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Unveiling belief and pedagogical content knowledge of prospective secondary mathematics teachers

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Abstract

Prospective teachers' beliefs have a beneficial effect on their knowledge and actions. Furthermore, teachers' pedagogical content knowledge is an essential contributor to teachers' impact on students' academic outcomes. However, examining how prospective teachers' beliefs influence their pedagogical content knowledge is still unclear. The current study investigates prospective mathematics teachers' beliefs and pedagogical content knowledge. We recruited three undergraduate students in the mathematics department consisting of one male and two female participants in this study. By assigning two interviews, a test, and observation, we examined three prospective mathematics teachers' pedagogical content knowledge and their beliefs about the nature of mathematics, mathematics learning, and mathematics teaching. Prospective mathematics teachers' test, interview, and observation results were analyzed in three stages: data condensation, presentation, and conclusion drawing and verification. The findings point out that traditional beliefs (instrumentalist and platonist views) are more dominant in prospective teachers' beliefs about the nature of mathematics. Moreover, the more sophisticated prospective teachers' beliefs about the nature of mathematics, mathematics learning, and mathematics teaching are, the higher one's content knowledge and pedagogical content knowledge levels are. Implications of this study, mathematics teacher educators should assist and foster prospective teachers in developing constructivist or problem-solving beliefs.

Keywords: belief; pedagogical content knowledge; prospective mathematics teacher

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Introduction

Pedagogical Content Knowledge was introduced by Shulman (1986), which was further developed by Ball and colleagues (2008) with the framework Mathematical Knowledge for Teaching (MKT or Mathematics knowledge for teaching. This MKT includes knowledge about content and students and knowledge about content and teaching. Furthermore, Hill, Rowan, and Ball (2005) define MKT as mathematical knowledge used to teach mathematics.

Meanwhile, PCK is a combination of pedagogical knowledge and content knowledge. Pedagogical knowledge is associated with planning and organizing learning; representations are used to illustrate a concept, examples and assignments, planning methods, and evaluation techniques (Ma'Rufi et al., 2017). At the same time, content knowledge is knowledge about the material or content being taught, including facts, concepts, principles, and procedures (Hill, Blunk, et al., 2008; Nur'aini & Pagiling, 2020). Content knowledge is actual knowledge about the material, the lesson to be learned or taught, including the main idea, theory, and organizing or linking ideas. Meanwhile, pedagogical knowledge is knowledge about teaching and learning processes and practices or methods, including classroom management, lesson plans development and implementation, and student assessment (Hill et al., 2005; Kim, 2018)

Knowledge of pedagogical content is knowledge of pedagogy that can be applied and is suitable for teaching certain content. Thus, pedagogical content knowledge (PCK) combines or compounds content knowledge with pedagogical knowledge (Ball et al., 2008). Furthermore, several researchers scrutinized that PCK is knowledge about teaching concepts (including giving feedback), students' understanding of concepts, task-level knowledge, and teaching knowledge (Ekawati et al., 2015; Kersting et al., 2012; Mitchell et al., 2014).

The results of MKT research have provided a foundation for improving the quality of mathematics learning and teacher professional development (An et al., 2004; Copur-Gencturk & Lubienski, 2013; Friesen & Kuntze, 2020; Hill, Ball, et al., 2008). To create rich and meaningful mathematics learning, prospective teachers should have in-depth knowledge of mathematics for teaching.

One of the factors that influence the knowledge of prospective teachers to teach mathematics is belief. Belief is an individual's psychological conception or understanding of the world and contains affective elements based on assessment and evaluation (Cross, 2009; Smith et al., 2016). Teacher beliefs are basic understandings, thoughts, conceptions, or propositions about something that is considered trustworthy, whether realized or not realized by a teacher (Philipp, 2007). The prospective teachers' beliefs can predict, reflect, and determine their actual teaching practice when they become teachers.

