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FACTOR DEMAND EQUATIONS AND INPUT-OUTPUT ANALYSIS

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The distinctive feature of multi-sectoral economic models which use an input-output framework is the detailed analysis of industrial interdependence: but when such a model is used for medium-term projections and for analysis of the structural effects of changes in government policy, it is the relationship between industrial outputs and the use of primary factors of production, particularly labour, which becomes of paramount importance. In the Cambridge growth model the attainable growth rate of output and consumption is constrained by the balance of payments and by a projection of the target year labour force derived from demographic and social considerations. During the past ten years various U.K. governments have introduced policies to stimulate industrial investment by offering generous tax incentives and cash grants, and to induce structural shifts in the labour force by discriminatory taxes and subsidies on employment, frequently justifying such measures by their favourable effects on the growth of labour productivity. The use of such policies indicates a belief on the part of the government that producers' decisions about primary inputs into the productive process are responsive to changes in the relative prices. This paper attempts to provide a framework within which it is possible to test the validity of this belief and to incorporate the effects of such policies in a multi-sectoral model.

The current version of the Cambridge model, which is outlined in the paper by Barker [3], does allow for the effect of gross investment in increasing productivity. This is done by adopting a vintage model of production of the type put forward by Wigley [17]: the resulting equation for the rate of growth of productivity implies that productivity growth for any industry is positively related to the level of investment and negatively related to the growth rate of output. But since the level of gross investment responds directly only to changes in the level of gross output for the industry, the only way in which government policies designed to promote investment or structural changes in the pattern of employment can affect productivity is indirectly, through the effects on demand from purchasers at home and abroad for the industry's product. The current structure of the model only allows for such indirect substitution effects in private consumption and foreign trade; and, in any case, there will only be a significant response if industries pass on the net costs and benefits in setting their prices. It seems probable that both direct and indirect effects are important: thus the evidence of the First Report on the Selective Employment Tax [12] indicates that a considerable proportion, but by no means all, of the effect of the tax on retailers' margins was counterbalanced by reduced labour input and higher productivity.

It would of course be possible to estimate suitable behavioural equations for investment goods and employment, while retaining the conventional assumption that intermediate goods are always required in fixed proportion to output. This is the approach which was originally envisaged for the Cambridge model in [10], and which has been adopted in the U.S.A. by Almon [1]. Alternatively it is possible, using time series data, to estimate a set of demand equations for all inputs simultaneously. The estimation of such equations for intermediate inputs as well as for labour and capital goods does not necessarily imply any disbelief in the conventional fixed-coefficient hypothesis of input-output analysis. Indeed one of the purposes of this paper is to set out a framework within which the conventional hypothesis can be subjected to statistical testing. A second aim is to provide a model of input demands in which it is possible to incorporate easily detailed estimates for individual sectors: thus work done in Cambridge on the fuel sector by Wigley [18] has shown a significant response of inter-industrial demand to relative prices. Finally it is possible that the estimation of sets of demand functions for intermediate inputs would simplify the task of projecting input-output tables for future years, and thus reduce the amount of information required if an input-output model is used to project the economy over a sequence of years.

The hypothesis that input coefficients are variable in response to price changes is clearly more likely to be acceptable if it is applied only to newly installed equipment: it is this view of investment as embodying new techniques which is used, for example, in the study of input requirements by Carter [5]. It is assumed that the objective of producers in each time period is the minimisation of cost on newly installed capacity, where the amount of new capacity (measured in terms of output to be produced from it, and not in terms of the value of investment undertaken, since this is a choice variable) is determined by considerations outside the cost minimisation model. There is a considerable problem involved in aggregating the cost and input demand functions of the individual firms, particularly if there are economies of scale or if the proportion of output supplied by different firms is changing over time, but for the purpose of empirical work this difficulty must be neglected.

The objective of the industry is thus the minimisation of the total cost incurred by production on newly installed capacity, subject to a constraint given by the production function.

(1) $\quad \text{Min } C = w'v \qquad \text{subject to} \qquad q = f(v).$

Solution of this problem implies the existence of a set of input demand functions

(2) $v = g(w, q)$

which satisfy certain restrictions. These restrictions are merely stated here, since their derivation is well known, and can be found, for example, in Samuelson [13].

(3) $w'v_q = \lambda$

(4) $w'V_w = 0'$

(5) $V_w = V_w'$

(6) $z'V_w z \leq 0$

In empirical application it is convenient to characterise the technology not by the production function, but by the corresponding cost function

(7) $C^* = C^*(w, q)$

which represents the minimum cost of producing q attainable at the set of input prices w and is derived by substituting the set of input demand functions (2) into equation (1). It has been shown by Shephard [14] that since the cost function and the production function are dual to each other, these two approaches are equivalent. Empirically it is convenient to describe the technology by the cost function since the vector of input demands v can easily be derived as

[1] (8) $v = C^*_w$

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1 This result was originally derived by Hotelling: a simple proof is given in [6].

