left[\log(R_{t+\triangle}/R_t)^6|R_t = r\right] \approx 15\lambda(r_{t-})\sigma_z^6.$$

Using the moment estimates $m_j,\ j = 1, 2, 4, 6$, defined in equation (4), the estimate for σ_z^2, denoted by $\hat{\sigma}_z^2$, can be calculated by first calculating a rate dependent estimate of σ_z^2, given by

$$\hat{\sigma}_z^2(r) = \frac{1}{5}\frac{m_6(r)}{m_4(r)},$$

and then taking the mean over a grid of values for r. The estimate for $\lambda(r_{t-})$ is then given by

$$\hat{\lambda}(r) = \frac{m_4(r)}{3\hat{\sigma}_z^4}.$$

Therefore, an estimate for $\sigma^2(r_t)$ is given by

$$\hat{\sigma}^2(r) = m_2(r) - \hat{\lambda}(r)\hat{\sigma}_z^2.$$

Note that if $m_2(r) \geq \hat{\lambda}(r)\hat{\sigma}_z^2(r)$, the estimate for the variance driven by the diffusion part of the model is negative, with the result of complex valued diffusion coefficients. This numerical error occurs in both sets of sample data for some values of r when fitting a jump diffusion model. Thus, we estimate $\sigma(r_t)$ by

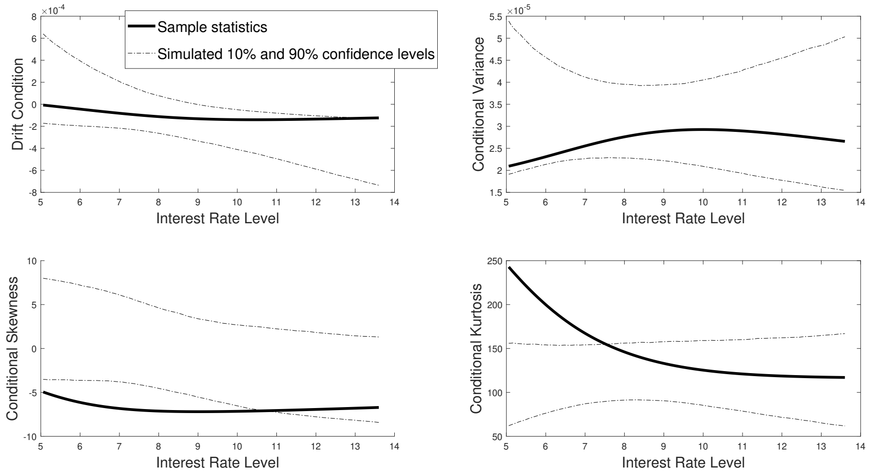
$$\hat{\sigma}(r) = \sqrt{\max\left(0, m_2(r) - \hat{\lambda}(r)\hat{\sigma}_z^2(r)\right)}.$$

However, this leads to the estimated conditional variance from our simulations overestimating the true conditional variance, as seen in the top right window of Figure 3. Nevertheless, the second and fourth row of windows from Figure 3 show that the observed level of skewness and excess kurtosis can be replicated under this model.

The conclusion is that jumps should be included and the question becomes whether a diffusion component should be included. It is shown in Figure 4 that the conditional sample variance of the log returns for the T-bill rate is approximately equal to the conditional sample variance generated from the jump component of the model. For the JIBAR, the variance generated from the jump component far outweighs the variance generate by the diffusion component. Therefore, we make the conclusion that there is an absence of a diffusion component in the South African short rate.

Including a diffusion component seems unnecessary, as the model contains more parameters than needed to estimate the underlying variance. To confirm that a pure jump model is appropriate, we need to investigate whether similar higher order conditional moments of interest rate returns are replicated when assuming a model with no diffusion component.

