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# Evaluation of Probabilistic Models for Characterizing Design Low-Flows of River Ogun, Southwest Nigeria

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**Abstract**

*Information of low-flow is important for maintaining instream flow, conserving biodiversity, enhancing food production, industrial abstraction, tourism and dilution of effluents from industries and households. This study establishes suitable probabilistic models for characterizing different durations of design low-flows of Ogun River. The adequacy of fit of four probability distributions, namely Reversed Generalized Extreme Value (GEVR) distribution, Generalized Normal (GNO) distribution, Generalized Logistic (GLO) distribution and Pearson Type III (PE3) were evaluated using the Anderson-Darling ($A^2$) goodness-of-fit statistic and the D-index diagnostic test. The study revealed that GLO is best suited for predicting the annual minimal, 3-day minima, 7-day minima, 10-day minima, 15-day minima and 30-day minima based on the $A^2$ and D-index values. Six mathematical models derived from probability plots were established to relate the different low-flow series to their non-exceedance probability. The models could be used for characterizing low-flows and for water resources management of Ogun River Basin.*

**Keywords:** Design low-flow, frequency Analysis, low-flow probabilistic models, water resources management.

## 1.0 INTRODUCTION

Engineers, policymakers and the general public need reliable intelligence to manage water resources sustainably. Models are proficient decision support outfits, and in recent times, insights generated from models are deployed for the development of smart systems that are capable of supporting timely decision-making in the water sector. The increase in water stress due to climate extremes, increasing population growth and industrial advancement has invigorated the need to pay close attention to minimum-flow or low-flow.

The International Glossary of hydrology specifies low-flow as the drift of water in a stream during a persistent dry atmospheric condition [1]. It is imperative to evaluate low-flows when initiating a water resources scheme to determine the viability of the scheme for the intended purpose before development, and in the case of huge capital investment, a detailed design low-flow frequency analysis must be undertaken [2].

Low-flow estimation is also expedient at the phase of operation of a water scheme to determine the means of precisely managing day-to-day water abstraction with the consciousness of not infringing the legal requirements of downstream users. Low-flow information will influence the decision-making of operators to maintain sufficient water flow for other important purposes such as conserving biodiversity, food production, household uses, industrial abstraction, tourism and the dilution of effluents from industries and households [2]. Low-flow estimations are also required to make timely operational decisions in cases of long-term planning [2].

Different guides have been established in the past to characterize low-flows [3]. According to Caruso [4], the methods for assessing low-flows for efficacious water management are the exceedance method such as frequency analysis using probability models, Tennant methods, hydraulic method, habitat methods such as the Incremental Flow Instream Method (IFIM) and the regional method involving regression analysis. The exceedance method which is also called frequency analysis encompasses the use of statistical models that are applied in the evaluation of risk or probability of low-flows for a given duration and return periods [4]. The low-flow frequency curve (LFFC) which is derived from the frequency analysis of low-flows depicts the proportion of years when a low-flow is not exceeded. That is the average year interval (return periods or reoccurrence interval) that a river falls below a specified discharge [5].

LFFC is typically designed according to a set of

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yearly flow minimums or volumes derived from the initial continuous flow sequence available on a daily or monthly (one value from every year of record) scale. The low-flow sequence of various average intervals can be used to create LFFCs. For daily records, minimum levels of 1, 3, 7, 10, 15, 30, 60, 90, or 100 can be analyzed as well as those of 120, 150, 180 or 183, 273 and 284 [3,4,5]. Whereas for monthly series averaging intervals such as 1, 3, 6 and 9 months may be selected [6].

Different low-flow indices have been adopted for different locations, for instance, 7-day 10-year low flow (7Q10) and 7-day 2-year low flow (7Q2), which are distinct as the lowest average occurring flows for a successive 7-day period at the return period of 10 and 2 years respectively, have been used in the USA. UK has been reported to have adopted the average of the annual series of minimum 7-day average flows which is known as Dry Weather Flow or Mean Annual 7-day Minimum flow (MAM7) [5]. On the other hand, the 1-day and 30-day summer and winter low-flows (either means or flows with an exceedance probability of 50%, 80%, 90%, 95%) have been adopted in Russia and Eastern Europe [5-7].

Estimations from low-flows frequency analysis are found to be useful in drought investigations because the information from such estimates is critical for various water-based operations and must be included in strategic short- and long-term water-resource management plans [8]. Low-flow analysis has also played a critical role in ensuring the reliability of water supply from streams and in identifying the magnitude of hydrological droughts [9]. Low-flows frequency analysis is equally important for water supply systems design, safe surface water extraction, the assimilative capacity of stream investigations, regulation of waste disposal and preservation of definite in-stream discharges among others. However, they have not received as much attention that has been given to flood frequency analysis.

Most of the river basins in Nigeria are partially gaged and characterized by insufficient data, and due to the stochastic nature of hydrologic engineering parameters such as stream discharge. It is only imperative that to achieve reliable frequency estimations of extreme low-flow events, the available data must be extrapolated beyond its years of record using theoretical or probabilistic models. This involves fitting the empirical distribution of the low-flows of different durations to a probability distribution to predict low-flows in the future. Some of the most wildly used probability distributions for modeling low-flows are Weibull, Gumbel (Extreme Value type 1), log-normal and Pearson Type 3 distributions [2]. The parameters of the fitted distributions are estimated and thereafter the relationship between the return period and low-flows are determined [9, 10].

There is presently a paucity of information on the applications of probability models in the assessment of low-flows of Ogun River Basin, Southwest Nigeria. While design maxima flow investigations have gained the attention of various researchers in Nigeria [11-15] and Globally [16-19], the frequency analysis of low-flow in design consideration for sustainable water resources management in the Ogun River Basin has not been investigated. In addition, the true probability distribution of low-flows of Ogun River Basin is unknown, therefore this study was conducted to fill the existing gap in the application of probabilistic models for evaluating low-flow of Ogun River as well as to establish the most suitable models for quantifying different average minimum daily series.

## 2.0 DESCRIPTION OF THE STUDY AREA

Ogun River basin is one of the five major river basins in the southwestern region of Nigeria, others being Yewa, Ona, Oshun and Sasa (Figure 1). It is located within latitudes 6° 33' N and 8° 58' N; and longitudes 2° 28' E and 4° 8' E and encompasses a spatial extent of approximately 23,700 square kilometers [20]. River Ogun takes its source from the Iganran Hills at an altitude of about 503 m east of Saki and flows for about 410 km before discharging into the Lagos Lagoon. In terms of physiography, the basin area is distinguished as the upper-central basin to the north and lower basin to the south of Abeokuta. It is characterized by a tropical climate, two maxima rainfall patterns [21], and high varying temperatures both spatial and temporally.