Since the aim of this paper is partly to test how far the conventional assumption of fixed input coefficients for intermediate goods is justified, the functional form specified for the cost function and hence for the set of demand equations should be one which includes this assumption as a special case. It is well known that one cost function which possesses this property is that which corresponds to the multi-factor generalisation of the C.E.S. production function due to Uzawa [16],

$$(9) \qquad f(v) = \prod_I \left(\left[\sum_{j \in I} d_j v_j^{e_I} \right]^{\frac{1}{e_I}} \right)^{\beta_I}$$

This function implies that the inputs can be grouped in such a way that the share of each group is constant: furthermore the number of independent substitution responses is limited by the number of groups. An alternative is the generalised Leontief function suggested by Diewert [6]

$$(10) \qquad C^*(q, w) = h(q) \sum_i \sum_j w_i^{\frac{1}{2}} b_{ij} w_j^{\frac{1}{2}}$$

From (8) the input demand functions are

$$(11) \qquad v_i = h(q) w_i^{-\frac{1}{2}} \sum_j b_{ij} w_j^{\frac{1}{2}} .$$

The typical term of the substitution matrix V_w is

$$(12) \qquad \frac{\partial v_i}{\partial w_j} = \frac{h(q)}{2} \left[w_i^{-\frac{1}{2}} b_{ij} w_j^{-\frac{1}{2}} - \delta_{ij} w_i^{-3/2} \sum_j b_{ij} w_j^{\frac{1}{2}} \right]$$

If all off-diagonal terms of the matrix $\left[b_{ij} \right]$ are zero, and the function $h(q)$ displays constant returns, then clearly this function reduces to the conventional fixed coefficient hypothesis of input-output analysis. The partial elasticities of substitution for this technology are given by

$$(13) \quad \sigma_{ij} = \frac{C^*(q, w)\dfrac{\partial v_i}{\partial w_j}}{v_i \; v_j} = \frac{b_{ij} \sum_k \sum_l w_k^{\frac{1}{2}} b_{kl} w_l^{\frac{1}{2}}}{\sum_k b_{ik} w_k^{\frac{1}{2}} \sum_l b_{jl} w_l^{\frac{1}{2}}}$$

Thus this production technology does not possess constant elasticities of substitution (except in the special Leontief case when all off-diagonal terms are zero), but there are no additional constraints to restrict the relationships between the various elasticities.

However it is not feasible to estimate the full set of input demand functions given by (11), since even when the symmetry constraint (5), which takes the form of a linear restriction $b_{ij} = b_{ji}$, is imposed, there are still $\frac{1}{2}n(n+1)$ independent price responses, in addition to returns to scale and technical progress terms: all of these must be estimated from n T observations. In the Cambridge model each industry can have inputs of up to 45 intermediate goods, though for any individual industry many of these are always zero and can be disregarded. In addition the model contains three principal investment assets, and it is hoped that in the future male and female employment will also be distinguished. In contrast a suitable time series of inputs is only available for the 15 years 1954-68. To solve this problem and also to reduce the computational burden we assume that the cost function (10) can be written in a form which is weakly separable. This implies that the process of cost-minimisation which the firm undertakes can be broken down into a two-stage procedure: in the first stage producers decide on their aggregate inputs of new investment goods, fuels, materials, services and labour, basing their decision on the expected level of output and a set of group price indices, while in the second stage they fix their demands for individual inputs, taking account of their first-stage decision and the individual input prices.

Although this hierarchic approach has frequently been used in the estimation of consumer demand systems, it is equally applicable to the theory of production. Indeed the original paper by Leontief [7] which introduced the concept to economics began with an example of the production of steel by means of labour and two intermediate products, coal and iron ore, which were themselves produced by 'lower-level' intermediate goods. In this type of multi-stage procedure suitable group price and quantity indices for the inputs will only exist everywhere if the cost function is strongly separable, or if the lower-level cost functions are homogeneous of degree 1. The assumption of strong separability is extremely restrictive, since it implies that the substitution matrix $\frac{\partial v_i}{\partial w_j}$ can be transformed to a block-diagonal form: this can be shown to imply that the production function is a two-level C.E.S. function similar to (9).[1] However since the aggregate cost function (10) is a separable function of input prices and output, it is easy to find suitable indices. Let the cost function be written as

$$(14) \qquad C^*(q, w) = H(q) \sum_I \sum_J \left(\sum_{i \in I} \sum_{j \in I} w_i^{\frac{1}{2}} d_{ij} w_j^{\frac{1}{2}} \right)^{\frac{1}{2}} b_{IJ} \left(\sum_{i \in J} \sum_{j \in J} w_i^{\frac{1}{2}} d_{ij} w_j^{\frac{1}{2}} \right)^{\frac{1}{2}}$$

and define the price index for the Ith group, w_I, as

$$(15) \qquad w_I = \sum_{i \in I} \sum_{j \in I} w_i^{\frac{1}{2}} d_{ij} w_j^{\frac{1}{2}}$$

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1 This result is proved by Berndt and Christensen [4].