Jump diffusion fitted on JIBAR



Jump diffusion fitted on T-bill

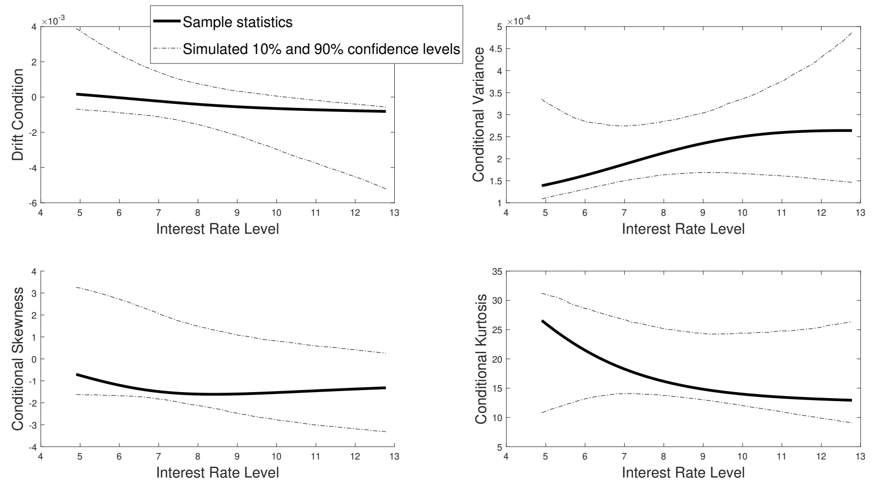


Figure 3. The simulated null distributions of the conditional drift and variance in the two top windows as well as the conditional normalised skewness and kurtosis in the two windows below them, compared with the matching test statistics. The null distributions were generated by semiparametric jump diffusion models, fitted on the historical 3-month JIBAR log returns as well as 91-day T-bill log returns.

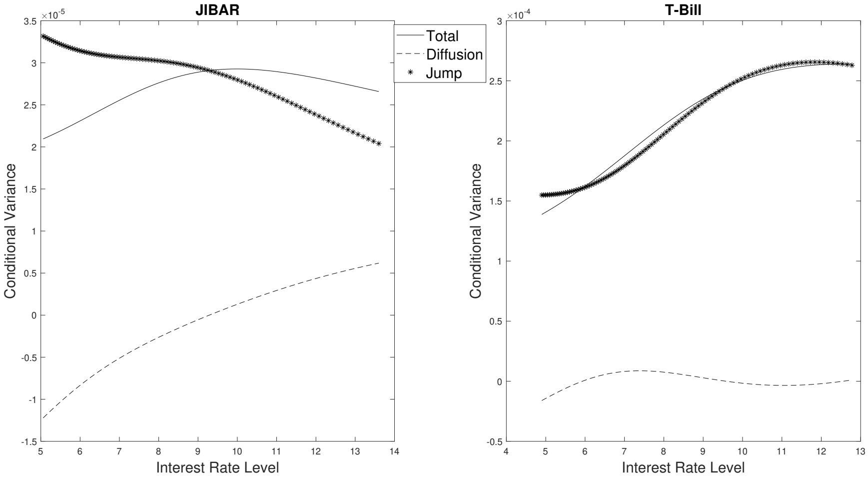


Figure 4. In these two graphs the estimated total conditional variance of a nonparametric pure diffusion model with normally distributed jumps of the log returns is shown. In the left window the result is shown from the model fitted to 3-month JIBAR returns, while in the right window the model was fitted to the 91-day T-bill returns. The conditional variance generated from the diffusion component as well as the conditional variance generated from the jump component are also shown.

Pure jump model

We now test the statement that the South African short rate is driven by a pure jump model. To do this we extend the method from Johannes (2004) to specify a pure jump model under the null hypothesis. We therefore assume the diffusion coefficient $\sigma(r_t)$ in (5) to be equal to zero, to get

$$\mathrm{d}\log r_t = \mu(r_t)\mathrm{d}t + \int_{-\infty}^{\infty} yN(\mathrm{d}t, \mathrm{d}y).$$

The conditional moments and their relationship to the parameters of the pure jump model are

$$\frac{1}{\triangle}E\left[\log(R_{t+\triangle}/R_t)|R_t = r\right] \approx \mu(r_t),$$
$$\frac{1}{\triangle}E\left[\log(R_{t+\triangle}/R_t)^2|R_t = r\right] \approx \lambda(r_{t-})\sigma_z^2,$$

and

$$\frac{1}{\triangle}E\left[\log(R_{t+\triangle}/R_t)^4|R_t = r\right] \approx 3\lambda(r_{t-})\sigma_z^4.$$

In order to estimate the parameters μ, λ and σ_z, we use the moment estimates m_j, $j = 1,2,4$, defined in equation (4), to estimate the terms on the left side of the set of equations above. This results in a system of equations, which is solved by first calculating an estimate for σ_z^2 by taking the mean over a grid of values r of the estimate

$$\hat{\sigma}_z^2(r) = \frac{1}{3}\frac{m_4(r)}{m_2(r)},$$

denoted by $\hat{\sigma}_z^2$. The estimate for $\lambda(r_{t-})$ is then given by

$$\hat{\lambda}(r) = \frac{m_2(r)}{\hat{\sigma}_z^2}.$$

In this case, we use only the conditional moment estimates of order one, two and four. As a result, as seen in Figure 5, better estimates of the conditional variance and kurtosis are obtained, compared to those in Figure 3.

