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Research on the Complexity Characteristics of Urban Metro Network Based on Complex Network Theory

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Abstract: It is to provide decision support for later planning of metro network. Firstly, the space-L method is used to model the metro network topology. Secondly, four different indicators are used to analyze the complexity of metro network. The results show that the degree of metro network nodes in Xuzhou is generally low, and the degree distribution and power distribution are quite different. The network has no scale network properties. In Xuzhou metro network, the path between random station pairs is long, and the degree of node aggregation is low. There is a positive correlation between degree and betweenness, which can make more accurate importance assessment of the site.

Keywords: Complex Network; Metro Network; Scale-Free Network

Introduction

In recent years, subway traffic has attracted more and more cities to start the construction of subway traffic facilities because of its characteristics of safety, speed and price. It has also attracted many domestic and foreign scholars to carry out a series of studies on urban metro network. LATORA[1] studied the characteristics of Boston metro network. Based on the reliability theory, GAO[2] established a reliability model based on the simulated rail transit network, and systematically evaluated the invulnerability of the network structure. YE[3] quantitatively analyzed the vulnerability characteristics of Chongqing rail transit network, and identified key subway stations according to the quantitative results. YUAN[4] believed that the interference of the external environment will also increase the complexity of the metro network. DENG[5] found that Nanjing rail transit network has the characteristics of scale-free network and small world network. WANG[6] analyzed the vulnerability of rail transit network by removing subway stations. Based on three different indicators, Zhang[7] made a comparative analysis of the evolution process of metro network complexity in Shanghai, Beijing and Guangzhou. LAI[8] verified that Fuzhou metro network is a scale-free network based on the complex network theory, and the network satisfies the related properties of the small-world network. ZHENG[9] verified the complex characteristics of Shanghai metro network according to the parameters such as node degree, average shortest path of network and clustering coefficient.

In summary, the research on metro network is of certain significance, but there are few studies on Xuzhou metro network. Under this background, this paper analyzes the complexity of Xuzhou metro network, which can provide reference for the planning and maintenance of Xuzhou metro network.

1. Construction of Urban Metro Network Model

1.1 Metro network topology modeling

According to the theory of complex network, the structure of urban metro network is abstracted to form a simple and clear two-dimensional graph. At present, the main topology is space-L, space-P, space-R and so on. In order to better reflect the connection and structure between urban subway stations, this paper uses the space-L method to construct the structural topology of urban subway traffic network.

1.2 Statistical parameters of metro network complexity

1.2.1 Degree indicators

The degree index of complex network refers to the scale of the direct connection between the node and other nodes, which can also be defined as the number of neighbors of the node. The more neighbors of the node, the greater the degree.

The degree can directly reflect the importance of the node in the network, and there are also indicators such as average degree and cumulative degree. The definition of degree is as follows:

$$k_i = \sum h_{ij} \qquad \text{* MERGEFORMAT (1)}$$

In the formula: when h_{ij} values 1, the representative node is the neighbor of the node, otherwise 0.

According to the degree index data of all nodes in the network, the average degree index value can be further calculated, which is expressed as follows:

$$\langle k \rangle = \frac{\sum_{i=1}^{N} k_i}{N} \qquad \text{* MERGEFORMAT (2)}$$

The cumulative degree index is the probability that a node in the network is not less than a certain degree value, which can reflect the degree distribution of nodes in the network and is a global index. It is defined as follows:

$$P_k\left(K > k\right) = \sum_{k_t > k}^{\infty} p(k_t) \qquad \text{* MERGEFORMAT (3)}$$

In the formula: $p(k)$ is the distribution probability of nodes when the degree value is k.

1.2.2 Shortest path index

Among the multiple routes between node pairs, the route with the least number of edges is the shortest path $d_{i\,i}$, and the corresponding number of edges is the length.

1.2.3 Intermediaries

In the network, the larger the betweenness, the more can reflect the influence of the relevant network nodes. The size of the betweenness depends on the size of the shortest path on the corresponding node. The specific calculation formula is as follows:

$$B_i = \sum_{\substack{t \neq j \neq i \\ t < j}} \frac{N_d^{\,tj}(i)}{N_d^{\,tj}} \qquad \text{* MERGEFORMAT (4)}$$

In the formula: molecules and denominators are the shortest path number between nodes and without nodes.

1.2.4 Clustering coefficient index

In order to reflect the distribution of nodes in the network, the clustering coefficient index is used to evaluate the closeness of links between network nodes, which is defined as follows:

$$C_i = \frac{2E_i}{k_i(k_i - 1)} \qquad \text{* MERGEFORMAT (5)}$$

In the formula: where k_i is the number of neighbors of node i and E_i is the number of edges connecting node i to all neighbors at present.