The corresponding quantity index, v_I, is given by

$$(16) \qquad v_I = \frac{\partial C^*}{\partial w_I} = h(q)\, w_I^{-\frac{1}{2}} \sum_J b_{IJ}\, w_J^{\frac{1}{2}} ,$$

while the demand equation for an individual input in group I, v_i, is given by

$$(17) \qquad v_i = \frac{\partial}{\partial w_i} (w_I\, v_I) = w_i^{-\frac{1}{2}} \sum_{j \in I} d_{ij}\, w_j^{\frac{1}{2}}\, v_I$$

The consistency of the two stage procedure is shown by the fact that the solution for v_i given by equations (16) and (17) is the same as that obtained directly if (14) is differentiated by w_i. Since the parameters of the 'true' price index (15) depend on the parameters of the lower level demand equation (17), an exact solution would require iteration between the two levels. However if the set of coefficients d_{ij} for each group is constrained by the normalisation rule

$$(18) \qquad \sum_{i \in I} \sum_{j \in I} d_{ij} = 1$$

then it can easily be shown that for small movements in relative prices there will be little error involved if the implicit price deflator for each group is used instead of the 'true' index. This use of an approximation to the 'true' price index implies a corresponding approximation on the quantity side, which results in the sum of the individual inputs given for any group by (17) not adding up exactly to the group total given by the aggregate cost-minimisation problem.

The estimation of a set of equations based on (16) and (17) in the context of a vintage model of production poses certain problems. Since when producers install new equipment they must irrevocably select the input requirements per unit of output for the entire period during which the equipment will be used, the prices relevant to the cost-minimisation problem are not the current input prices, but the discounted present values of the time paths for the various prices which the producers expect over the anticipated life time of the equipment. Thus it is necessary to make some assumption about the way in which producers form expectations about relative prices in the future. In this work the simplest possible assumption, that producers are completely myopic and hence that only current prices are relevant, is used. Secondly a vintage model predicts the technical characteristics only of new equipment, a set of variables which in the U.K. is not directly observable. However if it is assumed that in any year a constant proportion of the previous year's capacity (for which output can be used as a proxy) is replaced, then newly installed capacity, q_t, can be written as

$$(19) \qquad q_t = q(t) - (1 - \delta)\, q\,(t - 1)$$

An estimate of the depreciation parameter δ can be derived from data on the length of life of equipment.[1] If it is supposed that, on average, the principal differences between newly installed capacity and that which has just been scrapped are the result of technical change and trend price movements (as opposed to short-term fluctuations in input prices) then it is possible to transform the model so that it relates to observable variables.[2] If equipment life is denoted by T, then

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1 In this paper the equipment life used was taken from U.S. data given in Appendix C of Ture [15]. There is some reason to believe that these estimates are preferable to those used by the C.S.O. in calculating capital stock figures [9], since the C.S.O. figures have not been updated since the work of Redfern [11], whereas equipment life has probably fallen. The resulting value of δ for the U.K. engineering industry was 0.0477.

2 This transformation is, of course, not necessary for the investment equation.

$$(20)\quad v_I(t) - v_I(t-1) = v_{It} - v_{It-T}$$

$$= v_{It}\left\{1 - \left(\frac{v_{It-T}}{h(q_{t-T})}\Big/\frac{v_{It}}{h(q_t)}\right)\frac{h(q_{t-T})}{h(q_t)}\right\}$$

$$= v_{It}\left\{1 - e^{-g_{IT}}\ \frac{h(q_{t-T})}{h(q_t)}\right\}$$

Technical progress can be incorporated in such a model by treating it as a process which increases the number of 'efficiency units' represented by a given quantity of any input: thus in terms of a vintage model 1968 equipment is more productive partly because for a given level of output, less of the equipment is needed, and partly because, with this smaller amount of equipment, it may be possible to use less labour and raw materials. Accordingly exponential time trends were added to each equation. The form selected for the returns to scale function $h(q_t)$ implied a constant elasticity of input demand with respect to output. Thus the equations actually estimated were, for the group inputs,

$$(21)\quad v_I(t) - v_I(t-1) = \exp(g_I t)\ \gamma_I(t)\ \left[q(t) - (1-\delta)\ q(t-1)\right]^{\alpha}$$

$$W_I(t)^{-\frac{1}{2}} \sum_J b_{IJ}\ w_J(t)^{\frac{1}{2}}$$

$$\text{where}\quad \gamma_I(t) = \left[1 - \exp(-g_I T)\ \{(1-\delta)q(t-1)/q(t) - (1-\delta)q(t-1)\}^{\alpha}\right]$$

and for the individual inputs

$$(22)\quad v_i(t) - v_i(t-1) = \exp(g_i t)\ \{v_I(t) - v_I(t-1)\}\ w_i(t)^{-\frac{1}{2}} \sum_{j\,I} d_{ij}\ w_j(t)^{\frac{1}{2}}$$

Equations (21) and (22) were estimated for the U.K. engineering industry. This particular industry was chosen because it is the largest manufacturing industry distinguished in the Cambridge model, and uses a wide range of inputs. These inputs were divided into the five categories