Mathematics teachers' beliefs have been studied for decades and show that teachers' beliefs have a crucial role in learning strategies in the classroom. The instrumentalist view is that mathematics is a compilation of facts, rules, and abilities that are employed to accomplish some external goal (Ernest, 1989; Safrudiannur & Rott, 2019). Thus, mathematics is an isolated set of rules and facts, b) the Platonist view of mathematics as a particular static but the unified body of knowledge. Mathematics is discovered, not created,c) The view of solving mathematical problems as a dynamic and growing domain of human creation and discovery and a cultural product. Mathematics is a process of inquiry and knowing, not a finished product, as the results remain open to revision.

Constructivist teachers organize student-centered learning activities that encourage independent learning and group discussions. Moreover, teachers with *constructivist or problem-solving views* encourage students to engage in environmental learning. In comparison, teachers who have traditional beliefs prefer direct teaching strategies and position themselves as the sole knowledge providers (Kim, 2018).

Several studies have shown that belief is closely related to the knowledge of teachers or prospective teachers in teaching mathematics and demonstrate that beliefs influence teaching practice in the context of a supportive learning environment (Lui & Bonner, 2016; Safrudiannur & Rott, 2019; Stipek et al., 2001). Meanwhile, Muhtarom's (2019) research indicates the critical role of belief and PCK in influencing prospective teachers' teaching and learning practices. Research findings by Dede & Uysal (2012) reveal that primary school teachers have a student-centered view of the nature of mathematics and mathematics teaching. Previous research has primarily focused on the impact of teacher belief (Cross, 2015; Safrudiannur & Rott, 2020) or professional knowledge (Charalambous & Hill, 2012; Ekawati et al., 2015) in teaching approaches. Furthermore, most previous studies have focused on primary school teachers or future primary school teachers (Felbrich et al., 2012; Lui & Bonner, 2016; Wilkins, 2008)**.** Therefore, the study investigating prospective teachers' beliefs and pedagogical content knowledge is essential to provide a lens for professional development. The current study explores prospective mathematics teachers' beliefs and pedagogical content knowledge.

Methods

This study uses a qualitative approach to investigate prospective mathematics teachers' beliefs and pedagogical content knowledge. Three final-year mathematics education department students were recruited as participants in this study, including a male and two females at a university in Merauke Regency. Final year students are selected since they have accomplished most pedagogy and mathematics courses. In addition, they have completed the microteaching and practical field experiences courses, which are mandatory courses for prospective teacher students.

The researchers collected data using a belief interview guide adapted from the instrument (Kim, 2018), task-based interviews, and observation. Two mathematics education experts validated the construct validity of these instruments. Belief interviews were used to measure prospective mathematics teachers' beliefs about the nature of mathematics, mathematics learning, and mathematics teaching. Furthermore, task-based interviews, which include the content of the Pythagorean Theorem, Combinatorics, and Geometric Series, each task is carried out to measure the pedagogical content knowledge of prospective mathematics teachers. In this article, the researcher only presents the PCK of prospective teachers on the content of the Pythagorean theorem. Each participant was asked to accomplish all three tasks, and then participants were interviewed about their content knowledge and strategies in teaching the material.

Data were analyzed qualitatively, namely by reducing data, presenting data, and verifying (Miles et al., 2014). Researchers conducted data reduction by coding the beliefs and PCK of prospective teachers based on the results of belief interviews and PCK task-based interviews. Furthermore, the researcher presents the data in the form of a matrix and describes it narratively. Researchers categorize the PCK level of prospective teachers into three levels, namely beginner, intermediate, and advanced levels adapted from (Kim, 2018). To ensure the validity of the data, the researcher triangulated investigators (Rothbaeur, 2008). Both researchers were actively involved in collecting and interpreting the data. In addition, researchers conducted member checks to accurately describe prospective mathematics teachers' beliefs and PCK. In detail, the researchers initially classify prospective teachers' beliefs, which contain three dimensions (the nature of mathematics, mathematics learning, and teacher role based on Table 1.