2.2 Tests based on unconditional moments

In this section, we apply the formal hypothesis test from Johannes (2004) based on the unconditional sample kurtosis defined by

$$\hat{k} = \frac{\kappa_4}{\kappa_2^2}, \tag{6}$$

where

$$\kappa_2 = \frac{n}{n-1} m_2$$

and

$$\kappa_4 = \frac{n^2((n+1)m_4 - 3(n-1)m_2^2)}{(n-1)(n-2)(n-3)},$$

with m_2 the sample variance, m_4 the fourth central sample moment and n the sample size. The disadvantage of this method, compared to the informal tests from the previous section is that the kurtosis seems to be rate dependent. Generally, the kurtosis is lower at higher interest rates. However, the results from this section add to the set of evidence upon which we base our conclusions.

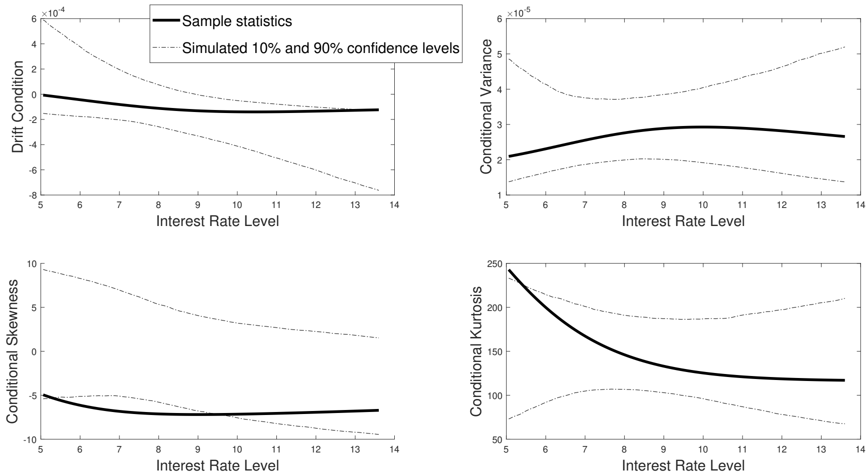
To apply the method, interest rate paths are again simulated from the model stipulated under the null hypothesis to get a null distribution of the unconditional sample kurtosis calculated from equation (6). Based on the percentiles of the simulated null distribution, we then decide whether it is likely that the model under the null hypothesis could generate the observed sample kurtosis. This decision is made at a significance level of $\alpha = 5\%$.

Tables 3 and 4 summarise the percentiles of the sample kurtosis under each null hypothesis as well as the test statistics. Neither the pure jump, nor the jump diffusion model is rejected, while the pure diffusion model is rejected at the given significance level. The conclusion from these tests are that jumps should be included in a model for the short rate.

2.3 Other tests for jumps

Several tests for jumps exist, with some of them summarised by Hong and Zou (2015). A test for jumps was first introduced by Aït-Sahalia (2002), after which many similar tests were developed. Carr and Wu (2003) developed a test in which the dynamics of an asset can be identified as either a diffusion, jump diffusion or pure jump model. This is done by analysing the behaviour of short-maturity options. However, due to the nature of the South African interest rate option market, data are not freely available to apply this test. It is advantageous to apply a number of tests to high-frequency data of the underlying asset. For example, the nonparametric test to identify isolated jumps by Lee and Mykland (2008) is an asymptotic test. This implies that if the data are available at low frequencies, then the test may have a high misclassification error in identifying jumps. This