1. Labour

2. Fuels (coal, coke, oil, gas, electricity)

3. Other materials (36 commodities, aggregated into 5 sub-groups)

4. Services (construction, transport, distribution, other services)

5. Investment goods (buildings, plant, vehicles)

The data used covered the years 1954-68. It should be made clear that the detailed input-output data is in a sense synthetic: as part of its work the Cambridge project has constructed commodity flow tables at current and constant prices linking the published tables for 1954 and 1963, using a modified version of the RAS method to preserve consistency with national accounts data [2]. The price of investment goods is defined by assuming a required instantaneous rate of return of 15% on the total cost of capital goods, net of the present value of tax incentives and grants: similarly the labour cost variable includes employers' social security contributions and the premiums paid to manufacturing employers after the introduction of S.E.T. in 1966 In view of the limited number of degrees of freedom, and the aim of modelling the long-term structure of the economy rather than its short-term response, no attempt was made to introduce lagged variables or a dynamic error specification. The estimation method used was Full Information Maximum Likelihood, since this provides a framework within which the validity of alternative specifications of a non-linear model such as (21) can be tested.[1]

The resulting values of the criterion function are set out in Table 1. For each pair of comparisons Model B, which represents the

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[1] I am grateful to Dr. A.S. Deaton for providing the algorithm used.

Table 1

Model		2 Log Likelihood	No. of constraints
Aggregate I	A	-542.88	15
	B	-579.28	24
Aggregate II	A	-521.35	15
	B	-578.35	24
Fuels	A	-56.96	16
	B	-91.07	26
Materials	A	-345.99	16
	B	-354.87	26
Services	A	-171.80	11
	B	-188.12	17
Investment	A	-132.10	7
	B	-137.18	10

Model A allows full price substitution between inputs, but imposes symmetry.
Model B assumes no substitution except for investment-labour substitution in the aggregate equation.
Aggregate model I permits non-constant returns to scale, subject to the rule $\sum_I \sum_J b_{IJ} = 1$: aggregate model II is constant returns.

conventional fixed coefficient of input-output analysis, is nested within the more general model A and hence the likelihood ratio test [19, chapter 13] can be applied.[1] This is an asymptotic test, but if the asymptotic results are acceptable as an approximation to small-

1 The likelihood ratio test is based on the fact that −2 Log Likelihood $\sim \chi^2(r)$, where r is the number of restrictions defining the parameter set: hence the effect of additional restrictions can be tested. The relevant values of χ^2 at the 5% significance level are

$\chi^2(10) = 18.31 \quad \chi^2(9) = 16.92 \quad \chi^2(6) = 12.59 \quad \chi^2(3) = 7.81.$

sample behaviour then it seems that at the aggregate level the hypothesis that intermediate inputs are required in fixed proportions can be rejected. On the other hand when the disaggregated equations, which 'share out' the effects of any change in aggregate inputs, are considered, price substitution is only significant within the two groups, fuels and services, and it is only for fuels that a considerable gain materialises.[1] The fact that it is not possible to reject the hypothesis of fixed coefficients for other materials is disappointing: however it is worth remembering that in the case of this group it was necessary to aggregate no less than 36 intermediate inputs into five sub-groups.

Further information about the results is given in tables 2-4, which present coefficient estimates and asymptotic standard errors for one of the aggregate models (Model IA in table 1) and for the disaggregated models for fuels and materials.[2,3] Since the value of all the price indices used was one in 1963, the off-diagonal terms of the matrix of price coefficients can be interpreted as the responses $\frac{\partial v_i}{\partial w_j}$: the table also gives the own price term for that year, calculated from (12). Graphs of the predicted and actual values for the aggregate equation are given in figure 1. From these results it can be seen that the main source of difficulty comes in the investment equa-

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1 The data for the service inputs is probably exceptionally poor, so that little weight can be placed on the results for this sector.

2 Constant returns to scale, as implied by (22), was imposed on the disaggregated models.

3 The values of R-squared are for first differences: negative values are possible because constraints have been imposed across equations.

tion. This is a very poor fit, and the coefficients imply an implausible pattern of substitution, since investment turns out to be extremely complementary to labour and substitutable for material inputs: the own price term also has the wrong sign. A possible explanation of this failure is the lack of any sophisticated dynamic structure in the investment equation: both a priori considerations, and the earlier work on investment functions (as well as the study of integrated sets of input demand functions by Nadiri and Rosen [8]) indicate that producers are much slower in adjusting their inputs of capital goods to desired levels. Clearly the development of a satisfactory model of input demand for dynamic multi-sectoral models will have to take account both of the wide range of feasible substitution possibilities and of the fact that producers adjust to the desired levels of different inputs at widely differing speeds.