Table 1. Prospective teachers' beliefs

Dimensions of Beliefs	Classification of Beliefs	Description
Nature of mathematics	Instrumentalist	Mathematics is a set of facts and formula
	Platonist	Mathematics as a specific, unchanging unit of knowledge
	Constructivist	Mathematics as a human creation that is constantly changing
Mathematics learning	Passive Knowledge Reception	Students exhibit respectful behaviour and master skills. Students passively receive knowledge from the teacher
	Active Knowledge Construction	Students actively construct understanding. Students explore their interests independently
Teacher role	Instructor	The aim of teaching is for students to master skills and perform them correctly
	Explanation	The purpose of teaching is for students to develop understanding conceptual understanding of an integrated collection of knowledge
	Facilitator	The purpose of teaching is for students to become confident problem solvers.

Next, the researcher coded the results of task-based interviews based on the scoring rubric to map prospective teachers' CK and PCK levels, which are shown in Table 2.

Table 2. Level of content knowledge and pedagogical content knowledge

Indicators of Content Knowledge	Indicators of Pedagogical Content Knowledge
A. Prospective teachers are able to prove the Pythagorean theorem using the concepts of the area of a square and the area of a triangle.	A. Prospective teachers are able to teach the proof of the Pythagorean theorem using the concept of the area of a square and the area of a triangle.
B. Prospective teachers are able to prove the Pythagorean theorem using the concepts of the area of a square and the similarity of a triangle.	B. Prospective teachers are able to teach the proof of the Pythagorean theorem using the concept of the area of a square and similarity.
C. Prospective teachers are able to prove the Pythagorean theorem using the concept of the area of a square and the side-angles theorem.	C. Prospective teachers are able to teach the proof of the Pythagorean theorem using the concept of the area of a square and the side-angles theorem.
One of A-C fulfilled is coded beginner Two of A-C fulfilled is coded intermediate Three of A-C fulfilled is coded advanced.	One of A-C fulfilled is coded beginner Two of A-C fulfilled is coded intermediate Three of A-C fulfilled is coded advanced.

Results

Prospective mathematics teachers' beliefs about the nature of mathematics, mathematics learning, and mathematics teaching

The following is dialogue 1, showing the transcript of an interview between the researcher and subject 1.

Dialogue 1

R : *What do you think mathematics is?*
I : *Mathematics is the science of numbers useful for humans in solving everyday problems.*
R : *According to Igor, what is the essential thing in mathematics as a formula and problem-solving?*
I : *Mathematics as rules or algorithms.*
R : *Why does Igor think like that?*
I : *Because since elementary school to high school, mathematics subjects mainly discuss rules and procedures*
R : *What subjects are most similar to mathematics?*
I : *Science especially Physics*
R : *Why do you think like that?*
I : *Because Physics also contains rules or formulas and in general, the concepts in Physics are the application of mathematics, such as the concepts of velocity, derivative, and integral*

R : Why do we need to learn mathematics?
I : Mathematics helps us in problem-solving.
R : How can mathematics be helpful in your daily life?
I : In my opinion, mathematics is fundamental in everyday life, such as applying mathematics in knowing profit and loss, architecture such as building construction.
R : According to Igor, how do students learn mathematics?
I : Students learn mathematics by exploration or investigation so that learning becomes meaningful
R : Why?
I : Because with exploration, students will find out to find, work on math problems so that the student's learning experience becomes meaningful.
R : According to Igor, what is the role of a mathematics teacher?
I : In my opinion, a mathematics teacher is more of a facilitator.
R : Why think like that?
I : Teachers need to facilitate students to think mathematically.
R : To be a good math teacher, what do you think is the most important thing a teacher should do?
I : Teachers need to know the characteristics of students, students' initial abilities, learning strategies, updating teaching materials, professional development training.
R : Why are student characteristics the most important thing?
I : To ensure student-focused learning.
R : To be a good math teacher, what do you think is the most important thing a teacher should master?
I : Materials, learning strategies, lesson plans, and software mathematics
R : Why do teachers need to use technology?
I : To make mathematics learning interactive.

