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Multivariate Cognitive Walkthrough of QubitVR: An Educational Quantum Computing, Virtual Reality Application

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MULTIVARIATE COGNITIVE WALKTHROUGH OF QUBITVR: AN EDUCATIONAL QUANTUM COMPUTING, VIRTUAL REALITY APPLICATION

by

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B.S., Computer Science, University of Central Florida, 2021

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ABSTRACT

Quantum computing is a promising field but understanding how it works can be challenging for a beginner. There are also not many educational resources to visualize and learn about quantum computing. To advance knowledge in this area, we have created QubitVR, which employs a Bloch sphere representation of a qubit, and supports trajectory visualizations and state equations in a virtual reality (VR) setting. We also conducted a multivariate cognitive walkthrough with subject matter experts (SMEs) on QubitVR to assess the effectiveness of trajectory visualizations and state equations in learning about quantum gates. The results were that trajectory visualizations aided users in understanding and learning the outcomes of quantum gate applications, while state equations did not aid users in understanding or learning the outcomes of gate applications. This implies that when designing quantum educational applications, researchers should consider focusing on visual enhancements like trajectory visualizations while avoiding mathematical representations accompanying the Bloch sphere.

ACKNOWLEDGMENTS

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Finally, I would like to thank my family and friends for their unwavering love, support and encouragement.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1: INTRODUCTION

1.1 Motivation

Quantum computing is a very promising but notoriously difficult subject. The subject's basis in linear algebra and quantum mechanics makes becoming familiar with quantum computing an intimidating prospect for many. In addition, quantum mechanical concepts such as superposition and entanglement are by nature unintuitive, as they are contradictory to the physical properties of the world around us.

There is a decent variety of tools that are able to simulate quantum computer and quantum circuitry. IBM Quantum Computing [15] and Quirk [11] are two tools that support dragging and dropping gates to build multi-qubit quantum circuits. IBM Quantum Computing also supports coding using the Qiskit [4] software development kit. Microsoft's Quantum Development Kit [2] is a proprietary SDK that uses the Azure Quantum cloud platform to run its quantum language. QuTiP [16] and Quantum++ [10] are libraries that simulate quantum systems, written in Python and C++ respectively. QuEST [17] is a high-performance quantum simulation toolkit that can be used inside C or C++ code for simulating quantum operations as well.

However, there are not many educational tools out there that actually teach quantum computing. Zable [24] created an educational quantum computing program to test the effectiveness of VR as a mode of education. Hello Quantum [1] is a mobile application game that uses a puzzle format to teach quantum gates in an entertaining way for beginners. EntangleVR [8] teaches quantum entanglement through the generation of interdependent scenes. Finally, SpinQ [13] is an inexpensive, portable quantum computer able to simulate two qubits, aimed for use in K-12 and higher education programs. While the current educational solutions are impressive, there is definitely a need for more progress in this area.

1.1.1 Bloch Sphere Representation

The Bloch sphere is a model of a qubit that geometrically represents the pure state space of a qubit as points on a sphere [12]. Bloch spheres are a convenient tool because they convey the same information that a qubit's mathematical state does, but in a more intuitive way. They can also be directly manipulated through rotations.

Unfortunately, there are even fewer examples of quantum computing education tools that employ the Bloch sphere representation to teach quantum information science. In addition, many of these tools rely on math to convey state information and merely use Bloch spheres as a display, such that they are not a primary focus of the application. For example, the IBM Quantum Computing platform uses the Q-sphere, which is a way to visualize n-qubit states (as opposed to the Bloch sphere, which shows a single qubit state), but the rotational effects of applying gates to the Q-sphere are not shown. Quirk utilizes a Bloch sphere display, but again the focus of the software lies on building circuits with different combinations of quantum gates. EntangleVR employs a Bloch sphere representation to help the user understand the state of a scene-building block. The QuTiP library also uses Bloch sphere representation as a display. There are two notable exceptions. ShorVis [22] recognizes the potential of Bloch spheres for making quantum concepts more digestible and teaches them throughout the application. Zable et. al also capitalizes on the Bloch sphere representation more than other quantum simulators for education. That being said, both of these tools still put a significant focus on numerical displays in addition to the Bloch sphere.

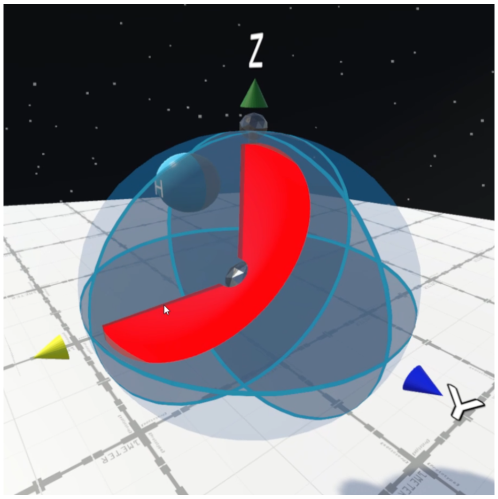


Figure 1.1: Trajectory Visualization (in red) follows the Bloch vector rotation of Hadamard gate

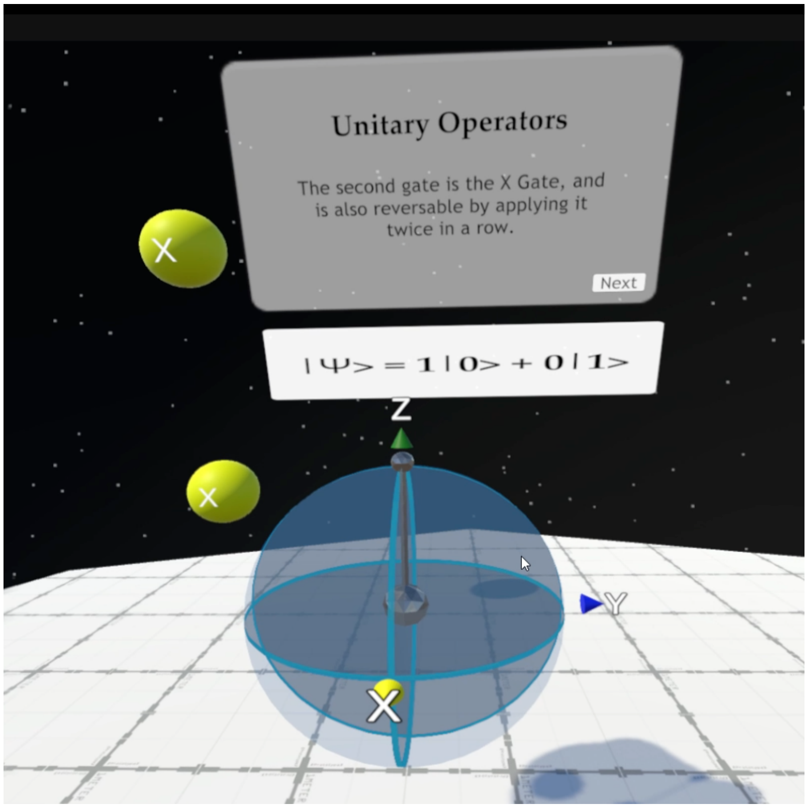
1.1.2 Trajectory Visualizations

A trajectory visualization is the path rendered for the Bloch vector's motion as a gate is applied, as shown in Figure 1.1. It allows a learner to see the rotational path that the vector takes. In QubitVR, the red trajectory visualization lingers until the Bloch vector is done rotating in order to display all the time and positional information of the vector's motion for the applied gate.

A few researchers have investigated trajectory visualizations of quantum gates/operations for scientific visualizations. A few examples of this are the Q-CTRL Visualizer Python package [6] and a Bloch Sphere Animation Software using a Three Dimensional Java Simulator [14]. However, trajectory visualizations for educational purposes have not been explored.

1.1.3 State Equations

A state equation delineates the current state of one or more qubits. For a single qubit, the state equation describes a qubit state as a linear combination of the basis states $|0\rangle$ and $|1\rangle$ (more ex-



Unitary Operators

The second gate is the X Gate, and is also reversable by applying it twice in a row.

$|\Psi> = 1|0> + 0|1>$

Figure 1.2: State Equation (in white box) is displayed above the Bloch sphere and below the prompt

plained in section 3.2.2), as shown in QubitVR in Figure 1.2.

Most educational quantum computing tools use state equations to describe a qubit state. However, state equations may not be necessary to understand concepts in quantum information science and engineering, as discussed in section 3.1.1. In fact, having the state equation present might actually hinder learning ability and lead to cognitive overload.

1.1.4 Virtual Reality

Quantum mechanical concepts such as superposition and entanglement are by nature unintuitive, as they are contradictory to the physical properties of the world around us. For this reason, virtual reality has great potential to help individuals learn and truly understand quantum mechanical concepts that quantum computing relies on. Virtual reality is "an approach that uses displays, tracking, and other technologies to immerse the user in a synthetic, spatial world, often seen from a first-person point of view that is under the real-time control of the user". [18] When a user puts on a VR headset, their visual field is entirely within the virtual learning environment and their movements are corresponded in virtual space. This allows a user to realistically perceive things that are not present in everyday life.

In this way, virtual reality is distinctive from other learning techniques. Instead of viewing two- and three-dimensional objects on a flat viewing mechanism like paper or a computer screen, virtual reality enables a user to view and manipulate objects that are actually in front of them, which allows them to learn through direct, captivating experience.

Despite VR's advantages for teaching quantum computing, only Zable et. al has used VR for quantum computing education thus far.

1.2 Definitions

The following definitions are used throughout the discussion and are provided here for easy reference.

- **Head-mounted display** - "A device worn on the head that provides a small display in front of both eyes or just one eye." [18]

- **Virtual reality** - "An approach that uses displays, tracking, and other technologies to immerse the user in a synthetic, spatial world, often seen from a first-person point of view that is under the real-time control of the user." [18]

- **Quantum computers** - A computer that "uses qubits and quantum operations and solves certain complex computational problems more efficiently than classical computers." [7]

- **Quantum information science** (QIS) - A field that "exploits quantum principles to transform how information is acquired, encoded, manipulated, and applied. Quantum information science encompasses quantum computing, quantum communication, and quantum sensing, and spurs other advances in science and technology." [7]

- **Quantum state** - "A mathematical representation of a physical system, such as an atom, and provides the basis for processing quantum information." [7]

- **Qubit** - The fundamental unit of quantum information in a quantum system.

- **Bloch sphere** - A model of a qubit that geometrically represents its pure state space as points on a sphere. [12].

- **Superposition** - A linear combination of a qubit's basis states $|0\rangle$ and $|1\rangle$, such that the qubit is in both of these states at the same time.

- **Quantum gate** - An operation that is performed on a qubit to modify its state.

- **Entanglement** - "An inseparable relationship between multiple qubits, and a key property of quantum systems necessary for obtaining a quantum advantage in most QIS applications." [7]

- **Cognitive walkthrough** - A usability testing method where evaluators "walk through" a product as if they were a user in the target audience and answer questions, determining how usable it the product is [5]. This paper uses a modified cognitive walkthrough to determine how effectively learning takes place instead.

- **Multi-variate testing** - A method of running a controlled experiment in which multiple factors each with multiple conditions are tested. [19]

1.3 Problem Statement

While there are various quantum circuit simulators and libraries, there are very few educational applications that teach concepts of quantum information science. There are even fewer that put an emphasis on teaching the Bloch sphere representation. Furthermore, there has been little research conducted to investigate the usefulness of Bloch sphere representations, particularly in VR. Finally, there has been no work to evaluate the usefulness of these elements for educational purposes, particularly with regard to state equations and trajectory visualizations.

1.4 Research Questions and Methodologies

To advance knowledge in these areas, we have created QubitVR, a virtual reality application that teaches principles of quantum computing. QubitVR shows learners how to read and interact with a Bloch sphere. They learn about quantum superposition, measurement, and gates using one qubit. They also learn about interactions between qubits using a two-qubit gate and quantum entanglement. QubitVR has a total of four different content sections, the first two being primarily focused on single-qubit concepts and the second on two-qubit concepts. The development and educational content of QubitVR will be discussed more in chapter 3.

In addition, we assessed the learning capability of trajectory visualizations and state equations in

QubitVR to explore the use of visual and mathematical enhancements on the Bloch sphere. During the development processes of QubitVR, we added state equations to investigate their usefulness in aiding user comprehension of quantum states. Using QubitVR and a multivariate cognitive walkthrough methodology with subject matter experts, we have addressed the following research questions in sections 1.4.1 and 1.4.2.

Here, we define understand as have a comprehension of how a quantum state is changing. If a user understands the result of applying a gate, they will know why the Bloch vector moves in a certain way after a gate is applied. If a user has learned, they have a more advanced understanding to where they can reproduce the action. If a user has learned, they will have a reasonable degree of confidence applying gates to reach a certain target state.

1.4.1 Trajectory Visualizations

RQ1. Do trajectory visualizations improve the learner's ability to understand the result of applying a gate?

H1. Yes, trajectory visualizations will improve the learner's ability to understand the result of applying a gate. Most quantum learning simulations that use qubits change the Bloch vector state instantly, such that it "snaps" to the new state. Trajectory visualizations, on the other hand, highlight the path that the Bloch vector is taking during a gate application, which can help a learner understand how the vector got to its final state. In QubitVR, trajectory visualizations allow a learner to see the entire rotational path that the vector took at a glance, up until the gate is done being applied. This offers additional information that a simple rotation of the Bloch vector does not. Trajectory visualizations might be cause slight cognitive overload, but the additional emphasis on the path will most likely be beneficial enough to outweigh that risk.

RQ2. Do trajectory visualizations improve the learner's ability to learn the result of applying a gate?

H2. Yes, trajectory visualizations will improve the learner's ability to learn the result of applying a gate. Because of the emphasized path visualization, the rotational path that each gate takes might be remembered better and can help a learner visualize sequences of gate applications. This in turn will increase the chance that a learner will be able to correctly apply a sequence of gates to get a desired final state.

