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Original article

# Behaviour of Magnetized Strange Quark Matter in f(R,T) Theory for General Kantowski-Sachs Model

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**Abstract**

In this study, we have investigated the behavior of magnetized strange quark matter (MSQM) in f(R,T) gravity for LRS Bianchi I, Bianchi III and Kantowski-Sachs (GKS) universe models with cosmological term. For the solutions of modified field equations, we have used linearly varying deceleration parameter (LVDP), anisotropy parameter and equation of state for strange quark matter. When the models goes to the isotropy magnetic field only occurs in Bianchi III and Kantowski-Sachs universe models. When $t\to\infty$, strange quark matter distribution behaves like dark energy. The $K(\theta)$ parameter, which allows us to obtain different universe models, is effective on the magnetic field, cosmological term and f(R,T) function. In addition, the graphics of the obtained results were examined in detail.

**Keywords:** f(R, T) gravitation theory, General Kantowski-Sachs universe, Magnetic field, Linear deceleration parameter.

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## INTRODUCTION

The quarks, which are the basic building blocks of the community of matter in the universe, have a very important role in examining the evolution of the universe. According to the standard model, there are six different kinds of quark, namely, top, bottom, up, down, charm and strange (Witten, 1984, Bodmer, 1971). The idea of quark was first introduced in the 1970s. There are two scenarios which are conversion of neutron stars into strange ones at ultra-high densities and the quark-hadron phase transition in the early universe, related to the formation of quark matter (Witten, 1984, Bodmer, 1971, Itoh, 1970, Aktaş and Yılmaz, 2007). As a result of the important researches carried out in Brookhaven National Laboratory, the energy momentum tensor of quark matter will be taken as perfect fluid since quark material appears to be in perfect fluid form (Back and Collaboration, 2005, Adams and Collaboration, 2005). In particular, we will investigate strange quark matter (SQM) solutions in this study. The equation of state (EoS) for SQM is as follows (Sotani et al., 2004).

$$p = \frac{\rho - 4B_c}{3} \tag{1}$$

where $B_c$ is bag constant and the value of $B_c$ is $60 - 80\frac{MeV}{fm^3}$, $p$ and $\rho$ are pressure and density of SQM, respectively (Sahoo et al., 2018). This model is defined as bag model. Massachusetts Institute of Technology (MIT) bag model is utilized for SQM in the magnetic field (Sahoo et al., 2018).

In this research, we will investigate MSQM distribution in $f(R,T)$ gravity. Because we do not have detailed information about the evolution of the magnetic field. Therefore, cosmologists and astrophysicists carry out various studies on both the observational and theoretical and simulation models in order to fully understand the source, evolution and effects of the magnetic field (Sahoo et al., 2018, Marinacci and Vogelsberger, 2016). However, observations and research have proven the presence of magnetic fields in many galaxy clusters, the Milky Way galaxy, Pulsars and Neutron stars (Marinacci and Vogelsberger, 2016, Peebles, 1967, Grasso and Rubinstein, 2001). Large scale fields of the universe can be explained by magnetic fields (de Gouveia dal Pino et al., 2005, Wolfe et al., 1992). These fields are the mysterious and common component of our universe. Therefore, it is very important to study magnetic fields as it informs us about the evolution of the universe (de Gouveia dal Pino et al., 2005, Wolfe et al., 1992).

However, another mystery of the universe is the expansion and acceleration. In order to solve this mystery, scientists observe and investigate CMB, supernovae-Ia and BAO in detail (Riess et al., 1998, Perlmutter et al., 1999). Alternative gravitation theories have recently been used to explain the accelerated expansion of the universe. One of these alternative gravitation theories is $f(R,T)$ theory (Harko et al., 2011), which has recently been used by scientists to explain the acceleration of the universe.

In this context, using time depend deceleration parameter, Nagpal et al. have researched FLRW metric in $f(R,T)$ gravity (Nagpal et al., 2019). Biswas et al. have studied strange stars in $f(R,T)$ gravity (Biswas et al., 2019). Godani has researched LRS Bainchi II space-time model in $f(R,T)$ gravitation theory (Godani, 2019). Sofuoğlu has obtained Bianchi IX and Gödel metric in $f(R,T)$ theory (Sofuoğlu, 2016, Sofuoğlu, 2019). Güdekli and co-authors have investigated $\mathrm{f(R,T)}$ gravity with various matter distributions (Güdekli and Çalışkan, 2018, Momeni et al., 2015). Keskin has obtained scalar field solutions in modified $\mathrm{f(R,T)}$ theory (Keskin, 2018).

Agrawal and Pawar have researched SQM solutions for plane symmetric universe model in $f(R,T)$ gravity (Agrawal and Pawar, 2017). Deb et al. have studied anisotropic strange stars and SQM distributions for spherically symmetric metric in $f(R,T)$ gravity (Deb et al., 2018). For Marder type universe model SQM solutions in $f(R,T)$ gravity have been researched by Aygün et al. (Aygün et al., 2016).

Besides, Aktaş et al. (Aktaş et al., 2018) have obtained relationship between anisotropy parameter and magnetic field in $f(R,T)$ theory. Using various deceleration parameters, Sahoo et al. (Sahoo et al., 2018, Sahoo et al., 2017) have obtained LRS Bianchi I solutions for MSQM distribution in $f(R,T)$ theory. Aktaş and Aygün (Aktaş and Aygün, 2017b) have investigated FRW, Einstein static universe and de-Sitter space-time models for MSQM with cosmological constant in $f(R,T)$ gravity. Using quark and MSQM distributions, Nagpal et al. (Nagpal et al., 2018) have studied FRW model with two different deceleration parameter in $f(R,T)$ theory. Aygün et al. (Aygün et al., 2017) have researched MSQM solution in $f(R,T)$ for Bianchi $VI_h$ universe model. Researching anisotropic and homogenous universe models is important to understand the evolution of the universe. For this aim, various studies have been carried out for the distribution of homogeneous and anisotropic Kantowski-Sachs model in $f(R,T)$ gravity (Zubair and Ali Hassan, 2016, Katore and Hatkar, 2016, Samanta, 2013).