Based on dialogue 1, it was evident that Igor had instrumentalist beliefs about *the nature of mathematics*. His statement indicates that mathematics is a collection of facts, formulas, rules, and skills useful for human life and used to solve everyday problems. In addition, he argues that the truth of mathematics content in schools is absolute so that mathematical content in schools is free from contradictory or contradictory interpretations. However, Igor believes in experiential learning of mathematics while learning in high school as an independent activity and exploration through rich discussions involving students and teachers to construct an understanding of mathematical ideas. For the dimension *of teaching mathematics*, Igor believes that a good mathematics teacher should act as a facilitator. He argues that teachers should encourage students to become reliable and confident problem solvers in teaching mathematics. Igor also believes that teachers need to know the characteristics of students, students' initial abilities, learning strategies, updating teaching materials, and professional development training. His statement indicates that Igor wants student-centered mathematics learning because each student has unique characteristics and different abilities. Therefore, the researcher concludes that Igor has a *problem solving* on the dimensions of mathematics teaching.

Below is presented dialogue 2, which displays a transcript of the interview between the researcher and subject 2, namely Tasya.

Dialogue 2

R : What do you think mathematics is?
T : Mathematics is an exact science of logic and measurement
R : Why?
T : Because mathematics contains an abstract object of study, understanding it requires reasoning or logic.
R : What subjects are most similar to mathematics?
T : Physics.
R : Why think like that?
T : Physics is the development of mathematical concepts.
R : Why do we need to study mathematics?
T : Because mathematics is essential in life and work, the spread of covid-19 can be made in mathematical models.
R : Specifically, how is mathematics practical in Tasya's daily life?
T : I am buying books with different brands and prices.
R : According to Tasya, how do students learn mathematics?
T : Students learn mathematics with the help of the teacher
R : Why?
T : In my view, math is complicated.
R : What do Tasya most want students to master when studying mathematics?
T : Concept understanding.
R : According to Tasya, what is the teacher's role in teaching mathematics?
T : In my opinion, a mentor or facilitator. Teachers need to be creative in choosing mathematics teaching strategies.
R : To become a mathematics teacher, what are the critical things teachers need to do?
T : Teachers need to evaluate, reflect, update knowledge and skills through seminars.
R : Apart from that, is there anything else?
T : We are updating teaching materials.
R : What is the most important thing for the teacher to do out of the four components?
T : Conduct evaluations, then attend seminars, reflect, and update teaching materials.
R : What things do mathematics teachers need to know for effective learning?
T : Learning strategies, materials, knowledge of learning evaluation, knowledge of mathematics media or software, general knowledge of technology
R : What are the learning strategies in teaching mathematics in terms of concepts?
T : If the concept is difficult, you need to use the help of media or software math. On the other hand, if the concept is easy enough to be given a brief explanation and examples.

Based on dialogue 2, it was found that she had platonist beliefs about *the nature of mathematics*. Tasya argues that mathematics is the science of definite, abstract, and absolute logic, so it is a static science. She believes that mathematics content or school material is connected and logically connected in an organized structure. However, for the truth value of mathematics, she argues that humans make mathematics to determine the truth value of mathematics content in schools. For the dimension *of learning mathematics*, she has the concept that in learning mathematics, students need to understand mathematical concepts and learn from the surrounding environment. In addition, students need to be accustomed to constructing understanding mathematical ideas and actively exploring their interests. As for the dimensions *of teaching mathematics*, she believes that teachers should be creative in choosing teaching strategies.

Furthermore, she asserted that teachers need to evaluate, reflect, and improve professional competence; they periodically improve their knowledge and skills through seminars. Tasya also has aspects that mathematics teachers need to know or master for effective learning: mathematics material or content, learning strategies, technology in general, software to support mathematics teaching, and evaluation. Her statement is associated with a platonist view emphasizing students' mastery of mathematical content.

The following shows dialogue 3, which displays a transcript of the researcher's interview with Ana.