1.4.2 State Equations

RQ3. Do state equations improve the learner's ability to understand the result of applying a gate?

H3. No, state equations will not improve the learner's ability to understand the result of applying a gate. Most educational quantum computing applications use state equations, so there is a possibility that a state equation will be helpful. However, we believe the mathematical notation will likely confuse the learner unless they are already well-versed in how changes in the state equation correspond to rotations in the Bloch sphere. Also, state equations can lead to cognitive overload, because the learner must focus on both a change in rotation and in the equation.

RQ4. Do state equations improve the learner's ability to learn the result of applying a gate?

H4. No, state equations will not improve the learner's ability to learn the result of applying a gate. It could be argued that the learner might develop an intuition of how the state equation will change after some practice. However, without a proper introduction to the linear algebra concepts that go into the formulation of a state equation, we predict that a learner will not be able to draw any useful conclusions from watching the state equation change. A proper explanation of the state equation could be difficult if the learner does not have the necessary prerequisite knowledge to understand linear algebra, while rotations of the Bloch sphere does not require this. In addition, we believe it is more likely that state equations will lead to cognitive overload because they are distracted by a complicated-looking equation near the qubit. This could cause a learner to give up, ceasing learning altogether. This is especially true if the state equation is not directly referenced or explained, which could make learners feel that they are incapable of understanding quantum computing concepts because the learning curve is simply too steep. Finally, a learner still has a complete knowledge of quantum gates without the state equation representation. [7]

1.5 Contributions

1. We have created QubitVR, an educational quantum computing application in virtual reality that utilizes Bloch sphere representations.
2. We have assessed the educational potential of state equations.
3. We have assessed the educational potential of trajectory visualizations.

1.6 Thesis Overview

This thesis has five chapters and two appendices.
Chapter 2 will go through the related works mentioned in the introduction and some others.
Chapter 3 will talk about QubitVR and the mathematical concepts behind the implementation.
Chapter 4 will discuss how the cognitive walkthrough was conducted and the results.
Chapter 5 will summarize our findings.
Appendix A provides some diagrams that outline QubitVR's components.
Appendix B provides a transcript of the subject matter experts feedback and answers to the questions asked during the cognitive walkthrough. Appendix C provides the Not Human Research

Determination by the Institutional Review Board.

CHAPTER 2: LITERATURE REVIEW

2.1 Overview

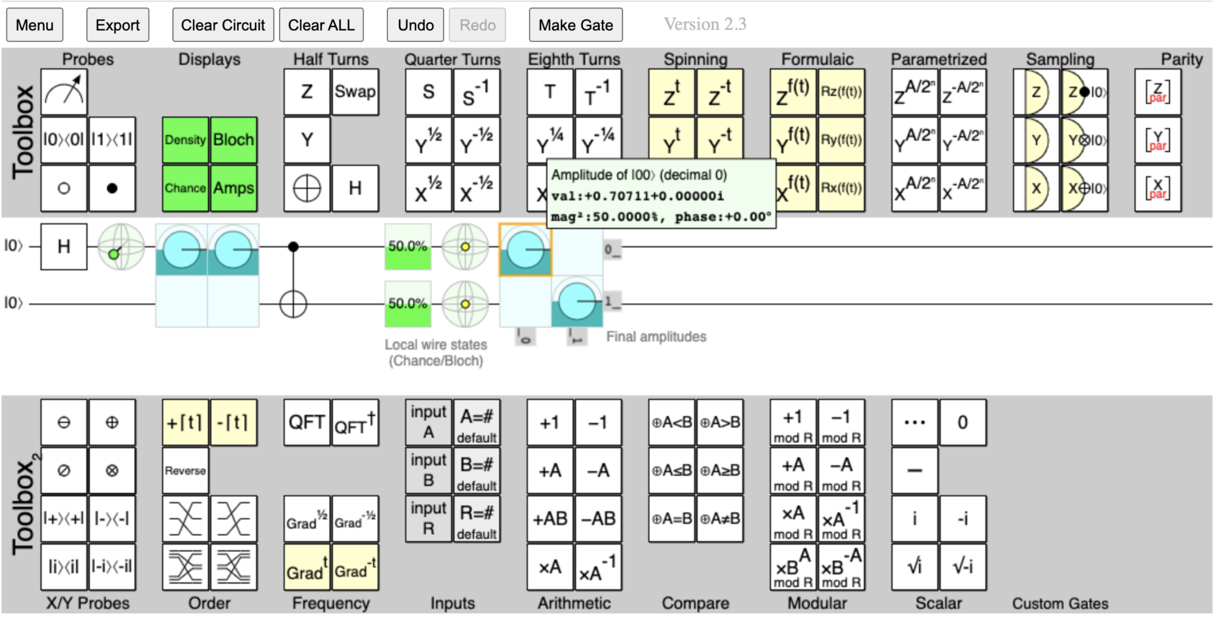
This chapter will elaborate more on the main technologies described in the introduction that are relevant to QubitVR and the multivariate cognitive walkthrough.

2.2 Quantum Computing Learning Applications

The following section will present different educational quantum computing applications that teach different aspects of quantum computing.

2.2.1 Quirk

Quirk describes itself as "a drag-and-drop quantum circuit simulator, great for manipulating and exploring small quantum circuits" [11]. Quirk is a very useful tool for building quantum circuits, and was actually used frequently in the development of QubitVR. Quirk allows a user to place different quantum gates onto a circuit and observe changes to qubit states after doing so. Two useful features of Quirk include different probe tools which allow for multi-qubit gates, and display tools which allows a user to drop and observe a system's state at different points in the circuit. Both of these can be seen in Figure 2.1. One of these display tools is as a Bloch sphere display which provides a visual representation of the Bloch vector and a coordinate representation in terms of (r, θ, and ϕ) and (x, y, z). Another useful display tool is the state matrix of the quantum circuit, which shows the quantum amplitude of each qubit basis state in matrix form.



Amplitude of |00⟩ (decimal 0)
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Figure 2.1: Quirk

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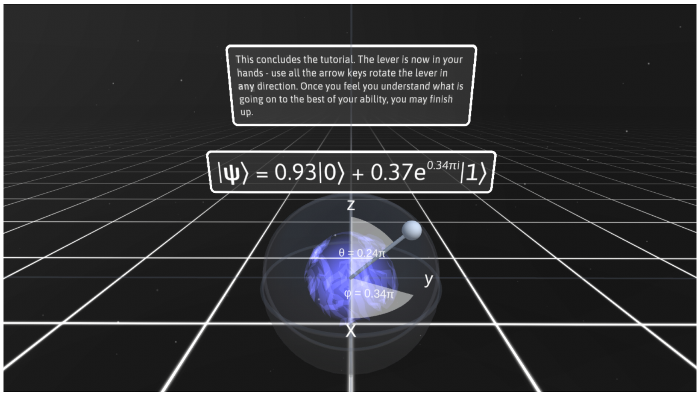
For QubitVR, Quirk was very useful in developing the quantum gates modules. Prior to the development of QubitVR, this tool was helpful for learning about the movement of the Bloch vector and the effect of different sequences of gate applications. While QubitVR was in development, Quirk allowed us to to verify the correctness of our own application after sequences of gate applications. For a person with some background knowledge of quantum computing concepts, Quirk is an excellent resource, as it provides a broad range of functionality for learning about different concepts. In addition, it does not require the user to have a linear algebra background. However, for someone with little to no knowledge of quantum computing, this tool has an arguably steep learning curve and could be confusing.



This concludes the tutorial. The lever is now in your hands - use all the arrow keys rotate the lever in any direction. Once you feel you understand what is going on to the best of your ability, you may finish up.

Figure 2.2: Bloch Sphere Representation

2.2.2 *Investigating Immersive Virtual Reality as an Educational Tool for Quantum Computing*

Investigating Immersive Virtual Reality as an Educational Tool for Quantum Computing and QubitVR explore very similar topics. Zabel et al. compared a VR and desktop quantum simulator, and aimed to see if there was a difference in learning outcomes between the two. In addition, the authors also observed whether or not the mathematical background of the participants had any impact on their performance. The only difference between each simulation was that desktop version did not have a stereoscopic display and navigation through the virtual environment was controlled by the mouse and keyboard [24]. To measure performance, the authors developed their own knowledge test, consisting of five questions about single-qubit systems and five questions about multi-qubit systems.

The Bloch sphere representation can be seen below. The Bloch sphere is superimposed over the blueish orb that represents the qubit. The state equation is included above the qubit to facilitate some kind of understanding of the relationship between a visual change in state and a change in the state equation. For multi-qubit states, more qubits were added and the state reflected that of a multi-qubit system.

2.2.3 Bloch Sphere Explorer

The Bloch Sphere Explorer is an application that simulates the process of applying quantum gates to a qubit. It was developed using Unity in IBM's Emerging Technology Lab [20]. The state appears in the top left corner of the application. A user is able to set the state to one of six states in the bottom left hand corner. They are also able to "roll" around a qubit represented by a Bloch sphere to collide into blocks representing different quantum gates around the environment. When the Bloch sphere intersects with one of these blocks, the quantum gate is applied to it, and the Bloch vector updates to reflect its new state.

2.3 Cognitive Walkthrough for Education Technologies

This section focuses on other cognitive walkthroughs that test educational technologies.

2.3.1 Cognitive Walkthrough for Learning Through Game Mechanics

Cognitive Walkthrough for Learning Through Game Mechanics puts forth a method of evaluating educational tools through the use of a cognitive walkthrough, then uses this method to evaluate a learning game [9]. The Cognitive Walkthrough for Learning Through Game Mechanics is a modification of the classical cognitive walkthrough, such that instead of putting the focus on whether or not a user will be able to use a system, it is whether or not a learner will be able to learn from a system. The framework used in this study was modified and used in our own research.

Farrell and Moffat argue that the motivation for creating a special kind of cognitive walkthrough to evaluate game-based learning is the lack of consistent standards for teaching with educational games. They elaborate that game designers often try and use their best judgement, but the reliance on intuition for game design creates a lot of risk and variability in the quality of games. A cognitive walkthrough can help to bring any assumptions about learner's knowledge and personal biases to light, which can then be used to evaluate why certain teaching methods in a game are effective or ineffective.

The CWLTGM method was used later on to evaluate the e-Bug Platform game. In summary, there were two sections, an effective one and an ineffective one, that had a very similar design. Through the use of CWLTGM, the researchers were able to ascertain that the ineffective section made unreasonable jumps in the logical steps that it took to learn a certain concept, while the effective section made more realistic logical jumps.

CHAPTER 3: QUBITVR

3.1 Overview

As stated previously, QubitVR is currently broken down into two modules. The first module covers single-qubit concepts and the second module teaches two-qubit concepts. QubitVR uses Bloch spheres to represent the state space of a qubit. In order to discuss how the Bloch sphere representation was implemented throughout QubitVR, more mathematically-intensive explanations of qubit states and operators will be given. While the level of detail this chapter goes into is not representative of what is taught during the modules, it is necessary to explain how QubitVR was created.

3.1.1 Thought Process Behind Educational Content Design

The design choices made during QubitVR's development were largely a function of the National Science Foundation's workshop outlining the most important concepts that one should know to gain a holistic, conceptual understanding of quantum computing. These concepts are

1. Quantum information science
2. Quantum states
3. Measurement
4. Quantum bit, or qubits
5. Entanglement
6. Coherence of quantum states
7. Quantum computers
8. Quantum communication

9. Quantum sensing

QubitVR mainly focuses on the first five principles, although it might be expanded to all 9 in the future.

The workshop has a noticeable lack of mention of linear algebra or mathematical concepts. This reinforces that a background of linear algebra or physics does not have to be a requirement for a conceptual understanding of quantum computing, in the same way that one does not need a mathematical background to conceptually understand kinematics. The linear algebra described here is mentioned to describe how QubitVR was developed, as it was required to code the back-end of the models.

3.2 Module 1: Single-Qubit Concepts

3.2.1 Introduction

The first module is divided into two sections. The first section covers quantum superposition and measurement and the second section goes over quantum logic gates. Since we are going more in depth than the educational content of QubitVR does, quantum states and the Bloch sphere representation of a qubit will be described first so that the implementation details of the modules are given a proper introduction.

3.2.2 The State of a Qubit

The fundamental unit of information in a classical computers is a bit. Combinations of bits are used to store the state of a computer at a given time. In a quantum computer, the fundamental unit of information is a **qubit**. The value of bits are controlled by transistors in classical computers, while qubits are represented by particles small enough to be subject to the laws of quantum physics. Typically, these tiny particles are electrons or photons [23]. Qubits are similar to binary bits in the sense that they will only ever output a binary value of 0 or 1 after being measured [3]. However, before they are measured, they can also exist in both of these states at the same time.

The two basis states for a single qubit are $|0\rangle$, or up, and $|1\rangle$, or down. A single qubit can be represented by a 2×1 matrix, which represents the linear combination of the states $|0\rangle$ and $|1\rangle$.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \; |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3.1}$$

The $|$ and $\rangle$ enclosing the state is **Dirac's notation** for column vectors, which is used to represent quantum states. A qubit system with n qubits can be demonstrated by the matrix representation of its linear combination i.e. a column vector with 2^n elements. For example, the basis states for a 2-qubit system are the following column vectors.

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \; |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{3.2}$$

These 2^n column vectors are used in QubitVR's quantum engine to represent a n-qubit system. Multi-qubit system states will be discussed more in section 3.3.2.

