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Active-passive dynamic consensus filters: Theory and applications

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ACTIVE-PASSIVE DYNAMIC CONSENSUS FILTERS: THEORY AND APPLICATIONS

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ABSTRACT

This dissertation presents a new method for distributively sensing dynamic environments utilizing integral action based system theoretic distributed information fusion methods. Specifically, the main contribution is a new class of dynamic consensus filters, termed active-passive dynamic consensus filters, in which agents are considered to be *active* if they are able to sense an exogenous quantity of interest and are considered to be *passive* otherwise, where the objective is to drive the states of all agents to the convex hull spanned by the exogenous inputs sensed by active agents. Additionally, we generalize these results to allow agents to locally set their *value-of-information*, characterizing an agents ability to sense a local quantity of interest, which may change with respect to time.

The presented active-passive dynamic consensus filters utilize equations of motion in order to diffuse information across the network, requiring continuous information exchange and requiring agents to exchange their measurement and integral action states. Additionally, agents are assumed to be modeled as having single integrator dynamics. Motivated from this standpoint, we utilize the ideas and results from event-triggering control theory to develop a network of agents which only share their measurement state information as required based on errors exceeding a user-defined threshold. We also develop a static output-feedback controller which drives the outputs of a network of agents with general linear time-invariant dynamics to the average of a set of applied exogenous inputs. Finally, we also present a system state emulator based adaptive controller to guarantee that agents will reach a consensus even in the presence of input disturbances.

For each proposed active-passive dynamic consensus filter, a rigorous analysis of the closed-loop system dynamics is performed to demonstrate stability. Finally, numerical examples and experimental studies are included to demonstrate the efficacy of the proposed information fusion filters.

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1. INTRODUCTION

Distributed dynamic information fusion is a task performed by a group of agents which use local peer-to-peer information exchange in order to achieve system level goals. In recent years, distributed information fusion has been identified as a major research thrust area owing to its envisioned use in a wide variety of applications in law-enforcement [1]–[3], military [4], [5], and scientific data gathering [6]–[17] to name but a few examples. At the core of distributed sensing networks is an information fusion algorithm for distributing and fusing the local information sensed by each agent.

Classical distributed sensing methods require large bandwidth, memory, and overhead for record keeping [9], [18]–[25]. One widely used classical distributed sensing method, *flooding* [9], requires all agents to store all information from all other agents in the network and forward all received information to neighbors, resulting in redundant information exchange and high memory usage. Motivated from this standpoint, this dissertation considers *system theoretic information fusion* methods where equations of motion are used to perform information fusion, requiring agents to only exchange their local information with their nearest neighbors.

1.1. SYSTEM THEORETIC INFORMATION FUSION

In contrast to classical distributed sensing methods where messages must be routed and agents must track individual messages, system theoretic information fusion utilizes equations of motion to *diffuse* information across the network. This means agents only fuse information from nearest neighbors, significantly reducing information exchange, memory, and computational costs. In addition, system theoretic methods are generally robust to noise and uncertainty and provide insight into the mechanics of the information fusion process [26], [27]. At the core of system theoretic information fusion processes is a consensus algorithm responsible for the information fusion step.

Among two important classes of consensus algorithms are *static* consensus algorithms and *dynamic* consensus algorithms. Much literature, notably [26], [28]–[31], focuses on static consensus, where agents reach an agreement on a fixed quantity of interest. However, this class of consensus algorithms is not suitable for agents sensing time-varying environments. To address the challenges of dynamic sensing several authors, most notably [6], [8], [10], [12]–[14], have studied dynamic consensus algorithms where agents reach an agreement on a *time-varying* quantity of interest. Owing to their versatility, this thesis will focus on dynamic consensus algorithms.

1.2. HETEROGENEITY IN SENSING

Existing dynamic consensus algorithms are suitable for applications where *all* agents are able to sense the same time-varying quantity of interest. From a practical standpoint, some agents may be able to sense a quantity of interest while others may not be able to sense any quantities. Furthermore, in many situations, agents may be heterogeneous with respect to which quantity of interest they are able to sense. Throughout this dissertation we will consider an agent to be *active* if it is able to sense a quantity of interest and consider an agent to be *passive* if it not able to sense any quantities of interest. To elucidate this point, consider the scenario in Figure 1.1 where a network of 7 agents are tracking 3 dynamic targets with the goal of building a time-varying environment map. Three mobile agents are active for one target each as follows: agent M1 is active for area T1, agent M2 is active for area T2, and agent M3 is active for area T3. In addition, four stationary agents are active for some targets and passive for others as follows: agent S2 is active for target T1, agent S3 is active for targets T1 and T2, agent S4 is active for targets T2 and T3, and agent S1

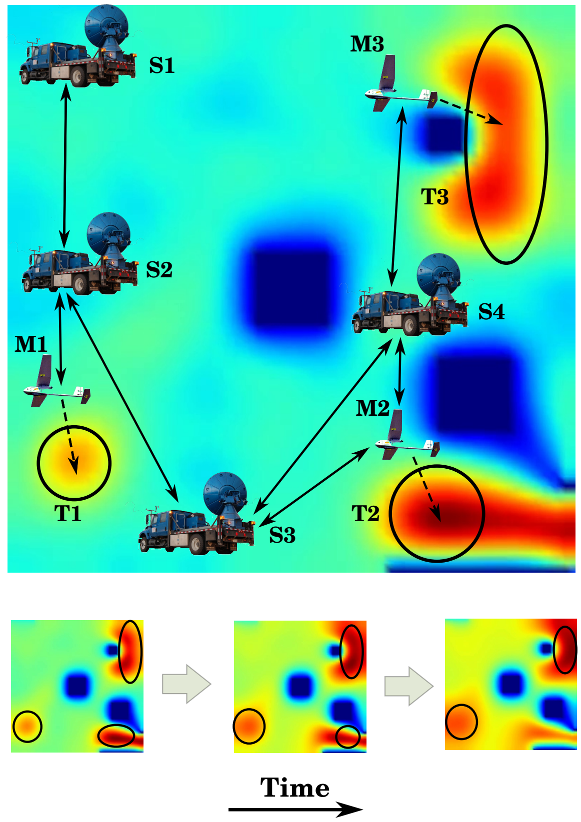

Figure 1.1. Distributed sensing network with 4 stationary ground agents (denoted by Sx, where x denotes the agent number) and 3 mobile agents (denoted by Mx) sensing a dynamic environment. Solid black lines denote communication links. Black circles (labeled Tx) denote areas of interested directly sensed by mobile agents.

is passive. Note that an agent is referred to as passive for targets it is not able to sense. For example, agent M1 is passive for targets T2 and T3. In order for all agents to build and maintain an environment map, agents must use information received from neighbors to learn about areas which they are not able to directly access. As an example, agents S1 and S2 must be able to learn about target T3 even though they are not able to directly access the state of this target.

The first contribution of this dissertation, provided in Paper I, is to introduce a new class of integral-action based dynamic consensus filters, *active-passive dynamic consensus filters*, in which the state of all agents is driven to a *user-adjustable* neighborhood of the average of the set of applied exogenous inputs sensed by active agents. Here, we would like to note that that some authors refer to integral-action consensus algorithms as PI consensus algorithms due to similarities in structure with standard PI controllers. Several other studies exist on integral-action based distributed control algorithms, notably [32]–[36]. However, the authors of [32], [33] only consider specific situations where all agents are active with respect to a set of applied exogenous inputs. The authors of [34] overview several dynamic consensus algorithms, but are commonly interested in optimizing the network feedback gains and do not consider exogenous inputs. In [35], the authors consider an integral-action based distributed control algorithm but only consider agents having no observations. The results of [36] present a distributed proportional-integral-derivative control algorithm, where agents track locally generated reference velocity and acceleration signals. Yet, all agents are only able to sense one corresponding velocity and acceleration pair, which may not be suitable for applications where agents are required to change their active and passive roles.

It is important to note that agents may also be heterogeneous with respect to their sensing power, measured by their *value of information*, due to variations in sensor capabilities, the quality of sensors, and/or simply distance to the target. To elucidate this point, consider that even though agents M3 and S4 are both active for target T3, agent M3 is clearly able to more accurately sense the target since it is closer to and able to move with the target. In addition, the sensing power of agents may vary with time. Motivated from this standpoint, the second contribution of this dissertation, presented in Paper IV, exploits heterogeneity in individual agents' value-of-information to reduce the sensing error owing to lower valued agents. Even though a few works exist on this subject, notably [15], [37]–[41], they are presented in the context of static consensus algorithms and/or only consider mission planning (i.e. sensor placement) scenarios. Here, we extend the proposed active-passive dynamic consensus filters algorithm to account for heterogeneity in the sensing capabilities of each agent. To demonstrate the efficacy of these results, Paper II and Paper III present two practical examples in which several ground sensors work cooperatively to reconstruct an environment even though each agent is passive for some parts of the environment. In addition, we draw parallels to existing classes of leaderless and leader-follower networks, and demonstrate that for static exogenous inputs, all agents asymptotically converge to the weighted average of the set of applied exogenous inputs.

1.3. REDUCING INTER-AGENT INFORMATION EXCHANGE

The results presented in Paper I and Paper IV utilize an integral-action based dynamic consensus filter as do the works presented in [32]–[36]. All these works, with the notable exception of [35], require that agents exchange their integral-action states with their neighbors in addition to their measurement state, incurring a high cost of inter-agent information exchange. In addition, due to the nature of system-theoretic dynamic consensus filters, agents must continuously exchange information in order to reach and maintain a consensus. While [35] takes steps to reduce inter-agent information exchange by considering agents which can estimate neighboring agents integral-action states, they only consider agents that have no observations, which may not be suitable for all situations. To address these challenges, the next contribution of this dissertation introduced in Paper V extends the proposed active-passive dynamic consensus filters approach to allow agents to exchange *only their measurement states* in a simple, isotropic manor, reducing the cost of inter-agent information exchange. Paper V also utilizes the ideas and results from event-triggering control theory, notably [42]–[46], to further reduce the cost of inter-agent information exchange. This generalization additionally allows agents to schedule information exchange based on errors exceeding a user-defined threshold. This eliminates the need for agents to synchronize their update intervals, which is important for practical implementations. Finally, we demonstrate that no Zeno behavior can occur as a result of our using an event-triggered controller.

1.4. EXTENSIONS TO LINEAR TIME-INVARIANT AGENTS

The active-passive dynamic consensus filters presented in Paper I-Paper V assume that all agents have, or can be made to act as having single integrator dynamics, which may not be practical in all situations. To elucidate this point, we again consider the target tracking scenario in Figure 1.1, specifically agents M3 ad S4. Since agent S4 is stationary, we may be able to decouple the sensed information from the agent dynamics. In contrast, agent M3 is mobile and moving with the target. Consider that agent M3 is a fixed-wing aircraft circling the target. In addition, as the target moves in space, the aircraft must adjust its course to continue tracking. Owing to the motion of the aircraft, the dynamics will inherently appear in the agent's measurement state and must be accounted for when fusing information with other agents. To this end, Paper VII contributes a static output feedback active-passive dynamic consensus filter for agents with homogeneous dynamics. While a few works exist for dynamic consensus filters with general linear time-invariant agents, notably [33], [47]–[53], the authors of [47]–[51] only consider cases where agents have no observations. The authors of [33], [52], [53] do consider agents which are subjected to exogenous inputs, however, only [53] considers agents which are active and passive. Yet, the authors of [53] only considers that all agents are sensing a target with known (or estimated) linear time-invariant dynamics where the sensing agents are following the target in a leader-follower paradigm.

To address this challenge, Paper VII utilizes the tools and ideas presented in Paper VI to extend the active-passive dynamic consensus filters approach to agents having dynamics of the form (A, B, C). In particular, we develop a static output feedback controller for a network of linear time-invariant agents which drives the output of each agent to a neighborhood of the weighted average of a set of applied exogenous inputs sensed by the active agents.

1.5. CONSENSUS IN THE PRESENCE OF DISTURBANCES

The active-passive dynamic consensus filters mentioned thus far have focused on fixed-gain distributed controllers which are unable to recover the desired performance in the presence of unknown exogenous disturbances as outlined in [54] and [55]. Specifically, these systems do not have a centralized mechanism to monitor for agent failures, malicious attacks, network link failures, and other disturbances, which can lead to system instability and failure to achieve the system-level goals as described in [54] and [56].

While several approaches have been presented in order to mitigate the effects of exogenous disturbances in distributed sensing, notably [57]–[60], they make the assumption that an agent's information is no longer usable and all information from the agent is ignored, which may not be appropriate in scenarios where the effect of the disturbance can be suppressed. The authors of [59] and [61] assume that the structure of the underlying communication network is known in order to mitigate disturbance effects, which is not practical for many situations. In addition, [58], [61], and [62] assume that a maximum number of agents are disturbed, which can be a strict assumption in hostile environments. Computationally expensive observer techniques are considered in [59] and [60]. In [63], the authors focus on discovering subsets of disturbed agents and require neighboring agents to mitigate the disturbance effects.

To address these challenges, the final contribution of this dissertation, given in Paper VI, considers a state emulator based adaptive control approach to mitigate the effects of an exogenous disturbance on a network of general linear time-invariant agents. Specifically, we utilize an undisturbed system state emulator as a reference trajectory to drive the states of all agents to a close neighborhood of the average of a quantity of interest. In contrast to the rest of this dissertation, the results of Paper VI only consider static consensus in order to more clearly demonstrate the outlined adaptive control approach.

1.6. ORGANIZATION

The organization of this dissertation is as follows. Paper I presents the proposed active–passive dynamic consensus filters approach to dynamic information fusion. In Paper II and Paper III, we demonstrate the efficacy of the presented active-passive dynamic consensus filter approach to distributed sensing with several experimental studies. Paper IV presents extensions to the active-passive dynamic consensus filters approach to dynamic information fusion to account for heterogeneity in the ability of agents to sense a time-varying quantity of interest. Next, Paper V draws on the ideas and tools of event-triggered control theory in order to reduce the cost of inter-agent information exchange and relax the requirement that agents synchronize their information update intervals. In Paper VI, we utilize a state emulator based adaptive information filter to allow agents to reach a consensus in the presence of exogenous disturbances, even if all agents are subjected to disturbances. Finally, Paper VII extends the presented active-passive dynamic consensus filters using the tools and ideas presented in Paper VI in order to account for agents with general linear time-invariant dynamics. Our conclusions and future research prospects are summarized in Section 2.

1.7. NOTATION

The necessary notations are introduced in the individual papers.

PAPER

I. DISTRIBUTED CONTROL OF ACTIVE–PASSIVE NETWORKED MULTIAGENT SYSTEMS

Tansel Yucelen and John Daniel Peterson

ABSTRACT

An active–passive networked multiagent system framework is introduced and analyzed, which consists of agents subject to exogenous inputs (active agents) and agents without any inputs (passive agents). Specifically, we propose an integral action-based distributed control approach and establish its transient time and steady state performance characteristics using results from graph theory, matrix mathematics, Lyapunov stability, and $\mathcal{L}$ stability. Apart from the existing relevant literature, where either none of the agents are subject to exogenous inputs (i.e., average consensus problem) or all agents are subject to these inputs (i.e., dynamic average consensus problem), the key feature of our approach is that the states of all agents converge to the average of the exogenous inputs applied only to the active agents. We further discuss the conditions when the performance of the proposed distributed controller specializes to the performance of standard distributed controllers used for average consensus and dynamic average consensus of leaderless networks, and also draw connections between pinning control and containment control of leader–follower networks. Several illustrative numerical examples are provided to demonstrate the efficacy of the proposed distributed control approach.

1. INTRODUCTION

The agreement of networked agents upon certain initial quantities of interest is called average consensus, which is a well studied class of leaderless networks (see, for example, [1]–[5] and references therein). Since this class of leaderless networks is insufficient for applications in dynamic environments, such as average consensus of distance measurements between static sensors and a moving target, [6]–[11] consider the dynamic average consensus problem – a problem that deals with the agreement of networked agents, where each agent is subject to an exogenous input. Throughout this paper, we say that an agent is active when it is subject to an exogenous input, otherwise we say that an agent is passive. From this point of view, the agents considered for the average consensus problem are all passive, whereas the agents considered for the dynamic average consensus problem are all active. However, it should be noted that it can be of practical importance to reach the average of the exogenous inputs only applied to a specific set of agents in the network. For example, a motivating scenario for this case include a distributed sensor network, where only a set of agents that are close to a target of interest can sense this target, and hence, they are active, whereas the rest are passive since they cannot sense this target and collect information. For such cases the networked multiagent system is in fact heterogenous and consists of both active and passive agents.

The contribution of this paper is to introduce and analyze an active–passive networked multiagent system framework. Specifically, we propose an integral action-based distributed control approach and establish its transient time and steady state performance characteristics using results from graph theory, matrix mathematics, Lyapunov stability, and $\mathcal{L}$ stability. Apart from the existing relevant literature on leaderless networks, where either none of the agents are subject to exogenous inputs or all agents are subject to these inputs, the key feature of our approach is that the states of all agents converge to the average of the exogenous inputs applied only to the active agents, where these inputs may or may not overlap within the active agents. We further show that the proposed algorithm acts as an average consensus algorithm only when there are no active agents, and as a dynamic average consensus algorithm only when all agents are active. In addition, we draw connections between leader–follower networks; for example, the proposed algorithm acts as a pinning control algorithm [12]–[17] only when there is one active agent or multiple active agents subject to same inputs, and as a containment control algorithm [18]–[23] only when the effect of the proposed distributed control approach's integral action is nullified – since in this case the states of passive agents converge to the convex hull spanned by those of the active agents. Several illustrative numerical examples are provided to demonstrate the efficacy of the proposed distributed control approach.

The organization of the paper is as follows. Section II introduces the notation used throughout the paper and recalls some of the basic notions from graph theory. Section III presents the problem formulation for active–passive networked multiagent systems along with necessary definitions and illustrations. Section IV and V analyze the proposed distributed control approach in the presence of constant and time-varying exogenous inputs, respectively. We draw connections between the proposed distributed control approach and others in Section VI and summarize conclusions in Section VII. Finally, it should be noted that an earlier version of this paper is appeared in [24]. This paper significantly goes beyond the results documented in [24] in that *a)* we propose and analyze a generalized integral action-based distributed control approach with added key design parameters to control the bandwidth of the overall network independently from the connectivity of a given graph topology (see Theorem 1), *b)* we analytically establish transient time performance characteristics of the proposed methodology and theoretically reveal the effect of design parameters on the overall network performance (see Corollary 1), *c)* we consider not only constant exogenous inputs case but also time-varying exogenous inputs case that is of practical importance for applications in dynamic environments (see Theorem 2 and Corollary 2), *d)* we comprehensively compare the proposed approach with the existing notable results in the related literature (see the detailed discussions in Section VI and Theorem 3), and *e)* we include several important details and key examples to further elucidate the results of our paper (see Remarks 1 and 2 and Examples 3, 4, and 5).

2. PRELIMINARIES

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbb{R}_+$ denotes the set of positive real numbers, $\mathbb{R}_+^{n\times n}$ (resp., $\overline{\mathbb{R}}_+^{n\times n}$) denotes the set of $n \times n$ positive-definite (resp., nonnegative-definite) real matrices, $\mathbb{S}_+^{n\times n}$ (resp., $\overline{\mathbb{S}}_+^{n\times n}$) denotes the set of $n \times n$ symmetric positive-definite (resp., symmetric nonnegative-definite) real matrices, $\mathbb{Z}$ denotes the set of integers, $\mathbb{Z}_+$ (resp., $\overline{\mathbb{Z}}_+$) denotes the set of positive (resp., nonnegative) integers, $\mathbf{0}_n$ denotes the $n \times 1$ vector of all zeros, $\mathbf{1}_n$ denotes the $n \times 1$ vector of all ones, $\mathbf{0}_{n\times n}$ denotes the $n \times n$ zero matrix, and I_n denotes the $n \times n$ identity matrix. In addition, we write $(\cdot)^{\mathrm{T}}$ for transpose, $(\cdot)^{-1}$ for inverse, $(\cdot)^{\dagger}$ for Moore-Penrose generalized inverse, $\|\cdot\|_2$ for the Euclidian norm, $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix A, $\lambda_i(A)$ for the i-th eigenvalue of A (A is symmetric and the eigenvalues are ordered from least to greatest value), $\mathrm{diag}(a)$ for the diagonal matrix with the vector a on its diagonal, and $[A]_{ij}$ for the entry of the matrix A on the i-th row and j-th column. Furthermore, for a signal $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$, defined for all $t \geq 0$, $\mathcal{L}_2$ norm and the $\mathcal{L}_\infty$ norm [25] are defined, respectively, as

$$\|x(t)\|_{\mathcal{L}_2} \triangleq \sqrt{\int_0^\infty \sum_{i=1}^n x_i^2(t)\mathrm{d}t}, \tag{1}$$

$$\|x(t)\|_{\mathcal{L}_\infty} \triangleq \max_{1\leq i\leq n}(\sup_{t\geq 0}|x_i(t)|). \tag{2}$$

Next, we recall some of the basic notions from graph theory, where we refer to [1], [26] for further details. In the multiagent literature, graphs are broadly adopted to encode interactions in networked systems. An *undirected* graph $\mathcal{G}$ is defined by a set $\mathcal{V}_{\mathcal{G}} = \{1, \ldots, n\}$ of *nodes* and a set $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$ of *edges*. If $(i, j) \in \mathcal{E}_{\mathcal{G}}$, then the nodes

i and j are *neighbors* and the neighboring relation is indicated with $i \sim j$. The *degree* of a node is given by the number of its neighbors. Letting d_i be the degree of node i, then the *degree* matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{n \times n}$, is given by

$$\mathcal{D}(\mathcal{G}) \triangleq \operatorname{diag}(d), \quad d = [d_1, \ldots, d_n]^{\mathrm{T}}. \tag{3}$$

A *path* $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1, \ldots, L$, and a graph $\mathcal{G}$ is *connected* if there is a path between any pair of distinct nodes. The *adjacency* matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{n \times n}$, is given by

$$[\mathcal{A}(\mathcal{G})]_{ij} \triangleq \begin{cases} 1, & \text{if } (i,j) \in \mathcal{E}_{\mathcal{G}}, \\ 0, & \text{otherwise}. \end{cases} \tag{4}$$

The *Laplacian* matrix of a graph, $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{n \times n}$, playing a central role in many graph theoretic treatments of multiagent systems, is given by

$$\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G}). \tag{5}$$

Throughout this paper, we model a given multiagent system by a connected, undirected graph $\mathcal{G}$, where nodes and edges represent agents and inter-agent communication links, respectively, and use the following lemmas.

Lemma 1 [1]**.** The spectrum of the Laplacian of a connected, undirected graph can be ordered as $0 = \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \leq \cdots \leq \lambda_n(\mathcal{L}(\mathcal{G}))$ with $\mathbf{1}_n$ as the eigenvector corresponding to the zero eigenvalue $\lambda_1(\mathcal{L}(\mathcal{G}))$ and $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$ and $\mathrm{e}^{\mathcal{L}(\mathcal{G})}\mathbf{1}_n = \mathbf{1}_n$.

Lemma 2 [1]**.** Let $K = \operatorname{diag}(k)$, $k = [k_1, k_2, \ldots, k_n]^{\mathrm{T}}$, $k_i \in \overline{\mathbb{Z}}_+$, $i = 1, \ldots, n$, and assume that at least one element of k is nonzero. Then, for the Laplacian of a connected, undirected graph, $\mathcal{F}(\mathcal{G}) \triangleq \mathcal{L}(\mathcal{G}) + K \in \mathbb{S}_+^{n \times n}$ and $\det(\mathcal{F}(\mathcal{G})) \neq 0$.

Lemma 3 [27]**.** The Laplacian of a connected, undirected graph satisfies $\mathcal{L}(\mathcal{G})\mathcal{L}^{\dagger}(\mathcal{G}) = \mathrm{I}_n - \frac{1}{n}\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}$.

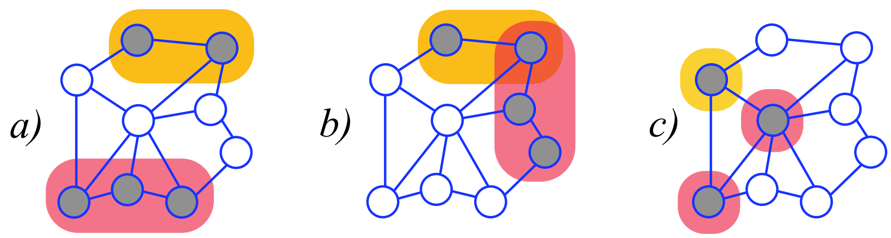

Figure 1. An active-passive networked multiagent system with *a)* two non-overlapping non-isolated inputs, *b)* two overlapping non-isolated inputs, and *c)* two non-overlapping inputs, where one of them is isolated and the other one is non-isolated (lines denote communication links, gray circles denote active agents, white circles denote passive agents, and shaded areas denote the exogenous inputs interacting with this system).

3. PROBLEM FORMULATION FOR ACTIVE–PASSIVE NETWORKED MULTIAGENT SYSTEMS

Consider a system of n agents exchanging information among each other using their local measurements according to a connected, undirected graph $\mathcal{G}$. In addition, consider that there exists $m \geq 1$ exogenous inputs that interact with this system.

Definition 1. If agent i, $i = 1, \ldots, n$, is subject to one or more exogenous inputs (resp., no exogenous inputs), then it is an active agent (resp., passive agent).

Definition 2. If an exogenous input interacts with only one agent (resp., multiple agents), then it is an isolated input (resp., non-isolated input).

An illustration of Definition 2 is given in Figure 1, where it also highlights that the exogenous inputs may or may not overlap within the active agents.

In this paper, we are interested in the problem of driving the states of all (active and passive) agents to the average of the applied exogenous inputs. Motivating from this standpoint, we propose an integral action-based distributed control approach given by

$$\dot{x}_i(t) = -\alpha \sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big) + \sum_{i\sim j}\Big(\xi_i(t) - \xi_j(t)\Big) - \alpha \sum_{i\sim h}\Big(x_i(t) - c_h(t)\Big), \quad x_i(0) = x_{i0}, \tag{6}$$

$$\dot{\xi}_i(t) = -\gamma \sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big), \quad \xi_i(0) = \xi_{i0}, \tag{7}$$

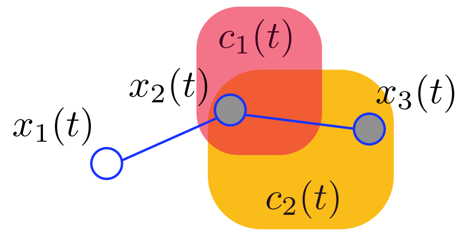

Figure 2. The active-passive networked multiagent system in Example 1 (lines denote communication links, two gray circles denote active agents $x_2(t)$ and $x_3(t)$, white circle denotes the passive agent $x_1(t)$, and two shaded areas denote applied two exogenous inputs $c_1(t)$ and $c_2(t)$).

where $x_i(t) \in \mathbb{R}$ and $\xi_i(t) \in \mathbb{R}$ denote the state and the integral action of agent i, $i = 1, \ldots, n$, respectively, $c_h(t) \in \mathbb{R}$, $h = 1, \ldots, m$, denotes an exogenous input applied to this agent, and $\alpha \in \mathbb{R}_+$ and $\gamma \in \mathbb{R}_+$. Similar to the $i \sim j$ notation indicating the neighboring relation between agents, we use $i \sim h$ to indicate the exogenous inputs that an agent is subject to. To elucidate this point, we give the following example.

Example 1. Consider the active-passive networked multiagent system given in Figure 2. It follows from (6) and (7) with $\alpha = \gamma = 1$ that three agents implement the following distributed control approaches locally

$$\dot{x}_1(t) = -\big(x_1(t) - x_2(t)\big) + \big(\xi_1(t) - \xi_2(t)\big), \tag{8}$$

$$\dot{\xi}_1(t) = -\big(x_1(t) - x_2(t)\big), \tag{9}$$

$$\begin{aligned} \dot{x}_2(t) = & -\big(x_2(t) - x_1(t)\big) - \big(x_2(t) - x_3(t)\big) + \big(\xi_2(t) - \xi_1(t)\big) + \big(\xi_2(t) - \xi_3(t)\big) \\ & -\big(x_2(t) - c_1(t)\big) - \big(x_2(t) - c_2(t)\big), \end{aligned} \tag{10}$$

$$\dot{\xi}_2(t) = -\big(x_2(t) - x_1(t)\big) - \big(x_2(t) - x_3(t)\big), \tag{11}$$

$$\dot{x}_3(t) = -\big(x_3(t) - x_2(t)\big) + \big(\xi_3(t) - \xi_2(t)\big) - \big(x_3(t) - c_2(t)\big), \tag{12}$$

$$\dot{\xi}_3(t) = -\big(x_3(t) - x_2(t)\big), \tag{13}$$

since agent 1 is passive, agent 2 is active and subject to $c_1(t)$ and $c_2(t)$, and agent 3 is active and subject to $c_2(t)$.

Next, let

$$x(t) = \left[x_1(t), x_2(t), \ldots, x_n(t)\right]^{\mathrm{T}} \in \mathbb{R}^n, \tag{14}$$

$$\xi(t) = \left[\xi_1(t), \xi_2(t), \ldots, \xi_n(t)\right]^{\mathrm{T}} \in \mathbb{R}^n, \tag{15}$$

$$c(t) = \left[c_1(t), c_2(t), \ldots, c_m(t), 0, \ldots, 0\right] \in \mathbb{R}^n, \tag{16}$$

where we assume $m \leq n$ for the ease of the following notation and without loss of generality. We can now write (6) and (7) in a compact form as

$$\dot{x}(t) = -\alpha \mathcal{L}(\mathcal{G})x(t) + \mathcal{L}(\mathcal{G})\xi(t) - \alpha K_1 x(t) + \alpha K_2 c(t), \; x(0) = x_0, \tag{17}$$

$$\dot{\xi}(t) = -\gamma \mathcal{L}(\mathcal{G})x(t), \quad \xi(0) = \xi_0, \tag{18}$$

where $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{n\times n}$ satisfies Lemma 1,

$$K_1 \triangleq \mathrm{diag}([k_{1,1}, k_{1,2}, \ldots, k_{1,n}]^{\mathrm{T}}) \in \overline{\mathbb{S}}_+^{n\times n}, \tag{19}$$

with $k_{1,i} \in \overline{\mathbb{Z}}_+$ denoting the number of the exogenous inputs applied to agent i, $i = 1, \ldots, n$, and

$$K_2 \triangleq \begin{bmatrix} k_{2,11} & k_{2,12} & \cdots & k_{2,1n} \\ k_{2,21} & k_{2,22} & \cdots & k_{2,2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{2,n1} & k_{2,n2} & \cdots & k_{2,nn} \end{bmatrix} \in \mathbb{R}^{n\times n}, \tag{20}$$

with $k_{2,ih} = 1$ if the exogenous input $c_h(t)$, $h = 1, \ldots, m$, is applied to agent i, $i = 1, \ldots, n$, and $k_{2,ih} = 0$ otherwise. Note that

$$k_{1,i} = \sum_{j=1}^{n} k_{2,ij}. \tag{21}$$

To visualize this representation, we now give an example.

Example 2. For the active-passive networked multiagent system given Figure 2, one can write

$$K_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \tag{22}$$

since $n = 3$, $m = 2$, and $c(t) = [c_1(t), c_2(t), 0]^{\mathrm{T}}$.

Since we are interested in driving the states of all (active and passive) agents to the average of the applied exogenous inputs, let

$$\delta(t) \triangleq x(t) - \epsilon(t)\mathbf{1}_n \in \mathbb{R}^n, \tag{23}$$

$$\epsilon(t) \triangleq \frac{\mathbf{1}_n^{\mathrm{T}} K_2 c(t)}{\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n} \in \mathbb{R}, \tag{24}$$

be the error between $x_i(t)$, $i = 1, \ldots, n$, and the average of the applied exogenous inputs $\epsilon(t)$. Based on (24), $\epsilon(t)$ can be equivalently written as

$$\epsilon(t) = \frac{k_{2,11}c_1(t) + k_{2,12}c_2(t) + \cdots + k_{2,21}c_1(t) + k_{2,22}c_2(t) + \cdots}{k_{2,11} + k_{2,12} + \cdots + k_{2,21} + k_{2,21} + \cdots}, \tag{25}$$

which clearly shows the average of the applied exogenous inputs. Furthermore, note for the special case of isolated exogenous inputs that

$$\epsilon(t) = \Big(c_1(t) + c_2(t) + \cdots + c_m(t))\Big)/m. \tag{26}$$

For specific applications with non-isolated exogenous inputs, if it is of interest to drive the states of all agents to (26) instead of (25), then agents subject to non-isolated exogenous inputs can communicate to achieve this objective by avoiding non-isolated exogenous inputs under the assumption that these agents are neighbors on a given graph topology as illustrated in Figure 1. Based on the problem formulation introduced in this section and the new proposed distributed control architecture, the next section and Section V analyze the stability properties of the error given by (23) in the presence of constant inputs and time-varying inputs, respectively.

4. CONSENSUS IN THE PRESENCE OF ACTIVE AGENTS WITH CONSTANT INPUTS

Let $c_h(t) = c_h$, $h = 1, \ldots, m$, be a constant exogenous input. Since $c(t) = c$, and hence, $\epsilon(t) = \epsilon$ from (24), it follows from (23) and $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$ of Lemma 1 that

$$\dot{\delta}(t) = -\alpha\mathcal{L}(\mathcal{G})\big[\delta(t) + \epsilon\mathbf{1}_n\big] + \mathcal{L}(\mathcal{G})\xi(t) - \alpha K_1\big[\delta(t) + \epsilon\mathbf{1}_n\big] + \alpha K_2 c(t)$$

$$= -\alpha\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})\xi(t) - \alpha\left[K_1\mathbf{1}_n\epsilon - K_2c\right]$$
$$= -\alpha\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})\xi(t) - \alpha\left[\frac{K_1\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}K_2c}{\mathbf{1}_n^{\mathrm{T}}K_2\mathbf{1}_n} - K_2c\right]$$
$$= -\alpha\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})\xi(t) - \alpha L_cK_2c, \tag{27}$$

where $\mathcal{F}(\mathcal{G}) \triangleq \mathcal{L}(\mathcal{G}) + K_1$ and

$$L_c \triangleq \frac{K_1\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}}{\mathbf{1}_n^{\mathrm{T}}K_2\mathbf{1}_n} - \mathrm{I}_n. \tag{28}$$

Note that $\mathcal{F}(\mathcal{G}) \in \mathbb{S}_+^{n\times n}$ from Lemma 2 and

$$\mathbf{1}_n^{\mathrm{T}}L_c = \mathbf{1}_n^{\mathrm{T}}\left[\frac{K_1\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}}{\mathbf{1}_n^{\mathrm{T}}K_2\mathbf{1}_n} - \mathrm{I}_n\right] = \frac{\mathbf{1}_n^{\mathrm{T}}K_1\mathbf{1}_n}{\mathbf{1}_n^{\mathrm{T}}K_2\mathbf{1}_n}\mathbf{1}_n^{\mathrm{T}} - \mathbf{1}_n^{\mathrm{T}} = 0, \tag{29}$$

since $\frac{\mathbf{1}_n^{\mathrm{T}}K_1\mathbf{1}_n}{\mathbf{1}_n^{\mathrm{T}}K_2\mathbf{1}_n} = 1$ from (21).

Next, letting

$$e(t) \triangleq \xi(t) - \alpha\mathcal{L}^{\dagger}(\mathcal{G})L_cK_2c, \tag{30}$$

and using (30) in (27) along with Lemma 3 yields

$$\dot{\delta}(t) = -\alpha\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})\left[e(t) + \alpha\mathcal{L}^{\dagger}(\mathcal{G})L_cK_2c\right] - \alpha L_cK_2c$$
$$= -\alpha\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})e(t) + \alpha\left[\mathrm{I}_n - \frac{1}{n}\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}\right]L_cK_2c - \alpha L_cK_2c$$
$$= -\alpha\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})e(t), \tag{31}$$

since $\frac{1}{n}\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}L_cK_2c = 0$, as a direct consequence of (29). In addition, differentiating (30) with respect to time yields

$$\dot{e}(t) = -\gamma\mathcal{L}(\mathcal{G})\left[\delta(t) + \epsilon\mathbf{1}_n\right] = -\gamma\mathcal{L}(\mathcal{G})\delta(t), \tag{32}$$

where $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$ of Lemma 1 is used. The next theorem, which can be viewed as a generalized version of Theorem 1 in [24], shows that the state of all agents $x_i(t)$, $i = 1, \ldots, n$ asymptotically converge to ϵ.

Theorem 4.1. *Consider the networked multiagent system given by (6) and (7), where agents exchange information using local measurements and with $\mathcal{G}$ defining a connected, undirected graph topology. Then, the closed-loop error dynamics defined by (31) and (32) are Lyapunov stable for all initial conditions and $\delta(t)$ asymptotically vanishes.*

Proof. Consider the Lyapunov function candidate given by

$$V(\delta, e) \;=\; \frac{1}{2\alpha}\delta^{\mathrm{T}}\delta + \frac{1}{2\alpha\gamma}e^{\mathrm{T}}e, \tag{33}$$

and note that $V(0,0) = 0$ and $V(\delta, e) > 0$ for all $(\delta, e) \neq (0,0)$. Differentiating (33) along the trajectories of (31) and (32) yields

$$\dot{V}(\delta(t), e(t)) \;=\; -\delta^{\mathrm{T}}(t)\mathcal{F}(\mathcal{G})\delta(t) \leq 0. \tag{34}$$

The rest of the proof is identical to the proof of Theorem 1 in [24], and hence, is omitted. ■

In Theorem 1, we establish the steady state performance characteristics of the proposed integral action-based distributed control approach, i.e., we show that all $x_i(t)$, $i = 1, \ldots, n$ asymptotically converge to ϵ. In the next corollary, we also establish the transient time performance characteristics of our approach using $\mathcal{L}$ stability.

Corollary 1. Consider the networked multiagent system given by (6) and (7), where agents exchange information using local measurements and with $\mathcal{G}$ defining a connected, undirected graph topology. Then, the $\mathcal{L}_2$ and $\mathcal{L}_\infty$ bounds of $\delta(t)$ in (23) are given as

$$\|\delta(t)\|_{\mathcal{L}_2}^2 \leq \frac{1}{2\alpha\lambda_{\min}(\mathcal{F}(\mathcal{G}))}\|\delta(t)\|_{\mathcal{L}_\infty}^2, \tag{35}$$

$$\|\delta(t)\|_{\mathcal{L}_\infty}^2 \leq \|x(0) - \epsilon\mathbf{1}_n\|_2^2 + \gamma^{-1}\|\xi(0) - \alpha\mathcal{L}^{\dagger}(\mathcal{G})L_cK_2c\|_2^2. \tag{36}$$

Proof. Consider the Lyapunov function candidate given by (33) and $\dot{V}(\delta(t), e(t)) \leq 0$ as a direct consequence of Theorem 1, where we can equivalently write

$$\begin{aligned} V(\delta(t), e(t)) &\leq V(\delta(0), e(0)) \\ &= \frac{1}{2\alpha}\|x(0) - \epsilon\mathbf{1}_n\|_2^2 + \frac{1}{2\alpha\gamma}\|\xi(0) - \alpha\mathcal{L}^{\dagger}(\mathcal{G})L_cK_2c\|_2^2. \end{aligned} \tag{37}$$

Noting

$$\frac{1}{2\alpha}\delta^{\mathrm{T}}(t)\delta(t) \leq V(\delta(t), e(t)), \tag{38}$$

it follows from (37) and (38) that (36) holds due to the fact that $\|\cdot\|_\infty \leq \|\cdot\|_2$ holds uniformly. To show (35), we first write $-\int_0^t \dot{V}(\delta(\tau), e(\tau))\mathrm{d}\tau = V(\delta(0), e(0)) - V(\delta(t), e(t)) \leq V(\delta(0), e(0))$ that yields

$$-\int_0^t \dot{V}(\delta(\tau), e(\tau))\mathrm{d}\tau = \frac{1}{2\alpha}\|x(0) - \epsilon\mathbf{1}_n\|_2^2 + \frac{1}{2\alpha\gamma}\|\xi(0) - \alpha\mathcal{L}^{\dagger}(\mathcal{G})L_c K_2 c\|_2^2. \tag{39}$$

Now, using $\dot{V}(\delta(t), e(t)) = -\delta^{\mathrm{T}}(t)\mathcal{F}(\mathcal{G})\delta(t)$ in the left hand side of (39) with $\lambda_{\min}(\mathcal{F}(\mathcal{G}))\delta^{\mathrm{T}}(t)\delta(t) \leq \delta^{\mathrm{T}}(t)\mathcal{F}(\mathcal{G})\delta(t)$ and then taking the limit as $t \to \infty$, (35) follows. ■

Remark 1. Letting $\xi(0) = 0$ for the ease of the exposition (this is doable since the initial condition of the integrators can be viewed as design parameters), it follows from (35) and (36) that

$$\|\delta(t)\|_{\mathcal{L}_2}^2 \leq \frac{1}{2\lambda_{\min}(\mathcal{F}(\mathcal{G}))}\left[\alpha^{-1}\|x(0) - \epsilon\mathbf{1}_n\|_2^2 + \gamma^{-1}\alpha\|\mathcal{L}^{\dagger}(\mathcal{G})L_c K_2 c\|_2^2\right], \tag{40}$$

$$\|\delta(t)\|_{\mathcal{L}_\infty}^2 \leq \|x(0) - \epsilon\mathbf{1}_n\|_2^2 + \gamma^{-1}\alpha^2\|\mathcal{L}^{\dagger}(\mathcal{G})L_c K_2 c\|_2^2. \tag{41}$$

If we choose α and γ such that both α^2/γ and $1/\alpha$ are small, then both $\mathcal{L}$ norms in (40) and (41) are minimized. This implies that agents can converge faster to a sufficiently close neighborhood of $\delta(t) = 0$ provided that α^2/γ and $1/\alpha$ are small (this is illustrated in the next example). Finally, it should be also mentioned that one can rewrite the closed-loop error dynamics given by (31) and (32) in a compact form as

$$\begin{bmatrix} \dot{\delta}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} -\alpha\mathcal{F}(\mathcal{G}) & \mathcal{L}(\mathcal{G}) \\ -\gamma\mathcal{L}(\mathcal{G}) & 0 \end{bmatrix} \begin{bmatrix} \delta(t) \\ e(t) \end{bmatrix}, \tag{42}$$

and relate the convergence rate of the closed-loop error dynamics to the eigenvalues of the matrix in (42).

Example 3. Consider a networked multiagent system represented by the graph shown in Figure 3 with 5 active agents and 20 passive agents. Let the active agents be subject to random and isolated constant inputs and let all agents have arbitrary initial conditions and

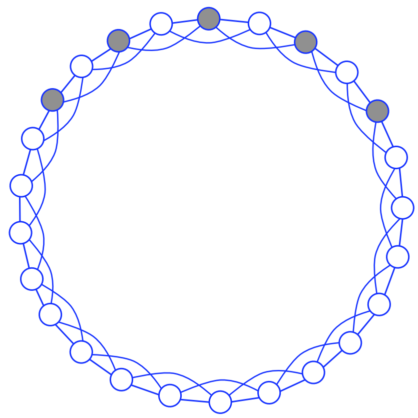

Figure 3. Graph used in Example 3 (lines denote communication links, gray circles denote active agents, and white circles denote passive agents).

$\xi_i(0) = 0$, $i = 1, \ldots, n$. Figure 4 shows the response with the proposed distributed controller given by (6) and (7) with $\alpha = 1$ and $\gamma = 1$, where all agents converge to the average of the applied exogenous inputs as expected. In this case, we have $\|\delta(t)\|_{\mathcal{L}_2} \leq 9.01$ and $\|\delta(t)\|_{\mathcal{L}_\infty} \leq 3.87$. Figure 5 shows the response with $\alpha = 5$ and $\gamma = 50$, where all agents converge to the average of the applied exogenous inputs significantly faster. Note that we choose α and γ to obtain smaller α^2/γ and $1/\alpha$ values for this case as they are compared with the ratios used to generate the results in Figure 4. In this case, we have $\|\delta(t)\|_{\mathcal{L}_2} \leq 3.74$ and $\|\delta(t)\|_{\mathcal{L}_\infty} \leq 3.59$. Even though the $\mathcal{L}_\infty$ norm is improved slightly in Figure 5 (this is expected since the selection of α and γ only affects the second term on the right hand side of (41)), the $\mathcal{L}_2$ norm is improved substantially as compared with the result in Figure 4 (this is also expected since the selection of α and γ affect both terms on the right hand side of (40)).

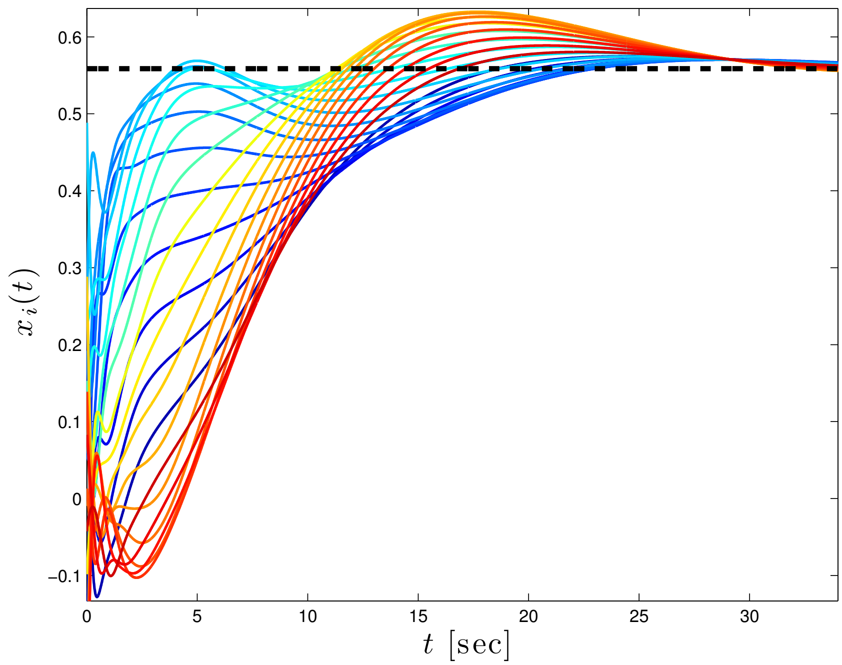

Figure 4. Response of the networked multiagent system in Example 3 with the distributed controller given by (6) and (7) with $\alpha = 1$ and $\gamma = 1$ (solid lines denote agent positions and dashed line denotes the average of applied constant inputs).

5. QUASI-CONSENSUS IN THE PRESENCE OF ACTIVE AGENTS WITH TIME-VARYING INPUTS

This section deals with the case when active agents are subject to time-varying exogenous inputs $c_h(t)$, $h = 1, \ldots, m$. We assume that both $c_h(t)$ and $\dot{c}_h(t)$ are bounded for each input h, $h = 1, \ldots, m$. In this case, we slightly modify the integral action-based distributed control approach in (6) and (7) to the following

$$\dot{x}_i(t) = -\alpha \sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big) + \sum_{i\sim j}\Big(\xi_i(t) - \xi_j(t)\Big) - \alpha \sum_{i\sim h}\Big(x_i(t) - c_h(t)\Big), \quad x_i(0) = x_{i0}, \tag{43}$$

$$\dot{\xi}_i(t) = -\gamma\Big[\sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big) + \sigma\xi_i(t)\Big], \quad \xi_i(0) = \xi_{i0}, \tag{44}$$

where $\alpha \in \mathbb{R}_+$, $\gamma \in \mathbb{R}_+$, and $\sigma \in \mathbb{R}_+$.