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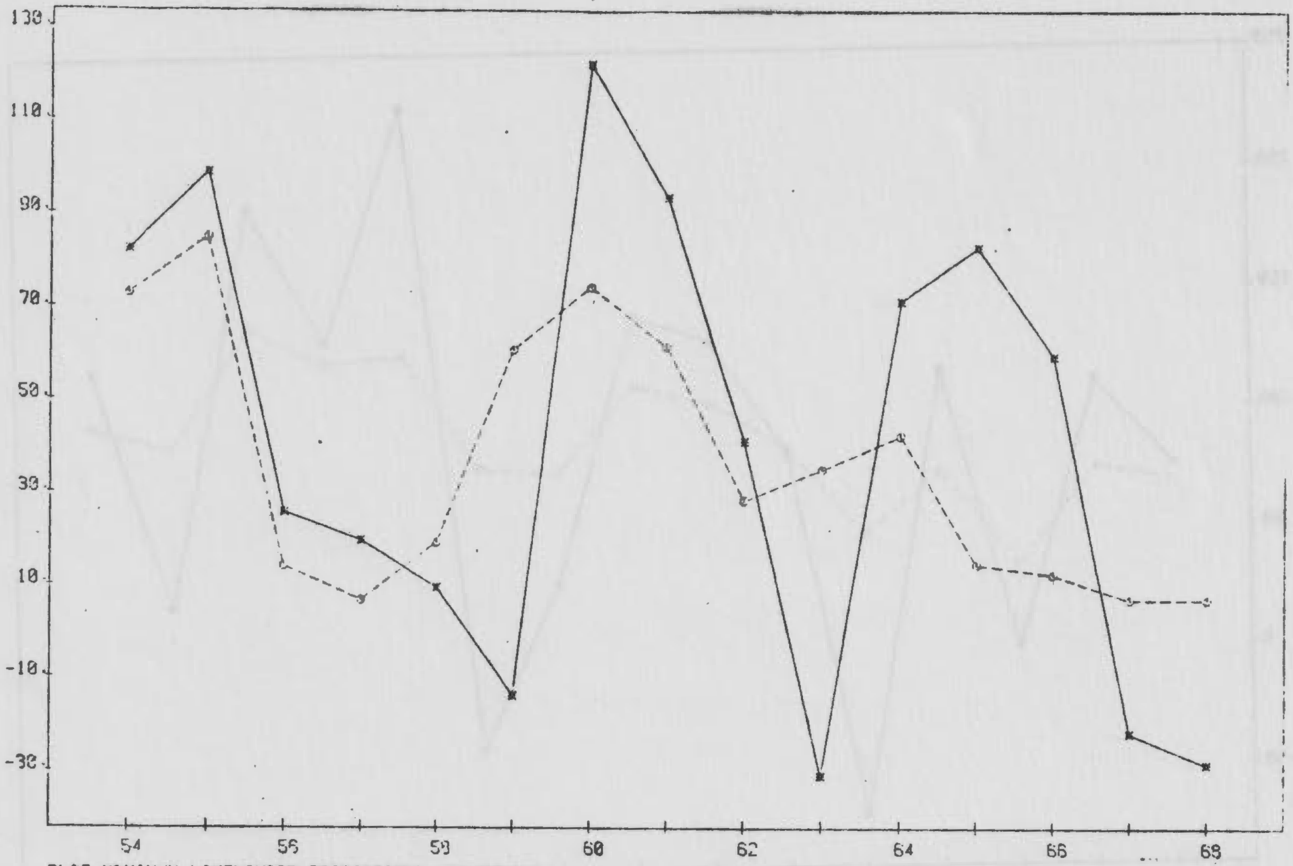
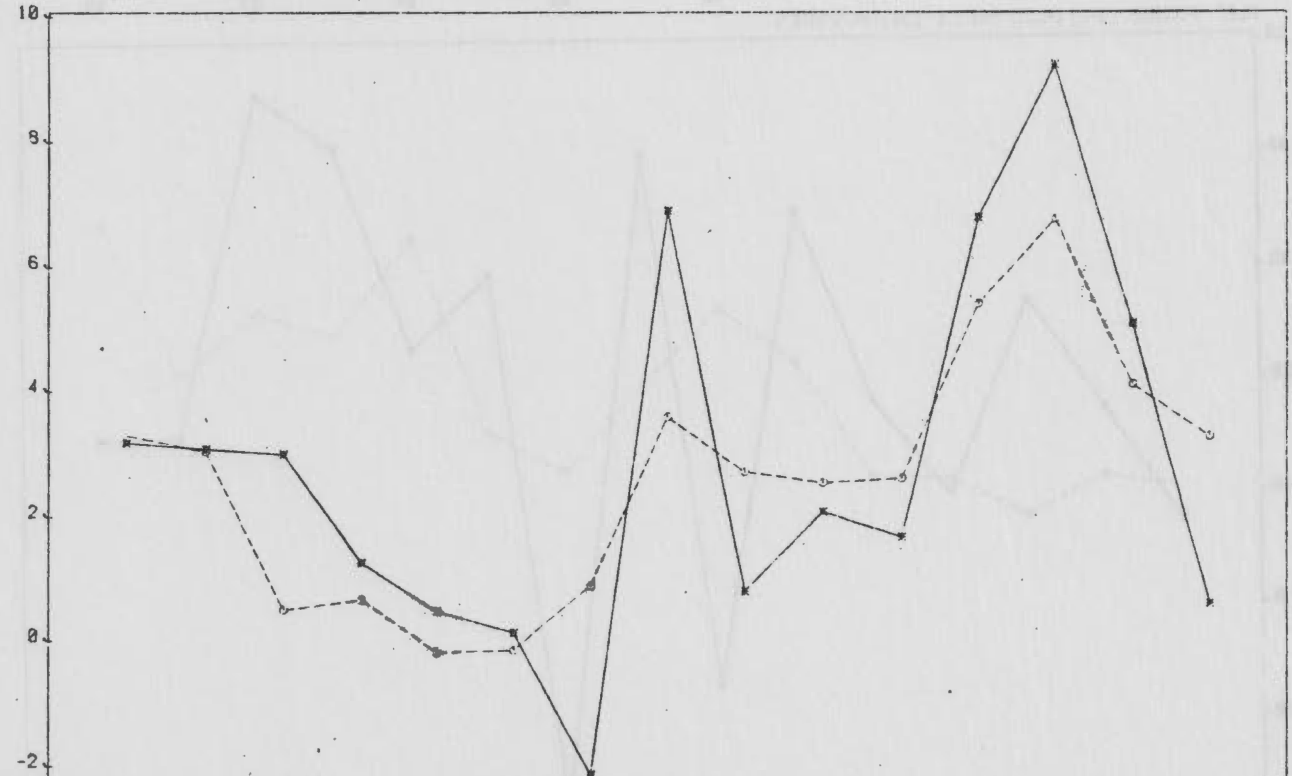