Pure jump fitted on JIBAR



Pure jump fitted on T-bill

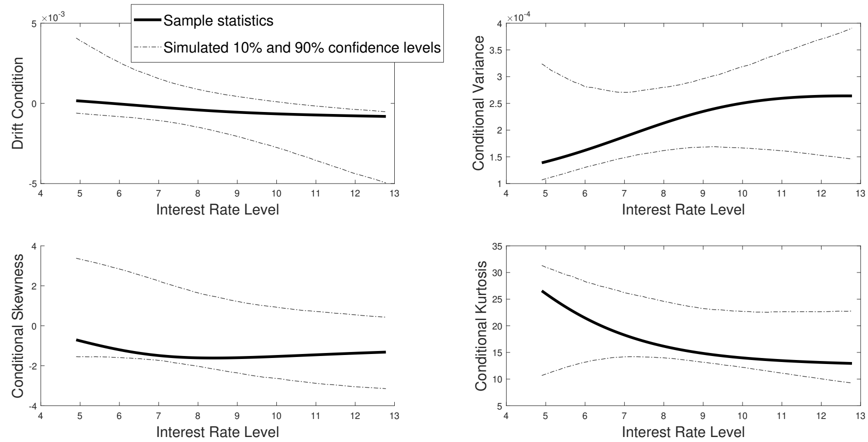


Figure 5. The simulated null distributions of the conditional drift and variance in the two top windows as well as the conditional normalised skewness and kurtosis in the two windows below them, compared with the matching test statistics. The null distributions were generated by nonstationary compound Poisson process with normally distributed jumps, fitted on the historical 3-month JIBAR log returns as well as 91-day T-bill log returns.

Table 3. Kurtosis percentiles JIBAR.

	Pure jump	**Jump Diffusion**	**Diffusion**
Test statistic values	159.16*	159.16*	141.22**
p-values	0.92	0.40	< 0.02
Percentiles		Simulated Kurtosis	
1	94.57	86.99	−0.04
5	106.21	96.57	0.05
10	114.98	101.41	0.13
25	131.82	110.92	0.33
40	147.38	121.46	0.50
50	155.34	128.92	0.62
60	165.45	136.96	0.74
75	184.08	151.09	0.97
90	218.02	174.58	1.28
95	242.40	201.11	1.51
99	309.51	247.41	1.87

* Calculated from log returns.
** Calculated from simple returns.

is also true for the nonparametric test for jumps by Aït-Sahalia and Jacod (2009) and its extension by Aït-Sahalia, Jacod and Li (2012), as well as the test by Jiang and Oomen (2008). Unfortunately, high-frequency data are not available for short term South African interest rates.

We test the hypothesis that the short rate follows a nonparametric diffusion process by applying a nonparametric test described by Lee and Mykland (2008). Although the frequency of data available is a concern due to possible high misclassification errors, Lee and Mykland (2008) specify how to implement their method with daily as well as weekly data. A random variable, $\mathcal{L}$, is defined as the realised return of the 3-month JIBAR divided by its realised instantaneous volatility. The method is based on whether $\mathcal{L}$ is within expected bounds if we assume the underlying process is a diffusion process. Lee and Mykland (2008) define a short rate diffusion process by

$$\mathrm{d}r_t = \mu(t)\mathrm{d}t + \sigma(t)\mathrm{d}W_t, \tag{7}$$

and a jump diffusion process by

$$\mathrm{d}r_t = \mu(t)\mathrm{d}t + \sigma(t)\mathrm{d}W_t + Y(t)N(\mathrm{d}t), \tag{8}$$

where N is a point process and Y is the jump size. This model differs slightly from the models defined in the previous section as the drift, diffusion and intensity λ are functions of t and not of r_t. We apply the method to test whether r_t has jumps. We will therefore only show the results for the models defined in equations (7) and (8).

We again denote the observed sample values by $R_1, R_2, \ldots, R_T$ and form a statistic to test for a jump from time t_{i-1} to t_i by

$$\mathcal{L}_i = \frac{R_{t_i} - R_{t_{i-1}}}{\hat{\sigma}_{t_i}},$$

Table 4. Kurtosis percentiles T-bill.

	Pure jump	**Jump Diffusion**	**Diffusion**
Test statistic values	14.43*	14.43*	16.64**
p-values	0.48	0.54	< 0.02
Percentile		Simulated Kurtosis	
1	10.25	9.70	−0.19
5	11.66	11.37	−0.01
10	12.56	12.24	0.11
25	14.54	14.26	0.39
40	16.24	15.90	0.61
50	17.29	17.03	0.77
60	18.82	18.63	0.93
75	22.08	21.27	1.21
90	28.79	25.67	1.77
95	32.97	29.45	2.14
99	47.42	40.41	2.96

* Calculated from log returns.
** Calculated from simple returns.