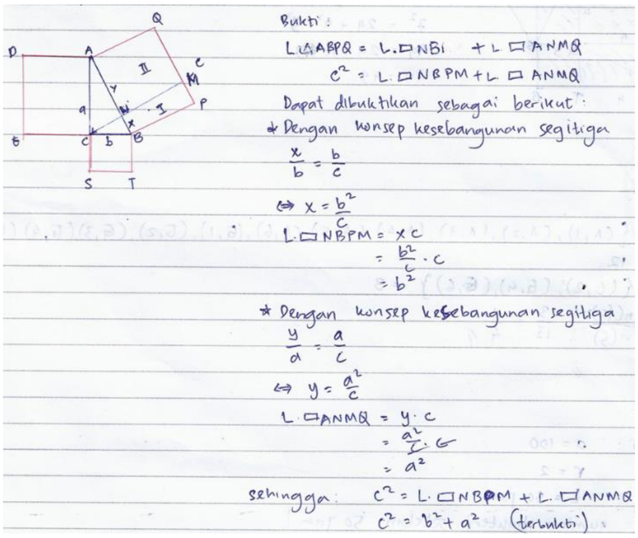
Dialogue 3

R : What do you think mathematics is?
A : Mathematics is an exact science about calculations
R : Why does Ana think like that?
A : Mathematics contains formulas and calculations.
R : What subjects are most similar to mathematics?
A : Physics
R : Why think like that?
A : Because physics also contains a lot of formulas and theorems.
R : Why do we need to study mathematics?
A : Mathematics is closely related to everyday life.
R : According to Ana, how do students learn mathematics?
A : Students learn mathematics with repeated exercises.
R : Why?
A : The more we practice, the more we will understand a mathematical concept.
R : The most important aspect of mathematics that students must master?
A : Concept understanding and problem-solving
R : Why think like that?
A : Because if you do not understand the concept, you will not be able to solve math problems.
R : What is the role or task of the mathematics teacher?
A : Facilitator
R : Why?
A : Students can learn independently and find solutions.
R : Can you give an example?
A : The teacher gives worksheets, and students are led to find the correct answer or solution.
R : To be a good math teacher, what are the essential things teachers need to do?
A : Designing creative teaching materials, teachers conduct evaluations, participate in professional development regularly, webinars
R : What things do mathematics teachers need to know or master for effective learning?
A : Teachers need to master the material, master technology, be creative in delivering material, and master mathematics software.

Based on dialogue 3, it is obtained that Ana has two different beliefs, namely instrumentalist and platonist, about mathematics's nature. Ana argues that mathematics is a science that contains formulas or rules for performing calculations and is a definite, abstract, and absolute science so that it is a static science. For the dimension *of mathematics learning*, she argues that students learn mathematics by repeated exercises to understand a mathematical concept associated with a platonist view. As for the dimension *of teaching mathematics*, she has a platonist view since she believes that the purpose of teaching mathematics is for students to develop a conceptual understanding of an integrated collection of knowledge. She believes the most important things that need to be mastered by teachers are teachers need to master the content, master technology, be creative in delivering material, and master *software* math to achieve this goal. Furthermore, she believes that teachers need to design creative teaching materials, conduct evaluations, regularly participate in professional development, and participate in webinars, especially during the pandemic.

Prospective mathematics teachers' content knowledge

The following is presented in Figure 1 as the results of Igor's work in solving the problem of proving the Pythagorean theorem.



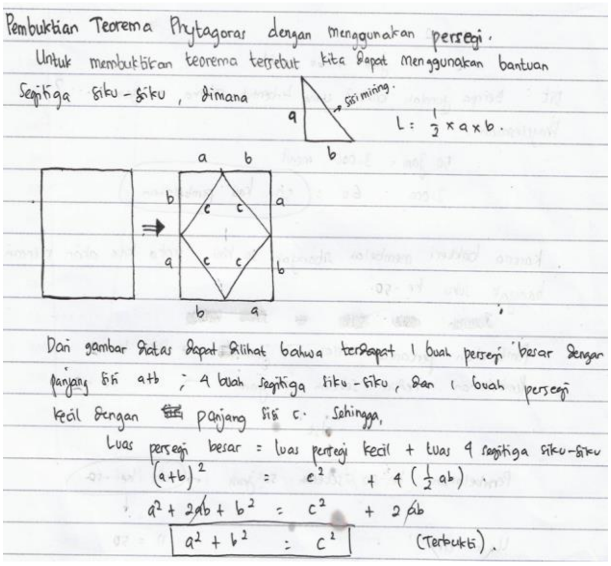
Translation:
The area of square ABPQ= the area of square NBPM+ the area of square ANMQ. c^2 = the area of square NBPM+ the area of square. It can be proven as follows. By using the concept of similarity triangle, we get $\frac{x}{b} = \frac{b}{c} \leftrightarrow x = \frac{b^2}{c}$. The area of square NBPM is equivalent to $xc = \frac{b^2}{c} .c = b^2$. By using the concept of similarity triangle, we get $\frac{y}{a} = \frac{a}{c} \leftrightarrow y = \frac{a^2}{c}$. The area of square ANMQ is equivalent to $yc = \frac{a^2}{c} .c = a^2$. Thus, we obtain $c^2 = a^2 + b^2$.

