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ULTIMATE STATE CRITERIA AND STRENGTH CHARACTERISTICS OF THE ROCK MASSIFS BEING UNDERMINED REPEATEDLY

Abstract. Methodological approaches to the selection of ultimate state criteria and strength characteristics of the repeatedly undermined rock massifs were developed. These approaches were designed to provide parametric support to the geomechanical modelling of the massif stress-strain state and the mining systems of the Starobin potash deposit mine fields planned for the additional mining of the mineral reserves left. It was established that a complex criterion must be used to study the massif ultimate state. Determination of such criterion can be carried out using the developed approaches. The first approach is to select several criteria that evaluate the massif ultimate state by certain types of the massif stress-strain state. These criteria are the following: the criterion of the maximum normal stresses, criterion of the maximum linear strains, the criterion of the maximum shear stresses and the Coulomb–Mohr failure criterion. The second approach is to construct an integrated failure state criterion for materials whose ultimate tensile and compressive stresses differ significantly. In this case, parameters characterizing the type of stress state and properties of the material are introduced. These parameters together determine the destruction character – tear or shear. To describe the rocks behavior in the extreme strength stage of deformation, it is proposed to apply deformation theory of strength using the developed strain failure criterion. When calculating the strength characteristics of the repeatedly undermined rock massif, it is recommended to use a structural attenuation coefficient as the product of several factors, taking into account various types of disturbances in the primary undermined massif and the time factor. The Coulomb–Mohr strength condition is recommended to be used taking into account the composite structural attenuation coefficient. Dependencies have been developed to describe the change in the strength characteristics of rocks in the undermined massif, considering the attenuation coefficient.

Keywords: layered rock massif, mechanical characteristics, stress-strain state, ultimate criteria

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КРИТЕРИИ ПРЕДЕЛЬНОГО СОСТОЯНИЯ И ПРОЧНОСТНЫЕ ХАРАКТЕРИСТИКИ ПОВТОРНО ПОДРАБАТЫВАЕМЫХ МАССИВОВ ГОРНЫХ ПОРОД

Аннотация. Разработаны методические подходы к выбору критериев предельного состояния и прочностных характеристик повторно подрабатываемых массивов горных пород, предназначенных для параметрического обеспечения геомеханического моделирования напряженно-деформированного состояния массива и горнотехнических систем участков шахтных полей Старобинского месторождения калийной руды, планируемых для доизвлечения оставленных запасов полезного ископаемого. Установлено, что для изучения предельного состояния массива необходимо применять комплексный критерий. Определение такого критерия можно выполнить с использованием разработанных подходов. Первый подход заключается в выборе нескольких критериев, оценивающих предельное состояние массива по отдельным типам напряженно-деформированного состояния массива: критерия наибольших нормальных напряжений, критерия наибольших линейных деформаций, критерия наибольшего касательного напряжения, критерия Кулона–Мора. Второй подход заключается в построении объединенного критерия предельного состояния предлагаемого вида для материалов, у которых предельные величины напряжений на растяжение и сжатие отличаются существенным образом. При этом вводятся параметры, характеризующие вид напряженного состояния и свойства материала, которые в совокупности определяют характер разрушения – отрыв или срез. Для описания поведения горных пород в запредельной по прочности стадии деформирования предлагается применять деформационную теорию прочности с использованием разработанной зависимости деформационного критерия прочности. При расчете прочностных характеристик повторно подрабатываемого массива горных пород рекомендуется применять коэффициент структурного ослабления в виде произведения нескольких коэффициентов, учитывающих различные типы нарушения первично подработанного массива и временной фактор. Условие прочности Кулона–Мора рекомендуется использовать с учетом составного коэффициента структурного ослабления. Разработаны зависимости, описывающие изменение прочностных характеристик пород в подработанном массиве с учетом коэффициента разрыхления.

Ключевые слова: слоистый массив горных пород, механические характеристики, напряженно-деформированное состояние

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Introduction. At the present time, additional mining of the previously mined out panels is actual for the Starobin potash salt deposit in order to extract the left ore reserves concentrated in the protective and inter-chamber pillars and the underworked sylvinite layer. Study of rock massifs stability, the stress-strain state of which was once disturbed by mining operations, must be done before the design excavation technology the leftover reserves and cavities protection methods. At the same time, the choice/construction of criteria, according to which the rock massif comes to the ultimate state, is one of the urgent problems for repeatedly undermined rock massifs.