PLOT MAXIMUM LIKELIHOOD PREDICTIONS:EMPLOYMENT



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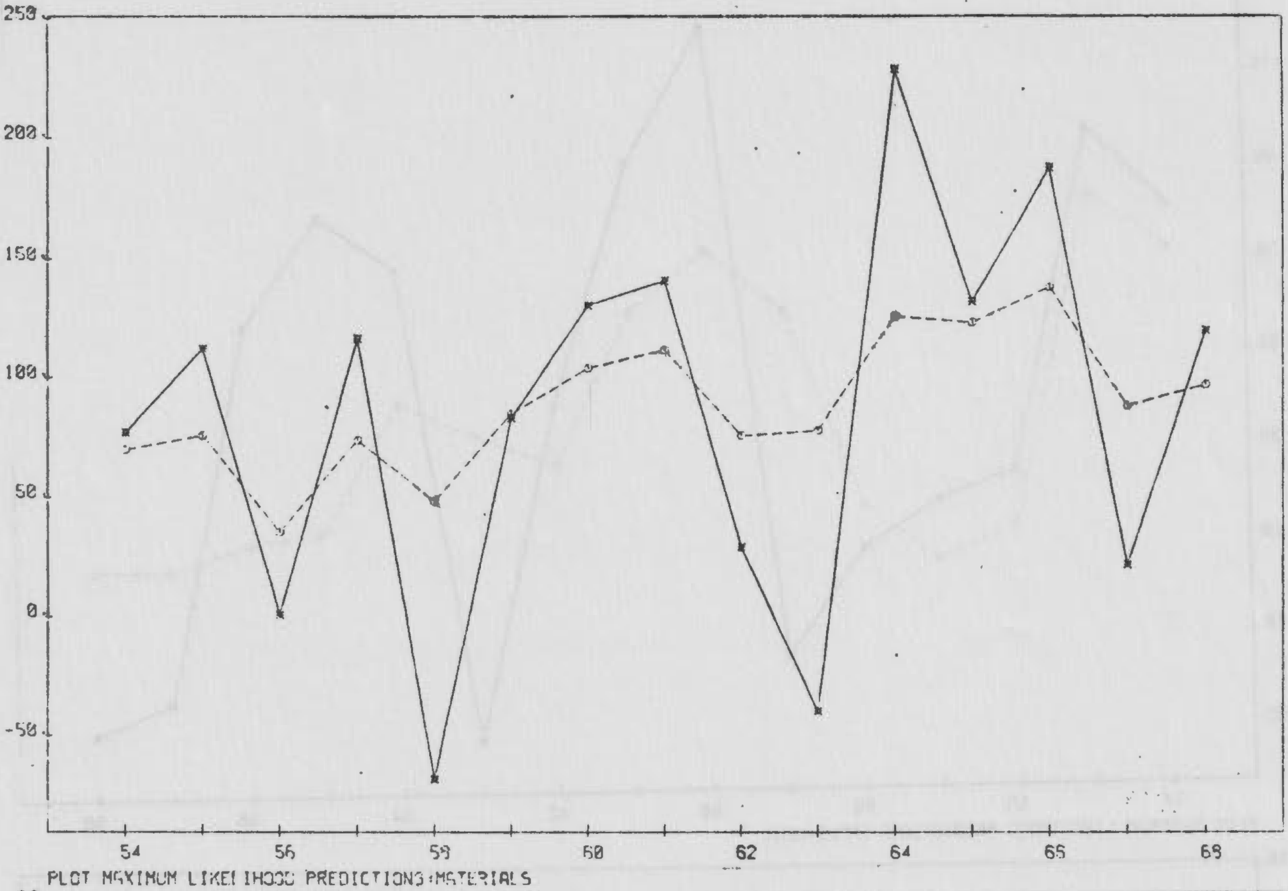