Figure 1. Igor's work

Based on Igor's work in Figure 1, it was demonstrated that Igor proved the Pythagorean theorem by using three squares. The square is constructed from the three sides of the triangle. He glued the three squares together so that two of their four corners coincided and formed a triangle inside. Furthermore, Igor uses the concept of a similarity triangle to show the area of a large square (ABPQ) equivalent to the area of a medium square (ADEC) plus the area of a small square (BCST). Next, Igor demonstrated that the area of the ABPQ square is equal to the area of the NBPM square added to the ANMQ area. He uses the concept of triangle similarity, and namely, triangle BNC is congruent with triangle ACB. The BN side of the BNC triangle corresponds to the BC side of the ACB triangle, while the BC side of the BNC triangle corresponds to the AB side of the ACB triangle so that $\frac{x}{b} = \frac{b}{c}$, consequently $x = \frac{b^2}{c}$. Furthermore, he managed to find the area of the square NBPM equivalent to $\underline{BN} \times \underline{BP} = x.c = \frac{b^2}{c}.c = b^2$. In the same way, he obtains the area of the square ANMQ equivalent to

$\underline{AN} \times \underline{NM} = y.c = \frac{a^2}{c}.c = a^2$. In the final stage, he succeeded in proving that the area of ABPQ = area of NBPM + area of ANMQ or $c^2 = a^2 + b^2$.

The following presents the results of Tasya's work in solving the problem of proving the Pythagorean theorem.


Pembuktian Teorema Phytagoras dengan menggunakan persegi.
Untuk membuktikan teorema tersebut kita dapat menggunakan bantuan segitiga siku-siku, dimana

Dari gambar diatas dapat dilihat bahwa terdapat 1 buah persegi besar dengan panjang sisi a+b ; 4 buah segitiga siku-siku, dan 1 buah persegi kecil dengan panjang sisi c. Sehingga,
Luas persegi besar = luas persegi kecil + luas 4 segitiga siku-siku
$(a+b)^2 = c^2 + 4(\frac{1}{2}ab)$
$a^2 + 2ab + b^2 = c^2 + 2ab$
$a^2 + b^2 = c^2$ (Terbukti).

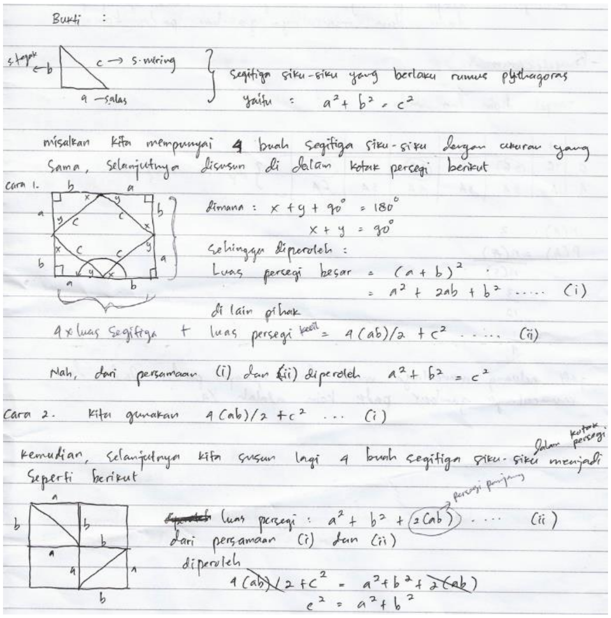
Translation:
To prove the Pythagorean theorem, we can use the help of area of a right triangle. From the picture above, there is one large square with side length a+b, four right triangles, and one small square with side length c. So, the area of the large square = the area of the small square + the area of 4 right triangles.