## 3.0 METHODOLOGY

Daily records of minimum flows of 1, 3, 7, 10, 15, and 30 series were constructed from a time series of 27 years (1987–2013). These records were collected from Ogun Oshun River Basin Development Authority (OORBDA) for Abeokuta gauging station. The probabilities of exceedance of the low-flows were computed using Weibull plotting position, while seven probability distributions, namely Reversed Generalized Extreme Value (GEVR) distribution, Generalized Normal (GNO) distribution, Generalized Logistic (GLO) distribution, Pearson Type III (PE3), Gumbel distribution (GUM), Weibull distribution (WEI), and Log-Normal (LN) distribution were preselected for the study. The seven distributions were selected because of their robustness and extensive applications in low-flow assessment. The L-moments method of parameter estimation was used for this study, and it was selected because of its widespread application in water resources engineering studies and its less susceptibleness to sample variability, unlike the product-moment estimators. Detailed descriptions of the L-moments can be accessed in the work of Izinyon and Ehiorobo [14]. Table 1 provides the mathematical expressions of the models.

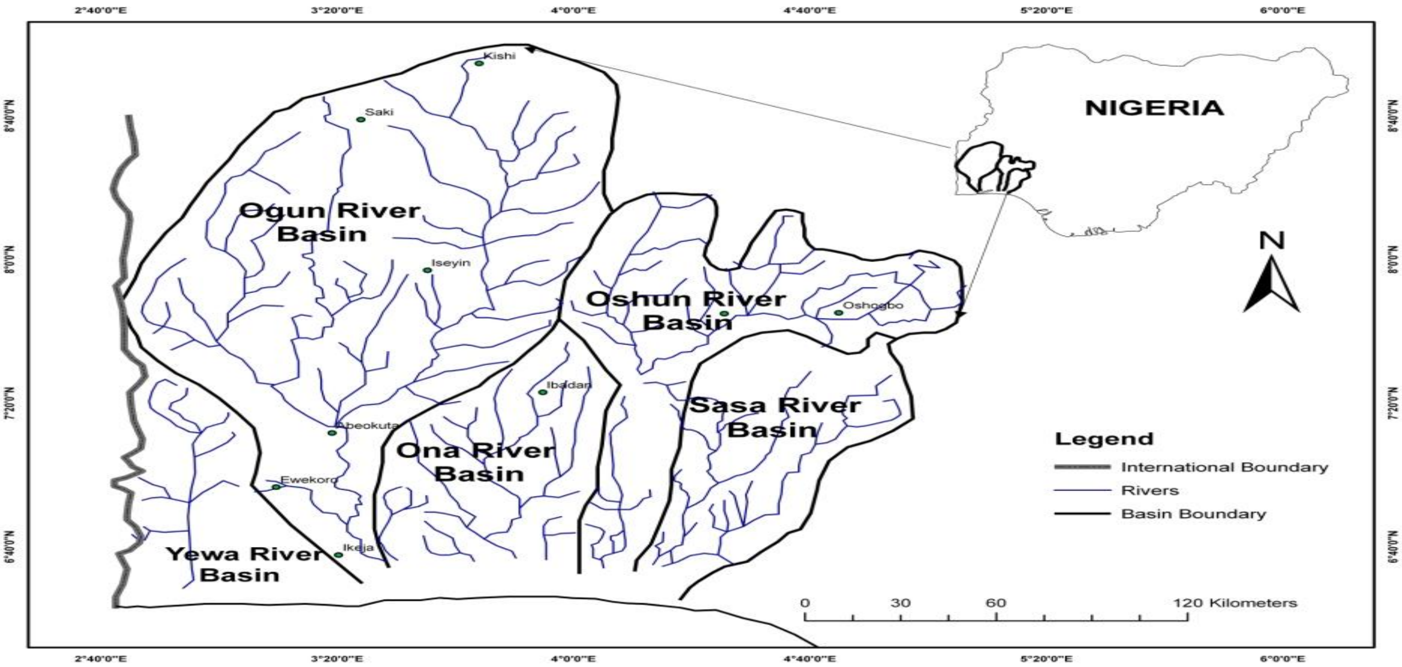

**Figure 1:** Location map of Ogun River Basin Area (Adapted from Oke et al. [22])

**Table 1:** Summary of Mathematical Description of the Selected Models

| **Model** | **Mathematical Formula** | **Description** |
|---|---|---|
| Weibull plotting position | $P_i = \frac{m}{n+1}$ | m is the order or rank while n is number of years of study for estimating exceedance probability $P_i$ |
| GEVR | $F(x) = 1 - \exp\left[-\left(\frac{x-\xi}{\alpha}\right)^{\kappa}\right]$ | location ($\xi$), scale parameter ($\alpha$) and the shape parameter ($\kappa$) |
| GNO | $f(x) = \frac{p}{2\sigma\Gamma\left(\frac{1}{p}\right)} \exp\left\{-\left\lvert\frac{x-\mu}{\sigma}\right\rvert^{p}\right\}$ | $\mu$, $\sigma$ and p in the equation is, respectively location, scale and the shape of the distribution |
| GLO | $f(x) = \frac{\alpha^{-1}\exp(-(1-\kappa)y)}{(1+\exp(-y))^2}$ | $\xi, \alpha, \kappa$ is respectively the location, scale and shape parameters |
| PE3 | $f(x) = \frac{1}{a\Gamma(b)}\left(\frac{x-c}{a}\right)^{b-1} \exp\left[-\left(\frac{x-c}{a}\right)\right]$ | $a > 0, b > 0$ and $0 < c < x$ are parameters that denote the location, scale, and shape of the distribution |
| GUM | $f(\mathrm{x}) = \frac{e^{-(x-\beta)/\alpha} e^{-e^{-(x-\beta)/\alpha}}}{\alpha}$ | $-\infty<x<\infty, \alpha>0$, $\alpha$ and $\beta$ in the equation is, respectively scale and location of the distribution |
| WEI | $f(\mathrm{x}) = \frac{\beta}{\alpha}\left(\frac{\alpha}{x}\right)^{\beta+1} e^{-(\alpha/x)^{\beta}}$ | $-\infty < x < \infty, \alpha > 0$, $\beta$ and $\alpha$ in the equation is, respectively shape and scale of the distribution |
| LN | $f(\mathrm{x}) = \frac{1}{\sigma_y x\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln(x)-\mu_y}{\sigma_y}\right)^2}$ | $\mu_y$ and $\sigma_y$ respectively represents the mean and standard deviation of the log-transformed series of low-flow |

