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# ECONOMIC OPTIMIZATION OF HEATPUMP ASSISTED SOLAR HEATING IN ILLINOIS

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## Abstract

This study undertakes the task of determing the optimal mix of solar and heat pump forms of heating. By installing a solar heating system a homeowner is considered to be an energy producer and thus to apply the least cost methods used by firms in the competitive market for any given level of fuel conservation. The study will examine the simulated performances of air and liquid circulating systems in conjunction with heat pumps, in parallel as well as combined fashion. Optimization is achieved by equating the present value of the cost of solar and heat pump heating systems at margin.

## 1. INTRODUCTION

This study examines economically optimal solar-heat pump heating systems for new residences in Illinois. The "economically optimal" heating system incurs the least present value of all costs over the life of the system in providing the annual heating load. Optimality of solar heating systems is examined with an electric pump in serial or parallel combination with solar as the supplemental heating source. Also, an economic analysis of the federal government's tax incentive for encouraging solar utilization by homeowners is presented.

There are powerful social and political forces at work in Illinois and the entire U.S. to shift toward an all electric base. With a diminishing supply of natural gas, low thermal energy requirements are being shifted from direct fossil fuel combustion to electricity* and renewable energy resources. Optimally designed heat pump/solar hybrid systems offer significant promise in the reduction of direct fossil fuel consumption in the area of space conditioning.

This study also investigates whether such a conservation of fuel, which is accepted as a social good, is an economic proposi-

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*It has been argued that utilization of electricity for meeting low thermal energy requirements does not allow for efficient use of electricity. See Barry Commoner, Poverty of Power (Alfred A. Knopf: New York, 1976). This study only evaluates the optimal microeconomic mix of solar and supplemental energy sources.

tion from the point of view of an average household. When one of our national goals is the conservation of conventional fuel, with minimum adverse impact on the economy, then the energy policy should be set such that the most conserving mode of heating becomes an economic proposition to the average household.

This report presents an optimization for solar-assisted space and domestic hot water heating for the new residential housing market, and is not concerned with space cooling. However, the hot, humid Illinois summer has made air-conditioning an accepted and desired social pattern. The heat pump offers the ability to mechanically cool space and will probably play an increasingly important role in the residential market.*

For the purposes of this analysis, the state of Illinois is divided into three climatic regions which reflect the variations in heating demands and solar radiation levels. Representative cities for each region were studied using climatic data and the estimated energy requirements for a typical family of four.

The "typical" 1500-square-foot house is a well-insulated, detached residence integrating no special design features to add passive solar heat gain except an east-west longitudinal orientation to accept adequate collector area. The assumption was made that generally the cost of the initial insulation measures is less than the present value of the lifetime cost of the fuel saved due to these measures; thus, the first consideration was to reduce the heating load to a level significantly less than that for an average home. Energy conserving features incorporated into the house included thermal pane windows and storm doors, R-30 ceiling insulation, R-19 wall insulation; R-11 floor insulation over an unheated basement (crawl space) and an infiltration factor of 1/2 air changes per hour. This implies a heat loss coefficient of 10,500 BTUs per degree day. The daily domestic hot water requirement was assumed to be 80 gallons delivered at 120 degrees F. Incorporating the monthly variations of inlet tap temperature and the weather data, total monthly and annual heat load of space and domestic hot water was determined for each region.

Two basic solar systems are evaluated in this analysis. One utilizes an air-cooled collector with rock storage. The other is a liquid-cooled collector with water storage. The performance for each collector type is typical for flat plate collectors readily available on the market.

Both collector systems were integrated into two designs of parallel and serial interface with the heat pump.

Since the availability of solar radiation is dependent on the unpredictable amount of cloud cover and seasonal weather patterns, solar heating systems without large seasonal heat storage usually are not capable of providing a given heat load during prolonged periods of severe cold or cloudiness. Therefore, solar heating systems should be designed such that they provide only a fraction of the load. The methodology used in this study was developed to find the optimal size of the solar system for each location. The optimal size is determined by the life cost of each

*Sales representatives from Commonwealth Edison Company and Illinois Power Company (Illinois' two largest distributors of electricity) report that over 10 percent of the new electrical residential market were installing heat pumps in 1975. This includes both single and multi-family residences.

system where the cost of the marginal contribution to the annual heat load is equal for all systems.