$$(a+b)^2 = c^2 + 4.\frac{1}{2}ab$$
$$a^2 + 2ab + b^2 = c^2 + 2ab$$
$$a^2 + b^2 = c^2 \text{ (Proved)}$$

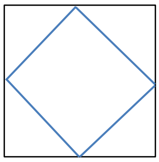
Figure 2. Tasya's work

Based on Tasya's work in Figure 2, it is evident that Tasya proves the Pythagorean theorem by arranging four congruent right triangles into a square such that the hypotenuse of the four right triangles forms a square. The perpendicular sides of the right triangle are a and b, respectively, while c is the triangle's hypotenuse. Subsequently, Tasya checked the truth of the Pythagorean Theorem by using the concept of area. She explained that the area of a large square is equivalent to the area of a small square added to the area of a small square so that we get $(a+b)^2 = c^2 + 4\,(\frac{1}{2}ab)$, $a^2 + 2ab + b^2 = c^2 + 2ab$, $a^2 + b^2 = c^2$.

The following presents Ana's work in solving the problem of proving the Pythagorean theorem. It can be proven as follows with the concept of similarity triangle. Suppose we have four right-angled triangles of the same size, then they are arranged in a square as follows so that we get. From equations 1 and 2, we get. Then, we arrange four right-angled triangles into a square as follows.



Translation:
Suppose we have four right-angled triangles of the same size, then they are arranged in a square as follows,



Thus, we get, the area of the large square is equivalent to $(a+b)^2 = a^2 + 2ab + b^2$ (i). On other hand, four multiply the area of the right triangle add the area of the small square = $4\frac{ab}{2} + c^2$ (ii). From equations (i) and (ii), we get $a^2 + b^2 = c^2$.

Figure 3. Ana's work

Based on Ana's work in Figure 3, it was evident that Ana proved the Pythagorean theorem by using two congruent squares and four triangles. Subsequently, Ana uses the concept of area, which is a large square area equivalent to $(a+b)^2 = a^2 + 2ab + b^2$. She wrote in detail the proof process by starting from the example of four congruent triangles. Subsequently, she drew a square containing four congruent triangles and one smaller square. She wrote down the rule for the sum of the angles in a triangle, which is 180°. However, this rule is not required for the next solution phase.

She wrote that the area of a large square is equivalent to $a^2 + 2ab + b^2$ as equation 1. Next, she wrote down the formula for the area of 4 congruent triangles and the area of a square that is $4\frac{ab}{2}$ and c^2 as the second equation, respectively. From the first equation and the second equation, she arranges equation $4.\frac{ab}{2} + c^2 = a^2 + 2ab + b^2$ $2ab + c^2 = a^2 + 2ab + b^2$ and prove that $a^2 + b^2 = c^2$.

Prospective mathematics teachers' pedagogical content knowledge

From the observation, we concluded that Ana could only teach the proof of the Pythagorean theorem using the concepts of the area of a square and the area of a triangle. She uses the lecture method by drawing a large square containing four right-angled triangles and one small square. Ana explains visually well, but the verbal explanation given is not systematic in finding the square of the hypotenuse of a right triangle equal to the sum of the squares of the right sides. Therefore, the researcher categorizes Ana's PCK as being at the beginner level. Meanwhile, Tasya teaches the proof of the Pythagorean theorem interactively using GeoGebra. She proved the Pythagorean theorem by using (1) the concepts of the area of a square and the area of a triangle and (2) the concepts of the area of a square and the similarity of a triangle. Thus, Tasya's PCK is at an intermediate level.

On the other hand, Igor teaches the proof of the Pythagorean theorem interactively using GeoGebra and concrete artifacts (origami paper). He was able to prove the Pythagorean theorem by using (1) the concept of the area of a square and the area of a triangle, (2) the concept of the area of a square and similarity of a triangle, and (3) the concept of the area of a square and the theorem of sides. Thus, PCK Igor is at an advanced level.