The stability and strength calculations of the surrounding rock massifs in the vicinity of underground workings are largely determined by the selected ultimate (limit) criteria. In turn, the acceptance of a certain criterion as an estimate of the ultimate (critical) rocks state must be reasonable and based on the regularities and features of the physical process being considered. Previously, no such studies were carried out at the Starobin deposit. The influence of structural heterogeneities on the repeatedly undermined potash rock massifs strength was also insufficiently studied. At the same time, many problems were investigated conceptually in a lot of works devoted to geomechanical modeling of rock ultimate states in a vicinity of mine cavities, laws of deformation processes and rocks fracture [1–10]. However, the problem of selecting the ultimate state criteria and strength characteristics of the rock massifs being undermined repeatedly was not solved as applied to the conditions of the Starobin deposit. In addition, reliable experimental data for the considered class of problems (stability of underground structures in repeatedly mined rock massifs) is also still insufficient.

The purpose of this study is to develop methodological approaches to the selection of ultimate state criteria and strength characteristics of the rock massifs being undermined repeatedly, as applied to the Starobin deposit conditions.

Results and discussion. Ultimate state criteria for the repeatedly undermined rock massifs. Under limiting state of rock massif we mean such a state when there are zones of significant rock failure, fracturing, displacements on sliding lines and other disturbances of continuity in the considered rock massif area. The ultimate state criterion must have a clear physical meaning. Out of a large number of factors directly or indirectly influencing the deformation process regularities and strength values of rock formations, it is very important to select those, which are determinative for the processes being considered. Therefore, in terms of practical use, the strength and fracture criterion should be expressed by an equation with a minimum number of material constants determined, in turn, from the simplest experiments.

The limit (extreme) equilibrium equation for the principal stress components (σ_1, σ_2, σ_3) is generally written as follows:

$$\psi(\sigma_1, \sigma_2, \sigma_3) = 0. \tag{1}$$

The ultimate state and destruction of geomaterials and rock massifs can follow different scenarios, depending on a large number of factors. Consequently, there is a rather wide range of explicit representations of the ultimate state equations (1).

The feature of the underground structures behavior is the fact that the ultimate equilibrium of rock massifs does not yet mean complete loss of the bearing capacity of an underground structure. Therefore, to describe the strength and stability of rock massifs, only the "limit relations" between the principal stresses are not sufficient [11, 12]. The "limit relations" must be supplemented by an indication of the ultimate (critical) deformations (strains or displacements) at which the bearing capacity of the object will be exhausted. The critical deformations provide information about the deformation process, especially at the stage of the "descending deformation curve" [13]. The bearing capacity of rock massifs depends significantly on the "extreme" deformed state. Consequently, ultimate stress state testing is insufficient to estimate the bearing capacity of many geomechanics objects. The rock massifs deformations (including residual deformation) depend significantly on the loading or deformation history. Therefore, for assessment of rock massifs destruction, the deformed state type should be taken into account. In this case, the critical deformations are one of the physically correct criteria for object destruction.

Thus, to describe the rocks behavior beyond the strength it is necessary to use the strength deformation theory. In accordance with this, we modify equations (1) of limit equilibrium taking into account massif strains on "descending branch of deformation curve" as follows:

$$\psi(\sigma_1, \sigma_2, \sigma_3, e_1, e_2, e_3) = 0, \tag{2}$$

where e_1, e_2, e_3 – are the principal strains.

Due to the formation of a complex stress state in the undermined rock massif, where generalized compression, tension and shear areas exist simultaneously, it is obvious that a complex criterion must be used to study the ultimate (critical) rock massifs state. The determining of such complex criterion can be done using two approaches.

First approach. Selection of several criteria assessing the limit massif state by the certain types of the massif stress-strain state. In the frame of first approach in our modeling studies, we had used the following ultimate criteria to investigate the undermined rock massif state with underground workings.

