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DESIGN GUIDE FOR COLD-FORMED STEEL PURLIN ROOF FRAMING SYSTEMS D111-09

design guide

Design Guide for Cold-Formed Steel Purlin Roof Framing Systems

DESIGN GUIDE D111-09

June 2009
with Errata Incorporated

Committee on Specifications
for the Design of Cold-Formed
Steel Structural Members

American Iron and Steel Institute

DESIGN GUIDE
FOR COLD-FORMED STEEL
PURLIN ROOF FRAMING SYSTEMS

Design Guide D111-09
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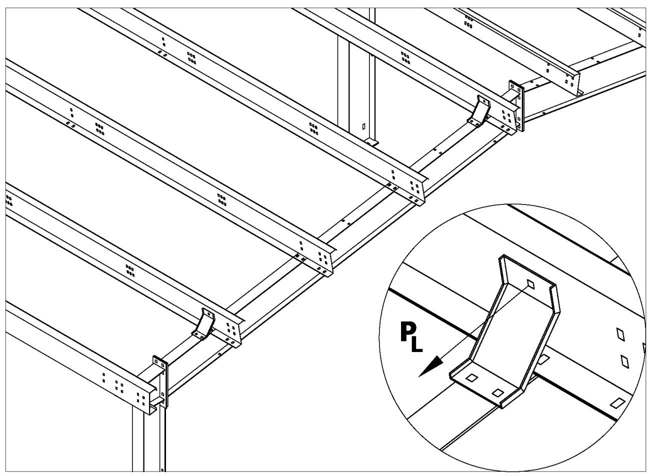

Committee on Specifications
for the Design of Cold-Formed
Steel Structural Members

American Iron and Steel Institute
1140 Connecticut Avenue, NW
Washington, DC 20036

The following Design Guide has been developed under the direction of the American Iron and Steel Institute Committee on Specifications for the Design of Cold-Formed Steel Structural Members. The development of the Guide was sponsored by the American Iron and Steel Institute (AISI) and the Metal Building Manufacturers Association (MBMA). The AISI Committee on Specifications and MBMA Technical Committee wish to acknowledge and express gratitude to Dr. Thomas Murray, Mr. Jeff Sears and Dr. Mike Seek who are the authors of this Guide.

With anticipated improvements in understanding of the behavior of cold-formed steel and the continuing development of new technology, this material might become dated. It is possible that AISI will attempt to produce updates of this Guide, but it is not guaranteed.

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First Printing – June, 2009
Second Printing – March 2011

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PREFACE

The *Design Guide for Cold-Formed Steel Purlin Roof Framing Systems* provides information for the designer of single span and continuous multiple span steel purlin-supported roof systems with an emphasis on the design anchorage systems. The Design Guide is based on the American Iron and Steel Institute (AISI) *North American Specification for the Design of Cold-Formed Steel Structural Members, 2007 Edition*. Where the *Specification* is silent on design issues, the procedures are based on published references and on the opinions of the authors.

The Design Guide was co-sponsored by the American Iron and Steel Institute (AISI) and the Metal Building Manufacturers Association (MBMA).

AISI and MBMA acknowledge the efforts of Dr. Thomas M. Murray, P.E., Emeritus Professor of Civil Engineering and Environmental Engineer, Virginia Tech; Dr. Michael W. Seek P.E., Walter Seek Engineering, Johnson City, Tennessee; and Mr. Jeff Sears, P.E., Kirkpatrick Forest Curtis PC, Oklahoma City, Oklahoma. The authors wish to acknowledge the financial assistance of Virginia Tech and Star Building Systems, Oklahoma City, Oklahoma, in sponsoring a large part of the background research for this Design Guide. The contributions of Dennis Watson, P.E., BC Steel, Oklahoma City, are also acknowledged.

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Design Guide for Cold-Formed Steel Purlin Roof Framing Systems
2009 Edition

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DESIGN GUIDE FOR COLD-FORMED STEEL PURLIN ROOF FRAMING SYSTEMS

CHAPTER 1 INTRODUCTION

A typical cold-formed steel purlin roof framing system consists of four primary components: roof sheathing (panels), purlins, purlin braces, and system anchorage. Such systems are truly structural systems: the roof sheathing supports both gravity and wind uplift loading while providing lateral support to the purlins. In turn, the purlins support the roof sheathing and provide lateral and, together with flange braces, flexural-torsional support to the supporting building frame members. The system anchorage restrains displacements of the purlin in the plane of the roof with the resulting forces resisted by anchorage devices (anti-roll clips) at the building frames or by restraint braces within the purlin spans. Optional in-plane or torsional braces provide lateral support to purlins at discrete locations. Some purlins, referred to as strut purlins, are required to carry axial force from the building end walls to the longitudinal bracing system. Figure 1.1 shows cold-formed steel roof framing for a typical metal building.

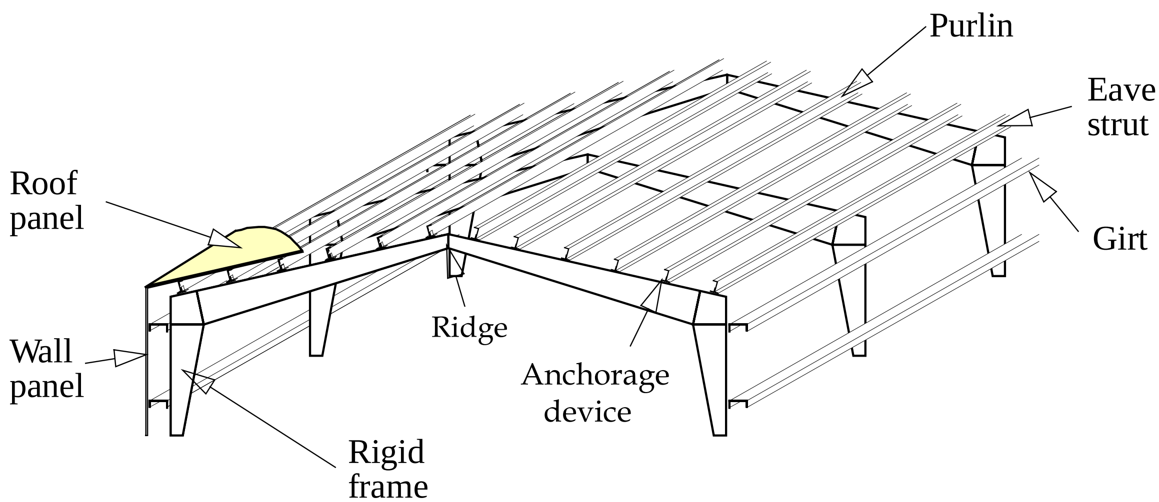
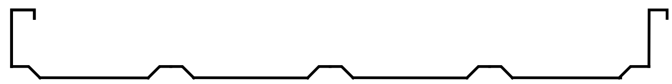


Figure 1.1 Typical Metal Building Framing

Steel roof panels serve as an environmental barrier as well as providing restraint to the supporting purlins. Roof panels are one of two basic types: through-fastened (sometimes referred to as screw-fastened) and standing seam. Panel profiles are commonly referred to as pan-type as shown in Figure 1.2(a), or rib-type as shown in Figure 1.2(b). Through-fastened panels are attached directly to the supporting purlins using self-drilling or self-tapping screws as shown in Figure 1.3(a). Where purlins are highly restrained, thermal movement of attached panels can potentially enlarge the screw holes resulting in roof leaks.

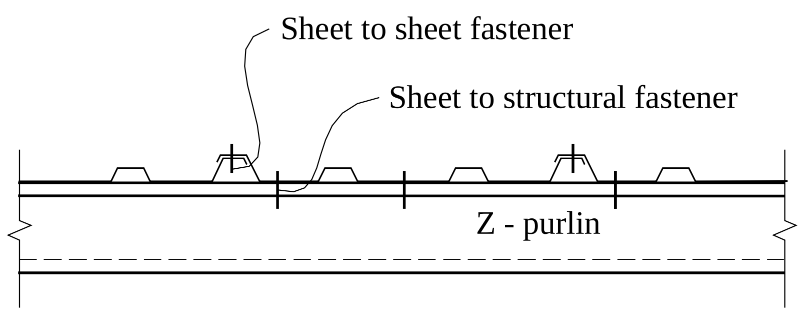
Standing seam roofing provides a virtually penetration-free surface resulting in a water-tight roof membrane. Except at the building eave or ridge and panel end laps, standing seam panels are attached to the supporting purlins using concealed clips as shown in Figure 1.3(b). The clips are attached to the purlin flange using self drilling screws, that combine drilling and tapping functions. The standing seam clips are specially designed connection elements that are embedded in the seam of the standing seam roof panels during field assembly. There are two basic clip types: fixed, Figure 1.4(a) and sliding or two-piece clip, Figure 1.4(b). Thermal movement is accounted for by movement between the roof panel and the fixed clip, by bending of the fixed clip, or by movement between the parts of the sliding clips. The lateral support provided by standing seam panels and clips is highly dependent on the panel profile and clip details. The design of standing seam roof panels is described in the American Iron and Steel Institute publication, *A Guide for Designing with Standing Seam Roof Panels* (AISI 1997).



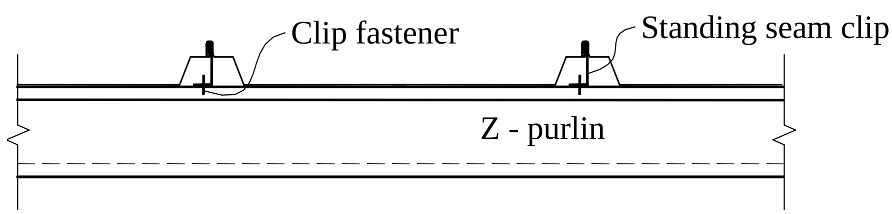
(a) Pan-Type Panel Profile

(b) Rib-Type Panel Profile

Figure 1.2 Panel Profiles

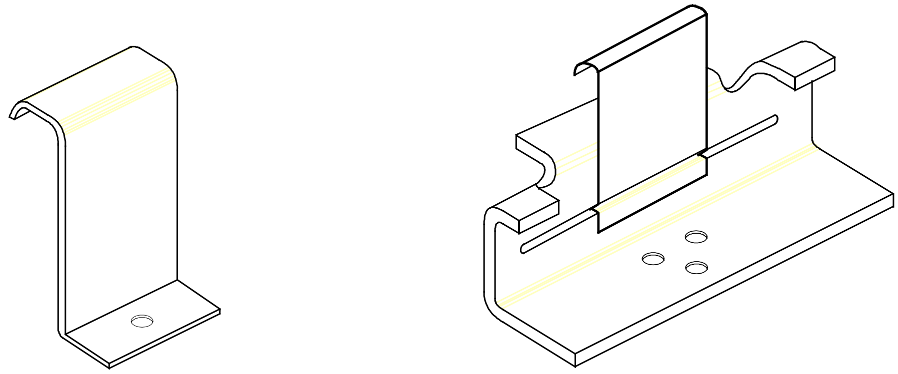


(a) Through-Fastened Panel



(b) Standing Seam Panel

Figure 1.3 Roof Panel Profiles



(a) Fixed Clip **(b) Sliding or Two-Piece Clip**

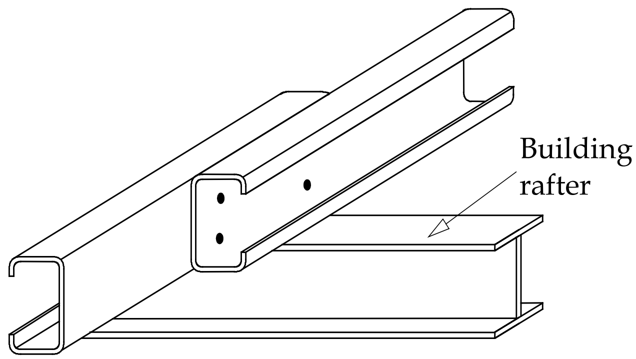
Figure 1.4 Types of Standing Seam Clips

Cold-formed steel roof framing systems are commonly constructed using cold-formed C- or Z-sections, referred to as purlins. Purlins are considered the primary load carrying components of the roof system but are commonly called secondary members with respect to the entire building system. Generally, purlins are lapped as shown in Figure 1.5 to provide continuity and therefore greater efficiency. Z-section purlins are essentially point-symmetric; however, some manufacturers produce Z-sections with slightly unequal width flanges to facilitate nesting in the lapped region. The lap connection is usually made with at least two machine-grade bolts through the webs of the lapped purlins near each end of the lap as shown in Figure 1.5. In addition, the purlins are either flange bolted to the supporting rafter or connected as shown in Figure 1.6.

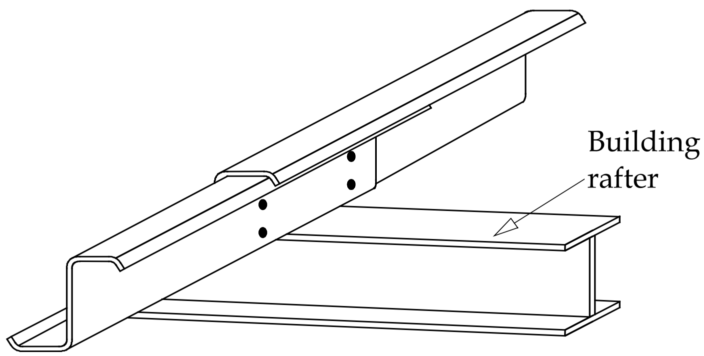
The effective lateral support provided to purlins by the panel and the system anchorage is a function of the purlin attachment and the system details as well as the loading direction. Generally, through-fastened sheathing is assumed to provide full lateral and torsional restraint for gravity loading in the positive moment region (the portion of the span where the panel is attached to the purlin compression flange). Design assumptions for the negative moment region (the portion of the span where the panel is attached to the purlin tension flange) vary from unrestrained to fully restrained. A common assumption is that the purlin is unbraced between the end of the lap and the adjacent inflection point (AISI 1997), but this assumption may be unduly conservative as is discussed in Chapter 2.

For uplift loading, through-fastened sheathing provides lateral but not full torsional restraint. Attempts have been made to develop test methods to determine the torsional restraint provided by specific panel profile/screw combinations. However, the variability of the methods and their complexity necessitated something simpler for routine use. Consequently, the empirical R-factor method was developed for determining the flexural strength of through-fastened roof purlins under uplift loading

The lateral and torsional restraint provided by standing seam roof systems varies considerably depending on the panel profile and the clip details. Consequently, a generic solution is not possible and the Base Test Method (AISI 2008) was therefore developed. Alternatively, the purlins can be designed as unbraced beams between restraint or brace locations.