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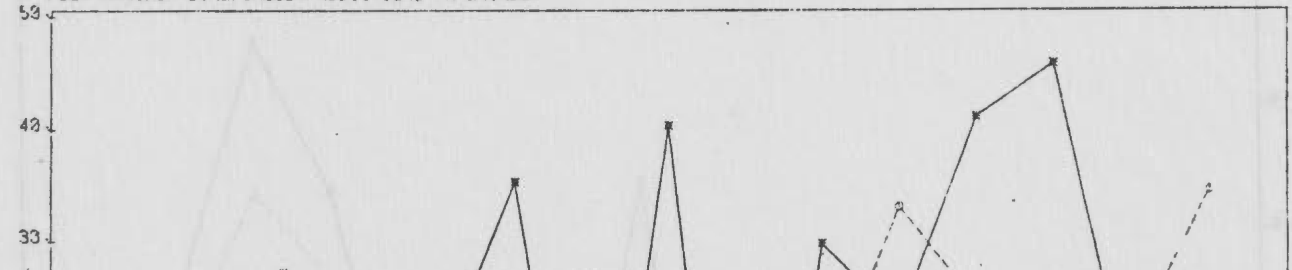

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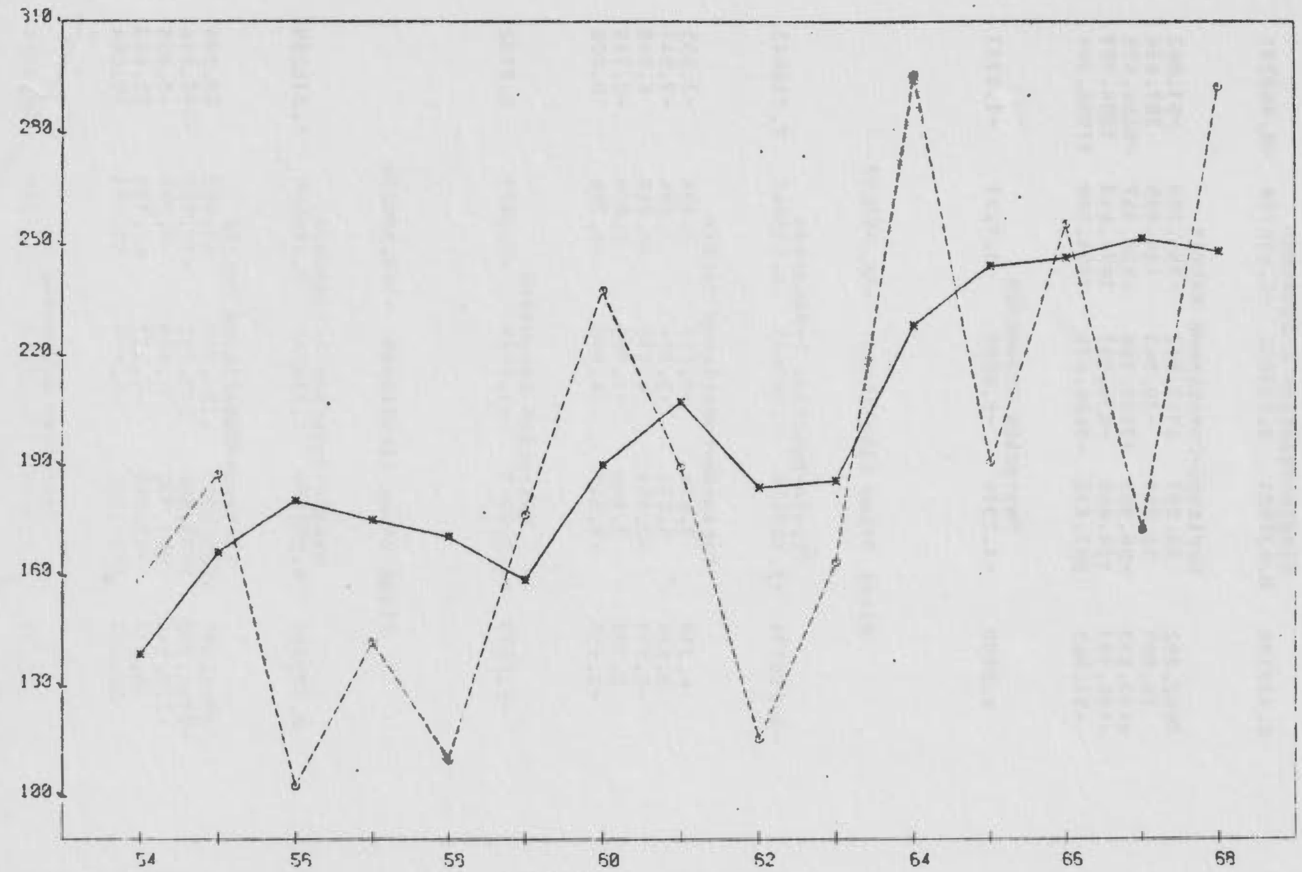

PLOT MAXIMUM LIKELIHOOD PREDICTIONS:INVESTMENT

Table 2.