In this study, we will investigate homogeneous and anisotropic GKS space-times for MSQM distribution in $f(R,T)$ and General Relativity theories. In section 2, we gave $f(R,T)$ formalism and modified field equations. In section 3, we solved GKS space-times for MSQM distribution with cosmological term in $f(R,T)$ theory. Also, in the last section we represented conclusions.

## FIELD EQUATIONS OF MSQM DISTRIBUTION IN $f(R,T)$ THEORY

The action integral of $f(R,T)$ gravitation theory is given by (Harko et al., 2011, Kömürcü and Aktaş, 2020)

$$S = \int \left(\frac{f(R,T)+2\Lambda}{16\pi} + L_m\right)\sqrt{-g}\,d^4x \tag{2}$$

here $L_m$ represents the matter of Lagrangian and $g$ shows the determinant of $g_{\alpha\beta}$ (Harko et al., 2011). Also, $T_{\alpha\beta}$ is defined by (Harko et al., 2011)

$$T_{\alpha\beta} = g_{\alpha\beta}L_m - \frac{2\,\partial L_m}{\partial g^{\alpha\beta}} \tag{3}$$

We can get equation (4) by variation equation (2)

$$f_R R_{\alpha\beta} - \frac{1}{2}f(R,T)g_{\alpha\beta} + (g_{\alpha\beta}\blacksquare - \nabla_\alpha\nabla_\beta)f_R = 8\pi T_{\alpha\beta} - f_T T_{\alpha\beta} - f_T\Theta_{\alpha\beta} + \Lambda g_{\alpha\beta} \tag{4}$$

where $f_R = \frac{\partial f(R,T)}{\partial R}$, $f_T = \frac{\partial f(R,T)}{\partial T}$, $\nabla_\alpha$ is the covariant derivative and $\blacksquare = \nabla_\alpha\nabla^\alpha$ (Harko et al., 2011). However, $\Theta_{\alpha\beta}$ is given by (Harko et al., 2011)

$$\Theta_{\alpha\beta} = -2T_{\alpha\beta} + g_{\alpha\beta}L_m - 2g^{\mu\nu}\frac{\partial^2 L_m}{\partial g^{\alpha\beta}\,\partial g^{\mu\nu}} \tag{5}$$

From equation (4), we get

$$f_R R + 3\blacksquare f_R - 2f(R,T) = (8\pi - f_T)T - f_T\Theta + 4\Lambda \tag{6}$$

where $\Theta = g^{\alpha\beta}\Theta_{\alpha\beta}$ (Harko et al., 2011). Using eqs. (4) and (6), we get field equations of $f(R,T)$ gravitation theory by (Harko et al., 2011)

$$f_R\left(R_{\alpha\beta} - \frac{1}{3}Rg_{\alpha\beta}\right) + \frac{1}{6}f(R,T)g_{\alpha\beta} = 8\pi\left(T_{\alpha\beta} - \frac{1}{3}Tg_{\alpha\beta}\right) - f_T\left(T_{\alpha\beta} - \frac{Tg_{\alpha\beta}}{3}\right) \tag{7}$$

$$+f_T\left(\Theta_{\alpha\beta} - \frac{\Theta g_{\alpha\beta}}{3}\right) + \nabla_\alpha\nabla_\beta f_R + \Lambda g_{\alpha\beta}$$

To solve new theory, Harko et al. have suggested three $f(R,T)$ functions as follows (Harko et al., 2011), $f(R,T) = R + 2f(T)$, $f(R,T) = f_1(R) + f_2(T)$ and $f(R,T) = f_1(R) + f_2(R)f_3(T)$. In this paper, we used first model ($f(R,T) = R + 2f(T)$) for solutions of modified field equations. From equations (1)-(7), we get

$$G_{\alpha\beta} = [8\pi + 2f'(T)]T_{\alpha\beta} + [2pf'(T) + f(T) + \Lambda]g_{\alpha\beta} \tag{8}$$

here the prime represents $f'(T)=\frac{df(T)}{dT}$. If we take $f(T)=\mu T$ (here $\mu$ is a constant) in equation (8), we obtain

$$G_{\alpha\beta}=(8\pi+2\mu)T_{\alpha\beta}+[\mu(\rho-p)+\Lambda]g_{\alpha\beta} \tag{9}$$

For $\mu=0$ in equation (9), we get the field equations of General Relativity Theory (GRT). The homogeneous and anisotropic general model for GKS space-time is defined by (Zubair and Ali Hassan, 2016)

$$ds^2=dt^2-\mathcal{A}(t)^2dr^2-\mathcal{B}(t)^2[d\theta^2+\mathcal{K}(\theta)^2d\phi^2] \tag{10}$$

where

$$\mathcal{K}(\theta)=\begin{cases}\theta & \text{LRS Bianchi I}\\ \sinh\theta & \text{Bianchi III}\\ \sin\theta & \text{Kantowski}-\text{Sachs}\end{cases} \tag{11}$$

