4-1-1974

Problems of Relations Among Balances in Various Classifications

Bohuslav Sekerka

C58 8

SIXTH INTERNATIONAL CONFERENCE
ON input-output techniques

(Vienna, 22 - 26 April 1974)

Session 3: Studies of national applications

PROBLEM OF RELATIONS AMONG BALANCES IN VARIOUS CLASSIFICATIONS

by

Bohuslav Sekerka
National Economy Computing Institute
of the State Planning Commission
Czechoslovakia

Received on 7.1.1974

id.74-242

Problem of Relations Among Balances in Various Classifications

by

Bohuslav Sekerka

1. Introduction

This paper deals with the problem of relations among balances in various classifications. We shall consider input-output balances, first with rows and columns corresponding to commodities, second with rows corresponding to commodities and columns corresponding to producers (enterprises) and finally with rows and columns corresponding to producers. When investigating the relations among these balances we shall use the methods of disaggregation, as described in paragraph 4.

All these questions are important e.g. for the middle-term planning of socialist economy, in which organization aspects for short-term planning and commodity aspects for long-term planning are considered. The described methods of disaggregation are important for analyses, when a certain phenomenon is investigated in various classifications. For example, when expressing quantities of a certain nomenclature in another nomenclature.

We denote that the development in computation technique, construction of information systems and databanks for central control enable us to introduce various classification aspects into the investigation of economic events.

We shall consider the economy in which m enterprises produce n commodities. We denote

x_{ij}^{st} quantity of commodity i, produced by enterprise s and used for the production of commodity j by enterprise t during a certain period of time;

y_i^s quantity of commodity i, produced by enterprise s during a certain period and used as final product;

z_i^s quantity of commodity i, produced by enterprise s during a certain period.

We shall assume the variables in monetary expression, so that all required aggregations are allowed. The length of time interval facilitates to assume that the produced commodity is spent during the same period or that initial and final stocks of a certain commodity approximately equal.

The symbol $\cdot$ on the place of index expresses the values aggregated by this index. We shall demand the natural relations be fulfilled, i.e. aggregation corresponds to addition. Then $z_i^{\cdot}$ denotes both the total production of commodity i and $\sum_s z_i^s$. In some cases the aggregation bias may appear. If we, for example, define for every enterprise the quantity of commodity which should be produced, and define the total production of commodity by input-output relations, then it is not necessary that $\sum_{s=1}^{m} z_i^s$ be equivalent to $z_i^{\cdot}$. The balance method secures that these values equal. So we shall assume the aggregated value as the sum of non-aggregated values. When a bias appears, it is necessary to remove it by the change of dependences, structures and so on. We denote that for values describing the realized events, the condition of nonbiased aggregation is always fulfilled.

The basic relation of distribution of commodity i produced by the enterprise s could be expressed

$$x_{i.}^{s.} + y_i^s = z_i^s \quad , \tag{1.0}$$

which could be also written

$$\sum_{j=1}^{n} x_{ij}^{s.} + y_i^s = z_i^s \quad , \tag{1.0a}$$

$$\sum_{t=1}^{m} x_{i.}^{st} + y_i^s = z_i^s \quad , \tag{1.0b}$$

where $i = 1,\dots,n, \quad s = 1,\dots,m.$

If we aggregate the relations (1.0a) in accordance with the enterprises, we shall obtain

$$\sum_{j=1}^{n} x_{ij}^{..} + y_i^{.} = z_i^{.} \quad , \quad i = 1,\dots,n \tag{1.1}$$

representing the balance of distribution of commodity i for the production of other commodities. Let us assume the consumption of a certain commodity for the production of another commodity be directly proportionate to the production of the certain commodity. It means that there exist the coefficients a_{ij} independent of the enterprise, so that

$$x^{\cdot t}_{ij} = a_{ij}\, z^{t}_{j} \qquad ,$$

for any $i, j = 1, \ldots, n, \quad t = 1, \ldots, m$.

These coefficients correspond to the technical coefficients.

From it follows

(1.1a) $$\sum_{j=1}^{n} a_{ij}\, z^{\cdot}_{j} + y^{\cdot}_{i} = z^{\cdot}_{i} \qquad , \qquad i = 1, \ldots, n$$

well known from the classic conception of intercommodity relations. In practice it is assumed that for any $j = 1, \ldots, n$ is

$$\sum_{i=1}^{n} a_{ij} < 1 \qquad ,$$

and so (1.1a) is the one to one relation among

$$y^{\cdot}_{i} \text{ and } z^{\cdot}_{i} \qquad i = 1, \ldots, n.$$

From (1.0b) follows, that for any $i = 1, \ldots, n$ could be written

(1.2) $$\sum_{t=1}^{m} x^{\cdot t}_{i.} + y^{\cdot}_{i} = \sum_{t=1}^{m} z^{t}_{i} \qquad .$$

The left side of the last relation represents the sum of consumption for further production of a certain commodity classified in accordance with the enterprises and the final consumption of a certain commodity. The right side represents the production of a certain commodity classified in accordance with the enterprises. Thus the commodity-producer model may be obtained.

