https://dx.doi.org/10.4314/dujopas.v8i2b.3
ISSN (Print): 2476-8316
ISSN (Online): 2635-3490

Dutse Journal of Pure and Applied Sciences (DUJOPAS), *Vol. 8 No. 2b June 2022*

Utilization of Fuzzy Critical Path Method and Fuzzy Program Evaluation and Review Technique for Building a Hydroelectric Power Station

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Abstract

In this paper, fuzzy critical path method and fuzzy program evaluation and review technique are used to calculate the earliest project completion time for constructing a hydroelectric power plant project. Fuzzy trapezoidal numbers are used to estimate the activity time and determine the range of pessimistic to optimistic variation of time. Furthermore, the minimum and maximum times of project completion duration were calculated by using arithmetic operations and ranking of fuzzy trapezoidal numbers. These hybrid methods are able to deal with the limitations associated with classical critical path method and program evaluation and review technique. The fuzzy techniques were applied to network activities in a manner similar to the classical methods for optimizing the project completion duration, thereby minimizing the cost of the project. Analysis was carried out to determine the critical path through the use of fuzzy critical path method. The fuzzy program evaluation and review technique was also used to determine the probability of completing the project at a scheduled time. These two methods were then compared and the most probable scenarios were analyzed. Finally, it was concluded that fuzzy program evaluation and review technique is better than fuzzy critical path method and more efficient in terms of early project completion time.

Keywords: Critical path method, Fuzzy sets, management of project, program evaluation and review technique, trapezoidal numbers.

INTRODUCTION

Hydroelectric power is a form of renewable source of energy that relies on the water cycle which is constantly renewed by the sun. Other renewable sources of energy are solar power, tidal power, geothermal, wind power and wave power (Britannica, 2021).

Hydroelectric power is generated by the conversion of energy from flowing water into electricity. The water flow is caused by the force of gravity on water runoff from streams, lakes and mountains. The flowing water is then used to turn turbines and generators that produce electricity. Since the initial source of this kind of electricity is water, it is called hydroelectric

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power or hydroelectricity. The facilities where hydroelectric power is generated are the hydroelectric power plants or hydroelectric power stations. Most hydroelectric power stations are located on canals, streams and rivers. However, for a reliable water supply dams are usually built. These dams are used to store water that is later released for domestic and industrial use, irrigation and generation of electricity.

Timely completion of the construction of hydroelectric power plants is indispensable due to their economic, political, and social impacts. Many of the factors considered in the construction of dams, especially those related to flowing water runoff are vague, subjective and difficult to quantify. In order to realistically estimate project durations, consideration must be made of the uncertainties and variations as well as create a robust and secured scheduling at the beginning of the project (Semsettin *et al.*, 2012).

Murty (2003) discuss techniques that help in planning, scheduling, and controlling of a hydroelectric dam building project. The study used CPM to break down the project into a number of activities, and the precedence relationships of these activities were represented through the use of project network. The network was then used to make a schedule for these activities over time that minimizes the project completion duration. The dynamic programming algorithm was used to determine the longest route applying on the project network.
AbdulRahman *et al*. (2010) discussed the techniques of project management for the construction of nuclear power plants in Malaysia. The objective of their study was to ensure that the construction of a reactor is completed within a stipulated time and not to exceed the estimated cost. CPM, Gantt chart, Microsoft Project and PERT were the techniques used in the management of the project. Findings from the study revealed that the estimated time, cost and quality of the project was managed more effectively through the use of these techniques.

Habibi *et al*. (2018) presents a step-by step technique for accurate estimation of time and cost of projects using PERT in conjunction with expert views as trapezoidal fuzzy numbers. Essentially, Fuzzy Delphi Method (FDM) and experts' opinions were used to reduce to a greater extent the effects of existing uncertainties on the final results. Finally, the project completion time and cost estimates were found to be more appropriate and an improvement over the classical PERT.

In this study, FCPM and FPERT are used to calculate the project completion time for the construction of a hydroelectric power plant. The project durations were analyzed with the help of these two techniques and comparison was done to ascertain which of the two methods is better.