According to the clustering coefficient index data of all nodes in the network, the average clustering coefficient index value of the network can be further calculated, which is expressed as follows:

$$C = \frac{\sum C_i}{N} \qquad \text{* MERGEFORMAT (6)}$$

2. Complexity analysis of Xuzhou metro network based on space-L model

2.1 Metro network topology modeling

According to the recent subway planning scheme of Xuzhou City, six subway lines will be completed, namely, Lines 1–6, among which Lines 1–3 have already realized actual operation. According to the recent planning of Xuzhou subway project, this paper selects subway lines 1–6 to construct the subway traffic network, and uses the space method to construct the two-dimensional topology of Xuzhou metro network, as shown in Figure 1. According to the node size of Xuzhou metro network, 100×100 adjacency matrix can be constructed.

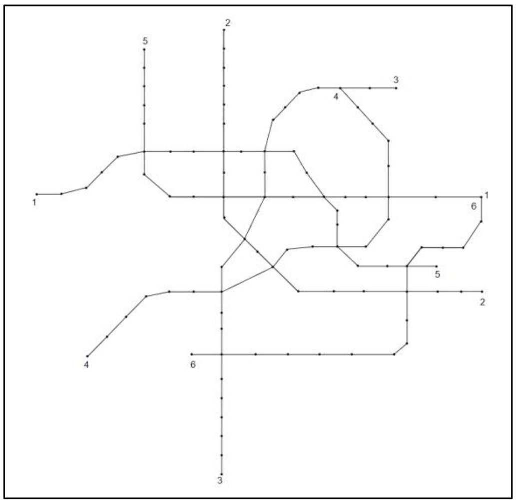

Fig.1 Topology of Xuzhou Metro Network

Description of topology modeling of Xuzhou metro network structure:

1) Xuzhou metro network is under construction. This paper takes the recent planning of Xuzhou metro network as the analysis object.

2) Without considering factors such as operation flow, train number and train type, only the structure of metro network itself is studied.

3) Although in the actual subway operation, the train in the line runs two-way, but taking into account the two-way symmetry of passenger flow, so the metro network to deal with the direction, the establishment of undirected network.

2.2 Subway complexity analysis

2.2.1 Degree distribution

By statistics, Xuzhou metro network contains 100 different stations, with 212 network edges, Figure 2 statistics the distribution probability of different degrees. There are more than 85 % of the nodes whose degree value is not higher than 3. Among them, the degree of 9 sites is 1, the degree of 76 sites is 2, the degree of 3 nodes is only 1, and the degree of 4 nodes is 14. The overall average degree of the network is 2.2.

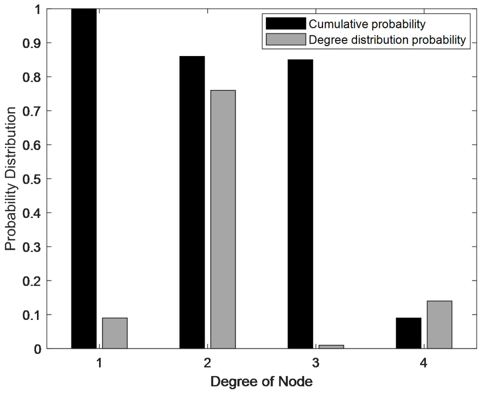

Fig.2 Statistics of probability distribution of node degree

In the study of complex networks, it is considered that if the node degree distribution of the network is similar to the power distribution, it has the property of scale-free network[10]. Therefore, the function fitting experiment is carried out on the cumulative degree distribution of metro network nodes in Xuzhou City. The results show that the fitting degree of Gaussian function is the highest, $R^2 = 0.8212$, and the fitting effect of power function is the worst, and R^2 is only 0.4953. The reason may be that in the site selection of subway stations, due to the influence of objective factors such as environment and economy, it cannot be carried out preferentially, so it does not have scale-free network characteristics.

2.2.2 Distance distribution

Figure 3 shows the distribution of the shortest path of Xuzhou metro network. According to the data in the figure, the longest and shortest path length passed between the two stations in Xuzhou metro network is 22 connections, which indicates that the diameter of the network is 22. According to MATLAB calculation, the average shortest path of the network is 9.1523, which has a large gap with the network diameter. In addition, 54 % of the shortest path length is lower than the average shortest path, which reflects the low traffic convenience of Xuzhou metro network.

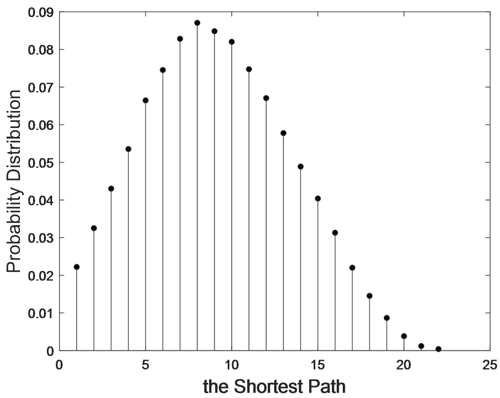

Fig.3 Statistical chart of shortest path probability distribution

2.2.3 Cluster analysis

When analyzing the clustering degree of metro network, it is found that the clustering coefficient of Xuzhou metro network is 0, and the distribution of nodes in the network is loose and the degree of aggregation is low. In order to analyze the aggregation degree of network nodes more accurately, the connectivity index is introduced, which is defined as follows:

$$z = \frac{E}{3(n-2)} \qquad \text{* MERGEFORMAT (7)}$$

In the formula: z is connectivity, molecular and denominator are the number of edges and maximum achievable edges of the metro network.