Criterion of maximum normal stresses, which in terms of principal stresses can be written in the following form:

$$\left[\begin{array}{l} \sigma_1 \geqslant \sigma_{\lim\text{ext}}, \sigma_1 > 0; \\ \sigma_3 \leqslant \sigma_{\lim\text{press}}, \sigma_3 \leqslant 0. \end{array}\right. \tag{3}$$

In (3) it is considered that $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$; $\sigma_{\lim\text{ext}}$ is ultimate tensile stress (positive value); $\sigma_{\lim\text{press}}$ is the ultimate compressive stress (negative value), where the ultimate compressive value may be the yield stress or the allowable stress. The ultimate values in (3) can be taken as strength for uniaxial, biaxial or volumetric stress states. The ultimate value for the uniaxial stress state is most often used.

Criterion of maximum linear strains. According to this criterion, the massif strength is effected if the greatest absolute value of the relative linear deformation exceeds some extreme value, independent of the type of stress state. A mathematical formulation of this criterion is as follows:

in terms of the deformed state:

$$F(\varepsilon_1, \varepsilon_3) = \left[\begin{array}{l} \varepsilon_{\max} = \varepsilon_1 \geqslant \varepsilon^{\text{el}}_{\lim\text{ ext}}; \\ \varepsilon_{\min} = \varepsilon_3 \leqslant \varepsilon^{\text{el}}_{\lim\text{ press}}, \end{array}\right. \tag{4}$$

where $\varepsilon^{\text{el}}_{\lim\text{ ext}}$ is ultimate strain on tension, it corresponds to the point of the elasticity limit (positive value); $\varepsilon^{\text{el}}_{\lim\text{ press}}$ is ultimate strain on compression, it corresponds to the point the elasticity limit (negative value);

in terms of the stress state:

$$F(\sigma_1, \sigma_2, \sigma_3) = \left[\begin{array}{l} \varepsilon_{\max} E = \sigma_1 - \nu(\sigma_2 + \sigma_3) \geqslant \sigma^{\text{el}}_{\lim\text{ext}}; \\ \varepsilon_{\min} E = \sigma_3 - \nu(\sigma_1 + \sigma_2) \leqslant \sigma^{\text{el}}_{\lim\text{press}}, \end{array}\right. \tag{5}$$

where $\sigma^{\text{el}}_{\lim\text{ext}}$ is the ultimate tensile stress (positive value); $\sigma^{\text{el}}_{\lim\text{ press}}$ is the ultimate compressive stress (negative value).

The use of the *maximum linear strains* criterion in the form (5) is preferable in practice. Firstly, during the deformation processes development over time, the calculated stresses are the true stresses. Secondly, the stress extreme values are easier to determine empirically than the strain extreme values.

The maximum shearing stress criterion. The limit state condition according to the maximum *shearing stress criterion*, written out in terms of principal stresses, is as follows:

$$\left[\begin{array}{l}\sigma_1-\sigma_3 \geqslant \sigma_{\lim \text{ext}};\\ \sigma_1-\sigma_3 \geqslant \left|\sigma_{\lim \text{press}}\right|.\end{array}\right. \tag{6}$$

It should be noted that there are quite a lot of modifications of the criterion (6) [13].

The Coulomb–Mohr criterion. The Coulomb–Mohr limit condition can be written out as follows:

$$\max\left(\left|\tau_n\right|-\left(\sigma_n \text{tg } \varphi+C\right)\right)=0, \tag{7}$$

where τ_n and σ_n are respectively the tangential and normal stress at the site with the normal line n; φ is the internal friction angle (respectively tgφ is internal friction coefficient); C is bonding strength.

It is important that the Coulomb–Mohr condition can be used as a boundary condition in the areas of significant horizontal layer dispacements. In addition, even if condition (7) is acceptable for irreversible deformation zones, it can also be used for elastic zones if the contact displacements in them are sufficiently large.

It is also possible to use the expression of equation (7) in principal stresses:

$$\sigma_1-(2\lambda+1)\sigma_3=\sigma_{\text{press}}, \tag{8}$$

where σ_1 and σ_3 are the maximum and minimum principal normal stresses respectively (taking into account the sign), $\lambda = \sin\varphi/(1 - \sin\varphi)$, σ_{press} is the ultimate strength of rocks in uniaxial compression.

The peculiarity of strength criterion (8) is that it takes into account both shear failure and tearing failure.

Second approach. Construction of a combined strength theory. When we were comparing the results of theoretical calculations with experimental data it was noticed that for each hypothesis there is an area of stressed states, in which theory is in the best agreement with experience. Therefore, it seems reasonable to construct a combined ultimate state criterion, which includes several criteria.