Maximum likelihood estimates of aggregate inputs for engineering

Constant	Time	Price terms				
0.8083	-0.3881	0.8831	0.0216	0.3798	0.5427	-5.2342
0.1816	0.2550	1.3419	0.2219	2.0046	1.7711	7.1363
0.8083	-0.0166	0.0216	-0.6815	-0.2737	0.7434	0.7457
0.1816	0.0713	0.2219	0.7546	0.4678	0.9114	0.8055
0.8083	0.0383	0.3798	-0.2737	-2.8130	1.3116	7.3909
0.1816	0.0901	2.0046	0.4678	6.7190	4.6945	9.7653
0.8083	0.0437	0.5427	0.7434	1.3116	-1.9149	-1.8707
0.1816	0.1319	1.7711	0.9114	4.6945	4.5894	3.6669
0.8083	0.0993	-5.2342	0.7457	7.3909	-1.8707	-2.0820
0.1816	-0.0630	7.1363	0.8055	9.7653	3.6669	4.6000

Final 2*Log Likelihood -542.884766

Single-Equation R-Squareds

0.337196	0.627591	0.433892	-0.016128	-0.868231

Variance-Covariance Matrix

7962.892	76.097	6143.513	-155.783	-51.862
76.097	16.069	-20.963	134.095	207.836
6143.513	-20.963	17939.354	-626.957	-9666.675
-155.783	134.095	-626.957	1471.643	1204.189
-51.862	207.836	-9666.675	1204.189	11796.344

Own-price responses

4.2900	-1.2370	-8.8086	-0.7271	-1.0317

Table 3.

Maximum likelihood estimates of fuel inputs for engineering

Constant	Time	Price terms				
1.0000	-0.2924	-0.8178	0.3826	0.5030	0.7839	-0.7822
0.0000	0.3503	2.1358	1.4804	0.7811	1.0707	1.1424
1.0000	2.0964	0.3826	0.4728	-0.4730	0.0978	-0.4981
0.0000	5.1514	1.4804	1.8170	1.8289	0.3854	1.9193
1.0000	-0.0164	0.5030	-0.4730	-0.8380	0.1301	1.0318
0.0000	0.1432	0.7811	1.8289	1.3616	0.2952	1.4427
1.0000	0.9697	0.7839	0.0978	0.1301	-0.3103	-0.6247
0.0000	0.2358	1.0707	0.3854	0.2952	0.3289	0.8987
1.0000	0.0903	-0.7822	-0.4981	1.0318	-0.6247	1.3908
0.0000	0.0925	1.1424	1.9193	1.4427	0.8987	2.8767

Final 2*Log Likelihood -56.957657

Single-Equation R-Squareds

-0.219774	-1.147651	0.340033	0.535160	0.118442

Variance-Covariance Matrix

4.319	3.834	-1.193	2.394	-3.355
3.834	5.011	-2.154	2.646	-3.541
-1.193	-2.154	4.278	-0.760	4.948
2.394	2.646	-0.760	2.808	-0.744
-3.355	-3.541	4.948	-0.744	8.004

Own-price responses

-0.8873	0.4907	-1.1919	-0.3871	0.8732

Table 4.

Maximum likelihood estimates of materials inputs for engineering

Constant	Time	Price terms				
1.0000	0.0048	3.6592	-2.7788	-0.4190	-0.0057	0.1825
0.0000	0.0201	5.4732	5.0223	2.3071	1.5489	1.3190
1.0000	-0.0064	-2.7788	1.8913	0.2931	0.5789	0.2620
0.0000	0.0462	5.0223	5.6868	0.8052	1.3346	0.8187
1.0000	-0.0872	-0.4190	0.2931	-0.0345	-0.2060	0.3831
0.0000	0.3014	2.3071	0.8052	3.0207	0.6315	1.2437
1.0000	0.0798	-0.0057	0.5789	-0.2060	0.0442	-0.3572
0.0000	0.3652	1.5489	1.3346	0.6315	1.3248	0.6089
1.0000	-0.1048	0.1825	0.2620	0.3831	-0.3572	-0.4256
0.0000	0.1433	1.3190	0.8187	1.2437	0.6089	1.1939

Final 2*Log Likelihood -345.990234

Single-Equation R-Squareds

0.772396	0.588534	0.534248	0.370590	0.236255

Variance-Covariance Matrix

2946.380	-2760.231	-220.214	-8.932	57.294
-2760.231	2935.836	207.732	-92.557	-196.446
-220.214	207.732	30.739	-8.277	-5.835
-8.932	-92.557	-8.277	60.739	55.693
57.294	-196.446	-5.835	55.693	95.084

Own-price responses

3.0211	1.6449	-0.0512	-0.0099	-0.4704