## 2. METHODOLOGY AND DATA

To provide an economic evaluation of various heating systems at the least cost condition one needs:

(i) to determine the heat load, i.e., the energy needed for space and water heating and the level of solar and heat-pump utilization, and

(ii) to ascertain the cost of alternative solar systems, and heat-pump heating systems.

### 2.1 THE LOAD AND SOLAR UTILIZATION

Due to the crucial effect of the load on the utilization level of available solar energy and the performance of solar heating systems, obtaining information on the load must precede any attempt to design a solar heating system.

Annual Space Heat Load =
(Annual Degree Days) (Energy/Degree Day)

The hot water heat load is a function of the supply water temperature and the living habits of the residents of the house. Since components of the total heat load, i.e., space heat and hot water supply, are functions of different variables, the dynamic interactions of these components are checked by setting a priority for space heating in the control strategy of solar systems. For the purposes of this analysis 80 gallons of domestic hot water at 120 degrees F. is assumed to be the typical load.

The quantity of heat supplied during a given time period by the solar system is calculated by solving the functional equations, or mathematical descriptions, of the components of the system. The supply of heat provided by the solar heating system is a function of time, i.e., meteorological data, as well as of parametric values of the components and the load, or heat requirement, of the house.

Computer simulation was used to generate data on the long-run average performance of the solar and back-up heating units. The simulation was done using TRNSYS Version 8.1 (1) which we modified in accordance with the design and control strategies chosen for this study. Since performance data was needed in order to derive cost curves for different heating modes, simulation runs of different combinations of collector and storage sizes were performed for each particular solar heating design. Insolation values measured at the Argonne National Laboratory were used for Chicago. Insolation data for Springfield were taken from <u>Dawning of an Age</u> by Barfield <u>et al.</u> (2) And insolation data for Carbondale was determined by the same method used by Barfield <u>et al.</u> for Springfield. Monthly maximum and monthly mean temperatures were obtained by using a cosine function to determine the daily variation of temperature.

In order to find the life cycle least cost heating condition of the alternative modes described in the design section, the costs of the systems have to be compared. To achieve the least cost condition the present value of all costs of alternative modes for a given period should be equal at margin; then, the different modes can be combined (e.g., the load is provided in part by a solar system and in part by a heat pump) and the least cost combination determined. In order to compare the marginal cost function of alternative modes we need to derive the cost function of each mode, from which the marginal cost functions are ascertained.

### 2.2 LIFE CYCLE COST FUNCTIONS OF HEATING SYSTEMS

Cost functions express the relation between output and costs of a production process. Cost functions are determined on the basis of prices of inputs and the production function which expresses the physical relation between outputs and inputs. The output (or the end product) of a heating system is taken to be the percentage of the heating load it provides. The inputs are expressed in terms of collector and storage tank sizes and other components of specified technical characteristics. Production of the same output can utilize a variety of inputs and processes. Therefore, to compare the competitiveness of different processes, economists give most of their attention to relating the cost function of varying production processes designed to provide the same output.

In finding the life cycle least cost heating condition, we compare the cost functions of the alternative heating systems described in the design section. In order to compare the marginal cost function of alternative modes we need to derive the cost function of each mode, from which the marginal cost functions are ascertained.

### 2.2.1 Life Cycle Cost Function of a Solar System

To determine the cost function the present value of all costs associated with systems with collectors of varying size (including maintenance, salvage value, and operational costs) is related to the portion of the heat load they furnish. The cost of a solar system is an increasing function of the load it furnishes. However, successive increments in solar collectors with adjustment of other components on a given load house make a diminishing contribution to the annual load.