(a) Lapped C-Purlins



(b) Lapped Z-Purlins

Figure 1.5 Purlin Continuity Laps

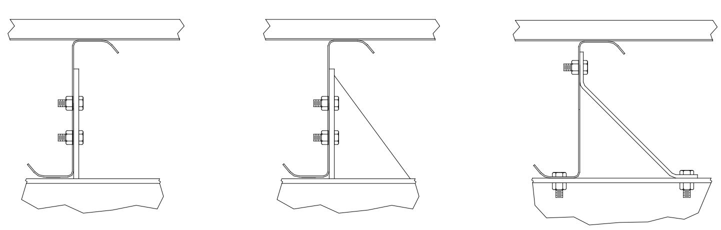
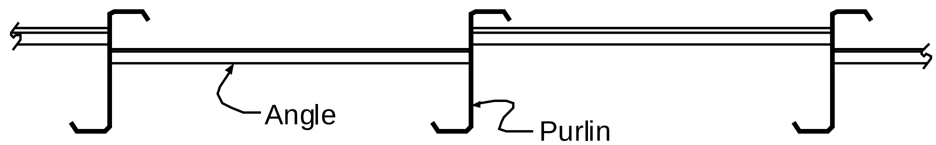
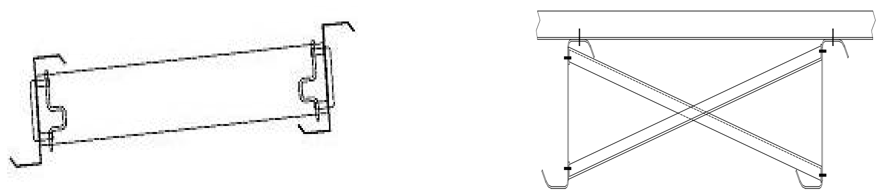


Figure 1.6 Purlin-to-Rafter Connections

Additional lateral restraint to the purlins is sometimes provided by discrete purlin braces. These braces may be horizontal angles capable of resisting compression or tension, angle X-braces, threaded rods, proprietary devices, or torsional braces. Examples of discrete and torsional braces are shown in Figure 1.7. Lateral bracing forces that accumulate either in the discrete braces or the diaphragm must be anchored to the primary lateral load resisting system; external anchorage is not needed for torsional braces.



(a) Discrete Brace



(b) Torsional Braces

Figure 1.7 Examples of Discrete and Torsional Braces for Purlin Stability

The use of C- or Z-section purlins leads to eccentric loading and bending oblique to the principal axes of the purlins, which cause the purlins to tend to twist. Typical engineering practice is to design the purlins for the component of the applied load normal to the plane of the roof using the purlin section properties about the centroidal axes that are perpendicular (x-axis) and parallel (y-axis) to the purlin web. This approach assumes that the purlin experiences fully constrained bending due to the load component parallel to the web and that the lateral and torsional effects due to any eccentricity and the down slope component are negligible or are resisted by some other means. It is common practice to assume that the partial lateral and torsional restraint provided by the roof sheathing is adequate to reduce the torsional effect to a negligible level provided the lateral displacements are held within prescribed limits. The torsional moments transferred into the sheathing tend to counteract and are resolved within the roof sheathing, and therefore do not typically require special consideration. However, the lateral forces will accumulate as a shear force in the plane of the diaphragm that must be removed by an external anchorage system. The behavior of Z-sections in roof systems is very complex and subject to many subtleties. On low slope roofs, Z-sections have the tendency to translate "uphill" towards the ridge; whereas, on roofs with steeper slopes, a Z-section will translate "downhill" towards the eave.

System anchorage is used to limit the lateral (uphill or downhill) displacement of the purlins. The most common system anchorage consists of devices or braces attached to purlin webs near the top flange and directly or indirectly connected to the primary structural framing to limit the lateral displacement of the purlins. Anchorage is most often applied at the purlin supports because of the ease in which the force can be transferred out of the system. When located at the frame lines, the anchorage devices (anti-roll clips) typically consist of either a web wing plate, a multi-piece welded assembly that is attached to the purlin web and to the top flange of the rafter, or a diagonal bent clip as shown in Figure 1.6. Or, discrete anchorage points may be located at various locations along the purlin span, similar to lateral restraint braces described above. Anchorage braces and lateral restraint braces may be the same unless the latter are "floating" braces. For anchorage braces within the purlin span, special detailing considerations need to be addressed with preferred details varying greatly.

The intent of this Design Guide is to provide a comprehensive review of C- or Z-purlin supported cold-formed steel purlin roof framing systems with emphasis on the design of anchorage. All design provisions are from the 2007 edition of the American Iron and Steel Institute *North American Specification for the Design of Cold-Formed Steel Structural Members - ANSI S100 2007 (*AISI *2007)*, referred to hereafter as the *Specification.* Chapter 2 is an overview of design methods for cold-formed purlin supported roof systems. The design of continuous purlin lines is discussed in Chapter 3, along with ASD and LRFD example calculations. The system anchorage requirements in the *Specification* are discussed in Chapter 4, as well as, simplified and matrix based solutions, along with an extensive set of ASD and LRFD examples. Chapter 5 includes alternate methods (the component stiffness method, frame element stiffness modeling, and finite element modeling) for analyzing complex anchorage systems

Anchorage configurations, applications, and the analysis procedures in order of increasing complexity are listed in Table 1.1 with references to the applicable sections of this Design Guide.

Table 1.1 Anchorage Analysis Procedures

Anchorage Configuration	Application	Simplified Solution (4.5.1)	Main Spec. Procedure (4.2)	Matrix Solution (4.5.2)	Component Stiffness (4.5.3 & 5.1)	Frame Element Stiffness Model (5.2)	Finite Element Stiffness Model (5.3)
Lateral Bracing							
Supports, 1/3 Points or Midpoint	(a)	X	X	X	X	X	X
	(b)		X	X	X	X	X
				X		X	X
1/4 Points or 1/3 Points + Supports	(b)				X	X	X
						X	X
Arbitrary Locations						X	X
Torsional Bracing							
1/3 Points + Supports	(b)				X		X
Arbitrary Locations							X

(a) Uniform Purlin Spaces, Uniform Load, Top Flanges Facing Upslope and Anchors Evenly Distributed

(b) Any "Reversed" Purlins Evenly Distributed

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CHAPTER 2 DESIGN METHODS FOR PURLINS

Symbols and Definitions Used in Chapter 2

b	Flange width of the purlin
d	Depth of the purlin
B	Purlin spacing
$C2$	= 8.3 from *Specification* Table D6.3.1-1
$C3$	= 28 from *Specification* Table D6.3.1-1
F_y	Yield stress for design
F_{yt}	Measured yield stress of tested purlin
I_x	Moment of inertia of full unreduced section about major centroidal axis
I_{xy}	Product of inertia of full unreduced section about major and minor centroidal axes
L	Span of the purlins tested, center to center of the supports
M_n	Nominal flexural strength of a fully constrained beam, S_eF_y
$\overline{M}_{nt_{min}}$	Average flexural strength of the thinnest sections tested
$\overline{M}_{nt_{max}}$	Average flexural strength of the thickest sections tested
M_{nt}	Flexural strength of a tested purlin, $S_{et}F_{yt}$
M_{ts}	Failure moment for the single span purlins tested, $w_{ts}L^2/8$
p_d	Weight of the specimen (force/area)
p_{ts}	Failure load (force/area) of the single span system tested
P_L	Lateral anchorage force in accordance with Section D6.3.1 of the AISI S100
r	Correction factor
R	Reduction factor computed for nominal purlin properties
R_t	Modification factor from test, M_{ts}/M_{nt}
$R_{t_{min}}$	Mean minus one standard deviation of the reduction factors of the three thinnest purlins tested
$R_{t_{max}}$	Mean minus one standard deviation of the reduction factors of the three thickest purlins tested
s	Tributary width of the purlins tested
S_e	Section modulus of the effective section
S_{et}	Section modulus of the effective section of the tested member using measured dimensions and the measured yield stress
t	Purlin thickness
t_i	Thickness of uncompressed glass fiber blanket insulation
w_{ts}	Failure load (force/length) of the single span purlins tested
ϕ	Resistance factor
Ω	Safety factor
σ_{max}	One standard deviation of the R_t factors of the thickest purlin tested
σ_{min}	One standard deviation of the R_t factors of the thinnest purlin tested

2.1 General

The *Specification* provides an empirical based method for determining the uplift loading strength of purlins in through-fastened panel roof systems (the R-Factor Method) and a test based method for both the gravity and uplift loading purlin strength in standing seam panel roof systems (the Base Test Method). Determination of the gravity loading strength of through-fastened panel systems is not specified. The industry practice is to assume full lateral and torsional support in the positive moment region and within the purlin lap, but no lateral support between an inflection point and the end of the lap in the negative moment region. However, recent testing (Bryant and Murray 2000) has shown that full lateral and torsional support can also be assumed in this region. Lateral and torsional support for cantilevers is a matter of engineering judgment.

Testing and rational engineering methods are permitted by the *Specification* for certain circumstances. For instance in *Specification* Section D6.1.1, Flexural Members Having One Flange Through-Fastened to Deck or Sheathing, it is stated that "if variables fall outside any of the above stated limits, the user shall perform full scale tests … or apply a rational engineering analysis procedure."

2.2 R-Factor Method for Purlins Supporting Through-Fastened Panel Systems with Uplift Loading

The design procedure for purlins subject to uplift loading in *Specification* Section D6.1.1, Flexural Members Having One Flange Through-Fastened to Deck or Sheathing, is based on the use of reduction factors (R-factors) to account for the flexural, torsional or nonlinear distortional behavior of purlins with through-fastened sheathing. The R-factors are based on tests performed on simple span and continuous span systems using both C- and Z-sections. All tests were conducted without intermediate lateral bracing.

The R-factor design method simply involves applying a reduction factor (R) to the full elastic bending section strength (S_eF_y) as given by *Specification* Equation (D6.1.1-1) to give the nominal member moment strength.

$$M_n = RS_eF_y \qquad \text{(Eq. D6.1.1-1)}$$

with

R = 0.6 for continuous span C-sections
= 0.7 for continuous span Z-sections
= values from *Specification* Table D6.1.1-1 for simple span C- and Z-sections.

The restraint provided to the purlin is dependent on the behavior of the panel-to-purlin connection, and the rotational stiffness of the connection is dependent on purlin thickness, panel thickness, fastener type and location, and insulation. Therefore, the reduction factors only apply for the range of sections, lap lengths, panel configurations, and fasteners tested as set out in *Specification* Section D6.1.1. For continuous span purlins, compressed glass fiber blanket insulation of thickness between zero and 6 in. does not measurably affect the purlin strength. The effect is greater for simple span purlins requiring that the reduction factor (R) be further reduced to rR, where

$$r = 1.00 - 0.01t_i \qquad \text{(Eq. D6.1.1-2)}$$

The resulting design strength moment (ϕ_bM_n) or the allowable stress design moment capacity (M_n/Ω_b) is compared with the maximum bending moment in the span determined from an elastic analysis. The resistance factor (ϕ_b) is 0.90 and the safety factor (Ω_b) is 1.67.

The *Specification* R-factor design method does not apply to the region of a continuous beam between an inflection point and a support nor to cantilever beams. For these cases, the design must explicitly consider lateral-torsional buckling.

If the section geometry, lap length, panel configuration, fastener or combinations thereof are outside the Section D6.1.1 limits, full-scale tests or a rational engineering analysis may be used to determine the design strength.

2.3 The Base Test Method for Standing Seam Panel Systems

The lateral and torsional restraint provided by standing seam sheathing and clips depends on the panel profile and clip details. Lateral restraint is provided by both friction in the clip and drape or hugging of the sheathing. Because of the wide range of panel profiles and clip details, a generic solution for the restraint provided by the system is impossible. For this reason the Base Test Method uses separate sets of simple span, two purlin line tests to establish the nominal moment strength of the positive moment regions of gravity loaded systems and the negative moment regions of uplift loaded systems. The results are then used to predict the strength of multi-span, multi-line systems for either gravity or wind uplift loadings.

The nominal moment strength of the positive moment regions for gravity loading or the negative moment regions for uplift loading is to be determined using *Specification* Equation D6.1.2-1 given as

$$M_n = RS_eF_y \tag{Eq. D6.1.2-1}$$

with R = the reduction factor determined in accordance with AISI S908 "Base Test Method for Purlins Supporting a Standing Seam Roof System" (AISI 2008). The resistance factor (ϕ_b) for LRFD design is 0.90 and the factor of safety (Ω_b) for ASD design is 1.67.

To determine the relationship for R, six tests are required for each gravity or uplift load case and for each combination of panel profile, clip configuration, and purlin profile, and lateral bracing layout. A purlin profile is defined as a set of purlins with the same depth, flange width, and edge stiffener angle, but with varying thickness and edge stiffener length. Three of the tests are conducted using the thinnest material and three using the thickest material used by the manufacturer for the purlin profile. All components used in the base tests must be nominally identical to those used in the actual systems. The purlins must be oriented in the same direction (purlin top flanges facing toward the building ridge or toward the building eave) as used in the actual building.

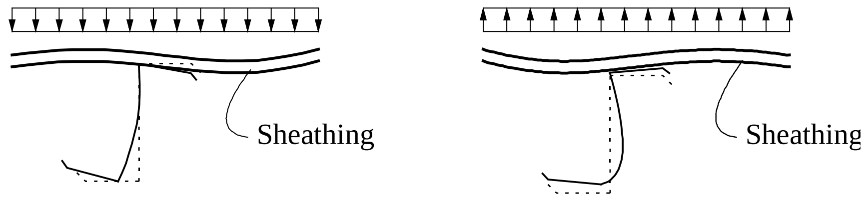
Results from the six tests are then used in Equation (2.3.1) to determine an R-factor relationship

$$R = \left(\frac{R_{t_{max}} - R_{t_{min}}}{\overline{M}_{nt_{max}} - \overline{M}_{nt_{min}}}\right)(M_n - \overline{M}_{nt_{min}}) + R_{t_{min}} < 1.0 \tag{Eq. 2.3.1}$$

The reduction factor for each test (R_t) is computed from

$$R_t = M_{ts} / M_{nt} \tag{Eq. 2.3.2}$$

Reported reduction factor values are generally between 0.40 and 0.98 for both gravity and uplift loading depending on the panel profile and clip details. Gravity loading tends to increase purlin rotation as shown in Figure 2.3.1(a), and uplift loading tends to decrease Z-purlin rotation as shown in Figure 2.3.1(b). For some standing seam Z-purlin systems, sufficient torsional restraint is provided by the panel/clip connection, so that a larger reduction factor may be obtained for uplift loading than for gravity loading.