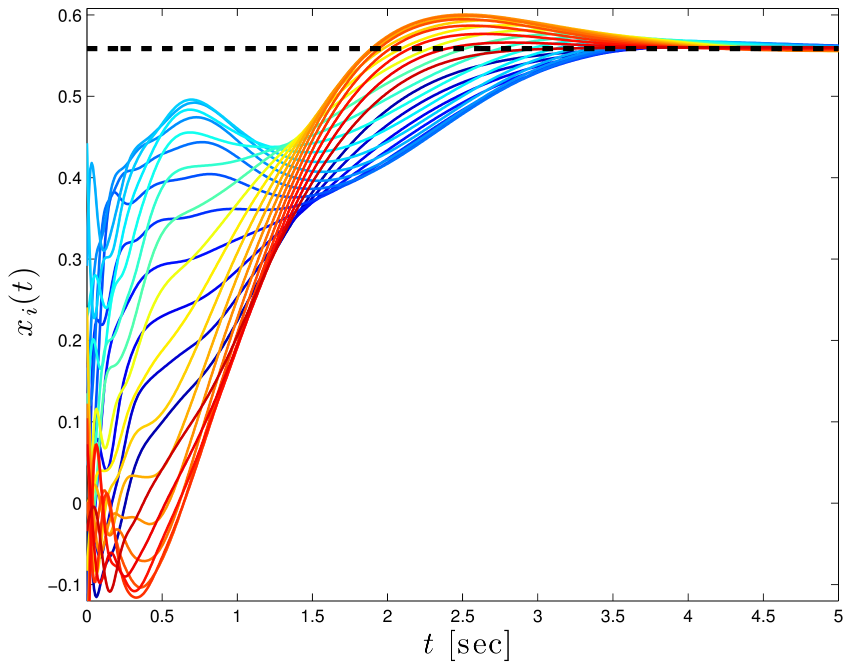

Figure 5. Response of the networked multiagent system in Example 3 with the distributed controller given by (6) and (7) with $\alpha = 5$ and $\gamma = 50$ (solid lines denote agent positions and dashed line denotes the average of applied constant inputs).

Next, using (23) and (30) and following the steps highlighted in the previous section, the closed-loop error dynamics are given by

$$\dot{\delta}(t) = -\alpha\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})e(t) + p_1(t), \tag{45}$$

$$\dot{e}(t) = -\gamma\big[\mathcal{L}(\mathcal{G})\delta(t) + \sigma e(t)\big] + p_2(t), \tag{46}$$

where $p_1(t)$ and $p_2(t)$ represent the perturbation terms of the form

$$p_1(t) \triangleq -\dot{\epsilon}(t)\mathbf{1}_n, \tag{47}$$

$$p_2(t) \triangleq -\alpha\mathcal{L}^{\dagger}(\mathcal{G})L_cK_2\big[\dot{c}(t) + \gamma\sigma c(t)\big], \tag{48}$$

that satisfy $\|p_1(t)\|_2 \le p_1^* \triangleq n\dot{\epsilon}^*$ and $\|p_2(t)\|_2 \le p_2^* \triangleq \alpha\|\mathcal{L}^{\dagger}(\mathcal{G})L_cK_2\|_{\mathrm{F}}\bar{c}^*$ with $\|\dot{\epsilon}(t)\|_2 \le \dot{\epsilon}^{*2}$ and $\|\dot{c}(t) + \mu c(t)\|_2 \le \bar{c}^*$. Notice that $p_1(t)$ and $p_2(t)$ are bounded since $c(t)$ and $\dot{c}(t)$ are assumed to be bounded. The next theorem presents the first result of this section.

Theorem 5.1. *Consider the networked multiagent system given by (43) and (44), where agents exchange information using local measurements and with $\mathcal{G}$ defining a connected, undirected graph topology. Then, the closed-loop error dynamics defined by (45) and (46) are bounded.*

Proof. Consider the Lyapunov function candidate given by (33) and note that $V(0,0) = 0$ and $V(\delta, e) > 0$ for all $(\delta, e) \neq (0,0)$. Differentiating (33) along the trajectories of (45) and (46) yields

$$\begin{aligned}\dot{V}(\delta(t), e(t)) =& -\delta^{\mathrm{T}}(t)\mathcal{F}(\mathcal{G})\delta(t) - \sigma\alpha^{-1}e^{\mathrm{T}}(t)e(t) + \alpha^{-1}\delta^{\mathrm{T}}(t)p_1(t) + \alpha^{-1}\gamma^{-1}e^{\mathrm{T}}(t)p_2(t)\\ \leq& -c_1\|\delta(t)\|_2^2 - c_2\|e(t)\|_2^2 + c_3\|\delta(t)\|_2 + c_4\|e(t)\|_2, \end{aligned} \tag{49}$$

where $c_1 \triangleq \lambda_{\min}(\mathcal{F}(\mathcal{G}))$, $c_2 \triangleq \sigma\alpha^{-1}$, $c_3 \triangleq \alpha^{-1}p_1^*$, and $c_4 \triangleq \alpha^{-1}\gamma^{-1}p_2^*$. Note that (49) can be rearranged as

$$\dot{V}(\delta(t), e(t)) \leq -c_1\|\delta(t)\|_2\Big(\|\delta(t)\|_2 - \frac{c_3}{c_1}\Big) - c_2\|e(t)\|_2\Big(\|e(t)\|_2 - \frac{c_4}{c_2}\Big). \tag{50}$$

Since $\dot{V}(\delta(t), e(t)) \leq 0$ when $\|\delta(t)\|_2 \geq c_3/c_1$ and $\|e(t)\|_2 \geq c_4/c_2$, it follows that the closed-loop error dynamics defined by (45) and (46) are bounded. ■

If the modified distributed control approach given by (43) and (44) is compared with its original version given by (6) and (7), it can be seen that we added a term to (7) that results in (44). From the proof of Theorem 2, the purpose of this added term in (44) is to keep $\xi_i(t)$ bounded in the presence of time-varying exogenous inputs.

In the next corollary presenting the second result of this section, we determine the bound of $\delta(t)$ for $t \geq T$ characterizing the ultimate distance between $x(t)$ and $\epsilon(t)\mathbf{1}_n$, which is important for the applications involving time-varying exogenous inputs.

Corollary 2. Consider the networked multiagent system given by (43) and (44), where agents exchange information using local measurements and with $\mathcal{G}$ defining a connected, undirected graph topology. Then, the bound of $\delta(t)$ for $t \geq T$ is given by

$$\|\delta(t)\|_2^2 \leq \frac{1}{\alpha^2}\left[\frac{n^2\dot{\epsilon}^{*2}}{\lambda_{\min}^2(\mathcal{F}(\mathcal{G}))}\right] + \frac{\alpha^2}{\gamma}\left[\frac{\|Q(\mathcal{G})\|_{\mathrm{F}}^2\bar{c}^{*2}}{\mu^2}\right], \tag{51}$$

where $\mathcal{Q}(\mathcal{G}) \triangleq \mathcal{L}^\dagger(\mathcal{G})L_cK_2$ and $\sigma = \mu/\gamma, \mu \in \mathbb{R}_+$.

Proof. In the proof of Theorem 1, we showed that $\dot{V}(\delta(t), e(t)) \le 0$ when $\|\delta(t)\|_2 \ge c_3/c_1$ and $\|e(t)\|_2 \ge c_4/c_2$. Note that this implies $\dot{V}(\delta(t), e(t)) \le 0$ outside the compact set

$$\mathcal{S} \triangleq \left\{(\delta(t), e(t)) : \|\delta(t)\|_2 \le \frac{c_3}{c_1}\right\} \bigcap \left\{(\delta(t), e(t)) : \|e(t)\|_2 \le \frac{c_4}{c_2}\right\}. \tag{52}$$

Since $V(\delta(t), e(t))$ cannot grow outside $\mathcal{S}$, the evolution of $V(\delta(t), e(t))$ is upper bounded by

$$\begin{aligned} V(\delta(t), e(t)) &\le \max_{(\delta(t),e(t))\in\mathcal{S}} V(\delta(t), e(t)) \\ &= \frac{1}{2\alpha}\frac{c_3^2}{c_1^2} + \frac{1}{2\alpha\gamma}\frac{c_4^2}{c_2^2}, \quad t \ge T. \end{aligned} \tag{53}$$

Using (38) in (53), (51) follows. ■

Remark 2. Corollary 2 implies that if we choose α and γ such that both $1/\alpha^2$ and α^2/γ are small, then (51) is small for $t \ge T$. Note that if $(\delta_0, e_0) \in \mathcal{S}$, then $T = 0$ in (51). In addition, if $(\delta_0, e_0) \notin \mathcal{S}$ and since $\dot{V}(\delta(t), e(t)) \le 0$ outside $\mathcal{S}$, then it similarly follows from Corollary 1 that small $1/\alpha^2$ and α^2/γ implies a tighter bound for $t < T$. This is illustrated in the next example.

Example 4. Consider a networked multiagent system represented by the graph shown in Figure 3 with 5 active agents and 20 passive agents. Let the active agents be subject to isolated time-varying inputs given by $c_1(t) = \sin(0.05t)$, $c_2(t) = \sin(0.1t)$, $c_3(t) = \sin(0.15t)$, $c_4(t) = \sin(0.2t)$, and $c_5(t) = \sin(0.25t)$, and let all agents have arbitrary initial conditions and $\xi_i(0) = 0$, $i = 1, \ldots, n$. Figure 6 shows the response with the proposed distributed controller given by (43) and (44) with $\alpha = 1$, $\gamma = 1$, and $\sigma = 0.1/\gamma$ (we have $\|\delta(t)\|_2 \le 52.39$ for this case), whereas Figure 7 shows the response with $\alpha = 5$, $\gamma = 50$, and $\sigma = 0.1/\gamma$ (we have $\|\delta(t)\|_2 \le 24.74$ for this case). Following the discussion in Remark 2, the networked multiagent system in Figure 7 is able to fuse and track the given time-varying inputs closely since we choose α and γ to obtain smaller $1/\alpha^2$ and α^2/γ values for this case.

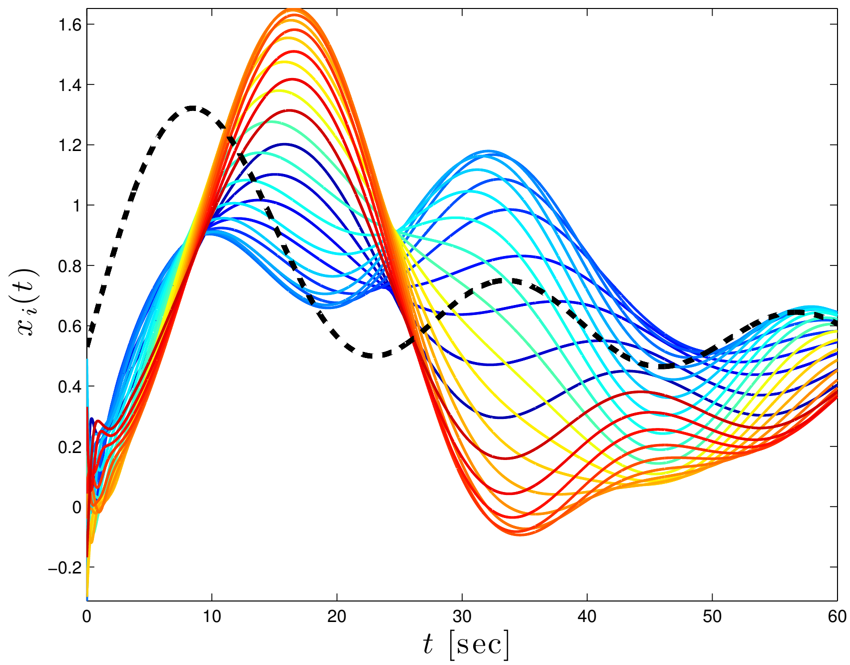

Figure 6. Response of the networked multiagent system in Example 4 with the distributed controller given by (43) and (44) with $\alpha = 1$, $\gamma = 1$, and $\sigma = 0.1/\gamma$ (solid lines denote agent positions and dashed line denote the average of applied time-varying inputs).

6. OBSERVATIONS AND DISCUSSION

We next draw connections between the proposed distributed control approach and other approaches to highlight why our methodology presents a unification among those approaches. Consider the proposed algorithm given by (6) and (7) and let $\alpha = \gamma = 1$ without loss of generality. As the first special case, assume that there does not exist any active agent in the network. In this case, the proposed algorithm acts as an integral-based average consensus algorithm, where all passive agents converge to the average of their initial conditions.

Theorem 6.1. *Consider the networked multiagent system given by (6) and (7) with $\alpha = \gamma = 1$, where agents exchange information using local measurements and with $\mathcal{G}$ defining a connected, undirected graph topology. In addition, assume that there does not exist any*

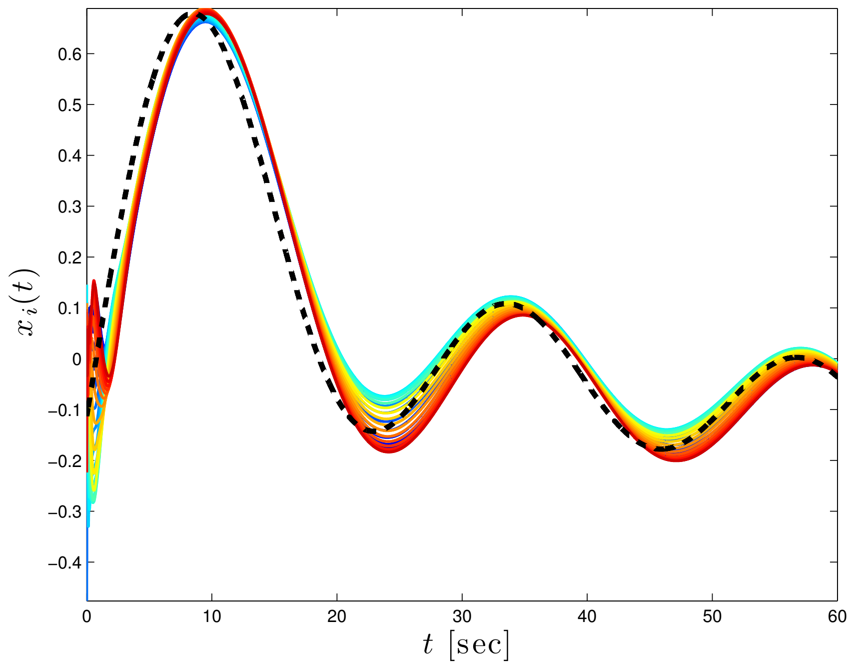

Figure 7. Response of the networked multiagent system in Example 4 with the distributed controller given by (43) and (44) with $\alpha = 5$, $\gamma = 50$, and $\sigma = 0.1/\gamma$ (solid lines denote agent positions and dashed line denote the average of applied time-varying inputs).

active agent in the network. Then, the closed-loop error dynamics defined by

$$\dot{\delta}(t) = -\mathcal{L}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})\xi(t), \tag{54}$$

$$\dot{\xi}(t) = -\mathcal{L}(\mathcal{G})\delta(t), \tag{55}$$

are Lyapunov stable for all initial conditions and $\delta(t) = x(t) - \mathbf{1}_n\bar{x}$ *asymptotically vanishes, where* $\bar{x} = \frac{1}{n}\mathbf{1}_n^{\mathrm{T}}x_0$ *denotes the average of all agents initial conditions.*

Proof. The time derivative of the average of the agents' states is zero in this case, i.e.,

$$\frac{1}{n}\mathbf{1}_n^{\mathrm{T}}\dot{x}(t) = \frac{1}{n}\mathbf{1}_n^{\mathrm{T}}\big[-\mathcal{L}(\mathcal{G})x(t) + \mathcal{L}(\mathcal{G})\xi(t)\big] = 0, \tag{56}$$

as a direct consequence of Lemma 1, and hence, $\frac{1}{n}\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}x(t) = \mathbf{1}_n\bar{x}$ holds for all $t \in \overline{\mathbb{R}}_+$. Note that if we show that

$$\lim_{t\to\infty}\delta(t) = 0, \tag{57}$$

then this proves the result since (57) implies [28]

$$\lim_{t\to\infty} x(t) = \frac{1}{n}\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}x(t) = \mathbf{1}_n\bar{x}. \tag{58}$$

For this purpose, consider the Lyapunov function candidate given by

$$V(\delta,\xi) = \frac{1}{2}\delta^{\mathrm{T}}\delta + \frac{1}{2}\xi^{\mathrm{T}}\xi, \tag{59}$$

and note that $V(0,0) = 0$ and $V(\delta,\xi) > 0$ for all $(\delta,\xi) \neq (0,0)$. Differentiating (59) along the trajectories of (54) and (55) yields

$$\dot{V}(\delta(t),\xi(t)) = -\delta^{\mathrm{T}}(t)\mathcal{L}(\mathcal{G})\delta(t) \leq 0. \tag{60}$$

This shows that the closed-loop error dynamics given by (54) and (55) are Lyapunov stable for all initial conditions. Now, let $\mathcal{R} \triangleq \big\{(\delta(t),\xi(t)) : \dot{V}(\delta(t),e(t)) = 0\big\}$ and let $\mathcal{M}$ be the largest invariant set contained in $\mathcal{R}$. Note that, in this case, since $\mathcal{L}(\mathcal{G})\delta(t) = 0$, it follows that $\sum_{i\sim j}\big(\delta_i(t) - \delta_j(t)\big) = 0$ for all $(i,j) \in \mathcal{E}_{\mathcal{G}}$. Using similar arguments as in Theorem 3 of [29], it follows from the connectivity of the graph $\mathcal{G}$ that $\delta_i(t) = \delta_j(t)$ for all $i,j \in \mathcal{V}_{\mathcal{G}}$. Furthermore, by $\delta(t) = x(t) - \mathbf{1}_n\bar{x}$, which implies that $\sum_{i=1}^{n}\delta_i(t) = 0$. Therefore, as noted, $x(t) \to \mathbf{1}_n\bar{x}$ as $t \to \infty$. ■

Example 5. Consider a networked multiagent system represented by the graph shown in Figure 3 without any active agents and let all agents have arbitrary initial conditions and $\xi_i(0) = 0$, $i = 1,\ldots,n$. Then, Figure 8 illustrates Theorem 3, where all agents converge to the average of their initial conditions.

In this case, it follows from (6) and (7) that

$$\dot{x}_i(t) = -\alpha\sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big) + \sum_{i\sim j}\Big(\xi_i(t) - \xi_j(t)\Big) - \alpha\Big(x_i(t) - c_i(t)\Big), \quad x_i(0) = x_{i0}, \tag{61}$$

$$\dot{\xi}_i(t) = -\gamma\sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big), \quad \xi_i(0) = \xi_{i0}, \tag{62}$$

where we recover a form of the dynamic average consensus algorithm in [6]–[11].

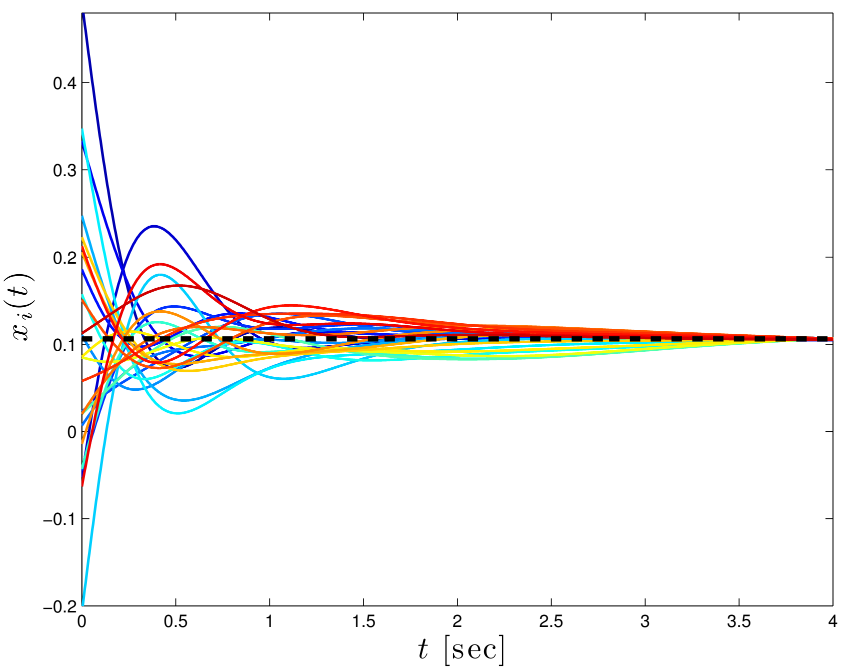

Figure 8. Response of the networked multiagent system in Example 5 with the distributed controller given by (54) and (55) without any inputs (solid lines denote agent positions and dashed line denote the average of the initial conditions of agents).

The first and second special cases can be viewed in the context of leaderless networks. In order to draw connections between leader–follower networks; for example, the proposed algorithm (6) and (7) acts as a pinning control algorithm [12]–[17] only when there is one active agent or multiple active agents subject to same inputs (e.g., pins). However, the algorithms in [12]–[17] do not have an integral action in their formulation, and hence, they do not provide robustness for instances when there are multiple pins that are not perfectly aligned with each other. Note that the proposed algorithm will still lead to an agreement, even if these pins are not exactly the same (Theorem 1).

It should be also noted that (6) and (7) acts as a containment control algorithm [18]–[23] only when the effect of the proposed distributed control approach's integral action is nullified – since in this case the states of passive agents converge to the convex hull spanned by those of the active agents. This is an important property and can be used to make a tradeoff between the containment problem (network segregation) and the consensus problem (network integration) via introducing a scalar switching between 0 to 1 that multiplies the integral action. This will be considered as a part of future research. To summarize, the proposed distributed control approach presents both unification and generalization to the results of several classes of leaderless and leader-follower network approaches.

Finally, we also note that the authors of [30], [31] propose a selective gossip algorithm in the context of efficiently computing spare approximations of network data via considering nodes that contain significant energy, which is similar in spirit to the problem considered in this paper. However, our results go beyond [30], [31] in that the proposed framework of this paper is applicable to situations where applied exogenous inputs may overlap within the active agents and, more importantly, such inputs can be time-varying, which is important for applications in dynamic environments. Furthermore, we not only show the steady state performance characteristics but also rigorously analyze the transient time performance characteristics of the proposed approaches, and the effect of design parameters on overall network performance.

7. CONCLUSION

To contribute to the previous studies in networked multiagent systems, we investigated a system consisting of agents subject to exogenous inputs (active agents) and agents without any inputs (passive agents). Specifically, we proposed and analyzed a new integral action-based distributed control approach that drives the states of all agents to the average of the exogenous inputs applied only to the active agents, where these inputs may or may not overlap within the active agents. In addition, we discussed in detail that the proposed approach not only generalizes but also unifies the results of several classes of leaderless and leader-follower network approaches. Illustrative examples indicated that the presented theory and its numerical results are compatible. Future research will include extensions of the proposed integral action-based distributed control approach given by (6) and (7) (and (43) and (44)) to agents having high-order (linear and nonlinear) dynamics and graph topologies that vary in time.

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II. AN ACTIVE–PASSIVE NETWORKED MULTIAGENT SYSTEMS APPROACH TO ENVIRONMENT SURVEILLANCE

John Daniel Peterson and Tansel Yucelen

ABSTRACT

This paper presents a novel application of multiagent systems to environment surveillance and develops a robust network wherein there is no need for a gateway node, which fuses information received from each node to construct a global map on a single location, and may cause a communication bottleneck. Recognizing the fact that network nodes may be heterogeneous with respect to the number of exogenous inputs such that a node may not sense a quantity or can sense multiple quantities for certain time instants, we utilize a recently developed active-passive networked multiagent systems approach. Specifically, this approach consists of agents subject to exogenous inputs (active agents) and agents without any inputs (passive agents). The key feature of this approach is that the states of all nodes converge to the average of the exogenous inputs, where these inputs may or may not overlap within the active agents. A detailed illustrative study is provided to demonstrate the efficacy of this approach as applied to environment surveillance.

1. INTRODUCTION

Distributed control of networked multiagent systems has attracted increasing attention from many multidisciplinary researchers in systems and control science, wireless communication networks, and computer science, due to their broad applications in surveillance, reconnaissance, collaborative processing of information, and gathering scientific data from spatially distributed sources (see, for example, [1]–[9] and references therein). Distributed control is performed by a few to several hundreds of agents, where each agent locally estimates the environment, processes information, and utilizes control algorithms via peer-to-peer communications to achieve a global objective. Distributed control offers significant advantages over centralized control such as robustness against agent failures and network bottlenecks, as it does not rely on any specific network topology or data fusion centers [4], [10]. Since distributed control does not require nodes with global information sharing ability, overall communication cost is also lower than that of centralized sensing in many situations [11], [12].

This paper presents a novel application of multiagent systems to environment surveillance and develops a robust network wherein there is no need for a gateway node, which fuses information received from each node to construct a global map on a single location, and may cause a communication bottleneck. Recognizing the fact that network nodes may be heterogeneous with respect to the number of exogenous inputs such that a node may not sense a quantity or can sense multiple quantities for certain time instants, we utilize a recently developed active-passive networked multiagent systems approach presented in [13]. Specifically, this approach consists of agents subject to exogenous inputs (active agents) and agents without any inputs (passive agents). The key feature of this approach is that the states of all nodes converge to the average of the exogenous inputs, where these inputs may or may not overlap within the active agents. A detailed illustrative study is provided to demonstrate the efficacy of this approach as applied to environment surveillance.

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbb{R}_+$ denotes the set of positive real numbers, $\mathbb{R}_+^{n\times n}$ (resp., $\overline{\mathbb{R}}_+^{n\times n}$) denotes the set of $n \times n$ positive-definite (resp., nonnegative-definite) real matrices, $\mathbb{S}_+^{n\times n}$ (resp., $\overline{\mathbb{S}}_+^{n\times n}$) denotes the set of $n \times n$ symmetric positive-definite (resp., symmetric nonnegative-definite) real matrices, $\mathbb{Z}$ denotes the set of integers, $\mathbb{Z}_+$ (resp., $\overline{\mathbb{Z}}_+$) denotes the set of positive (resp., nonnegative) integers, $\mathbf{0}_n$ denotes the $n \times 1$ vector of all zeros, $\mathbf{1}_n$ denotes the $n \times 1$ vector of all ones, $\mathbf{0}_{n\times n}$ denotes the $n \times n$ zero matrix, I_n denotes the $n \times n$ identity matrix, $(\cdot)^{\mathrm{T}}$ denotes transpose, $(\cdot)^{-1}$ denotes inverse, $(\cdot)^{\dagger}$ denotes generalized inverse, and $\|\cdot\|_2$ denotes the Euclidian norm.

Next, we recall some of the basic notions from graph theory, where we refer to [14] and [15] for further details. In the multiagent literature, graphs are broadly adopted to encode interactions in networked systems. An *undirected* graph $\mathcal{G}$ is defined by a set $\mathcal{V}_{\mathcal{G}} = \{1, \ldots, n\}$ of *nodes* and a set $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$ of *edges*. If $(i, j) \in \mathcal{E}_{\mathcal{G}}$, then the nodes i and j are *neighbors* and the neighboring relation is indicated with $i \sim j$. The *degree* of a node is given by the number of its neighbors. Letting d_i be the degree of node i, then the *degree* matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by $\mathcal{D}(\mathcal{G}) \triangleq \operatorname{diag}(d)$, $d = [d_1, \ldots, d_n]^{\mathrm{T}}$. A *path* $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1, \ldots, L$, and a graph $\mathcal{G}$ is *connected* if there is a path between any pair of distinct nodes. The *adjacency* matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by $[\mathcal{A}(\mathcal{G})]_{ij} = 1$ if $(i, j) \in \mathcal{E}_{\mathcal{G}}$ and 0 otherwise. The *Laplacian* matrix of a graph, $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_{+}^{n\times n}$, playing a central role in many graph theoretic treatments of multiagent systems, is given by $\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G})$. Throughout this paper, we model a given multiagent system by a connected, undirected graph $\mathcal{G}$, where nodes and edges represent agents and inter-agent communication links, respectively.

2. ACTIVE–PASSIVE NETWORKED MULTIAGENT SYSTEMS OVERVIEW

This section overviews the active-passive multiagent systems approach introduced in [13]. In particular, consider a system of n agents exchanging information among each other using their local measurements according to a connected, undirected graph $\mathcal{G}$. In addition, consider that there exists $m \geq 1$ exogenous inputs that interact with this system. We say that if agent i is subject to one or more exogenous inputs (resp., no exogenous inputs), then it is an *active agent* (resp., *passive agent*). In addition, we say that if an exogenous input interacts with only one agent (resp., multiple agents), then it is an *isolated input* (resp., *non-isolated input*). These definitions are illustrated in Figure 1.

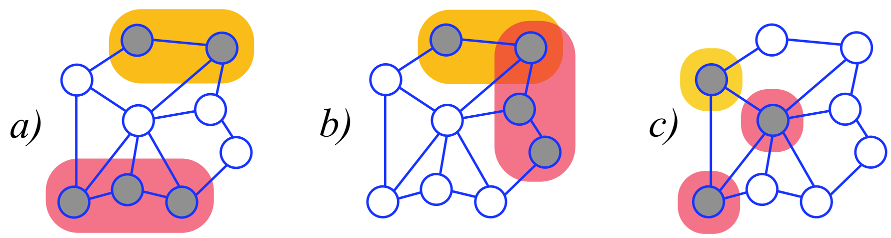


Figure 1. An active-passive networked multiagent system with *a)* two non-overlapping non-isolated inputs, *b)* two overlapping non-isolated inputs, and *c)* two non-overlapping inputs, where one of them is isolated and the other one is non-isolated (lines denote communication links, gray circles denote active agents, white circles denote passive agents, and shaded areas denote the exogenous inputs interacting with this system).

Considering environment surveillance, it is of interest to drive the states of all (active and passive) agents to the average of the applied exogenous inputs. Motivating from this standpoint, we use the integral action-based distributed control approach of [13] given by

$$\dot{x}_i(t) = -\sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big) + \sum_{i\sim j}\Big(\xi_i(t) - \xi_j(t)\Big) - \sum_{i\sim h}\Big(x_i(t) - c_h(t)\Big), \quad x_i(0) = x_{i0}, \tag{1}$$

$$\dot{\xi}_i(t) = -\sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big), \quad \xi_i(0) = \xi_{i0}, \tag{2}$$

where $x_i(t) \in \mathbb{R}$ and $\xi_i(t) \in \mathbb{R}$ denote the state and the integral action of agent i, $i = 1, \ldots, n$, respectively, and $c_h(t) \in \mathbb{R}$, $h = 1, \ldots, m$, denotes an exogenous input applied to this agent. Similar to the $i \sim j$ notation indicating the neighboring relation between agents, we use $i \sim h$ to indicate the exogenous inputs that an agent is subject to. Next, let $x(t) = \big[x_1(t), x_2(t), \ldots, x_n(t)\big]^{\mathrm{T}} \in \mathbb{R}^n$, $\xi(t) = \big[\xi_1(t), \xi_2(t), \ldots, \xi_n(t)\big]^{\mathrm{T}} \in \mathbb{R}^n$, and $c(t) = \big[c_1(t), c_2(t), \ldots, c_m(t), 0, \ldots, 0\big] \in \mathbb{R}^n$, where we assume $m \leq n$ for the ease of the following notation and without loss of generality. We can now write (1) and (2) in a compact form as

$$\dot{x}(t) = -\mathcal{L}(\mathcal{G})x(t) + \mathcal{L}(\mathcal{G})\xi(t) - K_1 x(t) + K_2 c(t), \quad x(0) = x_0, \tag{3}$$

$$\dot{\xi}(t) = -\mathcal{L}(\mathcal{G})x(t), \quad \xi(0) = \xi_0, \tag{4}$$

where $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{n\times n}$,

$$K_1 \triangleq \mathrm{diag}([k_{1,1}, k_{1,2}, \ldots, k_{1,n}]^{\mathrm{T}}) \in \overline{\mathbb{S}}_+^{n\times n}, \tag{5}$$

with $k_{1,i} \in \overline{\mathbb{Z}}_+$ denoting the number of the exogenous inputs applied to agent i, $i = 1, \ldots, n$, and

$$K_2 \triangleq \begin{bmatrix} k_{2,11} & k_{2,12} & \cdots & k_{2,1n} \\ k_{2,21} & k_{2,22} & \cdots & k_{2,2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{2,n1} & k_{2,n2} & \cdots & k_{2,nn} \end{bmatrix} \in \mathbb{R}^{n\times n}, \tag{6}$$

with $k_{2,ih} = 1$ if the exogenous input $c_h(t)$, $h = 1, \ldots, m$, is applied to agent i, $i = 1, \ldots, n$, and $k_{2,ih} = 0$ otherwise. Note that $k_{1,i} = \sum_{j=1}^{n} k_{2,ij}$.

Since we are interested in driving the states of all (active and passive) agents to the average of the applied exogenous inputs, let

$$\delta(t) \triangleq x(t) - \epsilon(t)\mathbf{1}_n \in \mathbb{R}^n, \tag{7}$$

$$\epsilon(t) \triangleq \frac{\mathbf{1}_n^{\mathrm{T}} K_2 c(t)}{\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n} \in \mathbb{R}, \tag{8}$$

be the error between $x_i(t)$, $i = 1, \ldots, n$, and the average of the applied exogenous inputs $\epsilon(t)$. Based on (8), $\epsilon(t)$ can be equivalently written as

$$\epsilon(t) = \frac{(k_{2,11}c_1(t) + k_{2,12}c_2(t) + \cdots + k_{2,21}c_1(t) + k_{2,22}c_2(t) + \cdots)}{(k_{2,11} + k_{2,12} + \cdots + k_{2,21} + k_{2,21} + \cdots)}, \tag{9}$$

which is the average of the applied exogenous inputs. Furthermore, note for the special case of isolated exogenous inputs that

$$\epsilon(t) = \frac{(c_1(t) + c_2(t) + \cdots + c_m(t))}{m}. \tag{10}$$

2.1. CASE 1: CONSTANT EXOGENOUS INPUTS IN ACTIVE AGENTS

Let $c_h(t) = c_h$, $h = 1, \ldots, m$, be a constant exogenous input. Since $c(t) = c$, and hence, $\epsilon(t) = \epsilon$ from (8), it follows from (7) and $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$ that

$$\begin{aligned} \dot{\delta}(t) &= -\mathcal{L}(\mathcal{G})\big[\delta(t) + \epsilon\mathbf{1}_n\big] + \mathcal{L}(\mathcal{G})\xi(t) - K_1\big[\delta(t) + \epsilon\mathbf{1}_n\big] + K_2 c(t) \\ &= -\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})\xi(t) - L_c K_2 c, \end{aligned} \tag{11}$$

where $\mathcal{F}(\mathcal{G}) \triangleq \mathcal{L}(\mathcal{G}) + K_1$ and

$$L_c \triangleq \frac{K_1 \mathbf{1}_n \mathbf{1}_n^{\mathrm{T}}}{\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n} - \mathrm{I}_n. \tag{12}$$

Note that $\mathcal{F}(\mathcal{G}) \in \mathbb{S}_+^{n\times n}$ from [13] and

$$\mathbf{1}_n^{\mathrm{T}} L_c = \mathbf{1}_n^{\mathrm{T}} \left[\frac{K_1 \mathbf{1}_n \mathbf{1}_n^{\mathrm{T}}}{\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n} - \mathrm{I}_n \right] = \frac{\mathbf{1}_n^{\mathrm{T}} K_1 \mathbf{1}_n}{\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n} \mathbf{1}_n^{\mathrm{T}} - \mathbf{1}_n^{\mathrm{T}} = 0, \tag{13}$$

since $(\mathbf{1}_n^{\mathrm{T}} K_1 \mathbf{1}_n)/(\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n) = 1$ from $k_{1,i} = \sum_{j=1}^{n} k_{2,ij}$. Next, letting

$$e(t) \triangleq \xi(t) - \mathcal{L}^{\dagger}(\mathcal{G}) L_c K_2 c, \tag{14}$$

and using (14) in (11) yields (see [13] for details)

$$\begin{aligned} \dot{\delta}(t) &= -\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})\big[e(t) + \mathcal{L}^{\dagger}(\mathcal{G}) L_c K_2 c\big] - L_c K_2 c \\ &= -\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G}) e(t). \end{aligned} \tag{15}$$

In addition, differentiating (14) with respect to time yields

$$\dot{e}(t) = -\mathcal{L}(\mathcal{G})\big[\delta(t) + \epsilon \mathbf{1}_n\big] = -\mathcal{L}(\mathcal{G})\delta(t), \tag{16}$$

where $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$ of Lemma 1 in [13] is used. The next theorem shows that the state of all agents $x_i(t)$, $i = 1, \ldots, n$ asymptotically converge to ϵ.

Theorem 2.1. *Consider the networked multiagent system given by* (1) *and* (2)*, where agents exchange information using local measurements and with $\mathcal{G}$ defining a connected, undirected graph topology. Then, the closed-loop error dynamics defined by* (15) *and* (16) *are Lyapunov stable for all initial conditions and $\delta(t)$ asymptotically vanishes.*

Proof. See the proof of Theorem 2.1 in [13]. ∎

A generalized version of the proposed integral action-based distributed control approach can be given by

$$\dot{x}_i(t) = -\alpha \sum_{i\sim j} \Big(x_i(t) - x_j(t)\Big) + \sum_{i\sim j} \Big(\xi_i(t) - \xi_j(t)\Big) - \alpha \sum_{i\sim h} \Big(x_i(t) - c_h\Big), \ x_i(0) = x_{i0}, \tag{17}$$

$$\dot{\xi}_i(t) = -\gamma \sum_{i\sim j} \Big(x_i(t) - x_j(t)\Big), \ \xi_i(0) = \xi_{i0}, \tag{18}$$

Corollary 1 in [13] proves that (17) and (18) asymptotically converges. In addition, Corollary 2 in [13] shows that if α and γ are chosen such that α^2/γ and $1/\alpha$ are small, then the transient response of the overall network is improved.

2.2. CASE 2: TIME-VARYING EXOGENOUS INPUTS IN ACTIVE AGENTS

This section deals with the case when active agents are subject to time-varying exogenous inputs $c_h(t)$, $h = 1, \ldots, m$. Let $c_h(t)$ and $\dot{c}_h(t)$ be bounded for each input h, $h = 1, \ldots, m$. In this case, we slightly modify the integral action-based distributed control approach in (17) and (18) to the following

$$\dot{x}_i(t) = -\alpha \sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big) + \sum_{i\sim j}\Big(\xi_i(t) - \xi_j(t)\Big) - \alpha \sum_{i\sim h}\Big(x_i(t) - c_h(t)\Big), \ x_i(0) = x_{i0}, \quad (19)$$

$$\dot{\xi}_i(t) = -\gamma\Big[\sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big) + \sigma\xi_i(t)\Big], \quad \xi_i(0) = \xi_{i0}, \quad (20)$$

where $\alpha \in \mathbb{R}_+$, $\gamma \in \mathbb{R}_+$, and $\sigma \in \mathbb{R}_+$. Next, the closed-loop error dynamics are given by

$$\dot{\delta}(t) = -\alpha\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})e(t) + p_1(t), \quad (21)$$

$$\dot{e}(t) = -\gamma\big[\mathcal{L}(\mathcal{G})\delta(t) + \sigma e(t)\big] + p_2(t), \quad (22)$$

where $p_1(t)$ and $p_2(t)$ represent the perturbation terms of the form

$$p_1(t) \triangleq -\dot{\epsilon}(t)\mathbf{1}_n, \quad (23)$$

$$p_2(t) \triangleq -\alpha\mathcal{L}^\dagger(\mathcal{G})L_cK_2\big[\dot{c}(t) + \gamma\sigma c(t)\big], \quad (24)$$

that satisfy

$$\|p_1(t)\|_2 \le p_1^* \triangleq n\dot{\epsilon}^*, \quad (25)$$

$$\|p_2(t)\|_2 \le p_2^* \triangleq \alpha\|\mathcal{L}^\dagger(\mathcal{G})L_cK_2\|_{\mathrm{F}}\bar{c}^*, \quad (26)$$

with $\|\dot{\epsilon}(t)\|_2 \le \dot{\epsilon}^{*2}$ and $\|\dot{c}(t) + \mu c(t)\|_2 \le \bar{c}^*$. Notice that $p_1(t)$ and $p_2(t)$ are bounded since $c(t)$ and $\dot{c}(t)$ are assumed to be bounded.

Theorem 2.2. *Consider the networked multiagent system given by (19) and (20), where agents exchange information using local measurements and with $\mathcal{G}$ defining a connected, undirected graph topology. Then, the closed-loop error dynamics defined by (21) and (22) are bounded.*

Proof. This can be easily shown by extending the proof of Theorem 2.1. ■

Similar to the constant exogenous inputs case, if α and γ are chosen such that α^2/γ and $1/\alpha$ are small, then the transient response of the overall network is improved (see Corollary 3 of [13]).

3. ILLUSTRATIVE ENVIRONMENT SURVEILLANCE STUDY

In this section, we present two illustrative examples of environment surveillance using the active-passive networked multiagent systems stated in Section 2. Our aim is to make each agent aware of an unknown global environment through local peer-to-peer communications. We consider the case in Section II.A where the environment is static with respect to time, as well as the case in Section II.B where the environment is dynamic with respect to time. To elucidate the application of the proposed approach to environment surveillance, consider Figure 2. In particular, the upper layer of this figure shows how agent $x_{i,j}$ (we use the subscript i, j to denote the state of an agent in this two-dimensional environment) uses the algorithm given by (1) and (2). This agent implements the proposed algorithm for its own (blue) environment (where it is subject to the exogenous input $c_{i,j}$, and hence, active) and exchanges its state associated with this environment, $x_{i,j}$, with its peers so that other neighboring agents can use the state of this agent to compute (1) and (2) simultaneously (note that neighbors of agent $x_{i,j}$ are passive with respect to this part of the environment). Likewise, this agent also utilizes (1) and (2) for the neighboring (green) environments, where it is passive, by using the state information received from its neighbors. Therefore, the stated results of Section 2 still hold for this agent as well as the entire multiagent system to achieve a global awareness of the unknown environment. A similar discussion can be given for the bottom layer of this figure for agent $x_{i+1,j+1}$.

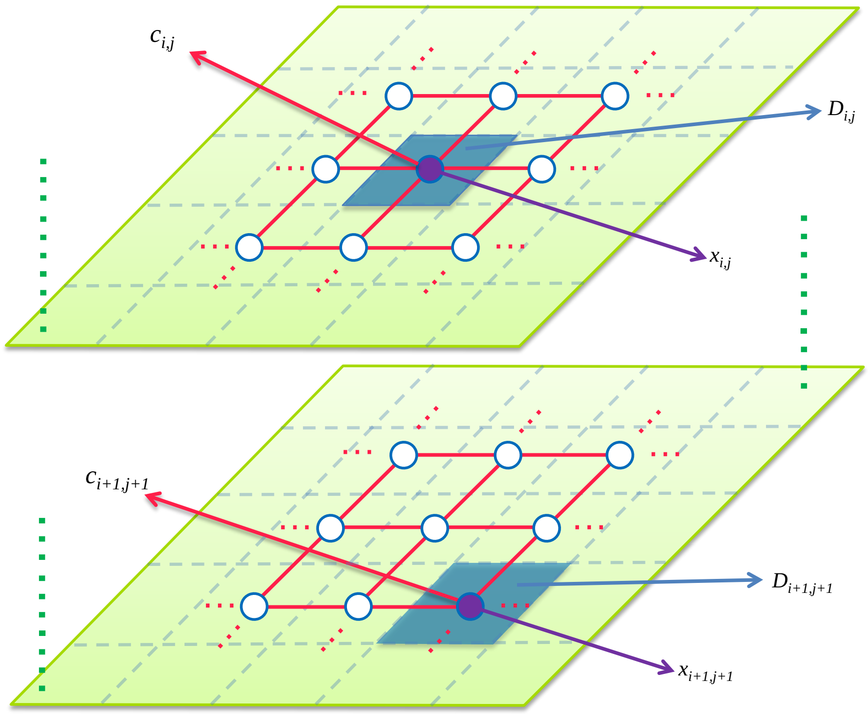


Figure 2. Agents arranged in a grid network sampling a plane. White circles denote passive agents with respect to a particular network layer. Purple circles denote active agents with respect to a particular network layer. Blue squares denote an agent's local environment. Red lines denote communication between nodes.

Case 1: For the unknown static environment depicted in Figure 3, a 16×16 sensor grid is placed to perform environment surveillance. Each agent uses (1) and (2) for their own local environment (where they act as active agents) as well as for other parts of the global environment (where they act as a passive agent since they do not have a priori knowledge on those parts). Figure 4 shows that nodes first become aware of the unknown neighboring environments during the transient response of the entire multiagent network and Figure 5 shows that all nodes achieve global awareness of the unknown environment in Figure 3 at their steady-state. Therefore, this study illustrates the efficacy of the proposed active-passive networked multiagent system framework for *static* environment surveillance.

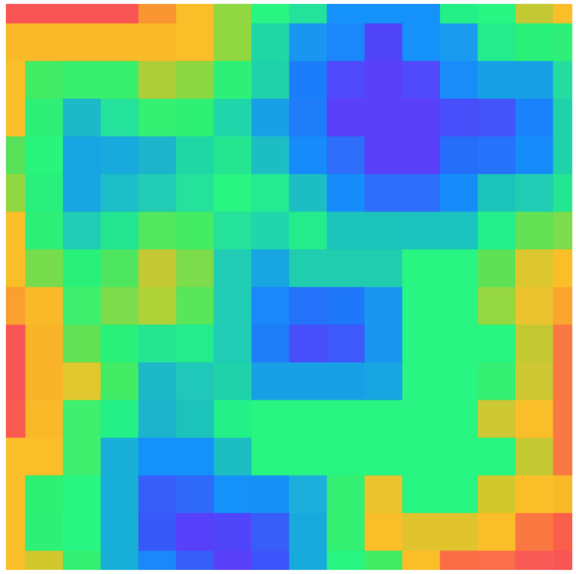


Figure 3. Unknown static environment used for surveillance.

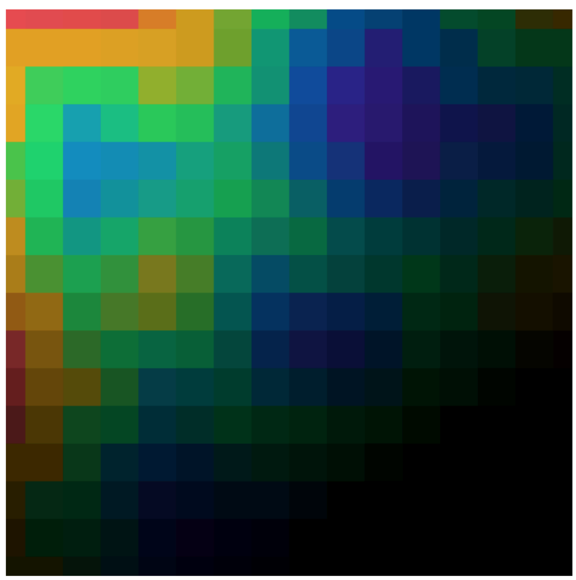


Figure 4. Environment as seen node 20 shortly after the simulation begins. Notice that the geographically close exogenous inputs are much clearer than the far inputs. Network gains are set as $\alpha = 1$ and $\gamma = 1$.

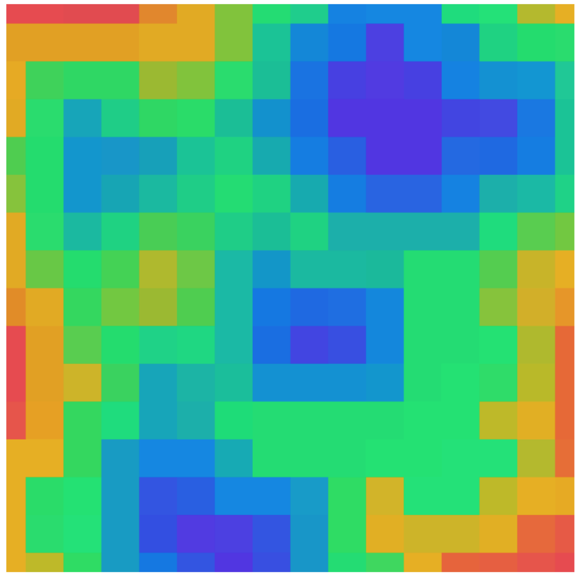


Figure 5. Final state of the network take from node 20.