The diminishing marginal contribution of the collectors and other components can be expressed in terms of an increasing marginal cost of solar heating with respect to a given heat load. In other words, the cost of solar heating is an exponential function of the solar contribution to the heat load at an increasing rate.

The increasing marginal cost of solar heating systems prohibits building units with the capacity to meet the total annual heat load and leads to the necessity of a backup heating unit.

In constructing a heating cost function the performance of alternative size heating systems and the present value of costs associated with them are found. Using the multiple regression technique on these points, one can fit a total cost curve for a particular heating system. In determining the costs of the solar unit, the present value of energy it saves in providing domestic hot water during the summer is also considered.

In calculating the present value of all costs incurred during the life of a heating system, one's financial condition, the expected rate of escalation in fuel prices, the maintenance cost, the income tax rate, and the general level of inflation have to be considered in the following pattern:

$$PV = C_s\left(1 + m\,\frac{(1+r)^N - 1}{r(1+r)^N}\right) + E\sum_{J=1}^{N}\left(\frac{1+i}{1+r}\right)^J - \frac{S}{(1+r)^N} \qquad (2\text{-}1)$$

where

$PV$ = present value of the system's cost over N years;

$C_s$ = initial capital costs of solar home above the costs of the same home with conventional heating system;

$m$ = estimated annual maintenance cost of solar equipment, expressed as a fraction (%) of $C_s$;

$r$ = annual discount rate in real terms which is considered to be the "real opportunity cost" of the money for the homeowner;*

N = expected life of the system in years;

E = net annual electricity cost of the system at present rates;

i = annual rate of escalation in price of electricity; and

S = estimated value of the system after N years; S may be considered the salvage value of the equipment or the value of the energy it will deliver after N years.

* To obtain r, first the effective discount rate has to be determined by:

$$r_e = (1 - t) \cdot r_m$$

where $r_m$ is the nominal (market) discount (interest) rate, and t is the income tax rate of the homeowner. The real discount rate is $r = (r_e - I)$. The real escalation rate in electricity prices is $i = i_m - I$, where I is the general level of inflation rate determined by the consumer price index, while $i_m$ and $r_m$ are market rates.

### 2.2.2 Life Cycle Cost Function of the Heat Pump System

The heat pump heating cost curve relates the present value of all costs associated with this mode of heating during the life of the solar system tested against it. These present values are measured for varying portions of the annual load. (A heat pump, in most regions, operates more hours on the heating cycle than an air-conditioner. But we have not been able to obtain data to determine the life of a heat pump based on the number of hours it runs.) Life cycle costs of a heat pump system depend on the longevity of the unit as well. The useful life of heat pumps is, at best, disputable and since most manufacturers are introducing new generations of heat pumps it will take some years for the longevity of their units to be tested in the field. However, manufacturers claim these new products are different in design than those with numerous failures and disappointing records marketed during the 1950s. Since the designed house is assumed to be air-conditioned, the costs of the heat pump are credited for the cost of a comparable air-conditioning unit. We assume the life of a heat pump to be different than that of the life of a comparable air-conditioner, and since heat pumps may last less than the expected life of the solar unit, the present value of renovation costs should be added to the capital cost of the original heat pumps.

We decided to determine the capital cost attributable to the heating function of the heat pump system during the life of the solar unit by:

$$K = D + \left[D + H\left(\frac{a-h}{h}\right)\right] \sum_{J=1}^{n-1} (1 + r)^{-Jh} + \frac{N-nh}{h} \left[D + H\left(\frac{a-h}{h}\right)\right] (1 + r)^{nh} \quad (2\text{-}2)$$

where:

K = PV of the capital costs of the heat pump system over N years;

D = the difference in installed cost between the heat pump unit and a comparable air conditioning unit;

H = the installed cost of the heat pump;

a = the expected life of the comparable air conditioner;

h = the expected life of the heat pump;

n = the number of times the heat pump unit has to be replaced over N years, or $n \leq \frac{N}{h}$ and

N = the expected life of the solar unit.