Analogically from (1.0c) follows the balance

$$\sum_{t=1}^{m} x_{..}^{st} + y_{.}^{s} = z_{.}^{s} \quad , \qquad s = 1,\dots,m, \tag{1.3}$$

which corresponds to the organization classification of the production. The value $z_{.}^{s}$ may characterize the total production of the enterprises. The last relation describes relations among different enterprises in the consumption for production.

There appears the question of transformation among relations (1.1), (1.2) and (1.3) or the methods of transformation of some of these relations to another ones. This problem will be solved in the next paragraphs.

Let us denote that besides the relations mentioned before, some other balance classification could be used, which however, is not the subject of this paper.

2. Balance in Commodity-Producer Classification

From the previous paragraph we easily see, that $z_{\cdot}^{t}$ can characterize the total production of any enterprise t. When the enterprise t produces commodity i = 1,...,n in the quantity z_i^t , then we put

$$(2.1) \qquad z_i^t = b_i^t \, z_{\cdot}^t \quad .$$

From it follows

$$(2.2) \qquad \sum_{i=1}^{n} b_i^t = 1 \quad , \qquad t = 1,\dots,n \quad .$$

We can see that the production programme of enterprise t could be characterized by b_i^t , i = 1,...,n and by $z_{\cdot}^t$. To a certain extent the programme of production could be defined in advance e.g. by allocating production tasks or by the decisions of the enterprise. It comes out that the coefficients b_i^t are frequently dependent on given decisions and so they cannot be considered invariable which means they represent only technological relations. In many a case some "inertia" exists in the production of enterprises and so these coefficients cannot be supposed a subject of short-term essential changes or rather their substantial changes are conditional on the methods of decision. Problems of the methods of decision are not the subject of this paper. Further we shall assume that b_i^t is given. When a disproportion apperars in calculation, these coefficients may be altered.

In accordance with the previous

$$x_{ij}^{\cdot t} = a_{ij} z_j^t$$

for any $i, j = 1,\ldots,n$, $t = 1,\ldots,m$,

which gives

$$x_{i.}^{\cdot t} = \sum_{j=1}^{n} a_{ij} z_j^t = \sum_{j=1}^{n} a_{ij} b_j^t z_{.}^t = r_i^t z^t \quad ,$$

where

$$(2.3) \qquad r_i^t = \sum_{j=1}^{n} a_{ij} b_j^t \qquad i = 1,\ldots,n \, , \quad t = 1,\ldots,m \, .$$

So (1.2) may be written as

$$(2.4) \qquad \sum_{t=1}^{m} r_i^t z_{.}^t + y_i^{\cdot} = \sum_{t=1}^{m} b_i^t z_{.}^t \quad , \qquad i = 1,\ldots,n$$

which are the basic relations for the next analysis.

Our task is - for given $y_i^{\cdot}$, $i = 1,\ldots,n$ - to find such production programmes of enterprises that the relation (2.4) is fulfilled. Generally this task needn´t have any solution if the coefficients r_i^t and b_i^t , $i = 1,\ldots,n$, $t = 1,\ldots,m$ are given before. The existence of solution depends on values of $y_i^{\cdot}$. Further on we shall show necessary and sufficient condition that the task should have a solution with given values $y_i^{\cdot}$.

Let y_i $i = 1,\ldots,n$ correspond to the values z_i° $i = 1,\ldots,n$ so that (1.1a) is valid. Let (2.1) - (2.3) be valid too. Then the necessary and sufficient condition, for the validity of relations (2.4) are valid relations

$$(2.5) \qquad \sum_{t=1}^{m} b_i^t \, z_{\cdot}^t = z_i^{\circ} \qquad , \qquad i = 1,\ldots,n \; .$$

Proof:

Using (2.3) for any $z_{\cdot}^t$ we can see that

$$\sum_{t=1}^{m} r_i^t \, z_{\cdot}^t = \sum_{t=1}^{m} \sum_{j=1}^{n} a_{ij} \, b_j^t \, z_{\cdot}^t = \sum_{j=1}^{n} a_{ij} \sum_{t=1}^{m} b_j^t \, z_{\cdot}^t \qquad i=1,\ldots,n,$$

and from it immediately follows (2.4).