Case 2: We now turn our focus to the case when agents are monitoring the dynamic environment in Figure 6. For this purpose, each agent utilizes (19) and (20) for their own local environment (where they act as active agents) and for the other parts of the global environment (where they act as passive agents since they have no real-time knowledge of those parts). An 8×8 sensor grid is placed to perform environment surveillance. In Figure 6 agents closely track the environment dynamics, demonstrating the efficacy of the proposed active-passive networked multiagent system framework for *dynamic* environment surveillance.

4. CONCLUSIONS

We proposed an active-passive networked multiagent systems approach to environment surveillance without requiring a gateaway node in the network, where this framework allows the states of all nodes to converge to the average of the exogenous inputs applied only to the active agents. Future work will concentrate on extending the proposed surveillance methodology to dynamic environments in the presence of nonstationary mobile nodes with system-theoretic guarantees.

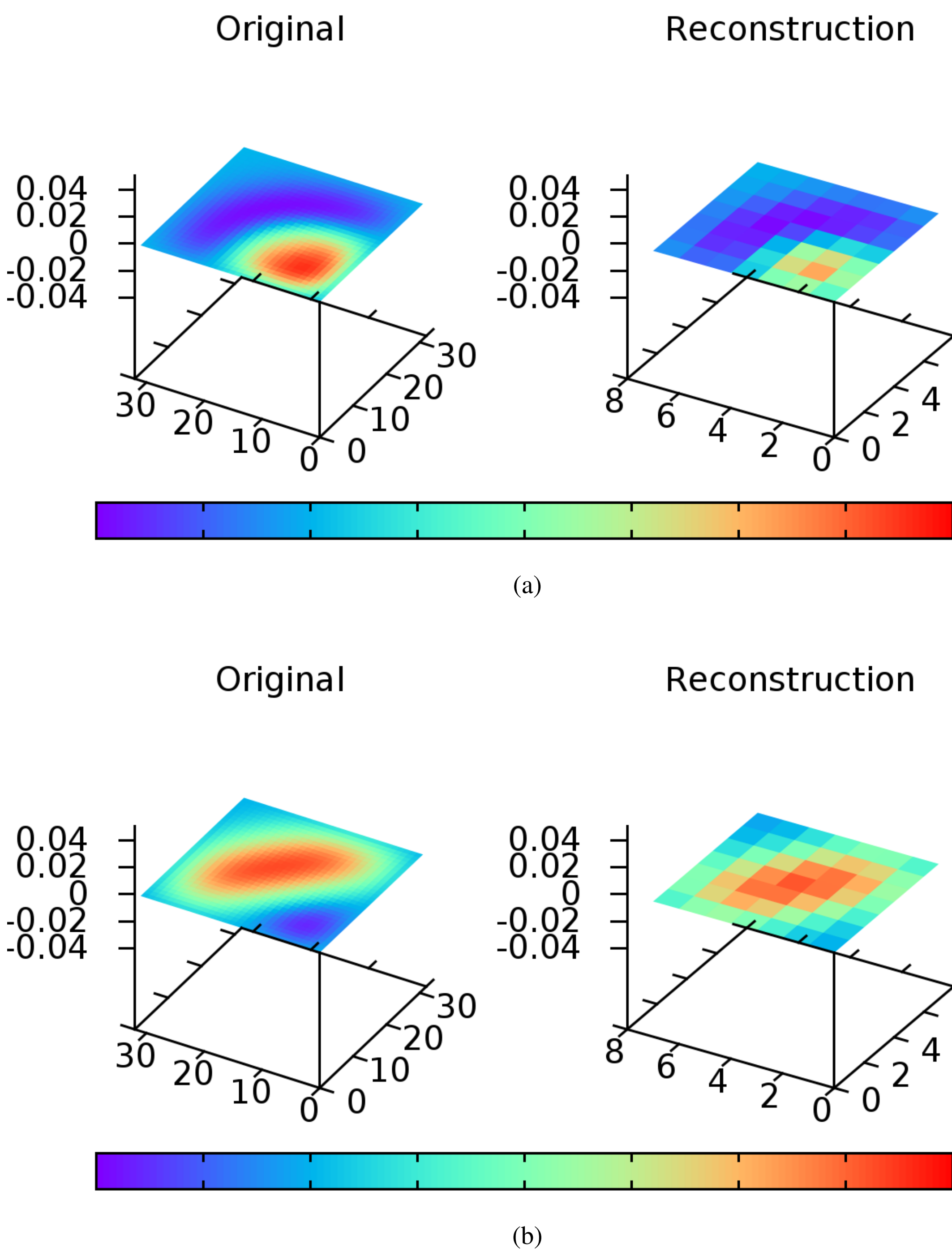
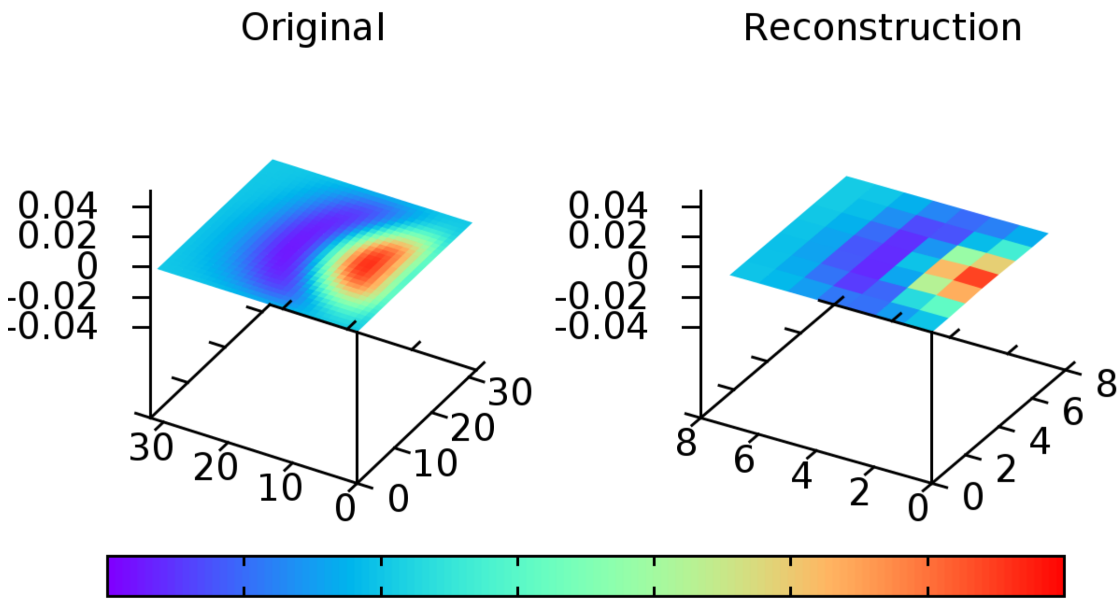
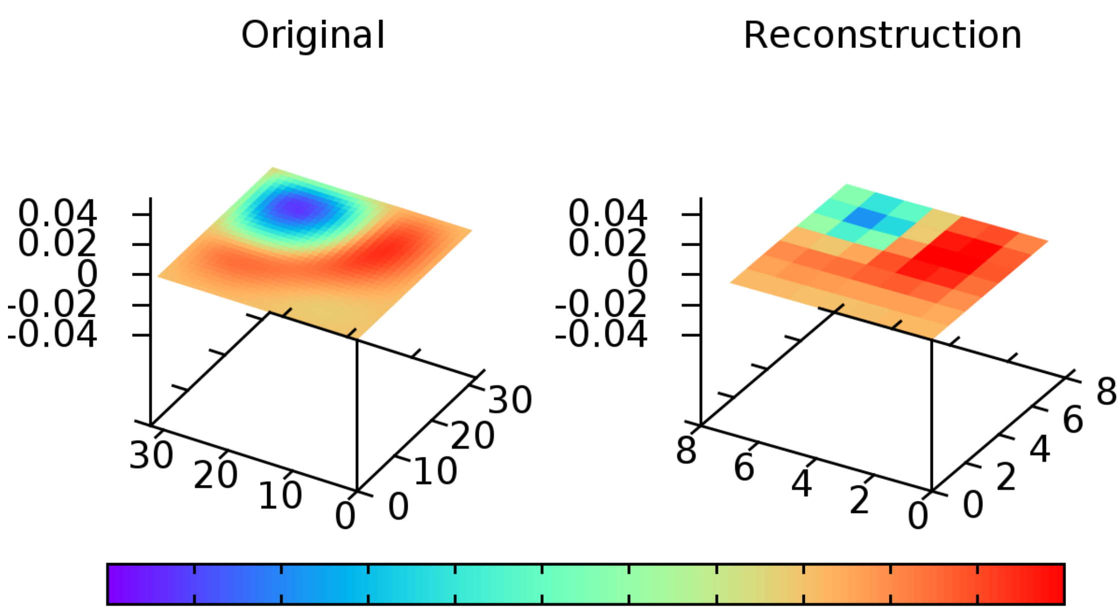


Figure 6. Comparison of the environment with the multiagent reconstruction where the network gains are $\alpha = 70$, $\gamma = 700$, and $\sigma = 0.1/\gamma$. The reconstruction is performed by the agent in the far west corner.



(c)



(d)

Figure 6. Comparison of the environment with the multiagent reconstruction where the network gains are $\alpha = 70$, $\gamma = 700$, and $\sigma = 0.1/\gamma$. The reconstruction is performed by the agent in the far west corner. (cont.)

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III. APPLICATION OF AN ACTIVE–PASSIVE DYNAMIC CONSENSUS FILTERS APPROACH TO THE MULTIAGENT TRACKING PROBLEM FOR SITUATIONAL AWARENESS IN UNKNOWN ENVIRONMENTS

John Daniel Peterson and Tansel Yucelen

ABSTRACT

In this paper, we present an application of multiagent systems to the multitarget tracking problem, where networked nodes exchange their local information to construct a global map of an unknown environment for situational awareness. Recognizing the fact that networked nodes can be heterogeneous with respect to the number of targets sensed in their respective local environments, we utilize a recently developed active–passive dynamic consensus filters approach ([1], [2]). Specifically, a node is considered active for targets it is able to sense and passive for targets it is unable to sense. We use two ground robots equipped with object detection sensors to track local targets in a global frame. Networked nodes use the active–passive dynamic consensus filters approach to distribute and fuse information to make all networked nodes aware of all targets in the environment.

1. INTRODUCTION

Distributed sensing has attracted much attention from researchers in recent years in multidisciplinary areas such as systems and control science, wireless communication networks, and computer science, due to their broad applications in surveillance, reconnaissance, collaborative processing of information, and gathering scientific data from spatially distributed sources (see, for example, [3]–[11]). In particular, multiagent systems are well suited for multitarget tracking ([12]–[17]). Target tracking is performed by several to a few hundred networked nodes, where each node locally estimates their environment and use local information exchange to become aware of a global environment. Distributed target tracking offers significant advantages over classical methods such as robustness to communication link and node failures, do not rely on specific network topologies or central fusion nodes, and allow for nodes to be added or removed form the network. In the absence of a central fusion node, dynamic distributed fusion algorithms are challenged by heterogeneity with respect to the number of targets each networked node can sense and overlap within the targets each node senses.

In this paper, we present an application of multiagent systems to multitarget tracking which is robust to link and node failures and does not require a gateway node. Recognizing that networked nodes may be heterogeneous with respect to the number of targets they are able to sense, we utilize a recently developed active–passive dynamic consensus filter approach ([1], [2]), where networked nodes are considered active for the targets they sense and passive for targets they cannot sense. This key feature makes nodes aware of the location of all targets in a global sense even though they cannot sense all targets, or multiple nodes sense the same target. We demonstrate the efficacy of our approach with a real world example. Specifically, we utilize a network of two mobile ground robots equipped with object tracking sensors to track multiple unknown static targets and build a global map of the unknown environment.

2. MATHEMATICAL PRELIMINARIES

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbb{R}_+$ denotes the set of positive real numbers, $\mathbb{R}_+^{n\times n}$ (resp., $\overline{\mathbb{R}}_+^{n\times n}$) denotes the set of $n \times n$ positive-definite (resp., nonnegative-definite) real matrices, $\mathbb{S}_+^{n\times n}$ (resp., $\overline{\mathbb{S}}_+^{n\times n}$) denotes the set of $n \times n$ symmetric positive-definite (resp., symmetric nonnegative-definite) real matrices, $\mathbb{Z}$ denotes the set of integers, $\mathbb{Z}_+$ (resp., $\overline{\mathbb{Z}}_+$) denotes the set of positive (resp., nonnegative) integers, $\mathbf{0}_n$ denotes the $n \times 1$ vector of all zeros, $\mathbf{1}_n$ denotes the $n \times 1$ vector of all ones, $\mathbf{0}_{n\times n}$ denotes the $n \times n$ zero matrix, I_n denotes the $n \times n$ identity matrix, $(\cdot)^{\mathrm{T}}$ denotes transpose, $(\cdot)^{-1}$ denotes inverse, $(\cdot)^{\dagger}$ denotes generalized inverse, and $\|\cdot\|_2$ denotes the Euclidian norm.

Next, we recall some of the basic notions from graph theory, where we refer to [18] and [19] for further details. In the multiagent literature, graphs are broadly adopted to encode interactions in networked systems. An *undirected* graph $\mathcal{G}$ is defined by a set $\mathcal{V}_{\mathcal{G}} = \{1, \ldots, n\}$ of *nodes* and a set $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$ of *edges*. If $(i, j) \in \mathcal{E}_{\mathcal{G}}$, then the nodes i and j are *neighbors* and the neighboring relation is indicated with $i \sim j$. The *degree* of a node is given by the number of its neighbors. Letting d_i be the degree of node i, then the *degree* matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by $\mathcal{D}(\mathcal{G}) \triangleq \mathrm{diag}(d)$, $d = [d_1, \ldots, d_n]^{\mathrm{T}}$. A *path* $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1, \ldots, L$, and a graph $\mathcal{G}$ is *connected* if there is a path between any pair of distinct nodes. The *adjacency* matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by $[\mathcal{A}(\mathcal{G})]_{ij} = 1$ if $(i, j) \in \mathcal{E}_{\mathcal{G}}$ and 0 otherwise. The *Laplacian* matrix of a graph, $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_{+}^{n\times n}$, playing a central role in many graph theoretic treatments of multiagent systems, is given by $\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G})$. Throughout this paper, we model a given multiagent system by a connected, undirected graph $\mathcal{G}$, where nodes and edges represent networked nodes and inter-agent communication links, respectively.

3. ACTIVE–PASSIVE DYNAMIC CONSENSUS FILTERS OVERVIEW

This section overviews active-passive dynamic consensus filters introduced in [1] and [2]. In particular, consider a system of n networked nodes exchanging information among each other using their local measurements according to a connected, undirected graph $\mathcal{G}$. In addition, consider that there exists $m \geq 1$ exogenous inputs that interact with this system. We say that if node i senses to one or more targets (resp., no targets), then it is an *active node* (resp., *passive node*). In addition, we say that if a target is tracked by only one node (resp., multiple nodes), then it is an *isolated target* (resp., *non-isolated target*). These definitions are illustrated in Figure 1.

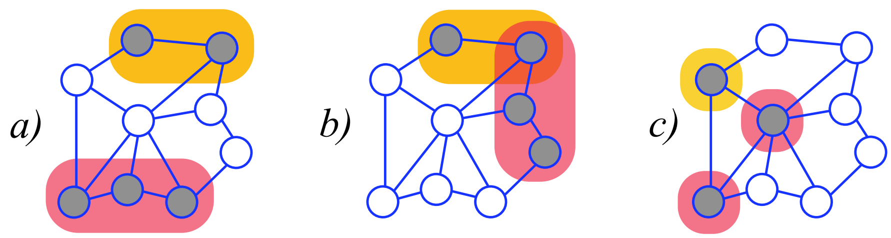


Figure 1. An active-passive dynamic consensus filter tracking *a)* two non-overlapping non-isolated targets, *b)* two overlapping non-isolated targets, and *c)* two non-overlapping targets, where one of them is isolated and the other one is non-isolated (lines denote communication links, gray circles denote active networked nodes, white circles denote passive networked nodes, and shaded areas denote the targets tracked by this system).

Considering the multiagent target tracking problem, it is of interest to drive the states of all (active and passive) networked nodes to the average of the locations of the global targets. Motivating from this standpoint, we use the integral action-based distributed control approach of [1] and [2] given by

$$\dot{x}_i(t) = -\sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big) + \sum_{i\sim j}\Big(\xi_i(t) - \xi_j(t)\Big) - \sum_{i\sim h}\Big(x_i(t) - c_h(t)\Big), \quad x_i(0) = x_{i0}, \tag{1}$$

$$\dot{\xi}_i(t) = -\sum_{i\sim j}\Big(x_i(t) - x_j(t)\Big), \quad \xi_i(0) = \xi_{i0}, \tag{2}$$

where $x_i(t) \in \mathbb{R}$ and $\xi_i(t) \in \mathbb{R}$ denote the state and the integral action of networked node i, $i = 1, \ldots, n$, respectively, and $c_h(t) \in \mathbb{R}$, $h = 1, \ldots, m$, denotes an tracked by this networked node. Similar to the $i \sim j$ notation indicating the neighboring relation between networked nodes, we use $i \sim h$ to indicate the targets tracked by that node. Next, let $x(t) = \big[x_1(t), x_2(t), \ldots, x_n(t)\big]^{\mathrm{T}} \in \mathbb{R}^n$, $\xi(t) = \big[\xi_1(t), \xi_2(t), \ldots, \xi_n(t)\big]^{\mathrm{T}} \in \mathbb{R}^n$, and $c(t) = \big[c_1(t), c_2(t), \ldots, c_m(t), 0, \ldots, 0\big] \in \mathbb{R}^n$, where we assume $m \leq n$ for the ease of the following notation and without loss of generality. We can now write (1) and (2) in a compact form as

$$\begin{aligned} \dot{x}(t) &= -\mathcal{L}(\mathcal{G})x(t) + \mathcal{L}(\mathcal{G})\xi(t) - K_1 x(t) + K_2 c(t), \quad x(0) = x_0, && (3)\\ \dot{\xi}(t) &= -\mathcal{L}(\mathcal{G})x(t), \quad \xi(0) = \xi_0, && (4) \end{aligned}$$

where $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{n\times n}$,

$$K_1 \triangleq \mathrm{diag}([k_{1,1}, k_{1,2}, \ldots, k_{1,n}]^{\mathrm{T}}) \in \overline{\mathbb{S}}_+^{n\times n}, \tag{5}$$

with $k_{1,i} \in \overline{\mathbb{Z}}_+$ denoting the number of targets tracked by node i, $i = 1, \ldots, n$, and

$$K_2 \triangleq \begin{bmatrix} k_{2,11} & k_{2,12} & \cdots & k_{2,1n} \\ k_{2,21} & k_{2,22} & \cdots & k_{2,2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{2,n1} & k_{2,n2} & \cdots & k_{2,nn} \end{bmatrix} \in \mathbb{R}^{n\times n}, \tag{6}$$

with $k_{2,ih} = 1$ if the target $c_h(t)$, $h = 1, \ldots, m$, is tracked by node i, $i = 1, \ldots, n$, and $k_{2,ih} = 0$ otherwise. Note that $k_{1,i} = \sum_{j=1}^{n} k_{2,ij}$.

Since we are interested in driving the states of all (active and passive) networked nodes to the location of global targets, let

$$\delta(t) \triangleq x(t) - \epsilon(t)\mathbf{1}_n \in \mathbb{R}^n, \tag{7}$$

$$\epsilon(t) \triangleq \frac{\mathbf{1}_n^{\mathrm{T}} K_2 c(t)}{\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n} \in \mathbb{R}, \tag{8}$$

be the error between $x_i(t)$, $i = 1, \ldots, n$, and the location of the global targets $\epsilon(t)$. Based on (8), $\epsilon(t)$ can be equivalently written as

$$\epsilon(t) = \frac{(k_{2,11}c_1(t) + k_{2,12}c_2(t) + \cdots + k_{2,21}c_1(t) + k_{2,22}c_2(t) + \cdots)}{(k_{2,11} + k_{2,12} + \cdots + k_{2,21} + k_{2,21} + \cdots)}, \tag{9}$$

which is the location of the global targets. Furthermore, note for the special case of isolated target that

$$\epsilon(t) = \frac{(c_1(t) + c_2(t) + \cdots + c_m(t))}{m}. \tag{10}$$

For the purposes of practical multitarget tracking, we consider static targets. We let $c_h(t) = c_h$, $h = 1, \ldots, m$, be a static target. Since $c(t) = c$, and hence, $\epsilon(t) = \epsilon$ from (8), it follows from (7) and $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$ that

$$\begin{aligned} \dot{\delta}(t) &= -\mathcal{L}(\mathcal{G})\big[\delta(t) + \epsilon\mathbf{1}_n\big] + \mathcal{L}(\mathcal{G})\xi(t) - K_1\big[\delta(t) + \epsilon\mathbf{1}_n\big] + K_2 c(t) \\ &= -\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})\xi(t) - L_c K_2 c, \end{aligned} \tag{11}$$

where $\mathcal{F}(\mathcal{G}) \triangleq \mathcal{L}(\mathcal{G}) + K_1$ and

$$L_c \triangleq \frac{K_1 \mathbf{1}_n \mathbf{1}_n^{\mathrm{T}}}{\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n} - \mathrm{I}_n. \tag{12}$$

Note that $\mathcal{F}(\mathcal{G}) \in \mathbb{S}_+^{n\times n}$ from [1] and

$$\mathbf{1}_n^{\mathrm{T}} L_c = \mathbf{1}_n^{\mathrm{T}} \left[\frac{K_1 \mathbf{1}_n \mathbf{1}_n^{\mathrm{T}}}{\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n} - \mathrm{I}_n \right] = \frac{\mathbf{1}_n^{\mathrm{T}} K_1 \mathbf{1}_n}{\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n} \mathbf{1}_n^{\mathrm{T}} - \mathbf{1}_n^{\mathrm{T}} = 0, \tag{13}$$

since $(\mathbf{1}_n^{\mathrm{T}} K_1 \mathbf{1}_n)/(\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n) = 1$ from $k_{1,i} = \sum_{j=1}^{n} k_{2,ij}$. Next, letting

$$e(t) \triangleq \xi(t) - \mathcal{L}^{\dagger}(\mathcal{G}) L_c K_2 c, \tag{14}$$

and using (14) in (11) yields (see [1], [2] for details)

$$\begin{aligned} \dot{\delta}(t) &= -\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G})\big[e(t) + \mathcal{L}^{\dagger}(\mathcal{G}) L_c K_2 c\big] - L_c K_2 c \\ &= -\mathcal{F}(\mathcal{G})\delta(t) + \mathcal{L}(\mathcal{G}) e(t). \end{aligned} \tag{15}$$

In addition, differentiating (14) with respect to time yields

$$\dot{e}(t) = -\mathcal{L}(\mathcal{G})\big[\delta(t) + \epsilon \mathbf{1}_n\big] = -\mathcal{L}(\mathcal{G})\delta(t), \tag{16}$$

where $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$ of Lemma 1 in [1] is used. The next theorem shows that the state of all networked nodes $x_i(t)$, $i = 1, \ldots, n$ asymptotically converge to ϵ.

Theorem 3.1 ([1])**.** *Consider the networked multiagent system given by* (1) *and* (2)*, where networked nodes exchange information using local measurements and with $\mathcal{G}$ defining a connected, undirected graph topology. Then, the closed-loop error dynamics defined by* (15) *and* (16) *are Lyapunov stable for all initial conditions and $\delta(t)$ asymptotically vanishes.*

A generalized version of the proposed integral action-based distributed control approach can be given by

$$\dot{x}_i(t) = -\alpha \sum_{i\sim j} \Big(x_i(t) - x_j(t)\Big) + \sum_{i\sim j} \Big(\xi_i(t) - \xi_j(t)\Big) - \alpha \sum_{i\sim h} \Big(x_i(t) - c_h\Big), \quad x_i(0) = x_{i0}, \tag{17}$$

$$\dot{\xi}_i(t) = -\gamma \sum_{i\sim j} \Big(x_i(t) - x_j(t)\Big), \quad \xi_i(0) = \xi_{i0}, \tag{18}$$

where it can be shown that $\delta(t)$ asymptotically vanishes. In addition, if α and γ are chosen such that α^2/γ and $1/\alpha$ are small, then the transient response of the overall network is improved (see [1]).

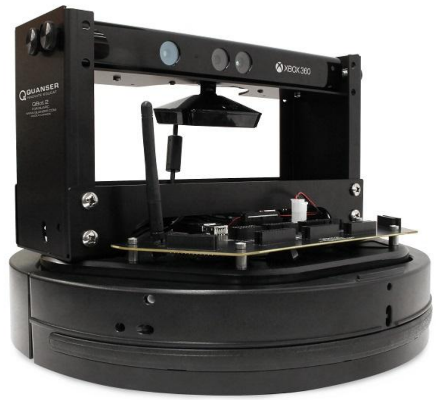
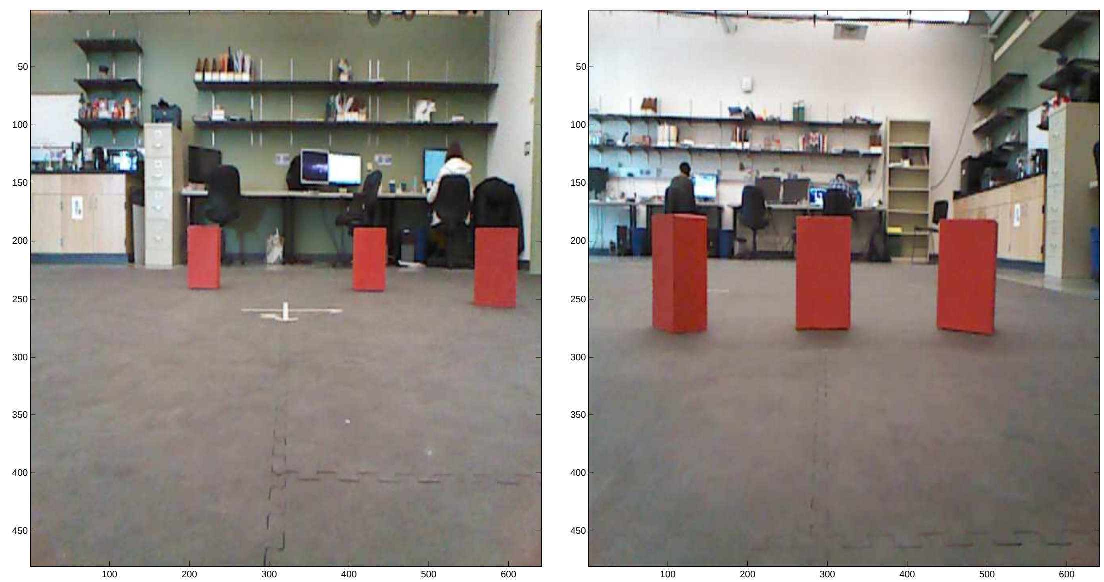


Figure 2. Quanser ground robot.

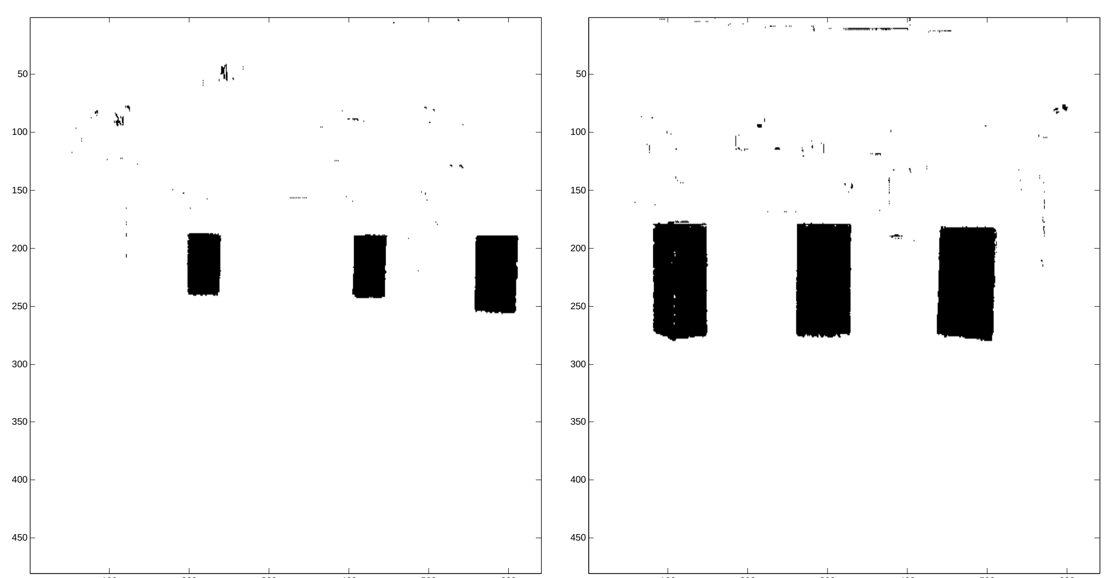
4. APPLICATION TO MULTITARGET TRACKING

In this section, we perform multitarget tracking utilizing the active–passive dynamic consensus filters approach overviewed in **??**. Using only local information exchange, all agents become aware of the global environment event thought they are only aware of their local information. Specifically, a network of two Quanser ground robots (Figure 2) track static targets. A motion capture system makes the ground robots aware of their global location. Through the use of a Microsoft Kinect, networked nodes track the location of red targets as seen in Figure 3 and Figure 4 and calculate their global position (Figure 5). Networked nodes then employ the active–passive dynamic consensus filters information fusion approach of **??**, which allows each node to know the global location of each target. Utilizing the proposed algorithm in (17) and (18), each networked node builds a map of the global environment as seen in Figure 6. Targets close to the robots match almost perfectly. Targets further from the robots do not match perfectly due to non-linearities in sensors that cause unreliable information, which will be accounted for un future work using the Active–Passive Dynamic Consensus Filters approach of [2] that specifically considers the value of sensed information to achieve better multitarget tracking performance.



(a) Environment sensed by ground robot. (b) Environment sensed by ground robot.

Figure 3. Environment sensed by ground robot.

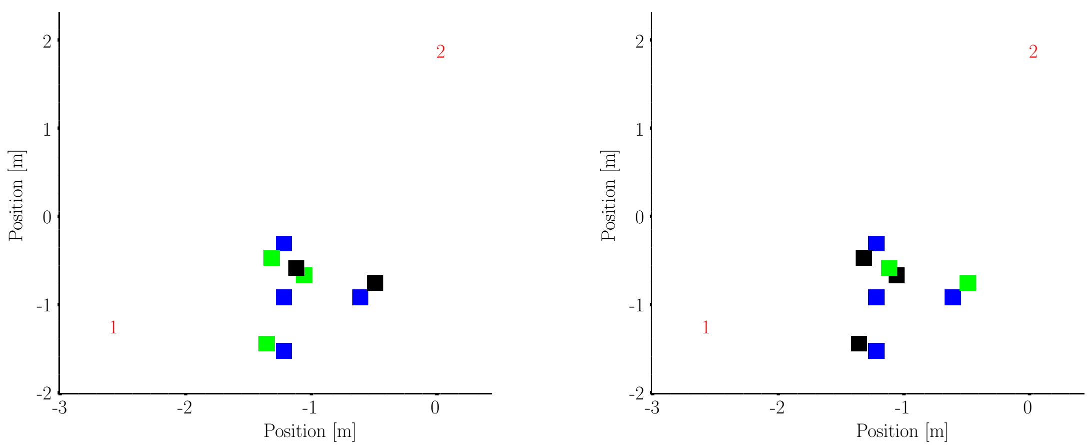


(a) Targets tracked by a ground robot. White squares indicate a target.

(b) Targets tracked by a ground robot. White squares indicate a target.

Figure 4. Targets tracked by ground robots.

Figure 5. Two Quanser ground robots tracking objects in a cluttered environment. Each networked node tracks the targets within its field of view and shares the target's global location with neighboring nodes. Locations are then fused using the active–passive dynamic consensus filters approach.



(a) Global map as constructed by robot 1.

(b) Global map as constructed by robot 2.

Figure 6. Global environment map as reconstructed by each robot. Red numbers denote robots. Blue squares denote true target positions. Green squares denote the position of targets as sensed by this robot. Black squares denote the position of targets sensed by neighboring robots.

5. CONCLUSION

We performed a multitarget tracking experiment using the active–passive dynamic consensus filters approach in [1] and [2]. We constructed a global map of an environment using two ground robots equipped with object tracking sensors. Networked nodes track targets they are able to locally sense, and build a global map using local information exchange. Building on these preliminary experimental studies, future research will involve a more comprehensive target tracking experiment involving more robotic nodes as well as time-varying targets.

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IV. EXPLOITATION OF HETEROGENEITY IN DISTRIBUTED SENSING: AN ACTIVE-PASSIVE NETWORKED MULTIAGENT SYSTEMS APPROACH

John Daniel Peterson, Tansel Yucelen, Suresh Kannan, and Girish Chowdhary

ABSTRACT

Current distributed sensing methods have a lack of insight and formal guarantees to deal with heterogeneity in a dynamic environment. These methods assume that the expected value of sensed information is same for all agents – ignoring differences in sensor capabilities due to, for example, environmental factors and sensors' quality and condition. Motivated from this standpoint, we present a distributed sensing framework, with system-theoretic performance guarantees, to exploit heterogeneity in information provided about a dynamic environment using an active-passive networked multiagent systems approach. Specifically, this approach consists of agents subject to exogenous inputs (active agents) and agents without any inputs (passive agents). In addition, if an active agent senses a quantity accurately (resp., not accurately), then it is weighted high (resp., low) in the network such that these weights can be a function of time due to varying environmental factors. The key feature of our approach is that the states of all agents converge to an adjustable neighborhood of the weighted average of the sensed exogenous inputs by the active agents. Illustrative numerical examples are further provided to demonstrate that utilizing heterogeneity allows the networked multiagent system to achieve better distributed sensing performance.

1. INTRODUCTION

1.1. BACKGROUND

Distributed sensing is performed by a few to several hundred agents, where each agent senses the environment and utilizes peer-to-peer communications to perform information fusion. It has broad applications in robotics, collaborative processing of information, and gathering of scientific data (see, for example, [1]–[12]). Classical distributed sensing methods assume instantaneous communication, which is not practical for situations involving a large number of agents, high-dimensional measurements, and unpredictable low-bandwidth networks [2], [13], [14]. Unlike classical methods, system-theoretical distributed sensing approaches involve equations of motion to describe dynamic behavior of the information fusion process, which allow one to understand overall network behavior, and can also have better robustness to uncertainties (e.g., asynchronous operations, time-varying link availability, and measurement noise) [1], [2], [13], [15]–[18]. A key component of system-theoretical distributed sensing is a consensus algorithm needed for the information fusion process. Among two widely-used classes of consensus algorithms, static and dynamic consensus algorithms, dynamic ones consider agreement upon time-varying quantities and are well-suited for dynamic environment applications.

Existing dynamic consensus algorithms are suitable for applications where each agent is subject to one input measurement [1], [3], [5], [7]–[9]. However, network nodes may differ in the number of input measurements; for example, one agent may not sense a quantity and another may sense multiple quantities for certain time instants. While [19]–[22] present methods that cover applications when a portion of the agents do not perform sensing (they still assume that the rest of the agents are subject to one input measurement only), [19], [20] consider static consensus algorithms (i.e., not suitable for dynamic environments) and [21], [22] consider homogenous sensing capability across nodes.

It is important to point out that sensing capability of each agent, measured by the *value of information*, may not be the same for all agents due to variations in quality and condition of sensors and/or simply distance to the available information, and therefore, some agents can have better sensing power and less sensing error than others. Consequently, heterogeneity in sensing capability needs to be considered to achieve reliable and correct network performance. Even though there exist a few works [10], [18], [21]–[24] that consider the value of information, these results either deal with analysis in the context of static consensus algorithms (e.g., assume that location of nodes and targets remain constant) or consider mission-specific scenarios (e.g., sensor placement, target tracking, or information broadcasting).

To elucidate the above points, consider two distributed sensing scenarios in Figure 1. Considering the load delivery scenario, six mobile ground robots (nodes) collaboratively hold a massive load for a delivery operation guided by two targets. In this case, targets 1 and 2 are only visible to nodes 1, 2, and 3, but the position vectors of all mobile ground robots (including nodes 4, 5, and 6) must agree and direct the centroid of the targets (otherwise the operation may fail). Furthermore, node 2 can sense both targets, and hence, is subject to more than one input. It is also clear that node 2 senses target 2 more accurately than target 1 since it is closer. Likewise, node 1 has a higher sensing value for target 1 than does node 2 for target 1. Considering surveillance, four quadcopters (nodes) sense a portion of an unknown environment and their aim is to construct a global map via local information exchange [25]. In particular, when two or more nodes move closer to one another, they form a network to share their local maps to reach an agreement on the global map. Note that when node 1 and 2 need to exchange their map information, node 1 (resp., node 2) has no sensor information (input) on C3, D3, E3, F3, D4, E4, and F4 (resp., A1, B1, C1, D1, E1, F1, A2, and B2) parts of the environment, and they are both subject to overlapping inputs

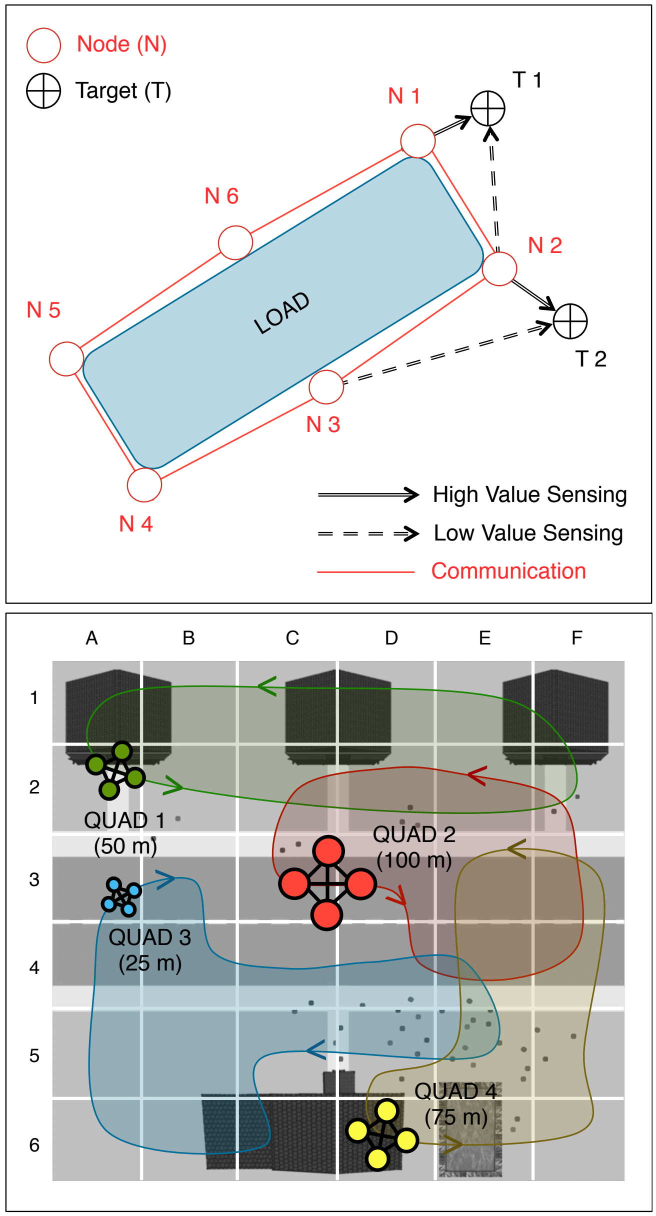

Figure 1. Load delivery (top) and surveillance (bottom) scenarios.

on C2, D2, E2, and F2 parts. In addition, node 1 senses C2, D2, E2, and F2 more accurately than node 2 since it flies closer to the ground than the other node. Similar situations hold when other nodes interact.

1.2. CONTRIBUTION

In this paper, we present a distributed sensing framework, with system-theoretic performance guarantees, to exploit heterogeneity in information provided about a dynamic environment using an active-passive networked multiagent systems approach. This approach is introduced in [26] to remove the assumption that each agent is subject to one input measurement, which is common among the class of dynamic consensus algorithms as discussed in Section I$-A$. In particular, the approach in [26] consists of agents subject to exogenous inputs (active agents) and agents without any inputs (passive agents), where these inputs may or may not overlap within the active agents.

This paper utilizes and generalizes the active-passive networked multiagent systems approach to account for heterogeneity in agents' sensing capability measured by the value of information, where the contribution of this paper versus the results in [26] are pictorially compared in Figure 2. Specifically, if an active agent senses a quantity accurately (resp., not accurately), then it reports a high (resp., low) value of information, and hence, it is weighted high (resp., low) in the network as compared to other agents. In addition, these weights can be a function of time due to varying environmental factors and/or changes in sensors' quality and condition. The key feature of our approach is that the states of all agents converge to an adjustable neighborhood of the weighted average of the sensed exogenous inputs by the active agents with heterogeneous sensing capabilities.

The organization of the paper is as follows. Section II states the notation used throughout the paper, recalls some of the basic notions from graph theory, and introduces several necessary lemmas. Section III overviews the active-passive networked multiagent systems approach introduced in [26]. Section IV presents the main contribution of this paper to exploit heterogeneity in distributed sensing. Illustrative numerical examples are provided in Section V to demonstrate that utilizing heterogeneity allows the networked multiagent system to achieve better distributed sensing performance. Finally, conclusions are summarized in Section VI.

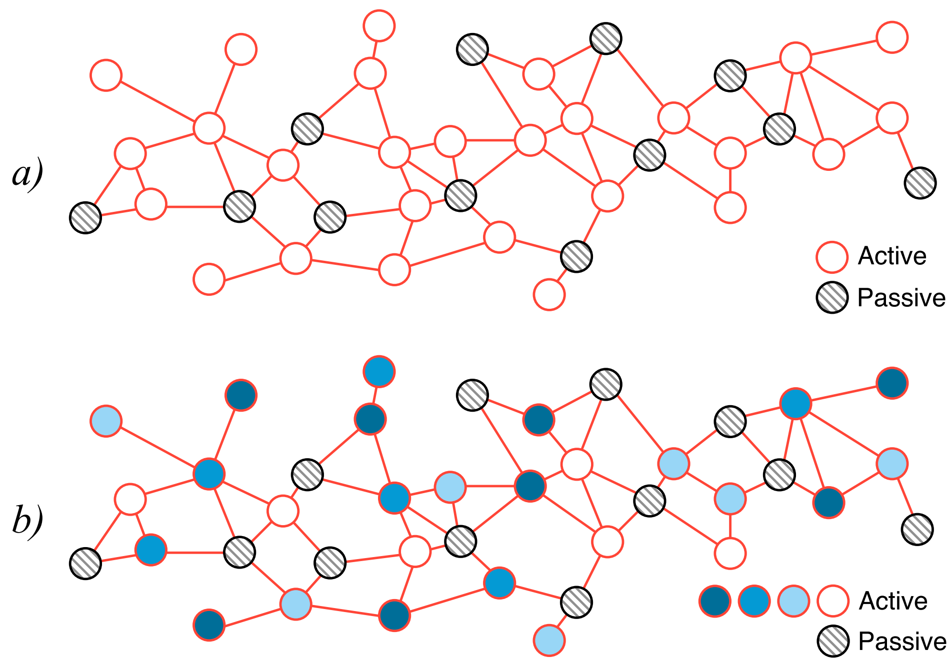

Figure 2. A networked multiagent system represented on a graph. Figure *a)* shows active and passive agents, where all active agents have the same value of sensed information, and hence, are weighted identically. Figure *b)* shows a more realistic situation, where active agents have heterogeneous value of sensed information, and hence, their weights differ due to varying environmental factors and/or changes in sensors' quality and condition.

2. MATHEMATICAL PRELIMINARIES

2.1. NOTATION

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbb{R}_+$ denotes the set of positive real numbers, $\mathbb{R}_+^{n\times n}$ (resp., $\overline{\mathbb{R}}_+^{n\times n}$) denotes the set of $n \times n$ positive-definite (resp., nonnegative-definite) real matrices, $\mathbb{S}_+^{n\times n}$ (resp., $\overline{\mathbb{S}}_+^{n\times n}$) denotes the set of $n \times n$ symmetric positive-definite (resp., symmetric nonnegative-definite) real matrices, $\mathbb{Z}$ denotes the set of integers, $\mathbb{Z}_+$ (resp., $\overline{\mathbb{Z}}_+$) denotes the set of positive (resp., nonnegative) integers, $\mathbf{0}_n$ denotes the $n \times 1$ vector of all zeros, $\mathbf{1}_n$ denotes the $n \times 1$ vector of all ones, $\mathbf{0}_{n\times n}$ denotes the $n \times n$ zero matrix, and I_n denotes the $n \times n$ identity matrix.

Furthermore, we write $(\cdot)^{\mathrm{T}}$ for transpose, $(\cdot)^{-1}$ for inverse, $(\cdot)^{\dagger}$ for generalized inverse, $\|\cdot\|_2$ for the Euclidian norm, $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix A, $\lambda_i(A)$ for the i-th eigenvalue of A (A is symmetric and the eigenvalues are ordered from least to greatest value), $\mathrm{diag}(a)$ for the diagonal matrix with the vector a on its diagonal, and $[A]_{ij}$ for the entry of the matrix A on the i-th row and j-th column.

2.2. GRAPH THEORY

Next, we recall some of the basic notions from graph theory, where we refer to [14], [27] for further details. In the multiagent literature, graphs are broadly adopted to encode interactions in networked systems. An *undirected* graph $\mathcal{G}$ is defined by a set $\mathcal{V}_{\mathcal{G}} = \{1, \ldots, n\}$ of *nodes* and a set $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$ of *edges*. If $(i, j) \in \mathcal{E}_{\mathcal{G}}$, then the nodes i and j are *neighbors* and the neighboring relation is indicated with $i \sim j$. The *degree* of a node is given by the number of its neighbors. Letting d_i be the degree of node i, then the *degree* matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by $\mathcal{D}(\mathcal{G}) \triangleq \mathrm{diag}(d)$, $d = [d_1, \ldots, d_n]^{\mathrm{T}}$. A *path* $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1, \ldots, L$, and a graph $\mathcal{G}$ is *connected* if there is a path between any pair of distinct nodes. The *adjacency* matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by

$$[\mathcal{A}(\mathcal{G})]_{ij} \triangleq \begin{cases} 1, & \text{if } (i,j) \in \mathcal{E}_{\mathcal{G}}, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The *Laplacian* matrix of a graph, $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_{+}^{n\times n}$, playing a central role in many graph theoretic treatments of multiagent systems, is given by $\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G})$. Throughout this paper, we model a given multiagent system by a connected, undirected graph $\mathcal{G}$, where nodes and edges represent agents and inter-agent communication links, respectively.

2.3. NECESSARY LEMMAS

We now introduce several necessary lemmas used in the main results of this paper.

Lemma 1 ([14])**.** The spectrum of the Laplacian of a connected, undirected graph can be ordered as

$$0 = \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \leq \cdots \leq \lambda_n(\mathcal{L}(\mathcal{G})), \tag{2}$$

with $\mathbf{1}_n$ as the eigenvector corresponding to the zero eigenvalue $\lambda_1(\mathcal{L}(\mathcal{G}))$ and $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$ and $\mathrm{e}^{\mathcal{L}(\mathcal{G})\mathbf{1}_n} = \mathbf{1}_n$.