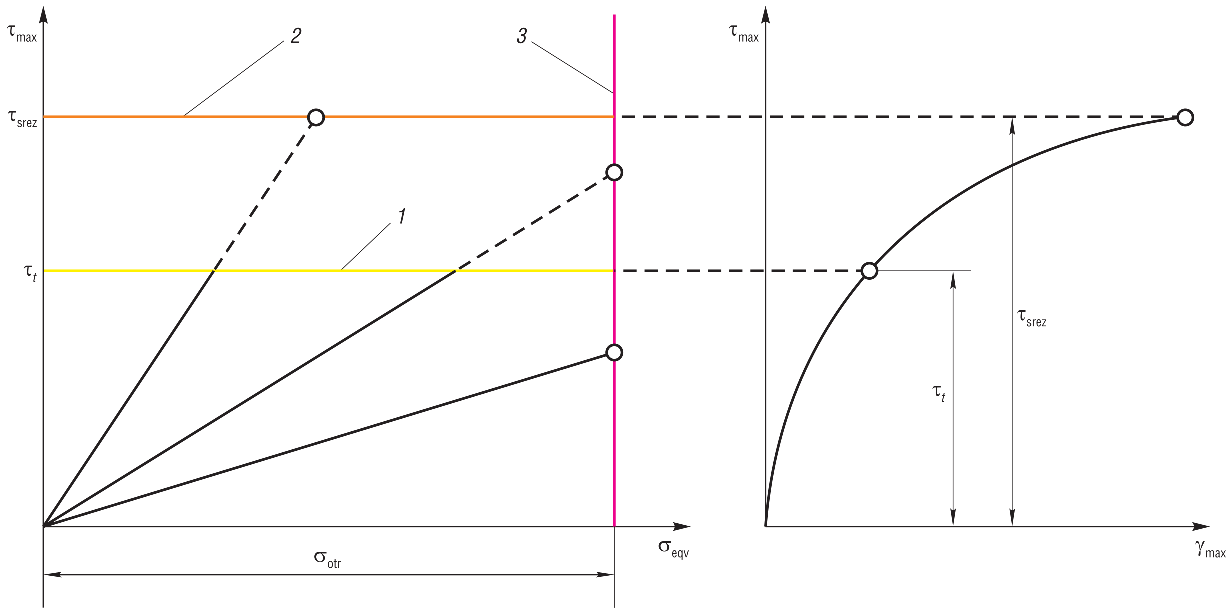
For materials in which the ultimate tensile and compressive stresses differ significantly, it is promising to use a combined criterion of the form [11]:

$$F(\sigma_1,\sigma_3)=\begin{cases}\sigma_1 \geqslant \sigma_{\lim \text{ext}}, \dfrac{\sigma_1+\sigma_3}{2}>0;\\ \dfrac{\sigma_1-\sigma_3}{2} \geqslant \dfrac{\sigma_{\lim \text{ext}}+\sigma_{\lim \text{press}}}{\sigma_{\lim \text{ext}}-\sigma_{\lim \text{press}}}\dfrac{\sigma_1+\sigma_3}{2}-\dfrac{\sigma_{\lim \text{ext}}\sigma_{\lim \text{press}}}{\sigma_{\lim \text{ext}}-\sigma_{\lim \text{press}}}, \ \dfrac{\sigma_1+\sigma_3}{2} \leqslant 0.\end{cases} \tag{9}$$

The physical meaning of (9) can be formulated as follows: material strength breaching occurs either when shearing stresses reach some critical value, depending on the normal stresses acting along the same sliding planes, or when the maximum normal stress σ_1 reaches the limit value for the material (tensile stress).

The use of the combined Davidenkov–Friedman strength criterion seems promising [14]. This criterion is based on the principle that the nature of material failure depends not only on its physical and mechanical properties and external operating conditions, but also on the stress-strain state scheme. It is taken into account that, depending on the nature of the stressed state, the material can fail both, from normal stresses and from tangential stresses. Thus, the Davidenkov–Friedman strength theory combines two classical strength theories: maximum tangential stresses and maximum relative elongations, which are important for geomaterials.