where R_{t_i} is the observed rate at time t_i and the realised bipower variation

$$\hat{\sigma}^2_{t_i} = \frac{1}{K-2} \sum_{j=i-K+2}^{i-1} |R_{t_j} - R_{t_{j-1}}||R_{t_{j-1}} - R_{t_{j-2}}|,$$

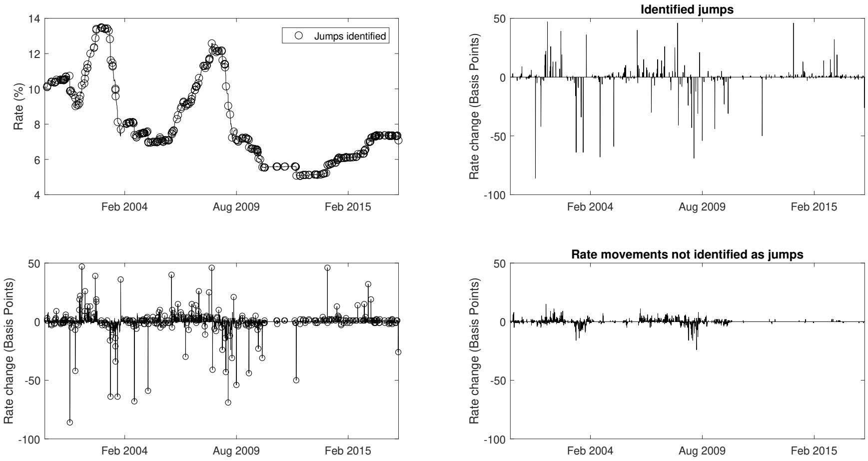
is a consistent estimator for the instantaneous volatility σ^2 (Barndorff-Nielsen and Shepard, 2004). The rationale behind this method, as described by Lee and Mykland (2008), is to compare the realised returns of the JIBAR with the local variation coming from the diffusion part of the process. Importantly, jumps do not affect the consistency of the estimate of $\sigma^2(t)$. Therefore, the statistic $\mathcal{L}_i$ should be able to distinguish whether the realised return is greater (or less) than what one would expect if the underlying process is a diffusion process.

Figure 6 shows which of the JIBAR changes in our dataset have been identified as jumps at a significance level of $\alpha = 5\%$. In total 29% of all JIBAR return data had non-zero movements, with 27% of those identified as jumps. Importantly, on 79% of all the trading days either a jump or no movement occurred. The conclusion is therefore made that the diffusion component has a small effect on the underlying process.

In Figure 6 we also present the results for the T-bill data. In this case a greater percentage of return data show nonzero movements (83%), with a smaller percentage being identified as jumps (9%). In this case 24% of all return data points were either no movements or identified as jumps, which is much lower than the corresponding percentage for the JIBAR data. However, the proportion of days on which jumps are deemed to have occurred is still substantial.

In Figure 6, for the JIBAR data, we clearly see a difference in the range of values for rate changes that

JIBAR



T-bill

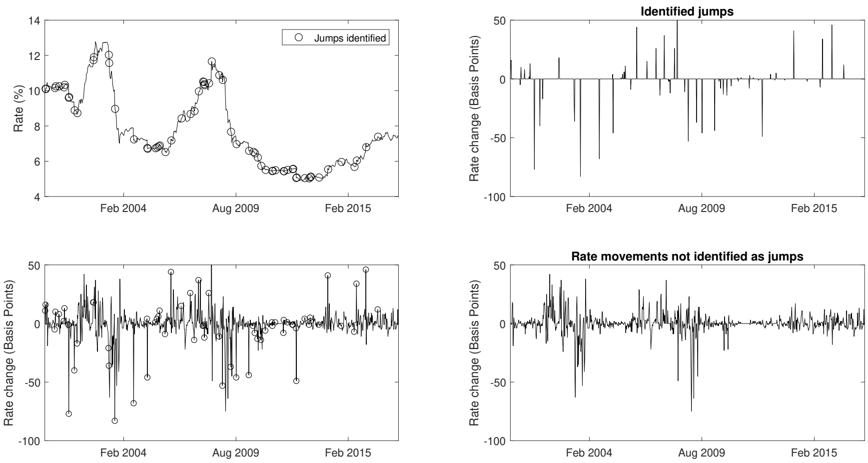


Figure 6. The historical JIBAR (top left graph) as well as the first difference (second from top left), with isolated jumps identified with a nonparametric test for jumps. The top two windows on the right split those movements identified as jumps, from those determined not to be jumps. The results for the T-bill dataset are also shown.