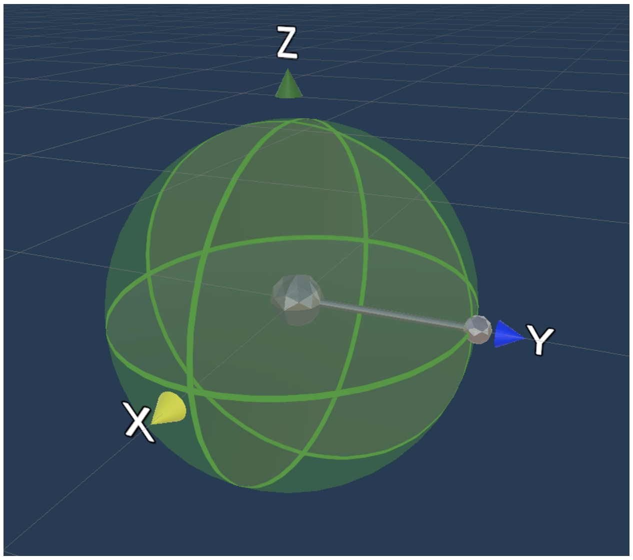


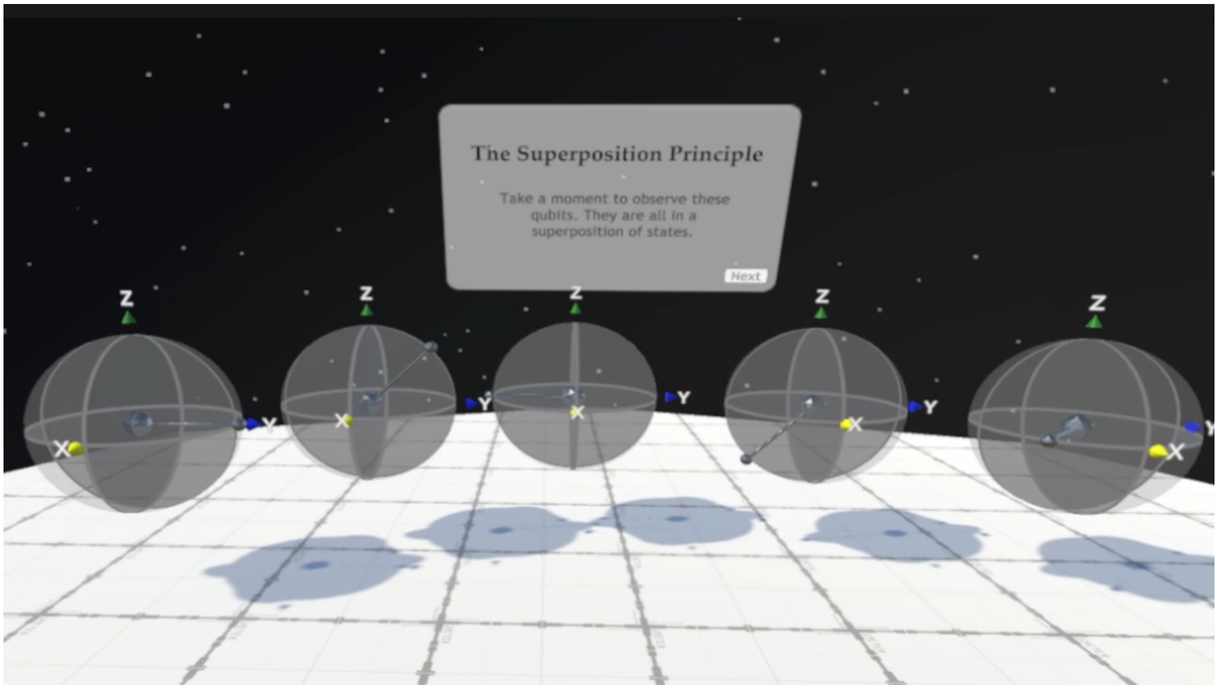
Figure 3.1: Qubit in QubitVR

3.2.3 Qubit Design

The design of the qubit took several iterations. The qubit is designed to model a Bloch sphere. The main parts of the qubit are the spherical shape and the vector inside. The only interactive part of the qubit is its state vector, the other visual elements being present to aid in the readability of the state. There are three planes inside the qubit that cut it into three portions along which the majority of states are aligned, an plane xy-, xz-, and yz-plane. The x-, y-, and z-axes are marked as well.

3.2.4 Section 1: Superposition and Measurement

This section discusses both the superposition principle and the measurement principle for qubits.



The Superposition Principle

Take a moment to observe these qubits. They are all in a superposition of states.

Figure 3.2: Qubits in Superposition in QubitVR

3.2.4.1 The Superposition Principle

The **superposition principle** is very important in quantum information science, as it is one of the properties that gives quantum computers their computational prowess. A single qubit can be in three different combinations of the basis states $|0\rangle$ and $|1\rangle$— $|0\rangle$, $|1\rangle$, or both $|0\rangle$ and $|1\rangle$. The last case is a linear combination of both its basis states $|0\rangle$ and $|1\rangle$, which is called a **superposition**. Figure 3.2 shows some examples of Bloch sphere representation of qubits in different superpositions in QubitVR.

The state of a quantum bit can be represented with the formula

$$|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \text{ such that } \alpha, \beta \in \mathbb{C} \text{ and } |\alpha|^2 + |\beta|^2 = 1 \tag{3.3}$$

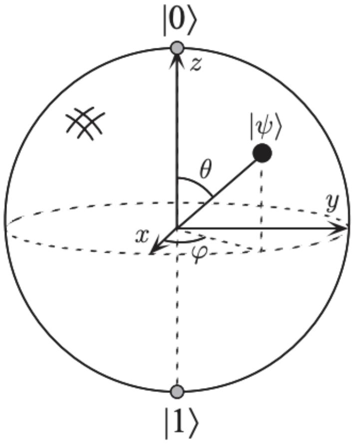


Figure 3.3: Bloch Sphere [21]

3.2.4.2 Bloch Sphere

All the possible states of a qubit, or a quantum bit, can be displayed on a **Bloch sphere**, as in Figure 3.3. All unentangled qubit states can actually be represented on the surface of a Bloch sphere. Entangled states will discussed more in section 3.3.5.

In QubitVR, the Bloch sphere model is used to visually represent the state of a qubit. In order to represent the state as a point on the surface of a Bloch sphere, we must translate $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ from the Cartesian coordinate system to the spherical coordinate system, which will be discussed in the following few paragraphs.

Since α and β are complex numbers (and therefore have imaginary components), they can be represented in Cartesian coordinates as $a + bi$ or polar coordinates as $re^{i\phi}$. Using the latter, we substitute $\alpha = re^{i\phi_\alpha}$ and $\beta = re^{i\phi_\beta}$ into the formula $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$, and we get $|\Psi\rangle = r_\alpha e^{i\phi_\alpha}|0\rangle + r_\beta e^{i\phi_\beta}|1\rangle$.

Since the global phase $e^{i\phi_\alpha}$ can be dropped from consideration, we can factor it out such that

$|\Psi\rangle = e^{i\phi_\alpha}(r_\alpha|0\rangle + r_\beta e^{i(\phi_\beta-\phi_\alpha)}|1\rangle)$. After dropping it, we get $|\Psi\rangle = r_\alpha|0\rangle + r_\beta e^{i(\phi_\beta-\phi_\alpha)}|1\rangle$.

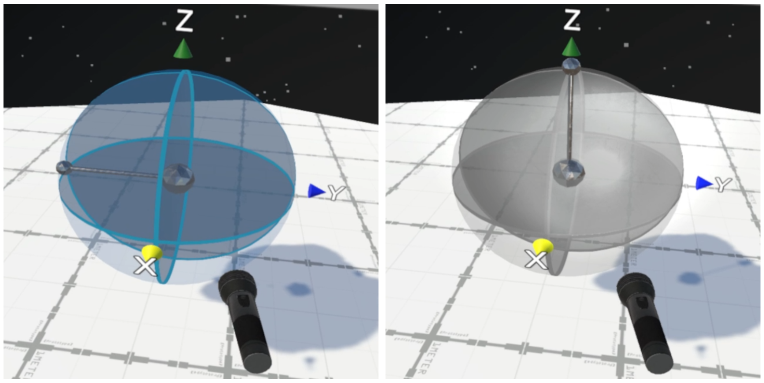
Since $r_{\alpha,\beta}$ are the the absolute values of the amplitudes of α and β, it follows that $r_\alpha, r_\beta \in [0,1]$. Also, since $r_{\alpha,\beta}$ describe the probability to measure in their corresponding basis states, and the state will definitely be in one of the two basis states, we can say $r_\alpha^2 + r_\beta^2 = 1$. Using the Pythagorean formula, we can represent the dependence between r_α and r_β on a unit circle, which can be used to write $r_\alpha = cos\frac{\theta}{2}$ and $r_\beta = cos\frac{\theta}{2}$, where $\theta \in [0,\pi]$. Using these new formulas, we can write $|\Psi\rangle = r_\alpha|0\rangle + r_\beta e^{i(\phi_\beta-\phi_\alpha)}|1\rangle$ as

$$|\Psi\rangle = cos\frac{\theta}{2}|0\rangle + sin\frac{\theta}{2}e^{i(\phi)}|1\rangle, \text{where } \phi \in [0,2\pi) \text{ and } \theta \in [0,\pi] \tag{3.4}$$

Looking at these value ranges, we can deduce that these angles can be used to determine the quantum state's location on a unit sphere, which means that we can use spherical coordinates to visualize a qubit.

3.2.4.3 The Measurement Principle

When a quantum system is measured, the state superposition of all qubits in the system collapse into either the up or down states. The **measurement principle** refers to the fact that one can't measure these superpositions without collapsing them. In other words, the state of a quantum computer can't be known until it is measured, at which point it is only possible to observe the measured outcomes. The total probability of a qubit must be equal to 1, which is why $|\alpha|^2 + |\beta|^2 = 1$. This is consistent with the fact that the sum of any set of probabilities must be equal to 1. Recall the state equation from above $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. α and β can be thought of as the "amplitude" of each basis state. Since the total probability here is equal to one, $1 = |\alpha|^2 + |\beta|^2$, where α corresponds to the probability of measuring $|0\rangle$ (up) and β corresponds to the probability



(a) Before Measurement (b) After Measurement

Figure 3.4: Measurement in QubitVR

of measuring $|1\rangle$ (down). In QubitVR, to measure a qubit, a random number is generated and compared to α^2. If the random number is less than α^2, the qubit measures up, and if it is greater than or equal to this probability, it measures down.

3.2.5 Section 2: Single-Qubit Gates

Quantum gates are operations that change the state of one or more quantum bits. In this section, our focus will be on single-qubit gates. The magic of the Bloch sphere model becomes clear when applying qubit gates, as matrix multiplications on paper come to life in through rotations of the Bloch sphere. Again, since QubitVR relies on a Bloch sphere to convey quantum state information to the user, the matrix representation of gates are left out. However, it was more advantageous for us to use matrix representations on the backend.

QubitVR teaches four different single-qubit gates and one two-qubit gate. The identity gate is used as well to perform different operations on the backend. The four single-qubit gates that are taught are the NOT gate, the Hadamard gate, and the S and T phase-shift gates. There are many more quantum gates, but these were chosen as the fundamentals because it is possible to get into any state on the Bloch sphere using a combination of the NOT, H, and T gates. Because of this, NOT, H, and T are a universal quantum gate set. While the S gate is equivalent to two T gates, and is therefore not a part of a universal quantum gate set, it is introduced to simplify some quantum circuits. [21]

Gates can be visually represented as a n° rotation around one or more axes, or mathematically through matrix multiplication. In the following sections, both the visual representation and the mathematical representations will be described. The learner is only exposed to the visual rotation of a qubit state, but the mathematical representation will be included to provide insight into how the quantum backend represents these operations. Note that both are correct and are expressing the same thing, as we saw the connection between the two in section 3.2.4.2.

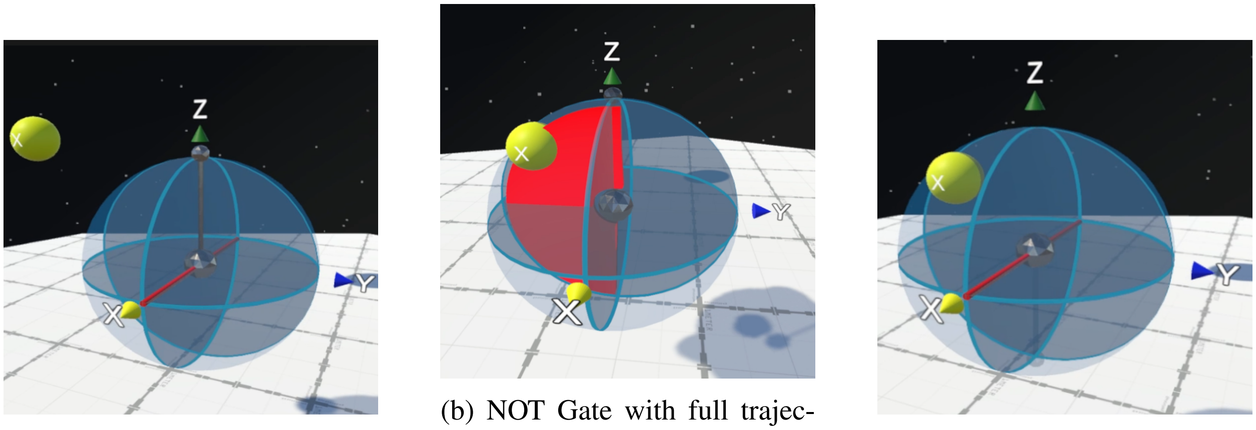
3.2.5.1 The NOT gate

The **NOT gate**, also known as the Pauli-X gate, corresponds to a rotation of π radians around the x-axis of a qubit, shown in Figure 3.5. This gate maps the basis states $|0\rangle$ to $|1\rangle$ and $|1\rangle$ to $|0\rangle$. The NOT gate can be represented by the matrix

$$\text{NOT} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

3.2.5.2 The Hadamard gate

The **Hadamard gate**, or H gate, corresponds to a rotation of π radians around the $\frac{(x+z)}{\sqrt{2}}$ axis of a qubit. The H gate can be represented by the matrix