It is easy to present this theory graphically in the form of a well-known mechanical state diagram, in which the basic properties of the material are reflected (Figure). The flow curve, independent of the stress state type, is placed in the right part of the diagram, and the yield and fracture ultimate states are represented in the left part by straight lines parallel to the coordinate axes of the system $\tau_{\max} - \sigma_{\text{eqv}}$. Here σ_{eqv} is an equivalent stress, determined according to the theory of maximum relative elongations (corresponding to the greatest linear positive strain). A ray starting from the coordinate origin corresponds to each stress state. Depending on which limit curve this ray crosses the type of failure (by tearing or by shear) and hence the strength theory is determined.



Material mechanical state diagram: *1* – beginning of yield strength, *2* – shear failure, *3* – tearing failure

The mechanical state diagram is based on the stresses limit values σ_{otr}, τ_{srez}, τ_t, which are assumed to be constant for the material and independent of the stress-strain state schemes. The object condition can be evaluated according to the mechanical state diagram (see Figure), representing the stress state in the diagram as a point with coordinates ($\tau_{max} = (\sigma_1 - \sigma_3)/2$, $\sigma_{eqv} = \sigma_1 - \nu(\sigma_2 + \sigma_3)$).

The mechanical meaning of the theory is well explained by the example of a simple loading. If a line is drawn through the origin and a given point, it is possible to predict which type of failure will correspond to the stress state being analyzed (see Figure). Thus, for line “2” there is a material failure by shear; for line “1” – a failure by tearing, which occurs after the plastic deformations development; for line “3” – a failure by tearing without development of plastic deformations.

According to the combined strength theory, the main characteristic of the stress state determining the nature of the failure can be the value α, which is equal to the following ratio:

$$\alpha = \frac{\tau_{max}}{\sigma_{eqv}} = \frac{\sigma_1 - \sigma_3}{2\left[\sigma_1 - \nu\left(\sigma_2 + \sigma_3\right)\right]}. \quad (10)$$

The parameter α characterizes the type of the stressed state. For example, if $\alpha = \nu/2$, then the straight line in the deformation diagram corresponds to the compression; if $\alpha = 1/(1 + \nu)$ – to torsion; and in case $\alpha = 1/2$ – to extension. We should note that instead of the parameter α, it seems promising to take the Nadai–Lode coefficient, which characterizes the type of the stress state.

In addition to the parameter α, we introduce a parameter β to evaluate material properties:

$$\beta = \tau_{srez}/\sigma_{otr}. \quad (11)$$

If $\beta \ll 1$, the material will most often fail by shearing; if $\beta < 1$, the material will generally fail by tearing; if $\beta \approx 1$, the material failure nature depends largely on the type of the stress state.

According to the introduced parameters, the destruction nature is determined both by the stress state (coefficient α) and by the nature of the material (parameter β). Namely, at $\alpha < \beta$ tearing takes place and at $\alpha > \beta$ we obtain shearing.

Before destruction, the behavior of the material is determined by the ratio of the coefficient α and the parameter $\eta = \tau_t/\sigma_{otr}$. If $\eta > \alpha$, the material fails without developing plastic deformations, i.e. becomes brittle; if $\eta < \alpha$, then the failure is preceded by the appearance of plastic deformations.

As noted earlier, the ultimate state of rock massifs according to the criteria including only components of the stress state does not yet mean a complete loss of bearing capacity of the underground structures. Therefore, the "limit relations" should be supplemented by an indication of the ultimate deformations values. In other words, the "deformation history" of the investigated rock massif area must be taken into account, i.e. the complete deformation diagram must be considered. Therefore, deformation theory of strength must be used to describe the rocks behavior beyond their extreme values.

Thus, at the stage of deformation up to the "critical point" (strength) the massif strength is considered according to a set of the described criteria. At the "beyond stage of deformation" equation (2) is written in the form corresponding to the deformation strength criterion, e.g. [8, 9]:

$$\sigma_1^* = (2\lambda + 1)\sigma_3 + \sigma_{\text{press}} - E^* e_1'. \tag{12}$$

Here σ_1^* is the value of the principal stress σ_1 at the beyond the limit branch of the deformation diagram (at points on the descending part of the curve $(\sigma_1 - e_1)$); e_1' is the value of principal strains increment e_1 at the beyond the limit stage of deformation; E^* is deformation module of the descending part of the full diagram curve; $\lambda = \sin\varphi/(1 - \sin\varphi)$.