(a) Gravity Loading **(b) Uplift Loading**

Figure 2.3.1 Purlin Rotation due to Gravity and Uplift Loading

The maximum single span moment (M_{nt}) is determined using the loading at failure determined from

$$w_{ts} = (p_{ts} + p_d)\ s + 2P_L\ (d/B) \qquad \text{(Eq. 2.3.3)}$$

where

$$P_L = 0.5\left(\frac{C2}{1000}\frac{I_{xy}L}{I_x d} + C3\frac{0.25bt}{d^2}\right)(p_{ts} + p_d)s$$

The expression $2P_L(d/B)$ in Equation (2.3.3) takes into account the effect of the overturning moment on the system due to the anchorage forces applied at the top flange of the purlin by the panel and resisted at the bottom flange of the purlin at the support. The expression $2P_L(d/B)$ is applied only to Z-sections under gravity loading when the purlin flanges are facing in the same direction, but is not to be included when discrete point braces are used and the braces are restrained from lateral movement. In addition, the expression $2P_L(d/B)$ is not to be applied unless the downslope (eave side) purlin is the first to fail.

The AISI Base Test procedure requires that the tests be conducted using a test chamber capable of supporting a positive or negative internal pressure differential. Figure 2.3.2 shows a typical chamber.

Construction of a test setup must match that of the field erection manuals of the standing seam roof system manufacturer. The lateral bracing provided in the test must match the actual field conditions. For example, if anchorage devices are installed at the rafter support of each purlin in the test then anchorage devices must be provided at every purlin support location in the actual roof. Likewise, if intermediate lateral support is provided in the test, the support configuration is required in the actual roof. If anti-roll clips are used at the supports of only one purlin in the test, the unrestrained purlin can be considered a "field" purlin as long as there is no positive connection between the two purlins. That is, if standing seam panels or an end-support angle is not screw-connected to both purlins, the actual "field" purlins need only be attached to the primary support member.

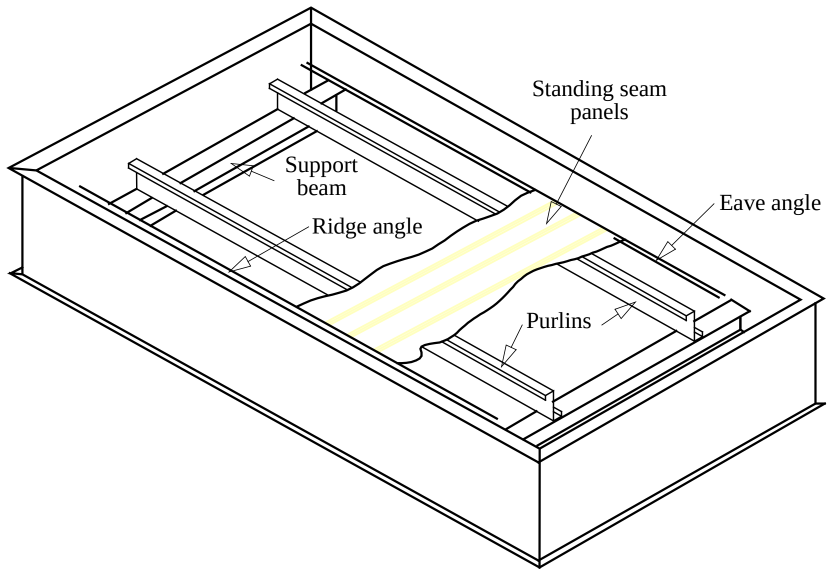


Figure 2.3.2 Base Test Chamber

Example 2.1. The R-value relationship for a set of gravity loading base test data is to be determined. The tests were conducted using Z-sections with nominal thicknesses of 0.060 in. and 0.095 in. and a nominal yield stress of 55 ksi. The span length was 22 ft 9 in. and intermediate lateral braces were installed at the one-third points. In the following, the total supported load (w_{ts}) is equal to the sum of the applied load (w) and the weight of the sheathing and purlins (w_d). The maximum applied moment is M_{ts}. The moment M_{nt} is calculated using an effective section modulus, S_{et}, determined using on the measured thickness, t, and the measured yield stress, F_y. The reduction factor for each test, R_t, is from Equation 2.3.2.

A. Summary of Test Loadings

Test Number	Max. Applied Load w (lb/ft)	Deck Weight (lb/ft)	Purlin Weight (lb/ft)	w_d (lb/ft)	Total Load, w_{ts} (lb/ft)	M_{ts} (kip-in)
1	91.8	4.0	3.10	7.10	98.9	76.8
2	86.3	4.0	3.14	7.14	93.4	72.5
3	81.8	4.0	3.10	7.10	88.9	69.0
4	186.5	4.0	5.05	9.05	195.4	151.7
5	189.1	4.0	5.01	9.01	198.1	153.8
6	184.5	4.0	4.91	8.91	193.4	150.2

B. Reduction Factor Data

Test Number	t (in.)	S_{etf} (in.)	F_y (ksi)	$M_{nt}=S_{eff}F_y$ (kip-in)	M_{ts} (kip-in)	R_t
1	0.059	1.88	60.0	112.8	76.8	0.681
2	0.059	1.90	59.2	112.5	72.5	0.645
3	0.060	1.92	57.3	110.0	69.0	0.628
Average				111.8		0.651
4	0.097	3.38	68.4	231.2	151.7	0.656
5	0.096	3.38	67.1	226.8	153.8	0.678
6	0.097	3.30	66.5	219.4	150.2	0.685
Average				225.8		0.673

Using the test data,

σ_{max} = one standard deviation of the modification factors of the thickest purlins tested = 0.0148

σ_{min} = one standard deviation of the modification factors of the thinnest purlins tested = 0.0271

$R_{t_{min}} = 0.651 - 0.0271 = 0.624$

$R_{t_{max}} = 0.673 - 0.0148 = 0.658$

$\overline{M}_{nt_{min}} = 111.8 \text{ kip-in}$

$\overline{M}_{nt_{max}} = 225.8 \text{ kip-in}$

C. Reduction Factor Relation

Using Equation (2.3.1):

$$R = \left(\frac{R_{t_{max}} - R_{t_{min}}}{\overline{M}_{nt_{max}} - \overline{M}_{nt_{min}}} \right) (M_n - \overline{M}_{nt_{min}}) + R_{t_{min}} < 1.0 \qquad \text{(Eq. 2.3.1)}$$

$$= \left(\frac{0.658 - 0.624}{225.8 - 111.8} \right) (M_n - 111.8) + 0.624 \le 1.0$$

$$= 0.298(M_n - 111.8)/1000 + 0.624 \le 1.0$$

The variation of R with purlin strength for the standing seam roof system tested is shown in Figure 2.3.3. Figure 2.3.3 shows the R-value line with slope upward to the right. For some standing seam roof systems, the line will slope downward to the right.

D. Application

For a purlin having the same nominal depth, flange width, edge stiffener slope, and material specification as those used in the above and with a nominal flexural strength $M_n = S_eF_y$ = 135 in-kips, the reduction factor is

R = 0.298 (135-111.8)/1000 + 0.624 = 0.631

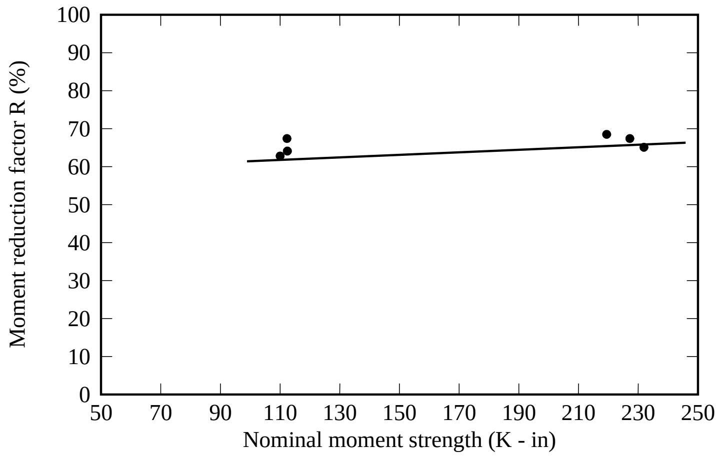


Figure 2.3.3 Reduction Factor versus Nominal Moment Strength

The positive moment design strength is then:

LRFD:

ϕ_b = 0.90

$\phi M_n = \phi_b R S_e F_y = 0.90 \times 0.631 \times 135 = 76.7$ kip-in.

ASD:

Ω_b = 1.67

$M = R S_e F_y / \Omega_b = 0.631 \times 135/1.67 = 51.0$ kip-in.

The Base Test Method requires that a set of six base tests must be conducted for each combination of purlin profile, deck panel profile, clip type, intermediate bracing configuration, and loading. This requirement can result in a large number of tests for a given manufacturer. For instance, if a manufacturer produces

- a Z-purlin profile with two flange widths,
- a standing seam deck profile with two thicknesses, and
- three clips types (low sliding, high sliding, and low fixed),

the required number of base tests is (2 x 2 x 3) times six base tests for each loading condition or 156 tests.

Trout and Murray (2000) found that, for a specific purlin depth, purlin flange width, clip type, and roof panel thickness have an effect on the strength of standing seam roof systems. By comparing the strength reduction factors obtained from tests using various roof components, the following trends were found: a single clip type produced the lowest results when compared to the other clips. Tests using purlins with a narrow flange width resulted in lower strengths for both thin and thick purlins of the same nominal cross-section. And, roof panel thickness was found to have no effect on the strength of systems constructed with 10 in. deep purlins but did affect the strength when 8 in. deep purlins were used. Although none of the roof components can be completely eliminated from a test matrix, by using trend relationships an acceptable test protocol was developed for reducing the number of tests required for the Base Test Method.

Assuming a manufacturer has three clip types, two flange widths for each purlin type of one depth, and two nominally identical standing seam panel profiles rolled in two gages, the following procedure will result in a R-value relationship (Equation 2.3.1) for all combinations with relatively few base tests. This procedure assumes that the combination of one panel thickness, one clip type, and the purlin cross-section with the narrower flange width results in the lowest R-value for all other combinations of parameters. The procedure is:

- The clip type, which is thought to result in the lowest R_t-value is selected. For illustration, type C3 (tall sliding) is assumed.
- Using this clip type, the thinner panel, and the purlin with the narrow flange width, two base tests are conducted for one depth purlin of the same nominal cross-section. One base test is conducted with the thinnest purlin and one test with the thickest purlin in the inventory.
- With the R_t-values from the two tests, a trend line is found as shown in Figure 2.3.4. Depending on the details of the system, the trend line can have either positive or negative slope as shown.
- To verify the choice of clip, two additional tests are conducted using the purlin thickness that resulted in the lower R_t-value, one with each of the other two clip types C1 and C2. In Figure 2.3.4, the thinner purlin controls for the solid trend line and the thicker purlin controls for the dashed trend line.
- If the original clip type does result in the lowest R_t-value, as shown in Figure 2.3.5, the choice of clip type is verified.
- If the original clip does not result in the lowest R_t-value, a test using the clip with the lowest R_t-value and the other purlin thickness is conducted. Figure 2.3.6 shows the resulting data, assuming clip type C2 is the controlling clip type.

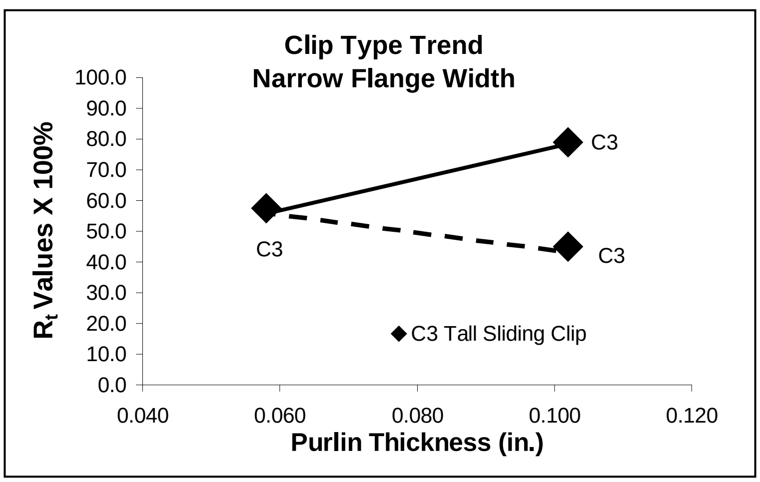


Figure 2.3.4 Possible Clip Type Trend Relationships

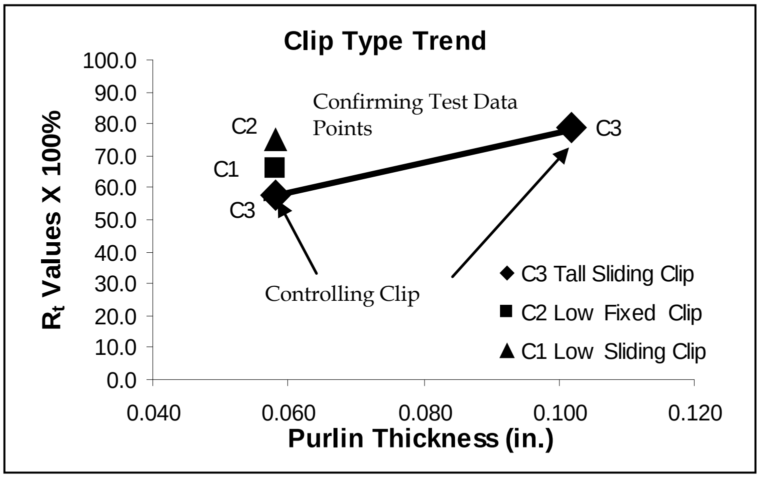


Figure 2.3.5 Confirmation of Initial Choice of Clip Type

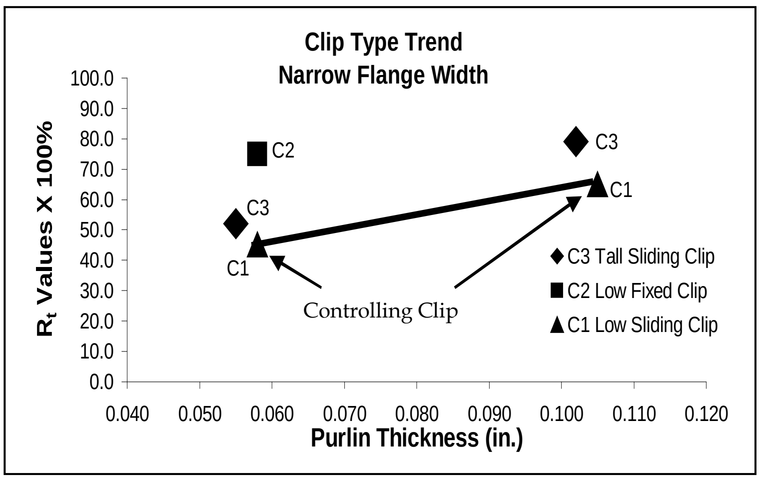


Figure 2.3.6 Confirmation of Clip Type

- Knowing the controlling clip type, two additional tests are required to validate the choice of panel thickness: one test is conducted using the controlling clip-type, the thinner purlin, and the other panel thickness; the other test is conducted using the controlling clip-type, the thicker purlin thickness and the other panel thickness.
- Using the combination of clip-type and panel thickness that resulted in the lowest R_t-value for the two purlin thicknesses, the remaining four tests required for the Base Test Method are then conducted and the R-value relationship is developed. Figure 2.3.7 shows the completed test sequence and the final R-value line.

