Estimation of Difficulty of Multidimensional Knapsack Problems with Demand Constraints

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1 Introduction

The development of computational complexity theory led to a fascinating insight into the inherent difficulty of computational optimization problems. Optimization problem of knapsack type involves various applications in engineering, science and economics, it is significant to solve various kinds of knapsack problems. Recently, an improved surrogate constraints method is proposed to solve multidimensional nonlinear knapsack problems of large size, witch have surrogate gaps. However the knapsack problems are of NP-hard. Furthermore, the knapsack problems with demand constraints are frequently quite difficult to obtain high quality solution because of loss of monotonicity of constraints. To obtain even feasible solution is extremely difficult. Application of the algorithms to such a problem directly is not effective.

In this report, by using entropy, a method for evaluating difficulty of multidimensional 0-1 knapsack problems with demand constraints is proposed. It is shown in chapter 5 that solving the problems with demand constraint is rather difficult than solving that with ordinary problems with no demand constraint. Computational experiments show that the proposed method is effective to evaluate the computational difficulty of the problems.

2 Formulation of Problems

Multidimensional Knapsack Problems with Demand Constraints can be formulated as follows;

$$[P] \quad \text{maximize} \quad f(\boldsymbol{x}) = \sum_{n=1}^{N} a_n x_n \qquad (1.a)$$

$$\text{subject to} \quad g_m(\boldsymbol{x}) = \sum_{n=1}^{N} c_{mn} x_n \leq b_m,$$

$$m = 1, 2, \ldots, m1, \qquad (1.b)$$

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$$m = m1+1, m1+2, \ldots, M \qquad (1.b)$$

$$\boldsymbol{x} (= (x_1, x_2, \ldots, x_N)^T) \in \mathbf{X} = \Pi_{n=1}^{N} \{0, 1\} \subseteq \mathbf{R}^N, \quad (1.c)$$

where, $\boldsymbol{x}$ denotes an N dimensional 0-1 valued decision variable, $f(\boldsymbol{x})$ denotes N dimensional vector-valued objective function $(g_1(\boldsymbol{x}), g_2(\boldsymbol{x}), \ldots, g_{m1}(\boldsymbol{x}))^T$ and $(g_{m1+1}(\boldsymbol{x}), g_{m1+2}(\boldsymbol{x}), \ldots, g_M(\boldsymbol{x}))^T$ denote $m1$ and $M - m1$ dimensional vector-valued constraint functions, respectively. The vector $\boldsymbol{b} = (b_1, b_2, \ldots, b_{m1}, b_{m1+1}, \ldots, b_M)^T$ denotes available amounts of resources. The constraint (1.b) is of ordinary type, and the constraint (1.c) is of demand type.

By using a surrogate multiplier $\boldsymbol{u} = (u_1, u_2, \ldots, u_M)^T$, the problem $[P]$ is translated into the surrogate demand constraints problem;

$$[P^{\mathrm{S}}(\boldsymbol{u})] \quad \text{maximize} \quad f(\boldsymbol{x}) \qquad (2.a)$$

$$\text{subject to} \quad \boldsymbol{u}^T \boldsymbol{h}(\boldsymbol{x}) \leq 0, \qquad (2.b)$$

$$\boldsymbol{x} \in \mathbf{X}, \qquad (2.c)$$

where,

$$\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{b}, \qquad (2.d)$$

$$\boldsymbol{u} \in \mathbf{U} = \{\boldsymbol{u} \in \mathbf{R}^M | \sum_{m=1}^{M} u_m \leq 1, \quad \boldsymbol{u} \geq 0\} \qquad (2.e)$$

Inequality $(2.b)$ is called a surrogate demand constraints equation.

A surrogate dual $[P^{\mathrm{SD}}]$ to the original problem $[P]$ is written as follows;

$$[P^{\mathrm{SD}}] \quad \min\{\mathrm{opt}[P^{\mathrm{S}}(\boldsymbol{u})] : \quad \boldsymbol{u} \in \mathbf{U}\} \qquad (3)$$

where, opt[$P^{\mathrm{S}}(\boldsymbol{u})$] denotes an optimal value of the objective function of the problem P^{S}.

3 Estimation of Difficulty of Problems

Let p_n be a probability such that

$$f^{\mathrm{UB}}(\boldsymbol{x})|_{x_n=0} \geq f^{\mathrm{UB}}(\boldsymbol{x})|_{x_n=1}$$

or

$$f^{\mathrm{UB}}(\boldsymbol{x})|_{x_n=1} < f^{\mathrm{UB}}(\boldsymbol{x})|_{x_n=0}$$

To estimate the probality p_n, the normalized difference of the upper bound;

$$\frac{d_n}{f^{\mathrm{REAL}}(\boldsymbol{x}) - f^{\mathrm{NEAR}}(\boldsymbol{x})}$$

$$d_n = |f^{\mathrm{UB}}(\boldsymbol{x})|_{x_n=0} - f^{\mathrm{UB}}(\boldsymbol{x})|_{x_n=1}|$$

The following entropy is used to estimate the difficulty of the problem;

$$H = \sum_{n=1}^{N} h_n$$

$$h_n = -p_n \log_2(p_n) - (1-p_n)\log_2(1-p_n)$$

4 Computational Experiments

The results obtained from computational experiments to 60 problems shown in the references Chu et.al.(12). The problems tested are generated by using random number generator such that the objective functions and constraint functions are correlated to mutually.