(a) Before NOT gate application (b) NOT Gate with full trajectory visualized (c) After NOT Gate application

Figure 3.5: NOT Gate in QubitVR

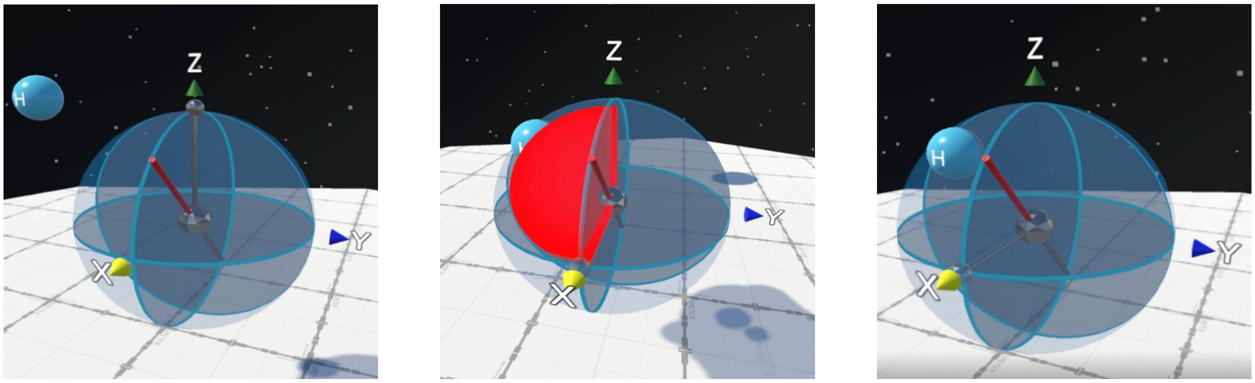
$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

This gate is the the only gate in our universal gate set that puts a vector in one of the basis states ($|0\rangle$ and $|1\rangle$) into superposition, allowing for a diverse range of states. This gate is also very important for quantum entanglement, which will be discussed in section 3.3.5.

3.2.5.3 The Phase Shift gates

The last two gates in our set are the S and T gates. These are phase shift gates, which rotate the Bloch vector along the z-axis. In general, phase shift gates can be represented by the matrix

$$P(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$



(a) Before Hadamard gate application

(b) Hadamard Gate with full trajectory visualized

(c) After Hadamard gate application

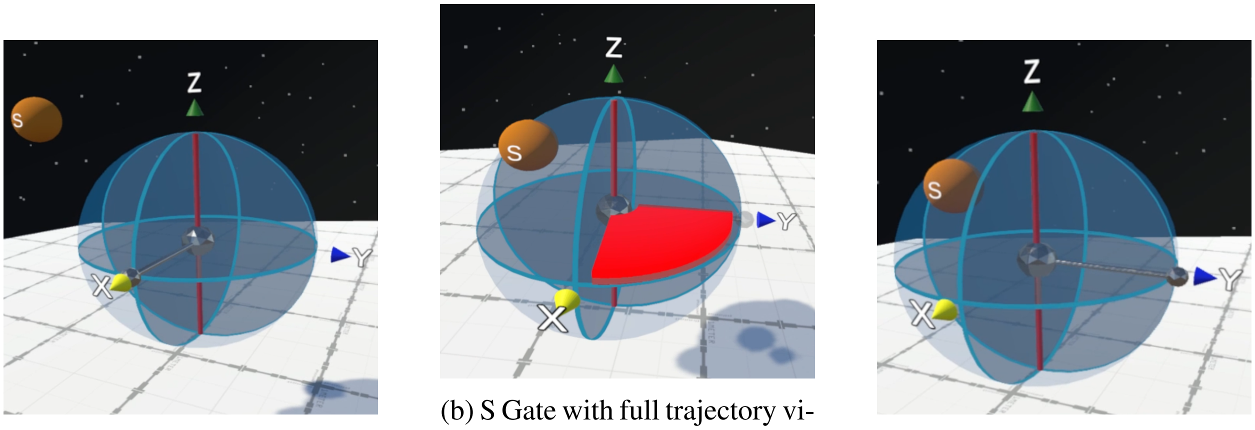
Figure 3.6: Hadamard Gate in QubitVR

where ϕ represents phase shift with a period (the distance a function takes to repeat itself) of 2π.

The **S gate**, or $\frac{\pi}{2}$ phase shift gate, corresponds to a rotation of $\frac{\pi}{2}$ radians along the z-axis. By plugging in the phase shift of $\frac{\pi}{2}$ into the phase shift gate matrix and simplifying with Euler's identity, we can obtain the following S gate matrix representation

$$\mathrm{S} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

The **T gate**, or $\frac{\pi}{4}$ phase shift gate, corresponds to a rotation of $\frac{\pi}{4}$ radians along the z-axis. The T gate is actually the square root of an S gate, so the S gate is not necessary for a universal quantum gate set. However, there are many cases where using a single S gate is more practical than applying two T gates. Since the phase shift of the T gate is $\frac{\pi}{4}$, the T gate can be represented by the matrix

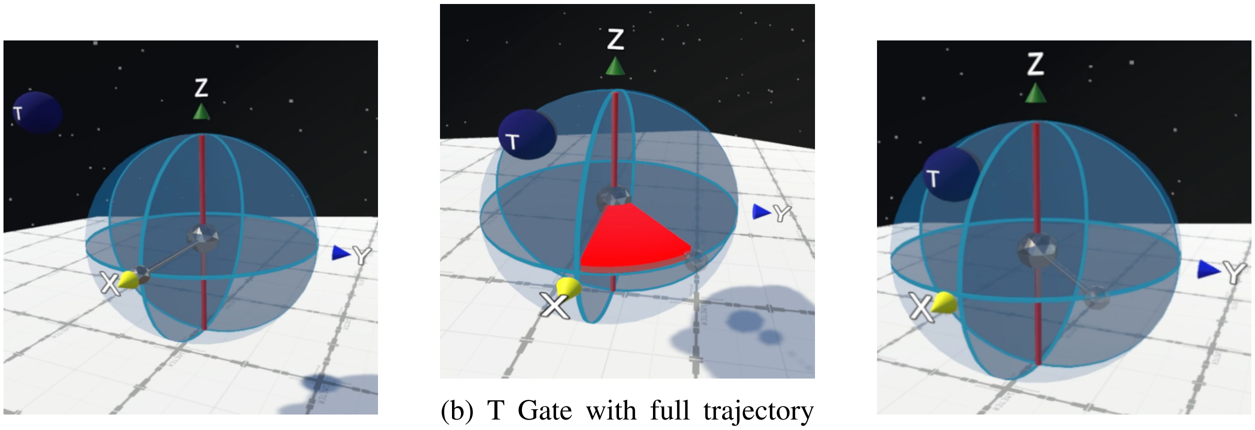


(a) Before S gate application

(b) S Gate with full trajectory visualized

(c) After S gate application

Figure 3.7: S Gate in QubitVR



(a) Before T gate application

(b) T Gate with full trajectory visualized

(c) After T gate application

Figure 3.8: T Gate in QubitVR

$$\mathrm{T} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \end{pmatrix}$$

3.2.5.4 The Identity gate

The **identity gate** does not change the qubit state, so it does not rotate the Bloch vector. While this gate is not introduced in the modules, it comes in handy when we are applying single or two-qubit gates to multi-qubit systems. By multiplying the identity matrix with other gates, we can expand these gates without changing its value, making it possible to multiply the resulting matrix to the system of n qubits. This will be covered more in the next section. The identity gate can be represented by the matrix

$$\mathrm{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

3.3 Module 2: Two-Qubit Concepts

3.3.1 Introduction

Within this module there are two sections, the CNOT gate and quantum entanglement. Like the previous section, a few concepts will be introduced to provide a background for these sections and how they were implemented.

3.3.2 Composite Qubit Systems

A **composite qubit system**'s state matrix is made up of qubits 0 to $n-1$, in the same way that a system of bits is made up of bits 0 to $n-1$. The combination of multiple qubits in superposition can represent exponentially more states than combinations of bits in a classical computer can. In this section, we discuss how QubitVR represents all the individual state matrices of qubits that make up a qubit system as a single system state matrix.

As introduced in the previous section, a qubit system can be represented by a $2^n \times 1$ matrix. When there are two qubits i.e. when $n = 2$, the size of the matrix is 4×1. This is because there are four different basis states, which are $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$. The state equation for a two-qubit system is $\psi = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$, which is equivalent to the matrix

$$|q\rangle = q_{00}|00\rangle + q_{01}|01\rangle + q_{10}|10\rangle + q_{11}|11\rangle = \begin{pmatrix} q_{00} \\ q_{01} \\ q_{10} \\ q_{11} \end{pmatrix} \tag{3.5}$$

Even though QubitVR has the potential to represent a large multi-qubit system, since Module 2's learning content currently only covers two-qubit systems, Equation 3.5 is all we'll need for now.

Qubits can be meaningfully represented in their individual states until two or more qubits are entangled with each other. In a system that does not have any entangled qubits, this matrix is the Kronecker product of all the individual states of the qubits in order. Once qubits have been entangled, this state matrix can no longer be separated out into individual qubits, as some of the qubits states cannot be differentiated from each other. For example, two qubits with unentangled states

$$q_0 = |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, q_1 = |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

can be represented by taking the Kronecker product like so

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Figure 3.9: Example of Single-Qubit Gate

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Figure 3.10: Example of Two-Qubit Gate

$$q_0 \otimes q_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

3.3.3 Expanding Single-Qubit gates using Kronecker Product

A quantum gate acting on n qubits is represented by a $2^n \times 2^n$ matrix. For example, the size of a matrix for a quantum gate acting on one qubit is 2×2, as shown in Figure 3.9, and the size of a gate acting on two qubits is 4×4, as shown in Figure 3.10

Even though single qubit gates can only act on one qubit, they can still be applied to a composite, or multi-qubit system. In order to do so, the gate's matrix has to be expanded to be big enough to be multiplied with the qubit system's state matrix. This is consistent mathematically, as one cannot multiply a 2×2 single-qubit gate matrix with a two qubit system's 4×1 state.

The Kronecker product can also be used to expand single-qubit gates in order to work on a multi-qubit system. For example, the matrix for H gate

$$CNOT\,|01\rangle = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

Figure 3.11: CNOT gate where control qubit is $|0\rangle$

$$CNOT\,|10\rangle = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Figure 3.12: CNOT gate where control qubit is $|1\rangle$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$$

cannot be multiplied with the state matrix in equation 3.5. To expand this matrix using the Kronecker product, take note of the indices of the qubit positions. First, take the Kronecker product of an identity matrix for each index of the non-affected qubits before the target qubit. Then, take the Kronecker product of the resulting matrix and the single-qubit gate matrix. Finally, take the Kronecker product of this resulting matrix and an identity matrix for each index of the non-affected qubits after the target qubit. The generalized formula for expanding a gate like this is shown in equation 3.6.

$$New\ Gate = \bigotimes_{i=0}^{j} I \otimes Gate_j \otimes \bigotimes_{i=j+1}^{n-1} I \tag{3.6}$$

3.3.4 Section 1: Controlled NOT Gate

The controlled NOT gate, or the **CNOT gate**, is a type of controlled gate that can entangle two qubits. In a CNOT gate, one qubit acts as the "control" and the other as the "target" that is affected by the gate. If the control qubit is $|0\rangle$, or off, the target qubit will not be affected by the gate. If the state of the control qubit is $|1\rangle$, or on, a NOT gate will be applied to the target qubit. When the control qubit is in superposition, applying a CNOT gate entangles the control and target qubits. This will be discussed further in section 3.3.5.

Since the composite qubit system can have n qubits, the CNOT gate must be able to expand past its 4×4 size. In addition, since one must pick the control and target qubits, there has to be a generalized way to create a CNOT gate matrix on the fly. The general formula for a CNOT gate is

$$CNOT = \pi_{up} \otimes I + \pi_{down} \otimes NOT, \text{ where } \pi_{up} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } \pi_{down} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.7}$$

The indices of the control and target qubit are used to generate a CNOT matrix, while the parts of the matrix that would affect inapplicable qubits are taken care of by taking the Kronecker product of the identity matrix where they would be affected, as in equation 3.6

One important example of a CNOT gate application is on a two-qubit system where the first qubit is pointing up and the second qubit is pointing down. For the CNOT gate, let's have the control qubit be the first qubit and the target be the second one. To build this matrix, use the formula to get

$$\begin{aligned} CNOT_{control=0,target=1} &= \pi_{up} \otimes I + \pi_{down} \otimes NOT \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{aligned} \tag{3.8}$$

In Figure 3.11, the result of multiplying the final matrix in equation 3.8 and the state $|01\rangle$ is shown. Since the control qubit is in the state $|0\rangle$, there is no change to the Bloch state and the state remains $|01\rangle$. In Figure 3.12, the result of multiplying the same matrix with the state $|10\rangle$ is shown. Since the control qubit is in the state $|1\rangle$, the state actually changes and becomes $|11\rangle$. Figure 3.13 shows an example of this in QubitVR.