The energy momentum tensor for MSQM is given by (Barrow et al., 2007, Tsagas and Barrow, 1997),

$$T_{\alpha\beta}=(p+\rho)u_\alpha u_\beta-\mathrm{p}g_{\alpha\beta}+\frac{h^2}{2}\left(2u_\alpha u_\beta-g_{\alpha\beta}\right)-h_\alpha h_\beta \tag{12}$$

here $p$ represents the pressure, $\rho$ shows the energy density, $h^2$ is the magnetic field and $u^\alpha$ indicates the four velocity vector. Since it should be $h_\alpha u^\alpha=0$, the magnetic field must be taken in the radial direction (Barrow et al., 2007, Tsagas and Barrow, 1997). From eq. (10), we get some kinematical quantities as follows,

$$a^3=\mathcal{A}\mathcal{B}^2 \tag{13}$$

$$\theta=\frac{\dot{\mathcal{A}}}{\mathcal{A}}+\frac{2\dot{\mathcal{B}}}{\mathcal{B}} \tag{14}$$

$$\sigma^2=\frac{1}{3}\left(\frac{\dot{\mathcal{A}}}{\mathcal{A}}-\frac{\dot{\mathcal{B}}}{\mathcal{B}}\right)^2 \tag{15}$$

$$H=\frac{\theta}{3} \tag{16}$$

and

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1 \tag{17}$$

here the dot represents w.r.t time. Using equations (9), (10) and (11), we obtain MSQM equalities for GKS universe models with $\Lambda(t)$ in $f(R,T)$ gravitation theory as follows

$$\frac{2\ddot{\mathcal{B}}}{\mathcal{B}}+\left(\frac{\dot{\mathcal{B}}}{\mathcal{B}}\right)^2-\frac{\delta}{\mathcal{B}^2}=4\pi(h^2-2p)+\mu(h^2-3p+\rho)+\Lambda \tag{18}$$

$$\frac{\ddot{\mathcal{A}}}{\mathcal{A}}+\frac{\dot{\mathcal{A}}\dot{\mathcal{B}}}{\mathcal{A}\mathcal{B}}+\frac{\ddot{\mathcal{B}}}{\mathcal{B}}=-4\pi(h^2+2p)-\mu(h^2+3p-\rho)+\Lambda \tag{19}$$

$$\left(\frac{\dot{\mathcal{B}}}{\mathcal{B}}\right)^2+\frac{2\dot{\mathcal{A}}\dot{\mathcal{B}}}{\mathcal{A}\mathcal{B}}-\frac{\delta}{\mathcal{B}^2}=4\pi(h^2+2p)+\mu(h^2+3\rho-p)+\Lambda \tag{20}$$

here

$$\delta=\frac{K''}{K}=\begin{cases}0 & \text{LRS Bianchi I}\\ 1 & \text{Bianchi III}\\ -1 & \text{Kantowski}-\text{Sachs}\end{cases} \tag{21}$$

## MSQM SOLUTIONS In $f(R,T)$ THEORY

From equations (18)-(20), we have six unknowns i.e. $\mathcal{A}$, $\mathcal{B}$, $p$, $\rho$, $h^2$ and $\Lambda$. For the solutions of the modified field equations, we will use three additional equations which are equation of state for MSQM, anisotropy parameter and deceleration parameter (DP). For this aim, firstly we will use anisotropy parameter. Because, our universe models are anisotropic. The definition of anisotropy parameter is given by

$$\frac{\sigma}{\theta}=\xi \tag{22}$$

here $\xi$ is a constant. From equation (22), we get following result

$$\mathcal{A}=c_1\mathcal{B}^n \tag{23}$$

here $c_1$ and $n=\frac{\sqrt{3}-6\xi}{\sqrt{3}+3\xi}$ are constants. Without loss of generality, we can choose $c_1=1$. Secondly, we will use the DP that is responsible for the expansion and evolution of the universe (Adhav, 2011). There are many DP models in the literature. In this study, we will use time varying deceleration parameter (Akarsu and Dereli, 2012) model which is often studied widely.

$$q=-kt+m-1 \tag{24}$$

hhere $k$ and $m$ are connstants. If we solve equation (24), it is obtained that

$$\mathcal{B}=c_3e^{\frac{6\tanh^{-1}\left(\frac{(n+2)(kt-m)}{\sqrt{(n+2)(m^2n-6c_2k+2m^2)}}\right)}{\sqrt{(n+2)(m^2n-6c_2k+2m^2)}}} \tag{25}$$

here $c_2$ and $c_3$ are constants. If $c_2=0$ and $c_3=1$ are taken to simplify solutions, $\mathcal{B}$ obtained as,

$$\mathcal{B} = \left(\frac{kt}{2m-kt}\right)^{\frac{3}{(n+2)m}} \tag{26}$$

Finally, using equations (1), (18)-(20), (23) and (26), we find $\mathcal{A}$, $\rho$, $p$, $h^2$ and $\Lambda$ for general anisotropic universe models as follows,

$$\mathcal{A} = \left(\frac{kt}{2m-kt}\right)^{\frac{3n}{(n+2)m}} \tag{27}$$

From equations (13), (26) and (27) we get scale factor is as follows

$$a = a_1\left(\frac{kt}{2m-kt}\right)^{\frac{1}{m}} \tag{28}$$

where $a_1$ is a constant. Also, from equations (14), (16), (26) and (27) we obtain Hubble parameter is as follows,

$$\mathrm{H} = \frac{2}{\mathrm{t}(2m-kt)} \tag{29}$$

Also, energy density, pressure, magnetic field and cosmological term are given by

$$\rho = \frac{9[3(n-1)-(n+2)(kt-m)]}{(n+2)^2(4\pi+\mu)(kt-2m)^2t^2} + B_c \tag{30}$$