Besides the capital cost, the costs of maintenance and required electricity of the heat pumps over N years of life of the solar system must be determined.

The present value of the maintenance costs on the heat pump unit is attributed both to its heating and cooling modes. The relative share of maintenance costs for each mode is assumed to be the inverse of the expected life of the heat pump and the comparable air conditioner. If $R_1$ is the relative maintenance cost of the heating mode and $R_2$ is the relative maintenance cost of the air conditioning mode, then:

$$R_1 + R_2 = 1$$

assume

$$\frac{R_1}{R_2} = \frac{a}{h}$$

thus,

$$R_1 = \frac{a}{a+h} \text{ and } R_2 = \frac{h}{a+h}$$

The PV of the life cycle maintenance cost of the heat pump is:

$$M = m\, P\left(\frac{a}{a+h}\right)\frac{(1+r)^N-1}{r(1+r)^N} \qquad (2\text{-}3)$$

where:

m = estimated annual maintenance cost of the heat pump, expressed as a fraction (%) of P, and

P = capital cost of a heat pump unit.

The present value of the electricity consumed by the heat pump during N years of operation, G, is determined by:

$$G = F \sum_{J=1}^{N}\left(\frac{1+i}{1+r}\right)^J \qquad (2\text{-}4)$$

where:

F = the annual cost of electricity required for operating the heat pump during the heating season at present electricity rates.

The total life cycle cost of the heat pump system is determined by:

$$C_{hP} = K + M + G$$

Since heat pump heating systems are considered to be alternatives to solar heating, the cost function for the heat pump is developed for comparison with the solar cost function. But, solar heating systems in this study are designed to supply energy for preheating the domestic hot water, as well as for space heating. Hence, in deriving the cost functions for heat pumps their total costs are increased by the present value of the electricity required to supply hot water by electric resisters during N years of life of the solar unit.

Since electricity is used for lighting and other purposes in a home, a given monthly life line electricity consumption of 500 KWH was assumed. Thus, throughout this study monthly electricity costs for the various systems were based on the utility rates for usage beyond 500 KWH.

## 2.3 LEAST COST HEATING CONDITION

In heating his house a homeowner has the choice of many combinations of solar and other heating systems which will satisfy the load. However, for any combination of two systems, the homeowner must consider two constraints. First, the sum of the output of the two systems should equal his heating load; any extra heat obtained by the solar unit must be dumped. Second, the total cost of the combined units over the life of the solar system should not exceed the cost of the non-solar heating system. The combination of solar and non-solar systems which meets the heating load and costs the least provides the optimal mix.

The difference between the present value of the electricity conserved by a combined system and the capital cost of the solar heating system determines the net gain (loss) to the homeowner who installs a solar heating system to provide a portion of the heat load. The optimal solution is found by solving the corresponding functions for highest possible net gain, indicated by the maximum vertical distance between the solar and savings in fuel curves where the marginal cost of solar equals the cost of fuel. (For a detailed treatment of the marginal cost of a solar unit versus the value of the fuel it saves, see The Feasibility of Solar House Heating: A Study in Applied Economics, unpublished doctoral dissertation by Ali Shams, Southern Illinois University, May 1977.)