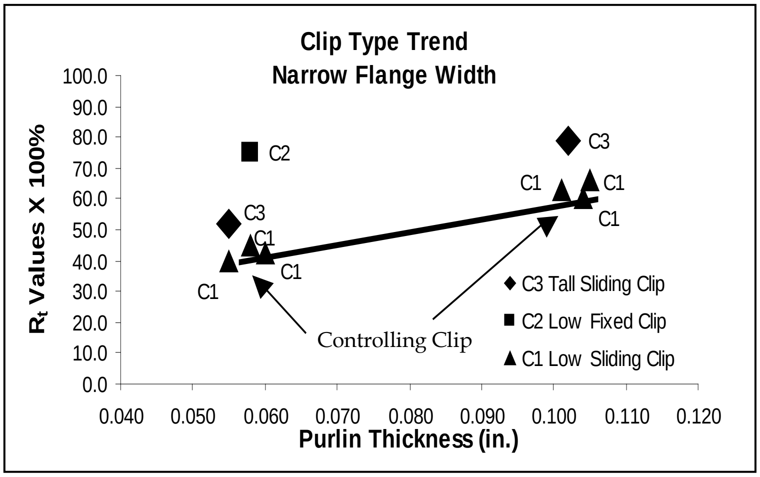


Figure 2.3.7 Final Results

Using the reduction procedure and assuming only one purlin depth and one purlin cross-section, the minimum required number of tests, for an inventory with three clip types, two flange widths, and two panel thicknesses is:

- Two tests with the initial clip type assumption to determine slope of the trend line (one thin and one thick purlin).
- Two tests to confirm initial clip-type selection (two remaining clip types).
- One test to determine panel thickness trend (with controlling clip type).
- Four tests required to meet the requirements of the Base Test Method.

That is, 2 + 2 + 1 + 4 or 9 tests. Thus, the required number of base tests for the loading condition (gravity or uplift) being considered is reduced from 72 tests (see Section 1.1) to 9 tests in the best-case scenario. The worst-case scenario requires 14 tests.

Example 2.2. This example demonstrates the above procedure using actual test data. The following components were used in the tests:

- Three clip types: low sliding clip (C1), low fixed clip (C2), and tall sliding clip (C3).
- 8 in. deep Z-purlins with two thicknesses, thinnest and thickest in the inventory.
- Two flange widths: 2-½ in. and 3-½ in.

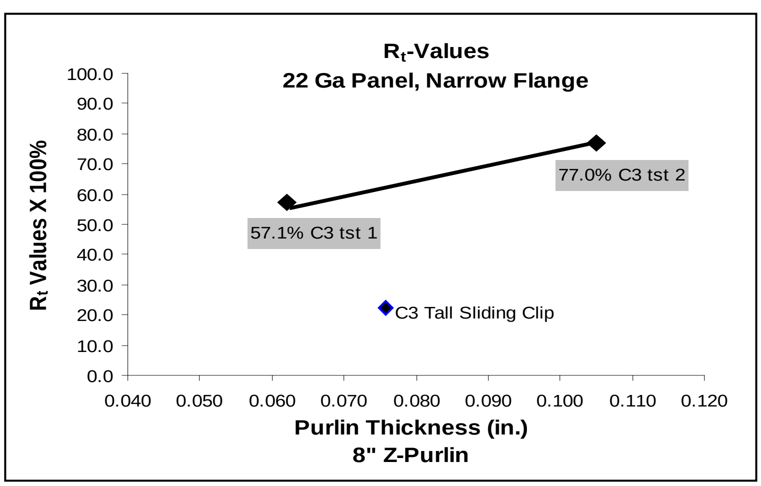


Figure 2.3.8 Initial test with Assumed Controlling Clip Type

- 22 ga. and 24 ga. standing seam roof panel thicknesses having nominally identical profiles.

It was initially assumed that the tall sliding clip, C3, controls, and the initial two tests were conducted using the thinnest and thickest 8 in. deep Z-purlins with 2-½ in. flange width (narrow flange) and the 22 ga. roof panel. The results of these tests are shown in Figure 2.3.8. The thinner purlin gives the lower R_t-value of 0.571 (57.1 percent).

Based on these results, tests were conducted using the other two clip types, low sliding, C1, and low fixed, C2, and tested using the thinner purlin thickness, with all other roof components remaining nominally the same. The resulting R_t-values for the tests are shown in Figure 2.3.9. The R_t-values obtained where 60.2 percent (C1) and 61.4 percent (C3), confirming that the high sliding clip controls. If the original clip selection did not result in the lowest R_t-value, a test using the clip with the lowest R_t-value and the other purlin thickness would be conducted.

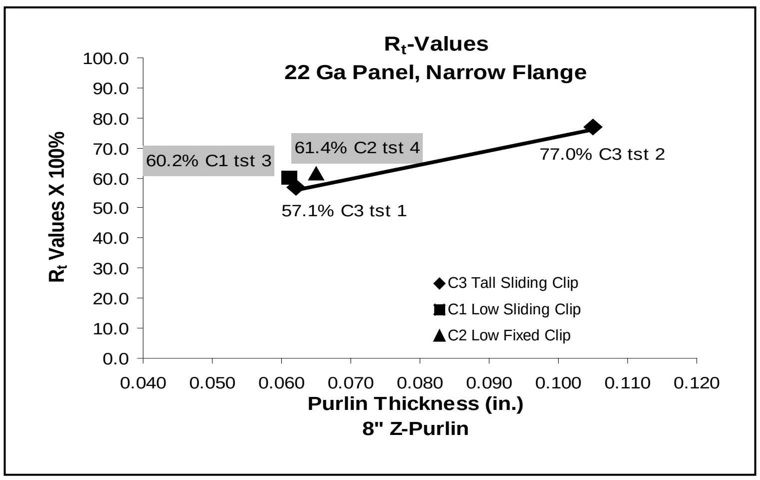


Figure 2.3.9 Controlling Clip Type Verification

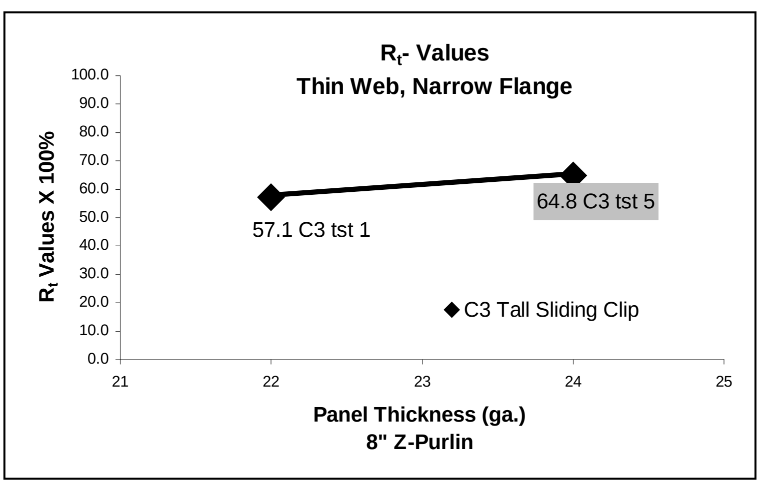


Figure 2.3.10 Roof Panel Trend Verification

After finding the controlling clip type and purlin thickness that results in the lowest R_t-value, the next step was to validate the panel thickness assumption. To do this, a test constructed using the same clip type and purlin thickness with the other roof panel thickness was conducted. From Figure 2.3.10, the high sliding clip, C3, and the thinner purlin resulted in the lowest R_t-value. Therefore, a test was conducted using the same clip type and purlin thickness but with 24 gage deck material. The resulting R_t-value is 64.8 percent, which is greater than the R_t-value of 57.1 percent when the 22-gage roof panel was used as shown in Figure 2.3.10. Thus, the remaining tests were conducted using the 22-gage panel.

Knowing the combination of clip-type and panel thickness, which results in the lowest R_t-value for the two-purlin thicknesses, four additional tests were required to satisfy the requirements of the Base Test Method. Figure 2.3.11 shows the results of the completed test sequence.

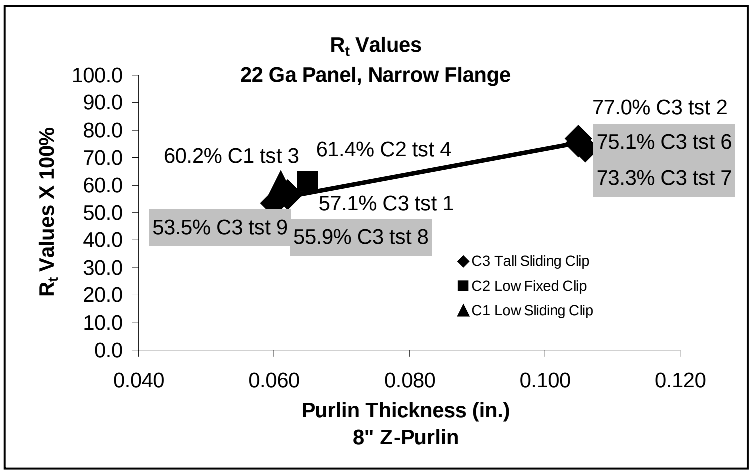


Figure 2.3.11 Final Verification

From the test data generated, the expression for the reduction factor (Equation 2.3.1) is developed as follows. From data in Table 2.3.1,

$$R_{t_{min}} = 0.555 - 0.018 = 0.537$$

$$R_{t_{max}} = 0.751 - 0.019 = 0.732$$

Table 2.3.1 Summary of Gravity Loading Base Test Results

Test No.	Purlin Thickness (in.)	R_t-value	$M_{nt}=S_tF_y$ (K-in.)
1	0.062	0.571	110.5
8	0.062	0.559	118.3
9	0.060	0.535	116.3
Average		0.555	115.0
Standard Deviation		0.018	
2	0.105	0.770	215.3
6	0.105	0.751	218.3
7	0.106	0.733	219.7
Average		0.751	217.8
Standard Deviation		0.019	

$\overline{M}_{nt_{min}} = 115.0\,\text{kip} - \text{in}$

$\overline{M}_{nt_{max}} = 217.8\,\text{kip} - \text{in}$

Thus, the reduction factor equation, in terms of the nominal flexural strength of the section, M_n, for the tested purlins is:

$$R = \left(\frac{R_{t_{max}} - R_{t_{min}}}{\overline{M}_{nt_{max}} - \overline{M}_{nt_{min}}}\right)(M_n - \overline{M}_{nt_{min}}) + R_{t_{min}} < 1.0 \tag{2.3.1}$$

$$= \left(\frac{0.732 - 0.537}{217.8 - 115.0}\right)(M_n - 115.0) + 0.537 \le 1.0$$

$$= 1.897(M_n - 115.0)/1000 + 0.537 \le 1.0$$

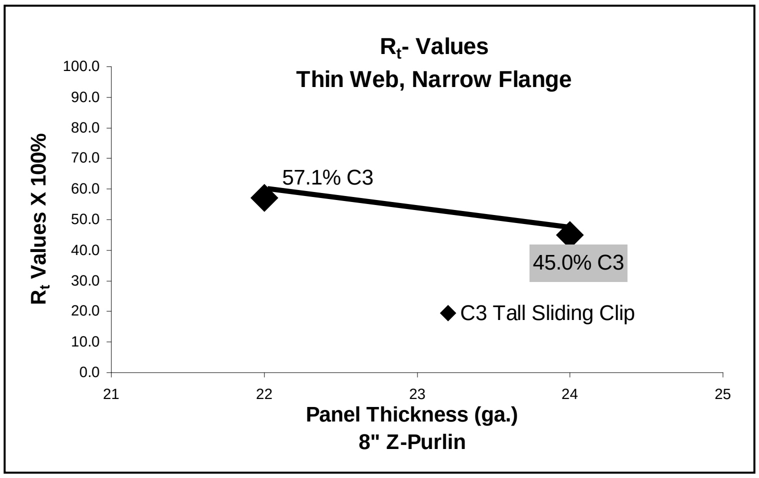


Figure 2.3.12 Roof Panel Thickness Trend

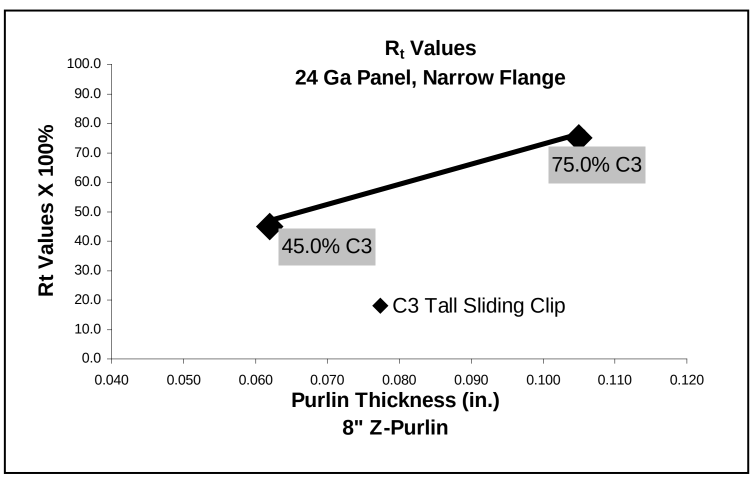


Figure 2.3.13 Determination of Panel Thickness Trend

If the initial choice of roof panel thickness did not result in the lowest R_t-value, as shown in Figure 2.3.10, three additional tests are required to determine the controlling combination. For example, suppose that the use of a 24–gage panel resulted in an R_t-value of 45 percent, as shown in Figure 2.3.12. A controlling purlin thickness, with the now controlling panel thickness, would need to be determined. A test is constructed using the same clip type and the now controlling panel thickness with the thicker purlin. With the R_t-values from the two tests a trend line is found as shown in Figure 2.3.13. The thinner purlin gives the lower R_t-value of 45.0 percent, meaning the thinner purlin is the controlling purlin thickness for the roof system constructed with the 24-gage roof panel.