$$p = \frac{3[3(n-1)-(n+2)(kt-m)]}{(n+2)^2(4\pi+\mu)(kt-2m)^2t^2} - B_c \tag{31}$$

$$h^2 = \frac{6(1-n)(kt-m+3)}{(n+2)(4\pi+\mu)(kt-2m)^2t^2} - \frac{\delta}{2(4\pi+\mu)}\left(\frac{2m-kt}{kt}\right)^{\frac{6}{(n+2)m}} \tag{32}$$

and

$$\Lambda = \frac{6(kt-m)(4\pi(n+2)+\mu(n+3))}{(n+2)(4\pi+\mu)(kt-2m)^2t^2} + \frac{72\pi(n^2+2n+3)+18\mu(n^2+n+4)}{(n+2)^2(4\pi+\mu)(kt-2m)^2t^2}$$

$$-\frac{\delta}{2}\left(\frac{2m-kt}{kt}\right)^{\frac{6}{(n+2)m}} - 4(2\pi+\mu)B_c \tag{33}$$

Using the Ricci scalar $R$ and the trace of $T_{\alpha\beta}$, we get

$$R = \frac{24\left((n+2)^2(m-kt)-3(n^2+2n+3)\right)}{(2m-kt)^2(n+2)^2t^2} + 2\delta\left(\frac{2m-kt}{kt}\right)^{\frac{6}{m(n+2)}} \tag{34}$$

$$T^{MSQM} = 4B_c \tag{35}$$

from equations (32) and (33), we obtain $f(R,T) = R + 2\mu T$ function for MSQM model as follows

$$f(R,T) = \frac{24\left((n+2)^2(m-kt)-3(n^2+2n+3)\right)}{(2m-kt)^2(n+2)^2t^2} + 2\delta\left(\frac{2m-kt}{kt}\right)^{\frac{6}{m(n+2)}} + 8\mu B_c \tag{36}$$

In figures (1)-(2), the variation of the energy density and pressure with time are given.

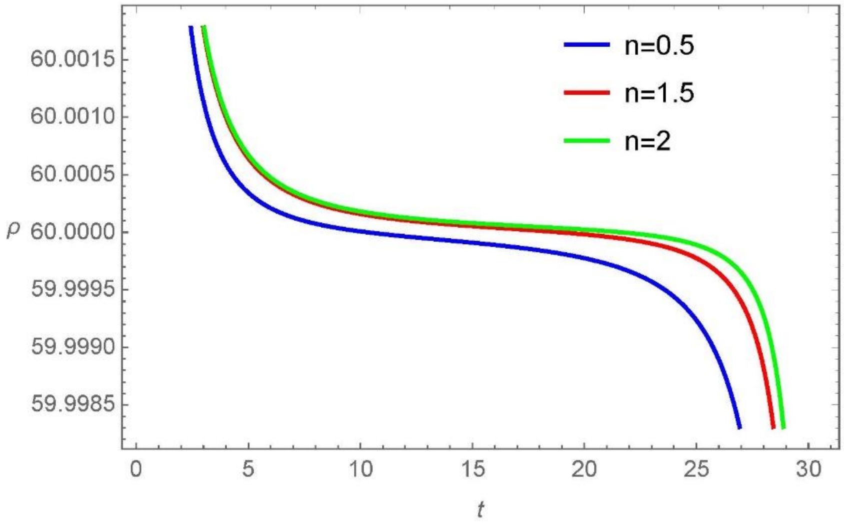



**Figure 1.** Variation of energy density ($\rho$) against time with n = 0.5, n = 1.5 and n =2.

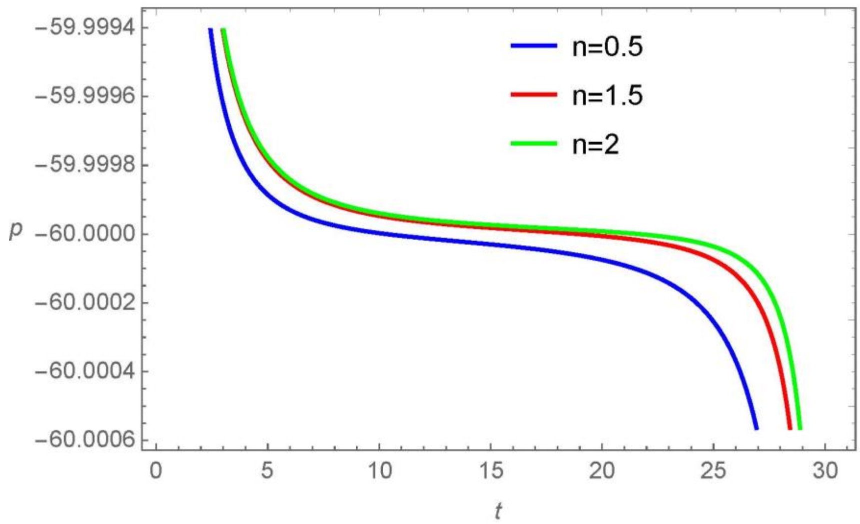



**Figure 2.** Variation of pressure (p) against time with n = 0.5, n =1.5 and n =2.

As mentioned in the equation (21), $\delta$ can take three different values. Various solutions have been obtained for each value of $\delta$. As can be seen from the equations (26)-(36), $\delta$ affects only the $h^2$, $\Lambda$ and $f(R,T)$ function. $\delta$ does not affect metric potentials, pressure and energy density.