## 2.4 SUMMARY DESCRIPTION OF COMBINED SOLAR AND HEAT PUMP SYSTEM

The configuration of solar heating with conventional furnaces is simpler in design than a system which combines a heat pump and a solar unit. Since the performance of the heat pump unit varies with ambient air temperature, one may try to utilize the heat pump section of a solar heat pump system when the ambient temperature is most favorable. Also, since the heat pump can draw energy from a cooler environment to a warmer one, the system may be designed such that the heat pump can use the solar heat storage after it has exhausted its ability to deliver direct solar heat to the house. This is done by a serial configuration of the storage tank and outdoor unit of the heat pump. This mode is called the heat-pump-assisted solar system throughout this study and its merits are tested against a solar heat pump system with parallel design, where the difference in capital (for extra ducts and storage area) and operational costs of these systems are compared with the gain in conserving electricity. For each parallel and serial mode of heat pump operation, in conjunction with each air and liquid solar heating unit at each location, a heat pump cost curve is constructed. This offers ten heat-pump cost curves to be solved with their solar counterparts. A simultaneous solution of each pair of curves at their greatest vertical distance provides the least cost heating condition, where the vertical distance between the two curves provides the present value of net savings (if any) accrued during the life of the solar system to the homeowner due to the utilization of solar energy. A comparison of net savings at the least cost heating condition of different modes will determine the optimal heating mode. The optimal mode is chosen where the maximum net savings is available between the least cost heating conditions of different modes and designs. This calculation will be made first on the basis of the market price of the selected solar systems. Then, since expected federal tax incentives suggested in President Carter's Energy Bill of April 20, 1977 will change the slope and the vertical intercept of the solar cost curve, thus reducing its marginal cost, the effect of such incentives on the cost of the different heating modes will be examined.

## 2.5 AVERAGE DAY HOURLY INSOLATION ON A TILTED SURFACE AND HOURLY TEMPERATURE

The average daily insolation on a horizontal surface was used to determine average hourly values. The hourly horizontal insolation values were then used to compute the insolation on the collector surface tilted at an angle equal to the latitude plus 15 degrees for each of the cities studies.

A diurnal variation of temperature was determined using the maximum and mean daily average temperatures. The hourly insolation values and temperatures were determined for an "average day" for September through May for each city.

## 3. TECHNICAL CHARACTERISTICS OF THE SOLAR DESIGNS

This study investigated many different configurations and sizes of solar energy systems. For convenience in referring to the various systems the following terminology will be used.

"Liquid system" refers to a solar energy system that uses an antifreeze-water mixture as the working fluid in the collectors and water for the storage medium. "Air system" refers to a solar energy system that uses air as the working fluid in the collectors and a rock bed for the storage medium. "Heat-pump-assisted solar heating system" refers to either a liquid system or an air system in which one of the possible modes of operation is the use of the storage as a heat source for the heat pump. (This is accomplished through the "basement room" to prevent excessive heat input to the heat pump.

"Parallel solar heating system" refers to either a liquid system or an air system which utilizes the "solar portion" of the system in parallel with the "conventionally fueled portion" of the system. This means that either solar energy is used for space heating <u>or</u> a heat pump is used to heat the space. In this case, there is no mode in which the two portions are used together as in the "heat-pump-assisted solar heating system." The "solar portion" refers to those circuits which operate to collect solar energy, such as the collector-storage circuit, and to those which supply solar energy for heating. This latter group of circuits includes the storage-preheat, storage-load, and storage-heat pump circuits in the liquid system, and the collector-load, collector-preheat, storage-load, and storage-heat pump circuits in the air system. The "conventionally fueled portion" refers to the use of a heat pump, and electric furnace, an oil furnace, or an LP (liquefied propane) furnace to provide supplementary heating for that portion of the space heating load that cannot be met by the "solar portion" of the system.

A three-letter code is used for ease and convenience in system identification. The first letter of the code will be C, S, or A, referring to Chicago, Springfield, and Carbondale, respectively. The second letter will be either L or A, referring to the "liquid system" or the "air system," respectively. And the third letter will be either H or P, referring to the "heat-pump-assisted solar heating system" or the "parallel solar heating system", respectively. For example, CLP refers to a "liquid" "parallel" solar heating system in Chicago. Similarly, SAH refers to an "air" "heat-pump-assisted" solar heating system in Springfield. In addition, the following terminology is used.

<u>Solar-Electricity</u> refers to the electricity required to operate the pumps and fans of the "solar portion" of any system. <u>Heat Pump-Electricity</u> refers to the electricity required to operate the compressor, indoor unit fan, and outdoor unit fan of the heat pump of any system.