METHODOLOGY

It will be helpful to review some of the concepts of fuzzy sets before explaining the nitty-gritty of FCPM and FPERT. A fuzzy set was born out of the effort to describe the real world that is characterized by vague definition. Given a universe of discourse X , a fuzzy subset A of X is defined by a membership function $f_A(x)$ which associates with each element x in X a real number in the interval $[0,1]$. The function $f_A(x)$ represents the value of the degree of membership of x in A. A trapezoidal fuzzy number is a fuzzy number whose membership function $f_A(x)$ is as described below:

$$f_A(x) = \begin{cases} (x-a)/(b-a), & a \le x \le b \\ 1, & b \le x \le c \\ (x-d)/(c-d), & c \le x \le d \\ 0, & otherwise \end{cases} \tag{1}$$

The trapezoidal fuzzy number is usually represented by a quadruple (a,b,c,d), where a, b, c and d are real values (Dubois and Prade, 2000). It is also significant to note that if $a=b$ and $c=d$, then A is called a crisp interval, and if $a=b=c=d$, then A is a crisp value. The fuzzy number A is called a triangular fuzzy number instead of trapezoidal if $b=c$. It is important to note that membership grades are not the same as probabilities though, the membership grades are in the interval $[0,1]$.

In this paper, only the arithmetic operations of Addition $(\oplus)$ and subtraction (Θ) on trapezoidal fuzzy numbers are considered. These basic operations on two trapezoidal fuzzy numbers $X_1=(a_1,b_1,c_1,d_1)$ and $X_2=(a_2,b_2,c_2,d_2)$ are done as in (2) and (3).

$$X_1 \oplus X_2 = (a_1,b_1,c_1,d_1) \oplus (a_2,b_2,c_2,d_2) = (\mathrm{a}_1+a_2, \mathrm{b}_1+b_2, \mathrm{c}_1+c_2, \mathrm{d}_1+d_2) \tag{2}$$

$$X_1 \Theta X_2 = (a_1,b_1,c_1,d_1) \Theta (a_2,b_2,c_2,d_2) = (\mathrm{a}_1-d_2, \mathrm{b}_1-d_2, \mathrm{c}_1-b_2, \mathrm{d}_1-a_2) \tag{3}$$

Procedure for classical Critical Path Analysis

Finding the critical path of a project involves modeling the project by using the following steps:

1. Develop a list of all activities required to complete the project.
2. Show the activity duration that each activity will take to complete.
3. Determine the dependency relationship between the various activities in the project. Most of these activities are dependent upon the completion of others.
4. Draw a project network chart which shows the various activities and the dependency relationship of the activities. Each activity with its completion time may also be indicated in the network.
5. Determine the earliest start time and the earliest finish time for each activity using the activity time estimates by making a forward movement through the network. The total time required to complete the project is equivalent to earliest finish time for the last activity in the project.
6. The project completion time in step 5 is the latest finish time for the last activity. The earliest finish time for the last activity is the same as the latest finish time. This is achieved by making a backward movement through the network to identify the start and latest finish time for each activity.
7. To calculate the slack time available for the activities use the difference between the latest start time and the earliest start time for each activity. The activities with zero slack are the critical path activities.

Procedure for FCPM

Before the consideration of steps involved in the determination of FCPM and FPERT, the following notations used in this work are explained.

N: All nodes in a project network,

A_{ij}: The activity between two nodes (from i to j)
FAT_{ij}: Fuzzy activity time of A_{ij}
EFT: Earliest fuzzy time
LFT: Latest fuzzy time
LFT: Latest fuzzy time
SFT_{ij}: The total fuzzy slack time of A_{ij}
$FCPM(\mathrm{P}_n)$: The fuzzy completion time of path P_n.
t: Number of activities in a project network
S(j): The set of all activities after node j.
NS(j): The set of all nodes connected to all activities after node j $=\{k / A_{jk} \in S(j), \mathrm{k} \in \mathrm{N}\}$
P(j): The set of all activities before node j.
NP(j): The set of all nodes connected to all activities before node j $=\{\mathrm{i}/ A_{ij} \in P(j), \mathrm{i} \in \mathrm{N}\}$.

The following steps can be used to calculate the fuzzy critical path, where the completion time of each activity in the network is represented by a trapezoidal fuzzy number.

