TIDAL CHARACTERISTICS AND A NUMERICAL MODEL FOR THE M_2 TIDE AT THE ESTUARINE COMPLEX OF THE BAY OF PARANAGUÁ, PARANÁ, BRAZIL

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INTRODUCTION

The Estuarine Complex of the Bay of Paranaguá (ECBP), Figure 1, is located in Paraná State, southern Brazil. It presents a semidiurnal tide with diurnal inequalities. The biases from the semidiurnal pattern during neap tides are explained here as non-linear effects. The prominent tidal constituent is M_2 (lunar semidiurnal), and tidal non-linear effects such as asymmetry and double high and low waters can be observed in the records.

The sparse works and results on the tidal regime and dynamics of the ECBP (Knoppers *et. al.*, 1987) agree that there is a mainly semidiurnal tide with diurnal inequalities, suggesting a mixed type propagation wave.

Several authors, modelling the tidal wave propagation (Camargo, 1991; Harari, 1984, 1985; Harari & Camargo, 1994) have studied the south-eastern Brazilian continental shelf. Only recently, studies for the Paranaguá area account for the problem of meteorological forcing on sea level (Camargo, 1996, 1997) and a numerical model for the current circulation was applied by Costa Tenório and Rosman (1997), which does not reproduce this mixed type propagation regime. Camargo & Harari (1994) Pereira (1987) and Stech & Lorenzzetti (1992) studied storm surge effects and modelling for the area.

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In the absence of strong meteorological effects, the estuarine circulation at the ECBP is forced, mainly, by tidal dynamism. Therefore, correct knowledge about tidal forcing would aid in describing the main circulation patterns and the astronomical tidal constituents could be used as forcing in numerical models.

The results for the M_2 tide were used to feed the "variable boundary numerical model" developed by Mungall and Matthews in the 70's.

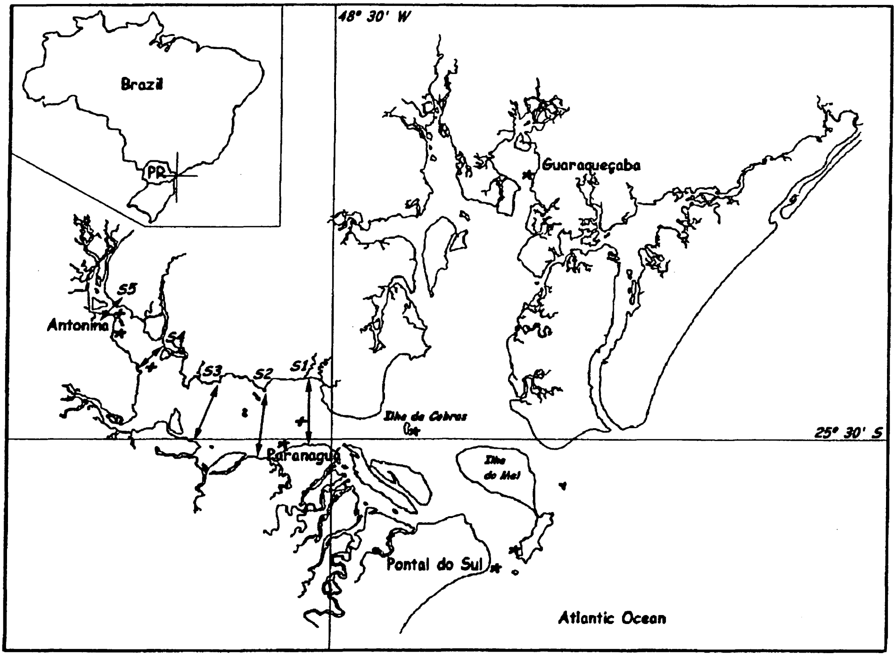

Fig. 1
The Estuarine Complex of the Bay of Paranaguá (ECBP). Points where tidal observations were obtained are indicated with asterisks (*). The model domain is between section S1 and the head of the bay (S5). Points indicated with a plus (+) were used as control and validation points for the numerical model.