As an example, the code ALH-Solar-Electricity would refer to the electricity required to operate the pumps and fans of the "solar portion" of the "liquid" "heat-pump-assisted" solar heating system in Carbondale. And ALH-Heat Pump-Electricity would refer to the electricity required to operate the "heat pump portion" of the said ALH system.

## 4. THERMAL SIMULATION OF A SOLAR HEATING SYSTEM

To make an economic evaluation of solar heating based on the methodology developed in Section 2, one must derive a solar cost function. The amount of heat delivered to the load is the output of the homeowner's enterprise, or investment, in a solar heating unit. However, the homeowner needs information about the market value of his heat output while the solar heating system is in operation, i.e., while his capital is being amortized. This ex-ante information depends not only on economic variables, such as inflation in fuel prices, discount rates, and income and property tax rates, but also on the heat transfer ability of the planned solar heating unit, which is based on hourly and seasonal meterological data, i.e., levels of solar insolation and the ambient temperature of the locale. Tracing the heat element of solar insolation from the collector surface to the heat exchanger to the storage tank to the load provides the information needed by the solar heat producer. To obtain the ex-ante information on the output of solar heating systems required for investment decisions, the economic analysis in this work is based on solar outputs simulated by TRNSYS, which was developed by the Solar Energy Laboratory of the University of Wisconsin.

To determine the total electrical energy required by the heat pump the simulation model was used with the heat pump as the sole heating source. The results were as follows: annual electricity required by the heat pump is 8889 kwh in Chicago, 8034 kwh in Springfield, and 6085 kwh in Carbondale.

The parameters used in the simulation of the liquid and air systems are listed in Table 1. The collector used as the model for the liquid system is manufactured by Chamberlain Manufacturing Corporation in Elmhurst, Illinois, and that for the air system is manufactured by Amcon Solar in Carbondale, Illinois. Since equal numbers of panels were simulated for the liquid and air systems of eight different collector sizes, the collector square footage in the air system size is slightly larger than that in the comparable liquid system size.

TABLE 1: TERMINOLOGY FOR VARIOUS SYSTEM SIZES

| | Collector Area, $m^2$ ($ft^2$) | |
|---|---|---|
| | Liquid System | Air System |
| Size 1 | 11.1 (120) | 11.7 (126) |
| Size 2 | 16.7 (180) | 17.6 (189) |
| Size 3 | 22.3 (240) | 23.4 (252) |
| Size 4 | 27.9 (300) | 29.3 (315) |
| Size 5 | 33.4 (360) | 35.1 (378) |
| Size 6 | 39.0 (420) | 41.0 (441) |
| Size 7 | 44.6 (480) | 46.8 (504) |
| Size 8 | 55.7 (600) | 58.5 (630) |
| Single Collector Panel Size | 1.86 (20) | 1.95 (21) |

Performance was calculated in terms of the solar energy provided for space and domestic hot water heating. The percentage of the total heat load supplied by solar was calculated for the average day and therefore for the entire month. The "monthly" totals of heat load and solar heat supplied were then used to calculate the annual percentages. These performances are reported in Table 2.

TABLE 2: PERCENTAGE OF ANNUAL HEAT LOAD PROVIDED BY SOLAR ENERGY

| SIZE | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| CLP | 19.5 | 26.5 | 32.3 | 38.8 | 42.1 | 49.0 | 53.3 | 61.6 |
| CLH | 19.9 | 27.2 | 32.6 | 39.7 | 48.2 | 51.9 | 56.3 | 64.5 |
| CAP | 17.1 | 22.7 | 26.7 | 32.4 | 36.0 | 44.3 | 46.2 | 54.3 |
| CAH | 17.8 | 25.7 | 30.6 | 37.7 | 39.4 | 47.4 | 50.4 | 57.3 |
| SLP | 18.8 | 27.1 | 32.2 | 37.6 | 42.8 | 48.7 | 53.0 | 60.5 |
| SLH | 19.9 | 27.5 | 31.7 | 38.7 | 44.2 | 49.8 | 54.4 | 61.6 |
| SAP | 19.1 | 21.1 | 24.8 | 28.1 | 32.5 | 40.6 | 43.8 | 53.1 |
| SAH | 21.1 | 26.9 | 28.7 | 34.6 | 39.4 | 43.1 | 47.4 | 56.0 |
| ALP | 22.0 | 33.1 | 39.9 | 46.4 | 53.5 | 59.1 | 64.1 | 72.2 |
| AAP | 19.7 | 24.2 | 35.8 | 36.5 | 38.0 | 44.6 | 53.6 | 63.6 |