Lemma 2 ([28])**.** The Laplacian of a connected, undirected graph satisfies $\mathcal{L}(\mathcal{G})\mathcal{L}^{\dagger}(\mathcal{G}) = \mathrm{I}_n - \frac{1}{n}\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}$.

Lemma 3. Let $K = \mathrm{diag}(k)$, $k = [k_1, k_2, \ldots, k_n]^{\mathrm{T}}$, $k_i \in \overline{\mathbb{R}}_+$, $i = 1, \ldots, n$, and assume that at least one element of k is nonzero. Then, for the Laplacian of a connected, undirected graph,

$$\mathcal{F}(\mathcal{G}) \triangleq \mathcal{L}(\mathcal{G}) + K \in \mathbb{S}_+^{n\times n}, \tag{3}$$

and $\det(\mathcal{F}(\mathcal{G})) \neq 0$.

Proof. Consider the decomposition $K = K_1 + K_2$, where $K_1 \triangleq \mathrm{diag}([0, \ldots, 0, \phi_i, 0, \ldots, 0]^{\mathrm{T}})$ and $K_2 \triangleq K - K_1$, where ϕ_i denotes the smallest nonzero diagonal element of K appearing on its i-th diagonal, so that $K_2 \in \overline{\mathbb{S}}_+^{n\times n}$. From the Rayleigh's Quotient [29], the minimum eigenvalue of $\mathcal{L}(\mathcal{G}) + K_1$ can be given by

$$\lambda_{\min}(\mathcal{L}(\mathcal{G}) + K_1) = \min_x\{x^{\mathrm{T}}\big(\mathcal{L}(\mathcal{G}) + K_1\big)\, x \mid x^{\mathrm{T}}x = 1\}, \tag{4}$$

where x is the eigenvector corresponding to this minimum eigenvalue. Note that since $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{n\times n}$ and $K_1 \in \overline{\mathbb{S}}_+^{n\times n}$, and hence, $\mathcal{L}(\mathcal{G}) + K_1$ is real and symmetric, x is a real eigenvector. Now, expanding (4) as

$$x^{\mathrm{T}}\left(\mathcal{L}(\mathcal{G}) + K_1\right) x = \sum_{i\sim j} a_{ij}(x_i - x_j)^2 + \phi_i x_i^2, \tag{5}$$

and noting that the right hand side of (5) is zero only if $x \equiv 0$, it follows that $\lambda_{\min}(\mathcal{L}(\mathcal{G}) + K_1) > 0$, and hence, $\mathcal{L}(\mathcal{G}) + K_1 \in \mathbb{S}_+^{n\times n}$. Finally, let λ be an eigenvalue of $\mathcal{F}(\mathcal{G}) = \mathcal{L}(\mathcal{G}) + K_1 + K_2$. Since $\lambda_{\min}(\mathcal{L}(\mathcal{G}) + K_1) > 0$ and $\lambda_{\min}(K_2) = 0$, it follows from Fact 5.11.3 of [30] that $\lambda_{\min}(\mathcal{L}(\mathcal{G}) + K_1) + \lambda_{\min}(K_2) \leq \lambda$, and hence, $\lambda > 0$, which implies that (3) holds and $\det(\mathcal{F}(\mathcal{G})) \neq 0$. ■

3. OVERVIEW OF ACTIVE-PASSIVE NETWORKED MULTIAGENT SYSTEMS

In this section, we briefly overview the active-passive networked multiagent systems approach of [26]. In particular, consider a system of n agents exchanging information among each other using their local measurements according to a connected, undirected graph $\mathcal{G}$. In addition, consider that there exists $m \geq 1$ exogenous inputs that interact with this system.

Definition 1. If agent i, $i = 1, \ldots, n$, is subject to one or more exogenous inputs (resp., no exogenous inputs), then it is an active agent (resp., passive agent).

Definition 2. If an exogenous input interacts with only one agent (resp., multiple agents), then it is an isolated input (resp., non-isolated input).

The approach presented in [26] deals with the problem of driving the states of all (active and passive) agents to the average of the applied exogenous inputs. For this purpose, the following integral action-based distributed sensing algorithm is proposed

$$\dot{x}_i(t) = -\alpha \sum_{i\sim j} \big(x_i(t) - x_j(t)\big) + \sum_{i\sim j} \big(\xi_i(t) - \xi_j(t)\big) - \alpha \sum_{i\sim h} (x_i(t) - c_h(t)),\ x_i(0) = x_{i0}, \quad (6)$$

$$\dot{\xi}_i(t) = -\gamma \sum_{i\sim j} \big(x_i(t) - x_j(t)\big),\ \xi_i(0) = \xi_{i0}, \quad (7)$$

where $x_i(t) \in \mathbb{R}$ and $\xi_i(t) \in \mathbb{R}$ denote the state and the integral action of agent i, $i = 1, \ldots, n$, respectively, $c_h(t) \in \mathbb{R}$, $h = 1, \ldots, m$, denotes an exogenous input sensed by this agent, $\alpha \in \mathbb{R}_+$, and $\gamma \in \mathbb{R}_+$. Note that $i \sim h$ notation indicates the exogenous inputs that an agent is subject to, which is similar to the $i \sim j$ notation indicating the neighboring relation between agents.

Remark 3. Theorem 4.1 of [26] shows that the states of all agents converge to the average of the exogenous inputs applied to active agents under the assumption that all active agents have the same value of sensed information, and hence, are weighted identically.

Remark 4. The proposed integral action-based distributed algorithm in (6) is applied to agents having dynamics of the form $\dot{x}_i(t) = u_i(t)$, where $u_i(t) \in \mathbb{R}$ denotes the input of agent i, $i = 1, \ldots, n$, satisfying the right hand side of (6) along with (7). For agents having complex dynamics, one can design low-level feedback controllers (or assume their existence) for suppressing existing dynamics and enforcing $\dot{x}_i(t) = u_i(t)$ (see, for example, Example 6.3 of [31]).

4. EXPLOITATION OF HETEROGENEITY

The value of sensed information is not necessarily identical for all active agents due to environmental factors and/or sensors' quality and condition, as previously discussed. To that end, this section utilizes and generalizes the active-passive networked multiagent systems to account for heterogeneity in active agents' sensing capability.

4.1. PROBLEM SETUP

We begin with proposing the following integral action-based distributed sensing algorithm

$$\dot{x}_i(t) = -\alpha \sum_{i\sim j} \big(x_i(t) - x_j(t)\big) + \sum_{i\sim j} \big(\xi_i(t) - \xi_j(t)\big) - \alpha \sum_{i\sim h} w_{ih}(t)\,(x_i(t) - c_h(t)),\ x_i(0) = x_{i0}, \tag{8}$$

$$\dot{\xi}_i(t) = -\gamma \left[\sum_{i\sim j} \big(x_i(t) - x_j(t)\big) + \sigma \xi_i(t) \right],\ \ \xi_i(0) = \xi_{i0}, \tag{9}$$

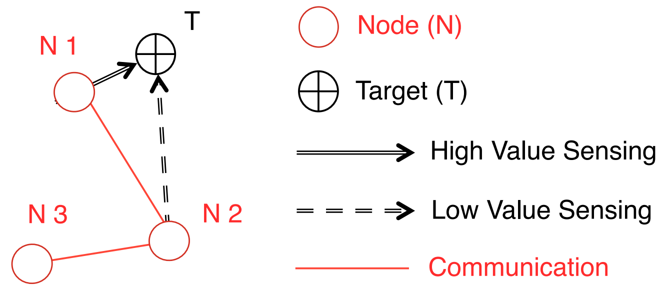

Figure 3. Active-passive networked multiagent system with one target and three agents with nodes 1 and 2 being active and node 3 being passive.

where $x_i(t) \in \mathbb{R}$ and $\xi_i(t) \in \mathbb{R}$ denote the state and the integral action of agent i, $i = 1, \ldots, n$, respectively, $c_h(t) \in \mathbb{R}$, $h = 1, \ldots, m$, denotes an exogenous input sensed by this agent, $\alpha \in \mathbb{R}_+$, $\gamma \in \mathbb{R}_+$, and $\sigma \in \mathbb{R}_+$. Note that $w_{ih}(t) \in \mathbb{R}_+$ is a weight capturing the expected value of information of the exogenous input with respect to agent i.

Remark 5. To elucidate (8) and (9), consider the active-passive networked multiagent system given in Figure 3 with $\alpha = 1$, $\gamma = 1$, and $\sigma = 1$. Let q_{T} be the actual target quantity sensed by active nodes 1 and 2. In addition, for the ease of exposition, let the value of information and the exogenous inputs be time-invariant, the location of nodes be fixed at point p_{x_i}, and the target be stationary at point p_{T}. Assume that the accuracy of the sensed target quantity decreases (resp., increases) as the distance between a node and the target gets larger (resp., smaller). In this scenario, it follows from (8) and (9) that

$$\dot{x}_1(t) = -\big(x_1(t) - x_2(t)\big) + \big(\xi_1(t) - \xi_2(t)\big) - w_{11}(x_1(t) - c_1), \tag{10}$$

$$\dot{\xi}_1(t) = -\big(x_1(t) - x_2(t)\big) - \xi_1(t), \tag{11}$$

$$\begin{aligned}\dot{x}_2(t) = &-\big(x_2(t) - x_1(t)\big) - \big(x_2(t) - x_3(t)\big)\\ &+\big(\xi_2(t) - \xi_1(t)\big) + \big(\xi_2(t) - \xi_3(t)\big) - w_{22}(x_2(t) - c_2),\end{aligned} \tag{12}$$

$$\dot{\xi}_2(t) = -\big(x_2(t) - x_1(t)\big) - \big(x_2(t) - x_3(t)\big) - \xi_2(t), \tag{13}$$

$$\dot{x}_3(t) = -\big(x_3(t) - x_2(t)\big) + \big(\xi_3(t) - \xi_2(t)\big), \tag{14}$$

$$\dot{\xi}_3(t) = -\big(x_3(t) - x_2(t)\big) - \xi_3(t), \tag{15}$$

where $c_1 = f(q_{\mathrm{T}}, ||p_{x_1} - p_{\mathrm{T}}||_2)$ represents a higher value sensing than $c_2 = f(q_{\mathrm{T}}, ||p_{x_2} - p_{\mathrm{T}}||_2)$, since $||p_{x_1} - p_{\mathrm{T}}||_2 \leq ||p_{x_2} - p_{\mathrm{T}}||_2$. Therefore, $w_{11} > w_{22}$ for this example.

Remark 6. The focus of this paper is to develop a distributed sensing framework, with system-theoretic performance guarantees, to exploit heterogeneity in sensed information. For this reason, we implicitly assume that the value of sensed information is already modeled and known by respective agents in the network. Note that each sensor can obtain the value of information through analysis of its own measurement using relative entropy measures such as Kullback-Liebler divergence [22]. In addition, considering the scenario highlighted in Remark 3, the Fisher information metrics [10], [23] can be used to model the value of information in active nodes 1 and 2 as a function of node-target distance.

Next, let

$$x(t) = \big[x_1(t), x_2(t), \ldots, x_n(t)\big]^{\mathrm{T}} \in \mathbb{R}^n, \tag{16}$$

$$\xi(t) = \big[\xi_1(t), \xi_2(t), \ldots, \xi_n(t)\big]^{\mathrm{T}} \in \mathbb{R}^n, \tag{17}$$

$$c(t) = \big[c_1(t), c_2(t), \ldots, c_m(t), 0, \ldots, 0\big] \in \mathbb{R}^n, \tag{18}$$

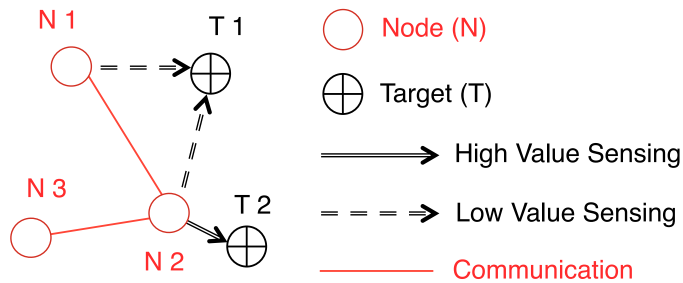

Figure 4. Active-passive networked multiagent system with two targets and three agents with nodes 1 and 2 being active and node 3 being passive.

where $m \leq n$ is assumed to ease notation without loss of generality. We can now rewrite (8) and (9) in the compact form given by

$$\dot{x}(t) = -\alpha\mathcal{L}(\mathcal{G})x(t) + \mathcal{L}(\mathcal{G})\xi(t) - \alpha K_1(t)x(t) + \alpha K_2(t)c(t), \quad x(0) = x_0, \tag{19}$$

$$\dot{\xi}(t) = -\gamma\mathcal{L}(\mathcal{G})x(t) - \gamma\sigma\xi(t), \quad \xi(0) = \xi_0, \tag{20}$$

where $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{n\times n}$ satisfies Lemma 1,

$$K_1(t) \triangleq \mathrm{diag}([k_{1,1}(t), \ldots, k_{1,n}(t)]^{\mathrm{T}}) \in \overline{\mathbb{S}}_+^{n\times n}, \tag{21}$$

with $k_{1,i} \in \overline{\mathbb{R}}_+$ denoting the number of the exogenous inputs applied to agent i, $i = 1, \ldots, n$, and

$$K_2(t) \triangleq \begin{bmatrix} k_{2,11}(t) & \cdots & k_{2,1n}(t) \\ k_{2,21}(t) & \cdots & k_{2,2n}(t) \\ \vdots & \ddots & \vdots \\ k_{2,n1}(t) & \cdots & k_{2,nn}(t) \end{bmatrix} \in \mathbb{R}^{n\times n}, \tag{22}$$

with

$$k_{1,i}(t) = \sum_{j=1}^{n} k_{2,ij}(t). \tag{23}$$

Remark 7. In (19) and (20), the elements of $K_1(t)$ and $K_2(t)$ are related to the weights $w_{ih}(t) \in \mathbb{R}_+$ capturing the expected value of information. To elucidate this point, consider the active-passive networked multiagent system given in Figure 4. Let $c = [c_1, c_2, 0]^{\mathrm{T}}$, where $c_1 = f(q_{\mathrm{T}_1}, ||p_{x_1} - p_{\mathrm{T}_1}||_2) = f(q_{\mathrm{T}_1}, ||p_{x_2} - p_{\mathrm{T}_1}||_2)$ (assuming $||p_{x_1} - p_{\mathrm{T}_1}||_2 = ||p_{x_2} - p_{\mathrm{T}_1}||_2$)

and $c_2 = f(q_{\mathrm{T}_2}, ||p_{x_2} - p_{\mathrm{T}_2}||_2)$, with q_{T_i}, $i = 1, 2$, representing the actual target quantities and p_{x_i}, $i = 1, 2, 3$, representing the location of nodes. In this case, if $w_{11}(t)$, $w_{21}(t)$, and $w_{22}(t)$ respectively denote the value of information of input c_1 with respect to agent 1, input c_1 with respect to agent 2, and input c_2 with respect to agent 2 (note that $w_{11}(t) = w_{21}(t)$), then one can write

$$K_1(t) = \begin{bmatrix} w_{11}(t) & 0 & 0 \\ 0 & w_{21}(t) + w_{22}(t) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{24}$$

$$K_2(t) = \begin{bmatrix} w_{11}(t) & 0 & 0 \\ w_{21}(t) & w_{22}(t) & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{25}$$

As an another example, consider the case $||p_{x_1} - p_{\mathrm{T}_1}||_2 \neq ||p_{x_2} - p_{\mathrm{T}_1}||_2$ and let $c = [c_1, c_2, c_3]^{\mathrm{T}}$, where $c_1 = f(q_{\mathrm{T}_1}, ||p_{x_1} - p_{\mathrm{T}_1}||_2)$, $c_2 = f(q_{\mathrm{T}_1}, ||p_{x_2} - p_{\mathrm{T}_1}||_2)$, and $c_3 = f(q_{\mathrm{T}_2}, ||p_{x_2} - p_{\mathrm{T}_2}||_2)$. In this case, if $w_{11}(t)$, $w_{22}(t)$, and $w_{23}(t)$ respectively denote the value of information of input c_1 with respect to agent 1, input c_2 with respect to agent 2, and input c_3 with respect to agent 2, then one can write

$$K_1(t) = \begin{bmatrix} w_{11}(t) & 0 & 0 \\ 0 & w_{22}(t) + w_{23}(t) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{26}$$

$$K_2(t) = \begin{bmatrix} w_{11}(t) & 0 & 0 \\ 0 & w_{22}(t) & w_{23}(t) \\ 0 & 0 & 0 \end{bmatrix}. \tag{27}$$

Without loss of generality, assume that $k_{2,ij} \in [0, 1]$, which corresponds to the fact that each value of information can take values from the scaled interval $[0, 1]$ (i.e., as agents' value of information increase about exogenous inputs, $k_{2,ij}$ increases from 0 toward 1). Since we are interested in driving the states of all (active and passive) agents to an adjustable neighborhood of the weighted average of the exogenous inputs applied to the active agents, let

$$\delta(t) \triangleq x(t) - \epsilon(t)\mathbf{1}_n \in \mathbb{R}^n, \tag{28}$$

$$\epsilon(t) \triangleq \frac{\mathbf{1}_n^T K_2(t)c(t)}{\mathbf{1}_n^T K_2(t)\mathbf{1}_n} \in \mathbb{R}, \tag{29}$$

be the error between $x_i(t), i = 1, \ldots, n$, and the weighted average of the applied exogenous inputs $\epsilon(t)$. Based on (29), $\epsilon(t)$ can be equivalently written as

$$\epsilon(t) = \frac{k_{2,11}(t)c_1(t) + k_{2,12}(t)c_2(t) + \cdots + k_{2,21}(t)c_1(t) + k_{2,22}(t)c_2(t) + \cdots}{\big(k_{2,11}(t) + k_{2,12}(t) + \cdots + k_{2,21}(t) + k_{2,22}(t) + \cdots}. \tag{30}$$

Note that the denominator of (30) is nonzero, since we assume that there exists $m \geq 1$ exogenous inputs, and hence, there exists at least one nonzero value of information weight. Furthermore, let this nonzero weight be lower bounded by $\phi_i \in \mathbb{R}_+$ and consider the decomposition

$$K_1(t) = K_0 + \tilde{K}(t), \tag{31}$$

where $K_0 \triangleq \mathrm{diag}([0, \ldots, 0, \phi_i, 0, \ldots, 0]^{\mathrm{T}})$ and $\tilde{K}(t) \triangleq K_1(t) - K_0$ such that $\tilde{K} \in \overline{\mathbb{S}}_+^{n \times n}$. Note that for situations where there exists more than one nonzero value of information weights, then we can either include the lower bounds of these nonzero weights to K_0 or we simply let ϕ_i to represent the smallest lower bound of these nonzero weights, and hence, we can perform this decomposition. This concludes the setup of our problem. Next, we present the stability and performance guarantees of the distributed sensing algorithm given by (8) and (9).

4.2. STABILITY AND PERFORMANCE GUARANTEES

Consider the error between $x_i(t), i = 1, \ldots, n$, and the weighted average of the sensed exogenous inputs $\epsilon(t)$ given by (28). Using Lemma 1, the time derivative of (28) can be given by

$$\dot{\delta}(t) = -\alpha \tilde{F}(\mathcal{G}, t)\delta(t) + \mathcal{L}(\mathcal{G})\xi(t) - \alpha L_c(t)K_2(t)c(t) - \dot{\epsilon}(t)\mathbf{1}_n, \quad \delta(0) = \delta_0, \tag{32}$$

where

$$\tilde{F}(\mathcal{G},t) \triangleq \mathcal{L}(\mathcal{G}) + K_1(t), \tag{33}$$

$$L_c(t) \triangleq \frac{K_1(t)\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}}{\mathbf{1}_n^{\mathrm{T}}K_2(t)\mathbf{1}_n} - \mathrm{I}_n. \tag{34}$$

Now consider

$$e(t) \triangleq \xi(t) - \alpha\mathcal{L}^{\dagger}(\mathcal{G})L_c(t)K_2(t)c(t). \tag{35}$$

Using Lemma 2 and noting that $\mathbf{1}_n^{\mathrm{T}}L_c(t) = \mathbf{1}_n^{\mathrm{T}}\left[K_1(t)\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}/(\mathbf{1}_n^{\mathrm{T}}K_2(t)\mathbf{1}_n) - \mathrm{I}_n\right] = 0$, (32) can be rewritten by

$$\dot{\delta}(t) = -\alpha\tilde{F}(\mathcal{G},t)\delta(t) + \mathcal{L}(\mathcal{G})e(t) - \dot{\epsilon}(t)\mathbf{1}_n. \tag{36}$$

Furthermore, applying the decomposition (31) to (36), it follows that

$$\dot{\delta}(t) = -\alpha F(\mathcal{G})\delta(t) - \alpha\tilde{K}(t)\delta(t) + \mathcal{L}(\mathcal{G})e(t) - \dot{\epsilon}(t)\mathbf{1}_n, \tag{37}$$

where $F(\mathcal{G}) \triangleq \mathcal{L}(\mathcal{G}) + K_0 \in \mathbb{S}_+^{n\times n}$ is a direct consequence of Lemma 3. Finally, the time derivative of (35) can be given by

$$\begin{aligned}\dot{e}(t) = & -\gamma\mathcal{L}(\mathcal{G})\delta(t) - \gamma\sigma e(t) - \alpha\gamma\sigma\mathcal{L}^{\dagger}(\mathcal{G})K_c(t)c(t) - \alpha\mathcal{L}^{\dagger}(\mathcal{G})\dot{K}_c(t)c(t) \\ & - \alpha\mathcal{L}^{\dagger}(\mathcal{G})K_c(t)\dot{c}(t),\ e(0) = e_0, \end{aligned} \tag{38}$$

where $K_c(t) \triangleq L_c(t)K_2(t)$.

Next, the closed-loop error dynamics given by (37) and (38) can be rewritten as

$$\dot{\delta}(t) = -\alpha F(\mathcal{G})\delta(t) - \alpha\tilde{K}(t)\delta(t) + \mathcal{L}(\mathcal{G})e(t) + s_1(t), \tag{39}$$

$$\dot{e}(t) = -\gamma\mathcal{L}(\mathcal{G})\delta(t) - \gamma\sigma e(t) + s_2(t), \tag{40}$$

where the perturbation terms are given by $s_1(t) \triangleq -\dot{\epsilon}(t)\mathbf{1}_n$ and $s_2(t) \triangleq -\alpha\gamma\sigma\mathcal{L}^{\dagger}(\mathcal{G})K_c(t)c(t) - \alpha\mathcal{L}^{\dagger}(\mathcal{G})\dot{K}_c(t)c(t) - \alpha\mathcal{L}^{\dagger}(\mathcal{G})K_c(t)\dot{c}(t)$. For the following results, we assume $||s_1(t)||_2 \leq s_1^*$ and $||s_2(t)||_2 \leq s_2^*$.

Theorem 4.1. *Consider the networked multiagent system given by* (8) *and* (9)*, where agents exchange information using local measurements through a connected and undirected graph topology. Then, the closed-loop error dynamics given by* (39) *and* (40) *are bounded.*

Proof. Consider the Lyapunov function candidate given by

$$V(\delta, e) = \frac{1}{2\alpha}\delta^{\mathrm{T}}\delta + \frac{1}{2\alpha\gamma}e^{\mathrm{T}}e, \tag{41}$$

and note that $V(0,0) = 0$ and $V(\delta, e) > 0$ for all $(\delta, e) \neq (0,0)$. Time derivative of (41) along the trajectories of (39) and (40) yields

$$\begin{aligned}\dot{V}(\cdot) = {} & \frac{1}{\alpha}\delta^{\mathrm{T}}(t)\Big[-\alpha F(\mathcal{G})\delta(t) - \alpha\tilde{K}(t)\delta(t) + \mathcal{L}(\mathcal{G})e(t) + s_1(t)\Big],\\ & + \frac{1}{\alpha\gamma}e^{\mathrm{T}}(t)\Big[-\gamma\mathcal{L}(\mathcal{G})\delta(t) - \gamma\sigma e(t) + s_2(t)\Big],\\ = {} & -\delta^{\mathrm{T}}(t)F(\mathcal{G})\delta(t) - \delta^{\mathrm{T}}(t)\tilde{K}(t)\delta(t)\alpha^{-1}\sigma e^{\mathrm{T}}(t)e(t) + \alpha^{-1}\delta^{\mathrm{T}}(t)s_1(t) + \alpha^{-1}\gamma^{-1}e^{\mathrm{T}}(t)s_2(t),\end{aligned}$$

which implies

$$\begin{aligned}\dot{V}(\cdot) \leq {} & -\delta^{\mathrm{T}}(t)F(\mathcal{G})\delta(t) - \alpha^{-1}\sigma e^{\mathrm{T}}(t)e(t) + \alpha^{-1}\delta^{\mathrm{T}}(t)s_1(t) + \alpha^{-1}\gamma^{-1}e^{\mathrm{T}}(t)s_2(t)\\ \leq {} & -d_1\|\delta(t)\|_2\Big(\|\delta(t)\|_2 - \frac{d_3}{d_1}\Big) - d_2\|e(t)\|_2\Big(\|e(t)\|_2 - \frac{d_4}{d_2}\Big),\end{aligned} \tag{42}$$

where $d_1 \triangleq \lambda_{\min}(\mathcal{F}(\mathcal{G}))$, $d_2 \triangleq \sigma\alpha^{-1}$, $d_3 \triangleq \alpha^{-1}s_1^*$, and $d_4 \triangleq \alpha^{-1}\gamma^{-1}s_2^*$. Since $\dot{V}(\delta(t), e(t)) \leq 0$ when $\|\delta(t)\|_2 \geq d_3/d_1$ and $\|e(t)\|_2 \geq d_4/d_2$, it follows that the closed-loop error dynamics given by (39) and (40) are bounded. ■

In the next theorem, we determine the bound of $\delta(t)$ for $t \geq T$ characterizing the ultimate distance between $x(t)$ and $\epsilon(t)\mathbf{1}_n$, which is of practical importance for distributed sensing applications.

Theorem 4.2. *Consider the networked multiagent system given by (8) and (9), where agents exchange information using local measurements through a connected and undirected graph topology. Then, the bound of $\delta(t)$ for $t \geq T$ is*

$$\|\delta(t)\|_2^2 \leq \frac{1}{\alpha^2}\left[\frac{n^2\dot{\epsilon}^{*2}}{\lambda_{\min}^2(\mathcal{F}(\mathcal{G}))}\right] + \frac{\alpha^2}{\gamma}\left[p_1^{*2} + \frac{2p_1^*p_2^*}{\gamma\sigma} + \frac{p_2^{*2}}{\gamma^2\sigma^2}\right], \tag{43}$$

where $||\dot{\epsilon}(t)||_2 \leq \dot{\epsilon}^*$, $||\mathcal{L}^{\dagger}(\mathcal{G})K_c(t)c(t)||_2 \leq p_1^*$, and $||\mathcal{L}^{\dagger}(\mathcal{G})\dot{K}_c(t)c(t) + \mathcal{L}^{\dagger}(\mathcal{G})K_c(t)\dot{c}(t)||_2 \leq p_2^*$.

Proof. In the proof of Theorem 4.1, we showed that $\dot{V}(\delta(t), e(t)) \leq 0$ when $\|\delta(t)\|_2 \geq d_3/d_1$ and $\|e(t)\|_2 \geq d_4/d_2$. Note that this implies $\dot{V}(\delta(t), e(t)) \leq 0$ outside the compact set $\mathcal{S} \triangleq \{(\delta(t), e(t)) : \|\delta(t)\|_2 \leq \frac{d_3}{d_1}\} \bigcap \{(\delta(t), e(t)) : \|e(t)\|_2 \leq \frac{d_4}{d_2}\}$. Since $V(\delta(t), e(t))$ cannot grow outside $\mathcal{S}$, the evolution of $V(\delta(t), e(t))$ is upper bounded by $V(\delta(t), e(t)) \leq \max_{(\delta(t),e(t))\in\mathcal{S}} V(\delta(t), e(t)) = \frac{1}{2\alpha}\frac{d_3^2}{d_1^2} + \frac{1}{2\alpha\gamma}\frac{d_4^2}{d_2^2}$, $t \geq T$, where using $\frac{1}{2\alpha}\delta^{\mathrm{T}}(t)\delta(t) \leq V(\delta(t), e(t))$ in this expression, (43) follows. ■

Remark 8. Theorem 4.2 implies that if we choose α and γ such that both $1/\alpha^2$ and α^2/γ are small, then (43) is small for $t \geq T$.

4.3. SPECIAL CASE COROLLARIES

It is shown in Theorem 4.1 and Theorem 4.2 that the states of all agents can be driven to an adjustable neighborhood of the weighted average of the sensed exogenous inputs by the active agents for the case of time-varying sensed exogenous inputs, which is due to a dynamic environment, with time-varying value of information. In this section, several corollaries are stated that present special cases of Theorem 4.1 and Theorem 4.2. We begin with the case of time-varying sensed exogenous inputs and constant value of information.

Corollary 3. Consider the networked multiagent system given by (8) and (9), where agents exchange information using local measurements through a connected and undirected graph topology. Let the value of information be constant. Then, the closed-loop error dynamics given by (39) and (40) are bounded and the bound of $\delta(t)$ for $t \geq T$ is given by (43) with $\dot{\epsilon}^* = \dot{c}^*||\mathbf{1}_n^{\mathrm{T}}K_2||_2/(\mathbf{1}_n^{\mathrm{T}}K_2\mathbf{1}_n)$, $p_1^* = c^*||\mathcal{L}^{\dagger}(\mathcal{G})K_c||_{\mathrm{F}}$, and $p_2^* = \dot{c}^*||\mathcal{L}^{\dagger}(\mathcal{G})K_c||_{\mathrm{F}}$, where $||c(t)||_2 \leq c^*$ and $||\dot{c}(t)||_2 \leq \dot{c}^*$.

Proof. See the proof of Theorem 4.1 and Theorem 4.2. ■

The next corollary presents the case of constant sensed exogenous inputs and time-varying value of information.

Corollary 4. Consider the networked multiagent system given by (8) and (9), where agents exchange information using local measurements through a connected and undirected graph topology. Let the sensed exogenous inputs be constant. Then, the closed-loop error dynamics given by (39) and (40) are bounded and the bound of $\delta(t)$ for $t \geq T$ is given by (43) with $\dot{\epsilon}^* \geq ||\mathbf{1}_n^{\mathrm{T}} K_2(t)\left[\mathbf{1}_n \mathbf{1}_n^{\mathrm{T}} \dot{K}_2(t) c - c\mathbf{1}_n^{\mathrm{T}} \dot{K}_2(t)\mathbf{1}_n\right] / (\mathbf{1}_n^{\mathrm{T}} K_2(t)\mathbf{1}_n)^2||_2$, $p_1^* = ||\mathcal{L}^{\dagger}(\mathcal{G})||_{\mathrm{F}}||c||_2 k_c^*$, and $p_2^* = ||\mathcal{L}^{\dagger}(\mathcal{G})||_{\mathrm{F}}||c||_2 \dot{k}_c^*$, where $||K_2(t)||_{\mathrm{F}} \leq k_c^*$ and $||\dot{K}_2(t)||_{\mathrm{F}} \leq \dot{k}_c^*$.

Proof. See the proof of Theorem 4.1 and Theorem 4.2. ∎

The bounds included in Corollaries 1 and 2 clearly show how the time rate of change of the exogenous inputs and the value of information affect (43), respectively. We now present the final case of constant sensed exogenous inputs and constant value of information.

Corollary 5. Consider the networked multiagent system given by (8) and (9) with $\sigma = 0$, where agents exchange information using local measurements through a connected and undirected graph topology. Let both the value of information and the sensed exogenous inputs be constant. Then, the closed-loop error dynamics given by (39) and (40) are Lyapunov stable for all initial conditions and $\delta(t)$ asymptotically vanishes.

Proof. We first note that the closed-loop error dynamics in (39) and (40) simplify to

$$\dot{\delta}(t) = -\alpha F(\mathcal{G})\delta(t) - \alpha\tilde{K}\delta(t) + \mathcal{L}(\mathcal{G})e(t), \tag{44}$$

$$\dot{e}(t) = -\gamma\mathcal{L}(\mathcal{G})\delta(t), \tag{45}$$

since $\sigma = 0$ and both the value of information and the exogenous inputs are constant, that is $s_1(t) = s_2(t) = 0$. It now follows from the Lyapunov function candidate given by (41) that $\dot{V}(\cdot) \leq -d_1\|\delta(t)\|_2^2$, $d_1 \triangleq \lambda_{\min}(\mathcal{F}(\mathcal{G}))$, which shows the Lyapunov stability of the closed-loop error dynamics for all initial conditions. Because $\ddot{V}(\delta(t), e(t))$ is bounded for all $t \in \overline{\mathbb{R}}_+$, it follows from Barbalat's lemma [32] that $\lim_{t\to\infty} \dot{V}(\delta(t), e(t)) = 0$, which consequently shows that $\delta(t)$ asymptotically vanishes. ∎

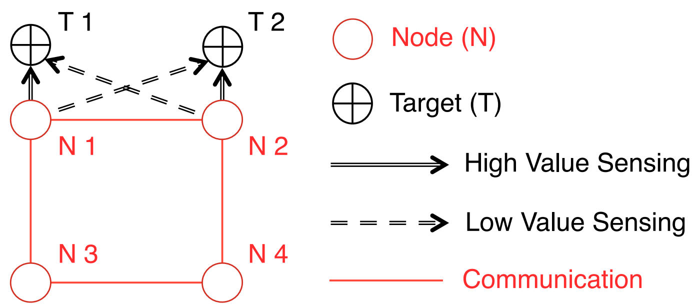

Figure 5. Active-passive networked multiagent system with two targets and four agents with nodes 1 and 2 being active and nodes 3 and 4 being passive.

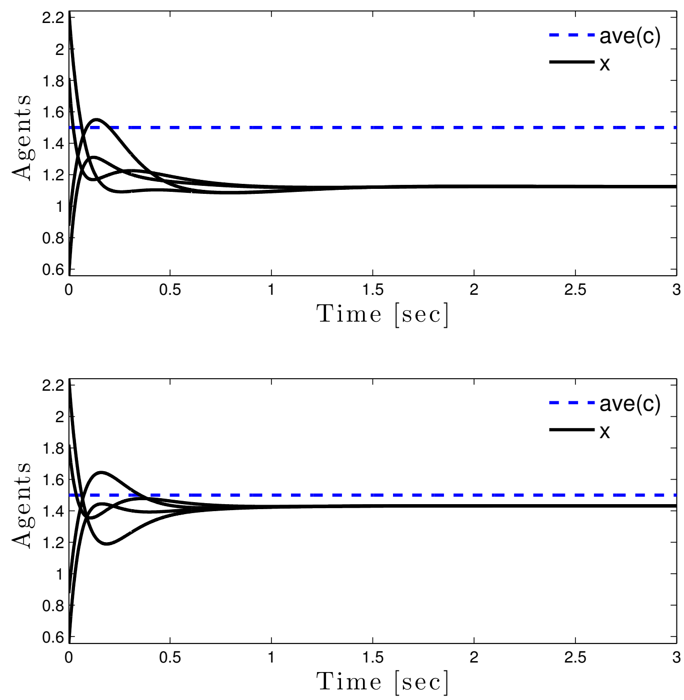

Figure 6. Response of the networked multiagent system in Figure 5 with identical value of information (top) and heterogeneous value of information (bottom) (dashed lines denote the actual average of the target quantities and solid lines denote the agent states).

5. ILLUSTRATIVE NUMERICAL EXAMPLES

In this section, the efficacy of the proposed distributed sensing algorithm given by (8) and (9) is illustrated using two examples. For the first example, consider the active-passive networked multiagent system given in Figure 5. Let the value of information and the exogenous inputs be constant, and the accuracy of the sensed target quantity decreases (resp., increases) as the distance between a node and the target gets larger (resp., smaller). In addition, let nodes 1 and 2 be sense targets 1 and 2 with perfect accuracy, respectively, and sense targets 2 and 1 with 50% accuracy, respectively. For this purpose, we set $c_1 = 1$ (node 1 measure of target 1 with perfect accuracy), $c_2 = 1$ (node 1 measure of target 2 with 50% accuracy), $c_3 = 0.5$ (node 2 measure of target 1 with 50% accuracy), $c_4 = 2$ (node 2 measure of target 2 with perfect accuracy). Figure 6 presents the results with $\alpha = 5$, $\gamma = 10$, and $\sigma = 0$ in (8) and (9). Specifically, the top figure shows the network response with identical value of information (i.e., we set $w_{11} = w_{12} = w_{23} = w_{24} = 1$) and the bottom figure shows the same response with heterogeneous value of information (i.e., we set $w_{11} = 1$, $w_{12} = 0.1$, $w_{23} = 0.1$, and $w_{24} = 1$). As expected, utilizing heterogeneity in the value of information allows network to converge a close neighborhood of the actual average of the target quantities sensed by agents.

For the second example, we consider a networked multiagent system tracking a moving target shown in Figure 7. Each agent has a sensing radius, where the value of information obtained by the agents decrease as the target moves away from them. In this study, we set $\alpha = 20$, $\gamma = 150$, and $\sigma = 0.1$ in (8) and (9). Figure 8 shows the network response both with identical value of information and with heterogeneous value of information. Once again, utilizing heterogeneity in the value of information allows the network to sense the actual trajectory with improved tracking accuracy.

6. CONCLUSION

In this paper, we utilized the active-passive networked multiagent systems approach to develop a distributed sensing framework that accounts for the heterogeneity in agents' sensing capability measured by the value of information. Specifically, we showed that the states of all agents converge to an adjustable neighborhood of the weighted average of the sensed exogenous inputs by the active agents when there exists time-variation in

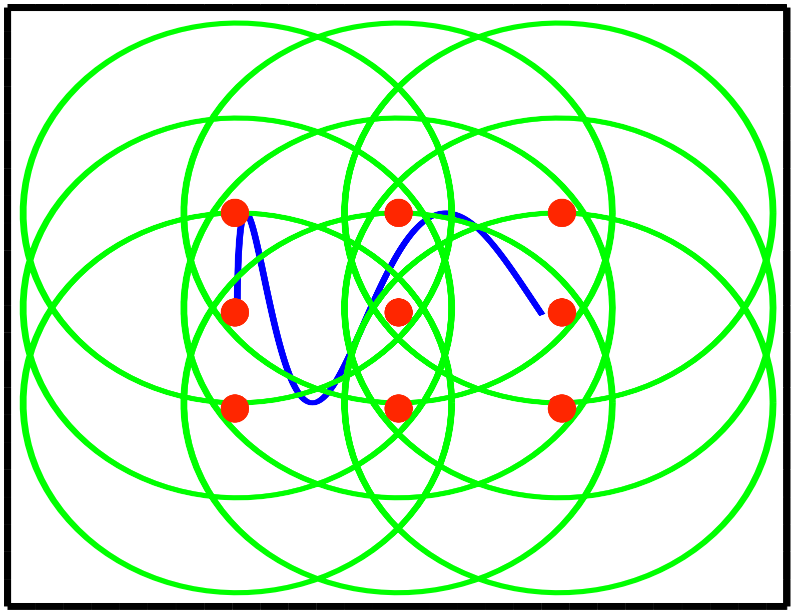

Figure 7. Active-passive networked multiagent system with one non-stationary target and nine fixed agents (dots denote the agents, circles denote the sensing radius of agents, and solid line denote the actual target trajectory).

both agents' sensing capability and the sensed exogenous inputs. In addition, we formally discussed several cases when agents' sensing capability and the exogenous inputs are time-invariant, which yields asymptotic stability of the error dynamics between the states of all agents and the weighted average of the sensed exogenous inputs. Illustrative examples indicated that utilizing heterogeneity allows the networked multiagent system to achieve better distributed sensing performance.

As noted in Remark 6, it was assumed in this paper that the value of sensed information is modeled and known by respective agents in the network. Future research will include the integration of the proposed framework exploiting heterogeneity in sensed information with the metrics that model the value of sensed information.

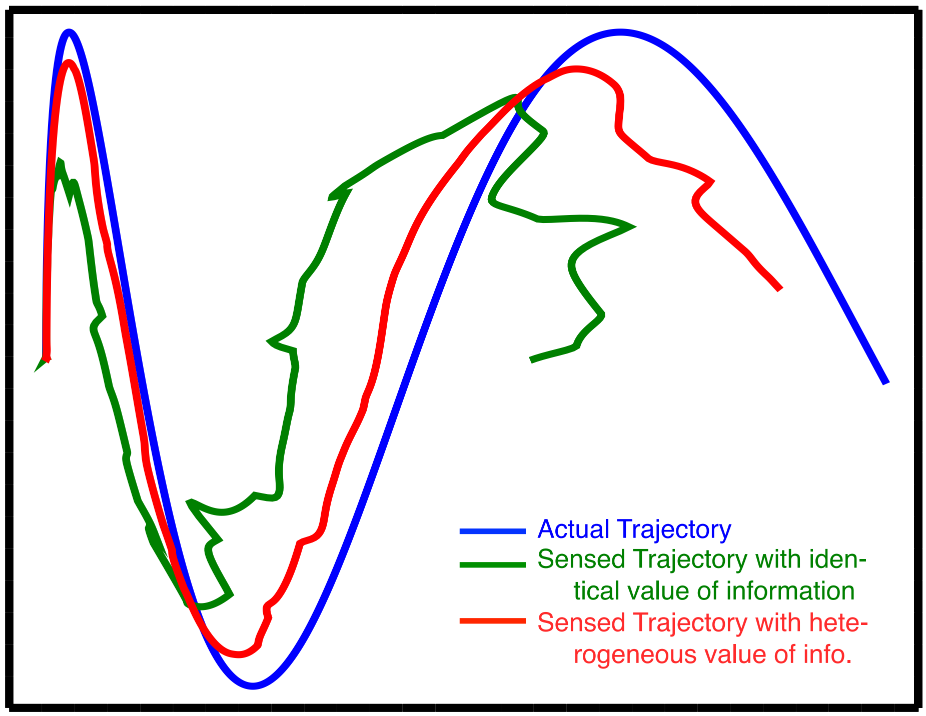

Figure 8. Response of the networked multiagent system in Figure 7 with identical value of information and heterogeneous value of information.

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V. ACTIVE-PASSIVE DYNAMIC CONSENSUS FILTERS WITH REDUCED INFORMATION EXCHANGE AND TIME-VARYING AGENT ROLES

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ABSTRACT

Active-passive dynamic consensus filters consist of agents subject to local observations of a process (i.e., *active* agents) and agents without any observations (i.e., *passive* agents). The key feature of these filters is that they enable the states of all agents to converge to the average of the observations *only* sensed by the active agents. Two sweeping generalizations can be made about existing active-passive dynamic consensus filters: *i)* They utilize integral action-based distributed control algorithms such that each agent is required to continuously exchange both its *current state* and *integral state* information with its neighbors. *ii)* They assume that the roles of active and passive agents are *fixed*; hence, these roles do *not* change with respect to time.

The contribution of this paper is to introduce and analyze a new class of active-passive dynamic consensus filters using results from graph theory and systems science. Specifically, the proposed filters *only* require agents to exchange their *current state* information with neighbors in a simple and isotropic manner to reduce the overall information exchange cost of the network. In addition, we allow the roles of active and passive agents to be *time-varying* for making these filters suitable for a wide range of multiagent systems applications. We show that the proposed active-passive dynamic consensus filters enable the states of all agents to converge to an *user-adjustable* neighborhood of the average of the observations sensed by a time-varying set of active agents. We also generalize our results using event-triggered control theory such that agents schedule information exchange dependent on errors exceeding user-defined thresholds (*not* continuously). This generaliza-

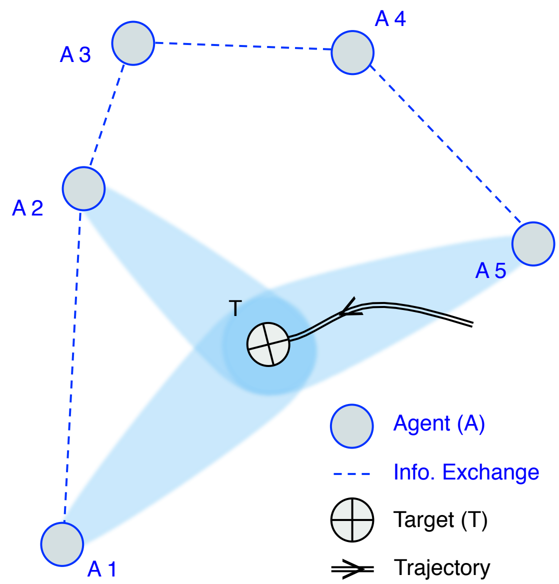

Figure 1. Illustration of heterogeneity in a sensor network for a target tracking problem.

tion allows agents to further reduce the overall cost of interagent information exchange and to determine when to broadcast their information to their neighbors thus eliminating the need to synchronize their states. Four illustrative numerical examples and one experimental study are also presented to demonstrate our theoretical findings.

1. INTRODUCTION

1.1. LITERATURE REVIEW

Distributed information fusion is a task performed by a group of agents that locally exchange information with each other. Owing to the distributed nature, it can impact a wide array of applications that range from surveillance and reconnaissance to guidance and control of autonomous vehicles. Classical distributed information fusion methods assume instantaneous communication, rely on high bandwidth networks, and do not necessarily account for system dynamics. For example, one common method for classical distributed information fusion is flooding, where agents broadcast all their information to their neighbors and store all the relayed information. It clearly requires significant memory to handle large amounts of information exchange [1]. On the other hand, system theoretic methods are based on the dynamics of the information fusion process (see, for example, [2], [3]), where each agent only shares its current information with its neighbors. In addition, system theoretic methods can naturally provide a level of robustness against modeling errors, changes in information paths, and measurement noise [4], [5]. The key component of system-theoretic distributed algorithms is a consensus or consensus-like algorithm needed for the information fusion process. Among two classes of consensus algorithms, static and dynamic consensus algorithms, the latter consider agreement upon time-varying quantities, which is more appropriate for applications driven by dynamic data.

Existing dynamic consensus filters (see, for example, [6]–[11]) are suitable for multiagent systems applications, where each agent is active in the sense that it is subject to an observation of a process of interest. From a practical standpoint, however, an agent can be passive for certain time instants owing to its heterogeneous sensing capability in that it may not be able to sense the process and collect information. To elucidate this point, consider a target tracking problem in Figure 1. In particular, agents 1, 2, and 5 are active in the sense that they are subject to observations of the target for the time instant depicted in this figure, whereas other nodes are passive as they have no observations. Consequently, a dynamic consensus algorithm needs have the capability to account for heterogeneity resulting from active and passive roles of networked agents.

Distributed control algorithms proposed in [12]–[17] make notable contributions to address this problem. While the authors of [12]–[14] present methods that cover specific applications, where a portion of the networked nodes are passive (and the remaining of nodes are active), their results are in the context of static consensus. In other words, their distributed control algorithms may not be suitable in their current form for dynamic environments as in the motivating example depicted in Figure 1. More recently, the authors of [15]–[17] propose *active-passive dynamic consensus filters*, where these filters not only allow the states of all agents to converge to the average of the observations *only* sensed by the active agents, but are also suitable for multiagent systems applications in *dynamic* environments.

The results in [15]–[17] utilize integral-action based distributed control algorithms for dynamic information fusion. While several other studies exist on integral-action based distributed control algorithms, notably [18]–[22], the authors of [18], [19] only consider specific cases where *all agents are active* with respect to a set of applied exogenous inputs. In addition, agents are required to have knowledge of the derivative of the applied exogenous inputs. The authors of [20] overview several dynamic consensus algorithms, but are commonly interested in *optimizing the network feedback gains* and do *not* consider exogenous inputs. In [21], the authors consider an integral-action based distributed control algorithm, where agents are able to estimate their neighboring agents' integral-action terms to reduce the cost of network information exchange, but only consider *agents having no observations*. The results of [22] present a distributed proportional-integral-derivative control algorithm, where agents track locally generated reference velocity and acceleration signals. Yet, all agents are *only* able to sense *one* corresponding velocity and acceleration pair, which may *not* be suitable for applications where agents are required to change their active and passive roles. Finally, the results of [23], [24] address tracking a target in camera networks, where some cameras are naiive (unable to sense the target) for some time intervals, utilizing a discrete extended Kalman filter approach, which can require *large communication bandwidths* and agents to *synchronize* their states. To address these problems, this paper presents a new active-passive dynamic consensus filter approach to dynamic information fusion.