Contrarywise, from (2.4) follows

$$\sum_{j=1}^{m} a_{ij} \sum_{t=1}^{m} b_j^t \, z_{\cdot}^t + y_i^{\circ} = \sum_{t=1}^{m} b_i^t \, z_{\cdot}^t \quad , \qquad i = 1,\ldots,n \; .$$

Because to any y_i° $i = 1,\ldots,n$ by (1.1a) there exists only one z_i° $i = 1,\ldots,n$ we can easily see that (2.5) must be valid. That is the proof of our assertion.

Let us show the application of this statement.

We assume the coefficients in (1.1a) and (2.4) are given. The $z_{\cdot}^t$ which fulfill the (2.4) can be specified by (2.5). The right sides of (2.5) we obtain from (1.1a).

If the relations (2.5) have no solution, we have to alter the coefficients in (2.4) because the production programmes can teoretically be chosen as desired. In that case we specify such z_i^t , $i = 1,\dots,n$, $t = 1,\dots,m$ that

$$\sum_{s=1}^{m} z_i^s = z_i^{\bullet} \quad , \; i = 1,\dots,n \quad .$$

From z_i^t we can easily specify $z_\circ^t$ and b_i^t and with the use of a_{ij} and b_i^t we can specify the coefficients r_i^t and obtain the values suitable to (2.4).

3. Balance in Producer Classification and Disaggregation

We have seen in the previous text, that the balances in commodity classification and in commodity-producer classification are closely connected intermediary the structure for production of different enterprises and intermediary the technical coefficients. Let us deal with the problem of relations between the balances in commodity classification and the balance in producer classification. For this reason let us assume the values $x_{\bullet\bullet}^{st}$, $z_\bullet^s$ and $y_\bullet^s$, $t,s=1,\dots,m$ to fulfill (1.3).

Let us also assume that we know the balance in commodity classification i.e. the values $x_{ij}^{\bullet\bullet}$ or a_{ij} , $z_i^{\bullet}$ and $y_i^{\bullet}$ $i,j = 1,\dots,n$ which fulfill the relations (1.1) or (1.1a). The question is, whether it is possible to specify the values x^{st}_{ij} , z^{s}_{i} and y^{s}_{i} as well as to fulfill the relations (1.0a) and (1.0b).

From the formal point of view there exists an infinite number of possible solutions of this question. So it is clear that we have to use further conditions.

Let us see to the quantities describing production and final consumption. In many cases the values z^{s}_{i} and y^{s}_{i} are know and that´s why it is possible to leave the question of their obtaining out.

Let only the values $z^{\bullet}_{i}$ and $z^{s}_{\bullet}$ be known. In that case we are searching such quantities z^{s}_{i} that $\sum_{i=1}^{n} z^{s}_{i} = z^{s}_{\bullet}$ and $\sum_{s=1}^{n} z^{s}_{i} = z^{\bullet}_{i}$.

From the previous text comes out that these values can be specified optionally with regard to the technological depèndance of commodity. However, it is clear that the production usage or any further conditions must be taken into account. If these conditions are formulated as the values realized before, then it is possible to use any disaggregation method (see paragraph 4).

If we accept the assumption that the structure of production of commodity of different enterprises is relatively steady then it is convenient to use the first disaggregation method. In this case the weight will be b^{s}_{i}.

Thus we obtain

$$\sum_{s=1}^{m} b_i^s = 1, \quad \sum_{s=1}^{m} b_i^s z_{\cdot}^s = z_i^{\circ} \quad , \quad i = 1,\dots,n \quad .$$

The bias will appear if the second relation is not fulfilled. In that case we have to change either $z_{\cdot}^{s}$ and $z_i^{\cdot}$ or the coefficients b_i^s . Then z_i^s can be extimated by disaggregation if they are not given.

Analogically the values of final consumption can be treated.

Let us study now the consumption for production. From z_i^s and y_i^s it is possible to obtain $x_{i\cdot}^{s\cdot}$ by (1.0).

If $x_{i\cdot}^{s\cdot}$ and $x_{ij}^{\cdot\cdot}$ are known, we can specify the value of $x_{ij}^{s\cdot}$ by their disaggregation.

If $x_{i\cdot}^{s\cdot}$ and $x_{\cdot\cdot}^{st}$ are known, we can specify the value of $x_{i\cdot}^{st}$ by their disaggregation.