A preliminary visual inspection was conducted to discard any candidate probability distributions with lack-of-fit in the left and right tails. Although in the context of minima flow, the left tail is more critical than the right tail, therefore all probability distributions whose left tail was visually far from the left tail of the empirical distribution were considered unfit and were discarded. Further analysis was performed on the remaining probability distribution to ascertain their fit to the empirical distribution of the minimum flow for different durations. Thereafter, the adequacy of fit of the retained probability distributions was assessed using the Anderson-Darling ($A^2$) goodness-of-fit statistic and the D-index diagnostic test. The Anderson-Darling statistic was selected to ascertain the fit of the retained probability distribution because it gives more emphasis to the tails as well as the body of distribution in comparison to other available goodness-of-fit statistics (Delignette-Muller, 2015) [23]. While D-index has been popularly applied to validate the competence of the goodness-of-fit statistics and for selecting the most suitable probability distributions (Feyissa et al, 2019; Vivekanandan, 2015) [24, 25]. The retained probability distributions with the lowest $A^2$ value are more robust in fitting low-flow, while distributions with the least D-index value were selected as the most suitable distribution for characterizing the low-flow of Ogun River.

The relationship between design low-flows and their respective non-exceedance probabilities was established using the most suitable probability models. Thereafter, the lines of best fit and their respective mathematical models were developed for the six durations of low-flow (daily, 3-day, 7-day, 10-day, 15-day, and 30-day) to serve has LFFC. Finally, the strength of each mathematical model in explaining the variations of distribution quantiles based on non-exceedance probabilities was ascertained using the coefficient of determination ($R^2$) and the adjusted coefficient of determination (adjusted-$R^2$). $R^2$ and adjusted-$R^2$ values that are close to 1 indicate that the points of the distribution quantiles lie close to the theoretical curves.

**Table 2:** Summary of Mathematical Description of the Goodness of fit Statistics

| **Statistic** | **Mathematical Formula** | **Description** |
|---|---|---|
| $A^2$ | $-N-\frac{1}{N}\sum_{i=1}^{N}(2i-1)*\left(\ln\ F_e(Q_i)\right)+\left(\ln\left(1-F_D(Q_i)\right)\right)$ | m is the order or rank while n is the number of years of study for estimating exceedance probability $P_i$ |
| D-index | $(1/\bar{Q})\sum_{i=1}^{6}\lvert Q_i-Q_i^*\rvert$ | Q̄ is the average (or mean) of the recorded AMD, Qi's (i = 1 to 6) are the first six highest sample values in the series and Q∗ i is the estimated value by the probability distribution. |
| $R^2$ | $\left(\frac{\sum_{i=1}^{n}(y_i-\overline{y})(\tilde{y}_i-\overline{\tilde{y}}_i)}{\sqrt{\sum_{i=1}^{n}((y_i-\overline{y}))^2}\sqrt{\sum_{i=1}^{n}(\tilde{y}_i-\overline{\tilde{y}}_i)^2}}\right)^2$ | $y_i$ is empirical low-flows, $\tilde{y}_i$ is predicted low-flow. $\overline{y}$ and $\overline{\tilde{y}}_i$ indicates the average $y_i$ and $\tilde{y}_i$ respectively. |
| Adjusted $R^2$ | $1-\frac{(1-R^2)(N-1)}{N-p-1}$ | $R^2$ is defined above, p number of predictors, N is the total sample size. |

# 4.0 RESULTS

## *4.1 Statistical Description of Low-Flow Series*

The summary of the statistical description of the low-flow series of River Ogun is presented in Table 2. As shown in the table the average low-flow increases from the annual average daily flow of 14.14 $m^3/s$ to the 30-day series of 14.97 $m^3/s$. It can be further observed from the table that the 30-day series of minimum flow has the least deviation of 0.73 $m^3/s$ from the mean of 14.97 $m^3/s$, implying that the higher the series the less susceptibility to variation.

In contrast to standards deviation, it could be observed that the coefficient of variation decreases with an increase in minimum flow series from average annual minima to 30-day minima. This implies that the regime's inter-annual variability of low-flow decreases with an increase in the flow duration. The negative skewness in all the selected series indicates that the low-flows of Ogun River are asymmetrical with a long tail to the left, while only the 7-day minimal has a distribution that is close to normal.

**Table 2:** Statistical Description of Low-flow Series

| | **Annual Minima** | **3-Day Minima** | **7-Day Minima** | **10-Day Minima** | **15-Day Minima** | **30-Day Minima** |
|---|---|---|---|---|---|---|
| Mean | 14.14 | 14.45 | 14.64 | 14.71 | 14.78 | 14.97 |
| Standard Deviation | 2.02 | 1.24 | 1.08 | 1.02 | 0.93 | 0.73 |
| Coefficient of Variation (CV) | 0.14 | 0.09 | 0.07 | 0.07 | 0.06 | 0.05 |
| Skewness | -2.69 | -1.06 | -1.50 | -1.51 | -1.17 | -0.63 |
| Kurtosis | 8.17 | 0.62 | 3.14 | 3.34 | 1.81 | 0.12 |

Table 3 displays the summary of the estimated parameters of the L-Moments. The table shows that the L-mean, being the position metric, is increasing as minimum low-flow rises from a normal annual sequence to a 30-day minimum. The L-CV is the scale measure or the variability and the L-kurtosis decreases from a daily minimum to a 30-day minimum. This reveals that from the estimated parameters based on L-moment, daily annual minima are prone to outliers and a very large degree of variability, which could have been the reason why they are mostly ignored when designing hydraulic structures for water resource management concerning low-flow.

This indicates that the annual minimum determined by the L-moment is vulnerable to outlying factors and a very high degree of uncertainty. Hence more attention should be given to low-flows of more than one-day average when designing hydraulic systems for water resources management in Ogun River basin.

**Table 3:** Summary of Estimated L-Moment Parameters

| | **L-mean** | **L-CV** | **L-skewness** | **L-kurtosis** |
|---|---|---|---|---|
| Annual Minima | 14.137 | 0.895 | -0.415 | 0.372 |
| 3-Day Minima | 14.454 | 0.677 | -0.246 | 0.168 |
| 7-Day Minima | 14.645 | 0.564 | -0.214 | 0.207 |
| 10-Day Minima | 14.711 | 0.530 | -0.202 | 0.196 |
| 15-Day Minima | 14.779 | 0.500 | -0.185 | 0.167 |
| 30-Day Minima | 14.967 | 0.405 | -0.098 | 0.075 |

### *4.2 Preliminary Investigation of Candidate Probability Distributions*

The preliminary assessment through visual check was conducted to identify the extent to which the probability distributions fit the empirical distributions of the different durations of low-flow series.