Step 1: Set initial node $EFT_1 = (0,0,0,0)$.

Step 2: Evaluate β, the risk factor for each A_{ij} using (4).

$$\beta = \sum_i \sum_j \left(\frac{(b_{ij} - a_{ij})}{(b_{ij} - a_{ij}) + (\mathrm{d}_{ij} - c_{ij})} \right) / t \tag{4}$$

The analysis of the risk factor of the project is calculated as follows: the situation is risky if $\beta < 0.5$, neutral if $\beta = 0.5$, and more risky if $\beta > 0.5$.

Step 3: Determine the EFT for individual node using (5).

$$EFT_j = EFT_i \oplus FAT_{ij} \tag{5}$$

Step 4: Take EFT_j with the highest value for each node after comparing the EFT_j s for intersecting nodes using (6).

$$EFT_j = \max\left\{EFT_i \oplus FAT_{ij}\right\} = \max\left\{(a_x, b_x, c_x, d_x), (a_y, b_y, c_y, d_y)\right\} \tag{6}$$

4(a): Evaluate the values of x_1 and x_2 using (7)

$$x_1 = \min\left\{a_x, b_x, c_x, d_x, a_y, b_y, c_y, d_y\right\}, x_2 = \max\left\{a_x, b_x, c_x, d_x, a_y, b_y, c_y, d_y\right\} \tag{7}$$

4(b): Determine the values of $\mathrm{R}((a_x, b_x, c_x, d_x))$ and $\mathrm{R}((a_y, b_y, c_y, d_y))$ using (8)

$$\mathrm{R}((a_i, b_i, c_i, d_i)) = \beta[(\mathrm{d}_i - \mathrm{x}_i) / (\mathrm{x}_2 - \mathrm{x}_1 - \mathrm{c}_i + \mathrm{d}_i)] + (1-\beta)[1-(\mathrm{x}_2 - \mathrm{a}_i)/(\mathrm{x}_2 - \mathrm{x}_1 + \mathrm{b}_i - \mathrm{a}_i)] \tag{8}$$

4(c): Rank the results of $\mathrm{R}((a_i, b_i, c_i, d_i))$ and look for the maximum value and take it.

Step 5: Determine LFT of individual node using (9)

$$LFT_j = EFT_k \Theta FAT_{jk} \tag{9}$$

Step 6: Consider LFT_j with the lowest value for each node after comparing the $LFT_j s$ for intersecting nodes using (10)

$$LFT_j = \min\left\{EFT_k \Theta FAT_{jk}\right\} = \min\left\{(a_x, b_x, c_x, d_x), (a_y, b_y, c_y, d_y)\right\} \tag{10}$$

The sub-steps of step 6 are generated in a similar way to the sub-steps of step 4 above.

Step 7: Determine SFT of every activity using (11)

$$SFT_{ij} = LFT_j \Theta (EFT_i \oplus FAT_{ij}) \tag{11}$$

Step 8: Examine all the paths in order to determine the FCPM values of all possible paths using (12).

$$FCPM(P_n) = \sum SFT_{ij} \tag{12}$$

Step 9: Consider all possible FCPMs values using (13) and rank the results of $\mathrm{R}((a_i, b_i, c_i, d_i))$ and take the one that has the lowest value.

$$FCPM(P_n) = \min \{ FCPM(P_i) \mid \mathrm{i} = 1, 2, 3, ..., \mathrm{n} \} \tag{13}$$

Step 10: finally, the path with the lowest FCPM value is considered as the critical path.

Procedure for FPERT

The use of FPERT requires finding a generalized mean and calculating the standard deviation based on trapezoidal fuzzy numbers. The generalized mean is calculated using (14).

$$\mathrm{M}(a_1, b_1, c_1, d_1) = \frac{a_1 + b_1 + c_1 + d_1}{4} \tag{14}$$

The standard deviation for the trapezoidal fuzzy numbers is calculated using (15).

$$\mathrm{D}(a_1, b_1, c_1, d_1) = \frac{[3(a_1^2 + b_1^2 + c_1^2 + d_1^2) - 2(a_1 b_1 + a_1 c_1 + b_1 c_1 + a_1 d_1 + b_1 d_1 + c_1 d_1)]}{36} \tag{15}$$

After the critical path is determined, the next thing to do is to find the probability of meeting the scheduled time of the project completion. When the project activities are large, as with the PERT projects then the expected time of individual activities has random probability distribution and the variation of the expected time for the project is completely assumed to have a normal distribution. This assumption of normal probability distribution is based on central limit theorem (Punmia & Khandelwal, 2006), and is achieved by finding the probability factor.