1.2. CONTRIBUTION

The contribution of this paper is to present and analyze a new class of *active-passive dynamic consensus filters* using methods from graph theory and systems science, which goes beyond the existing state-of-the-art literature results overviewed in the above subsection. Specifically:

- In contrast to the most closely related literature results in [15]–[17], the proposed filters presented in this paper *only* require agents to exchange their *current state* information with neighbors in a simple and isotropic manner to reduce the overall information exchange cost of the network.

- The authors of [15]–[17] require that agents are fixed with respect to their active and passive roles, which can be impractical for many multiagent systems applications where it is necessary to allow the roles of active and passive agents to change with respect to time as in the motivating example depicted in Figure 1. The proposed active-passive dynamic consensus filters allows the roles of active and passive agents to be *time-varying*. In addition, we show that the proposed active-passive dynamic consensus filters enable the states of all agents to converge to an *user-adjustable* neighborhood of the average of the observations sensed by a time-varying set of active agents.

- We also generalize our results here using event-triggered control theory (see, for example, [25]–[29] and references therein) such that agents can schedule information exchange dependent on errors exceeding user-defined thresholds (*not* continuously). This generalization allows agents to further reduce the overall cost of interagent information exchange and to determine when to broadcast their information to their immediate neighbors thus eliminating the need to synchronize their states.

- Finally, we present numerical and experimental results to demonstrate the efficacy of our theoretical findings.

Note that two preliminary conference versions of this paper appeared in [30], [31]. The present paper considerably expands on [30], [31] by providing system-theoretical proofs of the results with additional motivation, examples, and an experimental study.

1.3. CONTENTS

The contents of this paper are as follows. Section 2 defines the notations used throughout this paper, recalls some basic results from graph theory, and introduces several necessary lemmas. Section 3 reviews the active-passive dynamic consensus filters presented in [15]–[17]. In Section 4, we present and analyze the new class of active-passive dynamic consensus filters. In Section 5, we then generalize the results of Section 4 using event-triggered control theory. Four illustrative numerical examples and one experimental study are also presented respectively in Section 6 and in Section 7 to demonstrate our theoretical findings, and our conclusions are summarized in Section 8.

2. MATHEMATICAL PRELIMINARIES

The notations used in this paper are fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbb{R}_+$ denotes the set of positive real numbers, $\mathbb{R}_+^{n\times n}$ (resp., $\overline{\mathbb{R}}_+^{n\times n}$) denotes the set of $n \times n$ positive-definite (resp., nonnegative-definite) real matrices, $\mathbb{S}_+^{n\times n}$ (resp., $\overline{\mathbb{S}}_+^{n\times n}$) denotes the set of $n \times n$ symmetric positive-definite (resp., symmetric nonnegative-definite) real matrices, $\mathbb{N}$ denotes the set of natural numbers, $\mathbf{0}_n$ denotes the $n \times 1$ vector of all zeros, $\mathbf{1}_n$ denotes the $n \times 1$ vector of all ones, $\mathbf{0}_{n\times n}$ denotes the $n \times n$ zero matrix, and I_n denotes the $n \times n$ identity matrix. In addition, we write $(\cdot)^{\mathrm{T}}$ for transpose, $(\cdot)^{-1}$ for inverse, $(\cdot)^{\dagger}$ for generalized inverse, $\|\cdot\|_2$ for the Euclidian norm, $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix A, $\lambda_i(A)$ for the i-th eigenvalue of A (A is symmetric and the eigenvalues are ordered from least to greatest value), $\mathrm{diag}(a)$ for the diagonal matrix with the vector a on its diagonal, and $[A]_{ij}$ for the entry of the matrix A on the i-th row and j-th column. Furthermore, each agent holds its local and neighboring agent's states in a zero-order-hold (ZOH) operator until a predefined event s_{ri} occurs with $r \in \mathbb{R}$ denoting the time at which the event occurs, $i = 1, 2, \ldots, n$ denoting an agent, and s denoting a predefined monotonic sequence $\{s_{ri}\}_{r=1}^{\infty} \in \mathbb{R}$ of events (see, for example, [29] as well as [25]–[28]).

Next, we concisely overview some necessary notions from graph theory and refer the readers to [4], [32] for further details. In particular, graphs are broadly adopted to encode interactions in multiagent systems. An undirected graph $\mathcal{G}$ is defined by a set $\mathcal{V}_{\mathcal{G}} = \{1, \ldots, n\}$ of nodes and a set $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$ of edges. If $(i, j) \in \mathcal{E}_{\mathcal{G}}$, then the nodes i and j are neighbors and the neighboring relation is indicated with $i \sim j$. The degree of a node is given by the number of its neighbors. Letting d_i be the degree of node i, then the degree matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by $\mathcal{D}(\mathcal{G}) \triangleq \mathrm{diag}(d)$, $d = [d_1, \ldots, d_n]^{\mathrm{T}}$. A path $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1, \ldots, L$, and a graph $\mathcal{G}$ is connected if there is a path between any pair of distinct nodes. We write $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{n\times n}$ for the adjacency matrix of a graph $\mathcal{G}$ defined by $[\mathcal{A}(\mathcal{G})]_{ij} \triangleq 1$ if $(i, j) \in \mathcal{E}_{\mathcal{G}}$ and $[\mathcal{A}(\mathcal{G})]_{ij} \triangleq 0$ otherwise. Furthermore, we write $\mathcal{B}(\mathcal{G}) \in \mathbb{R}^{n\times m}$ for the incidence matrix of a graph $\mathcal{G}$ defined by $[\mathcal{B}(\mathcal{G})]_{ij} \triangleq -1$ if node i is the tail of edge j, $[\mathcal{B}(\mathcal{G})]_{ij} \triangleq 1$ if node i is the head of edge j, and $[\mathcal{B}(\mathcal{G})]_{ij} \triangleq 0$ otherwise, where m is the number of edges, i is an index for the node set, and j is an index for the edge set (under the common consideration that directions have been arbitrarily assigned to label the edges). By definition, note that $\mathcal{B}^{\mathrm{T}}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_m$. The graph Laplacian matrix, denoted by $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_{+}^{n\times n}$, is defined by $\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G})$ or equivalently

$$\mathcal{L}(\mathcal{G}) \triangleq \mathcal{B}(\mathcal{G})\mathcal{B}^{\mathrm{T}}(\mathcal{G}), \tag{1}$$

and note the spectrum of $\mathcal{L}(\mathcal{G})$ for a connected, undirected graph can be ordered as

$$0 = \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \leq \cdots \leq \lambda_n(\mathcal{L}(\mathcal{G})), \tag{2}$$

with $\mathbf{1}_n$ as the eigenvector corresponding to the zero eigenvalue, $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$, and $\mathrm{e}^{\mathcal{L}(\mathcal{G})}\mathbf{1}_n = \mathbf{1}_n$. Throughout this paper, we model a given multiagent system by a connected, undirected graph $\mathcal{G}$, where nodes and edges represent agents and interagent communication links, respectively.

Finally, we present two necessary lemmas used in the main results of this paper.

Lemma 4 ([17])**.** Let $K = \mathrm{diag}(k)$, $k = [k_1, k_2, \ldots, k_n]^{\mathrm{T}}$, $k_i \in \overline{\mathbb{R}}_+$, $i = 1, \ldots, n$, and assume that at least one element of k is nonzero. Then, for a connected, undirected graph,

$$\mathcal{F}(\mathcal{G}) \triangleq \mathcal{L}(\mathcal{G}) + K \in \mathbb{S}_+^{n\times n}, \tag{3}$$

holds and $\det(\mathcal{F}(\mathcal{G})) \neq 0$.

Lemma 5 ([33])**.** For a connected, undirected graph, the following equality holds:

$$\mathcal{L}(\mathcal{G})\mathcal{L}^{\dagger}(\mathcal{G}) = \mathrm{I}_n - \frac{1}{n}\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}. \tag{4}$$

3. OVERVIEW OF ACTIVE-PASSIVE DYNAMIC CONSENSUS FILTERS

We now overview the active-passive dynamic consensus filter architecture introduced in [15]–[17], where we refer to these references for further details. Specifically, consider a system of n agents exchanging information among each other using their local measurements according to a connected, undirected graph $\mathcal{G}$. In addition, consider that there exists $m \geq 1$ inputs that interact with (i.e., are sensed by) this system in the sense that they represent local observations of agents.

Definition 1. If agent i, $i = 1, \ldots, n$, is subject to (i.e., senses) one or more inputs (resp., no inputs), then it is an *active* agent (resp., *passive* agent).

Definition 2. If an input interacts with (i.e., is sensed by) only one agent (resp., multiple agents), then it is an *isolated* input (resp., *nonisolated* input).

The active-passive dynamic consensus filter architecture focuses on the problem of driving the states of all (active and passive) agents to the average of the inputs *only* sensed by the active agents. For this purpose, the authors of [15]–[17] propose the integral action-based distributed control algorithm given by

$$\dot{x}_i(t) = -\alpha \sum_{i\sim j} \big(x_i(t) - x_j(t)\big) + \sum_{i\sim j} \big(\xi_i(t) - \xi_j(t)\big)$$
$$- \alpha \sum_{i\sim h} (x_i(t) - c_h(t)), \quad x_i(0) = x_{i0}, \tag{5}$$
$$\dot{\xi}_i(t) = -\gamma \sum_{i\sim j} \big(x_i(t) - x_j(t)\big), \quad \xi_i(0) = \xi_{i0}, \tag{6}$$

where $x_i(t) \in \mathbb{R}$ and $\xi_i(t) \in \mathbb{R}$ denote the state and the integral action of agent i, $i = 1, \ldots, n$, respectively, that are exchanged with the neighbors of this agent, $c_h(t) \in \mathbb{R}$, $h = 1, \ldots, m$, denotes an input sensed by this agent, $\alpha \in \mathbb{R}_+$, and $\gamma \in \mathbb{R}_+$. Note that $i \sim h$ notation indicates the exogenous inputs that an agent is subject to, which is similar to the $i \sim j$ notation indicating the neighboring relation between agents.

Remark 9. The results of [15]–[17] show that the states of all agents converge to (resp., converge to an user-adjustable neighborhood of) the average of the constant (resp., time-varying) inputs applied to active agents under the assumption that the roles of active and passive agents are *fixed*; hence, does *not* change with respect to time.

4. REDUCED INFORMATION EXCHANGE AND TIME-VARYING AGENT ROLES

In this section, we propose a new class of active-passive dynamic consensus filters that *only* require agents to exchange their *current state* information with neighbors in a simple and isotropic manner in order to reduce the overall information exchange cost of the network, which also allows the roles of active and passive agents to be *time-varying* (see

Section 4.1). In addition, we show that the proposed filters enable the states of all agents to converge to an *user-adjustable* neighborhood of the average of the observations sensed by a time-varying set of active agents (see Section 4.2).

4.1. PROPOSED ACTIVE-PASSIVE DYNAMIC CONSENSUS FILTERS

We propose the new integral action-based distributed control algorithm given by

$$\begin{aligned}\dot{x}_i(t) =& -\alpha\Bigg[\sum_{i\sim j}\big(x_i(t)-x_j(t)\big)+\beta_i x_i(t)\Bigg]+p_i(t)-e^{-\gamma\sigma t}p_i(0)\\ &-\alpha\sum_{i\sim h}k_{ih}(t)\left(x_i(t)-c_h(t)\right), \qquad x_i(0)=x_{i0}, \qquad (7)\\ \dot{p}_i(t) =& -\gamma\Bigg[\sum_{i\sim j}\big(x_i(t)-x_j(t)\big)+\sigma p_i(t)\Bigg], \qquad p_i(0)=p_{i0}, \qquad (8)\end{aligned}$$

where $x_i(t)\in\mathbb{R}$ and $p_i(t)\in\mathbb{R}$ denote the state and the integral action of agent i, $i=1,\ldots,n$, respectively, with $x_i(t)$ being the only information exchanged with the neighbors of this agent, $c_h(t)\in\mathbb{R}, h=1,2,\ldots,m$ denotes an input sensed by this agent, $\alpha\in\mathbb{R}_+$, $\gamma\in\mathbb{R}_+$, $\sigma\in\mathbb{R}_+$, and $\beta_i\in\overline{\mathbb{R}}_+$. Here, we require that there exists at least one positive β_i, $i=1,\ldots,n$. In (7), $k_{ih}(t)$ denotes a smooth function[1] varying on the interval $[0,1]$.

Remark 10. From a theoretical standpoint (see the next subsection), the term "$e^{-\gamma\sigma t}p_i(0)$" is needed to preserve the generality of the proposed algorithm. From a practical standpoint, however, this term can be removed by simply initializing (8) as $p_i(0)=0$ for all agents.

Remark 11. While (7) and (8) require the knowledge of common parameters α, γ, and σ, the execution of (7) and (8) is clearly distributed in practice. In addition, for applications when the considered graph provides sufficient network bandwidth, one can simply set α and γ to one for all agents in order to remove these two common gains from (7) and (8). Moreover, the other common parameter σ is associated with the leakage term in (8) that is used to preserve the stability of the proposed algorithm (see the next subsection). As such,

[1]Considering the target tracking problem given in Figure 1 as an illustrative example, this function approaches to one as the target gets closer to an agent and otherwise it approaches to zero.

it can be set to a sufficiently small number for all agents by default if a control designer wants to avoid this particular common term. Here, we also would like to mention that the authors of [34] propose a new distributed control method using networks having multiple layers (multiplex networks) to avoid similar common terms in distributed system-theoretical algorithms. While it is outside the scope of this current paper, one may use the ideas and tools from [34] in order to completely avoid α, γ, and σ in (7) and (8) with a multiplex networks approach.

Remark 12. The integral action-based distributed control algorithm given by (7) is applied to agents with dynamics of the form $\dot{x}_i(t) = u_i(t)$, where $u_i(t) \in \mathbb{R}$ denotes the control signal of agent i, $i = 1, \ldots, n$, that satisfies the right hand side of (7) along with (8).

Next, let

$$x(t) \triangleq [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n, \tag{9}$$

$$p(t) \triangleq [p_1(t), p_2(t), \ldots, p_n(t)]^{\mathrm{T}} \in \mathbb{R}^n, \tag{10}$$

$$c(t) \triangleq [c_1(t), c_2(t), \ldots, c_m(t), 0, \ldots, 0]^{\mathrm{T}} \in \mathbb{R}^n, \tag{11}$$

where $m \leq n$ is considered to ease notation without loss of generality. We can now rewrite (7) and (8) in the compact form given by

$$\begin{aligned} \dot{x}(t) = &-\alpha\mathcal{L}(\mathcal{G})x(t) - \alpha K_1(t)x(t) - \alpha\beta x(t) + p(t) - e^{\gamma\sigma \mathrm{I}_n t}p_0 \\ &+ \alpha K_2(t)c(t),\ x(0) = x_0, \end{aligned} \tag{12}$$

$$\dot{p}(t) = -\gamma\mathcal{L}(\mathcal{G})x(t) - \gamma\sigma p(t),\ p(0) = p_0, \tag{13}$$

where $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{n\times n}$ satisfies (2),

$$\beta \triangleq \mathrm{diag}([\beta_1, \beta_2, \ldots, \beta_n]^{\mathrm{T}}) \in \overline{\mathbb{S}}_+^{n\times n}, \tag{14}$$

$$K_1(t) \triangleq \mathrm{diag}([k_{1,1}(t), \ldots, k_{1,n}(t)]^{\mathrm{T}}) \in \overline{\mathbb{S}}_+^{n\times n}, \tag{15}$$

with

$$k_{1,i}(t) \triangleq \sum_{i\sim h} k_{ih}(t) \in \overline{\mathbb{R}}_+, \tag{16}$$

denoting the number of the inputs applied to agent i, $i = 1, \ldots, n$, and

$$K_2(t) \triangleq \begin{bmatrix} k_{2,11}(t) & \cdots & k_{2,1n}(t) \\ k_{2,21}(t) & \cdots & k_{2,2n}(t) \\ \vdots & \ddots & \vdots \\ k_{2,n1}(t) & \cdots & k_{2,nn}(t) \end{bmatrix} \in \mathbb{R}^{n\times n}, \tag{17}$$

with

$$k_{1,i}(t) = \sum_{j=1}^{n} k_{2,ij}(t). \tag{18}$$

We refer the reader to [16], [17] for specific examples illustrating the construction of $K_1(t)$ and $K_2(t)$ matrices.

Since we are interested in driving the states of all (active and passive) agents to an user-adjustable neighborhood of the average of the inputs applied to the active agents, we define

$$\delta(t) \triangleq x(t) - \epsilon(t)\mathbf{1}_n \in \mathbb{R}^n, \tag{19}$$

$$\epsilon(t) \triangleq \frac{\mathbf{1}_n^T K_2(t) c(t)}{\mathbf{1}_n^T K_2(t) \mathbf{1}_n} \in \mathbb{R}, \tag{20}$$

where $\delta(t)$ is the error between $x_i(t), i = 1, \ldots, n$, and the average of the applied inputs $\epsilon(t)$. Using (20), $\epsilon(t)$ can be equivalently written as

$$\epsilon(t) = \frac{k_{2,11}(t)c_1(t) + k_{2,12}(t)c_2(t) + \cdots + k_{2,21}(t)c_1(t) + k_{2,22}(t)c_2(t) + \cdots}{k_{2,11}(t) + k_{2,12}(t) + \cdots + k_{2,21}(t) + k_{2,22}(t) + \cdots}, \tag{21}$$

that clearly shows the average of the applied inputs. Note that the denominator of (21) is nonzero, since we assume that there exists $m \geq 1$ inputs, and hence, there exists at least one nonzero value on the denominator of (21). This concludes the algorithmic setup of the proposed active-passive dynamic consensus filters.

4.2. STABILITY AND PERFORMANCE GUARANTEES

To analyze stability and performance guarantees of the proposed active-passive dynamic consensus filters introduced in Section 4.1, we first note that (13) can be rewritten as

$$\begin{aligned} p(t) &= e^{-\gamma\sigma \mathrm{I}_n t}p_0 + \int_0^t -\gamma e^{-\gamma\sigma \mathrm{I}_n (t-\tau)}\mathcal{L}(\mathcal{G})x(\tau)d\tau, \\ &= \mathrm{I}_n e^{-\gamma\sigma t}p_0 + \int_0^t -\gamma \mathrm{I}_n e^{-\gamma\sigma(t-\tau)}\mathcal{L}(\mathcal{G})x(\tau)d\tau, \end{aligned} \tag{22}$$

where $e^{\phi \mathrm{I}_n} = \mathrm{I}_n e^{\phi}$, $\phi \in \mathbb{R}$, is used [35]. In addition, using (22), (12) can be rewritten as

$$\begin{aligned} \dot{x}(t) =& -\alpha\mathcal{L}(\mathcal{G})x(t) - \alpha K_1(t)x(t) - \alpha\beta x(t) + \int_0^t -\gamma e^{-\gamma\sigma(t-\tau)}\mathcal{L}(\mathcal{G})x(\tau)d\tau \\ &+ \alpha K_2(t)c(t),\ x(0) = x_0. \end{aligned} \tag{23}$$

We now define

$$z(t) \triangleq \int_0^t -\gamma e^{-\gamma\sigma(t-\tau)}\mathcal{B}^{\mathrm{T}}(\mathcal{G})x(\tau)d\tau. \tag{24}$$

Since (1) holds, then

$$\dot{x}(t) = -\alpha\mathcal{L}(\mathcal{G})x(t) - \alpha K_1(t)x(t) - \alpha\beta x(t) + \mathcal{B}(\mathcal{G})z(t) + \alpha K_2(t)c(t),\ x(0) = x_0, \tag{25}$$

$$\dot{z}(t) = -\gamma\mathcal{B}^{\mathrm{T}}(\mathcal{G})x(t) - \gamma\sigma z(t),\ z(0) = 0. \tag{26}$$

follows from (23) and (24).

Next, using (25), the time derivative of the error (19) is given by

$$\begin{aligned} \dot{\delta}(t) =& -\alpha\mathcal{F}(\mathcal{G})\delta(t) - \alpha K_1(t)\delta(t) + \mathcal{B}(\mathcal{G})z(t) + L_c(t)K_2(t)c(t) - \alpha\beta\mathbf{1}_n\epsilon(t) - \mathbf{1}_n\dot{\epsilon}(t), \\ & \delta(0) = \delta_0, \end{aligned} \tag{27}$$

where

$$\mathcal{F}(\mathcal{G}) \triangleq \mathcal{L}(\mathcal{G}) + \beta \in \mathbb{S}_+, \tag{28}$$

as a direct consequence of Lemma 4, and

$$L_c(t) = \frac{K_1(t)\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}}{\mathbf{1}_n^{\mathrm{T}}K_2(t)\mathbf{1}_n} - \mathrm{I}_n. \tag{29}$$

Now, considering

$$e(t) \triangleq z(t) - \alpha \mathcal{B}^{\mathrm{T}}(\mathcal{G})\mathcal{L}^{\dagger}(\mathcal{G})L_c(t)K_2(t)c(t), \tag{30}$$

and noting from (18) that

$$\mathbf{1}_n^{\mathrm{T}} L_c(t) = \mathbf{1}_n^{\mathrm{T}}\left[K_1(t)\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}/(\mathbf{1}_n^{\mathrm{T}}K_2(t)\mathbf{1}_n) - \mathrm{I}_n\right] = 0, \tag{31}$$

(27) can be rewritten as

$$\dot{\delta}(t) = -\alpha\mathcal{F}(\mathcal{G})\delta(t) - \alpha K_1(t)\delta(t) + \mathcal{B}(\mathcal{G})e(t) - \alpha\beta\mathbf{1}_n\epsilon(t) - \mathbf{1}_n\dot{\epsilon}(t). \tag{32}$$

Finally, the time derivative of (30) is given by

$$\dot{e}(t) = -\gamma\mathcal{B}^{\mathrm{T}}(\mathcal{G})\delta(t) - \gamma\sigma e(t) - \alpha\gamma\sigma K_c(t)c(t) - \alpha\dot{K}_c(t)c(t) - \alpha K_c(t)\dot{c}(t),\ e(0) = e_0, \tag{33}$$

with

$$K_c(t) \triangleq \mathcal{B}^{\mathrm{T}}(\mathcal{G})\mathcal{L}^{\dagger}(\mathcal{G})L_c(t)K_2(t). \tag{34}$$

For the following result, the closed-loop error dynamics given by (32) and (33) can be rewritten as

$$\dot{\delta}(t) = -\alpha\mathcal{F}(\mathcal{G})\delta(t) - \alpha K_1(t)\delta(t) + \mathcal{B}(\mathcal{G})e(t) + s_1(t), \tag{35}$$

$$\dot{e}(t) = -\gamma\mathcal{B}^{\mathrm{T}}(\mathcal{G})\delta(t) - \gamma\sigma e(t) + s_2(t), \tag{36}$$

where the perturbation terms[2] are given by

$$s_1(t) \triangleq -\alpha\beta\mathbf{1}_n\epsilon(t) - \mathbf{1}_n\dot{\epsilon}(t), \tag{37}$$

$$s_2(t) \triangleq -\alpha\gamma\sigma K_c(t)c(t) - \alpha\dot{K}_c(t)c(t) - \alpha K_c(t)\dot{c}(t). \tag{38}$$

Theorem 4.1. *Consider the active-passive dynamic consensus filter given by (7) and (8), where nodes exchange information using local measurements though a connected, undirected graph topology. Then, the closed-loop error dynamics given by (35) and (36) are bounded.*

[2]It is assumed here that there exist positive constants s_1^* and s_2^* such that $\|s_1(t)\|_2^2 \le s_1^*$ and $\|s_2(t)\|_2^2 \le s_2^*$ hold.

Proof. Consider the Lyapunov-like function given by

$$V\left(\delta(t), e(t)\right) = \frac{1}{2\alpha}\delta^{\mathrm{T}}(t)\delta(t) + \frac{1}{2\alpha\gamma}e^{\mathrm{T}}(t)e(t), \tag{39}$$

and note that $V(0,0) = 0$ and

$$V\left(\delta(t), e(t)\right) > 0, \;\; \forall\left(\delta(t), e(t)\right) \neq (0,0)\,. \tag{40}$$

The time-derivative of (39) along the trajectories of (35) and (36) can be given by

$$\begin{aligned}
\dot{V}(\cdot) =& \frac{1}{\alpha}\delta^{\mathrm{T}}(t)\big[-\alpha\mathcal{F}(\mathcal{G})\delta(t) - \alpha K_1(t)\delta(t) + \mathcal{B}(\mathcal{G})e(t) + s_1(t)\big] \\
&+ \frac{1}{\alpha\gamma}e^{\mathrm{T}}(t)\big[-\gamma\mathcal{B}^{\mathrm{T}}(\mathcal{G})\delta(t) - \gamma\sigma e(t) + s_2(t)\big], \\
=& -\delta^{\mathrm{T}}(t)\mathcal{F}(\mathcal{G})\delta(t) - \delta^{\mathrm{T}}(t)K_1(t)\delta(t) + \frac{1}{\alpha}\delta^{\mathrm{T}}(t)s_1(t) - \frac{\sigma}{\alpha}e^{\mathrm{T}}(t)e(t) + \frac{1}{\alpha\gamma}e^{\mathrm{T}}(t)s_2(t), \\
\leq& -\delta^{\mathrm{T}}(t)\mathcal{F}(\mathcal{G})\delta(t) - \frac{\sigma}{\alpha}e^{\mathrm{T}}(t)e(t) + \frac{1}{\alpha}\delta^{\mathrm{T}}(t)s_1(t) + \frac{1}{\alpha\gamma}e^{\mathrm{T}}(t)s_2(t), \\
\leq& -d_1\|\delta(t)\|_2\left(\|\delta(t)\|_2 - \frac{d_3}{d_1}\right) - d_2\|e(t)\|_2\left(\|e(t)\|_2 - \frac{d_4}{d_2}\right),
\end{aligned} \tag{41}$$

where

$$d_1 \triangleq \lambda_{\min}\left(\mathcal{F}(\mathcal{G})\right),\; d_2 \triangleq \frac{\sigma}{\alpha},\; d_3 \triangleq \frac{s_1^*}{\alpha},\; d_4 \triangleq \frac{s_2^*}{\alpha\gamma}.$$

Since

$$\dot{V}\left(\delta(t), e(t)\right) \leq 0, \tag{42}$$

when $\|\delta(t)\|_2 \geq \frac{d_3}{d_1}$ and $\|e(t)\|_2 \geq \frac{d_4}{d_2}$, it follows that the closed-loop error dynamics given by (35) and (36) are bounded [36], [37]. ■

In the next corollary, we determine the bound of $\delta(t)$ for $t \geq T$ characterizing the ultimate distance between $x(t)$ and $\mathbf{1}_n\epsilon(t)$, which is of practical importance for multiagent systems applications.

Corollary 6. Consider the active-passive dynamic consensus filter given by (7) and (8), where nodes exchange information using local measurements though a connected, undirected graph topology. Then, the bound of $\delta(t)$ for $t \geq T$ is given by

$$\|\delta(t)\|_2^2 \leq \frac{\|\beta\|_2^2 \epsilon^{*2}}{\lambda_{\min}^2(\mathcal{F}(\mathcal{G}))} + \frac{2\|\beta\|_2 n \epsilon^* \dot{\epsilon}^*}{\alpha \lambda_{\min}^2(\mathcal{F}(\mathcal{G}))} + \frac{n^2 \dot{\epsilon}^{*2}}{\alpha^2 \lambda_{\min}^2(\mathcal{F}(\mathcal{G}))} + \frac{\alpha^2}{\gamma}\left[p_1^{*2} + \frac{2p_1^* p_2^*}{\gamma\sigma} + \frac{p_2^{*2}}{\gamma^2\sigma^2}\right], \tag{43}$$

where $\|\epsilon(t)\|_2 \leq \epsilon^*$, $\|\dot{\epsilon}(t)\|_2 \leq \dot{\epsilon}^*$, $\|\mathcal{B}^{\mathrm{T}}(\mathcal{G})\mathcal{L}^{\dagger}(\mathcal{G}) \cdot K_c(t)c(t)\|_2 \leq p_1^*$, and
$\|\mathcal{B}^{\mathrm{T}}(\mathcal{G})\mathcal{L}^{\dagger}(\mathcal{G})\dot{K}_c(t)c(t) + \mathcal{B}^{\mathrm{T}}(\mathcal{G})\mathcal{L}^{\dagger}(\mathcal{G})K_c(t)\dot{c}(t)\|_2 \leq p_2^*$.

Proof. In the proof of Theorem 4.1, we show that (42) holds when $\|\delta(t)\|_2 \geq d_3/d_1$ and $\|e(t)\|_2 \geq d_4/d_2$. Note that this implies

$$\dot{V}(\delta(t), e(t)) < 0, \tag{44}$$

outside the compact set defined by

$$\mathcal{S} \triangleq \left\{(\delta(t), e(t)) : \|\delta(t)\|_2 \leq \frac{d_3}{d_1}\right\} \bigcap \left\{(\delta(t), e(t)) : \|e(t)\|_2 \leq \frac{d_4}{d_2}\right\}. \tag{45}$$

Since $V(\delta(t), e(t))$ cannot grow outside $\mathcal{S}$, the evolution of $V(\delta(t), e(t))$ is upper bounded by

$$\begin{aligned} V(\delta(t), e(t)) &\leq \max_{(\delta(t), e(t)) \in \mathcal{S}} V(\delta(t), e(t)) \\ &= \frac{1}{2\alpha}\frac{d_3^2}{d_1^2} + \frac{1}{2\alpha\gamma}\frac{d_4^2}{d_2^2}, \quad t \geq T, \end{aligned} \tag{46}$$

where using

$$\frac{1}{2\alpha}\delta^{\mathrm{T}}(t)\delta(t) \leq V(\delta(t), e(t)), \tag{47}$$

in (46), (43) follows. ■

Remark 13. The bound of $\delta(t)$ given by (43) shows the effect of the design parameters α, γ, σ, and β_i of the active-passive dynamic consensus filter given by (7) and (8) on the overall network performance. In particular, it can be seen that when α and γ are chosen such that $\frac{1}{\alpha^2}$ and $\frac{\alpha^2}{\gamma}$ are small, then (43) is small for $t \geq T$. This provides a parameter tuning guideline for the proposed filter in (7) and (8).

5. EVENT-TRIGGERED ACTIVE-PASSIVE DYNAMIC CONSENSUS FILTERS

While the results of Section 4 go beyond [15]–[17] in terms of reducing the information exchange in active-passive dynamic consensus filters and allowing for time-varying agent roles, agents still need to *continuously* exchange information among themselves. This often incurs a high cost of interagent information exchange, which may *not* always be practical, especially for applications in low bandwidth environments. In addition, agents must *synchronously* update their states in (7) and (8). This requirement may *not* be trivially satisfied without an enforcing mechanism keeping the agent states synchronized.

For addressing these practical shortcomings, Section 5.1 generalizes the results of the previous section using event-triggered control theory such that agents schedule information exchange dependent on errors exceeding user-defined thresholds (*not* continuously). This generalization allows agents to further reduce the overall cost of interagent information exchange and to determine when to broadcast their information to their neighbors thus *eliminating* the need to synchronize their states. Furthermore, Section 5.2 shows that the proposed generalization still enables the states of all agents to converge to an *user-adjustable* neighborhood of the average of the exogenous inputs applied to a time-varying set of active agents.

5.1. PROPOSED ACTIVE-PASSIVE DYNAMIC CONSENSUS FILTERS WITH EVENT-TRIGGERING

Building on the results of Section 4, we now propose the new event-triggered integral action-based distributed control algorithm given by

$$\begin{aligned}\dot{x}_i(t) = & -\alpha \sum_{i\sim j} \big(\hat{x}_i(t) - \hat{x}_j(t)\big) - \alpha\beta_i \hat{x}_i(t) + p_i(t) - e^{-\gamma\sigma t} p_i(0) \\ & -\alpha \sum_{i\sim h} k_{ih}(t)\,(\hat{x}_i(t) - c_h(t)), \qquad \hat{x}_i(0) = x_{0i}, \qquad (48)\\ \dot{p}_i(t) = & -\gamma \sum_{i\sim j} \big(\hat{x}_i(t) - \hat{x}_j(t)\big) - \sigma\gamma p_i(t), \qquad p_i(0) = p_{0i}, \qquad (49)\end{aligned}$$

where $\hat{x}_i(t) \in \mathbb{R}$ denotes an event-triggered agent state that is the only information exchanged with the neighbors of this agent. Here, since we are interested in scheduling the exchanged information dependent on errors exceeding a user-defined threshold, let

$$\|x_i(t) - \hat{x}_i(t)\|_2 \leq \tau_i, \quad \tau_i > 0, \tag{50}$$

where τ_i is a user-defined local threshold (upper bound) on the distance between the agent's state and the agent's event-triggered state. An agent updates and broadcasts its event-triggered state to neighboring agents when the local event-trigger in (50) is violated.

Next, let

$$x(t) \triangleq [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n, \tag{51}$$

$$\hat{x}(t) \triangleq [\hat{x}_1(t), \hat{x}_2(t), \ldots, \hat{x}_n(t)]^{\mathrm{T}} \in \mathbb{R}^n, \tag{52}$$

$$p(t) \triangleq [p_1(t), p_2(t), \ldots, p_n(t)]^{\mathrm{T}} \in \mathbb{R}^n, \tag{53}$$

$$c(t) \triangleq [c_1(t), c_2(t), \ldots, c_m(t), 0, \ldots, 0]^{\mathrm{T}} \in \mathbb{R}^n, \tag{54}$$

where $m \leq n$. We can now rewrite (48) and (49) in the compact form given by

$$\begin{aligned} \dot{x}(t) = &- \alpha\mathcal{L}(\mathcal{G})\hat{x}(t) - \alpha K_1(t)\hat{x}(t) - \alpha\beta\hat{x}(t) + p(t) - e^{\gamma\sigma\mathrm{I}_n t}p_0 \\ &+ \alpha K_2(t)c(t),\ x(0) = x_0, \end{aligned} \tag{55}$$

$$\dot{p}(t) = -\gamma\mathcal{L}(\mathcal{G})\hat{x}(t) - \gamma\sigma p(t),\ p(0) = p_0. \tag{56}$$

In addition, we define

$$\Delta(t) \triangleq \hat{x}(t) - x(t) \in \mathbb{R}^n, \tag{57}$$

where $\Delta(t)$ is the error between the state $x(t)$ and the event-triggered state $\hat{x}(t)$. Using (19), note that $\Delta(t)$ can be equivalently written as

$$\Delta(t) = \hat{x}(t) - \delta(t) - \epsilon(t)\mathbf{1}_n. \tag{58}$$

This concludes the algorithmic setup of the proposed event-triggered active-passive dynamic consensus filters.

5.2. STABILITY AND PERFORMANCE GUARANTEES

Similar to Section 4.2, we first note that (56) has the solution given by

$$\begin{aligned} p(t) &= e^{-\gamma\sigma \mathrm{I}_n t} p_0 + \int_0^t -\gamma e^{-\gamma\sigma \mathrm{I}_n (t-\tau)} \mathcal{L}(\mathcal{G}) \hat{x}(\tau) d\tau, \\ &= \mathrm{I}_n e^{-\gamma\sigma t} p_0 + \int_0^t -\gamma \mathrm{I}_n e^{-\gamma\sigma (t-\tau)} \mathcal{L}(\mathcal{G}) \hat{x}(\tau) d\tau, \end{aligned} \tag{59}$$

Using (59), (55) can be given by

$$\begin{aligned} \dot{x}(t) =& -\alpha \mathcal{L}(\mathcal{G}) \hat{x}(t) - \alpha K_1(t) \hat{x}(t) - \alpha\beta \hat{x}(t) + \alpha K_2(t) c(t) \\ &+ \int_0^t -\gamma e^{-\gamma\sigma(t-\tau)} \mathcal{L}(\mathcal{G}) \hat{x}(\tau) d\tau, \ x(0) = x_0, \end{aligned} \tag{60}$$

Now, let

$$z(t) \triangleq \int_0^t -\gamma e^{-\gamma\sigma(t-\tau)} \mathcal{B}^{\mathrm{T}}(\mathcal{G}) \hat{x}(\tau) d\tau. \tag{61}$$

Since (1) holds, then

$$\dot{x}(t) = -\alpha \mathcal{L}(\mathcal{G}) \hat{x}(t) - \alpha K_1(t) \hat{x}(t) - \alpha\beta \hat{x}(t) + \mathcal{B}(\mathcal{G}) z(t) + \alpha K_2(t) c(t), \ x(0) = x_0, \tag{62}$$

$$\dot{z}(t) = -\gamma \mathcal{B}^{\mathrm{T}}(\mathcal{G}) \hat{x}(t) - \gamma\sigma z(t), \ z(0) = 0, \tag{63}$$

follows from (60) and (61). Using (62) and following the steps highlighted in Section 4.2, the closed-loop error dynamics can now be written as

$$\dot{\delta}(t) = -\alpha \mathcal{F}(\mathcal{G})(\Delta(t) + \delta(t)) - \alpha K_1(t)(\Delta(t) + \delta(t)) + \mathcal{B}(\mathcal{G}) e(t) + s_1(t), \tag{64}$$

$$\dot{e}(t) = -\gamma \mathcal{B}^{\mathrm{T}}(\mathcal{G})(\Delta(t) + \delta(t)) - \gamma\sigma e(t) + s_2(t). \tag{65}$$

In the following theorem, we demonstrate the boundedness of the error dynamics given by (64) and (65).

Theorem 5.1. *Consider the event-triggered active-passive dynamic consensus filter given by* (48) *and* (49) *subject to the event-triggering condition in* (50)*, where nodes exchange local information over a connected, undirected graph topology* $\mathcal{G}$*. Then, the error dynamics given by* (64) *and* (65) *are bounded.*

Proof. Consider the Lyapunov-like function given by (39). The time-derivative of (39) along the trajectories of (64) and (65) can be given by

$$\begin{aligned}
\dot{V}(\cdot) =& \frac{1}{\alpha}\delta^{\mathrm{T}}(t)\big[-\alpha\left(\mathcal{F}(\mathcal{G}) + K_1(t)\right)\left(\Delta(t) + \delta(t)\right) + s_1(t) + \mathcal{B}(\mathcal{G})e(t)\big] \\
&+ \frac{1}{\alpha\gamma}e^{\mathrm{T}}(t)\big[-\gamma\mathcal{B}^{\mathrm{T}}(\mathcal{G})(\Delta(t) + \delta(t)) - \gamma\sigma e(t) + s_2(t)\big], \\
\leq& -\delta^{\mathrm{T}}(t)\mathcal{F}(\mathcal{G})\delta(t) - \frac{\sigma}{\alpha}e^{\mathrm{T}}(t)e(t) + \frac{1}{\alpha}\delta^{\mathrm{T}}(t)s_1(t) - \frac{1}{\alpha}e^{\mathrm{T}}(t)\mathcal{B}^{\mathrm{T}}(\mathcal{G})\Delta(t) + \frac{1}{\alpha\gamma}e^{\mathrm{T}}(t)s_2(t) \\
&- \delta^{\mathrm{T}}(t)\left(\mathcal{F}(\mathcal{G}) + K_1(t)\right)\Delta(t), \\
\leq& -d_1\|\delta(t)\|_2\left(\|\delta(t)\|_2 - \frac{d_3}{d_1}\right) - d_2\|e(t)\|_2\left(\|e(t)\|_2 - \frac{d_4}{d_2}\right),
\end{aligned} \tag{66}$$

where $d_1 \triangleq \lambda_{\min}(\mathcal{F}(\mathcal{G}))$, $d_2 \triangleq \frac{\sigma}{\alpha}$, $d_3 \triangleq (\|\mathcal{F}(\mathcal{G})\|_{\mathrm{F}} + mn)\cdot\tau^* + \frac{s_1^*}{\alpha}$, and $d_4 \triangleq \frac{\tau^*}{\sigma}\|\mathcal{B}(\mathcal{G})\|_{\mathrm{F}} + \frac{s_2^*}{\sigma\alpha}$, with $\tau^* \triangleq n\max_i \tau_i$. Note that $\|\Delta(t)\| \leq \tau^*$ follows from (50) and $\|K_1(t)\|_{\mathrm{F}} < mn$ can be obtained by summing the entries of (18). Since

$$\dot{V}\left(\delta(t), e(t)\right) \leq 0, \tag{67}$$

when $\|\delta(t)\|_2 \geq d_3/d_1$ and $\|e(t)\|_2 \geq d_4/d_2$, it follows that the closed-loop error dynamics given by (64) and (65) are bounded ([36], [37]). ■

Next, we determine the bound of $\delta(t)$ for $t \geq T$ characterizing the ultimate distance between $x(t)$ and $\mathbf{1}_n\epsilon(t)$.

Corollary 7. Consider the active-passive dynamic consensus filter given by (48) and (49) subject to the event-triggering condition in (50), where nodes exchange information using local measurements though a connected, undirected graph topology. Then, the bound of $\delta(t)$ for $t \geq T$ is given by

$$\begin{aligned}
\|\delta(t)\|_2^2 \leq& \frac{p_1^{*2}\tau^{*2}}{\lambda_{\min}(\mathcal{F}((G))^2} + \frac{p_1^*\tau^* s_1^*}{\alpha\lambda_{\min}(\mathcal{F}((G))} + \frac{s_1^{*2}}{\alpha^2\lambda_{\min}(\mathcal{F}((G))^2} \\
&+ \frac{\alpha^2\tau^*\|\mathcal{B}(\mathcal{G})\|_{\mathrm{F}}^2}{\gamma} + \frac{2s_2^*\tau^*\|\mathcal{B}(\mathcal{G})\|_{\mathrm{F}}^2}{\gamma} + \frac{2\alpha s_2^{*2}}{\gamma},
\end{aligned} \tag{68}$$

where $\|\mathcal{F}(\mathcal{G}) + mn\|_{\mathrm{F}} \leq p_1^*$.

Proof. In the proof of Theorem 5.1, we showed that (66) holds when $\|\delta(t)\|_2 \geq d_3/d_1$ and $\|e(t)\|_2 \geq d_4/d_2$. Note that this implies $\dot{V}(\delta(t), e(t)) < 0$ outside the compact set

$$\mathcal{S} \triangleq \big\{(\delta(t), e(t)) : \|\delta(t)\|_2 \leq \frac{d_3}{d_1}\big\} \bigcap \big\{(\delta(t), e(t)) : \|e(t)\|_2 \leq \frac{d_4}{d_2}\big\}. \tag{69}$$

Since $V(\delta(t), e(t))$ cannot grow outside $\mathcal{S}$, the evolution of $V(\delta(t), e(t))$ is upper bounded by

$$\begin{aligned} V(\delta(t), e(t)) \leq & \max_{(\delta(t), e(t)) \in \mathcal{S}} V(\delta(t), e(t)) \\ = & \frac{1}{2\alpha} \frac{d_3^2}{d_1^2} + \frac{1}{2\alpha\gamma} \frac{d_4^2}{d_2^2}, \quad t \geq T, \end{aligned} \tag{70}$$

where using (47) in (70), (68) follows. ■

Remark 14. The bound of $\delta(t)$ given by (68) shows the effect of the design parameters α and γ of the event-triggered active-passive dynamic consensus filter given by (48) and (49) on the overall network performance. In particular, when the design parameters α and γ are chosen such that $\frac{1}{\alpha^2}$, $\frac{\alpha^2}{\gamma}$, and $\frac{1}{\gamma}$ are small, then (68) is small for $t \geq T$.

Remark 15. The bound of $\delta(t)$ given by (68) shows that if τ_i is chosen to be small, then $\|\delta(t)\|_2^2$ shrinks. However, choosing τ_i small causes the event-triggering condition in (50) to be violated more frequently, increasing the total cost of interagent information exchange. To this end, τ_i should be chosen such that the desired distance between the state of all agents and the average of the applied exogenous inputs is maintained while keeping the total cost of interagent information exchange low.

In the following corollary, we compute the lower bound of the event-triggered intersample time, demonstrating that no Zeno behavior can occur.

Corollary 8. Consider the active-passive dynamic consensus filter given by (48) and (49) subject to the event-triggering condition in (50), where agents communicate over a connected, undirected graph topology $\mathcal{G}$. Then, there exists positive scalars a_i such that

$$s_{i,k+1} - s_{i,k} \geq a_i, \; i = 1, 2, 3 \ldots, n, \; \forall k \in \mathbb{N}. \tag{71}$$

Proof. Consider the time derivative of (50) over the interval $t \in \left[s_{i,k}, s_{i,k+1}\right], \forall k \in \mathbb{N}$, given by

$$\frac{d}{dt}\|\hat{x}_i(t) - x_i(t)\| \leq \|\dot{\hat{x}}_i(t) - \dot{x}_i(t)\| = \|\dot{x}_i(t)\|, \tag{72}$$

which implies that

$$\begin{aligned}
\|\dot{x}_i(t)\| &= \left\|\alpha \sum_{i\sim j}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) - \alpha\beta_i \hat{x}_i(t) + p_i(t) - e^{-\gamma\sigma t} p_i(0) + \alpha \sum_{i\sim h} k_{ih}(t)\left(c_h(t) - \hat{x}_i(t)\right)\right\|, \\
&\leq \alpha \left\|\sum_{i\sim j} \hat{x}_i(t) - \hat{x}_j(t)\right\| + \|p_i(t)\| + \alpha\beta_i \|\hat{x}_i(t)\| \\
&\quad + \|e^{-\gamma\sigma t} p_i(0)\| + \left\|\alpha \sum_{i\sim h} k_{ih}(t)\hat{x}_i(t) - c_h(t)\right\|.
\end{aligned} \tag{73}$$

Next, let $\hat{x}_i(t) < x^*$ and $p_i(t) \leq p^*$ be the upper bounds of the the event-triggered state and integral action trajectories respectively, where the existence of positive constants x^* and p^* are guaranteed by Theorem 5.1. Using x^* and p^* in (73) gives the upper bound of derivative of the event-triggering as

$$\begin{aligned}
\frac{d}{dt}\|\cdot\| &\leq \alpha \left\|\sum_{i\sim j} \hat{x}_i(t) - \hat{x}_j(t)\right\| + \alpha\beta_i x^* + p^* + \|p_i(0)\| + \left\|\alpha \sum_{i\sim h} k_{ih}(t)\hat{x}_i(t) - c_h(t)\right\|, \\
&\leq 2\alpha N x^* + \alpha\beta_i x^* + 2\,p^* + \alpha h \left(\left\|\sum_{i\sim h} \hat{x}_i(t)\right\| + \left\|\sum_{i\sim h} c_h(t)\right\|\right), \\
&\leq \alpha\left(2Nx^* + \beta_i\right) x^* + 2\,p^* + \alpha h^2 x^* + \alpha h^2 c^*, \\
&\leq \alpha\left(2Nx^* + \beta_i + h^2\right) x^* + 2\,p^* + \alpha h^2 c^*.
\end{aligned} \tag{74}$$

Letting ϕ be an upper bound for (74) and since the initial conditions of each node satisfy $\lim_{t\to s_{i,k^+}} \|\hat{x}_i(t) - x_i(t)\| = 0$, it follows from (74) that

$$\|\hat{x}_i(t) - x_i(t)\| \leq \phi\left(t - s_{i,k}\right), \quad \forall t \in \left[s_{i,k}, s_{i,k+1}\right]. \tag{75}$$

Thus, if (50) is violated, then $\lim_{t\to s^-_{i,k+1}} \|\hat{x}_i(t) - x_i(t)\| = \tau_i$ and it follows from (75) that $s_{i,k+1} - s_{i,k} \geq a_i$. ■

Remark 16. Zeno behavior implies that an agent must update its local state infinitely fast. In the context of the integral action-based distributed control algorithm presented in (7) and (8), Zeno behavior also implies that agents must exchange information continuously. Corollary 8 demonstrates that the inter-sample times for information exchange between nodes are positive scalars; hence, the proposed event-triggered active-passive dynamic consensus filter given by (48) and (49) does not yield Zeno behavior and reduces the total cost of information exchange between nodes.