have been identified as jumps, compared to those not identified as jumps (second column of windows in Figure 6). However, this phenomenon is not as clear when observing similar graphs, when the test was applied to the T-bill data. This may be due to the high misclassification probability of jumps for low-frequency data. Although Lee and Mykland (2008) specifies the preferred parameter values of K for daily ($K = 16$) and weekly ($K = 7$) data, they report a high misclassification probability for relatively small jumps when using daily data. No results are reported when using weekly data, although we can only assume that the misclassification probability is much higher using weekly data. The results from this section should therefore be interpreted with caution, especially for the T-bill data.

3. Conclusion

In this paper we investigated whether jumps should be included in a South African short rate model. We applied two methods to test for jumps, keeping in mind that we had only low-frequency data available, due to the nature of the South African short dated interest rate market.

South African interest rate returns are leptokurtic. A pure diffusion model is not able to capture this property of observed data. We confirmed this by applying a Monte Carlo hypothesis test, which is the most appropriate test for low-frequency data. In addition to finding that jumps should be added to capture the leptokurtic nature of interest rate returns, we found that no diffusion component is evident at low interest rate levels. This result was found by adapting the Monte Carlo hypothesis test by Johannes (2004) to specify a pure jump model under the null hypothesis. Therefore, our main result is that a pure jump model is more appropriate for the South African short rate than a jump diffusion model.

Our results are only applicable to short dated interest rates where the underlying asset is not traded on an exchange. There are, however, other types of interest rate models where the underlying asset may be more liquid. This liquidity may influence our results and is a topic for further research.

In this paper we found a model which replicates leptokurtic interest rate returns. However, we did not address the question of how interest rate derivatives can be priced under the proposed model. This is also a topic for further research.

Acknowledgements. Special thanks to the South African National Research Foundation (NRF) for its part in funding this project as well as my Ph.D. supervisor, Prof. F. Lombard.

References

Aït-Sahalia, Y. (2002). Telling from discrete data whether the underlying continuous-time model is a diffusion. *Journal of Finance*, **57**, 2075–2112.

Aït-Sahalia, Y. and Jacod, J. (2009). Testing for jumps in a discretely observed process. *Annals of Statistics*, **37**, 184–222.

Aït-Sahalia, Y., Jacod, J., and Li, J. (2012). Testing for jumps in noisy high-frequency data. *Journal of Econometrics*, **168**, 207–222.

Aling, P. and Hassan, S. (2012). No-arbitrage one-factor models of the South African term structure of interest rates. *South African Journal of Economics*, **80**, 301–318.

Barndorff-Nielsen, O. and Shepard, N. (2004). Power and bipower variation with stochastic volatility and jumps. *Journal of Financial Econometrics*, **2**, 1–37.

Björk, T. (2004). *Arbitrage Theory in Continuous Time*. 2nd edition. Oxford University Press.

Carr, P. and Wu, L. (2003). What type of process underlies options? A simple robust test. *Journal of Finance*, **58**, 2581–2610.

Cox, J. C., Ingersoll, J. E., and Ross, S. A. (1985). A theory of the term structure of interest rates. *Econometrica*, **53**, 385–407.

Hong, L. and Zou, J. (2015). Jump tests for semimartingales. *South African Actuarial Journal*, **15**, 93–108.

Hull, J. and White, A. (1990). Pricing interest-rate-derivative securities. *Review of Financial Studies*, **3**, 573–592.

Jiang, G. J. and Oomen, R. C. A. (2008). Testing for jumps when asset prices are observed with noise – a swap variance approach. *Journal of Econometrics*, **144**, 352–370.

Johannes, M. (2004). The statistical and economic role of jumps in continuous-time interest rate models. *Journal of Finance*, **59**, 227–260.

Lee, S. S. and Mykland, P. A. (2008). Jumps in financial markets: A new nonparametric test and jump dynamics. *Review of Financial Studies*, **21**, 2535–2563.

Protter, P. E. (2005). Stochastic differential equations. *In Stochastic Integration and Differential Equations*. Springer, Berlin, 249–361.

Vasicek, O. (1977). An equilibrium characterisation of the term structure. *Journal of Financial Economics*, **5**, 177–188.

Manuscript received 2018-04-03, revised 2018-08-01, accepted 2018-10-12.