The probability factor called Z-Score is the standardized value that specifies the exact location of an X value within a distribution by describing its distance from the mean in terms of standard deviation σ units. Essentially, Z-Scores describe the exact location of a score within a distribution. Z-Score may be positive, zero or negative. The probability of completing a project in time is more than 0.5, if Z is positive. The probability of completing the project in time is 0.5, if Z is zero and the probability of completing the project in time is less than 0.5, if Z is negative.

The mean and standard deviation of the normal probability distribution are used to calculate the Z-scores using relation (16).

$$Z = \frac{X - \mu}{\sigma} \tag{16}$$

Illustration of Application of FCPM and FPERT

The application of FCPM and FPERT are illustrated using the building of a hydroelectric power plant project. The details of the various activities involved in the project, their dependency relationships, and the number of weeks estimated to complete each job are given in table 1. These expected completion time of each activity are given in form of fuzzy trapezoidal numbers.

Table 1: FATs for each activity of building a hydroelectric power plant

Activity	Description	Immediate Predecessor	Duration (Months)
A	Ecological Survey	-	6.2 6.4 7 7.2
B	File environmental impact report; get approval	A	9.1 9.5 9.6 10
C	Economic feasibility study	A	7.3 7.4 7.9 8
D	Preliminary design and cost estimation	C	4.2 5 5.5 6
E	Project approval and funding commitments	B, D	10.2 10.5 11 12
F	Call quotation for equipment (turbines, generators)	E	4.3 5 6 7
G	Select supplier for equipment	F	3.1 4 5 6
H	Final design of project	E	6.5 6.9 7 7.9
I	Select construction contractor	E	2.7 3 4 5
J	Arrange construction materials supply	H, I	5.2 5.5 6 6.5
K	Dam building	J	24.8 25 25.5 26
L	Power station building	J	18.4 19 19.5 20
M	Power lines erection	G, H	20.3 21 21.5 22
N	Turbines, generators installation	G, L	6.8 7 7.5 7.6
O	Build-up reservoir water level	K	2.1 2.3 2.5 3
P	Commission the generators	N, O	1.2 1.5 2 2.5
Q	Start supplying water	M, P	1.1 1.5 1.8 2

The first step is to draw the project network chart of the hydroelectric power plant for estimating the project completion time as shown in figure 1.
The next step is to calculate the earliest fuzzy time and latest fuzzy time for individual node using (5), (6) and (9), (10). These values are shown on table 2 and table 3 respectively.
The total fuzzy slack times are calculated for each activity using (11), and the results are shown on table 4.

Determination of Critical Paths

If there exists a path P_c in a project network such that $FCPM(P_c) = \text{minimum}\{FCPM(P_i) \mid P_i \in \mathrm{P}\}$, then the path P_c is a called fuzzy critical path. When all possible paths P are found, then FCPMs for each node are evaluated using (12). The FCPM (P_i) is calculated using (13) and the results obtained are compared using Step 9. Table 5 give summary of calculated values for FCPM and R[FCPM (P_i)] of every path obtained.