To verify the controlling clip, two additional tests are conducted using the purlin thickness, which resulted in the lower R_t-value, one with each of the other two clip types (C1 and C2). These two data points are plotted and used to verify that the initial controlling clip type continues to give the lowest R_t- value. The resulting R_t-values for the test with the low fixed and low sliding clip are shown in Figure 2.3.14. The R_t-values obtained were 55.0 percent and 65.0 percent, confirming that the high sliding clip controls.

Knowing the combination of clip type and purlin thickness that result in the lowest R_t-value for the 24-gage roof panel, the remaining four tests required by the Base Test Method are conducted, and the R-value relationship, Equation 2.3.1, is developed as shown above.

For this testing scenario, the required number of base tests increases from 9 to 11, but is less than the 72 tests required by the original test procedure 2, Component Stiffness Method.

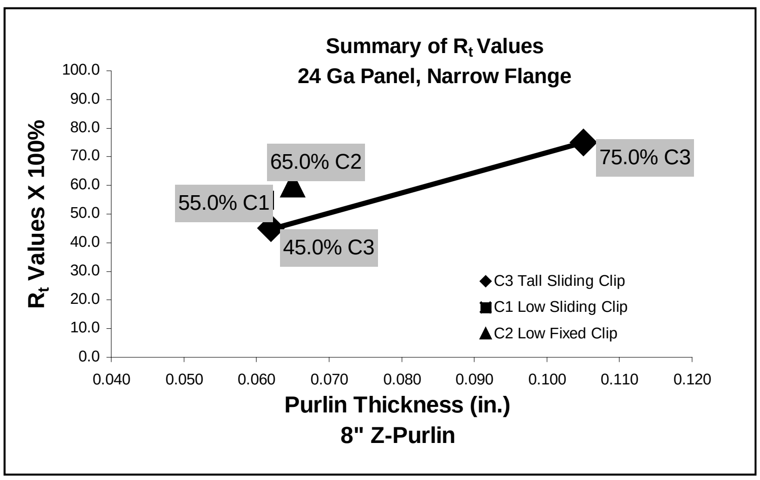


Figure 2.3.14 Controlling Clip Type Verification

CHAPTER 3 CONTINUOUS PURLIN DESIGN

Symbols and Definitions Used in Chapter 3

A_w	Area of web
b	Flange width
C	Coefficient from *Specification* Table C3.4.1-3
C_b	Bending coefficient dependent on moment gradient
C_h	Web slenderness coefficient
C_N	Bearing length coefficient
C_R	Inside bend radius coefficient
C_w	Torsional warping constant of cross-section
D	Dead load
d	Depth of section
E	Modulus of elasticity of steel, 29,500 ksi
F_d	Elastic distortional buckling stress
F_e	Elastic buckling stress
F_v	Nominal shear stress
F_y	Yield stress
G	Shear modulus of steel, 11,300 ksi
h	Depth of flat portion of web measured along plane
I_x, I_y	Moment of inertia of full unreduced section about principal axis
I_{yc}	Moment of inertia of compression portion of section about centroidal axis of entire section parallel to web, using full unreduced section
k_v	Shear buckling coefficient
K_y	Effective length factor for buckling about y-axis
L	Span length
L_r	Roof live load
L_y	Unbraced length of compression member for bending about y-axis
M	Required allowable flexural strength, ASD
M	Required flexural strength at, or immediately adjacent to, the point of application of the concentrated load or reaction, P
$\overline{M}$	Required flexural strength [factored moment]
M_D	Dead load bending moment
M_{Lr}	Roof live load bending moment
M_n	Nominal flexural strength [resistance]
M_{nxo}	Sum of nominal flexural strength about the centroidal x-axis of each purlin at the support determined in accordance with *Specification* Section C3.1.1
M_u	Required flexural strength for LRFD
M_w	Wind load bending moment
N	Actual length of bearing
P	Required allowable strength for concentrated load or reaction in presence of bending moment
$\overline{P}$	Required strength for concentrated load or reaction in presence of bending moment

P_D	Dead load reaction
P_n	Sum of nominal strength for concentrated load or reaction of each purlin at support in absence of bending moment determined in accordance with *Specification* Section C3.4
R	Reduction factor determined in accordance with AISI S908
R	Reduction factor determined from uplift tests in accordance with AISI S908
R	Inside bend radius
S_e	Elastic section modulus of effective section calculated relative to extreme compression or tension fiber at F_y
S_f	Elastic section modulus of full unreduced section relative to extreme compression fiber
t	Base steel thickness of any element or section
V	Required allowable shear strength for ASD
$\overline{V}$	Required shear strength
V_D	Shear force due to deal load
V_{Lr}	Shear force due to roof live load
V_n	Nominal shear strength
V_u	Required shear strength for LRFD
θ	Angle between web and bearing surface >45° but no more than 90°
θ	Angle between vertical and plane of web of Z-section
μ	Poisson's ratio for steel, 0.30
ϕ_b	Resistance factor for bending strength
ϕ_v	Resistance factor for shear strength
ϕ_w	Resistance factor for web crippling strength
Ω_b	Safety factor for bending strength
Ω_v	Safety factor for shear strength
Ω_w	Safety factor for web crippling strength

3.1 Design Assumptions

Because most Z-purlins are essentially point symmetric and the applied loading is generally not parallel to a principal axis, the response to both gravity and wind uplift loading is complex. The problem is somewhat less complex for continuous C-purlins since bending is about principal axes. Design is further complicated when standing seam roof panels, which may provide only partial lateral-torsional restraint, are used. In addition to *Specification* provisions a number of design and analysis assumptions are needed. Commonly used assumptions are:

1. Constrained bending, that is bending is about an axis perpendicular to the web.
2. Full lateral support is provided by through-fastened roof sheathing in the positive moment regions.
3. Partial lateral support is provided by standing seam roof sheathing in the positive moment regions, or the purlins are laterally unrestrained between intermediate braces. For the former assumption, the AISI Base Test Method is used to determine the level of restraint. For the latter assumption, *Specification* lateral buckling equations are used to determine the purlin strength.
4. An inflection point is a brace point.
5. For analysis, the purlin line is either considered prismatic, e.g. ignoring the increased stiffness because of the two cross-sections within the lap, or the purlin line is considered

non-prismatic, e.g. considering the increased stiffness within the lap.

6. Use of vertical short-slotted holes, which facilitate erection of the purlin lines, for the bolted lap web-to-web connection does not affect the strength of continuous purlin lines.
7. The critical location for checking combined bending and shear is immediately adjacent to the end of the lap in the single purlin.

Constrained bending implies that bending is about an axis perpendicular to the Z-purlin web and that there is no purlin movement perpendicular to the web. That is, all movement is constrained in a plane parallel to the web. Since a Z-purlin is point symmetric and because the applied load vector is not generally parallel to a principal axis of the purlin, the purlin tends to move out of the plane of the web and rotate. Constrained bending therefore is not the actual behavior. However, it is a universally used assumption and its appropriateness is implied in the *Specification*. For instance, in the nominal strength equations for Z-sections in *Specification* Section C3.1.2.1 Lateral-Torsional Buckling Strength (Resistance) of the *Specification*, the "x-axis shall be the centroidal axis perpendicular to the web." All of the analyses referred to in this Section are based on the constrained bending assumption.

It is also assumed that through-fastened roof sheathing provides full restraint support to the purlin in the positive moment region. It is obvious that this assumption does not apply equally to standing seam roof systems. The degree of restraint provided depends on the panel profile, seaming method, and clip details. The restraint provided by the standing seam system consists of panel drape (or hugging) and clip friction or lockup. Lower values are obtained when "snap-together" (e.g. no field seaming) panels and two-piece (or sliding) clips are used. Higher values are obtained when field seamed panels and fixed clips are used. However, exceptions apply to both of these general statements.

Specification Appendix A Section D6.1.2 Flexural Members Having One flange Fastened to a Standing Seam Roof System allows the designer to determine the design strength of C- and Z-purlins using (1) the theoretical lateral-torsional buckling strength equations in *Specification* Section C3.1.2, or (2) the Base Test Method as described in Section 2.3 of this Design Guide. The Base Test Method indirectly establishes the lateral-torsional restraint provided by a standing seam panel/clip/bracing combination.

If intermediate braces are not used, a lateral-torsional buckling analysis will predict an equivalent R-value in the range 0.12-0.20 for typical depth-to-span ratios. The corresponding Base Test generated R-value will tend to be three to five times larger, which clearly shows the beneficial effects of panel drape and clip restraint. If intermediate bracing is used, Base Test Method generated R-values will sometimes be less than that predicted by a lateral-torsional analysis with the unbraced length equal to the distance between intermediate brace locations. The latter results are somewhat disturbing in that panel/clip restraint is not considered in the analytical solution, yet the resulting strength is greater than the experimentally determined value. Possible explanations are that the intermediate brace anchorage in the base test is not as rigid as assumed in the *Specification* equations, or distortional buckling contributes to the failure mechanism. Also, the *Commentary* to *Specification* Section C3.1.2.1 states for Z-sections "a conservative design approach has been and is being used in the *Specification*, in which the elastic critical moment is taken to be one-half of that of for I-members" with no reference to test data given.

One of two analysis assumptions are commonly made by designers of multiple span, lapped, purlin lines: (1) prismatic (uniform) moment of inertia or (2) non-prismatic (non-uniform) moment of inertia. For the prismatic assumption, the additional stiffness caused by the increased moment of inertia within the lap is ignored. For the non-prismatic assumption, the additional stiffness is accounted for by using the sum of the moments of inertia of the purlins forming the lap. Larger positive (mid-span) moments and smaller negative (end-region) moments result when the first assumption is used with gravity loading. The reverse is true for the second assumption. For uplift loading the same conclusions apply except that positive and negative moment locations are reversed. It follows then that the prismatic assumption is more conservative if the controlling strength location is within the span (the positive moment region) and that the non-prismatic assumption is more conservative if the controlling strength location is at the supports, i.e. within or near the lap (the negative moment region).

Since the purlins are not continuously connected within the lap, full continuity will not be achieved, the degree of fixity is difficult to determine experimentally. However, experimental evidence indicates that the non-prismatic assumption is the more accurate approach (Murray and Elhouar 1994).

For many years it has been generally accepted that an inflection point is a brace point in a Z-purlin line; however, it is not so stated in the *Specification*. The American Institute of Steel Construction (AISC) *Specification for Structural Steel Buildings* (AISC 2005) states that an inflection point is not a brace point. In the AISI Design Guide *A Guide for Designing with Standing Seam Roof Panels* (AISI 1997) the inflection point is not considered as a brace point and the bending coefficient C_b is taken as 1.0. The inflection point has also been considered a brace point with C_b taken as 1.75 (CCFSS, 1992).

Because C- and Z-purlins tend to rotate or move in opposite directions on each side of an inflection point, tests were conducted by Bryant and Murray (2000) to determine if an inflection point can be safely assumed to behave as a brace point in continuous, gravity loaded, C- and Z-purlin lines of both through-fastened and standing seam roof systems. In the study, instrumentation was used to verify the actual location of the inflection point and the lateral movement of the bottom flange of the purlins on each side of the inflection point, as well as near the maximum moment location in an exterior span. The results were compared to movement predicted by finite element models of two of the tests. Both the experimental and analytical results showed that although lateral movement did occur at the inflection point, the movement was considerably less than at other locations along the purlins. The bottom flanges on both sides of the inflection point moved in the same direction and double curvature was not apparent from either the experimental or finite element results. The lateral movement in the tests using lapped C-purlins was larger than the movement in the Z-purlin tests, but was still relatively small. From these results it would seem that an inflection point is not a brace point.

The predicted and experimental controlling limit state for the three tests using through-fastened roof sheathing was shear plus bending failure immediately outside the lap in the exterior test bay. The experiment failure loads were compared to predicted values using provisions of the *Specification* and assuming (1) the inflection point is not a brace point, (2) the inflection point is a brace point, and (3) the negative moment region strength is equal to the effective yield moment, S_eF_y. All three analysis assumptions resulted in predicted failure loads less than the experimental failure loads: up to 23% for assumption (1), up to 11% for assumption (2), and approximately 8% for assumption (3). It is difficult to draw definite conclusions from this data. However, it is clear that the bottom flange of a continuous purlin line moves laterally in the same direction on both sides of an inflection point, but the movement is relatively small. It appears that assuming full lateral-torsional restraint for through-fastened roof systems is conservative.

The web-to-web connection in lapped Z-purlin lines is generally made with two, ½ in. diameter, machine bolts approximately 1½ in. from the end of the purlin as shown in Figure 1.5. To facilitate erection, vertical slotted web holes are generally used, which may allow slip in the lap invalidating the continuous purlin assumption. Murray and Elhouar (1994) analyzed 24 continuous span tests where vertical slotted holes were used in the lap connections. They found no indication in the data that the use of slots in the web connections of lapped purlins has any effect on the flexural strength of the purlins.

The moment gradient between the inflection point and rafter support of continuous purlin lines is steep. As a result, the location where combined bending and shear is checked can be critical. The industry practice is to assume the critical location is immediately outside of the lapped portions of continuous Z-purlin systems, that is, in the single purlin, as opposed to at the web bolt line. The rationale for the assumption is that for cold-formed Z-purlins, the limit state of combined bending and shear is actually web buckling. Near the end of the lap and especially at the web-to-web bolt line, out of plane movement is restricted by the non-stressed purlin section, thus buckling cannot occur at this location. Figure 3.1.1 verifies this contention. The corresponding assumption for C-purlin systems is that the shear plus bending limit state occurs at the web-to-web vertical bolt line.

Figure 3.1.1 Photograph of Failed Purlin at End of Lap

3.2 Four Span Continuous Z-Purlins Attached to Standing Seam Panels Design Example (Gravity and Uplift Loads) – ASD

Given:

1. Four span Z-purlin system using laps at interior support points to create continuity.
2. Roof covering is attached with standing seam panel clips along the entire length of the purlins.
3. Twelve purlin lines.
4. F_y = 55 ksi
5. Roof Slope = 0.5:12.
6. The top flange of each purlin is facing upslope except the purlin closest to the eave, which has its top flange facing downslope.

7. There are no discrete braces; anti-roll clips are provided at each support of every fourth purlin line.
8. Purlin flanges are bolted to a ¼-in. thick support member with a bearing length of 5 in.
9. Tested R-values using the AISI Base Test Method:
   For gravity loads:
   R = 0.85 for the 0.085 in. thick purlin.
   R = 0.90 for the 0.059 in. thick purlin.
   For uplift load:
   R = 0.70 for the 0.085 in thick purlin.
   R = 0.70 for the 0.059 in. thick purlin.
10. The loads shown are parallel to the purlin webs.

