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The effect of polluted samples on Bayesian Estimators of Burr type –XII distribution

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ABSTRACT

Bayesian estimators may be affected by the polluted samples, because these samples can lead to the influence of the estimation methods in general and the Bayesian methods in particular, and thus the deviation of the values of the distribution parameter from their real values, and this leads to the divergence of the capabilities of the Bayes survival estimators from the real values.
The results showed that the estimators of the parameters were affected by many factors (sample size, distribution parameter, number of outliers and the estimation method). Simulation experiment results also showed a difference in Mean Square Error (MSE) of the Bayes survival estimators for each different experiment. Bayesian methods can be compared with other estimation methods (Maximum likelihood Estimation (MLE), Moment estimation (MOM) and shrinkage method (SH)). Also, Bayesian methods can be used to estimate the survival function of other distributions (exponential, Gamma and mixed) to observe the estimation results with the presence of extreme values.

Keywords: Bayesian Estimator, Burr type –XII distribution, Polluted Distribution, Mean Square Error, Survival Function, Statistical Distribution

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1. Introduction

Polluted random samples may affect the statistical distribution and the estimation method may introduce bad estimators for these reasons a research to introduce the effect of polluted ratios on Bayesian Estimators of Burr type –XII distribution parameters ,the research introduce the theoretical approach for Burr type –XII distribution with its main properties with polluted and un polluted simulation experiments ,experimental results show that Bayesian Estimator with Jeffery information gives the best estimators for the simulation experiments ,experimental results also show that Bayesian Estimator effected with (sample size , true parameter values ,estimation method and number of outliers) [1-4].
In the concepts of studying estimation methods for Burr type –XII distribution many researches were done such that, And the research of Zeinhum and others in (2011) introduced research for Burr type XII estimators for simulated type-2 censoring data and compare results with mean square error criteria [6].
The research of Arwa and others in (2014) proposed an estimation method for marshall-olkin extended burr type XII distribution and compare maximum likelihood and Bayesian methods with experimental estimation results for electrical data [3].
The research of Mohamed Ibrahim and others in (2021) introduced research to compare moment and maximum likelihood methods by many simulated experiments and compare estimation methods by chi-squared and Nikulin-Rao-Robson tests [4].
The outliers may lead to the deviation of the estimated parameter distribution from the real values and the

deviation of the estimator of the survival function of the Burr type –XII distribution from the true value and inability to find the real survival function for the experiments.
The importance of the research lies in finding the method that is least affected by outliers and thus estimating survival functions that are closest to the real survival functions and helping to analyze different experiences. The research aims to reach the best Bayesian method by presenting an estimator of a survival function that is closest to the real function of the Burr type –XII distribution. The research also aims to find if survival function affected by outliers.

2. Method

2.1. Survival function $S(t)$

The survival function has a life time variable, which represents the period of time between the occurrences of the event such as (birth or beginning of disease diagnosis) until the occurrence of the end event such as (death or the end of the treatment period). The survival function is defined as the probability of survival of an organism after (t) time, and it is denoted by the symbol (S (t)), which takes the following form [2]:

$$S(t) = \Pr(T > t) \;\; \dots (1)$$

Whereas $S(t)$ represent Survival function, (T) Continuous variable represent time
The previous formula can be rewritten to be

$$S(t) = \int_{t}^{Max(t)} P(t)dt \;\; \dots (2)$$

The relation between cumulative distribution function ($F(t)$) and Survival function ($S(t)$) can be:

$$S(t) = 1 - F(t) \;\; \dots (3)$$

Since we have ($0 \leq F(t) \leq 1$) then ($0 \leq S(t) \leq 1$)

and ($F(t)$) Increasing function while ($S(t)$) decreasing function

2.2. Risk function ($h(t)$)

The risk function represents the death rate for the period between (t_1) and (t_2) and it can have the following formula [1]:

$$h(t) = \frac{f(t)}{S(t)} \;\; \dots (4)$$

Whereas ($f(t)$) represent probability density function, ($h(t)$) represent risk function