By disaggregation of $x_{i\cdot}^{st}$ and $x_{ij}^{s\cdot}$ we can specify the values of x_{ij}^{st}.

When all these dis~~s~~aggregations are non-biasing, we obtain the balanced systems.

We disclose further possibilities of specifying $x_{i\cdot}^{st}$. We shall make use of technical coefficients. Those are assumed to be independent of the enterprises, and so

$$x_{ij}^{\cdot t} = a_{ij}\, z_j^t$$

is valid for $i,j = 1,\dots,n$ and $t = 1,\dots,m$.

By aggregation of $x^{\cdot t}_{ij}$ it is possible to obtain $x^{\cdot t}_{i\cdot}$ and by disaggregation of $x^{\cdot t}_{i\cdot}$ and $x^{st}_{\cdot\cdot}$ we can obtain $x^{st}_{i\cdot}$. We denote, that we can modify the method of obtaining $x^{st}_{i\cdot}$. We can consider the relation

$$x^{st}_{i\cdot} = w^{st}_{i} \; x^{\cdot t}_{i\cdot} \quad , \; i = 1,\dots,n \qquad s,t = 1,\dots,m$$

where the weight w^{st}_{i} $s = 1,\dots,m$ indicates the structure of deliveries of commodity i from different producers to the enterprise t, which means that

$$\sum_{s=1}^{m} w^{st}_{i} = 1 \qquad i = 1,\dots,r \; , \; t = 1,\dots,m$$

is valid.

This modification assumes that the structure of deliveries of a certain commodity from different enterprises to a certain enterprise is relatively steady. This is similar to the case, when we consider the enterprises as consumers and not as suppliers.

The described method can also be modified so that we disaggregate $x^{s\cdot}_{i\cdot}$ with the use of the weights v^{st}_{i} and so

$$x^{st}_{i\cdot} = v^{st}_{i} \; x^{s\cdot}_{i\cdot} \quad , \; i = 1,\dots,n \; , \quad s,t = 1,\dots,m$$

where

$$\sum_{t=1}^{m} v^{st}_{i} = 1 \quad , \quad i = 1,\dots,n \qquad s = 1,\dots,m$$

In this case we prefer the idea that the enterprise (producer of commodity) keeps to the structure of distribution of this commodity to different enterprises.

Now we shall study the transformation of (1.1) to (1.3) and otherwise.

Let us assume we know the technical coefficients $(a_{\dot{i}\dot{j}})$ and the structure of production of commodity at different enterprises $(b_{\dot{i}}^{t})$ and the structure of deliveries of commodity i from different enterprises to the enterprise t $(w_{\dot{i}}^{st})$.

If we know the values of $y_{\dot{i}}^{\bullet}$, i = 1,...,n , then it is possible to obtain the values of $z_{\dot{i}}^{\bullet}$ and $x_{\dot{i}\dot{j}}^{\bullet\bullet}$, i,j = 1,...,n by using (1.1) and (1.1a). By the relation (2.5) we can estimate the values of $z_{\bullet}^{s}$, s = 1,...,m and put

$$x_{\dot{i}.}^{\bullet t} = r_{\dot{i}}^{t} z^{t} \quad ,$$

where all $r_{\dot{i}}^{t}$ are defined by (2.3).

When the structure of deliveries of commodity i from different enterprises to the enterprise t $(w_{\dot{i}}^{st})$ is known, then it is possible to put

$$x_{\dot{i}.}^{st} = w_{\dot{i}}^{st} r_{\dot{i}}^{t} z_{\bullet}^{t}$$

and obtain $x_{\dot{i}.}^{s\circ}$. From the difference between $z_{\dot{i}}^{s}$ and $x_{\dot{i}.}^{s\bullet}$ we obtain $y_{\dot{i}}^{s}$. It is easy to see that by this method we can obtain the values that fulfill (1.0b), from which (1.3) can be derived.

This description of the solution consists of transferring the balance in commodity classification to producer classification.

Let us deal with the solution of an opposite problem: how to obtain the balance in commodity classification out of the given balance in producer classification.

We shall start with the values of $z^s_{\cdot}$, $y^s_{\cdot}$ and $x^{st}_{\cdot\cdot}$ which fulfill the relations (1.3). If the coefficients a_{ij} and b^t_i are known then no problem with the solution, because it is easy to specify $z^t_i = b^t_i \, z^t_{\cdot}$ and from them to obtain $z^{\cdot}_i$.