Remark 17. The results of this section also demonstrate that a node broadcasts its local event-triggered state only when the distance between its current local state and its event-triggered state grows more than a local user-defined error threshold τ_i, which gives agents the ability to update their states asynchronously.

6. ILLUSTRATIVE NUMERICAL EXAMPLES

In this section, we present four illustrative numerical examples. Specifically, Examples 1 and 2 demonstrate the results of Section 4 for the active-passive dynamic consensus filter given by (7) and (8). In addition, Examples 3 and 4 demonstrate the results of Section 5 for the event-triggered active-passive dynamic consensus filter given by (48) and (49).

Example 6. We first consider a network with 25 agents exchanging information over a connected, undirected ring graph topology, where there are 20 passive agents and 5 active agents. Specifically, each active agent is subject to an input with these inputs given by $c_1(t) = \sin(0.1t)$, $c_2(t) = \cos(0.3t)$, $c_3(t) = \cos(0.5t) + 2\sin(0.01t)$, $c_4(t) = 0.5\sin(0.2t)+1.5\cos(0.1t)$, and $c_5(t) = 2$. Letting all agents have arbitrary initial conditions, the bottom of Figure 2 shows the response of this network with the active-passive dynamic consensus filter given by (7) and (8) subject to design parameters $\alpha = 1$, $\gamma = 1$, $\sigma = 0.1\gamma^{-1}$, and $\|\beta\|_2 = 0.001$. To elucidate the results of Corollary 6 and Remark 13, the top of Figure 2 shows the response of the network when we set α to 5 and γ to 50 such that $\frac{1}{\alpha^2}$

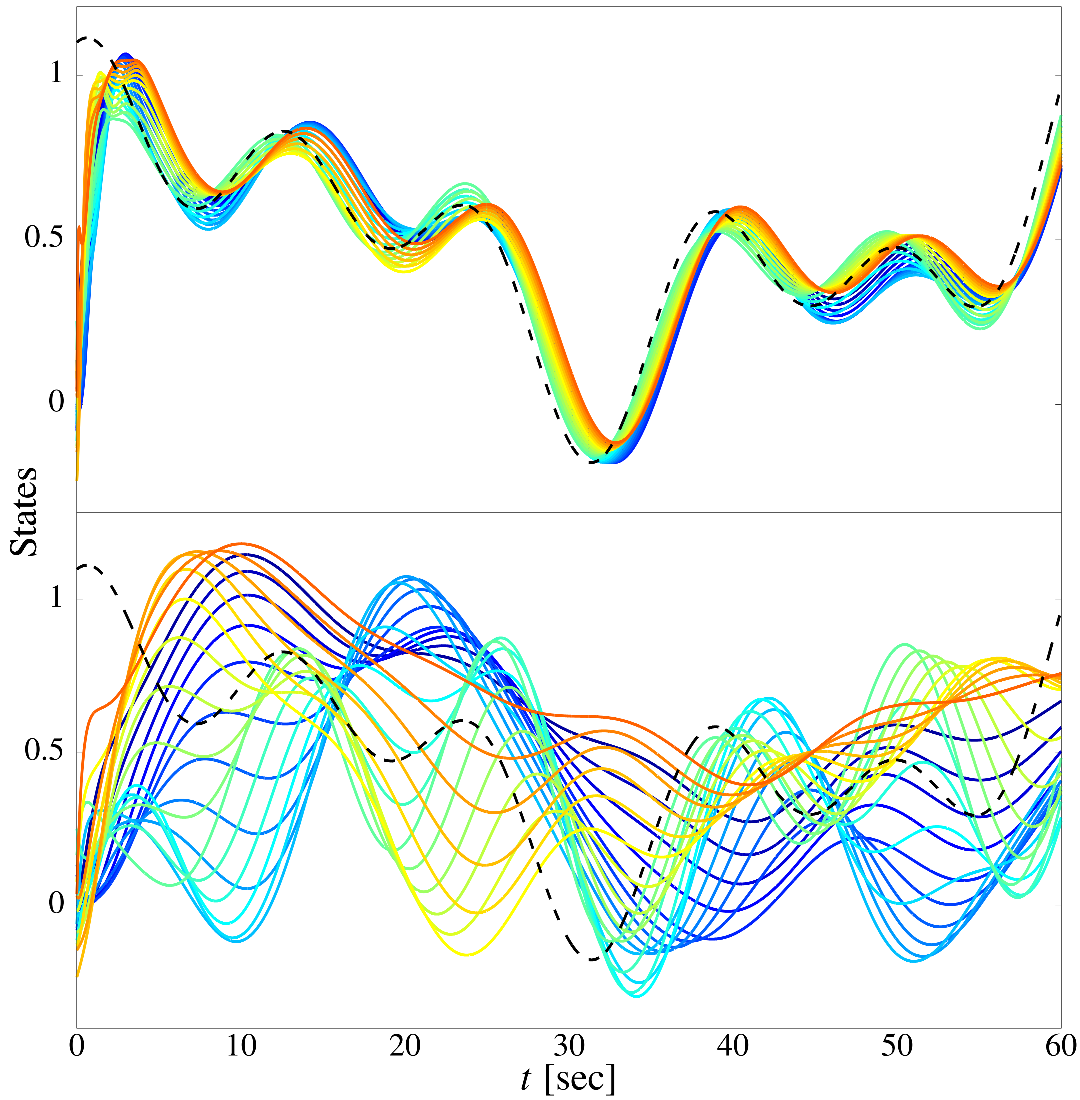

Figure 2. Response of a network with 25 agents communicating over a connected, undirected ring graph topology with the active-passive dynamic consensus filter given by (7) and (8). The bottom graph gives the agent response subject to design parameters $\alpha = 1$, $\gamma = 1$, $\sigma = 0.1\gamma^{-1}$, and $\|\beta\|_2 = 0.001$ (solid lines denote local node states and the dashed line denotes the actual average of the applied inputs). The top graph gives the agent response subject to design parameters $\alpha = 5$, $\gamma = 50$, $\sigma = 0.1\gamma^{-1}$, and $\|\beta\|_2 = 0.001$.

and $\frac{\alpha^2}{\gamma}$ are small; hence, (43) is small for $t \geq T$. To summarize, (43) provides a systematic way to tune the design parameters of the proposed active-passive dynamic consensus filter as demonstrated in this example.

Example 7. Consider a network with 24 agents exchanging information over four different connected, undirected graph topologies, where there are 5 active agents and 19 passive agents. Specifically, each active agent is subject to one of the inputs given by $c_1(t) = \sin(0.1t)$, $c_2(t) = \cos(0.3t)$, $c_3(t) = \cos(0.5t) + 2\sin(0.01t)$, $c_4(t) = 0.5\sin(0.2t) + 0.3\cos(0.1t)$, and $c_5(t) = 2$. Letting all agents have arbitrary initial conditions, Figure 3 shows the response of the event-triggered active-passive dynamic consensus filter given in (48) and (49) communicating over each network where all agents are subject to the design parameters $\alpha = 3$, $\gamma = 15$, $\tau = 0.03$, $\sigma = 0.1\gamma^{-1}$, and $\|\beta\|_2 = 0.01$. To elucidate the results of Corollary 8 and following well-known results in literature (see, for example, [38], [39]), Figure 3 demonstrates that networks with a high edge density, corresponding to a large Fielder eigenvalue (λ) of the graph Laplacian, drive the states of all agents to a close neighborhood of the weighted average of the applied exogenous inputs. In particular, the bottom of Figure 3 considers a network where agents are only able to exchange information with their nearest two neighbors in a ring topology, demonstrating that networks with low edge densities are not able to track the weighted average of the applied exogenous inputs in a sufficently close fashion. The top three networks in Figure 3 show that increasing the network edge density, increasing the Fielder eigenvalue of the graph Laplacian, allows agents to more closely track the weighted average of the set of applied exogenous inputs.

Example 8. We next consider a sensor network example with 9 agents tracking a dynamic target as illustrated in Figure 4. In particular, each agent has a sensing radius; hence, the roles of active and passive agents change with respect to time as the dynamic target moves in this planar environment. Figure 5 shows the response of the sensor network with the active-passive dynamic consensus filter given by (7) and (8) subject to design parameters $\alpha = 20$, $\gamma = 150$, $\sigma = 0.1\gamma^{-1}$, and $\|\beta\|_2 = 0.001$. In this figure, the sensor network reconstructs (i.e., estimates) the true trajectory of the target through local information exchange, where this clearly illustrates the efficacy of the proposed active-passive dynamic consensus filter.

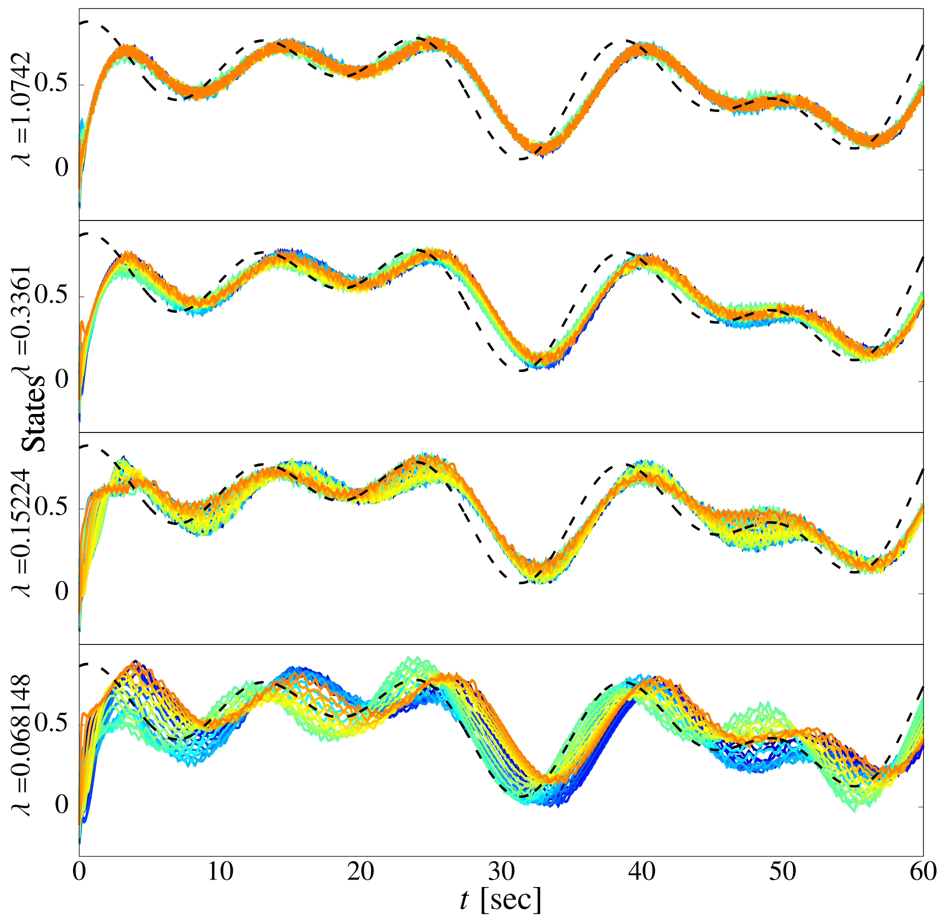

Figure 3. States of 24 agents communicating over connected, undirected graphs. Note that networks with a large edge density, corresponding to a large Fielder eigenvalue (λ) of the graph Laplacian, allows agents to converge to a close neighborhood of the weighted average of the applied exogenous inputs. Dashed lines denote the average of the set of applied exogenous inputs and colored lines denote the agent states.

Example 9. We now consider a network of four agents communicating over a connected, undirected Y graph topology using the event-triggered active-passive dynamic consensus filter given by (48) and (49). Three agents are subject to filtered square wave inputs with arbitrarily chosen frequencies and amplitudes, where agents are aiming to track the average of the applied exogenous inputs. Figure 6 demonstrates that all agents converge to a neighborhood of the average exogenous inputs for several different values of τ_i, $i = 1, 2, 3, 4$. Note that as the value of τ_i decreases, the agents converge closer to the average of the

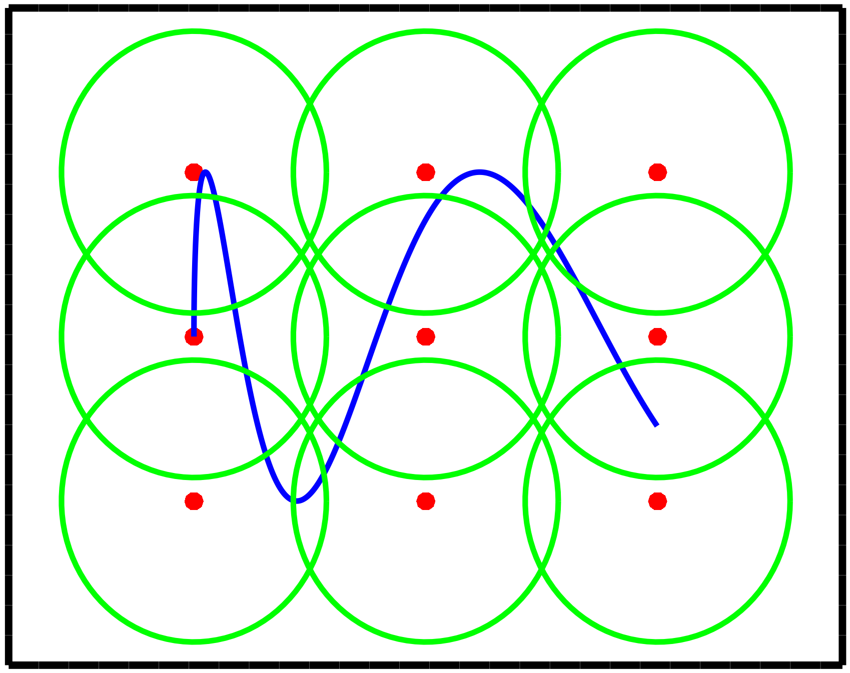

Figure 4. Sensor network with 9 agents tracking a dynamic target (dots denote the agents, circles denote the sensing radius of agents, and the solid line denotes the target trajectory).

exogenous inputs. Figure 7 contrasts the total cost of interagent information exchange required by the previous active-passive dynamic consensus filters in (7) and (8) with the total cost of interagent information exchange required by the proposed filter given by (48) and (49) for varying values of τ_i. As can be seen, as the value of τ_i increases, the total cost of information exchange decreases.

Example 10. Finally, we demonstrate the benefits of using the event-triggered active-passive dynamic consensus filters given by (48) and (49) as opposed to the active-passive dynamic consensus filter proposed in (7) and (8) for a multiple target tracking scenario. Specifically, consider a network of 19 agents, each responsible for sensing a portion of the environment given in Figure 8, where the red and green balls move around the environment. An agent is considered active for a certain ball if any portion of the ball is within the agent's sensing

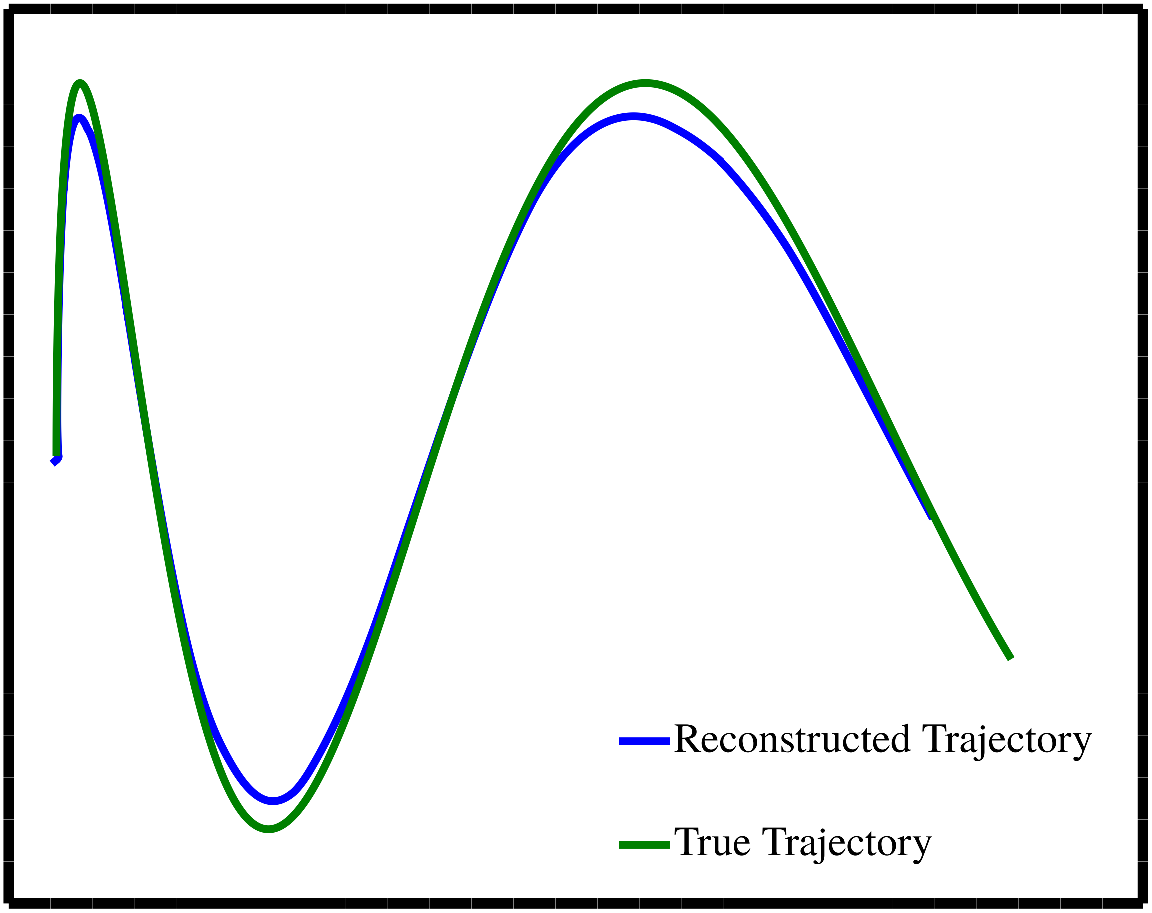

Figure 5. Response of the sensor network depicted in Figure 4 with 9 agents communicating over a connected, undirected graph topology with the active-passive dynamic consensus filter given by (7) and (8) subject to design parameters $\alpha = 20$, $\gamma = 150$, $\sigma = 0.1\gamma^{-1}$, and $\|\beta\|_2 = 0.001$.

field and passive otherwise. As can be seen from Figure 9, while both filters reconstruct the position of both balls, the cost of interagent communication is lower for the event-triggered filter[3].

[3] A supplementary video can be viewed at `https://youtu.be/fRdQ6if7fG8`.

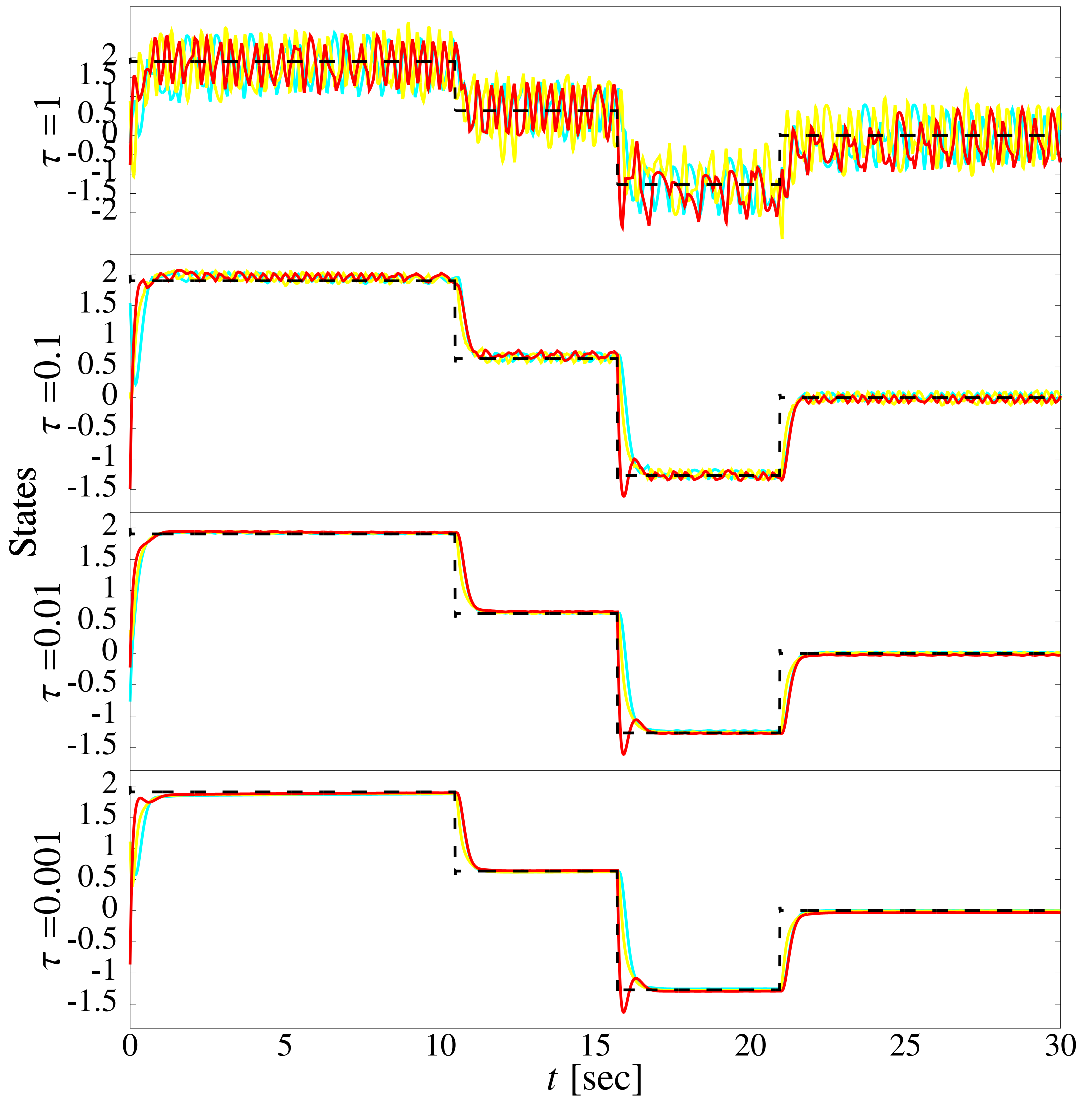

Figure 6. Event-triggered states of 4 agents communicating over a connected, undirected Y graph topology, where 3 agents are active and 1 is passive, with the values of τ_i, $i = 1, 2, 3, 4$ as indicated. Note that as τ_i increases, the error between each agent's event-triggered state and the actual average of the applied inputs increases (the black dotted line indicates the average of the actual applied exogenous inputs and solid colored lines represent the event-triggered state of an agent).

7. EXPERIMENTAL STUDY

In this section, we apply the event-triggered active-passive dynamic consensus filter given by (48) and (49) to a target tracking scenario. Specifically, we use a network of four Raspberry Pi B+ model computers with Raspberry Pi camera modules to track a predefined

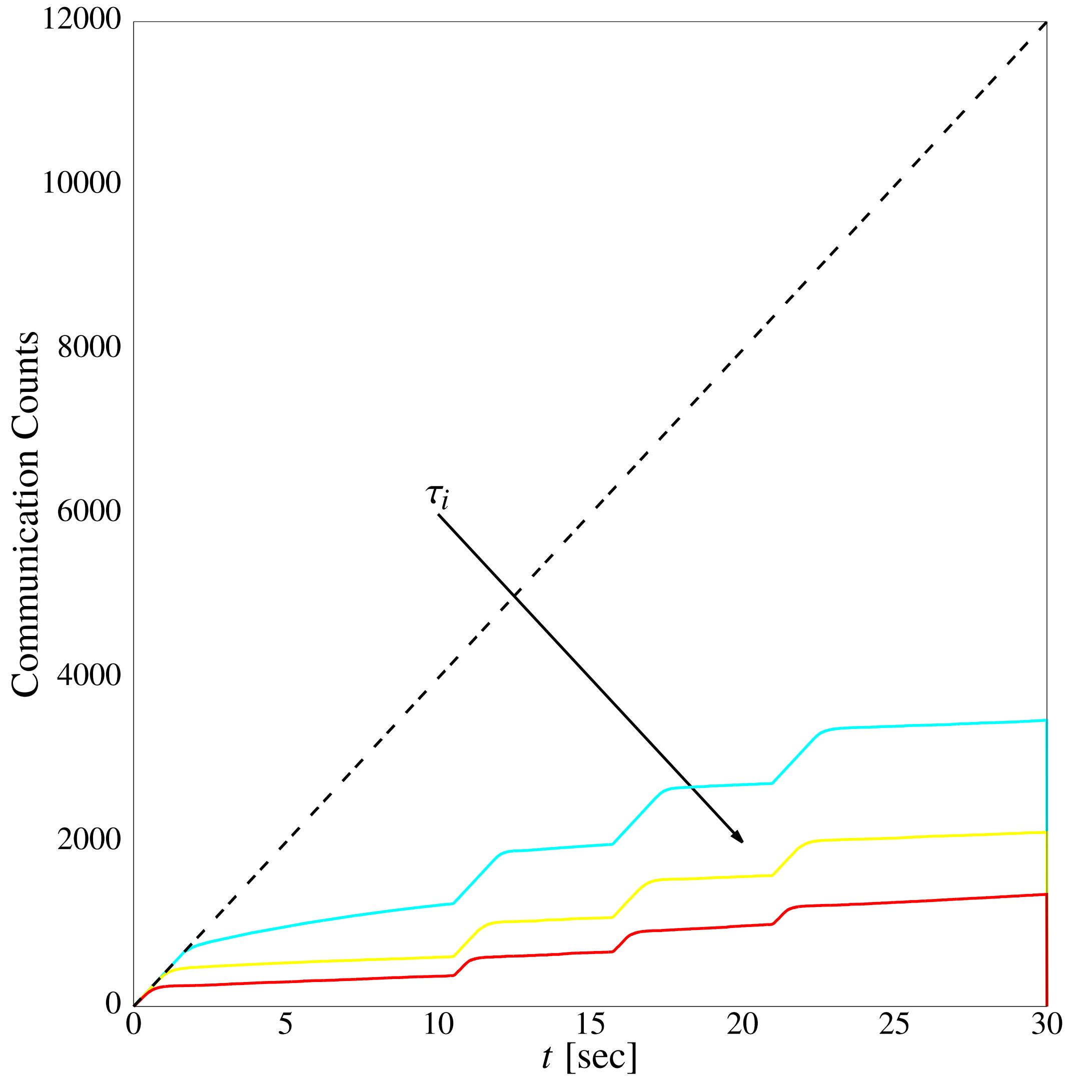

Figure 7. Total cost of communication for a network of 4 agents communicating over a connected, undirected ring graph with $\alpha = 5$, $\gamma = 50$, $\sigma = 0.1$, and $\beta = 0.001$. The black dotted line denotes the total cost of communication for the filter given by (7) and (8), and the solid colored lines denote the total cost of communication for the proposed filter given by (48) and (49) for a range of values of $\tau_i \in [0.0001, 1]$, $i = 1, 2, 3, 4$, with τ_i increasing as the total number of required communications decreases.

target as it moves through an environment, where a camera is considered to be an active agent if it is able to sense the target and passive if it is not able to sense the target. Each computer is a standard Raspberry Pi computer with a Broadcom BCM2835 CPU clocked at 700MHz and 1GB of RAM. Each computer has a dedicated 1080p camera module with

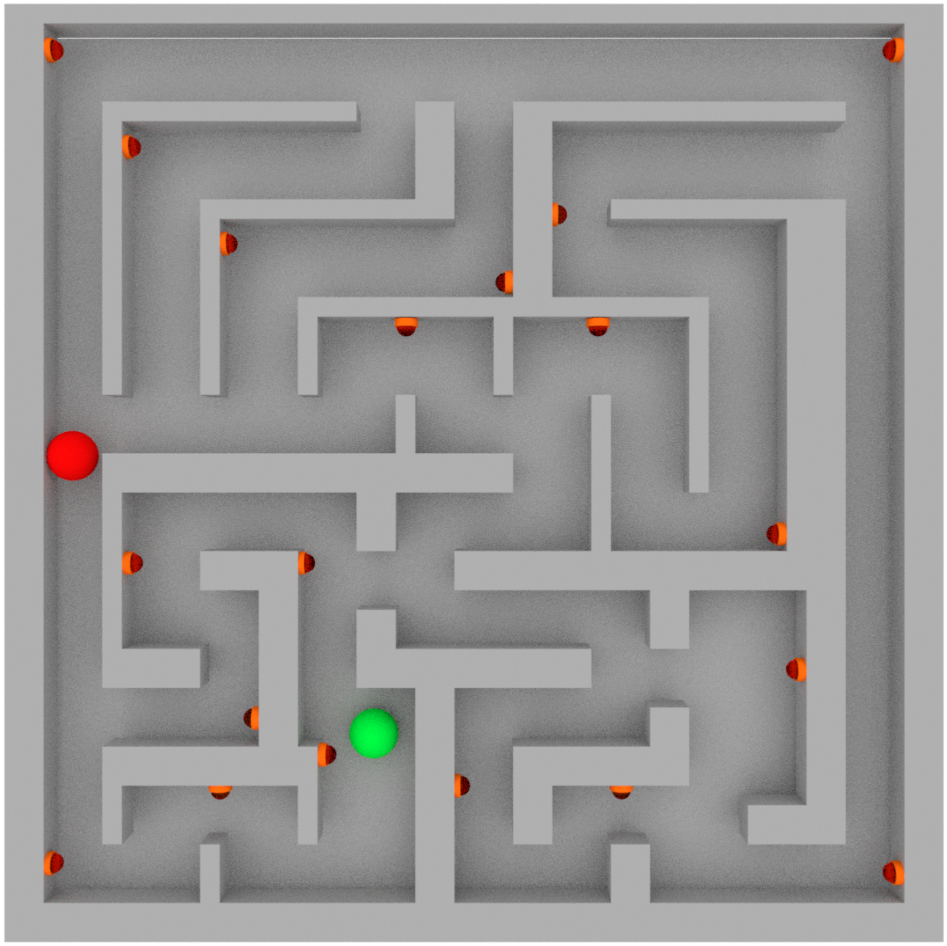

Figure 8. Placement of 19 cameras for tracking two balls, red and green, as they move through an environment. Orange dots denote cameras.

a 62 degree field of view, allowing four cameras to sense the entire test environment. The cameras capture 640x480 pixel images at a rate of 10 frames per second to prevent overloading the processor during image processing.

The OpenCV vision processing library was employed to track faces using the LBPH-FaceRecognizer algorithm. The system was trained to recognize two faces, labeled '0' and '1', and with '-1' indicating an unknown face. If a '-1' is present in the recognition results, the results are considered invalid and the agent is considered as passive in order to mitigate the effects of noise in the system. When a valid face is recognized, the corresponding label is used as an input into a Python implementation of the algorithm given in (48) and

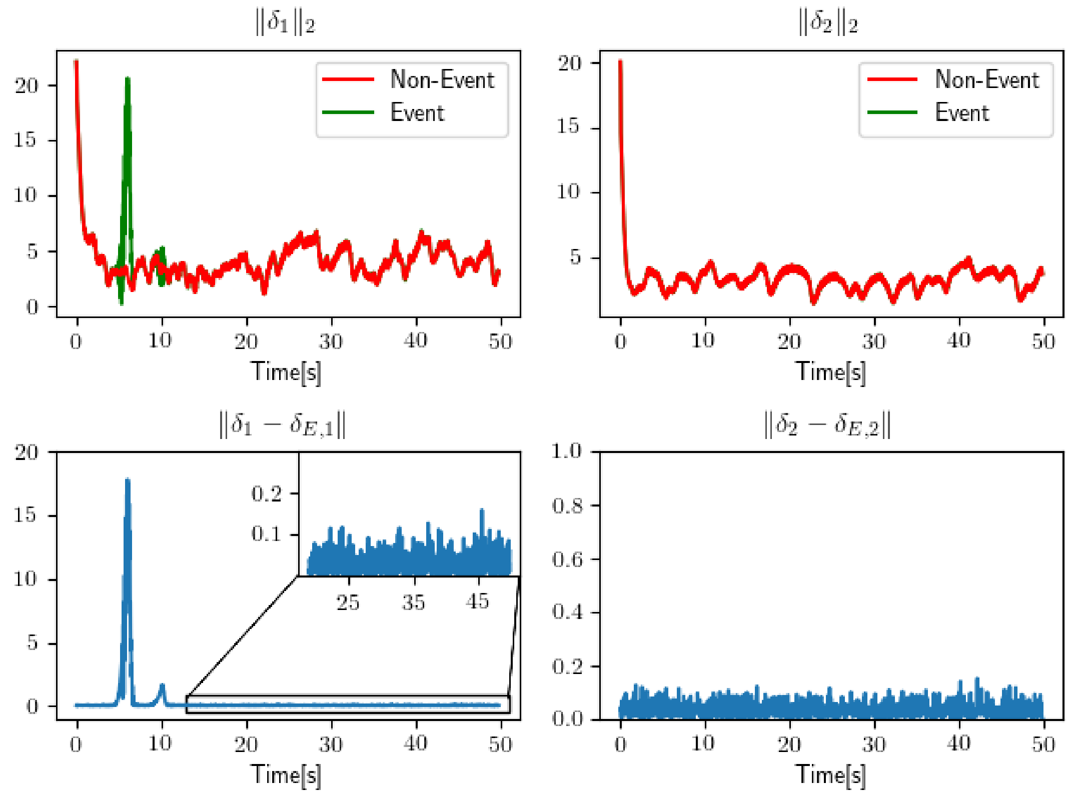

Figure 9. Two norm of the error trajectories of each ball. The top panels compare the error of the non-event-triggered trajectories with the error of the event-triggered trajectories. The bottom panels give the two norm of the difference between the non-triggered state trajectories and the triggered trajectories.

(49), where we have set $\alpha = 2$, $\gamma = 20$, $\sigma = 0.1\gamma^{-1}$, and $\tau = 0.1$. Note that since the event-triggered active-passive dynamic consensus filter is employed, agents do not need to synchronize their update times.

Agents communicate over an IPV4 network, where some agents are wireless and some agents are wired. Owing to the heterogeneous nature of the network, a Linksys WRT54G2 router is used for communication. Note that a mesh network could be employed for communication among agents, but was not utilized due to problems with the stock Raspberry Pi wireless adapters not supporting all communication modes. When an agent needs to broadcast new information, the information is sent to the network broadcast address, allowing all agents to listen for new information. Figure 10 shows the view of the

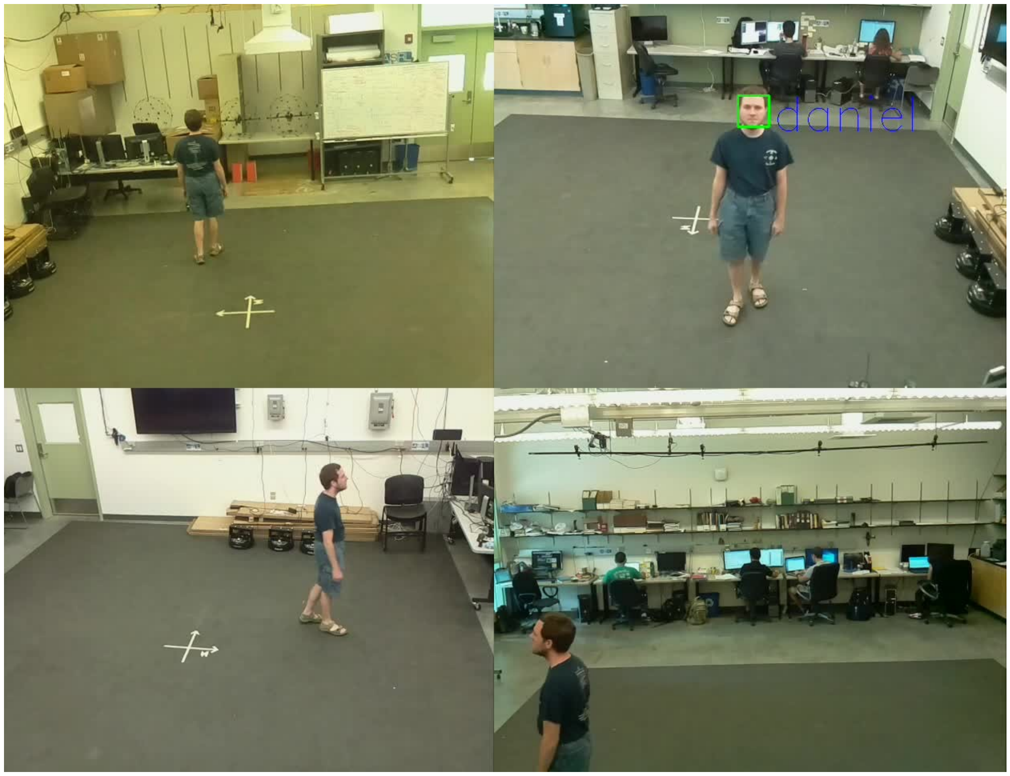

Figure 10. Four Raspberry Pi cameras sensing an environment, where one camera senses a predefined target, and is thus considered as active, and three cameras do not sense any targets, and are considered as passive. Note that the target is denoted with a green box around the face with a label indicating the target's name.

environment as seen from each camera, where one camera is active and the rest are passive. Figure 11 demonstrates that while only one agent is active, all agents are able to report if a target is sensed in the environment[4].

8. CONCLUSION

In this paper, we proposed a new class of active-passive dynamic consensus filters. The proposed filters only require agents to exchange their current state information with neighbors in a simple and isotropic manner and, importantly, allow the roles of active and passive agents to be time-varying for making them suitable for a wider range of

[4]A supplementary video can be viewed at `https://youtu.be/p0SCOh2J1_U`.

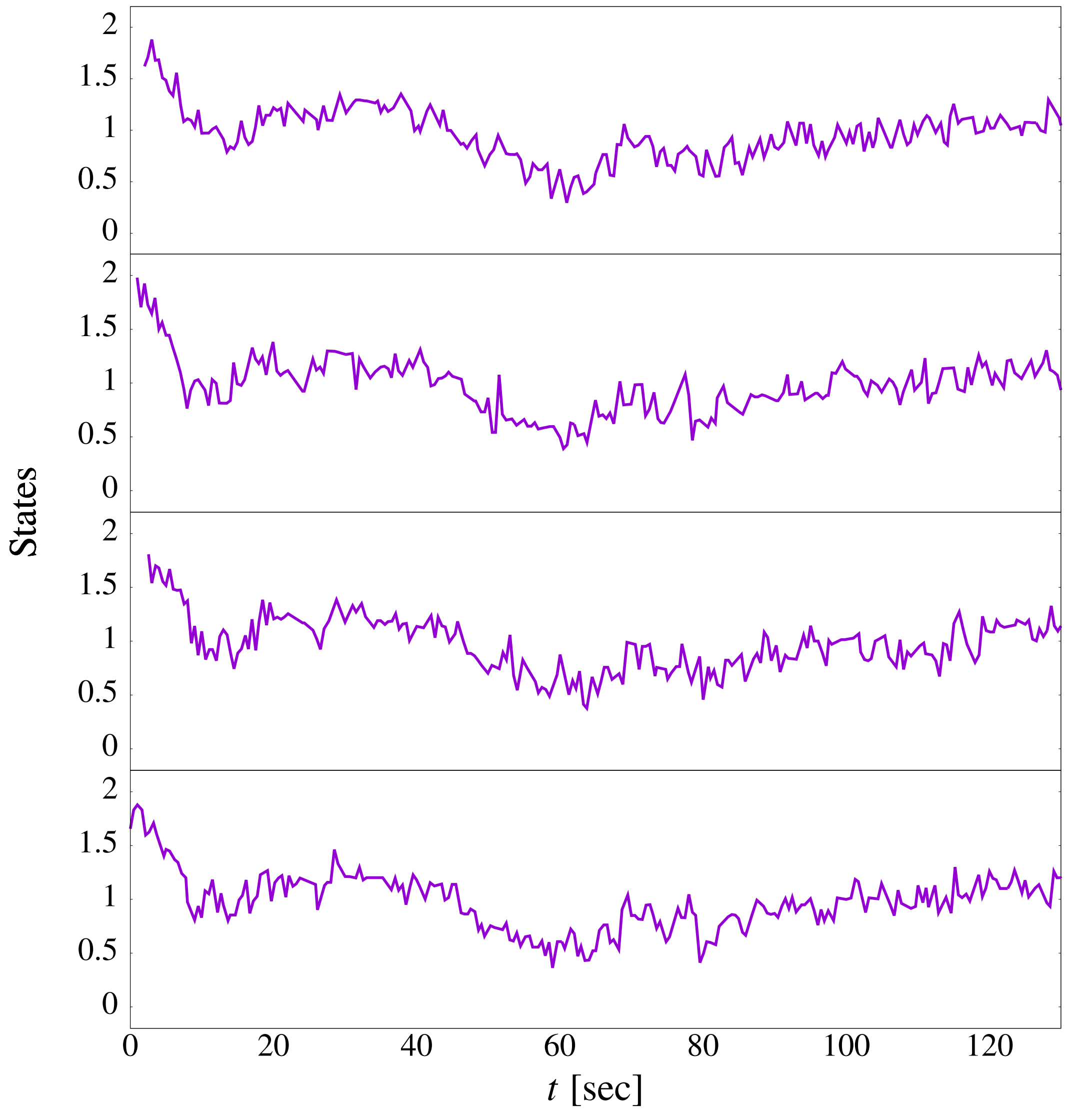

Figure 11. State responses of four Raspberry Pi cameras tracking the predefined target, where $\alpha = 2$, $\gamma = 20$, $\sigma = 0.1$, and $\tau = 0.1$. Agents exchange the ID of sensed target and reach an agreement on which target is identified using the event-triggered active-passive dynamic consensus filter given by (48) and (49) as seen in the 4 plots. Note that the target in the photos has ID number 1 and the agents converge near the value 1.

multiagent systems applications. Specifically, we showed that the proposed active-passive dynamic consensus filters enable the states of all agents to converge to a neighborhood of the average of the observations sensed by a time-varying set of active agents and we provided a systematic way to tune the design parameters of the proposed filters to make this neighborhood small for achieving a desired overall network performance. In addition, we extended our results using tools and methods from event-triggered control theory to further reduce the total cost of interagent information exchange and to remove the need for agents to synchronize their information update intervals. Finally, four illustrative numerical examples and an experimental study demonstrated the efficacy of the proposed theoretical contribution.

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VI. RESILIENT CONTROL OF LINEAR TIME-INVARIANT NETWORKED MULTIAGENT SYSTEMS

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ABSTRACT

A local state emulator-based adaptive control law is proposed for multiagent systems with agents having linear time-invariant dynamics. Specifically, we present and analyze a distributed adaptive control architecture, where agents achieve system-level goals in the presence of exogenous disturbances. Apart from existing relevant literature that makes specific assumptions on network topologies, agent dynamics, and/or the fraction of agents subjected to disturbances, the proposed approach allows agents to achieve system-level goals — even when all agents are subject to exogenous disturbances. Several numerical examples are provided to demonstrate the efficacy of our approach.

1. INTRODUCTION

The distributed control of networked multiagent systems, in which groups of agents work together to achieve a common goal through local peer-to-peer information exchange, has seen many advancements in the past decade (see, for example, [1], [2], and references therein). Such networks are envisioned for applications in demanding, human interactive, and safety critical systems where resilience in the presence of disturbances is required. Until recently, however, much work has focused on fixed-gain distributed controllers, which are unable to recover the desired performance in the presence of unknown exogenous disturbances as outlined in [3] and [4]. Specifically, these systems do not have a centralized mechanism to monitor for node failures, malicious attacks, network link failures, and other disturbances, which can lead to system instability and failure to achieve the system-level goals as described in [3] and [5].

Several approaches, most notably in [6]–[9], have been developed to detect node disturbances and mitigate their effects. These approaches simply assume that a node's information is no longer usable and all information from the node is ignored, which may not be appropriate in scenarios where the effect of the disturbance can be suppressed. The authors of [8] and [10] make assumptions on the network topology (other than the standard assumption of connectedness) requiring the underlying communication network to be known. In addition, [7], [10], and [11] assume that a maximum number of nodes are disturbed, which can be a strict assumption in hostile environments. Computationally expensive observer techniques are considered in [8] and [9]. In [12], the authors focus on discovering subsets of disturbed nodes and require neighboring nodes to mitigate the disturbance effects.

To address the short–comings of current approaches, we propose in this paper a distributed adaptive control approach for a benchmark consensus problem, without loss of generality, in the presence of exogenous disturbances for agents with linear time-invariant dynamics. Specifically, in order to achieve the desired network performance, an adaptive control approach utilizing local state emulators is employed. Similar approaches are reported in [13] and [14], but only for the case where agents have single integrator dynamics. While the authors of [15]–[17] consider the consensus problem for agents with disturbed linear time-invariant dynamics, [15] only considers disturbances which are polytopic in nature, [16] considers that agents must track an undisturbed leader, which is not practical for leader–less networks, and [17] requires that agents exchange their disturbance estimates as well as their state estimates, which incurs a higher communication cost, and which assumes that the communication channels are not disturbed. We show that for agents with linear time invariant dynamics, the effects of exogenous disturbances affecting any subset (or all) agents can be mitigated though local state information exchange.

The organization of this paper is as follows. First, we introduce some necessary notation from linear algebra and graph theory used throughout this paper. We then present the main results of this paper where we demonstrate the stability of the system in the presence of constant disturbances. Finally, we demonstrate the efficacy of our proposed control approach with several numerical examples.

2. MATHEMATICAL PRELIMINARIES

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}^n$ denotes the set of real $n \times 1$ column vectors, $\mathbb{R}^{m\times n}$ denotes the set of real $m \times n$ matrices, $\mathbb{R}_+$ denotes a set of positive real numbers, $\mathbb{R}_+^{m\times n}$ (resp., $\bar{\mathbb{R}}_+^{m\times n}$) denotes a set of real $m \times n$ positive definite (resp., nonnegative-definite) real matrices, $\mathbb{S}_+^{m\times n}$ (resp., $\bar{\mathbb{S}}_+^{m\times n}$) denotes a set of real, positive definite (resp., nonnegative definite) symmetric real matrices, $\mathbb{Z}$ the set of integers, $\mathbb{Z}_+$ (resp., $\bar{\mathbb{Z}}_+$) denotes the set of positive (resp., nonnegative) integers, $\mathbf{0}_n$ an $n \times 1$ vector with 0 entries, $\mathbf{1}_n$ an $n\times 1$ vector with all entries set to 1, $\mathbf{0}_{m\times n}$ a $m\times n$ matrix with all entries set to 0, $\mathbf{1}_{m\times n}$ a $m\times n$ matrix with all entries set to 1, and I_n denotes the $n\times n$ identity matrix. Furthermore, we write $(\cdot)^T$ for the transpose, $\|\cdot\|^2$ for the Euclidean norm, $\lambda_i(A)$ for the i-th eigenvalue of A (ordered from least to greatest), $\mathrm{diag}(a)$ for the diagonal matrix with the vector a on its diagonal, $[A]_{ij}$ for the entry of the matrix A on the i-th row and j-th column, $\mathrm{spec}(A)$ for the ordered spectrum of the matrix A, and J_A for the Jordan decomposition of the matrix A.