The considered approaches are based on the assumption that the ultimate strength state is practically independent of principal stress σ_2. The value of σ_2 determines the type of volumetric (3-Dimension) stress state. This fact can be clearly seen from the value of the Nadai–Lode parameter μ_σ, which characterizes the generalized stress state in the rock massif:

$$\mu_\sigma = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}, -1 \leqslant \mu_\sigma \leqslant 1.$$

Let us remind that for the principal stress components σ_1, σ_2, and σ_3 the inequality $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$ is fulfilled. Compressive stresses are taken as standard with a minus sign. The maximum compressive load applies along the vertical axis, therefore this axis is numbered "3" and the maximum compressive stress along this axis is indicated as σ_3.

The Nadai–Lode parameter varies from "–1" to "1" and characterizes the type of volumetric stress state: if $\mu_\sigma \in [-1; -0.5]$ – corresponds to a generalized tensile state; if $\mu_\sigma \in [-0.5; +0.5]$ – corresponds to a generalized shift state; if $\mu_\sigma \in [0.5; 1]$ – corresponds to a generalized compression state.

Today, there are quite a lot of recommendations for taking into account the value of the average principal stress σ_2. Thus, according to A. Nadai, instead of the ultimate state criterion in the form of the classical Coulomb–Mohr condition $\tau_n = f(\sigma_n)$, the condition for octahedral stresses should be written down:

$$\tau_{\text{okt}} = f(\sigma_{\text{okt}}). \tag{13}$$

Since the expressions for octahedral stresses include the value σ_2, the limit condition (13) depends on the stress σ_2. The expressions for the octahedral stresses are

$$\sigma_{\text{okt}} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3),\ \tau_{\text{okt}} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}.$$

The limit condition proposed by Professor A. I. Botkin, written out in terms of principal stresses, is [1]:

$$\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \frac{2\sqrt{2}\sigma_{\text{press}}\sigma_p}{\sigma_{\text{press}} + \sigma_p} + \frac{\sqrt{2}(\sigma_{\text{press}} - \sigma_p)}{\sigma_{\text{press}} + \sigma_p}(\sigma_1 + \sigma_2 + \sigma_3). \tag{14}$$

Comparing the equations (14) with the classical Coulomb–Mohr condition in the form (8), one can see that this limit condition extends the Coulomb–Mohr condition with respect to the 3D stress state.

For the 3D stress state, a generalized strength criterion can be proposed, which takes into account the material destruction as a result of both shearing and separation/gap:

$$\frac{\sigma_{\text{ext}}^2}{\sigma_{\text{press}}^2}\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] + \left(1 - \frac{\sigma_{\text{ext}}^2}{\sigma_{\text{press}}^2}\right)\sigma_1^2 \leqslant \sigma_{\text{ext}}^2. \tag{15}$$

In the case if $\sigma_{ext}/\sigma_{press} \sim 0$ (i.e. for ideally brittle materials), expression (15) is converted into the corresponding equation of the theory of maximum normal stresses.

Strength characteristics of the undermined repeatedly rock massifs. To estimate the rocks strength in the massif according to the known rock samples strength taken from core, the most common method is the use of a structural weakening coefficient μ_σ. It should be noted that the effect of structural heterogeneities on the massif strength is still poorly studied. Difficulties of the transition from sample strength to massif strength arises from the fact that the structural weakening coefficient μ_σ depends on a number of factors. The determination of μ_σ coefficients numerical values is carried out mainly by experimental methods, although approaches based on mathematical modelling are increasingly being used.

For the problem of the underground structures stability evaluation in an undermined repeatedly massif, one of the most important factors affecting the massif strength is the occurrence of fractures and slip lines due to primary excavation. Therefore, the structural weakening coefficient μ_σ can be a function of the relation between the bonding at the contacts of the weakenings in the undermined massif and the bonding in the undisturbed rock massif. In the absence of reliable representative experimental data, the value of this coefficient can be assumed to be in the range of 0.01–0.02 for a wide range of rock massifs [15]. Such value of the coefficient μ_σ indicates that the occurrence of slip and fracture lines in an undermined massif significantly affects the shear strength characteristics of a rock massif. Obviously, a reliable quantitative evaluation of the influence factors requires labour-intensive multiple in-situ tests.