From $z^{\cdot}_i$ and technical coefficients we can obtain $x^{\cdot\cdot}_{ij}$ and we can put $y^{\cdot}_i = z^{\cdot}_i - x^{\cdot\cdot}_{i\cdot}$ or we can find $y^{\cdot}_i$ by (1.1a).

All the previous subject-matters are the methodical instructions for practical application. We must realize that all the aspects were not taken into consideration and that the questions connected with economic interpretation were not the subject of the study.

4. Supplement - Disaggregation

Let us consider a quantity f, that resulted form aggregation of quantities f_i. From value of f along with some further information we want to derive f_i (disaggregation).

The most common disaggregation depends on the knowledge of hypothetical structure of quantity f, i.e. we know w_i such, that $\sum_i w_i = 1$ and we put for any i $f_i = w_i f$.

This method we can modify in following way: from indices i we create certain classes i^* and with the help of known or assumed laws we specify f_{i^*} such, that their amount equals f. These quantities may be disaggregated the same method as described before. The disaggregation is done with the use of hypothetical $w^i_{i^*}$ fulfilling the relation $\sum_{i \in i^*} w^i_{i^*} = 1$. Thus it is possible to write

$$f_i = w^i_{i^*} f_{i^*} \quad , \quad i \in i^* \quad .$$

Certain laws and relations can be used in disaggregation and so the disaggregated quantities are obtained by substitution of f or f_{i^*} into these relations.

Let us assume, now, that we know some quantities f_i and f^s, which are the amounts of certain f^s_i . From this follows

$$\sum_i f^s_i = f^s \quad , \quad \sum_s f^s_i = f_i \quad .$$

We want to estimate f^s_i on the basis of the known quantities. It is easy to see that the condition for the existence of such quantities is the relation $\sum_s f^s = \sum_i f_i$. Anyhow, it is clear there exists an infinite number of possibilities to specify the studied quantities.

In some cases it is possible to assume the relative stability in relations

(1a) $$w^s_i = \frac{f^s_i}{f^s} \quad .$$

If these relations are known, then

(2a) $$f_i^s = w_i^s f^s .$$

From basic conditions and definition of the values w_i^s follows

$$\sum_i w_i^s = 1 \quad , \quad \sum_s w_i^s f^s = f_i .$$

If those relations are not fulfilled, then disaggregation by (2a) is biasing from the mathematical point of view. In that case it is possible to remove the bias by changing the weights so that the relations should be fulfilled.

Anologically it is possible to assume the relative stability in relations

(1b) $$v_i^s = \frac{f_i^s}{f_i} .$$

When the relations are known then disaggregation can be done according to

(2b) $$f_i^s = v_i^s f_i .$$

From the conditions for addition and from the definition of v_i^s follows

$$\sum_s v_i^s = 1 \quad , \quad \sum_i v_i^s f_i = f^s .$$

If all those relations are not fulfilled, then the bias from disaggregation described by (2b) appears.

We shall describe another method of disaggregation. This method assumes that $\bar{f}_i^s$, $\bar{f}_i$ and $\bar{f}^s$ are given and that they fulfill the conditions for addition.

If the new values f_i and f^s are given, then it is possible to define the indices p_i and p^s with the help of formulas

$$p_i = \frac{f_i}{\bar{f}_i} \quad , \qquad p^s = \frac{f^s}{\bar{f}^s}$$

and disaggregate according to the formula

$$f_i^s = p_i \, \bar{f}_i^s \, p^s \quad .$$

We easily see that along with this method of disaggregation goes that

$$\sum_i f_i^s = p^s \sum_i p_i \, \bar{f}_i^s = p^s \sum_i \bar{v}_i^s \, f_i \quad ,$$

$$\sum_s f_i^s = p_i \sum_s p^s \, \bar{f}_i^s = p_i \sum_s \bar{w}_i^s \, f^s \quad ,$$

where $\bar{v}_i^s$ and $\bar{w}_i^s$ represent the weights defined by (1a) and (1b) with $\bar{f}_i^s$, $\bar{f}^s$ and $\bar{f}_i$. To prevent the bias at this type of disaggregation, the new given values f_i and f^s must fulfill the relation

$$\sum_i f_i \, \bar{v}_i^s = \bar{f}^s \quad , \quad \sum_s f^s \, \bar{w}_i^s = \bar{f}_i \quad .$$

For disaggregation we can use any of the mentioned methods, or choose the most well-founded one from the subject point of view. It is evident that all these methods can be modified like the first example mentioned in this paper. We shan´t deal with these problems.

Let us denote at the end that when some values are given in advance, which are to arise from disaggregation, it is advisable to exclude them from disaggregation or to designate propriete weights.