Next, we recall some of the basic notions from graph theory, where we refer to references [4] and [18] for further details. An *undirected* graph $\mathcal{G}$ is defined by a set $\mathcal{V}_\mathcal{G} = \{1, \ldots, N\}$ of *nodes* and a set $\mathcal{E}_\mathcal{G} \subset \mathcal{V}_\mathcal{G} \times \mathcal{V}_\mathcal{G}$ of *edges*. Furthermore, the number of agents, N, in the network is given by $N = |\mathcal{V}_\mathcal{G}|$. If $(i, j) \in \mathcal{E}_\mathcal{G}$, then the nodes i and j

are *neighbors* and the neighboring relation is indicated with $i \sim j$. The *degree* of a node is given by the number of its neighbors. Letting d_i be the degree of node i, then the *degree* matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by $\mathcal{D}(\mathcal{G}) \triangleq \mathrm{diag}(d)$, $d = [d_1, \ldots, d_N]^{\mathrm{T}}$. A *path* $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1, \ldots, L$, and a graph $\mathcal{G}$ is *connected* if there is a path between any pair of distinct nodes. The *adjacency* matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by

$$[\mathcal{A}(\mathcal{G})]_{ij} \triangleq \begin{cases} 1, & \text{if } (i,j) \in \mathcal{E}_{\mathcal{G}}, \\ 0, & \text{otherwise.} \end{cases}$$

The *Laplacian* matrix of a graph, $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_{+}^{n\times n}$, playing a central role in many graph theoretic treatments of multiagent systems, is given by $\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G})$. The spectrum of the Laplacian of a connected, undirected graph can be ordered as

$$0 = \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \leq \cdots \leq \lambda_n(\mathcal{L}(\mathcal{G})). \tag{1}$$

Furthermore, there exist $p,\ q \in \mathbb{R}^n$ such that

$$q^{\mathrm{T}}\mathcal{L}(\mathcal{G}) = 0, \quad \mathcal{L}(\mathcal{G})p = 0, \tag{2}$$

and $q^{\mathrm{T}}p = 1$. Note that q and p are normalized left and right eigenvectors associated with the zero eigenvalue of $\mathcal{L}(\mathcal{G})$, respectively. For the ease of exposition, we will assume $p = \mathbf{1}_N$ for the reminder of this paper without loss of generality. Throughout this paper, we model a given multiagent system by a connected, undirected graph $\mathcal{G}$, where nodes and edges represent agents and inter-agent communication links, respectively. Finally, the results of the following lemma will be used through this paper.

Lemma 6 (Theorem 2, [19])**.** Consider a group of agents communicating over a connected, undirected graph $\mathcal{G}$ where each agent has local dynamics given by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad x_i(t_0) = x_{i0}, \tag{3}$$

$$y_i(t) = Cx_i(t), \tag{4}$$

subject to the controller

$$u_i(t) = -\left[K \sum_{i \sim j} (y_i(t) - y_j(t))\right]. \tag{5}$$

If

$$\mathrm{Rank}(C) = \mathrm{Rank}\begin{pmatrix} C \\ B^{\mathrm{T}}\bar{P} \end{pmatrix}, \tag{6}$$

where $\bar{P} \in \mathbb{S}_{+}^{n\times n}$ satisfies

$$\bar{P}A + A^{\mathrm{T}}\bar{P} - 2\bar{P}BB^{\mathrm{T}}\bar{P} + \mathrm{I}_n = 0, \tag{7}$$

then, if the feedback gain matrix $K = \max\{1, \lambda_{k,min}^{-1}\}K_0$, where $\lambda_{k,min}$ is the minimum non-zero eigenvalue (Fielder eigenvalue) of the associated Laplacian, and K_0 is a solution of

$$K_0 C = B^{\mathrm{T}}\bar{P}, \tag{8}$$

the eignevalues of

$$A - \lambda_k(\mathcal{L}(\mathcal{G}))BKC, \quad k > 1, \tag{9}$$

$$A - d_i BKC, \quad i \in \mathcal{V}(\mathcal{G}), \tag{10}$$

lie in the open left half plane, that is $\mathrm{spec}(A-\lambda_k(\mathcal{L}(\mathcal{G}))BKC) < 0$, and $\mathrm{spec}(A-d_iBKC) < 0$, where $\lambda_k(\mathcal{L}(\mathcal{G}))$ are the non-zero eigenvalues of the associated Laplacian matrix.

3. RESILIENT NETWORKS FOR LINEAR TIME-INVARIANT SYSTEMS

In this section, we propose a networked control approach for coordination of agents with linear time-invariant dynamics in the presence of persistent local agent disturbances. First, we present the local agent system dynamics and propose a novel method for mitigating the effects of local agent disturbances. Next, we demonstrate that the agent dynamics converge to the designed local state emulator dynamics. Finally, we characterize and rigorously analyze the performance of the local agent state emulator.

3.1. PROBLEM FORMULATION

Consider a networked multiagent system whose agents are subject to disturbances such that their dynamics are given by

$$\dot{x}_i(t) = Ax_i(t) + B(u_i(t) + w_i),\ x_i(t_0) = x_{i0}, \tag{11}$$

$$y_i(t) = Cx_i(t), \tag{12}$$

where $x_i(t) \in \mathbb{R}^n$ denotes the state of agent $i, i = 1, 2, \ldots, N$, $u_i(t) \in \mathbb{R}^m$ denotes the control input to agent i, $w_i \in \mathbb{R}^m$ denotes the constant unknown disturbance affecting agent i, $y_i(t) \in \mathbb{R}^l$ denotes the output of agent i, $A \in \mathbb{R}^{n\times n}$ denotes the local agent state transition matrix of agent i, $B \in \mathbb{R}^{n\times m}$ denotes the control input matrix of agent i, $C \in \mathbb{R}^{l\times n}$ denotes the output matrix of agent i, and we assume the system (A, B, C) is stabilizable and detectable.

Remark 18. Note that a local controller may be used to place the eigenvalues of the local state transition matrix A to achieve a desired response.

Since our aim is to mitigate the effect of local disturbances in order to synchronize agent outputs, consider the relative output feedback controller

$$u_i(t) = -\left[K \sum_{i\sim j}(y_i(t) - y_j(t))\right] - \hat{w}_i(t), \tag{13}$$

where $K \in \mathbb{R}^{m\times m}$ is an output feedback gain matrix, $y_j(t)$ is the output of agent $j, j = 1, 2, \ldots, N$, and $\hat{w}_i(t)$ is the estimate of the disturbance of agent i to be designed. Using (12) and (13) in (11), the local agent dynamics can be rewritten as

$$\begin{aligned}\dot{x}_i(t) &= Ax_i(t) - B\left[K \sum_{i\sim j} \big(Cx_i(t) - Cx_j(t)\big) + \hat{w}_i(t) - w_i\right], \\ &= Ax_i(t) - B\left[KC \sum_{i\sim j} \big(x_i(t) - x_j(t)\big) + \hat{w}_i(t) - w_i\right]. \end{aligned} \tag{14}$$

Next, consider the local state emulator for agent i, characterizing the *desired* local behavior, given by

$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) - BKC \sum_{i\sim j}(\hat{x}_i(t) - x_j(t)), \quad \hat{x}_i(t_0) = x_{i0}, \tag{15}$$

where $\hat{x}_i(t)$ denotes the state emulator state of agent i, and note that while the state emulator has no local disturbance sources, disturbances may enter through information exchange.

Our next objective is to design a local weight update law $\hat{w}_i(t)$ to mitigate the effect of the local disturbance w_i. To this end, consider the weight update law given by

$$\dot{\hat{w}}_i(t) = \alpha B^{\mathrm{T}} P_i \tilde{x}_i(t), \quad w_i(t_0) = w_{i0}, \tag{16}$$

with $\alpha > 0$ being the system learning rate, $\tilde{x}_i(t)$ denoting the system error defined as

$$\tilde{x}_i(t) \triangleq x_i(t) - \hat{x}_i(t), \tag{17}$$

and $P_i \in \mathbb{S}_+^{n \times n}$ satisfies the Lyapunov equation

$$P_i(A - d_i BKC) + (A - d_i BKC)^{\mathrm{T}} P_i + Q_i = 0, \tag{18}$$

where $Q_i \in \mathbb{S}_+^{n \times n}$.

Remark 19. If K is chosen according to (8), then $\mathrm{spec}(A - d_i BKC) < 0$ as a direct result of Lemma 6, which implies solutions to (18) exist. Note that Lemma 6 only gives sufficient conditions for solutions to exist, and there may be other methods to choose a valid stabilizing matrix.

Now, using (14) and (15), the system error dynamics, characterizing the difference between the local agent dynamics and the desired local state emulator dynamics, can be given by

$$\dot{\tilde{x}}_i = (A - d_i BKC)\tilde{x}_i(t) + B\widetilde{w}_i(t), \ \tilde{x}_i(t_0) = 0, \tag{19}$$

where the weight update error is defined as

$$\widetilde{w}_i(t) \triangleq w_i - \hat{w}_i(t), \tag{20}$$

and

$$\dot{\widetilde{w}}_i(t) = -\dot{\hat{w}}_i(t). \tag{21}$$

Finally, using (19), the local agent state emulator in (15) can be rewritten as

$$\begin{aligned}\dot{\hat{x}}_i(t) &= A\hat{x}_i(t) - BKC\sum_{i\sim j}(\hat{x}_i(t) - x_j(t)) \pm BKC\sum_{i\sim j}\hat{x}_j(t),\\ &= A\hat{x}_i(t) - BKC\sum_{i\sim j}(\hat{x}_i(t) - \hat{x}_j(t)) + BKC\sum_{i\sim j}\tilde{x}_j(t).\end{aligned} \tag{22}$$

This concludes the setup of our problem. In the next section, we present the performance and stability guarantees for the system given by (11) and (12) subject to the controller (13).

3.2. PERFORMANCE AND STABILITY ANALYSIS OF THE CLOSED-LOOP ERROR DYNAMICS

In this section, we begin our analysis of the networked multiagent system. Specifically, we give sufficient conditions to demonstrate that the local agent dynamics converge to the desired state emulator dynamics. Note that we will discuss the stability and performance of the state emulator dynamics in the next section.

In the next theorem, we show that the system state $x_i(t)$ converges to the state emulator $\hat{x}_i(t)$.

Theorem 3.1. *Consider an agent with uncertain dynamics given by* (11) *and* (12)*, which satisfy condition* (6)*, subject to controller* (13)*, where the feedback gain has been chosen according to* (8)*, with state emulator given by* (15)*, and the adaptive feedback control law given by* (16)*, that exchange local information over a connected, undirected graph $\mathcal{G}$. Then the solution $(\tilde{x}_i(t), B\widetilde{w}_i(t))$ is uniformly exponentially stable for all $(0, B\widetilde{w}_{i0}) \in \mathbb{R}^n \times \mathbb{R}^n$.*

Proof. Consider the Lyapunov function candidate for an individual agent given by

$$V(\tilde{x}_i(t), \widetilde{w}_i(t)) = \tilde{x}_i^{\mathrm{T}}(t)P_i\tilde{x}_i(t) + \frac{1}{\alpha}\widetilde{w}_i^{\mathrm{T}}(t)\widetilde{w}_i(t), \tag{23}$$

and note that $V(0,0) = 0$ and $V(\cdot) > 0$, $\forall \tilde{x}_i(t), \widetilde{w}_i(t) \in \mathbb{R}\backslash\{0\}$. Differentiating $V(\cdot)$ along system trajectories (19) and (21) yields

$$\begin{aligned}
\dot{V}(\cdot) &= \dot{\tilde{x}}_i^{\mathrm{T}}(t)P_i\tilde{x}_i(t) + \tilde{x}_i^{\mathrm{T}}(t)P_i\dot{\tilde{x}}_i(t) + \frac{1}{\alpha}\dot{\widetilde{w}}_i^{\mathrm{T}}(t)\widetilde{w}_i(t), \\
&= \tilde{x}_i^{\mathrm{T}}(t)\left[(A - d_iBKC)^{\mathrm{T}}P_i + P_i(A - d_iBKC)\right]\tilde{x}_i(t) + 2x_i^{\mathrm{T}}(t)P_iB\widetilde{w}_i(t) - 2\tilde{x}_i^{\mathrm{T}}(t)P_iB\widetilde{w}_i(t), \\
&= -\,\tilde{x}_i^{\mathrm{T}}(t)Q_i\tilde{x}_i(t), \\
&\leq -\,\lambda_{\min}(Q)\|\tilde{x}_i(t)\|_2^2.
\end{aligned} \tag{24}$$

Hence, the closed loop error dynamics given by (19) and (21) are Lyapunov stable for all initial conditions. By evoking the Barbashin-Krasovskii-LaSalle Theorem ([20]), $\tilde{x}_i(t)$ uniformly asymptotically vanishes as $t \to \infty$, and as a result of (19), $B\widetilde{w}_i(t) \to 0$ as $t \to \infty$. Additionally, due to the system's linear time-invariant dynamics, since $(\tilde{x}_i(t), B\widetilde{w}_i(t))$ is uniformly asymptoticly stable, then it is also uniformly exponentially stable ([21]). ■

Remark 20. Theorem 3.1 demonstrates that the error system dynamics $(\tilde{x}_i(t), B\widetilde{w}_i(t))$ are exponentially stable, which is sufficient to show that the agent's states, $x_i(t)$, converge to the agent's state emulator, $\hat{x}_i(t)$. However, it is worth noting that, if, in addition to the assumptions outlined in Theorem 3.1, $B^{\mathrm{T}}B$ is invertible, then it can be shown that the solution $(\tilde{x}_i(t), \widetilde{w}_i(t))$ is uniformly exponentially stable for all $(0, \widetilde{w}_{i0}) \in \mathbb{R}^n \times \mathbb{R}^m$.

Remark 21. Note that Theorem 3.1 assumes the local agent dynamics satisfy condition (6), and the feedback gain matrix K has been chosen according to (8), which are sufficient conditions for the existence of $P_i \in \mathbb{S}_+$. If a feedback gain K can be found such that solutions to (18) exist, then the results of Theorem 3.1 hold regardless of the results of Lemma 6.

3.3. PERFORMANCE AND STABILITY ANALYSIS OF THE STATE EMULATOR

In this section, we rigorously analyze the response of the system state emulator. To begin, consider the aggregated state vectors given by

$$x(t) = [x_1^{\mathrm{T}}(t), x_2^{\mathrm{T}}(t), \ldots, x_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{nN}, \tag{25}$$

$$\hat{x}(t) = [\hat{x}_1^{\mathrm{T}}(t), \hat{x}_2^{\mathrm{T}}(t), \ldots, \hat{x}_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{nN}, \tag{26}$$

$$\tilde{x}(t) = [\tilde{x}_1^{\mathrm{T}}(t), \tilde{x}_2^{\mathrm{T}}(t), \ldots, \tilde{x}_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{nN}, \tag{27}$$

$$\widetilde{w}(t) = [\widetilde{w}_1^{\mathrm{T}}(t), \widetilde{w}_2^{\mathrm{T}}(t), \ldots, \widetilde{w}_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{mN}, \tag{28}$$

$$P = \mathrm{diag}([B^{\mathrm{T}}P_1, \ldots, B^{\mathrm{T}}P_N]) \in \mathbb{R}^{mN \times nN}, \tag{29}$$

and using (16), (19), and (22), the system dynamics can be written in the compact form

$$\dot{\hat{x}}(t) = U\hat{x}(t) + [\mathcal{A}(\mathcal{G}) \otimes BKC]\tilde{x}(t), \tag{30}$$

$$\dot{\tilde{x}}(t) = [\mathrm{I}_N \otimes A - \Delta \otimes BKC]\tilde{x}(t) + [\mathrm{I}_N \otimes B]\widetilde{w}(t), \tag{31}$$

$$\dot{\widetilde{w}}(t) = -\alpha P\tilde{x}(t), \tag{32}$$

with

$$U \triangleq \mathrm{I}_N \otimes A - \mathcal{L}(\mathcal{G}) \otimes BKC. \tag{33}$$

Since we are interested in synchronizing the outputs of all agents, we investigate the properties of the associated graph Laplacian as well as the local agent state transition matrix A. To this end, consider the Jordan decompositions

$$\mathcal{L}(\mathcal{G}) = RJ_{\mathcal{L}}(\mathcal{G})R^{-1}, \tag{34}$$

$$A = SJ_AS^{-1}, \tag{35}$$

where R and S are the transformation matrices of the associated graph Lapalcian v and the local agent state transition matrix respectively, and the first column of R is denoted as $p = \mathbf{1}_N$ and the first row of R^{-1} is denoted as q^{T}. Because $\mathcal{L}(\mathcal{G})$ is connected and undirected,

$$J_{\mathcal{L}} = \begin{bmatrix} \lambda_1 & \dots & 0 & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_N \end{bmatrix},$$

$$= \begin{bmatrix} 0 & \mathbf{0}_{N-1}^{\mathrm{T}} \\ \mathbf{0}_{N-1} & \bar{J}_{\mathcal{L}} \end{bmatrix}, \tag{36}$$

with λ_i being the i-th eigenvalue of the associated graph Laplacian ordered according to (1), and

$$\bar{J}_{\mathcal{L}} \triangleq \begin{bmatrix} \lambda_2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_N \end{bmatrix}. \tag{37}$$

Similarly,

$$J_A = \begin{bmatrix} J_A(0) & \mathbf{0}_{n-r\times r}^{\mathrm{T}} \\ \mathbf{0}_{n-r\times r} & \bar{J}_A \end{bmatrix}, \tag{38}$$

where $J_A(0) \in \mathbb{R}^{r\times r}$ are the aggregated Jordan blocks associated to the zero eigenvalue(s) of the state transition matrix (if A has a non-zero null space), r is the algebraic multiplicity of the zero eigenvalue of A and $\bar{J}_A \in \mathbb{R}^{n-r\times n-r}$ being the Jordan blocks associated with the non-zero eigenvalue(s) of A, which implies $\mathrm{spec}(\bar{J}_A) < 0$. The number of Jordan blocks associated to the zero eigenvalue is given by its geometric multiplicity.

Example 11. To elucidate this point, consider an agent who's dynamics have a zero eigenvalue with geometric multiplicity of 2 and algebraic multiplicity of 3, $J_A(0)$ can be given by

$$J_A(0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

▲

Using the decompositions given in (36) and (38), the state emulator transition matrix U given by (33) can be decomposed as

$$
\begin{aligned}
J_U &= (R^{-1} \otimes S^{-1})(\mathrm{I}_N \otimes A - \mathcal{L}(\mathcal{G}) \otimes BKC)(R \otimes S), \\
&= (R^{-1}\mathrm{I}_N R \otimes S^{-1}AS) - (R^{-1}\mathcal{L}(\mathcal{G})R \otimes K_S), \\
&= (\mathrm{I}_N \otimes J_A) - (J_{\mathcal{L}} \otimes K_S), \\
&= \begin{bmatrix} J_A & \dots & \mathbf{0}_n & \mathbf{0}_n \\ \mathbf{0}_n & J_A - \lambda_2 K_S & \dots & \mathbf{0}_n \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_n & \mathbf{0}_n & \dots & J_A - \lambda_N K_S \end{bmatrix}, \\
&= \begin{bmatrix} J_A(0) & \mathbf{0}^{\mathrm{T}}_{N(n-r)} \\ \mathbf{0}_{N(n-r)} & \bar{J}_U \end{bmatrix},
\end{aligned} \tag{39}
$$

where $K_S = S^{-1}BKCS$, $\bar{J}_U \triangleq \mathrm{I}_{N-1} \otimes J_A - \bar{J}_{\mathcal{L}} \otimes K_S$, $\lambda_k, \quad k = 1, 2, \dots, N$ are the non-zero eigenvalues of the associated Laplacian, and the system state emulator dynamics can be equivalently given by

$$\dot{\hat{x}}(t) = T^{-1}J_U T\hat{x}(t) + [\mathcal{A}(\mathcal{G}) \otimes BKC]\tilde{x}(t), \tag{40}$$

with $T \triangleq R \otimes S$.

Next, the system given by (40) can be broken into convergent and non-convergent dynamics, where the convergent dynamics exponentially decay to 0, and the non-convergent dynamics are driven to a solution dependent on the local agent dynamics given by A, which will be analyzed in Theorem 3.3. To this end, let

$$\dot{\hat{z}}(t) = J_A(0)\hat{z}(t) + A_1\tilde{x}(t), \quad \hat{z}(t_0) = \chi_{10}, \tag{41}$$

$$\dot{\hat{c}}(t) = \bar{J}_U\hat{c}(t) + A_2\tilde{x}(t), \quad \hat{c}(t_0) = \chi_{20}, \tag{42}$$

where $\hat{z}(t)$ denotes the non-convergent system dynamics, $\hat{c}(t)$ denotes the convergent system dynamics, $\chi_{10} \in \mathbb{R}^r$ is given by the first r elements of $\hat{x}_0$, $\chi_{20} \in \mathbb{R}^{Nn-r}$ is given by the last $Nn - r$ elements of $\hat{x}_0$, $A_1 \in \mathbb{R}^{r \times Nn}$ is given by the first r rows of $T^{-1}[\mathcal{A}(\mathcal{G}) \otimes BKC]T$, and $A_2 \in \mathbb{R}^{r \times Nn-r}$ is given by the last $Nn - r$ rows of $T^{-1}[\mathcal{A}(\mathcal{G}) \otimes BKC]T$.

Finally, the closed-loop system dynamics given by (31), (32), and (40) can be written in the compact form

$$\dot{\xi}(t) = M\xi(t), \quad \xi(t_0) = [\chi_{10}^{\mathrm{T}}, \chi_{20}^{\mathrm{T}}, 0, \widetilde{w}_0^{\mathrm{T}}]^{\mathrm{T}}, \tag{43}$$

with

$$\xi \triangleq \left[\hat{z}^{\mathrm{T}}(t), \hat{c}^{\mathrm{T}}(t), \tilde{x}^{\mathrm{T}}(t), \widetilde{w}^{\mathrm{T}}(t)\right]^{\mathrm{T}} \in \mathbb{R}^{2Nn+Nm}, \tag{44}$$

and

$$M = \left[\begin{array}{cc|cc} J_A(0) & \mathbf{0} & A_1 & \mathbf{0} \\ \mathbf{0} & \bar{J}_U & A_2 & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathrm{I}_N \otimes A - \Delta \otimes BKC & \mathrm{I}_N \otimes B \\ \mathbf{0} & \mathbf{0} & -\alpha P & \mathbf{0} \end{array}\right], \tag{45}$$

is the partitioned system matrix where the dimensions have been omitted for brevity. The next theorem demonstrates the stability of the closed-loop system dynamics given by (43).

Theorem 3.2. *Consider an agent with uncertain dynamics given by* (11) *and* (12)*, which satisfy condition* (6)*, subject to controller* (13)*, where the feedback gain has been chosen according to* (8)*, with state emulator given by* (15)*, and the adaptive feedback control law given by* (16)*, that exchange local information over a connected, undirected graph $\mathcal{G}$. Then, the convergent system dynamics given by* (42) *are exponentially stable.*

Proof. Consider the partitioned system transition matrix given by (45), which demonstrates that the convergent and non-convergent dynamics can be decoupled, and the stability of the convergent dynamics given by $\hat{c}(t)$ depend only on

$$\mathrm{spec}(\bar{J}_U) \bigcup \mathrm{spec}\left(\begin{bmatrix} \mathrm{I}_N \otimes A - \Delta \otimes BKC & \mathrm{I}_N \otimes B \\ -\alpha P & 0 \end{bmatrix}\right). \tag{46}$$

Theorem 3.1 showed that

$$\mathrm{spec}\left(\begin{bmatrix} \mathrm{I}_N \otimes A - \Delta \otimes BKC & \mathrm{I}_N \otimes B \\ -\alpha P & 0 \end{bmatrix}\right) < 0, \tag{47}$$

which implies that we only need demonstrate that $\mathrm{spec}(\bar{J}_U) < 0$. Consider that $\mathrm{spec}(\bar{J}_U)$ is given by

$$\begin{aligned}\mathrm{spec}(\bar{J}_U) &= \{\mathrm{spec}\,(J_A - \lambda_k(\mathcal{L}(\mathcal{G}))K_s) : k \in [2, N]\}\,, \\ &= \{\mathrm{spec}\,(A - \lambda_k(\mathcal{L}(\mathcal{G}))BKC) : k \in [2, N]\}\,, \\ &< 0, \end{aligned} \tag{48}$$

as direct consequence of Lemma 6, and it follows that the convergent mode system dynamics (42) are exponentially stable. ■

Remark 22. Note that Theorem 3.2 assumes the local agent dynamics given satisfy condition (6), and the feedback gain K has been chosen according to (8), which are sufficient conditions for $\mathrm{spec}(\bar{J}_U) < 0$. If a feedback gain K can be found such that $\mathrm{spec}(\bar{J}_U) < 0$ holds, then the results of Theorem 3.2 hold regardless of the results of Lemma 6.

Remark 23. Theorem 3.2 implies that the states of the local agent system dynamics corresponding to the negative eigenvalues of the local agent system state transition matrix A are exponentially stable and the corresponding shared states of $x_i(t)$ exponentially converge. Note that the stability of the system then depends only on the stability of the states corresponding to the zero eigenvalue(s) of the local agent system state transition matrix A.

In the next theorem, we analyze the stability of the system's non-convergent dynamics given by (41).

Theorem 3.3. *Consider an agent with uncertain dynamics given by* (11) *and* (12)*, which satisfy condition* (6)*, subject to controller* (13)*, where the feedback gain has been chosen according to* (8)*, with state emulator given by* (15)*, and the adaptive feedback control law given by* (16)*, that exchange local information over a connected, undirected graph $\mathcal{G}$. Then, all agents reach a consensus.*

Proof. Consider the non-convergent system dynamics given by (41). Then, the non-convergent state emulator dynamics can be given by

$$\dot{\hat{x}}_r(t) = (p^{\mathrm{T}} \otimes S^{-1})J_A(0)(q \otimes S)\hat{x}_r(t) + A_1\tilde{x}(t), \ \ \hat{x}_r(0) = \chi_{10}, \tag{49}$$

with $\hat{x}_r(t)$ denoting the local state emulator states corresponding to zero eigenvalues(s) of the local system transition matrix. Then, the solution to the system described by (49) can be given by

$$\hat{x}_r(t) = (p^{\mathrm{T}} \otimes S^{-1})e^{J_A(0)t}(q \otimes S)\chi_{10} + \int_0^t (p^{\mathrm{T}} \otimes S^{-1})e^{J_A(0)(t-\tau)}(q \otimes S)A_1\tilde{x}(t)d\tau. \tag{50}$$

In Theorem 3.1, we demonstrated $\tilde{x}(t)$ is uniformly exponentially stable, which implies

$$\int_0^\infty (p^{\mathrm{T}} \otimes S^{-1})e^{J_A(0)(t-\tau)}(q \otimes S)A_1\tilde{x}(t)d\tau = \theta, \tag{51}$$

such that $||\theta|| < \theta^*$where $\theta \in \mathbb{R}^{r\times 1}$ and θ^* is a computable upper bound. Then, the solution (50) implies that the non-convergent system dynamics can be given by

$$\hat{x}_r(t) \rightarrow (\mathbf{1}_N \otimes S)e^{J_A(0)t}(q^{\mathrm{T}} \otimes S^{-1})\hat{x}(t_0) + \theta, \tag{52}$$

and each individual agent's state emulator converges to

$$\hat{x}_i(t) \rightarrow \sum_{k\in\mathcal{V}(\mathcal{G})} q_k S e^{J_A(0)t} S^{-1}\hat{x}_k(t_0) + \theta_i. \tag{53}$$

Since θ is bounded, the non-convergent system dynamics are bounded and it follows that all agents reach a consensus as $t \rightarrow \infty$. ■

Remark 24. Theorem 3.3 demonstrates that all agents will reach a consensus as $t \rightarrow \infty$ on a quantity determined by the structure $Se^{J_A(0)t}S^{-1}$, which represents the average of each agent's response to the local system dynamics corresponding to the zero eigenvalue(s) of the local state transition matrix A, replicating well known results in literature (see, for example, [22],[23]). In addition, if the system is undisturbed, the non-convergent system dynamics will converge to the quantity given by

$$\hat{x}_i(t) \rightarrow \sum_{k\in\mathcal{V}(\mathcal{G})} q_k S e^{J_A(0)t} S^{-1}\hat{x}_k(t_0). \tag{54}$$

Note that (54) may have a non-zero steady state response as demonstrated in the following examples.

Example 12. Consider the case were each agent has first-order integrator dynamics given as $\dot{x}_i = u_i$. In this case, $S = 1$ and $J_A(0) = 0$ and, as a result, $x_i(t) \longrightarrow q^{\mathrm{T}}x(t_0)$ as $t \longrightarrow \infty$. Note this example only shows where the states of each agent corresponding to the zero eigenvalue of the state transition matrix A will converge and does not include disturbances. ▲

Example 13. Next, consider the case where local agent dynamics are second-order integrators given as $\ddot{x}_i = u_i$ with the Jordan decomposition of the local state transition matrix A given by $S = \mathrm{I}_2$ and $J_A(0) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. The solution (54) can be given as $\dot{x}_i(t) \longrightarrow q^{\mathrm{T}}\dot{x}_j(t_0)$, $x_i(t) \longrightarrow q^{\mathrm{T}}x_j(t_0) + q^{\mathrm{T}}\dot{x}_j(t_0)t$ as $t \longrightarrow \infty$. Note this example only shows where the states of each agent corresponding to the zero eigenvalues of the state transition matrix A will converge and does not include disturbances. ▲

Remark 25. Note that Theorem 3.3 assumes the local agent dynamics given satisfy condition (6), and the feedback gain matrix K has been chosen according to (8), which are sufficient conditions for the results of Theorem 3.1 to hold. If Theorem 3.1 holds, the results of Theorem 3.3 hold regardless of the results of Lemma 6.

Remark 26. As shown in equation (50), $\tilde{x}(t)$ acts as a vanishing disturbance to the system's non-convergent dynamics. Note, if $\|\theta\|_2$ is sufficiently small, then agents not only achieve consensus but consensus will occur near the undisturbed system consensus point, $\hat{x}_i(t) \longrightarrow \sum_{k\in\mathcal{V}(\mathcal{G})} q_k S e^{\theta t} S^{-1}\hat{x}_k(t_0)$. In addition, increasing α will decrease $\|\theta\|_2$. For proof, consider the summation of the Lyapunov function candidates in the proof of Theorem 3.1 given by

$$V(\tilde{x}(t), \tilde{w}(t)) = \sum_{i\in\mathcal{V}(\mathcal{G})} \tilde{x}_i^{\mathrm{T}}(t)P_i\tilde{x}_i(t) + \tilde{w}_i{}^{\mathrm{T}}(t)\tilde{w}_i(t)/\alpha. \tag{55}$$

Taking the time derivative yields

$$\dot{V}(\tilde{x}(t), \tilde{w}(t)) = -\sum_{i\in\mathcal{V}(\mathcal{G})} \tilde{x}_i^{\mathrm{T}}(t)Q_i\tilde{x}_i(t). \tag{56}$$

Therefore, $V(\tilde{x}(t), \tilde{w}(t)) \leq V(\tilde{x}(t_0), \tilde{w}(t_0)) = \tilde{w}_0^{\mathrm{T}}\tilde{w}_0/\alpha$ and $\sum_{i\in\mathcal{V}(\mathcal{G})} \tilde{x}_i^{\mathrm{T}}(t)P_i\tilde{x}_i(t) \leq \tilde{w}_0^{\mathrm{T}}\tilde{w}_0/\alpha$. As α is increased, the magnitude of the vanishing perturbation term $\|\tilde{x}\|$ becomes smaller, decreasing $\|\theta\|_2$. However, as with all adaptive control architectures, increasing the learning rate excessively may result in reduced time delay margins, highly oscillatory control inputs, and other implementation issues ([24], [25]).

4. ILLUSTRATIVE NUMERICAL EXAMPLES

In this section, we demonstrate the efficacy of our approach through several numerical examples. In particular, it is shown that agents achieve consensus in the presence of constant, randomly selected disturbances. In our first example, we consider agents with second order linear dynamics where output feedback is utilized to reach a consensus. In our second example, we reach a consensus on the states of three F-16 aircraft with full state feedback.

4.1. EXAMPLE 1: OUTPUT FEEDBACK

Consider a network of three agents whose dynamics are given by

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C = [2.99 \quad 1.94], \tag{57}$$

which communicate according to the connected, undirected graph described by

$$\mathcal{L}(\mathcal{G}) = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}. \tag{58}$$

Solving for (7) for this system yields,

$$\bar{P} = \begin{bmatrix} 2.6458 & 1.0000 \\ 1.0000 & 0.6458 \end{bmatrix}, \tag{59}$$

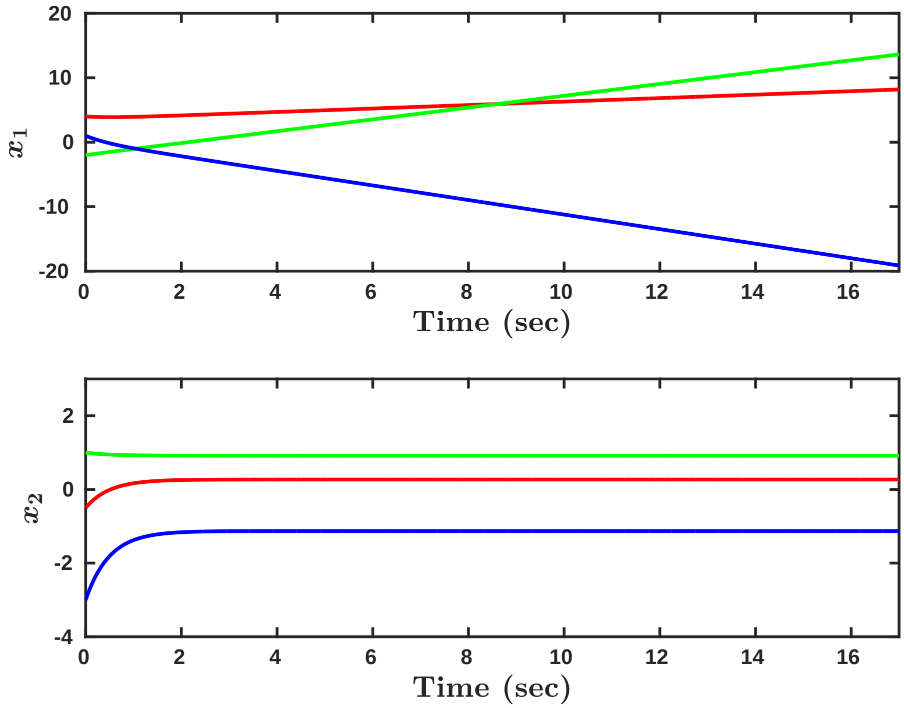

Figure 1. States of each agent subject to constant disturbances with only the standard consensus controller applied.

which satisfies (6) and gives $K = 0.3333$. Letting $x_1(0) = [4, -0.5]$, $x_2(0) = [-2, 1]$, and $x_3(0) = [1, -3]$, we see that without control, the states of each agent's dynamics naturally tends to $x_1(\infty) = [3.75, 0]$, $x_2(\infty) = [-1.5, 0]$, and $x_3(\infty) = [-0.5, 0]$. Noting that $q^T = \frac{1}{3}[1\ 1\ 1]$ is a left eigenvalue of (58) and solving (54), we find the undisturbed system approaches $x_{i1}(\infty) = 0.58$ and $x_{i2}(\infty) = 0$ as $t \to \infty$. With only the standard consensus algorithm, Figure 1 shows that the agents cannot reach a consensus in the presence of disturbances, where the constant disturbances are randomly selected as $w_i \in [-1, 1]$. Using the controller in (13) and (16) with the parameters designed above and $\alpha = 1$, the system reaches a consensus even in the presence of constant disturbances as seen in Figure 2. Increasing the learning gain α drives the system closer to the undisturbed system centroid as shown in Figure 3. ▲

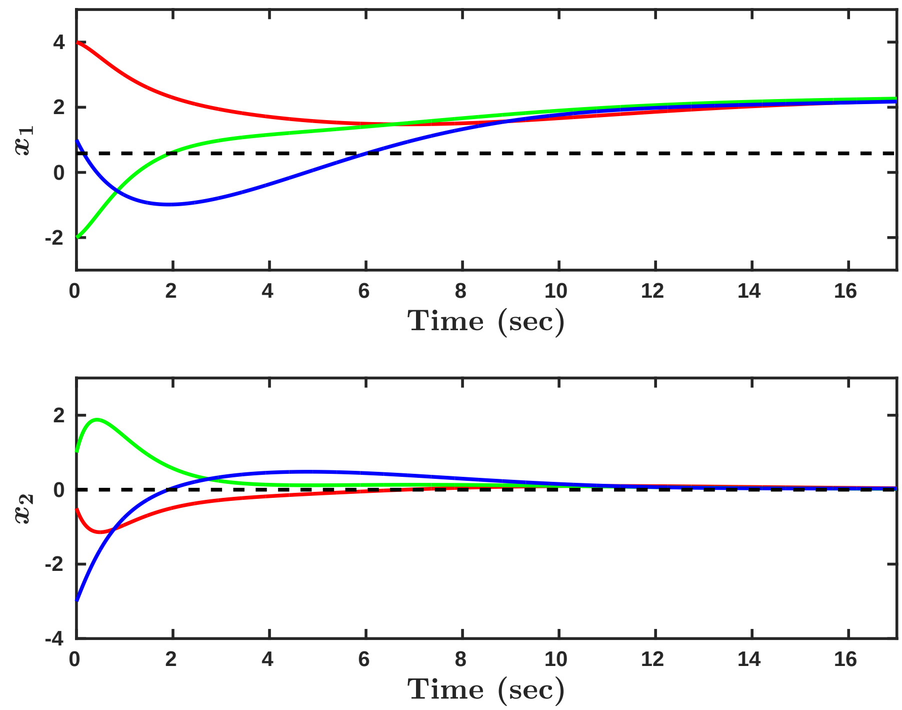

Figure 2. States of each agent with the controller in (13) and (16) applied where $\alpha = 1$. The dashed line indicates the undisturbed system centroid.

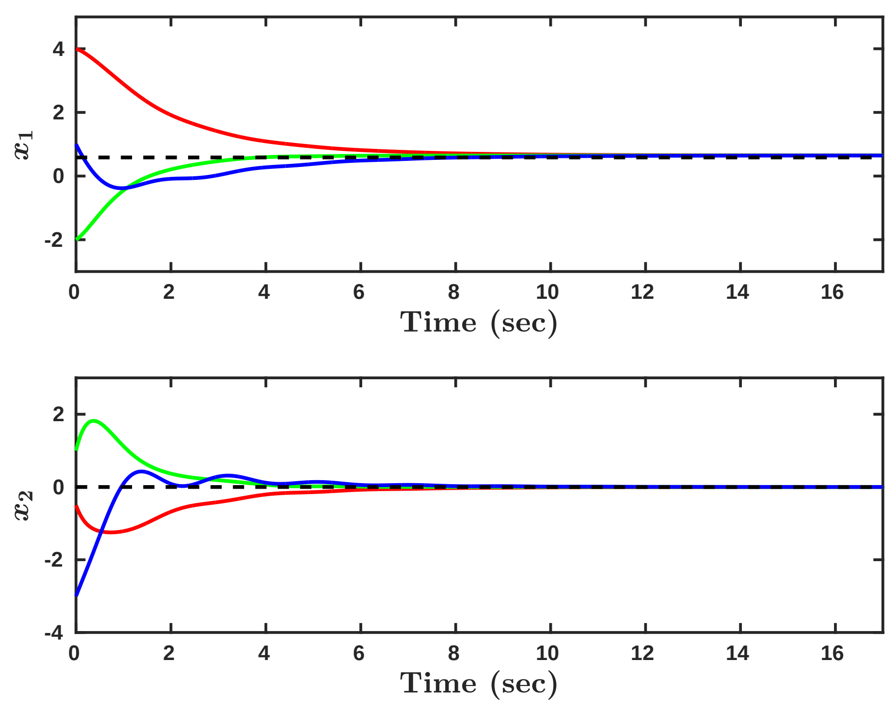

Figure 3. States of each agent with the controller in (13) and (16) applied where $\alpha = 15$. The dashed line indicates the undisturbed system centroid.

4.2. EXAMPLE 2: F-16 AIRCRAFT

In this example, we consider the longitudinal dynamics of three identical F-16 aircraft whose dynamics are described by

$$A = \begin{bmatrix} -0.0507 & -3.8610 & 0 & -32.2000 \\ -0.0012 & -0.5164 & 0.9283 & -0.0975 \\ -0.0001 & 1.4168 & -2.1382 & -2.2372 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}, \tag{60}$$

$$B = \begin{bmatrix} 0 \\ -0.0717 \\ -1.6450 \\ 0 \end{bmatrix}, \tag{61}$$

as given in [26], where the states μ, α, q, and θ are the *change in aircraft speed*, *angle of attack* (AOA), *pitch rate*, and *pitch*, respectively. Agents exchange their states over a connected, undirected line communication graph. Our aim is to synchronize the aircraft μ values in the presence of disturbances, where the constant disturbances are randomly selected as $w_i \in [-0.08, 0.08]$. Figure 4 demonstrates that the standard consensus controller is insufficient to synchronize the system outputs. In Figure 5, we show that using the controller in (13) and (16), the system converges to near the undisturbed system centroid given by (54). Increasing the learning gain α brings the convergence point closer to the undisturbed system centroid as shown in Figure 6. ▲

5. CONCLUSION

To contribute to resilient networked multiagent control, we have presented a novel state emulator based adaptive control architecture. In particular, we have demonstrated the proposed controller is able to mitigate the effects of constant disturbances and synchronize the outputs of each agent. Unlike previous studies, which make assumptions on agent

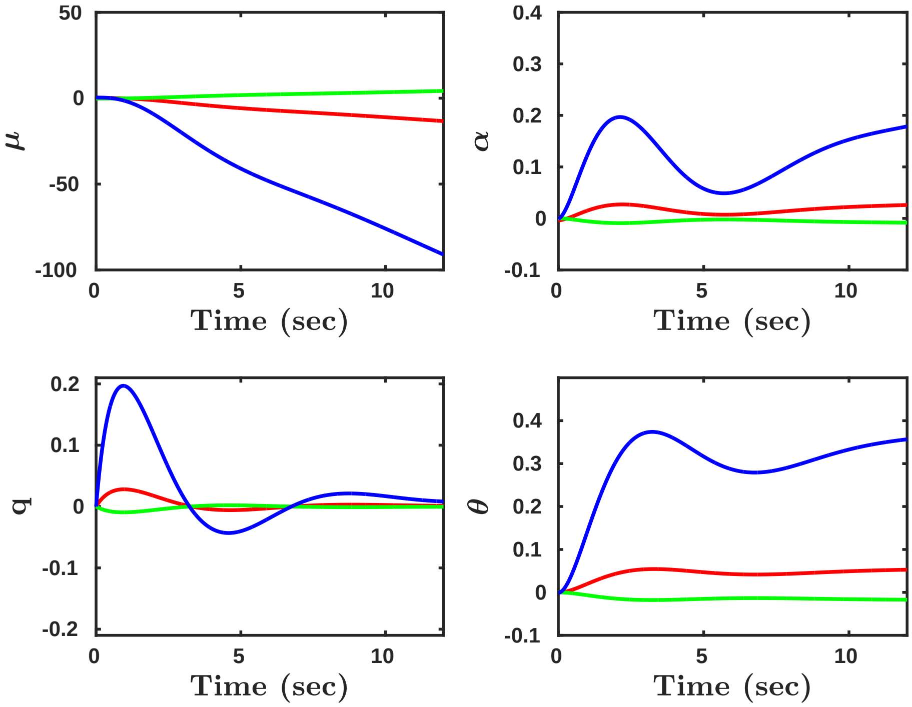

Figure 4. States of each aircraft subject to constant disturbances with only the standard consensus controller applied.

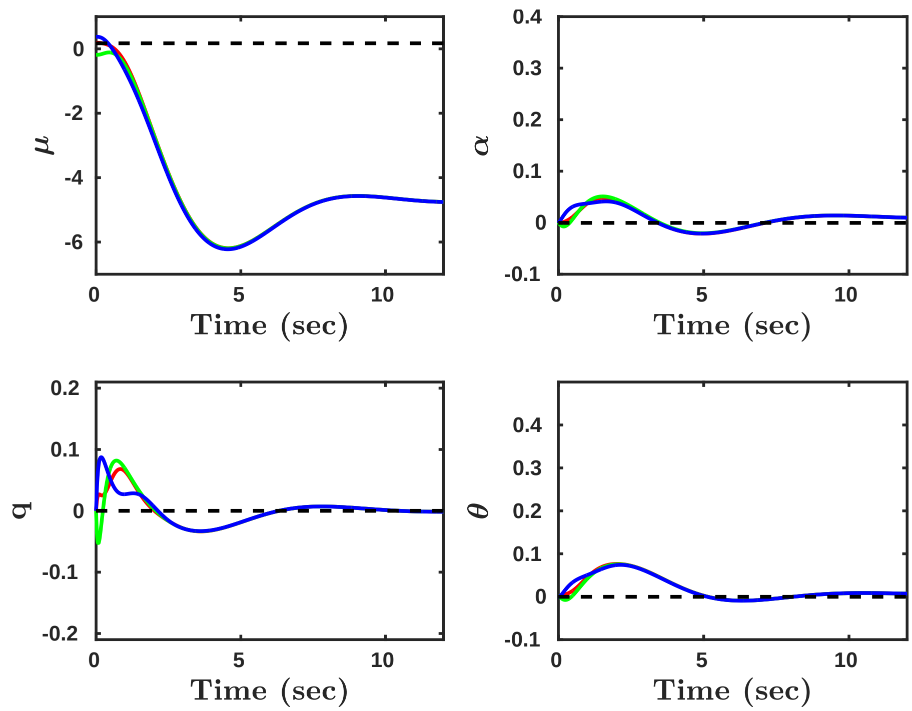

Figure 5. States of each aircraft with the controller in (13) and (16) applied where $\alpha = 1$. The dashed line indicates the undisturbed system centroid.

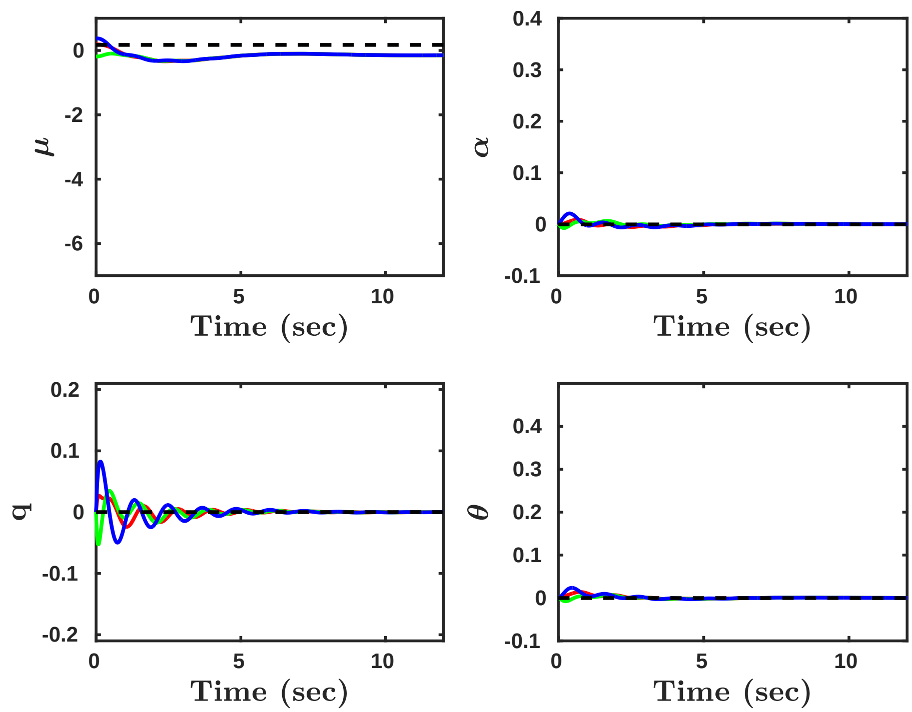

Figure 6. States of each aircraft with the controller in (13) and (16) applied where $\alpha = 15$. The dashed line indicates the undisturbed system centroid.

dynamics and network topologies, the presented results hold for agents with general linear time-invariant dynamics communicating over a connected undirected directed graph, even when all agents are subject to disturbances.