**LRS Bianchi I Solution in $f(R,T)$ Theory**

If we take $\mathcal{K}(\theta)=\theta \Rightarrow \delta=0$ in equations (32)-(36), we get magnetic field and Λ for MSQM distribution in LRS Bianchi I universe model as follows,

$$h^2=\frac{6(1-n)(kt-m+3)}{(n+2)(4\pi+\mu)(kt-2m)^2t^2} \tag{37}$$

$$\Lambda=\frac{6(kt-m)(4\pi(n+2)+\mu(n+3))}{(n+2)(4\pi+\mu)(kt-2m)^2t^2}+\frac{72\pi(n^2+2n+3)+18\mu(n^2+n+4)}{(n+2)^2(4\pi+\mu)(kt-2m)^2t^2}-4(2\pi+\mu)B_c \tag{38}$$

Also, $f(R,T)$ function for LRS Bianchi I universe ($f(R,T)^{B-I}$) is given by

$$f(R,T)^{B-I}=\frac{24\left((n+2)^2(m-kt)-3(n^2+2n+3)\right)}{(2m-kt)^2(n+2)^2t^2}+8\mu B_c \tag{39}$$

**Bianchi III Solution in $f(R,T)$ Theory**

If we take $\mathcal{K}(\theta)=\sinh\theta \Rightarrow \delta=1$ in equations (32)-(36), we obtain magnetic field and Λ for MSQM distribution in Bianchi III universe model as follows,

$$h^2=\frac{6(1-n)(kt-m+3)}{(n+2)(4\pi+\mu)(kt-2m)^2t^2}-\frac{1}{2(4\pi+\mu)}\left(\frac{kt-2m}{kt}\right)^{\frac{6}{(n+2)m}} \tag{40}$$

$$\Lambda=\frac{6(kt-m)(4\pi(n+2)+\mu(n+3))}{(n+2)(4\pi+\mu)(kt-2m)^2t^2}+\frac{72\pi(n^2+2n+3)+18\mu(n^2+n+4)}{(n+2)^2(4\pi+\mu)(kt-2m)^2t^2}$$

$$-\frac{1}{2}\left(\frac{kt-2m}{kt}\right)^{\frac{6}{(n+2)m}}-4(2\pi+\mu)B_c \tag{41}$$

However, $f(R,T)$ function for Bianchi III universe ($f(R,T)^{B-III}$) is given by

$$f(R,T)^{B-III}=\frac{24\left((n+2)^2(m-kt)-3(n^2+2n+3)\right)}{(2m-kt)^2(n+2)^2t^2}+2\left(\frac{2m-kt}{kt}\right)^{\frac{6}{m(n+2)}}+8\mu B_c \tag{42}$$

**Kantowski-Sachs Solution in $f(R,T)$ Gravitation Theory**

Using $\mathcal{K}(\theta)=\sin\theta \Rightarrow \delta=-1$ in equations (32)-(36), we find magnetic field and Λ for MSQM distribution in Kantowski-Sachs universe model as follows,

$$h^2=\frac{6(1-n)(kt-m+3)}{(n+2)(4\pi+\mu)(kt-2m)^2t^2}+\frac{1}{2(4\pi+\mu)}\left(\frac{kt-2m}{kt}\right)^{\frac{6}{(n+2)m}} \tag{43}$$

$$\Lambda=\frac{6(kt-m)(4\pi(n+2)+\mu(n+3))}{(n+2)(4\pi+\mu)(kt-2m)^2t^2}+\frac{72\pi(n^2+2n+3)+18\mu(n^2+n+4)}{(n+2)^2(4\pi+\mu)(kt-2m)^2t^2}$$

$$+\frac{1}{2}\left(\frac{kt-2m}{kt}\right)^{\frac{6}{(n+2)m}}-4(2\pi+\mu)B_c \quad (44)$$

and $f(R,T)$ function for Kantowski-Sachs universe ($f(R,T)^{KS}$) is given by

$$f(R,T)^{KS}=\frac{24\left((n+2)^2(m-kt)-3(n^2+2n+3)\right)}{(2m-kt)^2(n+2)^2t^2}-2\left(\frac{2m-kt}{kt}\right)^{\frac{6}{m(n+2)}}+8\mu B_c \quad (45)$$

In figures (3)-(5), the variation of the magnetic field, cosmological term and $f(R,T)$ function with time are given.