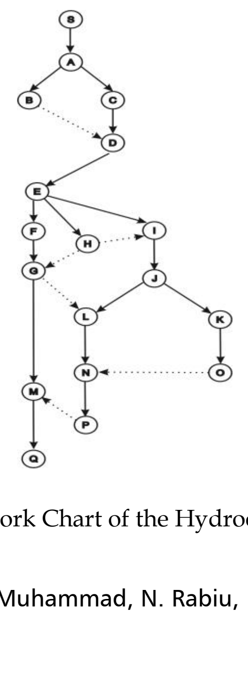

Figure 1: Project Network Chart of the Hydroelectric Power Plant

Table 2: EFT Values

NODE	EFT Values			
1	0	0	0	0
2	6.2	6.4	7	7.2
3	15.3	15.9	16.6	17.2
4	22.6	23.3	24.5	25.2
5	26.8	28.3	30	31.2
6	37	38.8	41	43.2
7	41.3	43.8	47	50.2
8	44.4	47.8	52	56.2
9	50.9	54.7	59	63.7
10	53.6	57.7	63	68.7
11	58.8	63.2	69	75.2
12	83.6	88.2	94.5	101.2
13	102	107.2	114	121.2
14	122.3	128.2	135.5	143.2
15	129.1	135.2	143	150.8
16	131.2	137.5	145.5	153.8
17	132.4	139	147.5	156.3
18	133.5	140.5	149.3	158.3

Table 3: LFT Values

NODE	EFT Values			
1	32.8	-8.6	8.6	24.8
2	40	-1.6	15.2	31
3	50	8	24.7	40.1
4	58	15.9	32.1	47.4
5	64	21.4	37.1	51.6
6	18.4	32.4	47.6	61.8
7	25.4	38.4	52.6	66.1
8	31.4	43.4	56.6	69.2
9	38.9	50.4	63.5	75.7
10	43.9	54.4	66.5	78.4
11	50.4	60.4	72	83.6
12	76.4	85.9	97	108.4
13	96.4	105.4	116	126.8
14	118.4	126.9	137	147.1
15	126	134.4	144	153.9
16	129	136.9	146.3	156
17	131.5	138.7	147.8	157.2
18	133.5	140.5	149.3	158.3

TABLE 4: SFT Values

SFT_{12}	32.8	-8.6	8.8	24.8
SFT_{23}	32.8	-8.6	8.8	24.8
SFT_{24}	42.8	1	18.3	33.9
SFT_{45}	32.8	-8.6	8.8	24.8
SFT_{35}	32.8	-8.6	8.8	24.8
SFT_{56}	-24.8	-8.6	8.8	24.8
SFT_{67}	-24.8	-8.6	8.8	24.8
SFT_{69}	-11.8	2.4	17.8	32.2
SFT_{610}	-4.3	9.4	24.7	38.7
SFT_{78}	-24.8	-8.6	8.8	24.8
SFT_{89}	-24.8	-8.6	8.8	24.8
SFT_{0910}	-11.8	2.4	17.8	32.2
SFT_{0813}	-24.8	-8.6	8.8	24.8
SFT_{1011}	-24.8	-8.6	8.8	24.8
SFT_{1113}	39.6	16.9	33.8	49.6
SFT_{1112}	-24.8	-8.6	8.8	24.8
SFT_{1315}	-2.8	12.9	29.8	45.1
SFT_{1216}	24.8	39.9	55.8	70.3
SFT_{1615}	-2.8	12.9	29.8	45.1
SFT_{1517}	-21.8	-6.3	11.1	26.9
SFT_{0814}	40.2	53.4	68.2	82.4
SFT_{1714}	40.3	53.4	68.2	82.4
SFT_{1418}	-11.7	3.2	19.6	34.9

TABLE 5: FCPM and R[FCPM (P_i)] values of each possible path

	POSSIBLE PATHS	FCM(P_i)				R
1	$1-2-4-5-6-10-11-12-16-15-17-14-18$	8.8	22	224.5	410.7	0.3277
2	$1-2-4-5-6-9-10-11-12-16-15-17-14-18$	39.1	65.9	282.4	481.9	0.2130
3	$1-2-4-5-6-9-10-11-13-15-17-14-18$	78.7	51.5	251.6	436.4	0.3792
4	$1-2-4-5-6-9-10-11-12-16-15-17-14-18$	39.1	65.9	282.4	481.9	0.4054
5	$1-2-4-5-6-7-8-14-18$	62.5	14.6	150.1	275.2	0.2599
6	$1-2-4-5-6-7-8-9-10-11-12-16-15-17-14-18$	-23.5	37.7	291	524.1	-3.4471
7	$1-2-4-5-6-7-8-9-10-11-13-15-17-14-18$	16.1	23.3	260.2	478.6	0.2874
8	$1-2-4-5-6-7-8-13-15-17-14-18$	13.1	-64.8	199.8	372	0.0624
9	$1-2-4-5-6-9-10-11-13-15-17-14-18$	78.7	51.5	251.6	436.4	0.8016
10	$1-2-4-5-6-7-8-14-18$	62.5	14.6	150.1	275.2	0.2199
11	$1-2-3-5-6-10-11-12-16-15-17-14-18$	-1.2	12.4	215	401.6	0.7651
12	$1-2-3-5-6-9-10-11-12-16-15-17-14-18$	40.8	53.1	253.3	437.9	0.3755