Through calculation, the connectivity of Xuzhou metro network is 0.7211, indicating that its connectivity level is high. In addition, the network data is imported into pajek software analysis, and it is found that Xuzhou metro network contains multiple circles, each circle structure contains at least four different subway stations, as shown in Figure 4. The triangle structure formed between nodes is beneficial to improve the stability and operation efficiency of the network[11]. Therefore, in the construction of Xuzhou subway, the existing circle structure can be adjusted to improve the connectivity and aggregation of the network.

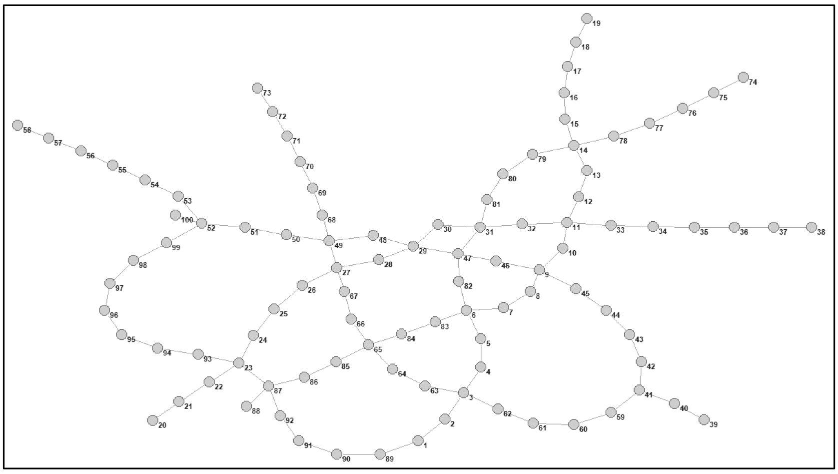

Fig.4 Node distribution map of Xuzhou metro network based on pajek software

2.2.4 Intermediary distribution analysis

The medians of Xuzhou metro network nodes are normalized and summarized in Table 1 below. The results show that 32% of the stations in the network fluctuate between 0-0.02, including 9 metro stations with medians of 0. Most of these nodes are far away from the central area of the metro network. 34% of the nodes fluctuate between 0.02-0.04, and 16% of the stations are in the interval of 0.04-0.06, There are only 5 nodes with an intermediate number of more than 0.1, and the proportion of nodes in the interval of 0.06-0.08 is the lowest, only 4%.

Table 1 Summary statistics of betweenness distribution

Betweenness	Node ratio	Betweenness	Node ratio
0-0.02	0.32	0.06-0.08	0.04
0.02-0.04	0.34	0.08-0.10	0.11
0.04-0.06	0.16	below 0.10	0.05

Figure 5 compares and analyzes the distribution of betweenness under different node degrees in Xuzhou metro network. It is found that the higher the degree value of the node, the higher the starting point of the corresponding betweenness. In addition, Figure 6 shows the data fitting curve of betweenness and degree. It can be seen that there is a positive correlation between the degree index and betweenness index of Xuzhou subway station, which reflects that the node degree and betweenness have certain similarity in the evaluation of metro network stations, and the comprehensive use of the two indicators can more objectively screen out key stations.

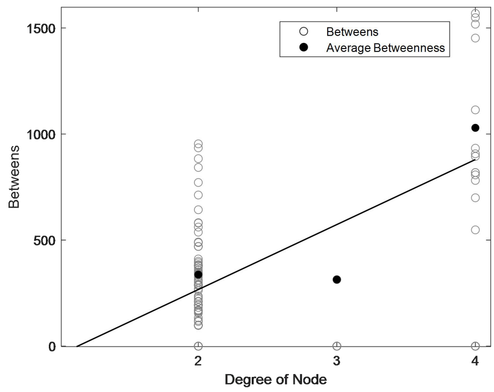

Fig.5 Intermediary Distribution Statistics under Different Degree Indicators

3. Conclusion

Based on the complex network, this paper analyzes the complexity of Xuzhou metro network. The results show that the degree of Xuzhou metro network does not meet the requirements of power distribution. From the data, Xuzhou metro network has no scale-free network characteristics. Xuzhou metro network has multiple circular structures, but each contains more stations, so the network structure has larger optimization space. With the continuous construction of subway engineering, the structure of metro network will become more and more complex. The study of metro network structure can provide reasonable reference information for subway planning.

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