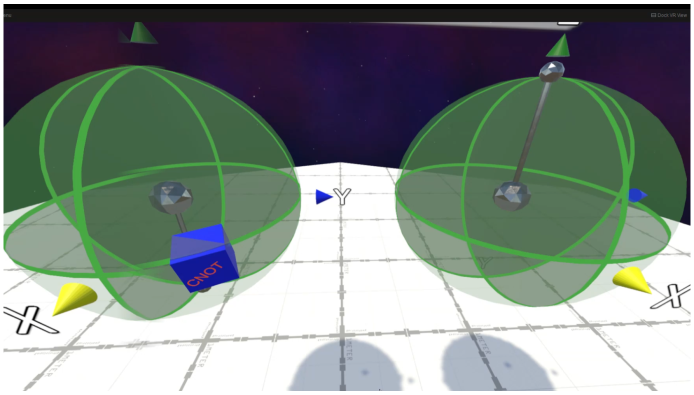
3.3.5 Section 2: Quantum Entanglement

A system of unentangled qubits can be represented by each qubit's own state matrix. However, once two or more qubits have been entangled, their measurement outcomes become intertwined, and their states no longer have any concrete meaning separately. In fact, the degree to which one can gather information from the Bloch vectors of entangled states is actually shown through the length of the Bloch vector. A Bloch vector partially shrinks when two qubits are entangled, and goes all the way to zero when maximally entangled. When two qubits are unentangled, the Bloch vector's length is one. An example of this in QubitVR can be seen in Figure 3.14.

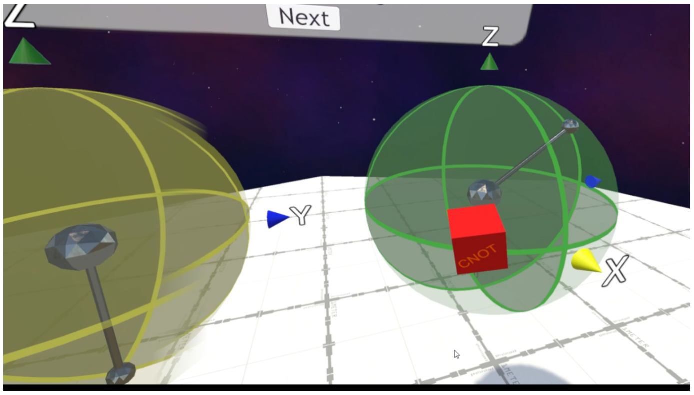
An important part of quantum gates is that they are unitary, meaning that they can be reversed by applying the mathematical inverse of the gate matrix. This inverse is also the complex conjugate transpose of the matrix. For the case of a CNOT, it is its own inverse. Therefore, a qubit can be unentangled by applying the CNOT gate to the same control and target qubits.

3.3.6 Gate Rotation

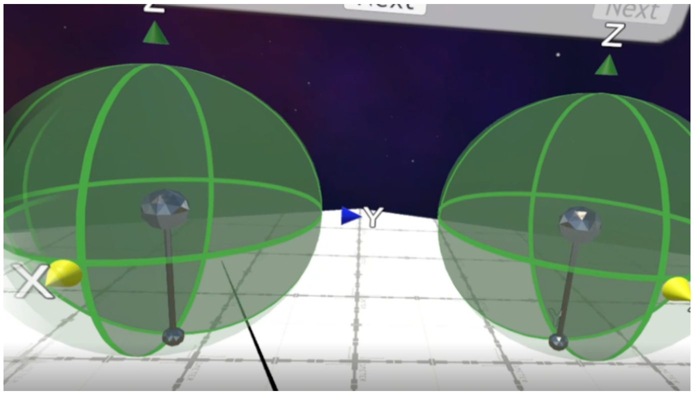
The rotation of a qubit can be determined by breaking up the gate matrix into smaller steps, by taking the nth root of the gate matrix. To achieve a smooth rotation, the n was set to 100, so the vector moves in relatively small increments at each step.



(a) Control portion of CNOT gate applied to qubit with state $|1\rangle$

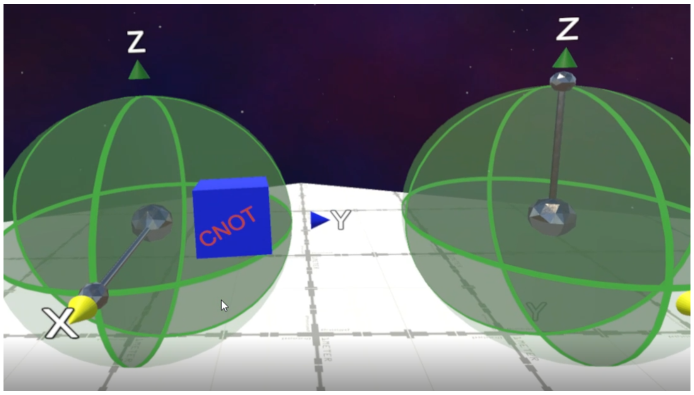


(b) NOT portion of CNOT gate being applied to second qubit with state $|0\rangle$, vector is rotating downwards to state $|1\rangle$

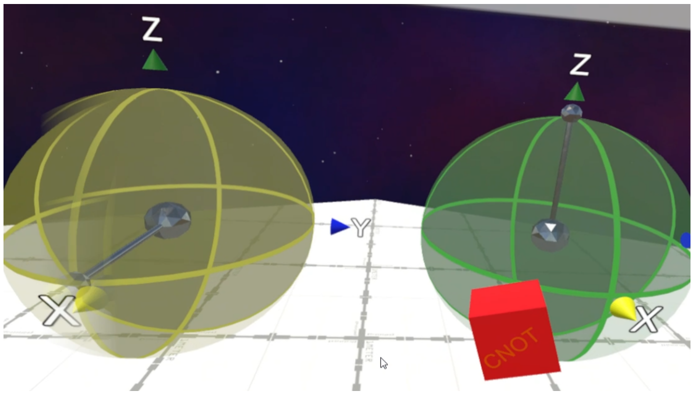


(c) Qubit system now in state $|11\rangle$

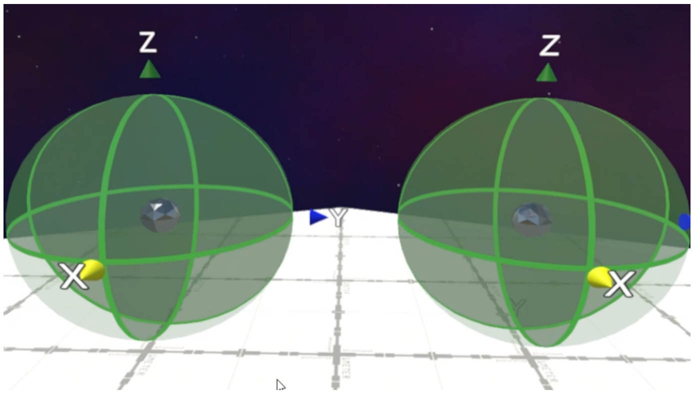
Figure 3.13: CNOT gate application in a system with initial state of $|10\rangle$, final state of $|11\rangle$



(a) Control Portion of CNOT Gate Applied To Qubit in $|+\rangle$ State



(b) NOT Portion of CNOT Gate Applied



(c) Two-Qubit System Becomes Entangled

Figure 3.14: Using a CNOT gate to entangle two qubits in QubitVR

The formula for the rotation is $R(\theta) = e^{-i\frac{\theta}{2}} e^{i\theta\sigma}$, where σ represents the gate that is being rotated. Since

$$e^{i\theta\sigma} = \sum_{n=0}^{\infty} \frac{i^n\theta^n\sigma^n}{n!} = \sigma^2(1 - \frac{\theta^2}{2} + \frac{\theta^4}{4} - ...) + i\sigma(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - ...) = cos(\theta) + i\sigma \sin(\theta),$$

through Taylor series expansion and since $\sigma^2 = \mathbf{I}$, we can simplify this down to $R(\theta) = e^{-i\frac{\theta}{2}}[cos(\theta) + i\sigma sin(\theta)]$.

For a gate that rotates the state by θ, we can plug that into the formula $R(\theta) = G$, where G is any gate. For example, if using the X gate, we can use $R(\frac{\pi}{2}) = X$, and to split it up in N steps, we have $R(\frac{\pi}{2N}) = X^{\frac{1}{N}}$. Then this value can be plugged into R in the formula above to get the rotation for each step.

3.4 Virtual Reality Design Choices

3.4.1 Background Design

The background that was chosen for QubitVR was designed as space-themed, in order to parallel the enigmatic properties of quantum physics itself, as well as provide a pleasing but non-distracting environment for the learner to reside in. This includes surroundings of "twinkling" stars (which unfortunately cannot be demonstrated on paper), dynamic color gradients to simulate nebulas, and a sun that gives qubits a shadow to increase the scenario fidelity, or how well real-world qualities are reproduced in virtual space. [18]

3.4.2 Virtual Travel

QubitVR was designed to be room-scale, which means a user is able to freely move around a designated "play area" in their space which is defined by them beforehand. Although it would have been possible to create a stationary experience, i.e. one where the user would not be able to move around the virtual environment, this would have made for a less immersive experience. One of the features of VR that is utilized in QubitVR is the ability to walk around and interact with the qubit as if it were a real object, and take advantage of the increased visibility that is a result of being able to move around and get different views of the Bloch vector.

There are two different modes of travel available to the user. The first technique is real walking, which uses an isomorphic mapping of the user's physical movements to their virtual movements [18] to move around the environment. The second technique is teleportation, which allows a user to choose a destination to teleport to in the environment by using a thumbstick on the VR controller to generate an arc to choose where they want to go. By using both these techniques, a user is able to have both high interaction fidelity through walking or trouble-free travel through teleportation (or in the case that they have a very small tracking space or just want to move to a location instantly).

3.4.3 Tool Belt For Qubit Interaction

The user interacts with a qubit using a "tool belt" attached to where their waist would be. This tool belt contains objects used to measure and manipulate a qubit's state. This includes a flashlight to measure a qubit state and different spheres and cubes representing the different quantum logic gates.

To measure the qubit with the flashlight, the user must reach down and grab the flashlight with either hand and shine the beam onto the qubit. To apply a gate, the user must reach down and grab

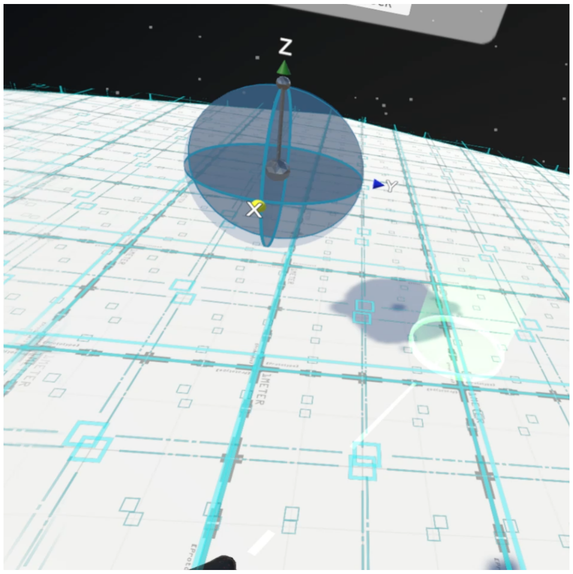


Figure 3.15: Teleportation

the gate they would like to apply, and drop it inside the qubit. The user is able to see a preview of what the vector's movement will be as soon as the gate touches the qubit. If dropped inside, the animation will finish and the state will be applied. If pulled out, the Bloch vector will resume its original position and no change will be made to the state.

Certain parts of the tool belt become available depending on your progression through the modules. This is done in two ways. If a piece of the tool belt is unrelated to the current module's content or hasn't been introduced yet, for example the CNOT gate in a module about measurement and superposition, you are unable to see it at all on the tool belt. If a part of the tool belt is currently being taught in the tutorial, but it is not quite yet ready for to you to apply it yet, for example, while the user is reading a description of what an H gate is before they can use it, it will be visible, but slightly transparent and unable to be picked up.

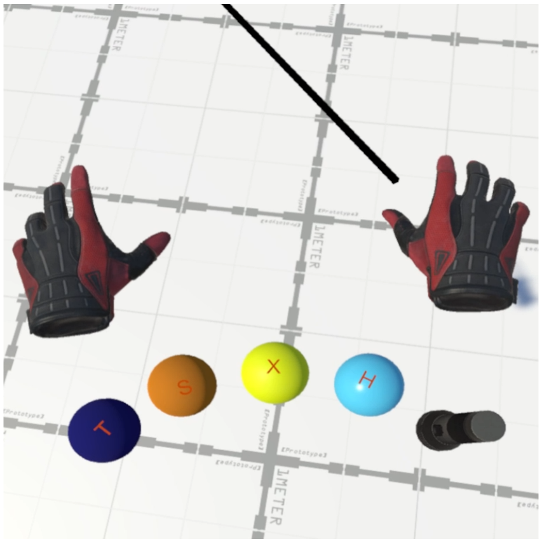


Figure 3.16: Toolbelt

3.4.4 Sound Design

Different music and interaction sounds were used in QubitVR to make it a more pleasant experience as well as to reinforce learning. There is different background music in each module to make the learning application a little more enjoyable. Different sound effects are used when objects on the tool belt are picked up and applied, so the user gets reinforcement that they are interacting with both the tool belt and the qubit alike. There are also different sound effects during module assessments based on whether the user got the question right or wrong in order to add further feedback to improve learning outcomes.

CHAPTER 4: MULTIVARIATE COGNITIVE WALKTHROUGH

4.1 Overview

In this chapter, we will discuss the multivariate cognitive walkthrough (MCW) of trajectory visualizations and state equations. The purpose of this evaluation is to determine the efficacy of trajectory visualizations and state equations in conveying and teaching the outcomes of quantum gates in QubitVR.

4.2 Motivation

As mentioned in the introduction, there are not many examples of consistent techniques to aid in the development and response to learner feedback of educational games. Also, as mentioned previously, QubitVR aims to set itself apart from other quantum computing learning applications by using visualizations and as little numerical displays as possible. This study intends on testing whether or not this approach actually performs well under scrutiny.

4.3 Experiment

The following section will describe the design and orchestration of the multivariate cognitive walkthrough.