ANALYTICAL STOCHASTIC MODEL

The harmonic analysis consists of a spectral analysis, via the Fast Fourier Transform, of the sea level time series record. Harmonic analysis is in essence a stochastic method used to determine the harmonic tidal constants, H (amplitudes) and G (phases), of the various tidal constituents. This method is based on the different phase variations of the constituents, resulting from the differences of their angular frequencies

(Franco, 1988). It not only provides the astronomical tidal constituents present in the tidal record, but also, the shallow water constituents that are generated by non-linear effects (Marone, 1991).

Six tidal records were analysed using the harmonic method (Franco, 1988). The six points are indicated in Figure 1 with asterisks (*). Figure 2 shows part of the used tidal records (two weeks for clarity, not simultaneous). The tidal records were obtained from the files of the Paranaguá Harbour Authority, the National Oceanographic Data Base of the Brazilian Hydrographic Directorate (DHN), and measured with the use of an Aanderaa tidal gauge by the Marine Physic Lab of the Centre for Marine Studies of the Federal University of Paraná.

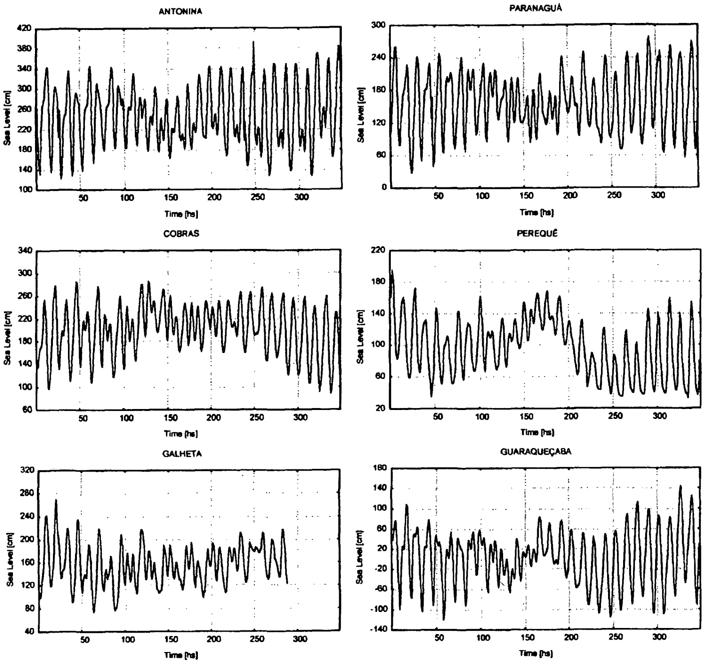

Fig. 2
Samples of tidal records for the indicated ports.

In Table 1, the ten most important harmonic tidal constants for the six ports are depicted. The length of each analysed tidal record is also indicated as well as the total number of harmonic constants accepted as significant using the signal-noise ratio test (Franco, 1988). Table 2 shows the main difference between the mouth of the bay (Galheta) and the end (Antonina), including records from Cobras and Paranaguá.

Table 1 - Ten main tidal harmonic constants for the analysed ports. The lengths of the records as well as the number of accepted tidal constituents for each port are also indicated. Phases are referred to local time