2.3. Burr type –XII distribution

It is one of the most important continuous statistical distributions, and it is sometimes called (Singh-Maddala distribution) and it is one of the twelve Burr families of distributions that were discovered by (Irving W. Burr) in (1942)
This distribution is used to describe many biological and clinical experiences. The distribution has the two location (α) and shape (β) parameters.
Burr type- XII distribution has the following properties [5]:-
The expected mean value ($E(t)$) can be

$$E(t) = \frac{\Gamma\left(\frac{1}{\alpha} + 1\right)\Gamma\left(\beta - \frac{1}{\alpha}\right)}{\Gamma(\beta)} \;\; \dots (5)$$

The median

$$M_e = \left(2\frac{1}{\beta} - 1\right)\frac{1}{\alpha} \;\; \dots (6)$$

The mode is as follows:

$$M_o = \left(\frac{\alpha - 1}{\alpha\beta + 1}\right)\frac{1}{\alpha} \quad ...(7)$$

The probability density function ($f(t)$)

$$f(t, \alpha, \beta) = \alpha\beta t^{\alpha-1}(1 + t^{\alpha})^{-(\beta+1)} \quad ...(8)$$

$$t > 0, \alpha > 0, \beta > 0$$

Figure (1) and (2) shows number of Burr type XII probability density function and cumulative distribution function

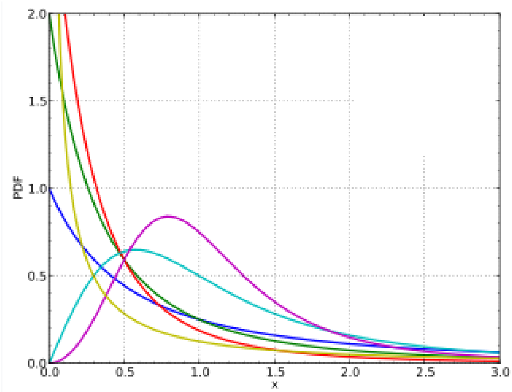

Figure 1. The probability density function ($f(t)$) of Burr type –XII with various (α and β) values

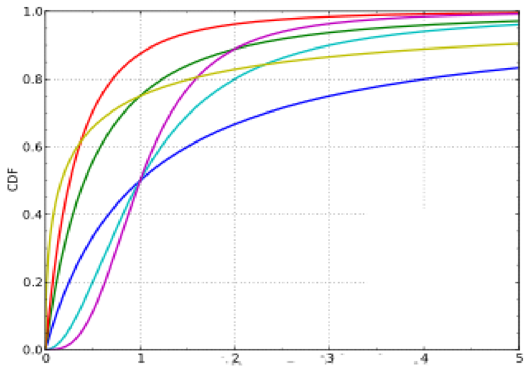

Figure 2. The cumulative distribution function ($F(t)$) of Burr type –XII with various (α and β) values

The cumulative distribution function can be

$$F(t) = \int_0^t f(u)du$$

$$F(t) = \int_0^t \alpha\beta t^{\alpha-1}(1 + t^{\alpha})^{-(\beta+1)}\ du$$

The integration can be solved by using partitioning integration method

$$\text{Let} = u^{\alpha} \ \Rightarrow \ u = y^{\frac{1}{\alpha}}$$

$$du = \frac{1}{\alpha} y^{\frac{1}{\alpha}-1}$$

So the cumulative distribution function will be

$$F(t) = u.v - \int_0^t v.du$$

$$F(t) = 1 - (1 + t^{\alpha})^{-\beta} \quad ...(9)$$

and the survival function will be

$$S(t) = (1 + t^{\alpha})^{-\beta} \quad \dots (10)$$

While risk function will be

$$h(t) = \frac{\alpha\beta t^{\alpha-1}(1 + t^{\alpha})^{-(\beta+1)}}{(1 + t^{\alpha})^{-\beta}} \quad \dots (11)$$

Simplified the previous function will be

$$h(t) = \frac{\alpha\beta t^{\alpha-1}}{(1 + t^{\alpha})} \quad \dots (12)$$

2.4. Bayesian estimation methods

In case of in full information for distribution parameter Jeffery method can be used to find Bayesian estimation methods with the two following basic rules [7,8]:

In case of Infinite space for the parameter (ϑ) with $(-\infty, \infty)$ interval then the prior distribution will be Uniform

In case of space for the parameter (ϑ) with $(0, \infty)$ interval then the prior distribution will be Logarithm Uniform

Jeffery suggests in case of no prior information about the distribution parameter then the Bayesian estimator function can be as follows:

1st -Bayesian Estimation methods

The method deboned on the following function

$g(\beta) = k.\beta$

With (k) constant

Then

$$H(\beta \backslash t_1, t_2, \dots, t_n) = \frac{L(t_1, t_2, \dots, t_n, \alpha, \beta).g(\beta)}{\int_0^{\infty} L(t_1, t_2, \dots, t_n, \alpha, \beta).g(\beta)\, d\beta} \quad \dots (13)$$

$$H(\beta \backslash t_1, t_2, \dots, t_n) = \frac{\alpha^n \beta^n \prod_{i=1}^{n} t_i^{\alpha-1} \prod_{i=1}^{n}(1 + t_i^{\alpha})^{-(\beta+1)} k.\beta}{\int_0^{\infty} \alpha^n \beta^n \prod_{i=1}^{n} t_i^{\alpha-1} \prod_{i=1}^{n}(1 + t_i^{\alpha})^{-(\beta+1)} k.\beta\, d\beta}$$

$$H(\beta \backslash t_1, t_2, \dots, t_n) = \frac{\beta^{n+1} \prod_{i=1}^{n}(1 + t_i^{\alpha})^{-(\beta+1)}}{\int_0^{\infty} \beta^{n+1} \prod_{i=1}^{n}(1 + t_i^{\alpha})^{-(\beta+1)}} \quad d\beta$$

The previous integration can be solved by using gamma function

$$Ln \prod_{i=1}^{n} (1 + t_i^{\alpha})^{-(\beta+1)} = -(\beta + 1) Ln \prod_{i=1}^{n} (1 + t_i^{\alpha})$$

$$Ln \prod_{i=1}^{n} (1 + t_i^{\alpha})^{-(\beta+1)} = -(\beta + 1) \sum_{i=1}^{n} (1 + t_i^{\alpha})$$

Tacking exponential for the previous function

$$H(\beta \backslash t_1, t_2, \dots, t_n) = \frac{\beta^{n+1} e^{-\beta \sum_{i=1}^{n} Ln(1+t_i^{\alpha})}}{\int_0^{\infty} \beta^{n+1} e^{-\beta \sum_{i=1}^{n} Ln(1+t_i^{\alpha})}\, d\beta}$$

According to Gamma function then

$$f(x,\alpha,\beta)=\frac{\beta^{\alpha}x^{\alpha-1}e^{-x\beta}}{\Gamma(\alpha)}$$

And

$$x^{\alpha-1}e^{-x\beta}=\beta^{n+1}e^{-\beta\sum_{i=1}^{n}Ln\left(1+x_i^{\alpha}\right)}$$

Let

$$z=\sum_{i=1}^{n}Ln(1+t_i^{\alpha})\quad ,\alpha-1=n+1$$

Since we have

$$\int_0^{\infty}\frac{\beta^{\alpha}x^{\alpha-1}e^{-x\beta}}{\Gamma(\alpha)}=1$$

Then we get

$$H(\beta\backslash t_1,t_2,\ldots,t_n)=\frac{\beta^{n+1}e^{-\beta\sum_{i=1}^{n}\left(1+t_i^{\alpha}\right)}}{\frac{\Gamma(n+2)}{z^{n+2}}}$$

$$H(\beta\backslash t_1,t_2,\ldots,t_n)=\frac{z^{n+1}e^{-\beta\sum_{i=1}^{n}\left(1+t_i^{\alpha}\right)}}{\Gamma(n+2)}$$