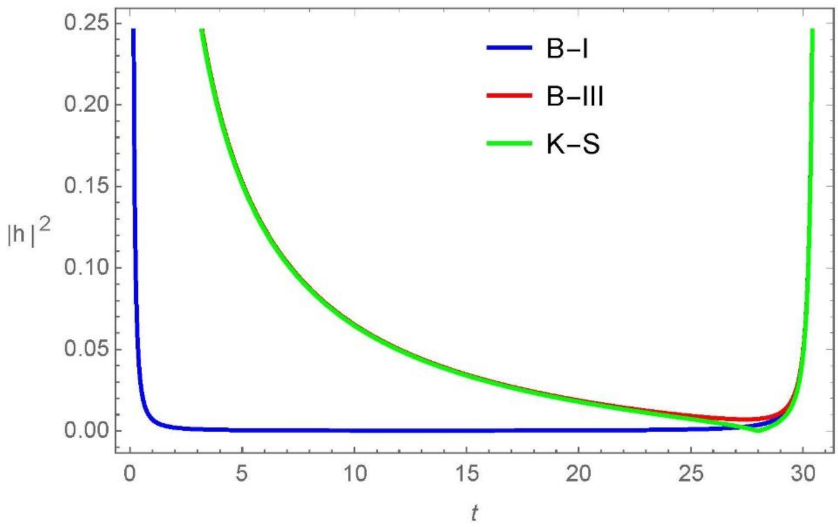



**Figure 3.** Variation of $h^2$ against time with n = 1.5 for various universe models

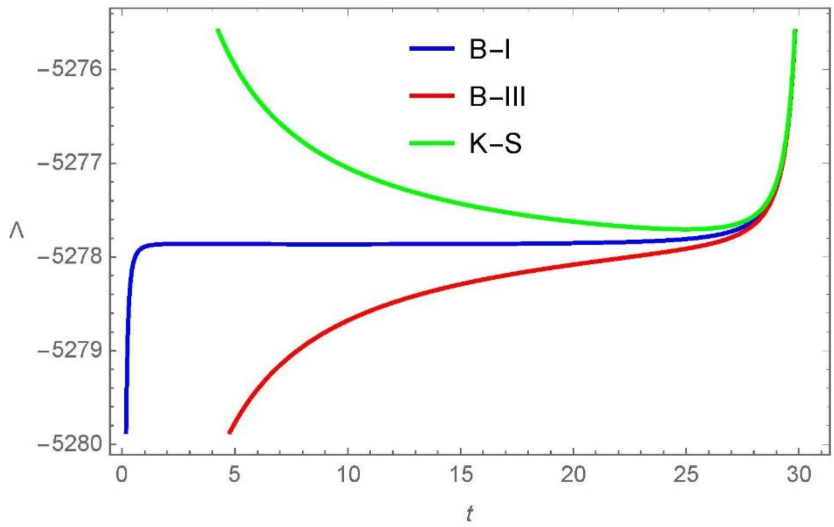



**Figure 4.** Variation of cosmological term (Λ) against time.

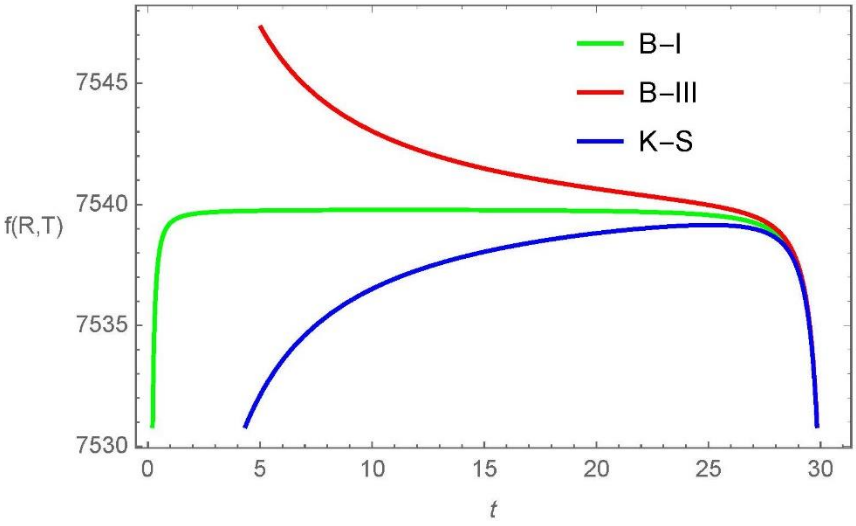
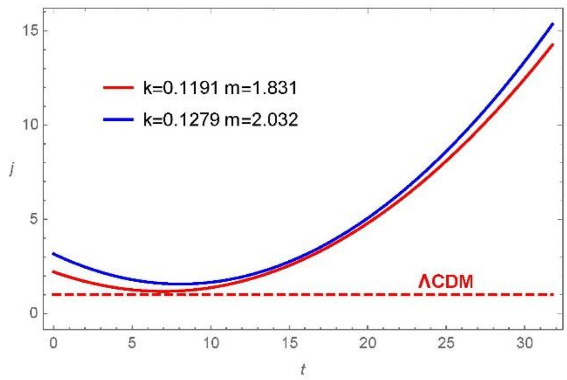



**Figure 5.** Variation of f(R,T) function against time.

From equations (24), (28) and (29), the jerk parameter (Akarsu et al. 2014) for this universe model is given by

$$j = \frac{\ddot{a}}{aH^3} = \frac{3}{2}k^2t^2 - 3(m-1)kt + (m-1)(2m-1) = \frac{3q^2+m^2-1}{2} \tag{46}$$

The variation of the Jerk parameter according to *t* and *q* is shown in the figures below.



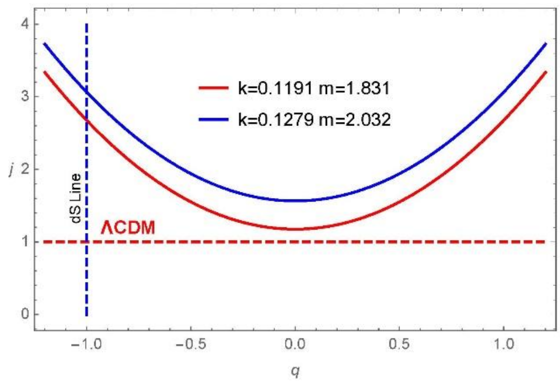




**Figure 6.** Variation of *j* against time.

**Figure 7.** Variation of *j* against *q*.

The snap jerk parameter (Akarsu et al. 2014) is given by

$$s=\frac{j-1}{3(q-1)}=-\frac{3kt(2-2m+kt)+2m(2m-3)}{6(2-m+kt)}=\frac{3q^2+m^2-3}{6(q-1)} \qquad (47)$$

The variation of the snap parameter according to $t$ and $q$ is shown in the figures below.