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VII. ACTIVE-PASSIVE DYNAMIC CONSENSUS FILTERS FOR LINEAR TIME-INVARIANT MULTIAGENT SYSTEMS

J. Daniel Peterson, Tansel Yucelen, and S. Jagannathan

ABSTRACT

Active-passive dynamic consensus filters consist of a group of agents, where a subset of these agents are able to observe a quantity of interest (*i.e. active agents*) and the rest are subject to no observations (*i.e. passive agents*). Specifically, the objective of these filters is that the states of all agents are required to converge to the weighted average of the set of observations sensed by the active agents. Existing active-passive dynamic consensus filters in the classical sense assume that all agents can be modeled as having single integrator dynamics, which may not always hold in practice. Motivating from this standpoint, the contribution of this paper is to introduce a new class of active-passive dynamic consensus filters, where agents have (homogeneous) linear time-invariant dynamics. We demonstrate that for output controllable agents, the *output* of all active and passive agents converge to a neighborhood of the weighted average of the set of applied exogenous inputs. A numerical example is also given to illustrate the efficacy of the presented theoretical results.

1. INTRODUCTION

Distributed information fusion is a task performed by a collection of agents, where agents communicate local information with neighbors to reach an agreement on a quantity of interest. Owing to their distributed attributes, it can impact a wide array of applications that range from mission planning to surveillance and reconnaissance to guidance and control of autonomous vehicles. At the core of many information fusion methods is a consensus or consensus-like algorithm needed for the information fusion process. Among two important classes of consensus algorithms, static and dynamic consensus algorithms, only dynamic consensus algorithms consider dynamic information, which is required for many applications. Existing dynamic consensus filters, notably [1]–[5], are suitable for applications where *all* agents are able to sense time-varying quantities of interest. From a practical standpoint, an agent may be passive (unable to sense a quantity of interest) for certain time instants. The distributed information fusion algorithms in [6]–[10] make notable contributions to address this problem. However, these works assume all agents can be modeled as single or double integrator dynamic systems which may not always hold in practice.

The authors of [2], [11]–[17] make notable contributions to information fusion for agents with linear time-invariant dynamics. However, only [2], [16], [17] consider agents which are active for (able to sense) exogenous quantities of interest. Of these works, only [17] considers the case where some agents are passive for certain time instants. The authors of [17] only considers the case where the linear time-invariant dynamics (A,B,C) of the exogenous system of interest are known. In addition, they consider their exogenous system as a leader in a leader-follower paradigm, where this paper considers that active agents are subjected to (possibly multiple) exogenous inputs with unknown dynamics.

The contribution of this paper is to present a new class of active-passive dynamic consensus filters, where all agents have linear time-invariant dynamics. Specifically, we build on the recent active-passive dynamic consensus filters presented in [6]–[10] as well as utilize the tools and ideas of presented in [12], [15] to develop a static output feedback algorithm for agents with linear time-invariant dynamics, which drives the *outputs* of all active and passive agents to a neighborhood of the weighted average of a set of applied exogenous inputs sensed by the active agents.

2. PRELIMINARIES

In this paper, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbb{R}_+$ denotes the set of positive real numbers, $\mathbb{R}_+^{n\times n}$ denotes the set of $n \times n$ positive-definite real matrices, $\mathbb{S}_+^{n\times n}$ denotes the set of $n \times n$ symmetric positive-definite real matrices, $\mathbf{0}_n$ denotes the $n \times 1$ vector of all zeros, $\mathbf{1}_n$ denotes the $n \times 1$ vector of all ones, $\mathbf{0}_{n\times n}$ denotes the $n \times n$ zero matrix, and I_n denotes the $n \times n$ identity matrix. We also write $(\cdot)^{\mathrm{T}}$ for transpose, $(\cdot)^{-1}$ for inverse, $(\cdot)^{\dagger}$ for generalized inverse, $\|\cdot\|_2$ for the Euclidian norm, $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of the matrix A, $\lambda_i(A)$ for the i-th eigenvalue of A (the eigenvalues of A are ordered from least to greatest value), $\mathrm{diag}(a)$ for the diagonal matrix with the vector a on its diagonal, and $[A]_{ij}$ for the entry of the matrix A on the i-th row and j-th column. The singular value decomposition of the positive definite matrix A is given by $A = SUS^{\mathrm{T}}$ where U is a diagonal matrix with the eigenvalues of A on the diagonal, the columns of S are the eigenvectors of A and $SS^{\mathrm{T}} = \mathrm{I}$.

Next, we concisely overview some key notions from graph theory (see, for example, [18], [19] for details). In particular, an undirected graph $\mathcal{G}$ is defined by a set $\mathcal{V}_{\mathcal{G}} = \{1, \ldots, n\}$ of nodes and a set $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$ of edges. If $(i, j) \in \mathcal{E}_{\mathcal{G}}$, then the nodes i and j are neighbors and $i \sim j$ indicates the neighboring relation. The degree of a node is the number of its neighbors. Letting d_i be the degree of node i, then the degree matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is $\mathcal{D}(\mathcal{G}) \triangleq \mathrm{diag}(d)$, $d = [d_1, \ldots, d_n]^{\mathrm{T}}$. A path $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1, \ldots, L$, and a graph $\mathcal{G}$ is said to be connected when there exists a path between any pair of distinct nodes. We write $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{n\times n}$ for the adjacency matrix of a graph $\mathcal{G}$, which is defined by $[\mathcal{A}(\mathcal{G})]_{ij} \triangleq 1$ if $(i, j) \in \mathcal{E}_{\mathcal{G}}$ and $[\mathcal{A}(\mathcal{G})]_{ij} \triangleq 0$ otherwise. Moreover, we write $\mathcal{B}(\mathcal{G}) \in \mathbb{R}^{n\times m}$ for the incidence matrix of a graph $\mathcal{G}$, which is defined by $[\mathcal{B}(\mathcal{G})]_{ij} \triangleq -1$ if node i is the tail of edge j, $[\mathcal{B}(\mathcal{G})]_{ij} \triangleq 1$ if node i is the head of edge j, and $[\mathcal{B}(\mathcal{G})]_{ij} \triangleq 0$ otherwise, where m is the number of edges, i is an index for the node set, and j is an index for the edge set (we assume that directions are arbitrarily assigned for labeling the edges). By definition, $\mathcal{B}^{\mathrm{T}}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_m$. The graph Laplacian matrix, denoted by $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_{+}^{n\times n}$, is defined by $\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G})$ or equivalently

$$\mathcal{L}(\mathcal{G}) \triangleq \mathcal{B}(\mathcal{G})\mathcal{B}^{\mathrm{T}}(\mathcal{G}). \tag{1}$$

The spectrum of $\mathcal{L}(\mathcal{G})$ for a connected, undirected graph can be ordered as

$$0 = \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \leq \cdots \leq \lambda_n(\mathcal{L}(\mathcal{G})), \tag{2}$$

with $\mathbf{1}_n$ as the eigenvector corresponding to the zero eigenvalue and $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$. The results of this paper assume a connected, undirected graph $\mathcal{G}$ with nodes and edges receptively denoting agents and interagent communication links. Next, we introduce several necessary lemmas.

Lemma 7 ([20])**.** The determinant of the block matrix

$$M \triangleq \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \tag{3}$$

can be given as

$$\det(M) = \det(D) \cdot \det(A - BD^{-1}C), \tag{4}$$

where D is a non-singular matrix.

Lemma 8 ([21], p. 742)**.** Consider a dynamic system whose dynamics are given by

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{5}$$

$$y(t) = Cx(t), \tag{6}$$

where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^p$ is the output, $u(t) \in \mathbb{R}^m$ is the input, $A \in \mathbb{R}^{n\times n}$ is a system state matrix, $B \in \mathbb{R}^{n\times m}$ is a system input matrix, and $C \in \mathbb{R}^{p\times n}$ is a system output matrix. The system (A, B, C) is said to be *output controllable* when the output $y(t)$ can be driven from any initial output $y(0)$ to any final output $y(t_f)$ in the finite time interval

$0 \le t \le t_f$. The system given by (5) and (6) is output controllable when the matrix

$$\begin{bmatrix} CB & CAB & CA^2B & \cdots & CA^{n-1}B \end{bmatrix}, \tag{7}$$

has rank p (i.e., has full row rank).

Lemma 9 ([22], Theorem 3.7.1)**.** For any matrix $C \in \mathbb{R}^{p\times n}$ with C having full row rank, there is a $n \times p$ matrix C_R^{-1} such that $CC_R^{-1} = \mathrm{I}_p$. Here, C_R^{-1} is termed the *right inverse* of C.

3. OVERVIEW OF ACTIVE-PASSIVE DYNAMIC CONSENSUS FILTERS

We now provide a concise an overview of the recent active-passive dynamic consensus filters (see [9], [10] for further details). In particular, N agents exchange information with each other through a connected, undirected graph $\mathcal{G}$, and that there are $l \ge 1$ inputs (i.e local observations) that are received (i.e., sensed) by the active subset of agents. For this setup, recall the following definitions from [9], [10].

Definition 3. If agent i, $i = 1, \ldots, N$, is subject to (i.e., senses) one or more inputs (resp., no inputs), then it is an *active* agent (resp., *passive* agent).

Definition 4. If an input interacts with (i.e., is sensed by) only one agent (resp., multiple agents), then it is an *isolated* input (resp., *nonisolated* input).

The active-passive dynamic consensus filter architecture focuses on the problem of driving the states of all agents, active and passive, to the weighed average of the exogenous inputs applied to active agents. For this purpose, the authors of [9], [10] propose the integral action-based distributed control algorithm given by

$$\dot{x}_i(t) = -\alpha \sum_{i\sim j} \big(x_i(t) - x_j(t)\big) - \alpha\beta_i x_i(t) + p_i(t) - e^{-\gamma\sigma t} p_i(0) - \alpha \sum_{i\sim l} k_{il}(t)\,(x_i(t) - r_l(t)),$$

$$x_i(0) = x_{i0}, \tag{8}$$

$$\dot{p}_i(t) = -\gamma \sum_{i\sim j} \big(x_i(t) - x_j(t)\big) - \gamma\sigma p_i(t),\ p_i(0) = p_{i0}, \tag{9}$$

where $x_i(t) \in \mathbb{R}$ denotes the state of agent i, $i = 1, \ldots, N$, that is exchanged with the neighbors of this agent, and $p_i(t) \in \mathbb{R}$ denotes the integral-action state of agent i, $r_l(t) \in \mathbb{R}$, $l = 1, \ldots, h$, denotes an input sensed by this agent, and α, σ, β_i, and $\gamma \in \mathbb{R}_+$. Note that $i \sim l$ notation indicates the exogenous inputs that an agent is subject to, which is similar to the $i \sim j$ notation defined in Section 2.

4. LINEAR TIME-INVARIANT ACTIVE-PASSIVE DYNAMIC CONSENSUS FILTER

In this section, we extend the active-passive dynamic consensus filters algorithm presented in [9], [10] using the tools and ideas from [12], [15], to remove the restriction that agents must be modeled as having single integrator dynamics. We demonstrate that, for agents with output controllable linear time-invariant agents, the proposed controller drives the *outputs* of all active and passive agents to a close neighborhood of the weighted average of a set of applied exogenous inputs.

4.1. PROBLEM STATEMENT

Consider a network of agents with (homogeneous) linear time-invariant dynamics given by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad x_i(0) = x_{i0}, \tag{10}$$

$$y_i(t) = Cx_i(t), \tag{11}$$

where $x_i(t) \in \mathbb{R}^n$ is the state of agent i, $i = 1, 2, \ldots, N$, $u_i(t) \in \mathbb{R}^m$ is the local input of agent i, $y_i(t) \in \mathbb{R}^p$ is the output of agent i, $A \in \mathbb{R}^{n \times n}$ is a system matrix, $B \in \mathbb{R}^{n \times m}$ is an input matrix, and $C \in \mathbb{R}^{p \times n}$ is an output matrix. In addition, consider the active-passive dynamic consensus filter given by

$$u_i(t) = H\Bigg[-\alpha\sum_{i\sim j}\big(y_i(t)-y_j(t)\big) - \beta y_i(t) + p_i(t) - e^{-\gamma\sigma t}p_{i0} - \sum_{i\sim l}k_{il}\left(y_i(t)-r_l(t)\right)\Bigg], \tag{12}$$

$$\dot{p}_i(t) = -\gamma\left[\sum_{i\sim j}\big(y_i(t)-y_j(t)\big) + \sigma p_i(t)\right], p_i(0) = p_{i0}, \tag{13}$$

where $H \in \mathbb{R}^{m\times p}$ is a static feedback gain matrix, $p_i(t) \in \mathbb{R}^p$ is the integral-action state of agent i, $\alpha, \sigma, \gamma, \beta \in \mathbb{R}_+$ are parameters, $k_{il} \in \{0, 1\}$ is the weight of input l, $l = 1, 2, \ldots h$ applied to agent i, and $r_l(t) \in \mathbb{R}^p$ is the l-th input applied to this agent. We also make the following assumptions that are necessary for the results of this paper.

Assumption 1. The output matrix C has full row rank.

Assumption 2. There exists a static feedback controller H that stabilizes the family of dynamic systems given by $A - \mu BHC$ for $\beta \le \mu \le \max_i d_i + Nh + \beta$.

Assumption 3. There is a (unknown) constant $r^* \in \mathbb{R}_+$ such that all of the applied exogenous inputs satisfy $\|r_l(t)\| \le r^*$.

Remark 27. This paper considers that the agent dynamics (A, B, C) are output controllable [21]. However, designing the static output feedback gain, H, is an open problem in literature and is beyond the scope of the results presented here. We refer the readers to [23]–[25] for further study.

Remark 28. Owing to the distributed nature of the proposed output feedback active-passive dynamic consensus filter, the stability of the proposed controller partially depends on agent interactions, requiring that the feedback gain stabilize the family of dynamic systems in assumption 2, where we discuss the necessity of assumption 2 below (see Remark 29).

Since we are interested in driving the outputs of all agents to a close neighborhood of the weighted average of a set of applied exogenous inputs, consider the error

$$\begin{aligned}\Delta_i(t) &\triangleq y_i(t) - \epsilon(t),\\ &= C(x_i(t) - C_R^{-1}\epsilon(t)),\end{aligned} \tag{14}$$

where C_R^{-1} is the right inverse of the output matrix C defined in Lemma 9 and $\epsilon(t)$ is the weighted average of the applied exogenous inputs given by

$$\begin{aligned}\epsilon(t) \triangleq \big[k_{11} + k_{12} + \cdots + k_{1h} + k_{21} + \cdots + k_{Nh}\big]^{-1}\big[k_{11}r_1(t) + k_{12}r_2(t) + \cdots\\ + k_{1h}r_h(t) + k_{21}r_1(t) + \cdots + k_{Nh}r_h(t)\big].\end{aligned} \tag{15}$$

Here, we consider that there are $h \geq 1$ inputs. Note that the denominator of (15) is nonzero owing to the existence of at least one exogenous input. In addition, it directly follows from assumption 3 that there is a constant $\epsilon^* \in \mathbb{R}_+$ such that $\|\epsilon(t)\|_2 \leq \epsilon^*$. This concludes our problem setup.

4.2. ANALYSIS OF THE PROPOSED ACTIVE-PASSIVE DYNAMIC CONSENSUS FILTERS

To analyze the proposed active-passive dynamic consensus filters, consider (14) in the following equivalent form

$$\Delta_i(t) = C\delta_i(t), \tag{16}$$

with

$$\delta_i(t) \triangleq x_i(t) - C_R^{-1}\epsilon(t). \tag{17}$$

Note that (16) implies when $\delta_i(t)$ is small, $\Delta_i(t)$ is small; hence all agents converge to a close neighborhood of the weighted average of the applied exogenous inputs. Motivating from this standpoint, let

$$x(t) \triangleq [x_1^{\mathrm{T}}(t), x_2^{\mathrm{T}}(t), \cdots, x_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{Nn}, \tag{18}$$

$$p(t) \triangleq [p_1^{\mathrm{T}}(t), p_2^{\mathrm{T}}(t), \cdots, p_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{Np}, \tag{19}$$

$$y(t) \triangleq [y_1^{\mathrm{T}}(t), y_2^{\mathrm{T}}(t), \cdots, y_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{Np}, \tag{20}$$

$$r(t) \triangleq [r_1^{\mathrm{T}}(t), \cdots, r_h^{\mathrm{T}}(t), \mathbf{0}_{p\times 1}^{\mathrm{T}}, \cdots, \mathbf{0}_{p\times 1}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{Np}, \tag{21}$$

$$\delta(t) \triangleq [\delta_1^{\mathrm{T}}(t), \delta_2^{\mathrm{T}}(t), \cdots, \delta_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{Nn}, \tag{22}$$

and consider $h \leq N$ without loss of much generality. Applying (12) to (10) and (11), one can compactly write

$$\begin{aligned}\dot{x}(t) = {} & (\mathrm{I}_N \otimes A)x(t) + (\mathrm{I}_N \otimes BH)\big[-(\beta \mathrm{I}_N \otimes C)x(t) - (K_1 \otimes C)x(t) - \alpha(\mathcal{L}(\mathcal{G}) \otimes C)x(t) \\ & + p(t) - e^{\gamma\sigma t}p_0 + (K_2 \otimes \mathrm{I}_p)r(t)\big],\ x(0) = x_0, \end{aligned} \tag{23}$$

$$\dot{p}(t) = -\gamma(\mathcal{L}(\mathcal{G}) \otimes C)x(t) - \gamma\sigma p(t),\ p(0) = p_0. \tag{24}$$

Next, $\delta_i(t)$ can be given in the compact form as

$$\delta(t) = x(t) - \mathbf{1}_N \otimes C_R^{-1}\epsilon(t), \tag{25}$$

whose time derivative is given by

$$\begin{aligned}\dot{\delta}(t) = {} & \big(\mathrm{I}_N \otimes A - \alpha\mathcal{L}(\mathcal{G}) \otimes BHC - \beta \otimes BHC - (K_1 \otimes BHC)\big)\delta(t) + (K_2 \otimes BH)\, r(t) \\ & + (\mathrm{I}_N \otimes BH)\left[\, \big(p(t) + e^{-\gamma\sigma t}p_0\big) - ((K_1 + \beta \mathrm{I}_N) \otimes C)\left(\mathbf{1}_N \otimes C_R^{-1}\epsilon(t)\right)\,\right] \\ & + (\mathrm{I}_N \otimes A)\left(\mathbf{1}_N \otimes C_R^{-1}\epsilon(t)\right),\ \delta(0) = \delta_0, \end{aligned} \tag{26}$$

where $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{N\times N}$ satisfies (2),

$$K_1 \triangleq \mathrm{diag}([k_{1,1}, \ldots, k_{1,N}]^{\mathrm{T}}) \in \overline{\mathbb{S}}_+^{N\times N}, \tag{27}$$

with

$$k_{1,i} \triangleq \sum_{i\sim l} k_{il} \in \overline{\mathbb{R}}_+, \tag{28}$$

denoting the total weight of the inputs applied to agent i, $i = 1, \ldots, N$, and

$$K_2 \triangleq \begin{bmatrix} k_{2,11} & \cdots & k_{2,1N} \\ k_{2,21} & \cdots & k_{2,2N} \\ \vdots & \ddots & \vdots \\ k_{2,N1} & \cdots & k_{2,NN} \end{bmatrix} \in \mathbb{R}^{N\times N}. \tag{29}$$

We refer the reader to [8], [26] for specific examples illustrating the construction of K_1 and K_2 matrices.

Next, consider the solution of (24) given by

$$\begin{aligned} p(t) - e^{-\gamma\sigma t} p_0 &= \int_0^t e^{-\sigma\gamma(t-\tau)} \left(-\gamma \left(\mathcal{L}(\mathcal{G}) \otimes C\right) x(\tau)\right) d\tau, \\ &= \gamma \left(\mathcal{B}(\mathcal{G}) \otimes I_p\right) q(t), \end{aligned} \tag{30}$$

where $\mathcal{B}(\mathcal{G})$ is the incidence matrix of the graph $\mathcal{G}$ satisfying (1), and

$$q(t) \triangleq -(\mathcal{B}^{\mathrm{T}}(\mathcal{G}) \otimes C) \int_0^t e^{-\sigma\gamma(t-\tau)} x(\tau) d\tau, \tag{31}$$

is the transformed integral action state.

Since we are interested in the stability of (26), consider

$$\zeta(t) \triangleq q(t) - \frac{1}{\gamma}(\mathcal{B}^{\mathrm{T}}(\mathcal{G})\mathcal{L}^{+}(\mathcal{G}) \otimes \mathrm{I}_p)\big((K_1 \otimes \mathrm{I}_p)\,(\mathbf{1}_N \otimes \epsilon(t)) - (K_2 \otimes \mathrm{I}_p) r(t)\big), \tag{32}$$

with a time derivative given by

$$\begin{aligned} \dot{\zeta}(t) &= \dot{q}(t) - \frac{1}{\gamma}(\mathcal{B}^{\mathrm{T}}(\mathcal{G})\mathcal{L}^{+}(\mathcal{G}) \otimes \mathrm{I}_p)\big((K_1 \otimes \mathrm{I}_p)\,(\mathbf{1}_N \otimes \dot{\epsilon}(t)) - (K_2 \otimes \mathrm{I}_p)\dot{r}(t)\big) \\ &= -\gamma\sigma\zeta(t) - (\mathcal{B}^{\mathrm{T}}(\mathcal{G}) \otimes C)\big(e^{-\sigma\gamma 0}\delta(t)) - e^{-\sigma\gamma(t-0)}\delta(0)\big) - \frac{1}{\gamma}(\mathcal{B}^{\mathrm{T}}(\mathcal{G})\mathcal{L}^{+}(\mathcal{G}) \otimes \mathrm{I}_p)\cdot \\ &\quad \big((K_1 \otimes \mathrm{I}_p)\,(\mathbf{1}_N \otimes \dot{\epsilon}(t)) - (K_2 \otimes \mathrm{I}_p)\dot{r}(t)\big) \\ &= -(\mathcal{B}^{\mathrm{T}}(\mathcal{G}) \otimes C)\delta(t) - \gamma\sigma\zeta(t) + d_1(t),\ \zeta(0) = 0, \end{aligned} \tag{33}$$

where

$$\begin{aligned} d_1(t) \triangleq{}& (\mathcal{B}^{\mathrm{T}}(\mathcal{G}) \otimes C)e^{-\gamma\sigma t}\delta(0) - \frac{1}{\gamma}(\mathcal{B}^{\mathrm{T}}(\mathcal{G})\mathcal{L}^{+}(\mathcal{G}) \otimes \mathrm{I}_p)\cdot \\ & \Bigg((K_2 \otimes \mathrm{I}_p)\dot{r}(t) - (K_1 \otimes \mathrm{I}_p)\,(\mathbf{1}_N \otimes \dot{\epsilon}(t))\Bigg). \end{aligned} \tag{34}$$

Using the transformation given by (30), (26) becomes

$$\begin{aligned} \dot{\delta}(t) &= \big(\mathrm{I}_N \otimes A - \alpha\mathcal{L}(\mathcal{G}) \otimes BHC - K_1 \otimes BHC - \beta\mathrm{I}_N \otimes BHC\big)\delta(t) \\ &\quad + (\mathrm{I}_N \otimes BH)\Big[\gamma\left(\mathcal{B}(\mathcal{G}) \otimes I_p\right) q(t) - \Big(\Big(\mathrm{I}_N - \mathbf{1}_N\mathbf{1}_N^{\mathrm{T}}\Big) \otimes \mathrm{I}_p\Big)\left[(K_1 \otimes \mathrm{I}_p)\,(\mathbf{1}_N \otimes \epsilon(t))\right. \\ &\quad \left. - (K_2 \otimes \mathrm{I}_p) r(t)\right]\Big] + d_2(t), \\ &= \mathcal{R}\,\delta(t) + \gamma\,(\mathcal{B}(\mathcal{G}) \otimes BH)\,\zeta(t) + d_2(t), \end{aligned} \tag{35}$$

where

$$\mathcal{R} \triangleq \mathrm{I}_N \otimes A - (\alpha\mathcal{L}(\mathcal{G}) + K_1 + \beta\mathrm{I}_N) \otimes BHC, \tag{36}$$

and

$$d_2(t) \triangleq \left(\mathrm{I}_N \otimes AC_R^{-1}\epsilon(t)\right) - (\beta\mathbf{1}_N \otimes BH\epsilon(t)). \tag{37}$$

In addition, it follows directly from assumption 3 that $\|d_1(t)\| \leq d_1^*$ and $\|d_2(t)\| \leq d_2^*$ for positive constants d_1^* and d_2^*.

Now, consider the compact form of the closed loop error dynamics in (35) and (33) given by

$$\dot{g}(t) = Fg(t) + d(t),\ g(0) = g_0, \tag{38}$$

where

$$g(t) = \begin{bmatrix} \delta(t) \\ \zeta(t) \end{bmatrix},\ d(t) = \begin{bmatrix} d_2(t) \\ d_1(t) \end{bmatrix}, \tag{39}$$

with $\|d(t)\|_2 \leq d^*$, and

$$F \triangleq \begin{bmatrix} \mathcal{R} & \gamma\left(\mathcal{B}(\mathcal{G}) \otimes BH\right) \\ -(\mathcal{B}^{\mathrm{T}}(\mathcal{G}) \otimes C) & -\gamma\sigma\mathrm{I}_{Np} \end{bmatrix}, \tag{40}$$

is a Hurwitz matrix for an output controllable system (A, B, C), where a static feedback gain H can be designed according to assumption 2.

Remark 29. A sketch of the proof that (40) is Hurwitz is given in Appendix A. Here, we would like to mention that F is Hurwitz when assumption 2 holds. In other words, one needs to seek the existence of a static feedback gain H such that the real part of the eigenvalues of

$$A - \mu BHC, \tag{41}$$

are negative where μ is an eigenvalue of the matrix $\mathcal{L}(\mathcal{G}) + K1(t) + \beta\mathrm{I}_N$. Owing to its dependency on the communication graph $\mathcal{G}$ and using the Gershgorin disk theorem ([22]), it can be shown that

$$\beta < \mu < (\max_i d_i + \beta + Nh), \tag{42}$$

under the assumption that $0 < \beta$. While the design of H relies on global information, the implementation of (12) and (13) is clearly distributed. Moreover, the parameter associated with the leakage term β that is used to preserve stability of the proposed algorithm may be set to a sufficiently small number.

In the next theorem, we demonstrate that the state of each agent $x_i(t)$ converges to a neighborhood of the weighted average of the set of applied exogenous inputs.

Theorem 4.1. *Consider a network of agents whose dynamics are given by* (10) *and* (11) *subject to* (12) *and* (13)*, communicating over a connected, undirected graph $\mathcal{G}$, where the local agents (A, B, C) are output controllable and a static feedback gain matrix H can be designed. Then, the closed loop error dynamics given by* (38) *are ultimately bounded.*

Proof. Consider the Lyapunov-like function candidate

$$V(g(t)) = g^{\mathrm{T}}(t)Pg(t), \tag{43}$$

where $V(g(t)) > 0, \forall g(t) \in \mathbb{R}^{N(n+p)\times 1} \setminus \{0\}$, $V(0) = 0$, and $P \in \mathbb{S}_+$ with appropriate dimensions is a positive definite solution to the Lyapunov equation

$$F^{\mathrm{T}}P + PF = -Q, \tag{44}$$

letting $Q = \mathrm{I}_{N(n+p)}$ without loss of generality. Then, the time derivative of (43) along the trajectories of (38) can be given by

$$\begin{aligned}
\dot{V}(g(t)) &= \dot{g}^{\mathrm{T}}(t)Pg(t) + g^{\mathrm{T}}(t)P\dot{g}(t), \\
&= g^{\mathrm{T}}(t)\left[F^{\mathrm{T}}P + PF\right]g(t) + 2g^{\mathrm{T}}(t)Pd(t), \\
&\leq -\lambda_{\min}(Q)\|g(t)\|_2^2 + \|g(t)\|_2\|P\|_{\mathrm{F}}\|d(t)\|_2, \\
&\leq -\lambda_{\min}(Q)\|g(t)\|_2\left(\|g(t)\|_2 - \frac{d^*\|P\|_{\mathrm{F}}}{\lambda_{\min}(Q)}\right).
\end{aligned} \tag{45}$$

Since

$$\dot{V}(g(t)) \leq 0, \tag{46}$$

outside the compact set given by $\|g(t)\|_2 > d^* \|P\|_\mathrm{F} / \lambda_{\min}(Q)$, the result follows from [27], [28]. ■

Remark 30. The proof of Theorem 4.1 demonstrates that the error trajectory $\delta(t)$ converges to a neighborhood of zero. In turn, the error (16) is bounded since

$$\|\Delta_i(t)\|_2 \leq \|C\|_\mathrm{F} \|\delta_i(t)\|_2, \tag{47}$$

and the outputs of all agents converge to a neighborhood of the weighted average of the set of applied exogenous inputs.

In the next corollary, we determine the bound of $g(t)$ for $t \geq T$ characterizing the ultimate distance between $x(t)$ and $\mathbf{1}_N \otimes \epsilon(t)$, and $q(t)$ and $d_2(t)$, which is of practical importance for multiagent systems applications.

Corollary 9. Consider a network of agents whose dynamics are given by (10) and (11) subject to the controller (12) and (13), communicating over a connected, undirected graph $\mathcal{G}$, where the local agents (A, B, C) are output controllable and a static feedback gain matrix H can be designed. Then, the ultimate bound of $g(t)$ for $t \geq T$ is given by

$$\|g(t)\|_2^2 \leq \left[\left(\frac{1}{\lambda_{\min}(Q)} \right)^2 (d_2^{*2} + d_1^{*2}) \|P\|_\mathrm{F}^2 \right] \lambda_{\max}(P), \tag{48}$$

where

$$d_2^* \leq \|\mathrm{I}_N \otimes A C_R^{-1}\|_\mathrm{F}\, \epsilon^* + \beta \|\mathbf{1}_N \otimes BH\|_\mathrm{F}\, \epsilon^* \tag{49}$$

$$d_1^* \leq \|(\mathcal{B}^\mathrm{T}(\mathcal{G}) \otimes C)\delta(0)\|_2 - \frac{1}{\gamma} \|(\mathcal{B}^\mathrm{T}(\mathcal{G}) \mathcal{L}^+(\mathcal{G}) \otimes \mathrm{I}_p)\|_\mathrm{F} \cdot \left(\|K_2 \otimes \mathrm{I}_p\|_\mathrm{F}\, r^* - \|K_1 \mathbf{1}_N \otimes \mathrm{I}_p\|_\mathrm{F}\, \dot{\epsilon}^* \right) \tag{50}$$

and

$$\lambda_{\max}(P) \leq \left| \sum^{Nn} \lambda(Q) \right| \cdot \left| \sum^{Nn} \lambda \left(F + F^\mathrm{T} \right) \right|^{-1}, \tag{51}$$

from the results of [29].

Proof. In the proof of Theorem 4.1, we show that (46) holds outside the compact set given by $\|g(t)\|_2 \geq d^*/\lambda_{\min}(Q)$, which implies the evolution of $V(g(t))$ is upper bounded by

$$\begin{aligned} V(g(t)) &\leq \max_{g(t)} V(g(t)), \\ &\leq \|g_{\max}\|_2^2 \lambda_{\max}(P), \end{aligned} \tag{52}$$

where using $\|g_{\max}\|_2 \leq d^* \|P\|_{\mathrm{F}}/\lambda_{\min}(Q)$, (48) follows. ■

Remark 31. The bound of $g(t)$ given by (48) shows the effects of the design parameters α, γ, β, and H of the active-passive dynamic consensus filters controller given in (12) and (13) on the overall network performance. In particular, it can be seen that if β and $1/\gamma$ are small, then (48) is reduced for $t \geq T$. Note that d_1^* and d_2^* both include uncontrolled terms, which implies $1/\gamma$ only has a small effect on the bound size. However, since $\lambda_{\max}(P)$ is directly proportional to $\left|\sum^{Nn} \lambda\left(F + F^{\mathrm{T}}\right)\right|^{-1}$, which is controlled by the static feedback control gain H and β. To this end, we choose $H = 1/\beta H_0$ for a suitable H_0, such that the eigenvalues of $A - BH_0C$ are large since we choose β small.

5. NUMERICAL EXAMPLE

We now present a numerical example to demonstrate the efficacy of the proposed active-passive dynamic consensus filters algorithm. Specifically, consider a network of 25 agents communicating over a connected, undirected ring graph topology with dynamics given by

$$A = \begin{bmatrix} 0 & 10 & 0 \\ 0 & -10 & 0 \\ 0 & -10 & -10 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \tag{53}$$

subject to the exogenous inputs $r_1(t) = \sin(0.2t), r_2(t) = \cos(0.3t), r_3(t) = -0.6 + \cos(0.5t) + 2\sin(0.01t), r_4(t) = 0.5\sin(0.2t) + 1.5\cos(0.1t)$, and $r_5(t) = 2$. Note that the system dynamics given in (53) are output controllable, satisfying all assumptions. Figure 1 demonstrates that with low gains, agents are not able to closely track the average of the applied exogenous

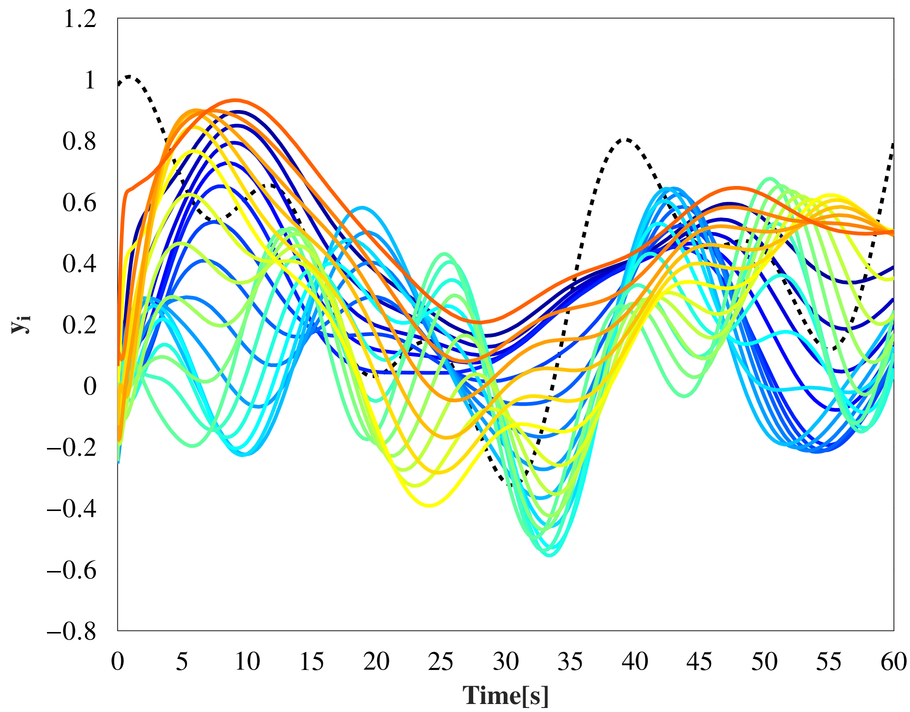

Figure 1. Output of 25 agents tracking 5 inputs subject to the gains $\alpha = 1$, $\gamma = 1, H = 1$, and $\beta = 0.1$ for all agents, where all inputs are weighted equally. The black dashed line indicates the average of the applied exogenous inputs and the color lines represent the agent outputs.

inputs. Figure 2 demonstrates that if α and H are chosen to be large, and $1/\gamma$ and β are chosen to be small, agents converge to a closer neighborhood of the average of the applied exogenous inputs as stated in Corollary 9.

6. CONCLUSION

To contribute to the state of the art in dynamic information fusion, we have presented a new active-passive dynamic consensus filters algorithm, where all agents have (homogeneous) linear time-invariant dynamics. We demonstrated that the outputs of all agents were able to closely track the weighted average of a set of applied exogenous inputs.

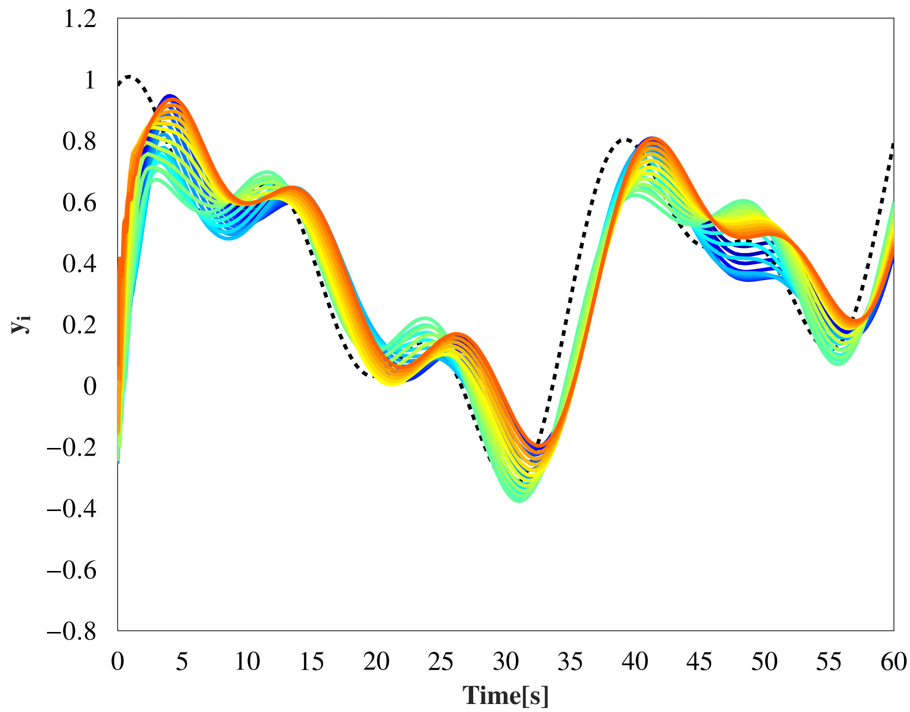

Figure 2. Output of 25 agents tracking 5 inputs subject to the gains $\alpha = 3$, $\gamma = 8$, $H = 3$, and $\beta = 0.001$ for all agents, where all inputs are weighted equally. The black dashed line indicates the average of the applied exogenous inputs and the color lines represent the agent outputs.

A future research direction can include investigation of conditions and algorithm structures that allow asymptotic convergence between the outputs of agents and the weighted average of a set of applied exogenous inputs.

APPENDIX: PROOF SKETCH ERROR SYSTEMMATRIX F IS HURWITZ

Here, we give an sketch of the proof that the error system matrix F is Hurwitz.

Proof. We begin by calculating the eigenvalues of F. To this end, consider

$$\left(F - \lambda \mathrm{I}_{N(n+p)}\right) v = 0, \tag{A.1}$$

where $\lambda \in \mathbb{C}$ is an eigenvalue and $v \in \mathbb{C}^{N(n+p)}$ is an eigenvector of the matrix F. Using the definition of a block matrix determinant in Lemma 7, the eigenvalues of (A.1) can be given by the roots of the polynomials

$$\gamma\sigma + \lambda = 0, \tag{A.2}$$

and

$$0 = \lambda^2 \mathrm{I}_{Nn} + \lambda \bar{F} + \gamma \hat{F}. \tag{A.3}$$

where

$$\bar{F} \triangleq \gamma\sigma \mathrm{I}_{Nn} - \mathcal{R}, \tag{A.4}$$

$$\hat{F} \triangleq \gamma((\mathcal{L}(\mathcal{G}) \otimes BHC) - \sigma\mathcal{R}), \tag{A.5}$$

and $\alpha, \gamma, \sigma, \beta, \in \mathbb{R}_+$ are the proposed active-passive dynamic consensus filters controller gains. Next, we demonstrate that if eigenvalues of $\bar{F}$ and $\hat{F}$ have positive real parts, F is Hurwitz. To this end, consider the eigendecomposition of the matrix

$$V \triangleq \alpha\mathcal{L}(\mathcal{G}) + K_1 + \beta \mathrm{I}_N = SUS^{\mathrm{T}}, \tag{A.6}$$

where $SS^{\mathrm{T}} = \mathrm{I}_N$ since V is symmetric and positive definite (see Lemma 1 of [7]), with $U = \mathrm{diag}([\mu_0, \mu_1, \ldots, \mu_N])$, where μ_i are the eigenvalues of V. Using (A.6) in (36), $\bar{F}$ becomes

$$\bar{F} = (S \otimes \mathrm{I}_n)\big[\gamma\sigma \mathrm{I}_{Nn} - \mathrm{I}_N \otimes A + U \otimes BHC\big](S^{\mathrm{T}} \otimes \mathrm{I}_n), \tag{A.7}$$

and eigenvalues of $\bar{F}$ are given by the block diagonal matrix $\gamma\sigma \mathrm{I}_{Nn} - \mathrm{I}_N \otimes A + U \otimes BHC$, where each block is given by $\gamma\sigma \mathrm{I}_n - A + \mu_i BHC$. Since the matrix $A - \mu_i BHC$ is Hurwitz for a suitable static feedback gain H (see assumption 2), the real part of all eigenvalues of $\bar{F}$ must be positive. A similar construction is used to demonstrate that the eigenvalues of $\hat{F}$ have positive real parts.

Next, consider $R \in \mathbb{S}_{+}^{Nn \times Nn}$ as a solution to the linear matrix inequalities

$$R(\epsilon_1 \mathrm{I}_{Nn} + \bar{F}) + (\epsilon_1 \mathrm{I}_{Nn} + \bar{F})^{\mathrm{T}} R \leq -\epsilon_1 \mathrm{I}_{Nn} \triangleq W_0,$$

$$R(\epsilon_2 \mathrm{I}_{Nn} + \hat{F}) + (\epsilon_2 \mathrm{I}_{Nn} + \hat{F})^{\mathrm{T}} R \leq -\epsilon_2 \mathrm{I}_{Nn} \triangleq W_1, \tag{A.8}$$

for a sufficient $\epsilon_1, \epsilon_2 \in \mathbb{R}_+$, and with negative definite matrices W_0 and W_1. The eigenvalues of F are given by the solutions λ of

$$\mathbf{0}_{Nn} = 2\lambda^2 R + \lambda(-W_1) + \gamma(-W_0), \tag{A.9}$$

where all solutions have negative real parts from [30]. Since the real part of all the roots of F lie in the open left half plane, F is Hurwitz. ■

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SECTION

2. CONCLUSIONS AND FUTURE WORK

2.1. CONCLUDING REMARKS

Existing system theoretic information fusion filters can generally be categorized as *static* where all agents are able to sense a constant exogenous input or *dynamic* where all agents are able to sense a time-varying quantity of interest. Such systems are not suitable for heterogeneous situations where some agents are *active* for (i.e able to sense) an exogenous quantity of interest and some agents are *passive* for (i.e. are not able to sense) an exogenous quantity of interest. Motivated from this standpoint, Paper I proposed a new active-passive dynamic information fusion controller where the states of all agents are driven to the average of the set of exogenous inputs sensed by the active agents. In addition, we discussed in detail that the proposed approach not only generalizes but also unifies the results of several classes of leaderless and leader-follower information fusion approaches approaches.

In Paper II and Paper III, we demonstrated the efficacy of the proposed active-passive dynamic information fusion controller presented in Paper I by performing environment surveillance, where this framework allows the states of all agents to converge to the average of the exogenous inputs applied only to the active agents. In particular, we utilize two ground robots equipped with Microsoft Kinect sensors to track objects in an environment and exchange information over a local network in order to produce a global map detailing object locations even though neither agent is able to sense all objects in the environment. Additionally, we provide several numerical studies detailing how networks of agents may reconstruct dynamic environments.

In addition to heterogeneity in each agents' ability to sense a quantity of interest, each agent may be heterogeneous with respect to its sensing power which is captured by adjusting the agent's *value-of-information* parameter. To address this challenge, Paper IV extended the active-passive dynamic information fusion controller presented in Paper I to enable agents to locally weigh their information based on their sensing ability. Specifically, we showed that the states of all agents converge to an adjustable neighborhood of the weighted average of the set of applied exogenous inputs sensed by the active agents when both the agents' value of information and the applied exogenous inputs are time-varying. In addition, we formally discussed several cases when agents' sensing capability and the exogenous inputs are time-invariant yielding asymptotic stability of the error dynamics between the states of all agents and the weighted average of the sensed exogenous inputs.

Next Paper V, proposed a new class of active-passive dynamic consensus filters, which only require agents to exchange their current measurement state information with neighbors in a simple and isotropic manner, and importantly, allow the roles of active and passive agents to be time-varying. This makes them suitable for a wider range of multiagent systems applications. Specifically, we showed that the proposed active-passive dynamic consensus filters enable the states of all agents to converge to a neighborhood of the average of the observations sensed by a time-varying set of active agents and we provided a systematic way to tune the design parameters of the proposed filters to make this neighborhood small for achieving a desired overall network performance. In addition, we extended our results using tools and methods from event-triggered control theory to further reduce the total cost of inter-agent information exchange and to remove the need for agents to synchronize their information update intervals.

Paper VI presented a new state emulator based adaptive control architecture which allows agents to reach an agreement on a quantity of interest even when all agents are subjected to exogenous disturbances. Unlike previous studies, which make assumptions on agent dynamics and network topologies, the presented results hold for agents with general linear time-invariant dynamics communicating over a connected undirected graph.

Finally, Paper VII presented a new active-passive dynamic consensus filters algorithm, where all agents have (homogeneous) linear time-invariant dynamics. We demonstrated that the outputs of all agents were able to closely track the weighted average of a set of applied exogenous inputs, allowing for agents where the sensor information and agent dynamics cannot be decoupled.

2.2. FUTURE WORK

To contribute to the state of the art in dynamic information fusion, this dissertation has presented a new class of dynamic consensus filters in which some agents are active (able to sense a quantity of interest) and some are passive (not able to sense any quantities), which make assumptions on the dynamics of local agents. Specifically, Paper VII presented a network of active and passive agents where agents have general linear time-invariant dynamics and fixed active and passive roles. We can improve on this control architecture by utilizing the ideas and tools from switched systems theory to allow agents to change their active and passive roles. In addition, using the results presented in [64], active-passive dynamic consensus filters may be extended to include agents with non-linear dynamics. We may also use the results of [65], [66] to consider agents which are heterogeneous with respect to their dynamics.

Paper II, Paper III, and Paper V present applications of the presented active-passive dynamic consensus filters in a highly controlled laboratory environment. To bridge the gap between theory and applications, one possible future research direction is to utilize the presented dynamic information fusion methods to allow agents to communicate across rapid-deployment emergency networks. As an example, consider mine/cave rescue networks where many repeater stations may be placed to overcome the difficultly of transmitting signals in underground environments. With current systems, operators must take care to place the stations in such a manner that a signal will be routed only one way through the network and that a signal may not be caught in a loop, which may compromise the entire network until the signal has been removed. In contrast, the proposed active-passive dynamic consensus filters approach allows for signal input at any node and does not make any assumptions on the underlying network topology, allowing for quick deployment and maintenance free operation.

In addition to utilizing active-passive dynamic consensus filters in place of traditional routing protocols, distributed sensing has been identified as a major research thrust area by the National Oceanic and Atmospheric Agency for use in collecting, processing, and analyzing large data sets [67]–[70]. Specifically, this research may be used to capture real-time data of large-scale events for real-time emergency operations and model-creation and validation for forecasting future events.

These examples are just a few research directions which may bridge the gap between theoretical and practical distributed sensing to further our scientific understanding of our environment.

APPENDIX

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