That represents Gamma function with
$Gamma(n+2,z)$
Tacking expectation then estimator of the parameter we get the 1st Bayesian estimator

$$\hat{\beta}_{j1}=\frac{n+2}{\sum_{i=1}^{n}Ln\left(1+t_i^{\alpha}\right)}\quad \ldots(14)$$

By using the same previous steps, we will get the following Bayesian estimator. Table 1 represents many Bayesian estimators according number of Jeffery Information functions

Table 1. Three Bayesian estimators depends on many Jeffery information

$Index$	Jeffery information	Bayesian estimator
1	$g(\beta)=k.\beta$	$\hat{\beta}_{j1}=\frac{n+2}{\sum_{i=1}^{n}Ln\left(1+t_i^{\alpha}\right)}$
2	$g(\beta)=k.\beta^2$	$\hat{\beta}_{j2}=\frac{n+3}{\sum_{i=1}^{n}Ln\left(1+t_i^{\alpha}\right)}$
3	$g(\beta)=k.\beta^{-2}$	$\hat{\beta}_{j3}=\frac{n-2}{\sum_{i=1}^{n}Ln\left(1+t_i^{\alpha}\right)}$

3. Results and discussion

3.1. Simulation experiments

In order to find the effect of outliers for the Bayesian survival estimators of Burr type –XII distribution many simulation experiments were made depends on the following information
$(sampel\ size=n, True\ Parameter\ Values=\beta, Outlier\ Ratios=R)$
The generation of (n) sample size with Burr type –XII distribution and (α,β) can be founded b the following steps [9]:
From equation (10) the cumulative distribution function can be

$$F(t)=1-(1+t^{\alpha})^{-\beta}$$

Replacing $(F(t))$ by (R) we get

$$R=1-(1+t^{\alpha})^{-\beta}$$

(R) represent random number in interval (0-1)

$$(1+t^{\alpha})^{-\beta}=1-R$$

Tacking logarithm, we get

$$-\beta \log(1 + t^{\alpha}) = \log(1 - R)$$

$$t^{\alpha} = e^{\frac{\log(1-R)}{-\beta}} - 1$$

$$t = \left(e^{\frac{\log(1-R)}{-\beta}} - 1\right)^{\frac{1}{\alpha}} \quad \dots (15)$$

Equation (15) represents simulated form to generate random variable (t) distributed Burr type –XII distribution Table (2) represent Simulation parameters with iterations (1000) times and ($n, \alpha, \beta\ and\ \gamma$) represent (sample size, location parameter, shape parameter and pollution ratio) respectively. The total number of simulation experiments were (27) non- polluted and (54) polluted experiments.

Table 2. Simulation parameters

i	n_i	α_i	β_i	γ_i
1	50	1	0.25	0
2	100	1.5	0.50	0.1
3	150	2	0.75	0.2

Comparing estimation results can be done by the Mean Square Error (MSE) and ($MMSE$) in the following forms [9]:-

$$MSE = \frac{\sum_{i}^{iter}(\hat{\beta}_i - \beta)^2}{iter} \quad \dots (16)$$

With

$\hat{\beta}_i$ *represent bayesian estimator*, β *represent true value*, *iter No. of iteration*

$$MMSE = \frac{\sum_{i}^{S} MSE_i}{S} \quad \dots (17)$$

With

MSE_i *reoresent best mean square error for simulation experiment i*

S No. of simulation experiments

3.2. Experimental results

After the (81) simulation experiments the estimation process and mean square error can be represented in the following tables and figures.

Table 3. The estimators and mean square error for these estimators for each Bayesian method and non-polluted data