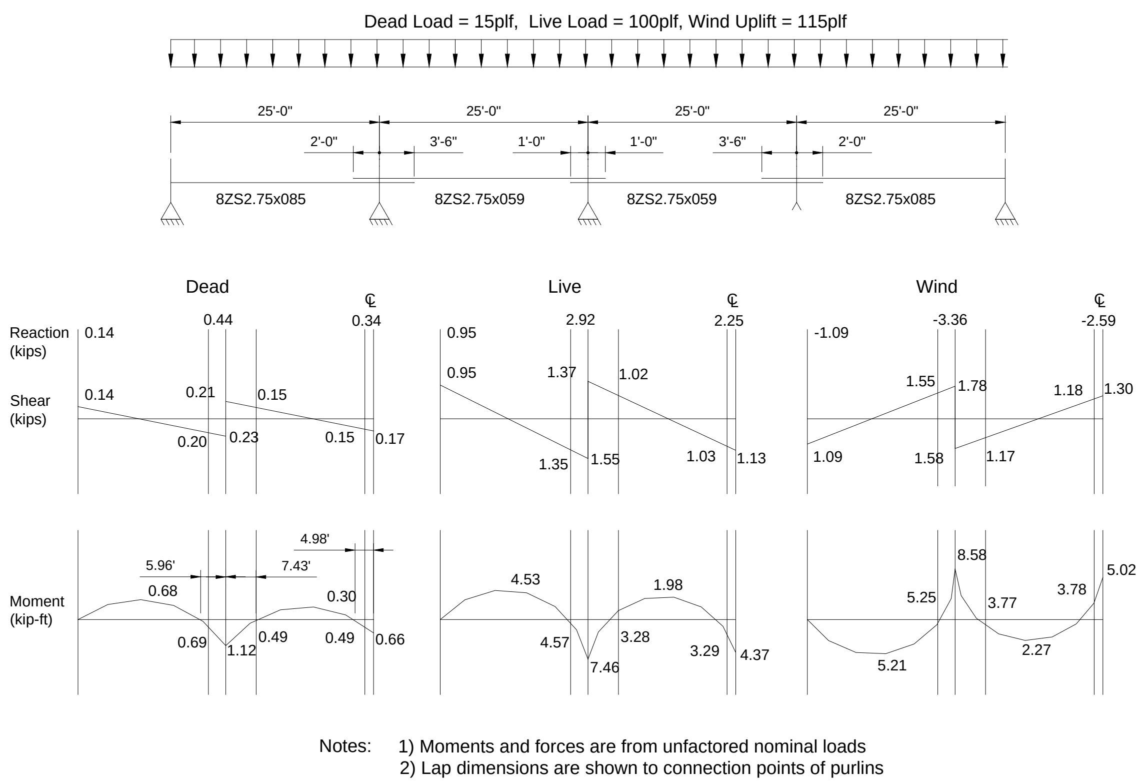


Figure 3.2.1 Shear and Moment Diagrams

Required:

1. Check the design using ASD with ASCE/SEI 7-05 (ASCE 2005) load combinations for Gravity Loads.
2. Check the design for uplift loads using ASD with ASCE/SEI 7-05 load combinations for Wind Uplift Loads.

Solutions:

Note: The equations referenced in the example refer to the *Specification* equation numbers. *Design Manual* refers to the *Cold Formed Steel Design Manual* (AISI 2008a).

1. *Assumptions for Analysis and Application of the Specification Provisions*

The *Specification* does not define the methods of analysis to be used; these judgments are the responsibility of the designer. The following assumptions are considered good practice but are not intended to prohibit other approaches:

a. The purlins are connected within the lapped portions in a manner that achieves full continuity between the individual purlin members.

b. The continuous beam analysis to establish the shear and moment diagrams assumes continuous non-prismatic members between supports in which I_x within the lapped portions is the sum of the individual members. Gross values of I_x are used for the beam analysis.

c. The strength within the lapped portions is assumed to be the sum of the strengths of the individual members.

d. For gravity loads, the region at and near the interior supports is assumed to be not subject to lateral-torsional or distortional buckling between the support and the ends of the laps.

e. Under uniform gravity loading, the negative moment region between the end of the lap and the inflection point is assumed to have an unbraced length for lateral-torsional and distortional buckling equal to the distance from the end of the lap to the inflection point.

f. Since the loading, geometry and materials are symmetrical; only the first two spans are checked.

2. *Section Properties*

The following sections properties are from the *Design Manual* Table I-4 and Table II-4.

Interior Bays	End Bays
For: 8ZS2.75x059	For: 8ZS2.75x085
t = 0.059 in.	t = 0.085 in.
I_x = 8.69 in.^{4}	I_x = 12.40 in.^{4}
S_f = 2.17 in.^{3}	S_f = 3.11 in.^{3}
S_e = 1.81 in.^{3}	S_e = 2.84 in.^{3}
I_y = 1.72 in.^{4}	I_y = 2.51 in.^{4}

Both sections have inside bend radius, R = 0.1875 in. and flange width, b = 2.75 in.

3. *Check Gravity Loads*

a. Strength for Bending Only (*Specification* Section D6.1)

Required Strength

ASD load combinations considered:

D

$D + L_r$

By inspection, $D + L_r$ controls:

$M = M_D + M_{Lr}$

End Span, from left to right:

Maximum positive moment:	M = 0.68 + 4.53 = 5.21 kip-ft.
Negative moment at end of right lap:	M = 0.69 + 4.57 = 5.26 kip-ft.
Negative moment at support:	M = 1.12 + 7.46 = 8.58 kip-ft.

Interior span, from left to right:

Negative moment at end of left lap:	M = 0.49 + 3.28 = 3.77 kip-ft.
Maximum positive moment:	M = 0.30 + 1.98 = 2.28 kip-ft.
Negative moment at end of right lap:	M = 0.49 + 3.29 = 3.78 kip-ft.
Negative moment at center support:	M = 0.66 + 4.37 = 5.03 kip-ft.

Allowable Design Flexural Strength

End Span:

At the location of maximum positive moment:

At the location of maximum positive moment, the section is assumed to be braced by the standing seam panel. The ability of the panel to brace the purlin has been quantified by the AISI Base Test Method (R = 0.85).

For the end span purlin, t = 0.085 in.

$M_n = RS_eF_y = (0.85)(2.84)(55) = 132.77$ kip-in. = 11.06 kip-ft. (Eq. D6.1.2-1)

$$\frac{M_n}{\Omega_b} = \frac{11.06}{1.67} = 6.63 \text{ kip-ft.} \geq 5.21 \text{ kip-ft. OK} \quad \text{(Eq. A4.1.1-1)}$$

In the region of negative moment between the end of the lap and the inflection point:

Determine the allowable moment using the distance from the inflection point to the lap as the unbraced length per *Specification* Section C3.1.2.(b).

$L_y = 5.96 - 2.00 = 3.96$ ft. = 47.5 in.

$$I_{yc} = \frac{I_y}{2} = \frac{2.51}{2} = 1.255 \text{ in.}^4$$

$K_y = 1.0$

$C_b = 1.67$ (Conservatively assumes linear moment diagram in this region).

$$F_e = \frac{C_b\pi^2 EdI_{yc}}{2S_f(K_yL_y)^2} = \frac{(1.67)\pi^2(29500)(8.0)(1.255)}{(2)(3.11)(47.5)^2} = 347.9 \text{ ksi.} \quad \text{(Eq. C3.1.2.1-15)}$$

$2.78F_y = (2.78)(55) = 153$ ksi.

Since $F_e > 2.78F_y$

The design flexural strength [moment resistance] shall be determined in accordance with *Specification* Section C3.1.1(a).

$M_n = S_eF_y = (2.84)(55) = 156.2$ kip-in. (Eq. C3.1.1-1)

$$\frac{M_n}{\Omega_b} = \frac{156.2}{1.67} = 93.5 \text{ kip-in.} = 7.79 \text{ kip-ft.} \geq 5.26 \text{ kip-ft. OK} \quad \text{(Eq. A4.1.1-1)}$$

At the negative moment at the support, the section is assumed to be fully braced:

Using allowable moments based on initiation of yielding per *Specification* Section C3.1.1(a), and summing the strength of the two overlapped purlins:

For the exterior purlin, t = 0.085 in.

$M_n = S_eF_y = (2.84)(55) = 156.2$ kip-in. or 13.02 kip-ft. (Eq. C3.1.1-1)

For the interior purlin, t = 0.059 in.

$M_n = S_eF_y = (1.81)(55) = 99.6$ kip-in. or 8.30 kip-ft. (Eq. C3.1.1-1)

Combined strength of purlins

$$\frac{M_n}{\Omega_b} = \frac{13.02 + 8.30}{1.67} = 12.77 \text{ kip-ft.} \geq 8.58 \text{ kip-ft. OK} \qquad \text{(Eq. A4.1.1-1)}$$

Interior Span:

In the region of negative moment between the end of the left lap and the inflection point:

Determine the allowable moment using the distance from the inflection point to the lap as the unbraced length per *Specification* Section C3.1.2.1(b) with C_b =1.67

L_y = 7.43 - 3.50 = 3.93 ft. or 47.2 in.

K_y = 1.0

$$I_{yc} = \frac{I_y}{2} = \frac{1.72}{2} = 0.86 \text{ in.}^4$$

$$F_e = \frac{(1.67)\pi^2(29500)(8.0)(0.86)}{(2)(2.17)(47.2)^2} = 346 \text{ ksi} \qquad \text{(Eq. C3.1.2.1-15)}$$

Since $F_e > 2.78F_y$

$M_n = S_eF_y = (1.81)(55) = 99.6$ kip-in. or 8.30 kip-ft. (Eq. C3.1.1-1)

$$\frac{M_n}{\Omega_b} = \frac{8.30}{1.67} = 4.97 \text{ kip-ft.} \geq 3.77 \text{ kip-ft. OK} \qquad \text{(Eq. A4.1.1-1)}$$

At the location of maximum positive moment, the section is assumed to be braced by the standing seam panel. The ability of the panel to brace the purlin has been quantified by the AISI Base Test Method (R = 0.90).

$M_n = RS_eF_y = (0.90)(1.81)(55) = 89.60$ kip-in. or 7.47 kip-ft. (Eq. D6.1.2-1)

$$\frac{M_n}{\Omega_b} = \frac{7.47}{1.67} = 4.47 \text{ kip-ft.} \geq 2.28 \text{ kip-ft. OK} \qquad \text{(Eq. A4.1.1-1)}$$

In the region of negative moment between the right lap and the inflection point:

Determine the allowable moment using the distance from the inflection point to the lap as the unbraced length per *Specification* Section C3.1.2.1(b).

L_y= 4.98 - 1.00 = 3.98 ft. or 47.8 in.

By inspection, the strength check for right lap will be satisfied, since the unbraced length and the required strength is about the same as those at the left support. Therefore the section is OK

At the negative moment at the center support, the section is assumed to be fully braced:

Use allowable moments based on initiation of yielding per *Specification* Section C3.1.1(a), summing the strength of the two overlapped purlins:

Combined strength of purlins

$$\frac{M_n}{\Omega_b} = \frac{8.30 + 8.30}{1.67} = 9.94 \text{ kip-ft.} \geq 5.03 \text{ kip-ft. OK} \qquad \text{(Eq. A4.1.1-1)}$$

b. Strength for Shear Only (*Specification* Section C3.2)

Required Strength:

By inspection, the load combination D + L_r controls:

$V = V_D + V_{Lr}$

End Span, from left to right:

At left support:	V = 0.14 + 0.95 = 1.09 kip
At end of right lap:	V = 0.20 + 1.35 = 1.55 kip
At first interior support:	V = 0.23 + 1.55 = 1.78 kip

Interior Span, from left to right:

At first interior support:	V = 0.21 + 1.37 = 1.58 kip
At end of left lap:	V = 0.15 + 1.02 = 1.17 kip
At end of right lap:	V = 0.15 + 1.03 = 1.18 kip
At center support:	V = 0.17 + 1.13 = 1.30 kip

Allowable Design Strength

End Span:

At the left support and right lap, t = 0.085 in. By inspection the end of the right lap controls.

For t = 0.085 in. and h = 7.455 in.

$$\frac{h}{t} = 87.7 > 1.51\sqrt{Ek_v / F_y} = 1.51\sqrt{(29500)(5.34)/55} = 80.8$$

$$F_v = \frac{\pi^2 E k_v}{12(1-\mu^2)(h/t)^2} = \frac{\pi^2 (29500)(5.34)}{12(1-0.3^2)(7.455/0.085)^2} = 18.51 \text{ ksi} \quad \text{(Eq. C3.2.1-4a)}$$

$$V_n = A_w F_v = (7.455)(0.085)(18.51) = 11.73 \text{ kip} \quad \text{(Eq. C3.2.1-1)}$$

$$\frac{V_n}{\Omega_v} = \frac{11.73}{1.60} = 7.33 \text{ kip} \geq 1.55 \text{ kip} \quad \text{OK} \quad \text{(Eq. A4.1.1-1)}$$

At the first interior support, sum the strength of the two overlapped purlins:

For t = 0.059 in. and h = 7.507 in., h/t = 127.2

$$\frac{h}{t} = 127.2 > 1.51\sqrt{Ek_v / F_y} = 1.51\sqrt{(29500)(5.34)/55} = 80.8$$

$$F_v = \frac{\pi^2 (29500)(5.34)}{12\left(1-0.3^2\right)(7.507/0.059)^2} = 8.79 \text{ ksi} \quad \text{(Eq. C3.2.1-4a)}$$

$$V_n = A_w F_v = (7.507)(0.059)(8.79) = 3.89 \text{ kip} \quad \text{(Eq. C3.2.1-1)}$$

For the combined section:

$$\frac{V_n}{\Omega_v} = \frac{3.89 + 13.15}{1.60} = 10.65 \text{ kip} \geq 1.78 \text{ kip} \quad \text{OK} \quad \text{(Eq. A4.1.1-1)}$$

Interior span:

By inspection of the left and right laps, the right lap controls

$$\frac{V_n}{\Omega_v} = \frac{3.89}{1.60} = 2.43 \text{ kip} \geq 1.18 \text{ kip} \quad \text{OK} \quad \text{(Eq. A4.1.1-1)}$$

At the center support, sum the strength of the two overlapped purlins:

For the combined section:

$$\frac{V_n}{\Omega_v} = \frac{3.89+3.89}{1.60} = 4.86 \text{ kip} \geq 1.30 \text{ kip} \quad \text{OK} \qquad \text{(Eq. A4.1.1-1)}$$

c. Strength for Combined Bending and Shear (*Specification* Section C3.3)

End Span:

$$\sqrt{\left(\frac{\Omega_b M}{M_{nxo}}\right)^2 + \left(\frac{\Omega_v V}{V_n}\right)^2} \leq 1.0 \qquad \text{(Eq. C3.3.1-1)}$$

where

M_{nxo} = M_n calculated on the initiation of yielding per *Specification* Section C3.1.1

Ω_b = 1.67

Ω_v = 1.60

At start of lap, t = 0.085 in.

$$\sqrt{\left(\frac{(1.67)(5.26)}{13.02}\right)^2 + \left(\frac{(1.60)(1.55)}{11.73}\right)^2} = 0.71 \leq 1.0 \quad \text{OK} \qquad \text{(Eq. C3.3.1-1)}$$

At interior support,

$$\sqrt{\left(\frac{(1.67)(8.58)}{13.02+8.30}\right)^2 + \left(\frac{(1.60)(1.78)}{11.73+3.89}\right)^2} = 0.70 \leq 1.0 \quad \text{OK} \qquad \text{(Eq. C3.3.1-1)}$$

Interior Span:

At end of laps, t = 0.059 in. Left lap controls by inspection.

$$\sqrt{\left(\frac{(1.67)(3.78)}{8.30}\right)^2 + \left(\frac{(1.60)(1.18)}{3.89}\right)^2} = 0.90 < 1.0 \quad \text{OK} \qquad \text{(Eq. C3.3.1-1)}$$

At center support,

$$\sqrt{\left(\frac{(1.67)(5.03)}{8.30+8.30}\right)^2 + \left(\frac{(1.60)(1.30)}{3.89+3.89}\right)^2} = 0.57 \leq 1.0 \quad \text{OK} \qquad \text{(Eq. C3.3.1-1)}$$

d. Web Crippling Strength (*Specification* Section C3.4.1)

Required Strength

By inspection, load combination D + L_r controls:

$P = P_D + P_{Lr}$

Supports, from left to right:

At left support:	P = 0.14 + 0.95 = 1.09 kip
At first interior support:	P = 0.44 + 2.92 = 3.36 kip
At center support:	P = 0.34 + 2.25 = 2.59 kip

Allowable Design Strength:

The bearing length is 5 inches.