Discussion

In this study, prospective secondary school mathematics teachers have different beliefs about the nature of mathematics, mathematics learning, and mathematics teaching. The findings of this study demonstrate that traditional beliefs (instrumentalist and platonist views) are more dominant in prospective teachers' descriptions of the nature of mathematics. The prospective teachers with traditional beliefs seemed to have incomplete mathematical knowledge and could not associate their knowledge to develop reasoning in poof problems (Kim, 2018). These findings corroborate previous studies (Muhtarom, Juniati, et al., 2019; Safrudiannur & Rott, 2019; Siswono et al., 2017), which pointed out that most prospective mathematics teachers have traditional beliefs (instrumental and platonists) about the nature of mathematics. Muhtarom, Juniati, & Siswono (2017) pointed out that most future mathematics teachers hold platonist views due to their school mathematics experiences, and university courses have also reinforced this belief. The three prospective teachers argue that mathematics is a science that contains a series of rules and formulas to help humans.

In the dimension of *mathematics learning*, only one mathematics teacher candidate has a *problem-solving* view which learning provides broad opportunities for students to learn according to their interests (Beswick, 2012; Safrudiannur & Rott, 2020). The prospective teacher emphasized students' active learning and believed students could learn mathematics through problem-solving, sharing, and negotiating their ideas with peers. Moreover, teachers should foster students to become flexible and confident problem solvers in learning mathematics. In comparison, the other two prospective teachers have the concept that students learn mathematics by repeated practice to understand a concept associated with a platonist view. They have conceptions that students exhibit respectful behavior and master skills to learn mathematics, which indicates that they passively receive knowledge from the teacher (Ernest, 1989; Kim, 2018). Thus, the prospective teacher who believed that students learn mathematics by acquiring abilities and repeating similar procedures had superficial pedagogical knowledge. On the other hand, the prospective teacher who considers mathematics learning as an active creation of knowledge may elicit meaningful and rich tasks and employ various methods for students to enhance their comprehension of the topic.

For the dimension *of teaching mathematics*, the three prospective teachers have a conception of the teacher's role as a facilitator. These results agree well with existing studies on prospective mathematics beliefs, which pointed out that as facilitators, the teachers are confident in engaging students in problem-solving and problem-posing activities (Muhtarom, 2020; Safrudiannur et al., 2021). Furthermore, findings in this study suggest that prospective teachers' beliefs and pedagogical content knowledge did influence their teaching approaches.

This result is aligned with Zhang's findings (2022) that the teachers' mathematical beliefs and professional knowledge influenced their instructional techniques, leading to a variety of instructional approaches based on those beliefs.

The study discovered that the more advanced prospective teachers' beliefs about the nature of mathematics, mathematics learning, and mathematics teaching are, the higher one's CK and PCK levels are. These findings are consistent with Kim's (2018) and Smith et al. s' (2016) studies. Both Kim (2018) and Smith et al. (2016) scrutinized that prospective mathematics teachers with problem-solving views of mathematics learning, and teaching had more excellent CK and PCK than prospective teachers with instrumentalist and platonist views of mathematics and learning.

Conclusion

Most of the prospective teachers in this study hold traditional beliefs (instrumentalist and platonist views) based on their descriptions of the nature of mathematics. Only one mathematics teacher candidate has a problem-solving view in the dimension of mathematics learning. In dimensions of mathematics teaching, prospective teachers have a conception of the teacher's role as a facilitator. The study's findings confirm an interaction between prospective teachers' beliefs and pedagogical content knowledge. The more advanced a teacher candidate's view about the nature of mathematics, mathematics learning, and mathematics teaching is, the more advanced a person's content knowledge and pedagogical content knowledge are. This study provides insight that mathematics teacher educators should consider promoting prospective teachers in developing constructivist or problem-solving beliefs by providing learning and teaching experiences with student-centered approaches. Moreover, mathematics teacher educators should enhance the prospective teachers' professional knowledge according to the requirements of the new Indonesian curriculum. This study only involved three prospective mathematics teachers. More studies with larger samples and more different mathematical situations are essential to explore the relationship between prospective teachers' beliefs and pedagogical content knowledge.

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Conflicts of Interest

The authors declare that no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely by the authors.

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