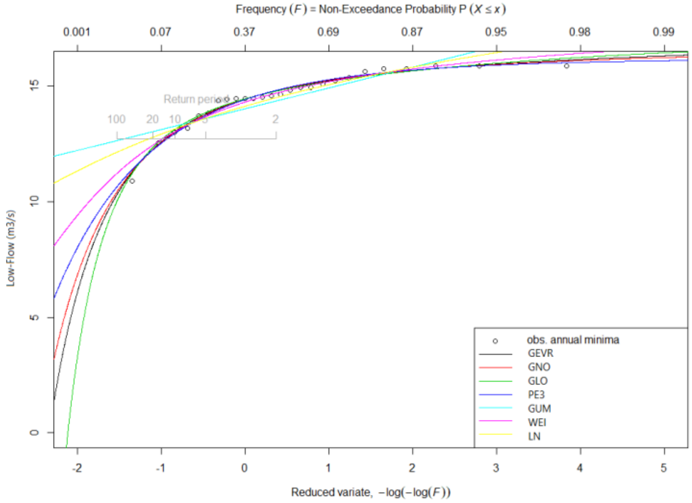

**Figure 2:** Low-flow Frequency Curve of Annual Minima

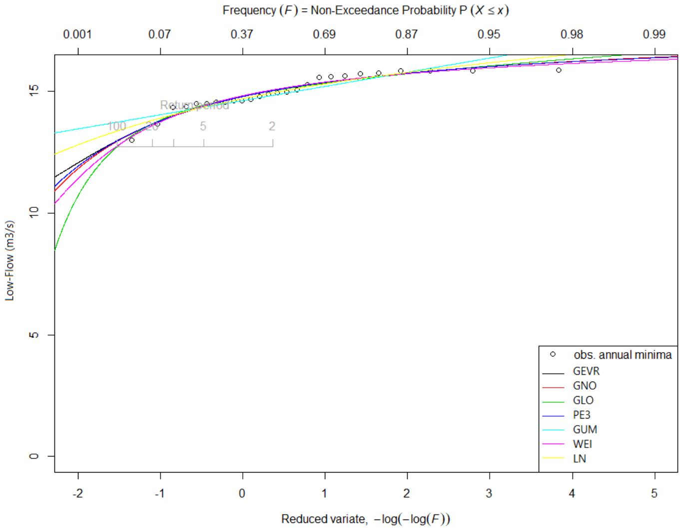

**Figure 3:** Low-flow Frequency Curve of 3-Day Minima

This qualitative assessment of the fit of the distributions was assessed by plotting the quantiles of low-flow using the Weibull plotting position and the probability distributions as shown in Figure 2 to Figure 7.

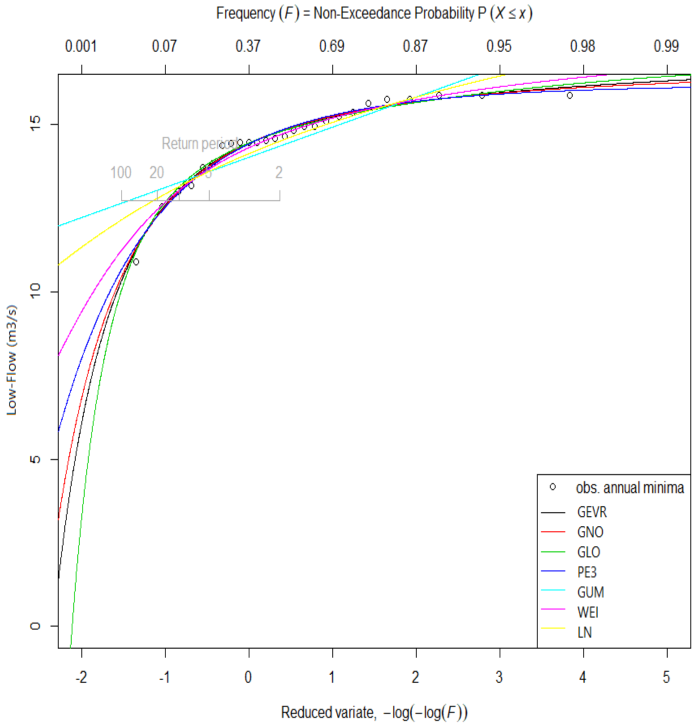

**Figure 4:** Low-flow Frequency Curve of 7-DAY Minima

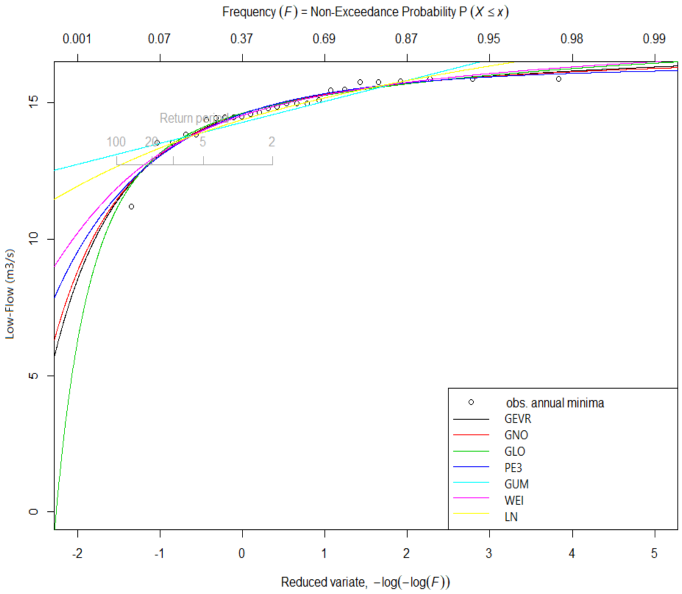

**Figure 5:** Low-flow Frequency Curve of 10-Day Minima

The figures show that GUM, WEI and LN distributions are not suitable for describing the characteristics of the low-flow series. Since compared to the other distributions, their lines of fit lie further away from the observed series. Indicating lack-of-fit at the center of the curve as well as at both right and left tails. As a result, GUM, WEI and LN were not retained for further assessment in the study.