Table 4.1: Multivariate Experimental Design

	Enabled	Disabled
Trajectory Visualization	Version 1	Version 2
State Equation	Version 3	Version 4

4.3.1 Experimental Design

Multivariate testing is a method of running a controlled experiment in which multiple factors, each with multiple conditions, are tested [19]. This study follows a 2x2 factorial design, as shown in Table 4.1. The two factors being tested are trajectory visualizations and state equations, and the two conditions are whether each factor is enabled or disabled. Each expert plays through each version shown in the above table.

A cognitive walkthrough is performed on each version of the application. A cognitive walkthrough has evaluators "walk through" a product as if they were a user in the target audience and answer questions in order to determine how usable the product is [5]. Since we wanted to assess how effectively learning takes place instead of how easily a user is able to carry out actions, we use Farrell and Moffat's cognitive walkthrough for learning through game mechanics (CWLTGM) methodology [9]. For our own cognitive walkthrough, we modify the original questions in Farrell and Moffat's approach to make them more applicable to QubitVR. Our questions are shown in Figure 4.1. The first two questions evaluate how well a user will understand the application of quantum gates and the last two questions evaluate how well a learner will learn the application of quantum gates.

After the cognitive walkthrough has been performed on all four versions, an additional set of questions, shown in Figure 4.2, are asked in order to evaluate trajectory visualizations and state

1. Will a student attempt to correctly answer the assessment questions?
2. Will a student understand what in-game actions would correctly answer the assessment questions?
3. Will a student associate their correct action as making progress towards correctly answering the assessment questions?
4. Assuming a student executes the correct in-game actions, is it reasonable to expect learning to take place?

Figure 4.1: Cognitive Walkthrough Questionnaire (CWQ)

1. Do you feel that the (trajectory visualization/state equation) presented redundant information regarding the outcome of applying a gate? Why or why not?
2. Do you feel that the (trajectory visualization/state equation) aided in understanding the outcome of applying a gate? Why or why not?
3. Do you feel that the (trajectory visualization/state equation) aided in remembering the outcome of applying a gate? Why or why not?
4. Do you recommend including the (trajectory visualization/state equation) in the assessment questions? Why or why not?

Figure 4.2: Multivariate Comparative Questionnaire (MCQ)

equations separately. Again, the first two questions evaluate how well a learner understands the outcome of applying gates and the last two question evaluate how well a user learned the outcome of applying gates.

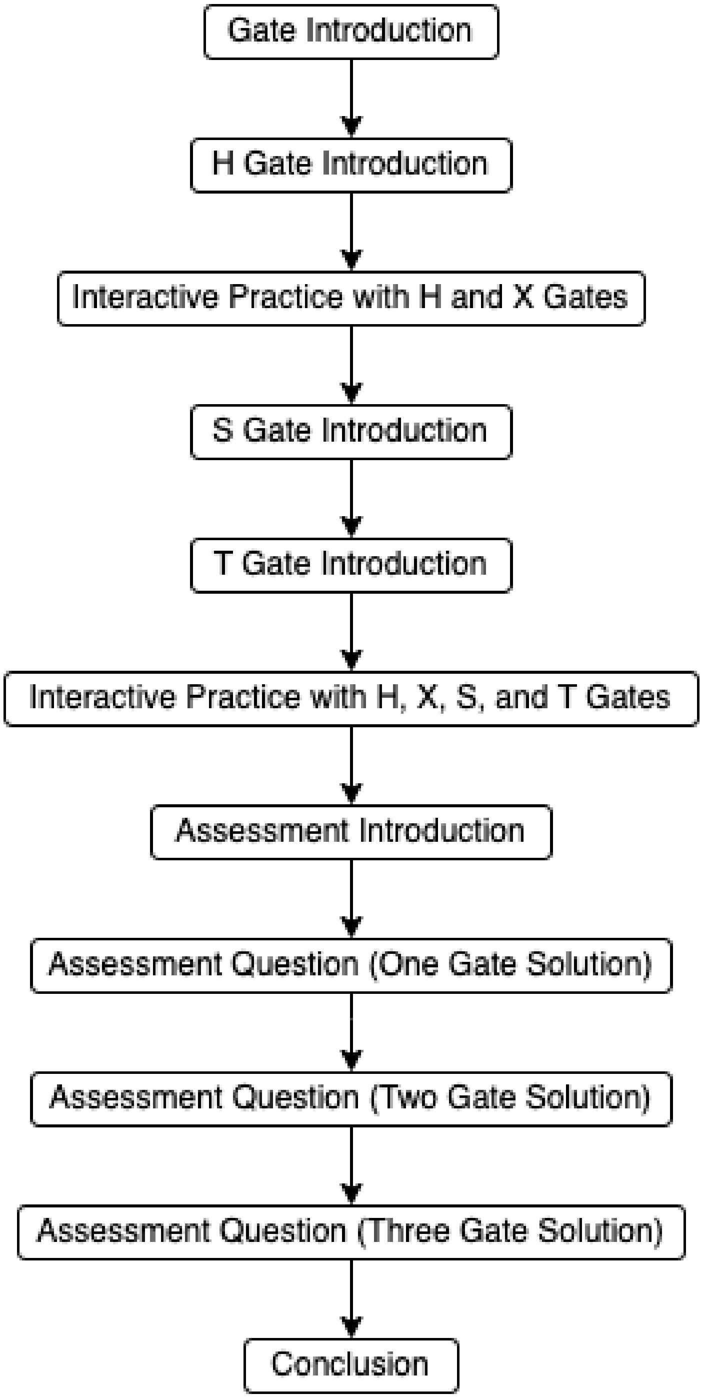


Figure 4.3: Flow through QubitVR section for Multivariate Cognitive Walkthrough

4.3.2 Materials

4.3.2.1 Software

A portion of the QubitVR application was modified to use in the cognitive walkthrough. The second section of module 1, single-qubit quantum gates, were selected to test trajectory visualizations and state equations. Figure 4.3 shows the content and flow through the modified application. The content of the tutorial for this section was not modified. However, the original ten question assessment was shrunk down to three questions.

To prepare QubitVR for feature testing, trajectory visualizations were used in the entire section. The tutorial contains interactive and non-interactive qubits and the assessment only contains interactive qubits. Originally, trajectory visualizations were turned on for the non-interactive qubits, but were not present when the user would apply gates to the interactive qubit. For this study, the tutorial and assessment were modified so that trajectory visualizations were always on (when their condition is enabled). It is important to note that the preview was turned off during the assessment, so the trajectory visualization was shown as the Bloch vector was rotating after it had already been applied, instead of before during a preview. The previews are off during the assessment so the learner cannot "cheat" by seeing where the Bloch vector will rotate before applying the gate, especially for the assessment questions with a one gate solution.

As mentioned in the experimental design, state equations were added to the entire section, and were positioned above the qubit as in [24]. The box with the state equation would resize based on the length of the equation, also as in [24]. Four different Unity scenes for each version were saved for each condition set.

4.3.2.2 Hardware

For the two in-person MCWs, the HTC Vive Virtual Reality System attached to a desktop PC with an Intel Core i7-6800K processor with 16GB RAM was used. The MCW conducted remotely was recorded over a Zoom call. For this MCW, the expert's personal HTC Vive Pro Eye attached to a desktop PC with an AMD Ryzen 7 5800X 8-Core Processor with 16GB RAM was used.

4.3.3 Experts

As mentioned above, a cognitive walkthrough requires experts to put themselves in user's shoes. The expert panel consisted of two quantum physics professors and one virtual reality professor with knowledge of quantum computing. The quantum physics experts were able to evaluate the correctness of the material QubitVR is teaching. In addition, since they are physics instructors, they are already familiar with teaching techniques for the subject. The virtual reality instructor was able to provide different feedback of the effectiveness of techniques in VR.

4.3.4 Procedure

First, the experts were told the purpose of the QubitVR application and that they were participating in a cognitive walkthrough. Next, they were informed of how the study was going to be conducted and verbally introduced to the VR interactions in QubitVR.

The study was conducted as follows. The version that the expert would run through a random permutation of the four versions, randomized based on the Latin squared design. Any comments the expert made during the cognitive walkthrough were recorded. After the version ended, the expert was instructed to take off their headset and answer the cognitive walkthrough questionnaire (Fig-

Table 4.2: Cognitive Walkthrough Questionnaire for Trajectory Visualization On, State Equation On

	CWQ1	CWQ2	CWQ3	CWQ4
Expert 1	1	1	1	1
Expert 2	1	1	1	1
Expert 3	1	0	1	0
Average	1	0.66	1	0.66

ure 4.1). After all four versions and all four CWQs questionnaires, the multivariate comparative questionnaire (Figure 4.2) was asked to evaluate the effectiveness of trajectory visualizations and state equations. Finally, the expert was asked if they had any closing thoughts.

4.4 Results

Both question sets aimed to gather feedback on the degree to which trajectory visualizations and state equations aided in both understanding and learning about quantum gates.

The first two cognitive walkthrough and multivariate comparative questions aimed to capture understanding capability and the last two questions aimed to capture learning capability.

Tables 4.2 through 4.5 show the results for the cognitive walkthrough questionnaire for each version. The majority of their responses can be found in Appendix B. The qualitative expert answers were codified and put in the following tables, where a 1 stands for a yes, a 0 stands for a maybe/inconclusive, and a -1 stands for a no. Tables 4.6 and 4.7 show the results for the multivariate comparative questionnaire. These results were codified in the same way, where a 1 stands for a yes, a 0 stands for a maybe/inconclusive, and a -1 stands for a no.

Overall, the CWQ results show that trajectory visualizations had a positive effect on both under-

Table 4.3: Cognitive Walkthrough Questionnaire for Trajectory Visualization On, State Equation Off

	CWQ1	CWQ2	CWQ3	CWQ4
Expert 1	1	1	1	1
Expert 2	1	1	1	1
Expert 3	1	1	1	1
Average	1	1	1	1

Table 4.4: Cognitive Walkthrough Questionnaire for Trajectory Visualization Off, State Equation On

	CWQ1	CWQ2	CWQ3	CWQ4
Expert 1	1	1	1	1
Expert 2	1	1	1	1
Expert 3	0	-1	-1	1
Average	0.66	0.33	0.33	1

Table 4.5: Cognitive Walkthrough Questionnaire for Trajectory Visualization Off, State Equation Off

	CWQ1	CWQ2	CWQ3	CWQ4
Expert 1	1	1	1	1
Expert 2	1	1	1	1
Expert 3	0	0	0	1
Average	0.66	0.66	0.66	1

Table 4.6: Multivariate Comparative Questionnaire for Trajectory Visualizations

	MCQ1	MCQ2	MCQ3	MCQ4
Expert 1	-1	0	-1	1
Expert 2	1	0	1	0
Expert 3	1	1	1	1
Average	0.33	0.33	0.33	0.66

Table 4.7: Multivariate Comparative Questionnaire for State Equations

	MCQ1	MCQ2	MCQ3	MCQ4
Expert 1	1	-1	-1	-1
Expert 2	1	-1	-1	-1
Expert 3	-1	0	-1	0
Average	0.33	-0.66	-1	-0.66

standing and learning the effects of quantum gates and that state equations had a negative effect on both understanding and learning the effects of quantum gates. It is important to note that the CWQ results were entirely affected by expert 3. The MCQ results show that trajectory visualizations had a positive effect on both understanding and learning the effects of quantum gates and that state equations had a negative effect on both understanding and learning the effects of quantum gates. The MCQ results were influenced by all three experts.

4.5 Discussion of Results

In this section, we will go more in depth with the results from the tables using excerpts from the expert feedback and our own interpretations.

One item that is important to address is the fact that the CWQ results were only influenced by one expert, since the other two experts said yes to every question in the questionnaire. We believe this positivity is because, in general, the experts liked QubitVR, not necessarily trajectory visualizations or state equations. Also, the experts did give feedback about the tested features in some of their answers to the cognitive walkthrough questionnaire, but their overall answer was always yes. It is telling that the experts differed in opinion for the MCQ, we believe means that the MCQ was a more effective in assessing the trajectory visualizations and state equations. This is most likely due to the fact that these questions directly referenced the features in their text.

4.5.1 Trajectory Visualizations

The experts generally agreed that trajectory visualizations were beneficial to understanding and learning the effects of quantum gates, to different degrees.

One way in which expert feedback differed was in what part of the application they found them useful. One expert said:

> *"It was very useful during the tutorial. During the assessment, not so much. It almost doesn't matter that it exists because it doesn't help you make a decision."*

However, another expert said:

> *"...during the assessment, for some of the more complicated gates, I found it to be useful because I could really see ... after I applied the gate exactly what was happening."*

Another way expert feedback different was for which gates trajectory visualizations were especially helpful on. Two experts explicitly said that the trajectory visualizations were useful for the H gate, since it is a more complex rotation. All three experts mentioned that the trajectory visualizations were useful in showing more complicated gate rotations.

One expert explained why he felt that trajectory visualizations were useful:

> *"Instead of having to remember a series of images, you only have to remember one single image about a rotation."*

4.5.2 State Equations

The experts unanimously agreed that state equations were not beneficial to understanding and learning the effects of quantum gates, to different degrees. During the CWQ with neither state equations nor trajectory visualizations, one expert said:

> *"I think the advantage of not putting the state vector equation is that you pay more attention to the actual Bloch sphere... one of the advantages of this instrument that you detach yourself from the math, so it builds intuition rather than associate things to mathematical symbols or expressions."*

Another expert gave feedback about how the display of state equations could be improved, saying a state equation should be included:

> *"Only if it is updated with the movement of the Bloch vector. I think if it was updated with the movement of the Bloch vector, it would allow people to perceptually map the number system to the surface of the sphere."*

4.6 Conclusion

Overall, the experts recommended trajectory visualizations to be included in the final version of QubitVR and did not recommend state equations to be included. The results were consistent with our hypotheses 1-4.