From the obtained results, the probability p_n in the problems with demand constraints may be estimated approximately as

$$2^{-\alpha d_n - \beta},$$

where the values of parameters α and β are -12.2 and 1.0, respectively.

In order to study effectiveness of the estimation method presented, the value of PGC(Percent Gap Closure) is introduced;

$$\frac{f^{\mathrm{REAL}} - f^{\mathrm{OPT}}}{f^{\mathrm{REAL}} - f^{\mathrm{OPT}}} \times 100$$

Table 1. Difference of upper bounds, probability p_n and its approximation

Difference	Problem				
δ_n	00～09	10～19	20～29	00～29	p_n
0.0～	0.5469	0.5517	0.4098	0.5027	0.503
0.01～	0.2800	0.2937	0.3085	0.2938	0.234
0.1～	0.1161	0.1008	0.1221	0.1129	0.100
0.2～	0.0421	0.0382	0.0490	0.0427	0.043
0.3～	0.0265	0.0167	0.0187	0.0206	0.018
0.4～	0.0074	0.0000	0.0000	0.0027	0.008
0.5～	0.0000	0.0000	0.0000	0.0000	0.003
0.6～	0.0000	0.0000	0.0000	0.0000	0.001
0.7～	0.0000	0.0000	0.0000	0.0000	0.001
0.8～	0.0000	0.0000	0.0000	0.0000	0
0.9～	0.0000	0.0000	0.0000	0.0000	0
1.0～	0.0000	0.0000	0.0000	0.0000	0

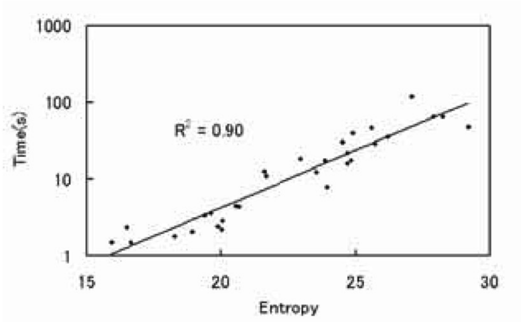

Fig.1. Entropy vs Computational Time

$(M = m1 = 5, N = 250)$

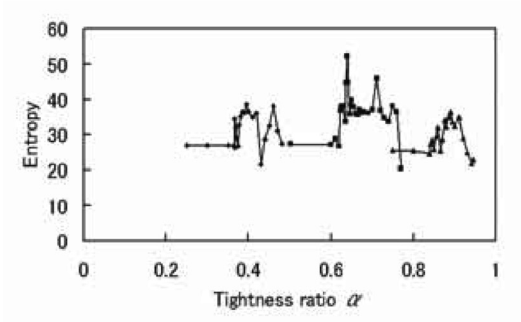

Fig.2. Entropy vs Tightness Ratio

$(m1 = 4, M = 5, N = 500$: Single Demand Constraint)

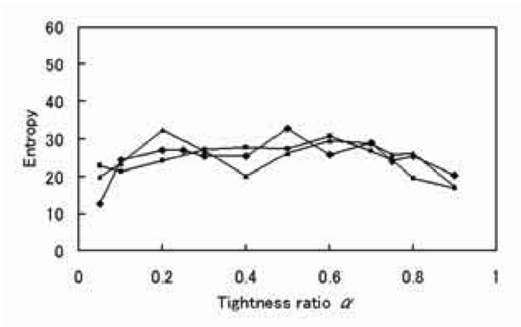

Fig.3. Entropy vs Tightness Ratio

$(m1 = M = 4, N = 500\ )$

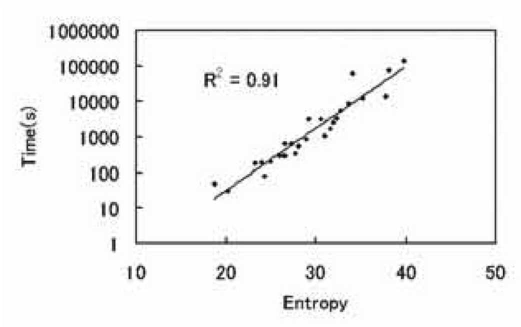

Fig.4. Computational Time vs Entropy

$(m1 = M = 5, N = 500\ )$

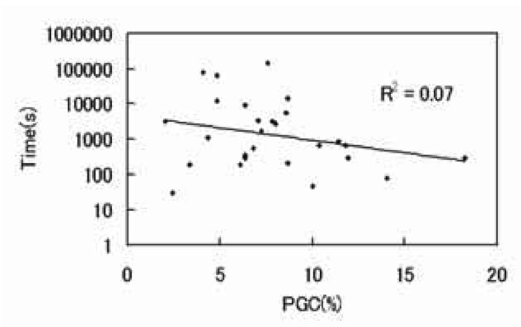

Fig.5. Computational Time vs PGC

$(m1 = M = 5, N = 500\ )$

5 Conclusions

Estimation method for difficulties of multidimensional 0-1 knapsack problems with multiple constraints including demand constraints is presented. From computational experiments, the probablity of p_n such that

$$f^{\mathrm{UB}}(\boldsymbol{x})|_{x_n=0} \geq f^{\mathrm{UB}}(\boldsymbol{x})|_{x_n=1}$$

can be approximately expreed as

$$2^{-\alpha d_n - \beta}$$

and the obtained computational results show that the approximated expression for the probability p_n is effective to estimate the entropy of difficulties of the tested problems.

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