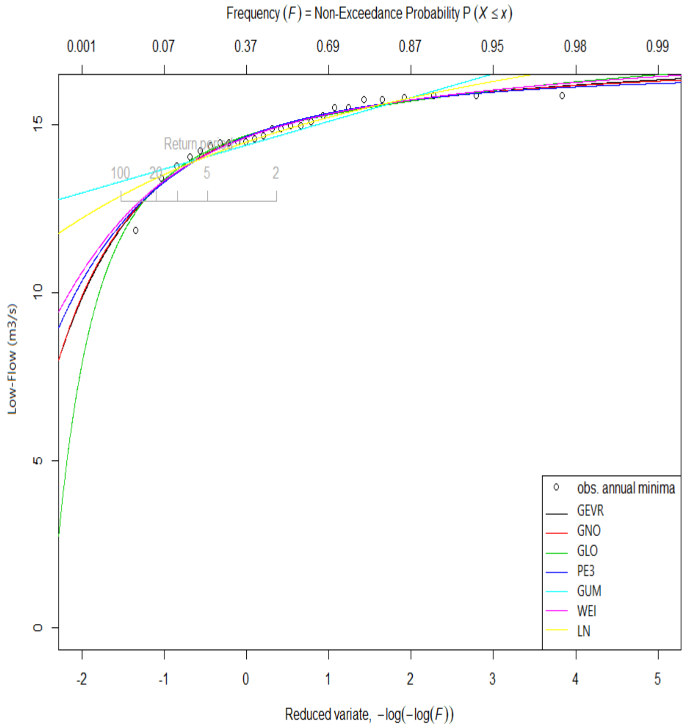

**Figure 6:** Low-flow Frequency Curve of 15-DAY Minima

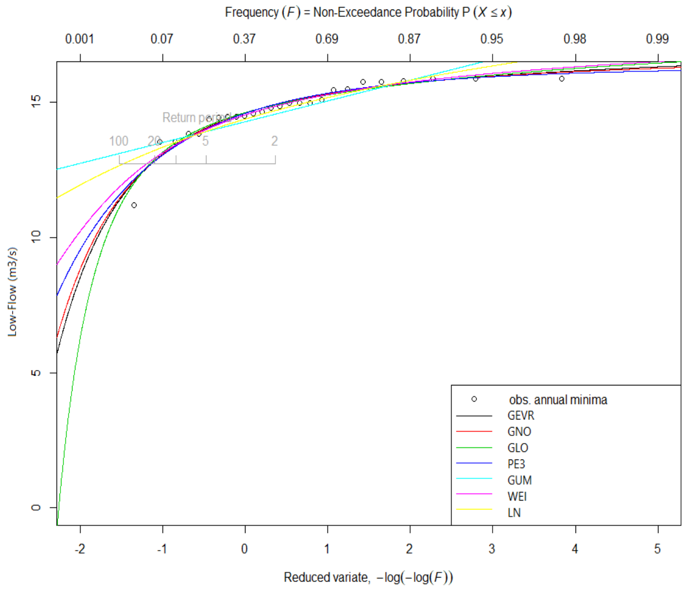

**Figure 7:** Low-flow Frequency Curve of 30-Day Minima

### *4.3 Performance Evaluation of Selected Probabilistic Models*

The estimated parameters of the retained probabilistic models are presented in Table 4. By supplying these parameters into the mathematical formulas of the models (Table 1), the theoretical quantities of the low-flow series were derived, which were further compared with the empirical quantiles using $A^2$ and D-index. The $A^2$ and D-index values of the probability distributions are presented in Table 5. As shown in table 5, GLO has the least $A^2$ value of 1.4414, 0.5359, 0.4460, 0.4187, 0.3201 respectively for annual, 3-day, 7-day 10-day, and 15-day series, while GNO has the least $A^2$ value of 0.1312 for the 30-day series. This implies that based on the $A^2$ goodness-of-fit statistic, GLO is best suited for the annual, 3-day, 7-day, 10-day, and 15-day series, while GNO is the most suitable distribution for the 30-day series. Furthermore, as depicted in Table 5, the D-index values validate the suitability of GLO for all the durations of low-flow. Compared to GNO which favors the 30-Day series based on the $A^2$ values, the D-index favors the GLO, indicating that although both GLO and GNO are suitable for the 30-day series, the GLO is preferable and hence selected for characterizing the 30-Day series.

Based on the selected suitable distributions, six probability plots were developed to relate the different low-flow quantiles to their non-exceedance probability. The developed probability plots are presented in Figure 8 to Figure 13. These figures show that design low-flows increases with non-exceedance probability; and their respective mathematical models could be used by engineers and researchers to predict the different average minima series for different non-exceedance probability.

The high $R^2$ values of the models are indications that each mathematical model is robust in explaining the variations of the theoretical quantiles based on non-exceedance probabilities. The high adjusted-$R^2$ values shown in Figure 14 further validate the robustness of the established mathematical model in characterizing design low-flows.

**Table 4:** Estimated Parameters of Candidate Stochastic Models

| | **Low-Flow Design** | **Location** | **Scale** | **Shape** |
|---|---|---|---|---|
| GEVR | Annual Minima | -15.044 | 0.825 | -0.350 |
| | 3-Day Minima | -15.066 | 0.869 | -0.114 |
| | 7-Day Minima | -15.139 | 0.761 | -0.068 |
| | 10-Day Minima | -15.169 | 0.729 | -0.050 |
| | 15-Day Minima | -15.203 | 0.706 | -0.024 |
| | 30-Day Minima | -15.272 | 0.642 | 0.115 |
| GNO | Annual Minima | 14.758 | 1.141 | 0.888 |
| | 3-Day Minima | 14.748 | 1.077 | 0.510 |
| | 7-Day Minima | 14.860 | 0.921 | 0.443 |
| | 10-Day Minima | 14.901 | 0.874 | 0.418 |
| | 15-Day Minima | 14.944 | 0.834 | 0.382 |
| | 30-Day Minima | 15.039 | 0.705 | 0.202 |
| GLO | Annual Minima | 14.699 | 0.662 | 0.415 |
| | 3-Day Minima | 14.720 | 0.612 | 0.246 |
| | 7-Day Minima | 14.839 | 0.522 | 0.214 |
| | 10-Day Minima | 14.884 | 0.495 | 0.202 |
| | 15-Day Minima | 14.929 | 0.473 | 0.185 |
| | 30-Day Minima | 15.032 | 0.398 | 0.098 |
| | Low-Flow Design | Location | Scale | Gamma |
| PE3 | Annual Minima | 14.137 | 1.906 | -2.508 |
| | 3-Day Minima | 14.454 | 1.284 | -1.480 |
| | 7-Day Minima | 14.645 | 1.053 | -1.295 |
| | 10-Day Minima | 14.711 | 0.985 | -1.223 |
| | 15-Day Minima | 14.779 | 0.922 | -1.123 |
| | 30-Day Minima | 14.967 | 0.725 | -0.601 |