CHAPTER 5: CONCLUSION

5.1 Overview

This thesis intended to explore the use of Bloch sphere representation, trajectory visualizations, and state equations in an education quantum computing VR application.

5.2 Research Contributions

Our research contribution are:

1. We have created QubitVR.
2. We have assessed the educational potential of trajectory visualizations.
3. We have assessed the educational potential of state equations.

5.2.1 QubitVR

QubitVR is a novel educational platform that puts an emphasis on teaching quantum computing with the Bloch sphere model in VR. The content of QubitVR was discussed in chapter 3. In summary, QubitVR had four different content sections. It covered single-qubit concepts such as quantum superposition, entanglement, and gates. It also taught two-qubit concepts such as the CNOT gate and quantum entanglement. QubitVR was used as the context for the development and research of trajectory visualizations and state equations.

5.2.2 Trajectory Visualizations

RQ1. Do trajectory visualizations improve the learner's ability to understand the result of applying a gate?

The results from the CWQ and MSQ support our hypothesis that trajectory visualizations would improve the learner's ability to understand the result of applying a gate.

RQ2. Do trajectory visualizations improve the learner's ability to learn the result of applying a gate?

The results from the CWQ and MSQ support our hypothesis that trajectory visualizations would improve the learner's ability to learn the result of applying a gate.

5.2.3 State Equations

RQ3: Do state equations improve the learner's ability to understand the result of applying a gate?

The results from the CWQ and MSQ support our hypothesis that state equations would not improve the learner's ability to understand the result of applying a gate.

RQ4: Do state equations improve the learner's ability to learn the result of applying a gate?

The results from the CWQ and MSQ support our hypothesis that state equations would not improve the learner's ability to learn the result of applying a gate.

5.3 Conclusion

To conclude, we have created QubitVR and have assessed the educational potential of trajectory visualizations and state equations. We found trajectory visualizations to be an effective aid in helping learners understand and learn the outcomes of applying gates. Based on qualitative feedback, we also found QubitVR to be an useful tool in teaching quantum gates.

5.4 Future Work

At this moment in time, QubitVR is still being developed. In the future, we hope to add more modules and educational content. Future work could include modifying trajectory visualizations and state equations based on the expert's feedback to see if an improved implementation of both features could influence our findings here.

APPENDIX A: HIGH-LEVEL DIAGRAMS

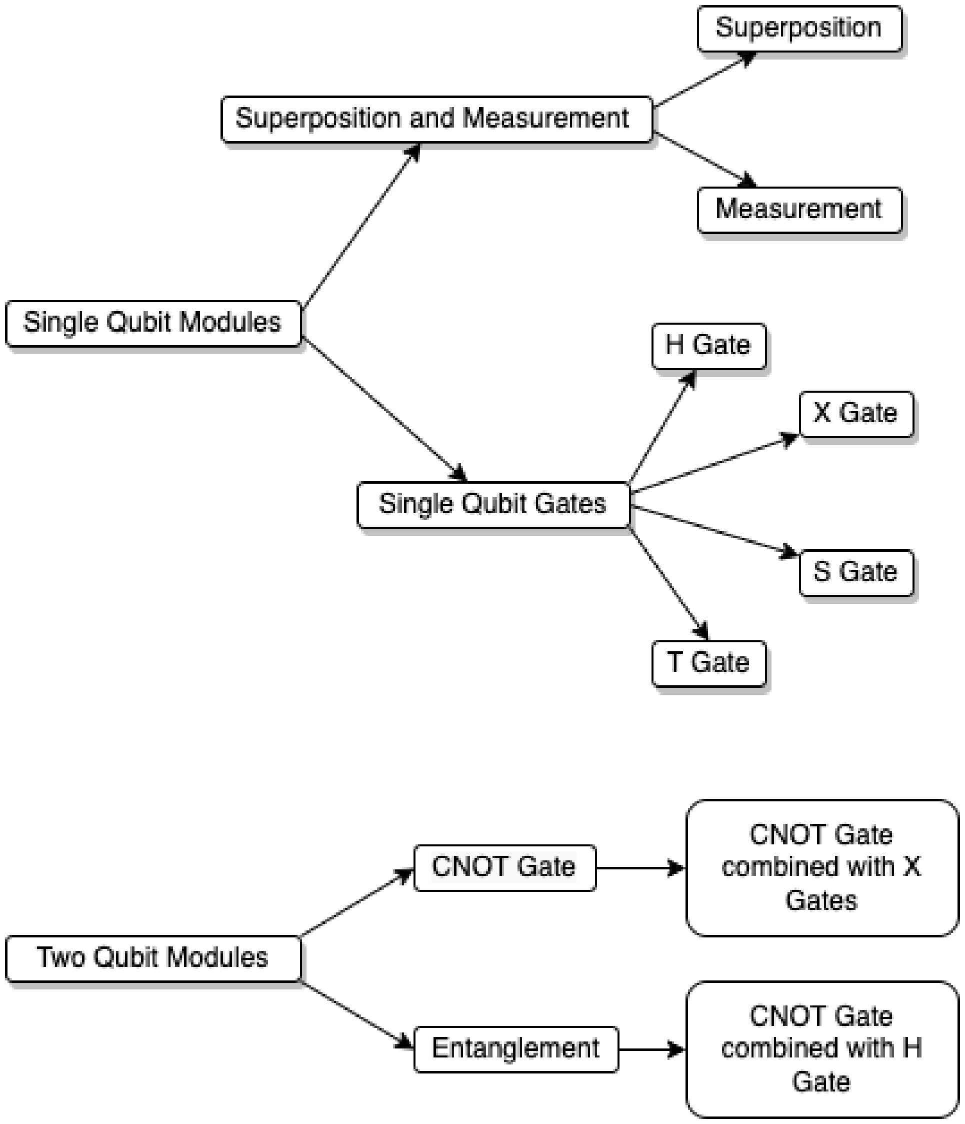


Figure A.1: Learning Module Content

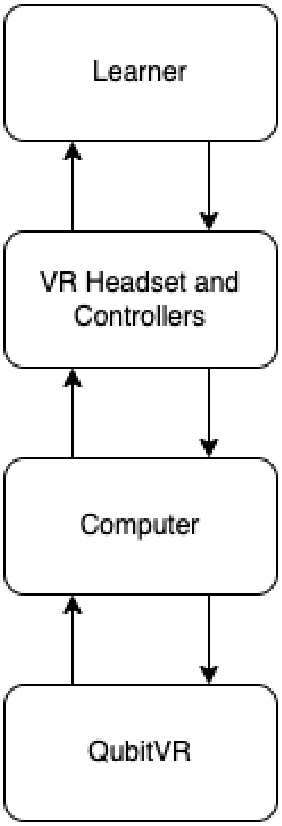


Figure A.2: Human Computer Interaction

APPENDIX B: SUBJECT MATTER EXPERT FEEDBACK TRANSCRIPTS

B.1 Subject Matter Expert #1

B.1.1 Cognitive Walkthrough Questionnaire Answers

B.1.1.1 Version 3

1. "Yes."
2. "If they get it right, they'll get it right because they understood why. They will basically logic it through. I would say if they get it wrong, they might not understand why. Probably some people would get lost and then go scrambling just trying whatever they want to."
3. "Yes."
4. "Yes, they'll definitely learn."

B.1.1.2 Version 1

1. "Yes, I think."
2. "I think so. So the case where the wedge vs non-wedge is most important is when you're rotating off of the principle axis. So like the T followed by a Hadamard. But I found it harder to see that rotation with the wedge. So without the wedge it was still a little hard to follow but without the wedge it was really quite difficult to follow. But then some other ones which I commented on in the live that were kind of in the back of the qubit was a little easier to see so I don't know if it's just the way the wedge is visualized. For the assessment, I think they would understand, but I think they would get a little confused about what the actual rotation is. It may just be a little bit better without it."
3. "Same as before."

4. "Yes."

B.1.1.3 Version 4

1. "Yes."
2. "Yeah, I think so."
3. "Yes, I mean I feel the same way."
4. "And again, yes. And with the same certainty as the previous visualizations."

B.1.1.4 Version 2

1. "Yes. I think this was probably true for all of the assessment, but when I let go of the gate and it showed me the rotation along with the wedge, I found that really helpful. So I'll answer the questions a little differently this time. Seeing the wedge as it was rotating during the assessment portion. If there was a student who was just randomly throwing gates at the wall and hoping for the best, they'll at least be able to see what it's exactly explicitly doing because the wedge emphasizes this is what's actually happening as opposed to if you just see the qubit rotating you think "oh, it's just going to its new state and I don't know what's actually happening" versus seeing the trajectory it's actually taking. I was surprised that I actually liked it. I didn't find it all that necessarily useful during the tutorial though."
2. "Yes."
3. "I think so."
4. "So what I would say is this is where the wedge was slightly better. If they do the right thing, then they actually see explicitly the rotation during the assessment. That I thought

was slightly better for learning. So I think that the wedge is slightly better for the learning aspect. And I would go back and change the other answer with the wedge."

B.1.2 Multivariate Summative Questionnaire Answers

B.1.2.1 Trajectory Visualizations

1. “I don’t feel that the information is redundant. It was just a different way of visualizing the motions. No, I don’t feel that it was redundant.”
2. “I didn’t feel that it was all that helpful during the tutorials stage, though I think that was mostly because, partially it was just really opaque so it was a little hard to see what was being traced out, and partially it’s just a three-dimensional object, it was hard to get a good angle where I could really see what the wedge looked like. It almost looked like a disk in most situations. So I would say that it was a little bit helpful, but not all that helpful. As I said, during the assessment, for some of the more complicated gates, I found it to be useful because I could really see after the fact, after I applied to gate exactly what was happening. It helped emphasize what the gate was doing during the assessment."
3. “No, not really. If they didn’t understand the rotation, they’re not going to remember it with the trajectory visualization.”
4. “So as I said, I definitely liked it after you apply the gates, seeing the trajectory visualizations. I liked it because I felt like it helped make sure the students were exactly clear on what was happening with the gate, instead of just seeing it lock into the second position. Every time I did the assessment, the first time that I tried to apply a gate but didn’t see a preview, it felt really weird. So, part of me feels like that really would have thrown some students off, that all of a sudden they’ve lost their preview.”

B.1.2.2 State Equations

1. “Yup, I feel it was redundant. Why is because it is redundant. We have the same information presented visually and algebraically. There’s no new information that the algebra adds, unless you want to do . . . physics which you’re not doing so.”

2. “Definitely no. If I were a student who wasn’t well versed in quantum mechanics, I would have completely ignored it. It was basically just meaningless text up there, unless it was very well explained. Even then, there was some confusing features that even I think some of my students would’ve been confused by. For example one thing I noticed was that when you go through the Hadamard, T gate process, they’ll start off pointing along the x-axis, you go through various Hadamard gates and S and T gates and you come back to a point along the x-axis, you end up with 0.707i + 0.707, which is correct, but I think the i factor doesn’t matter, but I think students, even quantum students, would get a little confused, because all of a sudden, it’s a different state, and they’re probably not going to be quick enough to understand that it’s just a phase rotation. It will not have helped the students, might have confused the students, but they’ll more likely than not just ignore it.”

3. “I would say no, def not. It didn’t affect my ability to understand the gates at all.”

4. “Again I would say no. It did not affect my ability to understand the solution."

B.2 Subject Matter Expert #2

B.2.1 Cognitive Walkthrough Questionnaire Answers

B.2.1.1 Version 1

1. "Yeah, I think so. I mean it's a lot of fun, it takes a little while to get hang of it, but yeah, it's very nice. I think for a lot of people, that's probably the best way to visualize things, I still like to think mentally about these types of things I think I'm still better in that instance but that's not the best for everyone. I think seeing things might actually be easier."
2. "I think so, when I think about students taking my class."
3. "I think so, so I mean it was well structured. The only thing that you may want to do is, I knew that I was wrong and I knew the solution was longer than it was supposed to be, but maybe you could let the player continue. Just say okay I know you didn't reach the goal but you got to the right final state, something like that."
4. "Oh yeah, I think so, I think this is good practice. One of the issues that we have when we teach that, and even I get confused sometimes that, I don't know how much you know about that, but these rotations of the Bloch sphere that are actually are sort of half an angle. For instance, the T gate and the S gate, sometimes they are called pi/8 instead of pi/4, so there's a factor of 2 that floats around, in the definition of the Bloch state. That's very confusing, even I sometimes make a mistake. So I think being able to actually visualize that helps to understand it. So instead, you detach yourself from the math, the mathematical formula, and you create intuition about the actual logic of the gate transformation and the angle that defines the state and all that. So I think it's a little easier in that way. Even I, when I gave an exam last week, when I was solving it, I forgot a factor of 2, which pops up on the Bloch

sphere when you write the … angles. So it's a common mistake. If you detach yourself from the formula and look at the actual Bloch sphere, I think it's actually better. Versus having to reconcile with these formulas that have a factor of 2, it's hard to remember, it's not intuitive."

B.2.1.2 Version 4

1. "Yes, same like previous one."
2. "Yes, yes."
3. "Yes, I think so."
4. "I think so. I think the advantage of not putting the state vector equation is that you pay more attention to the actual Bloch sphere. Yeah, I would actually not, if you were to shown that equation, put it in parallel with the actual activity. Maybe at the end you can say, you went from here to here and these were the conditions or the final state or something. But I, for instance, was so focused on trying to look at the Bloch sphere, I wasn't paying attention to the state equation at all, so I did not notice that it wasn't there. Yeah, because I think one of the advantages of this instrument that you detach yourself from the math, so it builds intuition rather than associate things to mathematical symbols or expressions, which I think is solid.

B.2.1.3 Version 3

1. "Yes, mhm."
2. "I think so."