## 5. COST FUNCTIONS OF HEATING SYSTEMS

To find the least cost heating mode a comparison must be made between costs of alternative solar heating systems. Costs associated with each unit must be converted to a common base. Such costs are calculated here on the basis of their present values using a time span equal to the life of the solar heating unit. In the following the costs of each unit of each heating system are reviewed.

## 5.1 COSTS OF SOLAR HEATING UNITS

Costs associated with solar heating systems consist of the initial capital outlay, maintenance costs, and the cost of the electricity required for operation of the system. The total present value of these costs was expressed in Equation 2-1 Using present market prices and given sets of rates for r and i the cost equations for solar units are obtained.

$$C = f(Q) = C_O + aQ + bQ^2 \qquad (5\text{-}1)$$

where C is the present value of total life cycle cost, $C_O$ is the constant factor, a and b are constants, and Q is the performance of the solar heating unit given in terms of the percentage of the annual heating load provided by the system. As an example, a yearly discount rate of 9 percent and 12 percent escalation in electricity prices results in the following estimated equations for the cost function of the liquid solar heating units in the CLP, CLH, SLP, SLH, and ALP heating systems.

CLP $C = 3886 + 30.94Q + 1{,}875Q^2$
CLH $C = 3243 + 118.8Q + 0.321Q^2$
SLP $C = 5271 - 35.63Q + 4.131Q^2$
SLH $C = 3444 + 149.9Q + 0.349Q^2$
ALP $C = 4278 + 25.52Q + 1.277Q^2$

## 5.2 COST OF HEAT PUMP HEATING UNITS

The solar heating system discussed in this study can be installed in new, as well as in existing, buildings. The homeowner must maintain a heat pump heating unit at full capacity in order to meet heating needs when severe weather conditions prevail. Therefore, a solar heating system offers savings only in terms of the electricity it conserves. These savings are an increasing function of the portion of fuel conserved. Comparing these savings with the capital cost of the solar heating system requires the derivation of a fuel cost curve based on the present value of fuel consumed by the heat pump system taken over the life of the solar heating system. The slope of such curves is a function of fuel price and the present rate of interest (r). In addition, the expected rate of increase in the price of fuel (i) must be taken into account. The present value of electricity consumed annually by a heat pump heating system is an increasing function of i and a decreasing function of r. In brief,

$$PV = \sum_{t=1}^{n} \left(\frac{1+i}{1+r}\right)^t \cdot B, \qquad (5\text{-}2)$$

where PV is the present value of fuel costs, i is the expected rate of increase in fuel price, r is the present discount rate, n is the expected life of the solar heating system, and B is the present annual heating bill of a house using a heat pump heating system.

Following the methodology discussed in Section 2, an expected life of eight years was chosen for the heat pump, while the life of a comparable air conditioner was taken to be twelve years. Using discount rates of 6 and 9 percent and electricity rate escalations of 12 and 15 percent and applying equation , forty cost equations for heat pumps used in ten heating systems of CLP, CLH, CAP, CAH, SLP, SLH, SAP, SAH, ALP, and AAP were found.

Solving corresponding cost functions for solar and heat pump units in each heating system, least heating cost prevails when the cost of solar heat and the savings it renders on the conserved electricity, at margin, are equal. Table 3 indicates these solutions tested for Chicago.