Place	**Antonina**		**Paranaguá**		**Cobras**		**Perequê**		**Galheta**		**Guaraqueçaba**	
Lat. S	25º26.5'		25º28.9'		25º28.9'		25º34.5'		25º32.0'		25º17.5'	
Long. W	48º41.6'		48º27		48º21.1'		48º19.2'		48º19.2'		48º19.5'	
# Data	1104		744		2979		1152		1152		5274	
# Tid. Cts.	27		18		24		10		10		41	
Tidal consts.	AMPL (cm)	PHASE (degr.)	AMPL (cm)	PHASE (degr.)	AMPL (cm)	PHASE (degr.)	AMPL (cm)	PHASE (degr.)	AMPL (cm)	PHASE (degr.)	AMPL (cm)	PHASE (degr.)
O_1	12.00	80.13	11.88	87.95	13.12	79.85	7.88	78.36	9.74	77.07	11.83	78.46
K_1	10.14	126.67	6.34	139.35	8.86	117.93	5.34	144.95	5.05	138.43	6.62	137.31
M_2	53.72	107.69	48.87	97.43	46.27	91.78	30.62	98.18	33.87	63.68	52.09	90.99
S_2	36.76	114.60	29.12	99.88	27.87	87.19	20.88	108.47	22.30	74.94	28.72	97.38
K_2	10.00	115.16	11.82	84.27	7.58	86.82	5.68	109.30	-	-	7.81	97.90
N_2	8.29	175.07	7.57	163.17	7.34	164.60	3.85	173.01	13.56	155.73	8.90	157.15
M_3	22.19	285.51	13.88	254.19	14.42	257.02	6.15	255.33	12.37	218.06	17.31	252.23
M_4	22.29	300.76	14.72	272.53	11.53	245.45	6.71	205.62	8.22	166.73	20.76	258.13
MS_4	10.88	56.27	8.23	354.10	4.91	340.73	1.24	232.34	4.41	276.76	9.24	355.34
MN_4	8.85	248.67	6.30	228.71	4.53	194.86	2.92	182.91	-	-	8.02	198.00

As one can see from Table 2, the tidal range is enhanced upstream, especially during spring tides, when it varies from 1.7 m in the entrance to nearly 2.7 m in Antonina. During neap tides, tidal range is reduced to less than 80% of the spring tidal range, and strong non-linear interaction between the tidal constituents allows for the formation of up to six high and low tides per day (Marone and Camargo, 1994). This higher frequency oscillation is ascribed to double high and double low waters, which results from a subtle interaction between the fundamental component (M_2) and its higher harmonics, especially M_6, M_8, M_{10}, etc. (Godin, 1993). For simplicity, Table 1 shows only the ten most significant tidal constants, but it is important to note that there is a great number of shallow water constituents, increasing from the mouth to the end of the bay. These non-astronomical tidal constituents are generated by strong non-linear interactions due to advection and friction inside the bay. Another important evidence of the non-linear interaction can be observed from the range amplification at higher harmonics. If M_2 is amplified 1.59 times from Galheta to Antonina, the higher non-linear components, like MS_4, are enlarged 2.5 times or more.

The tides also display an increasing asymmetry upstream, characterised by a shorter falling and a longer rising tide. The average ratio of flooding to ebbing time increases from 1.23 at Galheta Channel to 1.92 close to Antonina. Also, high and low water times in Antonina lag more than 1.5 hours behind those in Galheta Channel. Tidal intrusion is approximately 12 km, and flushing time is around 4 days (Marone et. al., 1995).

Table 2 - Tidal characteristics of the observed sea level records for the Bay of Paranaguá. Here, spring and neap phase differences are in minutes, considering Galheta as the zero origin. Ebb and flood times are indicated in hours and spring and tidal ranges in centimetres.

Place	Spring Phase Differences (min)		Neap Phase Differences (min)		Ebb Time (hs)	Flood Time (hs)	Spring Range (m)	Neap Range (m)
	high	low	high	low				
Galheta	-	-	-	-	6.8	5.5	1.74	1.30
Cobras	69	55	91	75	-	-	2.07	1.68
Paranaguá	88	68	106	93	7.5	4.8	2.09	1.70
Antonina	100	110	132	151	8.1	4.2	2.74	2.02

In spite of nearly equivalent distances between the Galheta, Cobras, Paranaguá and Antonina tidal gauges, the phase differences for M_2, are greater between Galheta and Cobras than between Paranaguá and Antonina, (note that this occurs for the other semidiurnal and quaterdiurnal constituents, but not for the diurnal and terdiurnal ones). This fact suggests that odd constituents, especially the diurnal components, but also M_3 propagate mainly as stationary waves along the whole bay. Even constituents show a progressive-like pattern at the mouth, evolving into a mixed type in the middle and, finally, becoming mainly stationary at the end of the bay.