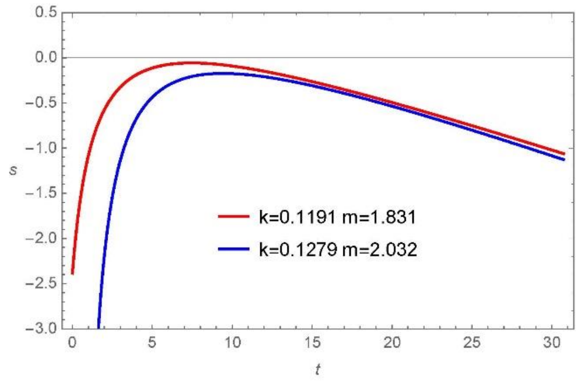



**Figure 8.** Variation of $s$ against time.

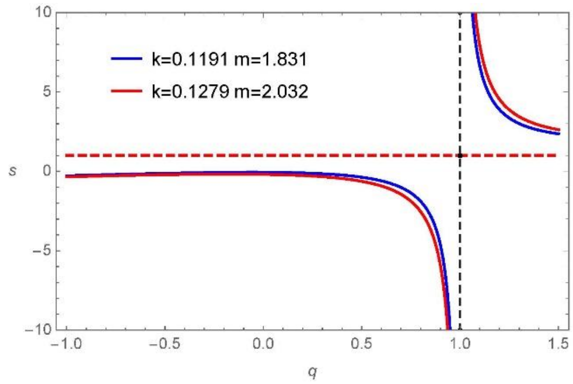



**Figure 9.** Variation of $s$ against $q$.

Also, the $s-j$ relation and $s-j$ trajectories figure are given by

$$j^2-2(3s^2-3s+1)j+3(m^2+2)s^2-6s+1=0 \qquad (48)$$

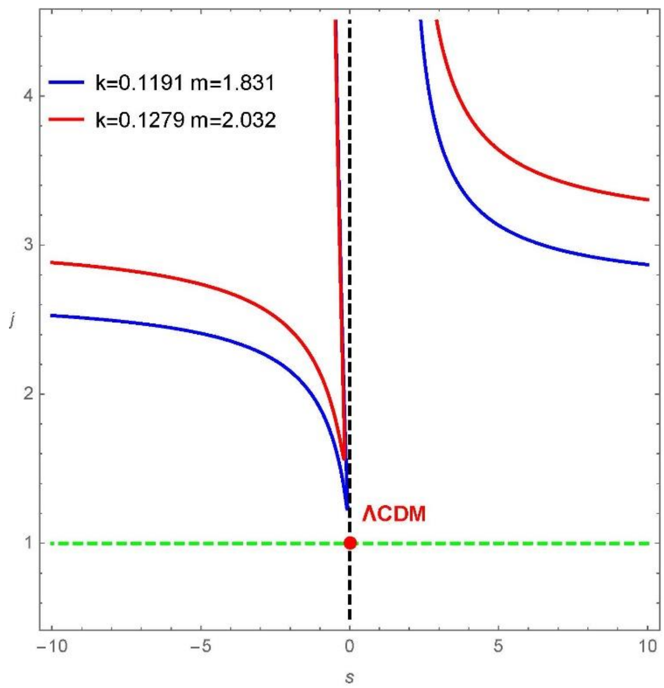



**Figure 10.** *s-j* trajectories.

Here $\Lambda CDM$ represents Λ Cold Dark Matter. The cosmic redshift $z$ is defined as (Akarsu et al. 2014)

$$1+z=\frac{a_0}{a}=\left[\left(\frac{m-q_0-1}{m+q_0+1}\right)\left(\frac{2m-kt}{kt}\right)\right]^{\frac{1}{m}}=\left[\left(\frac{(m-q_0-1)(m+q+1)}{(m+q_0+1)(m-q-q)}\right)\right]^{\frac{1}{m}} \tag{49}$$

The variation of the decerelation parameter according to $z$ is shown in the figure below.

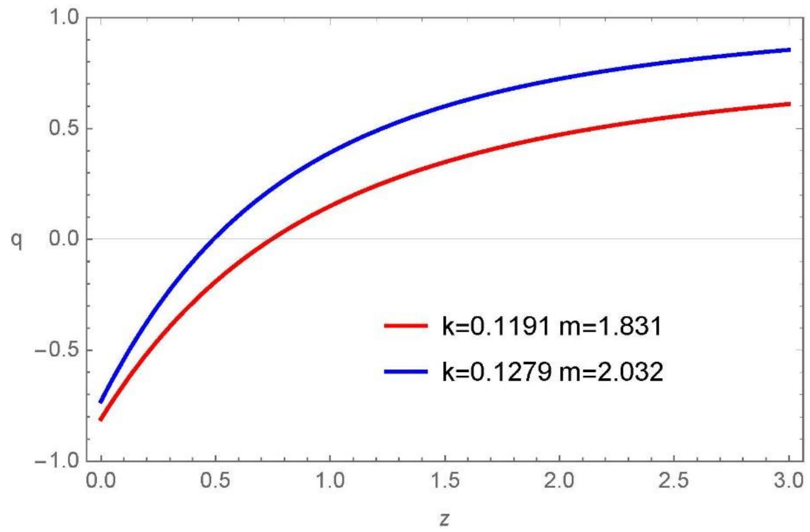



**Figure 11.** Variation of $q$ against $z$.

## RESULTS and DISCUSSION

Using $f(R,T)=R+2\mu T$ function, we get exact solution of MSQM distribution for GKS universe models with Λ in $f(R,T)$ theory. For MSQM, solutions are obtained by using anisotropy parameter, DP and EoS. For all three universe models, all physical parameters were obtained as nonzero and physically meaningful.