Simu. bar.			$\hat{\beta}_{\text{Bayesian1}}$	$\hat{\beta}_{\text{Bayesia2}}$	$\hat{\beta}_{\text{Bayesian3}}$	$MSE_{\hat{\beta}_{\text{Bayesian1}}}$	$MSE_{\hat{\beta}_{\text{Bayesian2}}}$	$MSE_{\hat{\beta}_{\text{Bayesian3}}}$	*Best*
n_1	α_1	β_1	0.264425	0.269511	0.244085	0.001477	0.001698	0.001116	0.001116
		β_2	0.535394	0.54569	0.49421	0.006645	0.007689	0.004628	0.004628
		β_3	0.794462	0.809741	0.73335	0.013225	0.015254	0.009861	0.009861
	α_2	β_1	0.271352	0.27657	0.250479	0.001844	0.002148	0.001183	0.001183
		β_2	0.538169	0.548518	0.496771	0.007533	0.008666	0.005187	0.005187
		β_3	0.801787	0.817206	0.740111	0.022355	0.024953	0.01686	0.01686
	α_3	β_1	0.262056	0.267096	0.241898	0.001515	0.001715	0.001233	0.001233
		β_2	0.520416	0.530424	0.480384	0.005524	0.006231	0.004736	0.004736
		β_3	0.803584	0.819038	0.74177	0.017023	0.019468	0.012126	0.012126
n_2	α_1	β_1	0.255956	0.258466	0.245919	0.00061	0.000658	0.000547	0.000547
		β_2	0.512924	0.517952	0.492809	0.002436	0.002636	0.002146	0.002146
		β_3	0.785826	0.79353	0.755009	0.006679	0.007397	0.005006	0.005006
	α_2	β_1	0.257281	0.259804	0.247192	0.000723	0.000779	0.000626	0.000626
		β_2	0.52184	0.526956	0.501376	0.0026	0.002892	0.001962	0.001962
		β_3	0.780744	0.788398	0.750126	0.005955	0.006583	0.004624	0.004624
	α_3	β_1	0.255426	0.25793	0.245409	0.00084	0.000889	0.000769	0.000769
		β_2	0.517742	0.522818	0.497439	0.00284	0.003096	0.002338	0.002338
		β_3	0.768718	0.776255	0.738572	0.006508	0.006968	0.005815	0.005815
n_3	α_1	β_1	0.252156	0.253815	0.245521	0.00046	0.000476	0.000452	0.000452
		β_2	0.512125	0.515494	0.498648	0.001687	0.001801	0.001462	0.001462
		β_3	0.76789	0.772942	0.747682	0.004476	0.004737	0.003946	0.003946

$Simu.bar.$			$\hat{\beta}_{\text{Bayesian1}}$	$\hat{\beta}_{\text{Bayesia2}}$	$\hat{\beta}_{\text{Bayesian3}}$	$MSE_{\hat{\beta}_{\text{Bayesian1}}}$	$MSE_{\hat{\beta}_{\text{Bayesian2}}}$	$MSE_{\hat{\beta}_{\text{Bayesian3}}}$	$Best$
	α_2	β_1	0.251214	0.252867	0.244603	0.000506	0.000519	0.000507	0.000506
		β_2	0.513139	0.516515	0.499635	0.00215	0.002277	0.001875	0.001875
		β_3	0.762304	0.767319	0.742243	0.003117	0.003305	0.002872	0.002872
	α_3	β_1	0.25681	0.2585	0.250052	0.000571	0.000604	0.000497	0.000497
		β_2	0.509173	0.512523	0.495773	0.001988	0.002086	0.001823	0.001823
		β_3	0.764146	0.769174	0.744037	0.00353	0.003741	0.003192	0.003192

Table 4. The estimators and mean square error for these estimators for each Bayesian method and polluted data

