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A semi-empirical stability criterion for real multi-planetary systems with eccentric orbits

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ABSTRACT
We test a crossing orbit stability criterion for eccentric planetary systems, based on Wisdom's criterion of first order mean motion resonance overlap (Wisdom 1980). We show that this criterion fits the stability regions in real exoplanet systems quite well. In addition, we show that elliptical orbits can remain stable even for regions where the apocenter distance of the inner orbit is larger than the pericenter distance of the outer orbit, as long as the initial orbits are aligned. The analytical expressions provided here can be used to put rapid constraints on the stability zones of multi-planetary systems. As a byproduct of this research, we further show that the amplitude variations of the eccentricity can be used as a fast-computing stability indicator.

Key words: celestial mechanics, methods: numerical, methods: analytical, (stars:) planetary systems, planets and satellites: individual: GJ 785 b, HD 114783 b, HD 7199 b, HD 85512 b, HD 47186 b, HD 47186 c, HD 51608 b, HD 51608 c, HD 134606 b, HD 134606 c, HD 134606 d

1 INTRODUCTION

Multi-planetary systems are discovered and confirmed with increasing frequency, with many of the newest planets in configurations called "compact systems" (where all possible stable regions are occupied and no additional bodies can be found). Usually the discoveries of new planetary systems are part of large projects, and it is important to know which systems are already "full" or not, in order to look for additional companions (e.g. Mayor et al. 2011). In addition, proposals for observations focus in discovering Earth-like planets in already confirmed exo-systems. It is thus important to have fast criteria to evaluate the stability of Earth-like planets in already confirmed exo-systems.

The Circular Restricted 3-Body Problem (CR3BP) consists of a particle moving under the gravitational influence of a primary and secondary masses. The primary and secondary move in a circular orbit about their common centre of mass and the particle is too small to affect the other two. There is a conserved quantity called *Jacobi constant*, that can be used to determine regions of allowed motion. The region of gravitational influence of the secondary mass is bounded by the Lagrange points, L_1 and L_2, forming the Hill sphere. The *Jacobi constant* can be interpreted as the "energy" of the test particle and defines the region of allowed motion. A particle can remain confined in orbit around the primary, or it can cross the Hill sphere through L_1, or even escape through L_2. A particle that cannot cross the Hill-sphere is called *Hill-stable* (see e.g. Murray & Dermott 1999).

In the late 19th century Henri Poincaré studied the stability of the three-body problem. He hinted at the complicated nature of the motion that can arise for some starting conditions. These chaotic trajectories diffuse in phase-space and can become unbounded, in contrast with regular trajectories (Poincaré 1899). In the CR3BP, orbits that are Hill-stable remain bounded even if they are chaotic, while those that do not obey Hill-stability, but are regular, are also bounded. Conversely, chaotic orbits that do not obey Hill-stability can become unbounded.

Wisdom (1980) deduced a criterion for the onset of chaos in the CR3BP based on the overlap of first order mean motion resonances. The overlap extends in a region around the planet of width

$$\delta = C\mu^{2/7}a, \tag{1}$$

where μ is the mass ratio between the planet and its parent star, a is the semi-major axis of the planet, and C is a constant value. Wisdom (1980) obtained a theoretical value $C = 1.33$, but using numerical simulations Duncan et al. (1989) estimated $C = 1.57$. The orbits of test particles in this region exhibit chaotic diffusion of eccentricity and semi-major axis until escape or collision occurs.

Marchal & Bozis (1982) obtained Hill-stability criteria in the general 3-body problem. These apply, in particular, to systems with two-planets orbiting a central star. In this case, the stability limit in terms of total angular momentum, c, and total energy, h, is:

$$-\frac{2M}{M_\star^3}\frac{c^2 h}{\mathcal{G}^2} > 1 + 3^{4/3}\frac{\mu_1\mu_2}{\alpha^{4/3}} , \tag{2}$$

where $\mathcal{G}$ is the gravitational constant, m_i is the planetary mass, $m_\star$