NUMERICAL MODE

The numerical model used for the area corresponds to the "variable boundary numerical model" developed by Mungall and Mathews (1970). The usefulness of this model, that does not include advective terms, was justified by its simplicity, and proved its ability to solve the tidal dynamics for semi-enclosed bays in different studies (Mungall and Mathews, op. cit.; Marone, 1982).

The capacity of the model to work with "variable boundaries" was not used in this simulation due to limitation of well represent tidal flats, as explained later in the text.

The basic hydrodynamic equations are:

$$\frac{\partial U}{\partial t} + \eta\,(U^2 + V^2)^{1/2}\,\frac{U}{H} - fV + g\,\frac{\partial Z}{\partial x} = 0$$

$$\frac{\partial V}{\partial t} + \eta\,(U^2 + V^2)^{1/2}\,\frac{V}{H} + fU + g\,\frac{\partial Z}{\partial y} = 0$$

and

$$\frac{\partial (HU)}{\partial x} + \frac{\partial (HV)}{\partial y} + \frac{\partial Z}{\partial t} = 0$$

where

$U = x$ -component of depth-mean velocity
$V = y$ -component of depth-mean velocity
$Z =$ vertical tide measured (positive upwards)
from mean sea level
D = depth of water beneath mean sea level
$H =$ total depth o $\eta =$ friction coefficient
$\eta =$ friction coefficient
$f =$ Coriolis parameter ($f = 2\Omega\sin(\text{latitude})$)
$g =$ acceleration due to gravity
$\Omega =$ angular rotational speed of the earth

The above equations were solved for the numerical model using the explicit method. The derivation of the finite difference form of the equations was covered, and the contours were digitised from bathymetric charts. The domain of the model corresponds to the liquid area between Paranaguá and the bottom of the bay (after Antonina) and it is depicted in Figure 3.

APPLICATION FOR THE ESTUARINE COMPLEX OF THE BAY OF PARANAGUÁ

The domain of the model corresponds to the ECBP area, which is about 21.6 km long, variable in width, with relatively shallow depths. Along the S3-line (Figure 1) the greatest depth is around 17.5 metres. At the S3-line the bay is 3.5 km wide, but narrows locally to 0.9 km along S5-line (see Figure 1). In Antonina the region becomes increasingly complicated (in the hydrodynamic sense) by the presence of shoals and muddy flats, with extensive areas being exposed at low tide. The bathymetry of the bay and the numerical grid are shown in Figure 3.

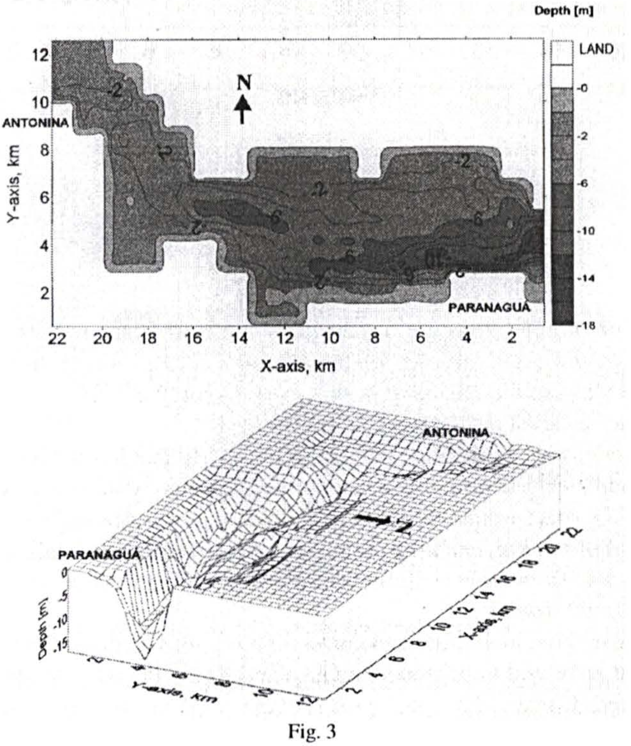

Fig. 3
Bathymetry and grid for the numerical model.