$Simu.bar.$				$\hat{\beta}_{\text{Bayesian1}}$	$\hat{\beta}_{\text{Bayesia2}}$	$\hat{\beta}_{\text{Bayesian3}}$	$MSE_{\hat{\beta}_{\text{Bayesian1}}}$	$MSE_{\hat{\beta}_{\text{Bayesian2}}}$	$MSE_{\hat{\beta}_{\text{Bayesian3}}}$	$Best$
n_1	γ_1	α_1	β_1	0.288334	0.293879	0.266155	0.00342	0.003952	0.001923	0.001923
			β_2	0.579091	0.590228	0.534546	0.013739	0.015915	0.00757	0.00757
			β_3	0.852022	0.868407	0.786482	0.025586	0.029787	0.014263	0.014263
		α_2	β_1	0.290138	0.295718	0.26782	0.003927	0.004496	0.002291	0.002291
			β_2	0.576067	0.587145	0.531754	0.011899	0.013944	0.006217	0.006217
			β_3	0.84987	0.866214	0.784495	0.02495	0.029063	0.01395	0.01395
		α_3	β_1	0.289739	0.295311	0.267451	0.003852	0.004414	0.002241	0.002241
			β_2	0.596811	0.608288	0.550902	0.019881	0.022643	0.011545	0.011545
			β_3	0.854108	0.870533	0.788407	0.02783	0.03218	0.015953	0.015953
	γ_2	α_1	β_1	0.327327	0.333622	0.302148	0.008126	0.009223	0.004549	0.004549
			β_2	0.637567	0.649828	0.588523	0.026878	0.030711	0.014613	0.014613
			β_3	0.925967	0.943774	0.854738	0.05132	0.058695	0.028315	0.028315
		α_2	β_1	0.327659	0.33396	0.302454	0.008811	0.009937	0.00512	0.00512
			β_2	0.637148	0.649401	0.588137	0.027625	0.031478	0.015279	0.015279
			β_3	0.934939	0.952919	0.863021	0.063724	0.071844	0.037928	0.037928
		α_3	β_1	0.333985	0.340407	0.308293	0.009539	0.010756	0.005516	0.005516
			β_2	0.635216	0.647432	0.586353	0.028175	0.032012	0.015885	0.015885
			β_3	0.943359	0.961501	0.870793	0.054452	0.062459	0.029131	0.029131
n_2	γ_1	α_1	β_1	0.284028	0.286812	0.272889	0.002104	0.00232	0.001397	0.001397
			β_2	0.563088	0.568609	0.541006	0.007314	0.008106	0.004759	0.004759
			β_3	0.836221	0.844419	0.803428	0.014334	0.015951	0.009224	0.009224
		α_2	β_1	0.284725	0.287516	0.273559	0.0021	0.002319	0.00138	0.00138
			β_2	0.562817	0.568335	0.540746	0.006795	0.007575	0.00429	0.00429
			β_3	0.842839	0.851102	0.809787	0.014897	0.016623	0.009369	0.009369
		α_3	β_1	0.287664	0.290484	0.276383	0.002276	0.002513	0.001487	0.001487
			β_2	0.569103	0.574682	0.546785	0.008404	0.009278	0.005539	0.005539
			β_3	0.849018	0.857341	0.815723	0.018025	0.019904	0.011908	0.011908
	γ_2	α_1	β_1	0.319624	0.322757	0.307089	0.006513	0.006992	0.004796	0.004796
			β_2	0.610701	0.616688	0.586751	0.016121	0.017558	0.011095	0.011095
			β_3	0.893519	0.902279	0.858479	0.03127	0.034072	0.021619	0.021619
		α_2	β_1	0.317431	0.320543	0.304982	0.005773	0.006227	0.004155	0.004155
			β_2	0.628184	0.634343	0.60355	0.022674	0.024413	0.016485	0.016485
			β_3	0.911824	0.920764	0.876066	0.033536	0.036654	0.022676	0.022676
		α_3	β_1	0.313881	0.316959	0.301572	0.00496	0.00538	0.003472	0.003472
			β_2	0.615409	0.621442	0.591275	0.017265	0.018771	0.011973	0.011973
			β_3	0.921564	0.930599	0.885424	0.039637	0.043019	0.027758	0.027758
n_3	γ_1	α_1	β_1	0.279615	0.281455	0.272257	0.001573	0.001695	0.001155	0.001155
			β_2	0.5598	0.563483	0.545069	0.006362	0.006853	0.004673	0.004673
			β_3	0.822377	0.827788	0.800736	0.009828	0.010701	0.006926	0.006926
		α_2	β_1	0.2786	0.280433	0.271268	0.001221	0.001335	0.000835	0.000835
			β_2	0.557811	0.56148	0.543131	0.005483	0.005949	0.00389	0.00389
			β_3	0.826731	0.83217	0.804975	0.009405	0.010316	0.006357	0.006357
		α_3	β_1	0.285213	0.287089	0.277707	0.001951	0.002096	0.001442	0.001442
			β_2	0.55768	0.561349	0.543004	0.005841	0.00631	0.004232	0.004232
			β_3	0.83299	0.83847	0.811069	0.012926	0.013945	0.009454	0.009454
	γ_2	α_1	β_1	0.310266	0.312308	0.302101	0.004611	0.004874	0.003642	0.003642
			β_2	0.608891	0.612897	0.592867	0.015162	0.016094	0.011758	0.011758
			β_3	0.905023	0.910977	0.881206	0.029709	0.031666	0.022597	0.022597
		α_2	β_1	0.311537	0.313587	0.303339	0.004467	0.004732	0.00349	0.00349
			β_2	0.616501	0.620557	0.600278	0.016734	0.017737	0.013053	0.013053
			β_3	0.892628	0.898501	0.869138	0.024812	0.026581	0.018431	0.018431
		α_3	β_1	0.314834	0.316905	0.306549	0.005128	0.005413	0.004074	0.004074
			β_2	0.622585	0.626681	0.606202	0.018492	0.019559	0.014564	0.014564
			β_3	0.905901	0.911861	0.882061	0.029951	0.031919	0.022793	0.022793