**Table 5:** Goodness of Fit and Diagnostic Test of Selected Probabilistic Models

| GOF | Model | Annual Minima | 3-Day Minima | 7-Day Minima | 10-Day Minima | 15-Day Minima | 30-Day Minima |
|---|---|---|---|---|---|---|---|
| $A^2$ | GEVR | 1.5218 | 0.5723 | 0.4753 | 0.4459 | 0.3386 | 0.1373 |
| | GNO | 1.6363 | 0.5801 | 0.4781 | 0.4473 | 0.3360 | 0.1312 |
| | GLO | 1.4414 | 0.5359 | 0.4460 | 0.4187 | 0.3201 | 0.1377 |
| | PE3 | 1.7582 | 0.6010 | 0.4928 | 0.4603 | 0.3436 | 0.1325 |
| D-Index | GEVR | 0.0850 | 0.1627 | 0.1471 | 0.0930 | 0.0800 | 0.0803 |
| | GNO | 0.0900 | 0.1631 | 0.1470 | 0.0890 | 0.0790 | 0.0789 |
| | GLO | 0.0820 | 0.1608 | 0.1469 | 0.0880 | 0.0750 | 0.0747 |
| | PE3 | 0.1050 | 0.1645 | 0.1470 | 0.0910 | 0.0800 | 0.0803 |

### *4.4 Discussion of Findings*

Various researchers have investigated the applicability of probabilistic models in predicting the design flood of most of the river basins in Nigeria. However, there is presently a paucity of information on the applications of these models in the characterization of low-flows. This study was developed to fill the existing knowledge gap as well as to evaluate the suitability of some selected probabilistic models in predicting design minima for different durations in Ogun River Basin.

The statistical description revealed that the higher the average minimum series the lesser its susceptibility to inter-annual variations. For instance, it was revealed that the daily average series had the least mean value of 14.14 $m^3/s$, highest values of the standard deviation of 2.02 $m^3/s$, highest coefficient of variation of 0.14 and highest skewness and kurtosis of -2.69 and 8.17 respectively. Generally, all the durations of the average low-flows of Ogun River were asymmetrical with a long tail to the left, whereas only the 7-day minimal has a distribution that is close to normal, which could have been the reason why it was chosen along with 30-day minima for climate change impact studies [3].

The L-moments parameters of the low-flow for different durations reveal further that the daily annual minimum is prone to outliers and a very large degree of variability, which could have been the reason why they are mostly Ignored when designing hydraulic structures for water resource management with respect to low-flow.

For instance, Smakthin [5] and Bormann and Pinter [26] discouraged the use of the daily average minima low-flow analysis because it is sensitive to measurement error or day-to-day variability.

Seven distributions were initially selected to characterize low-flow in the study area based on their robustness and applications in hydrological extremes. However, from visual inspection, GUM, WEI and LN distributions with two parameters were discarded due to their inability to fit the empirical distribution at the center and at the left and right tails (Figure 2 to Figure 7). The adequacy of fit of the retained distributions to the observed low-flow was assessed using the $A^2$ goodness-of-fit statistic, while D-index was used to select the most suitable distributions. Based on the $A^2$ and D-index, GLO was selected to be the most suitable model for characterizing low-flow for annual, 3-day, 7-day 10-day, and 15-day and 30-day durations.

Using the GLO, six probability plots were developed to relate the different low-flow series to their non-exceedance probability as presented in Figure 8 to Figure 13. The figure shows that design low-flows increase with increase non-exceedance probability. Based on the probability plots relating theoretical quantiles of low-flow to non-exceedance probability; mathematic models were established for the six series of low-flow. Engineers and researchers could use the models to predict the different average minima series for different non-exceedance probabilities.