Our universe models have singularity at $t=0$ and $t=\frac{2m}{k}$. Therefore, they should be $t\neq 0$ and $t\neq\frac{2m}{k}$. Also, $n$ and $\mu$ values must be $n\neq -2$ and $\mu\neq 4\pi$.

From equation (32), it is observed that the magnetic field is decreasing with time. From this result, it can be said that the magnetic field may lose its effect towards the end of the universe.

It is easily seen that, energy density is independent of $\mathcal{K}(\theta)$ and has same value for GKS universe models. However, when $t$ increases the energy density decreases with time. This shows that, quark matter transforms to elementary particles.

From equations (30)-(36), we have seen that $B_c$ Bag constant related with cosmic density, cosmological term and $f(R,T)$ function.

**Table 1.** The behaviour of geometrical parameters

| | $q$ | $a$ | $H$ |
|---|---|---|---|
| $t \to 0$ | $m-1$ | $0$ | $\infty$ |
| $t \to \infty$ | $-\infty$ | $\infty$ | $0$ |
| $t = \frac{2m}{k}$ | $-m-1$ | $\infty$ | $\infty$ |
| $t = \frac{m}{k}$ | $-1$ | $1$ | $\frac{2k}{m}$ |

In table (1), the behaviour of the geometric parameters for some critical values is given. Also, phase transition time $t_{tr} = \frac{m-1}{k}$.

When $t \to \infty$, we get $\rho \to B_c$, $p \to -B_c$ for GKS universe models. From these results, we conclude that strange quark matter behaves like dark energy. Because the density and pressure of strange quark stars are constant, our solutions also apply to strange quark stars in the case of limits. For $t \to \infty$, we obtain $\Lambda \to -4(2\pi + \mu)B_c$ for GKS universe models. If we take $\mu < -2\pi$, we find positive cosmological term, if we take $\mu > -2\pi$, we find negative cosmological term. These results agree with the studies of (Aktaş and Aygün, 2017a) also (Sahoo et al., 2017).

In all three anisotropic universe models, the $B_c$ value has an increasing effect on the $f(R,T)$ function. Specifically, the magnetic field is obtained as zero for $n = 1$ value in LRS Bianchi I universe model. Thus, we have obtained strange quark matter solutions in LRS Bianchi I universe model. Also, we obtain same metric potentials as $\mathcal{A} = \mathcal{B}$ and we get isotropic universe solutions for $n = 1$.

Energy conditions impose some restrictions on the pressure and energy density of the fluid. These conditions help us explain the evolution of the universe. Energy conditions for pressure and energy density are given as follows(see figure 12). Strong energy condition (SEC), dominant energy condition (DEC), null energy condition (NEC) and weak energy condition (WEC).

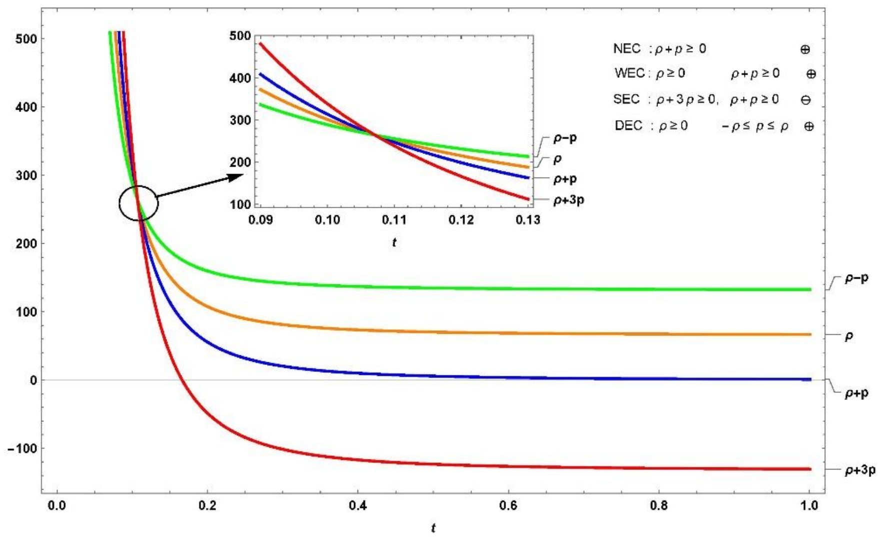



**Figure 12.** State of energy conditions for general Kantowski-Sachs model, in the graph ⊕ and ⊖ show satisfied and unsatisfied energy conditions, respectively.

If we take $\mu = 0$ in equation (9), $f(R,T)$ gravitation theory transforms to Einstein's GR. For this situation, we obtain $f(R,T) = R$ function and Einstein's GR solutions as follows

$$h^2 = \frac{3(1-n)(kt-m+3)}{2\pi(n+2)(kt-2m)^2t^2} - \frac{\delta}{8\pi}\left(\frac{kt-2m}{kt}\right)^{\frac{6}{(n+2)m}} \tag{44}$$

$$\rho = \frac{9[3(n-1)-(n+2)(kt-m)]}{4\pi(n+2)^2(kt-2m)^2t^2} + B_c \tag{45}$$

$$\Lambda = \frac{6(kt-m)}{(kt-2m)^2t^2} + \frac{18(n^2+2n+3)}{(n+2)^2(kt-2m)^2t^2} - \frac{\delta}{2}\left(\frac{kt-2m}{kt}\right)^{\frac{6}{(n+2)m}} - 8\pi B_c \tag{46}$$

For all three universe models, the results obtained in $f(R,T)$ gravitational theory also valid to Einstein's GR.

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