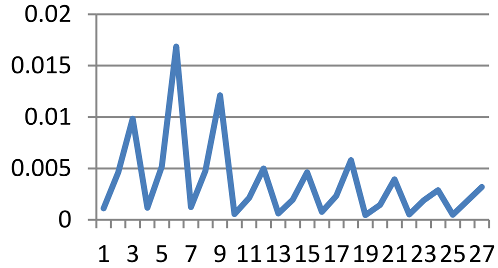

Figure 3. The best estimation method for the un-polluted simulation experiments

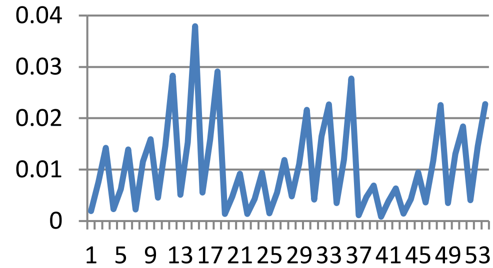

Figure 4. The best estimation method for the polluted simulation experiments

Compare simulation results in the previous tables and figures shows that $(\hat{\beta}_{j3})$ represent the best estimator for polluted and unpolluted experiments.

Tableau 5. MMSE for polluted and un-polluted simulation experiments

Polluted ratio	S	$\sum_i^s MSE_i$	$MMSE$
0	27	0.097388	0.003607
0.1	27	0.164271	0.006084
0.2	27	0.394767	0.014621

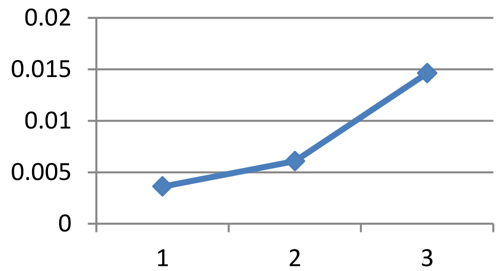

Figure 5. MMSE values with polluted ratios

Table 5 and Figure 4 show that MMSE best values effected with pollution ratios

1. Conclusions and suggestions

After (27) un-polluted and (54) polluted simulation experiments were applied each of them with (1000) iterations many conclusions and suggestions were established such that:

- Estimation method affected by sample size.
- Simulation experiments affected by true distribution parameters.
- Simulation experiments affected by number of outliers.
- Best estimators affected by pollution ratios.
- Other estimation methods such that (moment, shrinkage and white noise) can be compared with Bayesian estimation method.
- Adding other polluted ratios can be proposed as mixed distribution and other estimators can be compared

- Another simulation experiments for other statistical distributions can be proposed for Bayesian estimation method.

Declaration of competing interest

The authors declare that they have no any known financial or non-financial competing interests in any material discussed in this paper.

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