3. “Yes.”
4. “I think so. I mean this one had a harder question at the end, it was harder to visualize which direction it would go, That’s I was trying to remember, is it going to go clockwise or anticlockwise, I think I got it wrong so it sent me to the wrong side. But I think it’s pedagogical, I realized that I should have changed the order that it was applied.”

B.2.1.4 Version 2

1. “Yes.”
2. “Yeah.”
3. “Yes.”
4. “Yes.”

B.2.2 Multivariate Summative Questionnaire Answers

B.2.2.1 Trajectory Visualizations

1. “Yeah, I mean it is redundant but I can see one advantage. When you’re rotating like for instance the Hadamard, which is the one that’s hard to visualize, it does help a little bit to situate the axis rotation, because you can see the trajectory and you can say “Oh, it’s actually going around this axis”. Whereas the other ones are actually very simple, because they are always rotating around an axis that is actually vertical or horizontal. But the Hadamard, it does help to have the trajectory marked.”
2. “Partially, only really for the Hadamard I think it’s very useful. But the other ones are very simple. Maybe a little bit of an overkill.”

3. “Ah, yeah in some cases. When the Hadamard is involved, it helps a little bit.”
4. “I think it’s possible to live without it, but I guess one thing that could be done in the cases that the question is not answered, showing the visualization a posteriori would help the person to go back and say “Oh okay, now I see it.” Or maybe during the training but not during the assessment. Only a posteriori, so they can see what they missed and so on.”

B.2.2.2 State Equations

1. “Yes, mhm. I don’t think people are paying attention, well I wasn’t paying attention to them. I think there’s enough information between the slides and the actual object in front of you, the Bloch sphere, that you don’t need the equation. I don’t think they’re adding anything.”
2. “No. They are more distracting than actually helpful.”
3. “No. It’s just another element to think about. I think the goal is really to visualize the physical object and rotations rather than the math representations.”
4. “No, I don’t. I think they are just distracting.”

B.2.3 Follow-Up Questions

1. Did you feel like state equations helped develop an intuitive understanding of the math behind it, the linear algebra?
   "If you’re talking to someone who does not have any idea about linear algebra, then those equations are really not helpful at all. So I guess you could have an informative session at the start where you could show the equation and the rotation at the same time but the equation has to be really in a different format, it has to stand out. Because right now, you’re looking at the big Bloch sphere, that’s a big Bloch sphere, it’s like three feet wide, and then you have

this equation far away from your visual sight, and it’s hard to get both together. So I would say if you want to use the equations to show, they have to be proportional, they have to be large and nearby so you do not have to move your head too much. At least in my case, when I want to look at something, I focus my head and my sight, and if something is happening in the periphery, I’m not paying attention to it. So, yeah maybe, having one session where you show the initial state, I must say one thing that I suggest is when you start at the off state, when you write psi you should just put 0, don’t put 0 times ket 1. So when you do the X gate, you can show that the 0 state becomes the one state, the on state. Because looking at the states and then looking at the coefficients, it’s a lot of stuff you have to pick up right away. You’re excited about this Bloch sphere but now you have to look at the coefficients. And put it at a good angle so that you can look at both the equations and the Bloch sphere."

2. If we were talking about your own students, which I’m assuming have a linear algebra background, would they understand the state equations more?

   "Not if they haven’t been introduced to the Dirac notation. They may even have gone through linear algebra before, but haven’t been introduced to this notation. For a single qubit it’s pretty easy, it’s just 2x2 matrices. Before you do Dirac’s notation, you might be able to introduce that to them first."

3. Do you feel like the trajectory visualization ever blocked your field of view or obscured what you could see of the rotation?

   "Not so much. I mean it’s very thick, it’s a big thick thing, but it’s not so bad."

4. Do you have any final thoughts?

   "Sure, I like it. I wish I could just play around with it without having to answer questions. To apply gates endlessly and see what happens, trial and error. That’s what I was missing, having a practice section where I wasn’t following a particular instruction."

   “I think you have a fantastic instrument to build intuition about state movement in the Bloch sphere, which is really hard, and I mean I told you I get confused. If I look at the math I catch myself on the actual sphere. I can do the math without the actual sphere, because there's this damn factor of two. What you've got here is something really cool because you can build intuition without relying on the math, which is really hard. So I actually think it's fine to detach them [state equations] if you want to attach them, you need to work a little bit more on making the connection between the initial state and the final state with the equation. So they have to be in the same visual area, and you could go simple."

"I mean I found this amazing, I could play with it for hours, you know."

B.3 Subject Matter Expert #3

B.3.1 Multivariate Summative Questionnaire

B.3.1.1 Trajectory Visualizations

1. "I feel like in most cases, it was redundant for the X, S, and T gates. There was one instance where I thought it really kind of showed the X gate. It's the one that I solved that I was very proud of, where I used the X gate to flip it across to the correct answer and it created a conic rotation, which almost all the other X gates are like a slice of pie, slice of pizza kind of visualization. What's interesting is the S and the T gates, if I ever rotated the vector and it wasn't on the equator, I believe there would also be conic representations for those as well. I don't think I saw it in the assessment or tutorial but I think that that would have been useful as well to better understand that their rotation about the z-axis, not just that they rotate along the equator."
2. "Yes, especially for the H gate. Because the H gate is by default a conic rotation, or the rotation creates a conic volume, it is very useful when it comes to displaying that rotation."

3. “Yes, because I could actually recall a single instance or image of the rotation of the H gate, as opposed to having to just try to remember an animation right, like a series of images. Instead of having to remember a series of images, you only have to remember one single image about a rotation.”

4. “I think for the assessment, yeah it was useful after the fact to kind of see how you messed up, if you messed up. It was very useful during the tutorial. During the assessment, not so much, it almost doesn’t matter that it exists because it doesn’t help you make a decision. But during the tutorial it was very useful, particularly in regard to the H gate.“

B.3.1.2 State Equations

1. “Redundant information, no. Confusing information, especially when the numerical signs switch, like when the phase or whatever has switched, yes. And again, my biggest feedback is it would have been nice if there was a real-time update of the equation because it would allow someone to mentally map the sphere and assign values to the sphere.”

2. “A little bit. In terms of the X gate, no, not at all. I think initially, you get the sense that “Oh, the X gate just moves the numbers from the left to the right”, and maybe that’s kind of what it does. In terms of the X gate, it’s kind of just moving the units from the first parameter side to the second parameter side. But then you have this weird, when the numbers start to diverge, you have the imaginary numbers, and that’s not very clear. What about the S. The S in many cases seemed to just switch it from imaginary to non-imaginary and the sign. The T ended up seeming to take whatever the value was and split it in half, kind of dividing the number. Again, I feel like if I had the real time updates of the numbers transitioning, I would have understood better what was happening mathematically with the gates and the numbers, and how the numbers mapped to the surface of the sphere. As is, it was a little confusing.”

3. “No, not at all. Usually after applying the gate I was semi-confused about why then numbers changed the way they did anyway, so recalling the numbers. So basically, I would look at the numbers, yeah no.”

4. “Only if it is updated with the movement of the Bloch vector. I think if it was updated with the movement of the Bloch vector, it would allow people to perceptually map the number system to the surface of the sphere, as is, because of the jumping I don’t think people actually understand how the operators change the numbers of the state equations, so right now it’s more distracting and confusing and adds a lot more cognitive load to try and understand then if it just didn’t exist.”

B.3.2 Cognitive Walkthrough Questionnaire Answers

B.3.2.1 Version 4

1. "I think the first one, yes. I think the second one, most students will attempt it. I think the third one, will they attempt it? I think if they just had the option to skip it, some students would just be like "uhhh" and just skip it. So I guess my answer is, in some of the cases yes, not all students."

   [Researcher]: "Can you elaborate on if they had the option to skip it?"

   "Yeah, the last one where you have three gate applications, some students would be a little overwhelmed. They might try like one gate application, okay well maybe they would try it, but there is a small percentage of the student population that wouldn’t even attempt to apply the gate if they had the option to skip it."

   [Researcher]: "So in the current state, would that equate to just putting the headset down and quitting?"

"No, in its current state, I think everybody would attempt to apply some kind of gates but I think they would get that last question wrong."

2. "Not all students, say 60-75% of students, yes. I think the last question, some students will not understand that they should apply a Hadamard gate first."
3. "For the first two, yes. For the last one, if they got the Hadamard correct, they may or may not realize that they're on the right track. So yes for the first two, not necessarily for the third one."
4. "Yes. I feel like if they recognize that they should apply the Hadamard gate first and then the T gate, I think that they will have picked up on how the Hadamard gate– I think the most complex part of all this is the Hadamard gate, because of the dual axis rotation, it's, like I've done a lot of 3D rotation stuff, and that rotation itself is very hard for me to conceptualize. I feel like this might actually be one of the best times I've conceptualized the Hadamard gate, actually, in terms of rotations, probably just because instead of watching you guys on the Zoom screen, I'm more paying attention."

B.3.2.2 Version 1

1. "Yes, and they are more likely to attempt it in this condition because of the trajectory visualization."

   [Researcher]: "Why?"

   "Having the trajectory visualization during the tutorial made me feel confident about what the motion of each gate actually was, even though in the previous example I could see the animation of the vector itself, being able to see that trajectory mapped all out at once is useful for better understanding the path."

2. "No, still not necessarily. I think quantum is just hard. No, I think they are probably more likely to understand how to properly answer, but it's not 100

   [Researcher]: "And more likely to understand because why?"

   "Again, I think it has to do with understanding the motions better of the gates. Like, again, the Hadamard gate is the most complex and that's what I was playing with for a while. Whenever you have that off center axis it's hard to understand what a rotation around that axis looks like. Even with just the motion of the vector, like for example when you're on the 45 degree, so you've applied the Hadamard gate and then you apply the T gate and another Hadamard gate, that rotation is very off axis, very abnormal. Even with the motion of the vector in the previous one, I was still having a hard time visualizing what was going on. Even here, with the red trajectory, I was having a hard time a little bit really fully understanding how it mapped the way it did, but it made a lot more sense. I was able to line up that axis in the center of the red trajectory and actually see how it rotates about it."

3. "Yes. Again, because of the red visualization."

4. "I think understanding how the different quantum operators affect the Bloch vector, yes. Understanding the state equation, no. I was still trying to watch the state equation, and I don't know if it was because there were discrepancies in the state equation or like a bug or what, but it wasn't always clear to me what was going on with the state equations. Especially in the case of it being 0 and -1 when I expected it to be 0 and 1. And then just numerically too right. So like when it was on the equator, you get, what was it, like the 0.7141 number for both components. And then when it shifts around, it's like the number 1 component, its value changes a little bit, but it shifts up or down and the first component changes a little bit."

B.3.2.3 Version 2

1. "Yes, most students."
2. "Yes, more so than the first version. Again, because of the red visualization. I personally feel like it conveys a lot of information and it does a better job understanding the rotations. I actually like this condition better than the last condition because in the last condition I spent a lot of time trying to figure out what was going on with the state equation, and in this one I'm just like focused completely on the trajectory."
3. "Yeah. Because the red visualization further confirms what would happen in terms of the rotation."
4. "Yeah. Having seen the red trajectory visualizations, I was able to better kind of re-visualize them in place, to kind of figure out the chess moves. It's like you have to make multiple moves in your head before you make the first one because if the first one is wrong, basically you're not able to solve it. So, I was able to easily visualize because of the red visualizations, in my head I was able to visualize what happened if I was applied X, then H, and then I could see I had a solution in four steps but that wasn't the right step. I was also able to visualize the rotation about the x-axis, that was easier for me, although I will say one thing, I noticed that there might be something slightly wrong with your visualization for the T gate, whenever I applied the T, and it was over on that side, it skipped."

   [Researcher]: "Yeah that's after the H gate, there is a bug in the assessment specifically."

B.3.2.4 Version 3

1. "Most students will, some students will be overwhelmed."
2. "No."

[Researcher]: "Why?"

"In the case of the three quantum gate operation, it's very difficult, and I don't think the state equation helps at all."

3. "Not necessarily, because I thought I was on the right track for that last one, when I applied the Hadamard gate and it ended up on the wrong side of the sphere. So no, not necessarily. I think it will help in general, but again that's from the movement of the vector itself and not from the equation. Or I probably just didn't pay attention enough to the state equation. Maybe if I had paid more attention to the state equation, I would've realized that I was going to end up on the wrong part of the sphere after the H gate."

4. "Yes. Yes, for the most part. Again though, not from the state equation itself, but from how the operators change the position and orientation of the Bloch vector."

APPENDIX C: IRB NOT HUMAN RESEARCH DETERMINATION LETTER

UNIVERSITY OF CENTRAL FLORIDA

Institutional Review Board
FWA00000351
IRB00001138, IRB00012110
Office of Research
12201 Research Parkway
Orlando, FL 32826-3246

NOT HUMAN RESEARCH DETERMINATION

April 7, 2022

Dear Pauline Johnson:

On 4/7/2022, the IRB reviewed the following protocol:

Type of Review:	Initial Study
Title of Study:	Multivariate Cognitive Walkthrough of QubitVR
Investigator:	Pauline Johnson
IRB ID:	STUDY00004186
Funding:	None
Grant ID:	None
Documents Reviewed:	• HRP-251- FORM - Faculty Advisor Scientific-Scholarly Review RPM.pdf, Category: Faculty Research Approval; • HRP-250-FORM- Request for NHSR(2).docx, Category: IRB Protocol

The IRB determined that the proposed activity is not research involving human subjects as defined by DHHS and FDA regulations.

IRB review and approval by this organization is not required. This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are made and there are questions about whether these activities are research involving human in which the organization is engaged, please submit a new request to the IRB for a determination. You can create a modification by clicking **Create Modification / CR** within the study.

If you have any questions, please contact the UCF IRB at 407-823-2901 or irb@ucf.edu. Please include your project title and IRB number in all correspondence with this office.

Sincerely,

Katie Kilgore

Katie Kilgore
Designated Reviewer

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