TABLE 3: ECONOMIC OPTIMIZATION OF HEAT PUMP AND SOLAR HEATING CONFIGURATION

| System | Rates | % Load Solar | Net Savings ($) |
|---|---|---|---|
| CLH | r=9 i=12 | 10.1 | -5285 |
| | r=9 i=15 | 24.3 | -2504 |
| | r=6 i=12 | 17.5 | -4753 |

| System | Rates | % Load Solar | Net Savings |
|---|---|---|---|
| CLH | r=6 i=15 | 22.5 | -2542 |
| CLP | r=9 i=12 | 15.5 | -4395 |
| | r=9 i=15 | 23.5 | -3380 |
| | r=6 i=12 | 21.9 | -3628 |
| | r=6 i=15 | 33.3 | -1697 |
| CAH | r=9 i=12 | * | * |
| | r=9 i=15 | * | * |
| | r=6 i=12 | * | * |
| | r=6 i=15 | * | * |
| CAP | r=9 i=12 | 19.8 | -2741 |
| | r=9 i=15 | 27.0 | -1773 |
| | r=6 i=12 | 26.1 | -1867 |
| | r=6 i=15 | 35.9 | - 43 |

*This solution is outside of the meaningful range of performance.

## 5.3 THE EFFECT OF GOVERNMENT SUBSIDY ON THE UTILIZATION OF SOLAR HEATING

In its efforts to encourage energy conservation and the development of alternative sources of energy, the federal government has been actively promoting the use of solar energy. President Carter in his energy bill of April 20, 1977, asked for the passage of a 40 percent refundable tax credit on the first $1,600 and a 25 percent credit on the next $6,400 spent on purchasing and installing a solar heating system. The President also asked that these solar subsidies be considered retroactive from April 20, 1977. Such subsidies may well be required since other sources of energy have been subsidized in one form or another.

The subsidy outlined in the President's energy bill was applied to the costs of solar units in our designed heating systems. The subsidized costs were then used to find a new solar heating cost function. Fitting cost curves similar to those presented earlier, by using a multiple regression technique of least square estimates, results in new sets of cost equations. The tax effected optimization condition is reported in Table 4 for Chicago.

TABLE 4: ECONOMIC OPTIMIZATION OF HEAT PUMP AND SOLAR HEATING CONFIGURATION WITH FEDERAL SOLAR TAX CREDIT

| System | Rates | % Load Solar | Net Savings ($) |
|---|---|---|---|
| CLH | r=9 i=12 | 16.2 | -3234 |
| | r=9 i=15 | 24.0 | -2339 |
| | r=6 i=12 | 33.9 | -1103 |
| | r=6 i=15 | 33.2 | - 895 |
| CLP | r=9 i=12 | 21.8 | -2401 |
| | r=9 i=15 | 28.2 | -1168 |
| | r=6 i=12 | 26.9 | -1451 |
| | r=6 i=15 | 36.1 | 707 |
| CAH | r=9 i=12 | 12.5 | -1903 |
| | r=9 i=15 | 36.0 | - 995 |
| | r=6 i=12 | 32.6 | -1136 |
| | r=6 i=15 | 43.0 | 1611 |
| CAP | r=9 i=12 | 26.3 | -1190 |
| | r=9 i=15 | 32.2 | -1993 |
| | r=6 i=12 | 31.6 | - 98 |
| | r=6 i=15 | 39.8 | 1990 |

The effect of the proposed solar tax credit can be seen by comparing Table 3 with Table 4.

## 6. CONCLUSION

Our findings indicate that the solar-heat pump configuration can be an economical proposal if the price of electricity increases by nine percent in real term, i.e., i=15, or more per year. Also, the superiority of the parallel configuration over utilization of solar heat storage with a heat pump can be seen in the lower gains/higher losses of the latter with no improvement in solar utilization. The economic superiority of the air cooled solar system over the liquid system can be seen by comparing the corresponding columns of Tables 3 and 4 for liquid and air systems. Finally, the effect of the proposed federal solar tax credit is observed in both the enchancement of solar utilization and the improvement in the financial column.

## REFERENCES

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