13	$1-2-3-5-6-9-10-11-13-15-17-14-18$	68.7	41.9	242.1	427.3	0.3633
14	$1-2-3-5-6-9-10-11-12-16-15-17-14-18$	29.1	56.3	272.9	472.8	0.6131
15	$1-2-3-5-6-7-8-14-18$	51.7	5	140.6	266.1	0.2432
16	$1-2-3-5-6-7-8-9-10-11-12-16-15-17-14-18$	-33.5	28.1	281.5	515	0.3726
17	$1-2-3-5-6-7-8-9-10-11-13-15-17-14-18$	6.1	13.7	250.7	469.5	0.7931
18	$1-2-3-5-6-7-8-13-15-17-14-18$	3.1	3	190.3	362.9	0.2851
19	$1-2-3-5-6-9-10-11-13-15-17-14-18$	68.7	41.9	242.1	427.3	0.1076
20	$1-2-3-5-6-7-8-14-18$	52.5	5	140.6	266.1	0.2433

Using step 10 to generate table 5, and it is clear from the table that $R[\text{FCPM}(\text{P}_{16})]$ has the lowest value of all possible paths. Thus, the critical path is 1 – 2 – 3 – 5 – 6 – 7 – 8 – 9 – 10 – 11– 12 – 16 – 15 – 17 – 14 –18. It can then be concluded that the completion time of the project lies approximately between 140.5 and 149.3 weeks. i.e. (133.5, 140.5, 149.3, 158.3).

Use of FPERT

To use FPERT, standard deviation of the project is computed using the total values of variance of the activities on the critical path using (15). Using the values from table 6, the standard deviation is determined to be

$$\sigma = \left(\sum \sigma_{ij}^2\right)^{\frac{1}{2}} = (2.3966)^{\frac{1}{2}} = 1.548096\,.$$

The mean of the expected time for project completion is calculated using sum of mean of all activities on the critical path using (14). This is found from table 7 to be

$$\mu = \sum M_{ij} = 119.025$$

Table 6: Variance for each activity

Activity	B	C	D	Variance
A	6.4	7	7.2	0.0756
B	9.5	9.6	10	0.0456
C	7.4	7.9	8	0.0411
D	5	5.5	6	0.1964
E	10.5	11	12	0.2075
F	5	6	7	0.4631
G	4	5	6	0.5231
H	6.9	7	7.5	0.0564
I	3	4	5	0.3631
J	5.5	6	6.5	0.1089
K	25	25.5	26	0.0964
L	19	19.5	20	0.1564
M	21	21.5	22	0.1756
N	7	7.5	7.6	0.0497
O	2.3	2.5	3	0.0497
P	1.5	2	2.5	0.1089
Q	1.5	1.8	2	0.0511
TOTAL				2.7686
Activities of CP				2.3966

Table 7: Mean for each activity

Activity	B	C	D	Mean
A	6.4	7	7.2	6.700
B	9.5	9.6	10	9.550
C	7.4	7.9	8	7.650
D	5	5.5	6	5.175
E	10.5	11	12	10.925
F	5	6	7	5.575
G	4	5	6	4.525
H	6.9	7	7.5	6.975
I	3	4	5	3.675
J	5.5	6	6.5	5.800
K	25	25.5	26	25.325
L	19	19.5	20	19.225
M	21	21.5	22	21.200
N	7	7.5	7.6	7.225
O	2.3	2.5	3	2.475
P	1.5	2	2.5	1.800
Q	1.5	1.8	2	1.600
TOTAL				145.4
Activities of CP				119.025

The final step in the determination of FPERT is to calculate the Z-Score. The transformation of raw score (X value) to Z-Score is done using (16) and the results of calculation of the probabilities are given on table 8.