The change in massifs strength due to mining workings can be performed by introducing a discrete series of coefficients, depending on the irregularities present in the considered volume of the rock massif. For example, an effective way to account for the presence of low thickness clay interlayers is to introduce attenuation coefficients into the strength characteristics of the massifs, which is actively used in mining practice at the Starobin potash salt deposit.

An approach based on empirical dependencies of the form

$$\mu_\sigma = \eta_m \eta_\varphi / \eta_\alpha \tag{16}$$

can also be used to estimate the coefficient μ_σ. Here η_m is the strength reduction factor which depends on the ratio of the cavity size to the size l^* of the elementary block; η_φ and η_α take into account the rock strength reduction due to the spatial arrangement of the weakening planes in the elements of the underground structure (cavity walls and roofs).

Expression (16) can be supplemented by introduction of additional coefficients. For example, it is possible to take into account the reduction of the massif strength over time by means of the coefficient η_t. Structure of strength functional dependence on time factor can be as follows (based on the results of laboratory tests):

$$\eta_t = \eta_\infty + (1 - \eta_\infty)\exp(-\alpha t). \tag{17}$$

Here η_∞ is a parameter characterizing long-term strength; α is a parameter characterizing rate of strength reduction over time. In most cases it is correct to assume $\alpha = 0.02$ in the absence of experimental data (if time t is measured in days).

Taking into account the marked attenuation coefficients, the Coulomb–Mohr strength condition can be represented, for example, as [6]:

$$\tau_n - \sigma_n - \sin\varphi[\tau_n + \sigma_n + 2\Phi(k_\varphi) - k_t \operatorname{ctg}\varphi] = 0, \tag{18}$$

where $k_t = k \cdot \eta_t \cdot \eta_\sigma$; k and ctgφ are respectively, as before, bonding and coefficient of internal friction; $\mathrm{F}(k_\varphi)$ is a function describing the change in the bonding coefficient as a function of η_φ and η_α.

The ratio (18) can also be supplemented by the introducing a coefficient that takes into account the dynamic loading of the rock massif. Let $\sigma_{max.d}$ is the fatigue rock strength under dynamic loading, and $\sigma_{c.st}$ and $\sigma_{p.st}$ are the compressive and tensile strengths of the rock under static loading, respectively. The stress relation $k_d = \sigma_{max.d}/\ \sigma_{c.st(h.st)}$ is called the dynamic loading coefficient. In the article [16] it is shown that an approximate value of the dynamic loading coefficient for rocks can be calculated using the formulas: $k_{d.c} \cong 1.71 - 0.288\lg N$ for compression and $k_{d.p} \cong 1.24 - 0.240\lg N$ for tension. Here N is the number of load cycles.

One of the main consequences of the rock massif technological disturbance is a change in the rocks bonding coefficient. Therefore, the bonding coefficient can be represented as a distance function to the area of technological disturbance. In accordance with this hypothesis, it is necessary to introduce a strength reduction function into the strength condition, which takes into account the degree of the rocks deformation. Studies carried out by various authors, based on the processing of experimental data, have established that this dependence is highly nonlinear and can be assumed to be exponential [13].

At the limit of information about the extreme rock deformation, the degree of fragmentation can be taken as a destruction measure. Then the effect of rock strata technological heterogeneity on its strength properties can be taken into account by the degree of fragmentation K_p. The following law of change with distance from the technological disturbance (cavity contour) can be adopted for the degree of fragmentation K_p:

$$F\left(K_p\right)=\exp\left[-\alpha_k\left(K_p-K_p^{\text{H}}\right)\right].$$

Here K_p^{H} is the initial degree of rock fragmentation at the strength (determined experimentally); α_k is a parameter which values vary between 15 and 25. The specific value of α_k is determined from the condition that at $K_p \to K_p^{\Pi}$ the function is $F(K_p)\to 0$. In turn, here K_p^{Π} is the ultimate degree of fragmentation. The value K_p^{Π} is within a rather tight range from 1.1 to 1.3, with the lower value being typical for harder rocks. This value is also determined experimentally, but due to its small deviation from the one, it can be assumed with a safety factor that $K_p^{\Pi} = 1$.