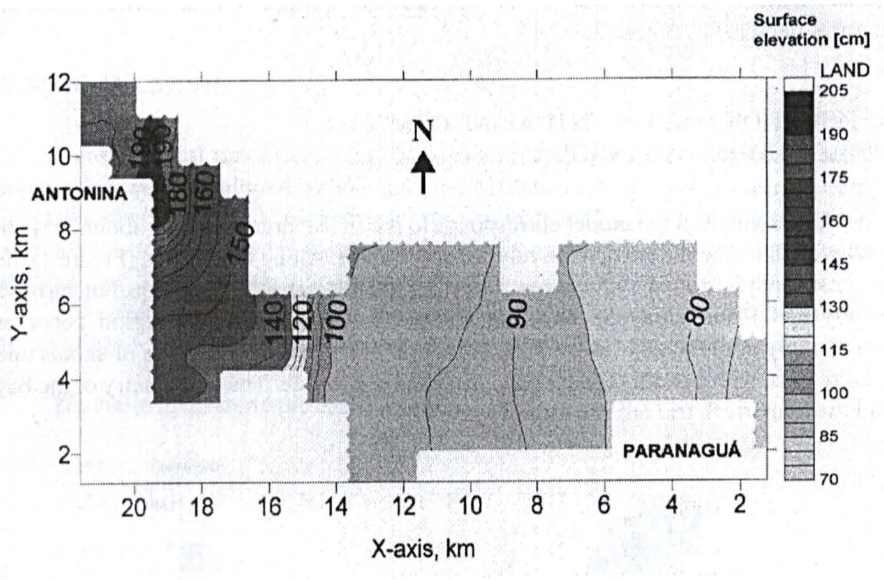

Fig. 4
Co-tidal lines from the numerical model.

The current conditions are of considerable practical interest in the case of the ECBP. Due to the predominance of the M_2 tide, it was used as the main forcing parameter. It was not the objective of this work to study the effect of meteorological forcing on the sea level and circulation at the ECBP.

The model was restricted to the part of the ECBP that has the highest number of inhabitants and where several important ports are located (Paranaguá, Antonina, etc.). To represent the coast around the numerical domain, it was deemed necessary to select a grid interval of 0.6 km, enabling the region of interest to be contained within a grid of 37x21 nodes. To comply with the accepted stability condition, a time interval of 31.05 seconds was found.

We chose three main tidal and current input points as indicated by a plus (+) in Figure 1, that were within the modelled area and where the tidal heights and currents were measured. Current data and sea level heights were used to validate the model results. An extrapolation was then performed to obtain the values at each input point. Because of the inability of such a model to handle muddy flats (i.e. region where the depth may occasionally become zero), all such regions were assigned with an arbitrary depth of 1 metre. The model is capable of handling variable boundaries when dynamical circulation does not suffer dynamic regime changes. In the case of channel circulation, a more "fluvial" dynamics occurs and when the waters circulate over the lateral and extensive muddy flats, during high tides, the circulation patterns change (lake kind) and the model does not respond if variable boundaries are used (Marone, 1982; Bonilha, 1974; Martins, 1992).

Five full tidal cycles were computed, after which dynamic conditions appeared steady. It becomes at once apparent that the tidal regime of the ECBP is divided into two distinctive regions, separated by an intermediate area. For the sake of convenience, the main areas may be called Paranaguá Inlet (delimited by the mouth - S1 - and the S3-line) and Antonina Inlet (delimited by the final coast contour and the S5-line). They are separated from each other by the natural feature of the narrow section that lies between the Ponta Grossa and Teixeira Islands at both sides of the bay that surround the S3 line.