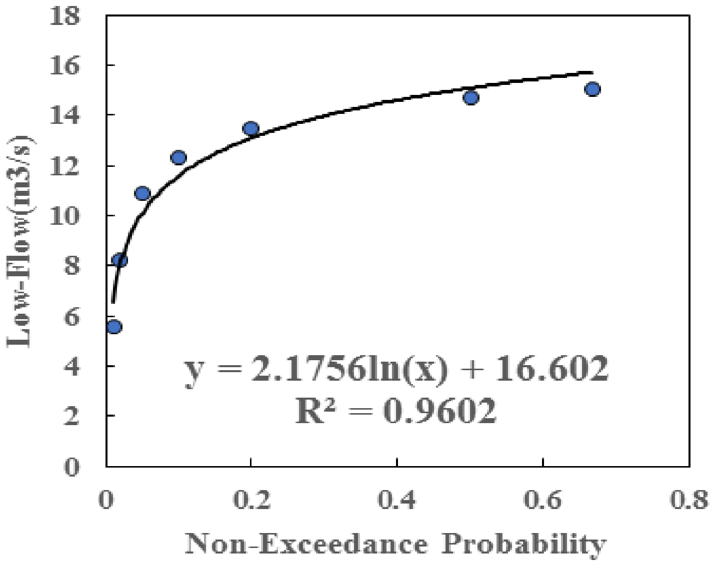

**Figure 8:** Probability Plot of Annual Minima Series

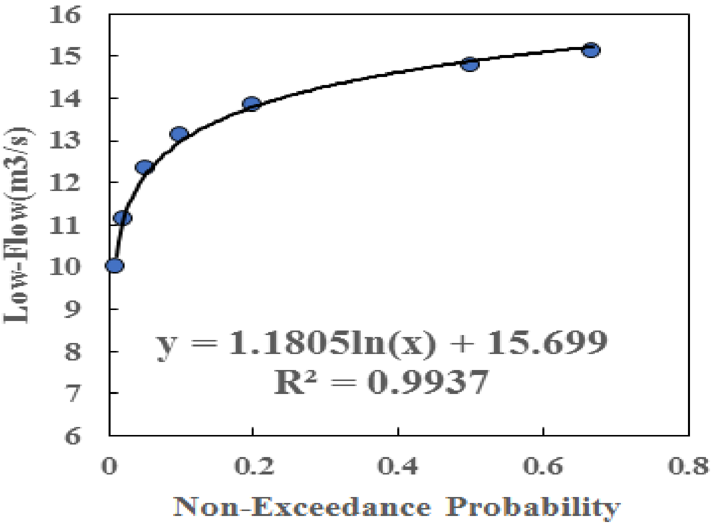

**Figure 9:** Probability Plot of 3-Day Minima Series

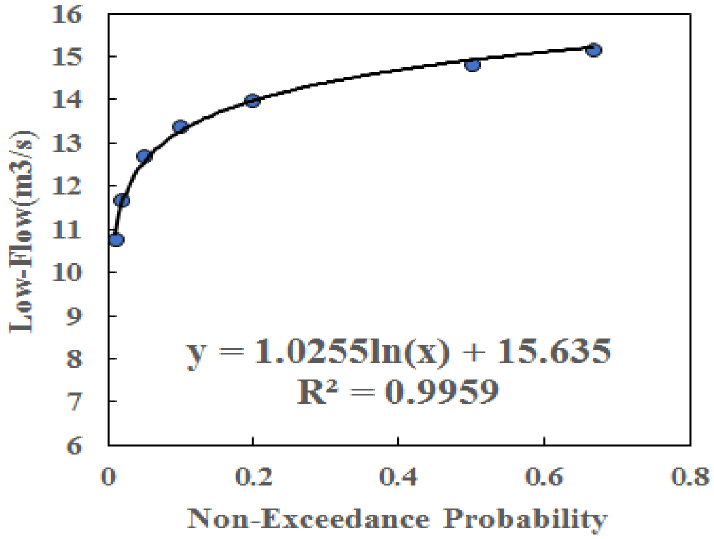

**Figure 10:** Probability of 7-Day Minima Series

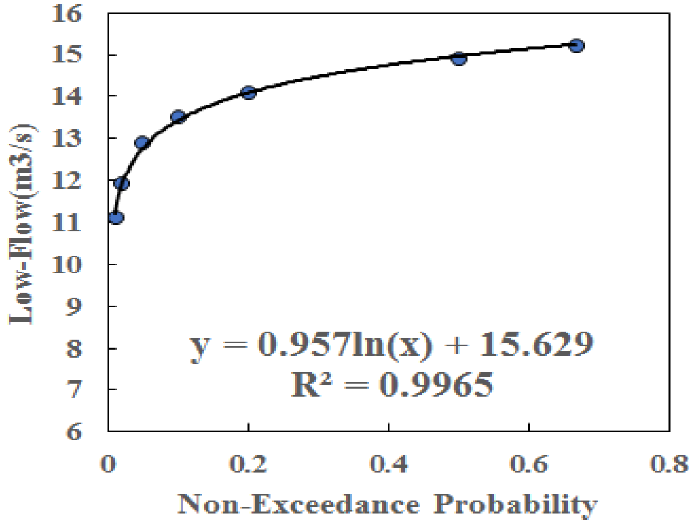

**Figure 11:** Probability Plot of 10-Day Minima Series

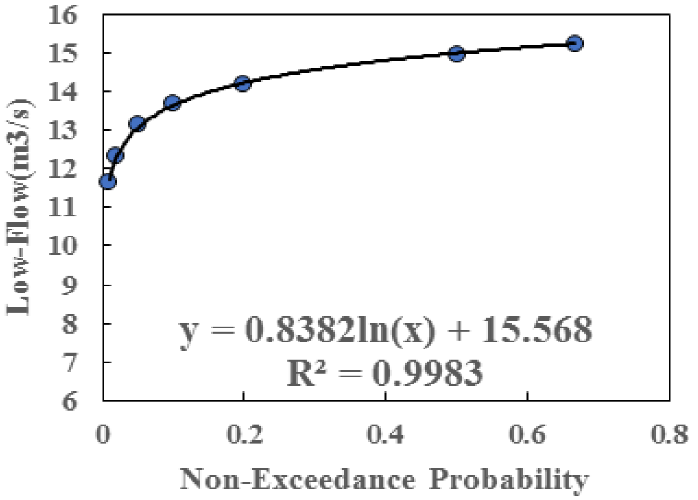

**Figure 12:** Probability Plot of 15-Day Minima Series

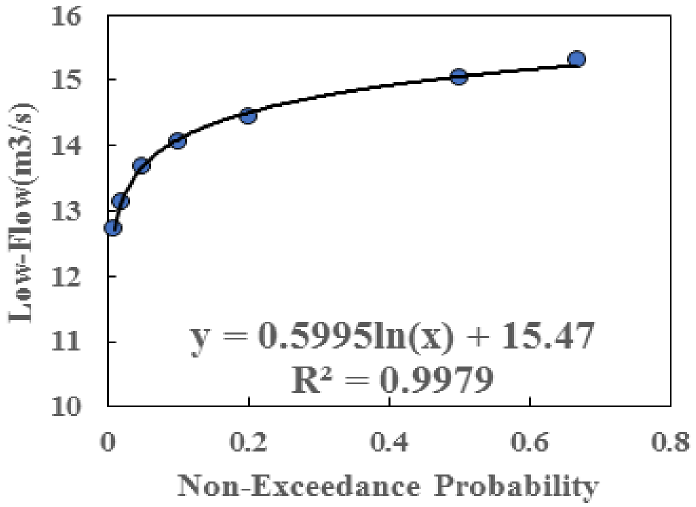

**Figure 13:** Probability Plot of 30-Day Minima Series

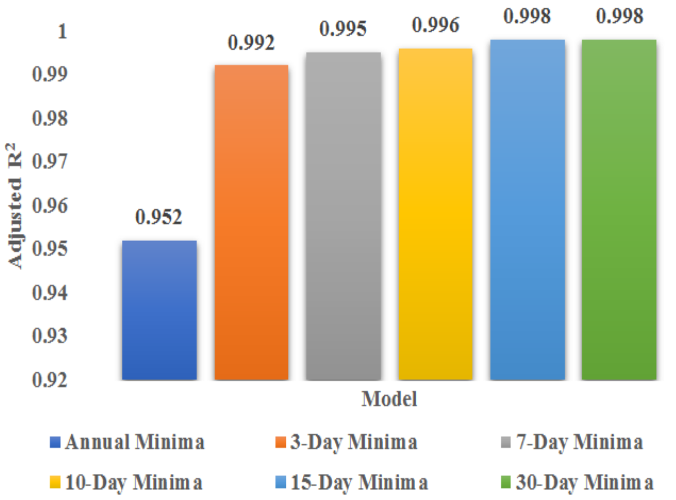

**Figure 14:** Performance Evaluation of Selected Probabilistic Model in Fitting Design Low-Flow

## 5.0 CONCLUSION

There is presently a paucity of information on the applications of probability models in the assessment of low-flows of Ogun River Basin, Southwest Nigeria. Key facts that emerged from this study are that the higher the average minimum series of Ogun River the lesser its susceptibility to inter-annual variations. All the durations of the average low-flows of Ogun River were asymmetrical in nature with a long tail to the left, whereas only the 7-day minimal has a distribution that is close to normal. GLO is suited for characterizing the average annual, 3-day, 7-day, 10-day, 15-day minima and 30-day minima. Six mathematical models based on probability plots have been established to relate the different low-flow series to their non-exceedance probability. The established models are useful for informed decision-making in water resources management of Ogun River Basin.