At outside supports use Eq. C3.4.1-1 of the *Specification.*

$$P_n = Ct^2F_y \sin\theta\left(1-C_R\sqrt{\frac{R}{t}}\right)\left(1+C_N\sqrt{\frac{N}{t}}\right)\left(1-C_h\sqrt{\frac{h}{t}}\right) \quad \text{(Eq. C3.4.1-1)}$$

where

F_y = 55 ksi.

θ = 90 degrees

R = 0.1875 in.

N =5.0 in.

h = 7.455 in.

t = 0.085 in.

From Table C3.4.1-3, using the coefficients for the case of:

Fastened to Support/One-Flange Loading or Reaction/End

C = 4

C_R = 0.14

C_N = 0.35

C_h = 0.02

Ω_w = 1.75

Check Limits: R/t = 0.1875/0.085 = 2.21 < 9 OK

h/t = 7.455/0.085 = 87.7 < 200 OK

N/t = 5.0/0.085 = 58.8 < 210 OK

N/h = 5.0/7.455 = 0.67 < 2.0 OK

$$P_n = (4)(0.085)^2(55)\sin 90\left(1-0.14\sqrt{\frac{0.1875}{0.085}}\right)\left(1+0.35\sqrt{\frac{5.0}{0.085}}\right)\left(1-0.02\sqrt{\frac{7.455}{0.085}}\right) \quad \text{(Eq. C3.4.1-1)}$$

=3.77 kip

$$\frac{P_n}{\Omega_w} = \frac{3.77}{1.75} = 2.15 \text{ kip} \geq 1.09 \text{ kip} \quad \text{OK} \quad \text{(Eq. A4.1.1-1)}$$

At interior supports use Eq. C3.4.1-1 of the *Specification.* For webs consisting of two or more sheets, the nominal strength is calculated for each individual sheet and the results are added to obtain the nominal strength of the full section.

$$P_n = Ct^2F_y \sin\theta\left(1-C_R\sqrt{\frac{R}{t}}\right)\left(1+C_N\sqrt{\frac{N}{t}}\right)\left(1-C_h\sqrt{\frac{h}{t}}\right) \quad \text{(Eq. C3.4.1-1)}$$

where

F_y = 55 ksi.

θ = 90 degrees

R = 0.1875 in.

N = bearing length = 5.0 in.

Exterior span:	Interior Span
h = 7.455 in.	h = 7.507 in.
t = 0.085 in.	t = 0.059 in.

From Table C3.4.1-3, using the coefficients for the case of:

Fastened to Support//One-Flange Loading or Reaction/Interior

C = 13

$C_R = 0.23$
$C_N = 0.14$
$C_h = 0.01$
$\Omega_w = 1.65$

Check Limits:

Exterior Span t = 0.085 in.	Interior Span t = 0.059 in.
R/t = 0.1875/0.085 = 2.21 < 9 OK	R/t = 0.1875/0.059 = 3.18 < 9 OK
h/t = 7.455/0.085 = 87.7 < 200 OK	h/t = 7.507/0.059 = 127 < 200 OK
N/t = 5.0/0.085 = 58.8 < 210 OK	N/t = 5.0/0.059 = 84.7 < 210 OK
N/h =5.0/7.455 = 0.67 < 2.0 OK	N/h= 5.0/7.507 = 0.67< 2.0 OK

Exterior Span t = 0.085 in.

$$P_n = (13)(0.085)^2(55)\sin 90\left(1-0.23\sqrt{\frac{0.1875}{0.085}}\right)\left(1+0.14\sqrt{\frac{5.0}{0.085}}\right)\left(1-0.01\sqrt{\frac{7.455}{0.085}}\right) \quad \text{(Eq. C3.4.1-1)}$$

$= 6.39$ kip

For t = 0.059 in.,

$$P_n = (13)(0.059)^2(55)\sin 90\left(1-0.23\sqrt{\frac{0.1875}{0.059}}\right)\left(1+0.14\sqrt{\frac{5.0}{0.059}}\right)\left(1-0.01\sqrt{\frac{7.507}{0.059}}\right) \quad \text{(Eq. C3.4.1-1)}$$

$= 2.98$ kip

At first interior support,

$$\frac{P_n}{\Omega_w} = \frac{6.39+2.98}{1.65} = 5.68 \text{ kip} \ge 3.36 \text{ kip OK} \quad \text{(Eq. A4.1.1-1)}$$

At center support,

$$\frac{P_n}{\Omega_w} = \frac{2.98+2.98}{1.65} = 3.61 \text{ kip} \ge 2.59 \text{ kip OK} \quad \text{(Eq. A4.1.1-1)}$$

e. Combined Bending and Web Crippling (*Specification* Section C3.5)

$$0.86\left(\frac{P}{P_n}\right)+\left(\frac{M}{M_{nxo}}\right)\le\frac{1.65}{\Omega} \quad \text{(Eq. C3.5.1-3)}$$

$\Omega = 1.70$

Ends of laps of each section are to be connected by a minimum of two 1/2 in. diameter A307 bolts through the web, the combined section is to be connected to the support by a minimum of two 1/2 in. diameter A307 bolts through the flanges, and the webs must be in contact. (Note: If the purlin webs are connected to a welded wing plate as shown in Figure 1.6, the limit state of combined bending and web crippling does not apply.)

Check Limits:

$F_y = 55$ ksi. ≤ 70 ksi.

$$\frac{t_{thick}}{t_{thin}} = \frac{0.085}{0.059} = 1.44 > 1.3 \text{ NG}$$

In this case, the strength of thicker purlin may be determined using a maximum thickness of (1.3)(0.059) = 0.076 or conservatively the strength of the combined section may be determined using the strength of the thinner purlin for both sections.

Exterior Span $t = 0.085$ in.	Interior Span $t = 0.059$ in.
$R/t = 0.1875/0.085 = 2.21 < 5.5$ OK	$R/t = 0.1875/0.059 = 3.18 < 5.5$ OK
$h/t = 7.455/0.085 = 87.7 < 150$ OK	$h/t = 7.507/0.059 = 127 < 150$ OK
$N/t = 5.0/0.085 = 58.8 < 140$ OK	$N/t = 5.0/0.059 = 84.7 < 140$ OK

At the first interior support, because the ratio of thickness of the thicker purlin to the thinner purlin, exceeds 1.3, the strength of the combined section is conservatively determined as two times the strength of the thinner purlin.

$$0.86\left(\frac{3.36}{2.98+2.98}\right)+\left(\frac{8.58}{8.30+8.30}\right) = 1.00 \approx 1.65/1.70 = 0.97 \text{ OK} \qquad \text{(Eq. C3.5.1-3)}$$

At second interior support

$$0.86\left(\frac{2.59}{2.98+2.98}\right)+\left(\frac{5.03}{8.30+8.30}\right) = 0.68 < 1.65/1.70 = 0.97 \text{ OK} \qquad \text{(Eq. C3.5.1-3)}$$

4. *Check Uplift Loads*

a. Strength for Bending Only (*Specification* Section C3.1.4)

Required Strength:

By inspection, ASD load combination $0.6M_D + M_W$ controls.

$M = (0.6M_D + M_w)$

End Span:

Moment near center of span: $M = [(0.6)(0.68) - 5.21] = -4.80$ kip-ft.

Interior Span:

Moment near center of span: $M = [(0.6)(0.30) - 2.27] = -2.09$ kip-ft.

Allowable Design Strength:

$M_n = RS_eF_y$ (Eq. D6.1.2-1)

$R = 0.70$ for both purlin thicknesses

End Span:

For $t = 0.085$ in.

$M_n = (0.70)(2.84)(55) = 109.3$ kip-in. or 9.11 kip-ft. (Eq. D6.1.2-1)

$$\frac{M_n}{\Omega_b} = \frac{9.11}{1.67} = 5.45 \text{ kip-ft.} > 4.80 \text{ kip-ft. OK} \qquad \text{(Eq. A4.1.1-1)}$$

Interior Span:

For $t = 0.059$ in.

$M_n = (0.70)(1.81)(55) = 69.7$ kip-in. or 5.81 kip-ft. (Eq. D6.1.2-1)

$$\frac{M_n}{\Omega_b} = \frac{5.81}{1.67} = 3.48 \text{ kip-ft.} > 2.09 \text{ kip-ft. OK} \qquad \text{(Eq. A4.1.1-1)}$$

b. Other Comments

Since the magnitude of the shears, moments and reactions are approximately 65 percent of those of the gravity case, it can be concluded that the design satisfies the *Specification* criteria for uplift.

3. System Anchorage

System anchorage is checked in Example 4.3.

3.3 Four Span Continuous C-Purlins Attached to Standing Seam Panels Design Example (Gravity and Uplift Loads) – LRFD

Given:

1. Four span C-purlin system using laps at interior support points to create continuity.
2. Roof covering is attached with standing seam panel clips along the entire length of the purlins.
3. Ten purlin lines.
4. F_y = 55 ksi
5. Roof Slope = 0.25:12.
6. Purlins are lapped back-to-back over interior supports but all face in the same direction in a given bay. The purlins in the left exterior bay face downslope.
7. There are no discrete braces; anti-roll clips are provided at each support of every fourth purlin line.
8. Purlin flanges are bolted to a ¼-in. thick support member with a bearing length of 5 in.
9. Tested R-values using the AISI Base Test Method:
   For gravity loads:
   R = 0.90 for the 0.070 in. thick purlin.
   R = 0.95 for the 0.059 in. thick purlin.
   For uplift load:
   R = 0.75 for the 0.070 in thick purlin.
   R = 0.75 for the 0.059 in. thick purlin.
10. The loads shown are parallel to the purlin webs.

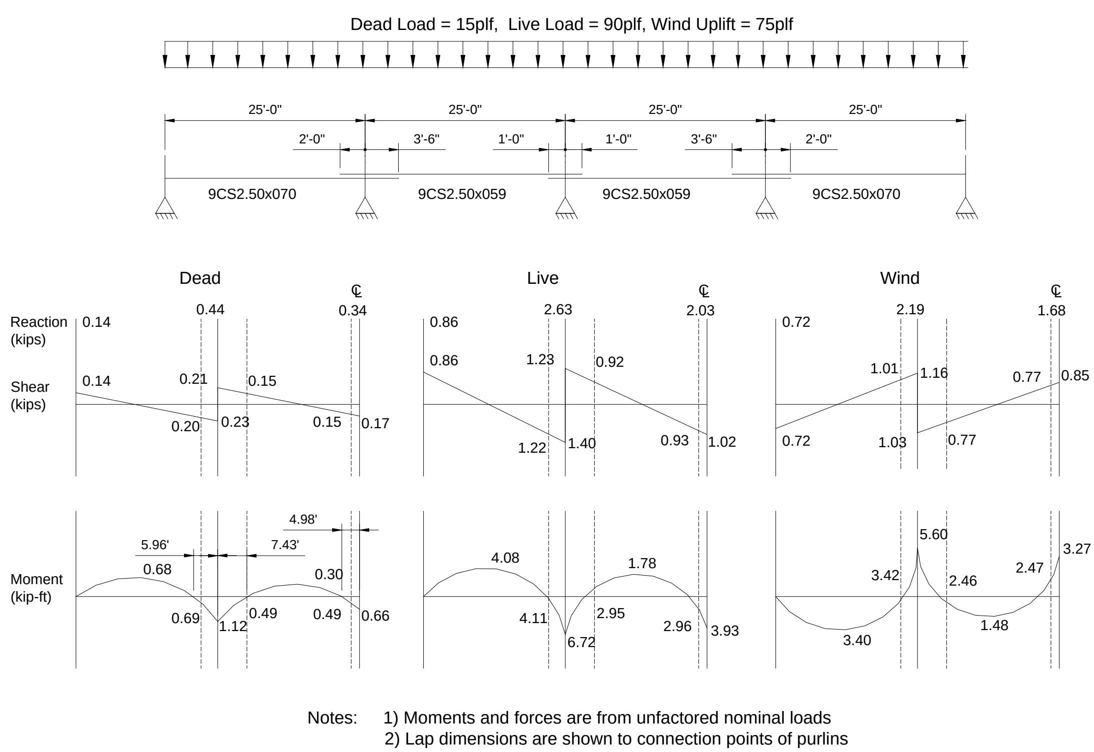


Figure 3.3.1 Shear and Moment Diagrams

Required:

Check the design using LRFD with the ASCE/SEI 7-05 (ASCE 2005) load combinations for:

a. Gravity Loads
b. Uplift Loads

Solution:

Note: The equations referenced in the example refer to the *Specification* equation numbers. *Design Manual* refers to the *Cold Formed Steel Design Manual* (AISI 2008a).