If the degree of fragmentation is taken as a measure of strength loss, then the two extremes of the strength reduction function can be assumed to be equal:

$$C(\rho)=C-C_1/\rho^n, \tag{19}$$

where C is the rocks bonding in the undisturbed massif, $\rho = R/R_0$, R is the radial coordinate, R_0 is the cavity equivalent radius, C_1, n are experimentally determined parameters.

Dependence (19) shows that the bonding function takes maximum value only at infinity (at $\rho\to\infty$).

From physical aspects, it seems more correct to consider that the bonding coefficient, depending on the current radius, increases from its minimum value C_0 at the cavity contour to its maximum value C_1 at the boundary separating the elastic region from the inelastic area. Due to this fact, when studying the mechanical processes in the vicinity of underground cavity, it is convenient to represent the variable coefficient $C(\rho)$ in the form:

$$C(\rho)=\begin{cases}(C_1-C_0)f(\rho)+C_0, & \text{if } 1\leqslant\rho\leqslant\rho_0\\ C, & \text{if } \rho>\rho_0\end{cases}. \tag{20}$$

Here C is a bonding in the undisturbed massif, $f(\rho)$ is a loss of strength function reflecting the heterogeneity nature of the rock massif properties when we are moving from the cavity contour deep into the rock massif.

The function $f(\rho)$ can be presented as follows:

$$f(\rho)=\frac{\rho_0^n}{\rho_0^n-1}\left(1-\frac{1}{\rho^n}\right);\ f(\rho)=\left(\frac{\rho^n-1}{\rho_0^n-1}\right)^m;\ f(\rho)=\log_{\rho_0}\rho;\ f(\rho)=\frac{l^{\rho-1}-1}{l^{\rho_0-1}-1},\ \text{etc.} \tag{21}$$

In these expressions, $\rho = r/R_0$ is the dimensionless current radius, r is the dimensional radial coordinate, R_0 is the cavity radius, $\rho = r_0/R_0$ is dimensionless external radius of the zone of non-elastic strains.

Thus, change of rock strength characteristics in the vicinity of the specific cavity when we are moving from the cavity contour can be described by dependence (20) and by a function (21). Accounting for several cavities placed arbitrarily in the rock massif can be performed using R-functions according to the method described in the [12].

Conclusion. Methodological approaches to the selection of ultimate state criteria and strength characteristics of the repeatedly undermined rock massifs were developed. These approaches were designed to provide parametric support to the geomechanical modelling of the massif stress-strain state and the mining systems of the Starobin potash deposit's mine fields planned for the additional mining of the mineral reserves left.

It was determined that due to formation of complex stress state in the undermined rock massif, where the areas of generalized compression, tension and shear simultaneously present, it is necessary to use complex criterion to study the ultimate state of the massif. Determination of such criterion can be carried out using several approaches. The first approach is to select some criteria that evaluate the massif ultimate state by certain types of its stress-strain state: the criterion of the *maximum* normal stresses (3), the criterion of the *maximum* linear strains (5), the criterion of the *maximum shearing* stress (6) and the Coulomb–Mohr criterion (8). The second approach is to construct a combined limit state criterion (9) for materials whose ultimate tensile and compressive stresses differ significantly. In this case, the parameter α (10) characterizing the type of stress state and the parameter β (11) characterizing the properties of the material are introduced, which together determine the character of failure – tear or shear. Thus, at the stage of deformation up to the "critical point" (ultimate strength), the strength of the massif should be considered by a set of the criteria described above, including stress-strain components. At the same time, in order to describe the behavior of rocks beyond the limit of strength (beyond stage of deformation) it is necessary to apply deformation theory of strength using the developed equation of the deformation strength criterion (12).

When calculating the strength characteristics of the repeatedly undermined rock massif, it is recommended to use a structural attenuation coefficient as the product of several factors (16), (17), taking into account different types of disturbance of the primary undermined massif and the time factor. The Coulomb–Mohr strength condition (with regard to the composite coefficient of structural attenuation) is recommended to use in the proposed form (18). The change of the rock's bonding coefficient is represented as a distance function to the area of technological disturbance as one of the main consequences of technological disturbance of the rock massif. It is shown that, taking into account the degree of fragmentation, the change in the strength characteristics of rocks in the undermined massif can be described by the proposed dependence (20) and the function (21).

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