## REFERENCES

[1] US EPA. “Definition and Characteristics of Low Flows”, US EPA. US EPA. Retrieved 1 April 2021, from https://www.epa.gov/ceam/definition-and-characteristics-low-flows. 2020

[2] WMO. “Manual on Low-Flow Estimation and Prediction”. *Koblenz, Germany: Chairperson, Publications Board*, 2008

[3] Sapač, K., Medved, A., Rusjan, S., & Bezak, N. “Investigation of Low- and High-Flow Characteristics of Karst Catchments under Climate Change” *Water,* 11(5), 2019, 925. https://doi.org/10.3390/w11050925

[4] Caruso, S.B. “Evaluation of Low-flow Frequency Analysis Methods”. *Journal of Hydrology,* 39(1), 2000, pp. 19-47

[5] Smakhtin, V. “Low flow hydrology: a review”. *Journal of Hydrology,* 240(3-4), 2001, pp 147-186. https://doi.org/10.1016/s0022-1694(00)00340-1

[6] Midgley, D.C., Pitman, W.V., Middleton, B.J. *“*Surface water resources of South Africa 1990*”. Water Research Commission Report*, No 298/5.1/94, Pretoria, South Africa, 1994.

[7] Sakovich, V.M. “The representation of minimum runoff series in Karelia and North-Western regio*n.” Transcaucasian Hydrometeorological Research,* 308, 1990, pp 129–136.

[8] Yürekli, K., Kurunç, A., & Selçuk, G. Ü. L. “Frequency analysis of low flow series from Çekerek stream basin”, *Journal of Agricultural Sciences*, 11(1), 2005, pp 72-78.

[9] Hasan, H. H., Razali, S. F. M., Muhammad, N. S., Mohamed, Z. S., & Hamzah, F. M. “Assessment of probability distributions and minimum storage draft-rate analysis in the equatorial region”, *Natural Hazards and Earth System Sciences Discussions,* 2020, pp 1-29.

[10] Bezak, N.; Brilly, M.; Šraj, M. “Comparison between the peaks-over-threshold method and the annual maximum method for flood frequency analysis”, *Hydrological* Sciences *Journal,* 59, 2014, pp 959–977.

[11] Karmakar, S.; Simonovic, S.P. “Bivariate flood frequency analysis: Part 1. Determination of marginals by parametric and nonparametric techniques”, *Journal of Flood Risk Management.* 1, 2008, pp 190–200.

[12] Adeboye, O. B. and Alatise, M. O. “Performance of Probability Distributions and Positions in Estimating the Flood of River Osun at Apoje Sub-basin Nigeria”. *Agricultural Engineering International: the CIGR Ejournal,* 9, 2007, pp 1-29.

[13] Agbede A. O and Abiona O. 2012. “Plotting Position Probability Fittings to Lagos Metropolitan Precipitation: Hydrological Tools for Hydraulic Structures Design in Flood Control”. *International Journal of Pure and Applied Sciences and Technology*, 10, 2012, pp 37- 43.

[14] Izinyon, O., & Ehiorobo, J. “L-moments approach for flood frequency analysis of river Okhuwan in Benin-Owena River basin in Nigeria”, *Nigerian Journal of Technology,* 33(1), 2014, pp 10-18. https://doi.org/10.4314/njt.v33i1.2

[15] Agbede, O.A and Aiyelokun, O.O. “Establishment of a Stochastic Model for Sustainable Economic Flood Management in Yewa Sub-Basin, Southwest Nigeria”, *Civil Engineering Journal,* 2(12), 2016, pp 646-655. DOI: 10.28991/cej-2016-00000065.

[16] Aiyelokun, O., Oyelakin, J., Ojelabi, A. and Agbede, O. “An Integration of Stochastic Models and GIS as Decision Support Tool for Regional Flood Management in Southwest Nigeria”. *Nigerian Institution of Water Engineers (NIWE) Water Journal,* 1(1), 2018, pp 18-29

[17] Vivekanandan, N. “Comparison of Probability Distributions for Estimation of Peak Flood Discharge”, *Open Access Library Journal*, 1, 2014

[18] Ahmad, I., Fawad, M., & Mahmood, I. “At-Site Flood Frequency Analysis of Annual Maximum Stream Flows in Pakistan Using Robust Estimation Methods”, *Polish Journal of Environmental Studies,* 24(6), 2015, pp 2345-2353. https://doi.org/10.15244/pjoes/59585

[19] Baidya, S., Singh, A. & Panda, S.N. “Flood frequency analysis”, *Natural Hazards,* 100, 2020, pp

1137–1158. https://doi.org/10.1007/s11069-019-03853-4

[20] Ogun River Basin Feasibility Study (OORBDA). “Report of Ogun-Oshun River Basin Development Authority”, 2, Ibadan. 1982

[21] Adediji A, Ajibade LT. “The change detection of major dams in Osun State, Nigeria using remote sensing (RS) and GIS techniques”, *Journal* of *Geography and Regional Planning*, 1(6), 2008, pp 110-115.

[22] Oke, M., Martins, O., Idowu, O., & Aiyelokun, O. “Comparative Analysis of Empirical Formulae Used in Groundwater Recharge in Ogun – Oshun River Basins”, *Ife Journal of Science*, 17(1), 2015, pp 53-63.

[23] Delignette-Muller, M.L.; Dutang, C. “Fitdistrplus: an R package for fitting distributions”, *Journal of Statistical Software,* 64(4), 2015, pp 1–34.

[24] Feyissa, T. A., & Tukura, N. G. “Evaluation of the best-fit probability of distribution and return periods of river discharge peaks. Case study: Awetu River, Jimma, Ethiopia”, *Journal of Sedimentary Environments*, 4(4), 2019, pp 361-368.

[25] Vivekanandan, N. “Flood frequency analysis using method of moments and L-moments of probability distributions”, *Cogent Engineering,* Vol. 2, Number 1, 2015, 1018704, DOI: 10.1080/23311916.2015.1018704

[26] Bormann, H.; Pinter, N. “Trends in low flows of German rivers since 1950: Comparability of different low-flow indicators and their spatial patterns”. *River Research and Applications.* 33, 2017, pp 1191–1204.