Comment on "Universal Out-of-Equilibrium Transport in Kondo-Correlated Quantum Dots: Renormalized Dual Fermions on the Keldysh Contour"

Two recent works attempt to extend results for the conductance G through a quantum dot described by the particle-hole symmetric (PHS) impurity Anderson model out of the PHS case using renormalized perturbation theory in U (RPTU) up to U^2 [1,2]. Contrary to what is stated in the "Note added" of Ref. [2], previous results for the PHS case (where the occupation of the dot is $n = 1$ by symmetry) and general coupling to the left and right leads $\Gamma_{L(R)}$ and chemical potentials $\mu_{L(R)} = \pm\alpha_{L(R)}$ eV [3] are recovered by the first approach [Eq. (30) of Ref. [1]]. The dependence on temperature T is also correct. Ward identities are trivially satisfied because they were used [see the paragraph above Eq. (23)]. A limitation of this approach is that out of the PHS case, the coefficients of the expansion of G in terms of T and voltage V contain derivatives of n or the real part of the retarded self-energy Σ^r, which for an interacting system seem to depend on high energy properties hard to capture in a Fermi liquid approach. Exceptions are the linear term in T for $V = 0$ (which vanishes) and the linear term in V for $T = 0$ (addressed below).

Instead, Ref. [2] provides explicit expressions for all coefficients up to second order in T and V for $\alpha_L\Gamma_R = \alpha_R\Gamma_L$ and $n \to 1$. Unfortunately, the authors have made mistakes in the evaluation of the lesser quantities Σ^{-+} and G^{-+} ($-\Sigma^<$ and $G^<$ in our notation) already in the PHS case. This implies that also the greater quantities are incorrect. While using conservation of the current (CC), lesser and greater functions can be eliminated from the expression of G, they play a crucial role precisely in this conservation [Eqs. (80)–(89) of Ref. [4]], and therefore the approach seems unreliable. One incorrect result is that $\Sigma^<_{\mathrm{MBK}}(\omega) = 2if_{\mathrm{eff}}(\omega)\mathrm{Im}[\Sigma^r(\omega)]$, where $f_{\mathrm{eff}}(\omega)$ is the average of the Fermi function at the two leads, weighted by the corresponding Γ_ν. In addition, the authors claim to demonstrate that the term proportional to the noninteracting lesser Green function $g^<$ in the expression for $G^<$ [first term in Eq. (73) of Ref. [4]] vanishes [although it can be written as $2if_{\mathrm{eff}}(\omega)\Delta|G^r|^2$ [Eqs. (7) and (8) of Ref. [5]] and uses this result to claim that $G^<_{\mathrm{MBK}} = -|G^r|^2\Sigma^<_{\mathrm{MBK}}$. CC would follow from the form of $G^<_{\mathrm{MBK}}$, $\Sigma^<_{\mathrm{MBK}}$, and known relations between the different Green functions.

Unfortunately, the demonstration is flawed because Eq. (76) of Ref. [4] is used, which misses the term $2i\Delta$. The correct form of this equation is $(G^r)^{-1} - (G^a)^{-1} = \Sigma^a - \Sigma^r + 2i\Delta$. This comes trivially from the definition of G^r [third line below Eq. (13) of Ref. [2]] and its complex conjugate G^a.

In Fig. 1, we compare $\Sigma^<_{\mathrm{MBK}}(\omega)$ at $T = 0$ with the correct one, obtained integrating numerically the RPTU expressions [5,6]. We also display the analytical result [1,6] up to total second order in ω and V [Eq. (20) or Ref. [1]]. As it is known [6], the correct result is continuous. Instead, $\Sigma^<_{\mathrm{MBK}}$ has jumps at μ_L and μ_R, and strongly disagrees with the correct result except at energies far away from both μ_ν.

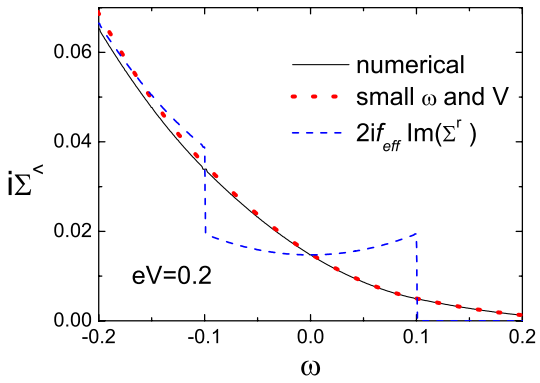

FIG. 1 (color online). Lesser self-energy as a function of frequency of the PHS impurity Anderson model for renormalized interaction $\tilde{u} = \tilde{U}/(\pi\tilde{\Delta}) = 1$, $\Gamma_R = \Gamma_L$, $\alpha_L = \alpha_R$, $T = 0$, and $eV = 0.2\tilde{\Delta}$. $\tilde{\Delta}$ is taken as the unit of energy.

It is difficult to say how these mistakes affect the reported expansion coefficients. The linear term in V can be written in the form $c_{VE_d} = 2(\alpha_L - \alpha_R)\cos(\pi n/2)$, which coincides with the result of Ref. [1], taking $\alpha_L\Gamma_R = \alpha_R\Gamma_L$ and $n \to 1$. In any case, CC is an essential requisite for any nonequilibrium theory. RPTU conserves the current in the PHS case and up to order V^3 in the general case for $T = 0$ [1].

This work was sponsored by PIP 1821 of CONICET and PICT R1776 of the ANPCyT, Argentina. A. A. A. is partially supported by CONICET.

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Received 28 January 2013; published 20 August 2013
DOI: 10.1103/PhysRevLett.111.089701
PACS numbers: 72.10.Bg, 72.15.Qm, 73.21.La, 75.30.Mb

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