Table 8: Some probabilities of the project completion time

X	Z Values	Probability
< 116	-1.9540	0.0256
< 117	-1.3081	0.0951
< 118	-0.6621	0.2546
< 119	-0.0161	0.4920
<120	0.6298	0.7324
< 121	1.2758	0.8997
< 122	1.9217	0.9726
< 123	2.5677	0.9949
< 124	3.2136	0.9993
< 125	3.8596	0.99994

RESULTS AND DISCUSSION

The construction of the hydroelectric power plant project has 17 activities. The different activities in the project, their dependency relationships and fuzzy time estimates of each activity were given on table 1.

Critical path analysis method utilizing FCPM and FPERT was used to determine the total completion time for the construction of a hydroelectric power plant project. The activities on a critical path are called the critical activities and the procedure is also called the Critical Path Method. This study found the critical path to be 1 – 2 – 3 – 5 – 6 – 7 – 8 – 9 – 10 – 11– 12 – 16 – 15 – 17 – 14 –18, since $R[\text{FCPM}(P_{16})]$ has the lowest value of all possible paths. Using FCPM, it is further found that the completion time of the project lies approximately between 140.5 and 149.3 weeks.

It is established that the critical path does not allow any flexibility in the critical activities (Punmia and Khandelwal, 2006). Thus, any slight change in any of the critical activities A, B, C, E, F, G , H, I, J, K, L, O, P, N or Q on the critical path will adversely affect the entire completion time of the hydroelectric power plant building project.

Table 8 gives the Z–Score values with some probabilities of the project completion time. The table indicated that the probability of project completion time in less than 120 weeks is 73.24%, in less than 121 weeks is 89.97%, in less than 122 weeks is 97.26%, in less than 123 weeks is 99.49%, in less than 124 weeks is 99.93%, and in less than 125 weeks is 99.99%.
According to the results in general, there are huge differences between the two methods. It is observed that even taking into consideration of the worst case scenario, the probability of completing the hydroelectric power plant building project is less than 125 weeks. This value is less than the probable range of 140.5 to 149.3 weeks obtained through the use of FCPM. This shows that the completion time of the project using FPERT is shorter than the one obtained using FCPM. Thus, FPERT is more efficient than FCPM.

CONCLUSION

One of the factors responsible for extreme poverty, especially in the most rural areas is lack of employment opportunities. Electricity is indispensable in order to boost economic activities that can lead to creation of employment. However, the provision of electricity will not automatically give rise to economic development. Unless communities are supported so that they can develop new businesses that will effectively use electricity. This study wishes to provide such support for the efficient development of electricity generation project.

Operations research techniques, CPM and PERT were used to manage a hydroelectric project in order to optimize the project duration. These methods are usually used to schedule and control projects from the beginning to completion within a specific time (Bagshaw, 2021). The traditional means of project management using CPM and PERT has limited success in project management. This is due to the inability of these methods to efficiently deal with imprecision or vagueness in project time estimates. Fuzzy sets in conjunction with the classical version of these techniques of project management are shown to efficiently deal with imprecision or vagueness in the activity time estimations.

In this paper, FCPM and FPERT were used to estimate the activity completion time of a hydroelectric power project. Trapezoidal numbers are used to represents the range of pessimistic to optimistic variation of time. The minimum and maximum time of project duration is calculated using arithmetic operations and ranking method among the fuzzy numbers. The study has shown that FCPM and FPERT may be used to find the solutions of many project related problems that involves the scheduling of large-scale projects such as hydroelectric power plant construction project. These two techniques provide great benefit to the decision makers for being analytical and easy to use. Furthermore, the study determined and compared project completion time and the most probable scenario were analyzed. Finally, it has shown that FPERT is more efficient than FCPM.

Investigations carried out in the course of this work have shown that CPM and PERT techniques blended with fuzzy sets provides a fertile ground for further research. Thus, it is recommended that CPM and PERT techniques blended with fuzzy sets should be exploited for gainful research in project management.

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