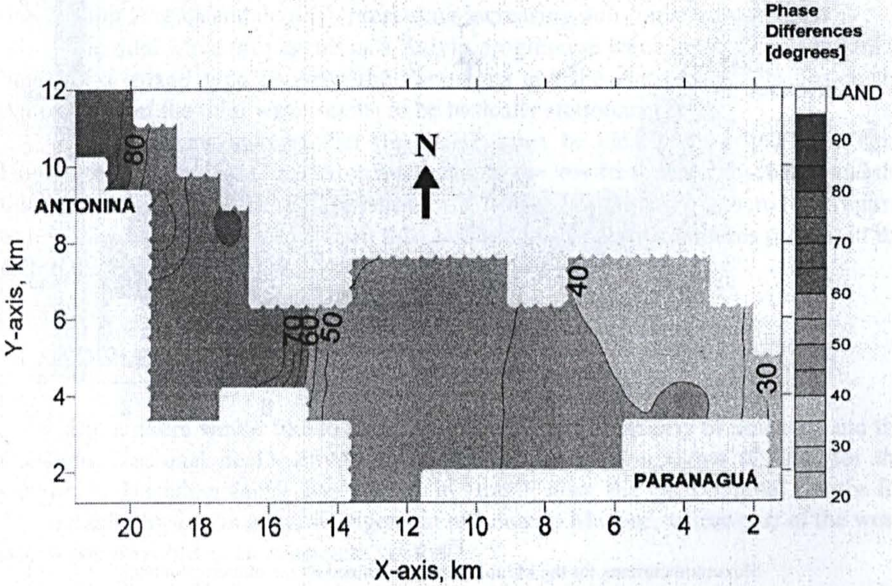

Fig. 5
Co-range lines as calculated by the numerical model.

The co-tidal and co-range lines are shown in Figures 4 and 5. The tides in the Paranaguá Inlet show the characteristic shape of a progressive Kelvin wave. The co-range lines lie along the length of the inlet and parallel to the co-tidal lines.

The co-tidal lines, in the region between Teixeira Island (S3) and the city of Paranaguá, lie generally perpendicular to the co-range lines, indicating that the wave is not entirely progressive but tends to be a mixed type of wave.

To the west of Teixeira and Ponta Grossa Islands (S3), the wave changes to one of standing wave characteristics. Considerable distortion from the frictionless case is present, as evidenced by the fact that the co-tidal lines are not perpendicular to the co-range lines, suggesting higher frictional activity.

The co-range lines, in addition, show that almost all the amplification of the tide occurs between Paranaguá and Antonina Inlets, the M2 range increasing from 70 to 200 cm.

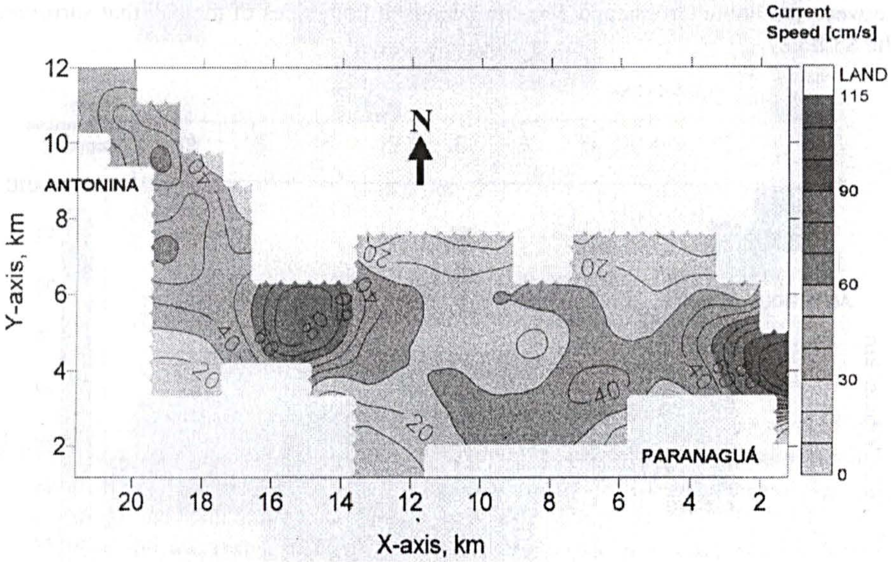

Fig. 6
Maximum currents for the M2 tide forcing obtained by the numerical model.

A look at the output of the current analysis shows that the maximum -mean-depth currents occur at the entrance of the Paranaguá section and Teixeira-Ponta Grossa narrows (S3 surroundings). They reach a maximum value of around 110 and 80 cm/sec, respectively, and rotate counterclockwise (see Figure 6).

Some eddy-like patterns were observed in several parts of the bay in the simulation. These patterns can be ascribed to the rotational effects of the earth and the transition from progressive to stationary propagation conditions. They can also explain the lateral inhomogeneities observed in the circulation regime (Marone *et. al.*, 1995). The model results were validated using two mooring arrays installed during one year in front of the Paranaguá harbour and in the middle of the channel in section S3. Both arrays used current metres and mini-std to record the appropriate variables in two layers (surface and bottom).

CONCLUSIONS

Tides at the studied area of the Estuarine Complex of the Bay of Paranaguá are mainly semidiurnal with diurnal inequalities. They present highly non-linear effects that create tidal asymmetry and double high and low water effects. The tidal ranges are amplified from the mouth to the end of the ECBP, mainly due to continuity effects (decreasing lengths and depths), presenting increasing non-linear interactions.

The tidal wave propagates as a Kelvin progressive wave in the Paranaguá Inlet, being of a mixed type between the Paranaguá and the Antonina inlets. Inside the Antonina Inlet the tidal wave seems to be basically stationary.

Tidal currents suggest that the ECBP must be clearly classified as a Tidal Dominated Estuary System, taking into account the low fresh water discharge and the tidal dynamics. The ECBP fails to be classified as having a micro or macro tidal regime as a whole, because of the different tidal ranges and circulation patterns present in the different sections and inlets.

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ABSTRACT

The tides at the estuarine complex of the Bay of Paranaguá (ECBP) were investigated using stochastic analysis and numerical modelling. Sea level records from six tidal locations were analysed using harmonic analysis, and showed that M_2 is the dominant tide. A numerical model was implemented based on the dominance of the M_2 component. The model uses finite differences to represent the hydrodynamic equations, with an explicit resolution scheme and non-slip contour conditions. The tidal regime is semidiurnal with pronounced diurnal inequalities. The overtides, the even, and the shallow water constituents are important, indicating strong non-linear interactions, which allow the formation of up to six high and low tides per day during neap cycles. This higher frequency oscillation is ascribed to double high and double low water effects. Tidal ranges increase from the mouth to the end of the complex, by 160%. Tidal phases and ranges indicate that the tidal wave propagates in a mixed form. The model confirms the mixed tidal wave regime.

Key-words: tides, analysis, numerical modelling, non-linear, estuary, Bay of Paranaguá.

RESUMO

As marés no Complexo Estuarino da Baía de Paranaguá são estudadas com o uso de um modelo estocástico de análise e da modelagem numérica. Registros das alturas do nível do mar em seis diferentes estações maregráficas foram analisadas pelo método harmônico, resultando num conjunto completo das componentes de maré para a área. Usando os valores da componente M_2, foi aplicado um modelo numérico bidimensional. O modelo usa o esquema de diferenças finitas para a representação das equações hidrodinâmicas, esquema explícito de resolução e condição de contorno não deslizante. Os resultados finais mostram uma descrição da dinâmica da componente M_2 para a região, incluindo o regime de correntes de marés. A partir do conjunto das componentes de maré é possível confirmar que o regime é principalmente semidiurno com desigualdades diurnas. As sobremarés, as componentes de maré de ordem par e as componentes de pequeno fundo são importantes, indicando a importância das interações não lineares, que provocam a formação de até seis preamares e baixa-mares por dia em períodos de quadratura. Estas oscilações de mais alta freqüência são conhecidas como duplas preamares e baixa-mares. A amplitude da maré aumenta da boca para montante do complexo, sendo amplificada quase duas vezes. As fases e amplitudes das marés indicam um regime misto para a propagação da onda de maré. O principal resultado do modelo foi confirmar numericamente o regime misto de propagação da onda de maré.

Palavras-chave: marés, análise, modelagem numérica, não linear, estuário, Baía de Paranaguá.

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