1. Assumptions for Analysis and Application of the Specification Provisions

The *Specification* does not define the methods of analysis to be used; these judgments are the responsibility of the designer. The following assumptions are considered good practice but are not intended to prohibit other approaches:

a. The purlins are connected within the lapped portions in a manner that achieves full continuity between the individual purlin members.
b. The continuous beam analysis to establish the shear and moment diagrams assumes continuous non-prismatic members between supports in which I_x within the lapped portions is the sum of the individual members. Gross values of I_x are used for the beam analysis.
c. The strength within the lapped portions is assumed to be the sum of the strengths of the individual members.
d. For gravity loads, the region at and near the interior supports is assumed to be not subject to lateral-torsional or distortional buckling between the support and the ends of the laps.
e. Under uniform gravity loading, the negative moment region between the end of the lap and the inflection point is assumed to have an unbraced length for lateral-torsional and distortional buckling equal to the distance from the end of the lap to the inflection point.
f. Since the loading, geometry and materials are symmetrical; only the first two spans are checked.

2. Section Properties

The following sections properties are from the *Design Manual* Table I-1 and Table II-1.

Interior Bays	End Bays
For: 9CS2.5x059	For: 9CS2.5x070
t = 0.059 in.	t = 0.070 in.
I_x = 10.3 in.^{4}	I_x = 12.20 in.^{4}
S_f = 2.29 in.^{3}	S_f = 2.71 in.^{3}
S_e = 1.89 in.^{3}	S_e = 2.47 in.^{3}
I_y = 0.698 in.^{4}	I_y = 0.828 in.^{4}

3. Check Gravity Loads

a. Strength for Bending Only (*Specification* Section D6.1)

Required Strength

LRFD load combinations considered

1.4D

1.2D + 1.6 L_r

By inspection, 1.2D + 1.6 L_r controls:

$M_u = 1.2M_D + 1.6M_{Lr}$

End Span, from left to right:

Maximum positive moment:	M_u = (1.2)(0.68) + (1.6)(4.08) = 7.34 kip-ft.
Negative moment at end of right lap:	M_u = (1.2)(0.69) + (1.6)(4.11) = 7.40 kip-ft.
Negative moment at support:	M_u = (1.2)(1.12) + (1.6)(6.72) = 12.1 kip-ft.

Interior span, from left to right:

Negative moment at end of left lap:	M_u = (1.2)(0.49) + (1.6)(2.95) = 5.31 kip-ft.
Maximum positive moment:	M_u = (1.2)(0.30) + (1.6)(1.78) = 3.21 kip-ft.
Negative moment at end of right lap:	M_u = (1.2)(0.49) + (1.6)(2.96) = 5.32 kip-ft.
Negative moment at center support:	M_u = (1.2)(0.66) + (1.6)(3.93) = 7.08 kip-ft.

Design Flexural Strength

End span:

At the location of maximum positive moment:

At the location of maximum positive moment, the section is assumed to be braced by the standing seam panel. The ability of the panel to brace the purlin has been quantified by the AISI Base Test Method (R = 0.90).

For the end span purlin, t = 0.070 in.

$M_n = RS_eF_y$ = (0.90)(2.47)(55) = 122.3 kip-in. = 10.2 kip-ft. (Eq. D6.1.2-1)

$\phi_b M_n$ = (0.90)(10.2) = 9.16 kip-ft > 7.34 kip-ft OK (Eq. A5.1.1-1)

In the region of negative moment between the end of the lap and the inflection point:

Determine the allowable moment using the distance from the inflection point to the lap as the unbraced length per *Specification* Section C3.1.2.(b).

L_y = 5.96 - 2.00 = 3.96 ft. = 47.5 in.

$$I_{yc} = \frac{I_y}{2} = \frac{0.828}{2} = 0.414 \text{ in.}^4$$

K_y = 1.0

C_b = 1.67 (Conservatively assumes linear moment diagram in this region).

$$F_e = \frac{C_b \pi^2 E d I_{yc}}{S_f (K_y L_y)^2} = \frac{(1.67)\pi^2 (29500)(9.0)(0.414)}{(2.71)(47.5)^2} = 296 \text{ ksi.} \quad \text{(Eq. C3.1.2.1-14)}$$

$2.78F_y$ = (2.78)(55) = 153 ksi.

Since $F_e > 2.78F_y$

The design flexural strength [moment resistance] shall be determined in accordance with *Specification* Section C3.1.1(a).

$M_n = S_eF_y = (2.47)(55) = 135.9$ kip-in. = 11.3 k-ft (Eq. C3.1.1-1)

$\phi_bM_n = (0.90)(11.3) = 10.2$ kip-ft > 7.40 kip-ft OK (Eq. A5.1.1-1)

At the negative moment at the support, the section is assumed to be fully braced:

Using allowable moments based on initiation of yielding per *Specification* Section C3.1.1(a), and summing the strength of the two overlapped purlins:

For the exterior purlin, t = 0.070 in.

$M_n = S_eF_y = (2.47)(55) = 135.9$ kip-in. or 11.3 kip-ft. (Eq. C3.1.1-1)

For the interior purlin, t = 0.059 in.

$M_n = S_eF_y = (1.89)(55) = 103.9$ kip-in. or 8.66 kip-ft. (Eq. C3.1.1-1)

Combined strength of purlins

$\phi_bM_n = (0.90)(11.3+8.66) = 18.0$ kip-ft > 12.1 kip-ft OK (Eq. A5.1.1-1)

Interior Span:

In the region of negative moment between the end of the left lap and the inflection point:

Determine the allowable moment using the distance from the inflection point to the lap as the unbraced length per *Specification* Section C3.1.2.1(b) with C_b =1.67

L_y = 7.43 - 3.50 = 3.93 ft. or 47.2 in.

K_y = 1.0

$$I_{yc} = \frac{I_y}{2} = \frac{0.698}{2} = 0.349 \text{ in.}^4$$

$$F_e = \frac{C_b\pi^2 E d I_{yc}}{S_f(K_yL_y)^2} = \frac{(1.67)\pi^2(29500)(9.0)(0.698)}{(2.29)(47.2)^2} = 598 \text{ ksi.} \quad \text{(Eq. C3.1.2.1-14)}$$

Since $F_e > 2.78F_y$

$M_n = S_eF_y = (1.89)(55) = 104.0$ kip-in. or 8.66 kip-ft. (Eq. C3.1.1-1)

$\phi_bM_n = (0.90)(8.66) = 7.80$ kip-ft > 5.31 kip-ft OK (Eq. A5.1.1-1)

At the location of maximum positive moment, the section is assumed to be braced by the standing seam panel. The ability of the panel to brace the purlin has been quantified by the AISI Base Test Method (R = 0.95).

$M_n = RS_eF_y = (0.95)(1.89)(55) = 98.75$ kip-in. or 8.23 kip-ft. (Eq. D6.1.2-1)

$\phi_bM_n = (0.90)(8.23) = 7.41$ kip-ft > 3.22 kip-ft OK (Eq. A5.1.1-1)

In the region of negative moment between the right lap and the inflection point:

Determine the allowable moment using the distance from the inflection point to the lap as the unbraced length per *Specification* Section C3.1.2.1(b).

L = 4.98 - 1.00 = 3.98 ft. or 47.8 in.

By inspection, the strength check for right lap will be satisfied, since the unbraced length and the required strength is about the same as those at the left support. Therefore the section is OK

At the negative moment at the center support, the section is assumed to be fully braced:

Use allowable moments based on initiation of yielding per *Specification* Section C3.1.1(a), summing the strength of the two overlapped purlins:

Combined strength of purlins

$\phi_b M_n$ = (0.90)(8.66+8.66) = 15.6 kip-ft > 7.08 kip-ft OK (Eq. A5.1.1-1)

b. Strength for Shear Only (*Specification* Section C3.2)

Required Strength:

By inspection, the load combination 1.2 D + 1.6 L_r controls:

$V = V_D + V_{Lr}$

End Span, from left to right:

At left support:	V_u = (1.2)(0.14) + (1.6)(0.86) = 1.54 kip
At end of right lap:	V_u = (1.2)(0.20) + (1.6)(1.22) = 2.19 kip
At first interior support:	V_u = (1.2)(0.23) + (1.6)(1.40) = 2.52 kip

Interior Span, from left to right:

At first interior support:	V_u = (1.2)(0.21) + (1.6)(1.23) = 2.22 kip
At end of left lap:	V_u = (1.2)(0.15) + (1.6)(0.92) = 1.65 kip
At end of right lap:	V_u = (1.2)(0.15) + (1.6)(0.93) = 1.67 kip
At center support:	V_u = (1.2)(0.17) + (1.6)(1.02) = 1.84 kip

Design Strength

End Span:

At the left support and right lap, t = 0.070 in. By inspection the end of the right lap controls.

For t = 0.070 in. and h = 8.485 in.

$$\frac{h}{t} = 121 > 1.51\sqrt{Ek_v/F_y} = 1.51\sqrt{(29500)(5.34)/55} = 80.8$$

$$F_v = \frac{\pi^2 E k_v}{12(1-\mu^2)(h/t)^2} = \frac{\pi^2(29500)(5.34)}{12(1-0.3^2)(8.485/0.070)^2} = 9.69 \text{ ksi} \qquad \text{(Eq. C3.2.1-4a)}$$

$V_n = A_w F_v$ = (8.485)(0.070)(9.69) = 5.76 kip (Eq. C3.2.1-1)

$\phi_v V_n = (0.95)(5.76) = 5.47 \text{ kip} \geq 2.19 \text{ kip}$ OK (Eq. A5.1.1-1)

At the first interior support, sum the strength of the two overlapped purlins:

For t = 0.059 in. and h = 8.507 in., h/t = 144

$$\frac{h}{t} = 144 > 1.51\sqrt{Ek_v/F_y} = 1.51\sqrt{(29500)(5.34)/55} = 80.8$$

$$F_v = \frac{\pi^2(29500)(5.34)}{12(1-0.3^2)(8.507/0.059)^2} = 6.85 \text{ ksi} \qquad \text{(Eq. C3.2.1-4a)}$$

$V_n = A_w F_v$ = (8.507)(0.059)(6.85) = 3.44 kip (Eq. C3.2.1-1)

For the combined section:

$\phi_v V_n = (0.95)(5.76+3.44) = 8.74 \text{ kip} \geq 2.52 \text{ kip}$ OK (Eq. A5.1.1-1)

Interior span:

The required strength is the same at both the right and left laps.

$\phi_v V_n = (0.95)(3.44) = 3.27 \text{ kip} \geq 1.67 \text{ kip}$ OK (Eq. A5.1.1-1)

At the center support, sum the strength of the two overlapped purlins:
For the combined section:

$$\phi_v V_n = (0.95)(3.44 + 3.44) = 6.54 \text{ kip} \geq 1.84 \text{ kip} \quad \text{OK} \qquad \text{(Eq. A5.1.1-1)}$$

c. Strength for Combined Bending and Shear (*Specification* Section C3.3)

End Span:

$$\sqrt{\left(\frac{\overline{M}}{\phi_b M_{nxo}}\right)^2 + \left(\frac{\overline{V}}{\phi_v V_n}\right)^2} \leq 1.0 \qquad \text{(Eq. C3.3.2-1)}$$

where

M_{nxo} = M_n calculated on the initiation of yielding per *Specification* Section C3.1.1

$\phi_b = 0.95$

$\phi_v = 0.95$

At start of lap, t = 0.070 in.

$$\sqrt{\left(\frac{(7.40)}{(0.95)(11.3)}\right)^2 + \left(\frac{(2.19)}{(0.95)(5.76)}\right)^2} = 0.79 \leq 1.0 \quad \text{OK} \qquad \text{(Eq. C3.3.2-1)}$$

At interior support,

$$\sqrt{\left(\frac{(12.1)}{(0.95)(11.3 + 8.66)}\right)^2 + \left(\frac{(2.52)}{(0.95)(5.76 + 3.44)}\right)^2} = 0.70 \leq 1.0 \quad \text{OK} \qquad \text{(Eq. C3.3.2-1)}$$

Interior Span:

At end of laps, t = 0.059 in. Left lap controls by inspection.

$$\sqrt{\left(\frac{(5.31)}{(0.95)(8.66)}\right)^2 + \left(\frac{(1.65)}{(0.95)(3.44)}\right)^2} = 0.82 \leq 1.0 \quad \text{OK} \qquad \text{(Eq. C3.3.2-1)}$$

At center support,

$$\sqrt{\left(\frac{(7.08)}{(0.95)(8.66 + 8.66)}\right)^2 + \left(\frac{(1.84)}{(0.95)(3.44 + 3.44)}\right)^2} = 0.51 \leq 1.0 \quad \text{OK} \qquad \text{(Eq. C3.3.2-1)}$$

d. Web Crippling Strength (*Specification* Section C3.4.1)

Required Strength

By inspection, load combination 1.2D + 1.6L_r controls:

$P_u = 1.2P_D + 1.6P_{Lr}$

Supports, from left to right:

At left support:	P_u = (1.2)(0.14) + (1.6)(0.86) = 1.54 kip
At first interior support:	P_u = (1.2)(0.44) + (1.6)(2.63) = 4.74 kip
At center support:	P_u = (1.2)(0.34) + (1.6)(2.03) = 3.66 kip

Design Strength:

At outside supports use Eq. C3.4.1-1 of the *Specification.*

$$P_n = Ct^2F_y \sin\theta\left(1 - C_R\sqrt{\frac{R}{t}}\right)\left(1 + C_N\sqrt{\frac{N}{t}}\right)\left(1 - C_h\sqrt{\frac{h}{t}}\right) \qquad \text{(Eq. C3.4.1-1)}$$

where

F_y = 55 ksi.

θ = 90 degrees

R = 0.1875 in.

N = bearing length = 5.0 in.

h = 8.485 in.

t = 0.070 in.

From Table C3.4.1-2, using the coefficients for the case of:

Fastened to Support/One-Flange Loading or Reaction/End

C = 4

C_R = 0.14

C_N = 0.35

C_h = 0.02

ϕ_w = 0.85

Check Limits: R/t = 0.1875/0.070 = 2.7 < 9 OK

h/t = 8.485/0.070 = 121 < 200 OK

N/t = 5.0/0.070 = 71.4 < 210 OK

N/h = 5.0/8.488 = 0.59 < 2.0 OK

$$P_n = (4)(0.070)^2(55)\sin 90\left(1 - 0.14\sqrt{\frac{0.1875}{0.070}}\right)\left(1 + 0.35\sqrt{\frac{5.0}{0.070}}\right)\left(1 - 0.02\sqrt{\frac{8.485}{0.070}}\right) \qquad \text{(Eq. C3.4.1-1)}$$

= 2.57 kip

$\phi_w P_n$ =(0.85)(2.57) = 2.19 kip ≥ 1.54 kip OK (Eq. A5.1.1-1)

At interior supports use Eq. C3.4.1-1 of the *Specification.* For webs consisting of two or more sheets, the nominal strength is calculated for each individual sheet and the results are added to obtain the nominal strength of the full section.

$$P_n = Ct^2F_y \sin\theta\left(1 - C_R\sqrt{\frac{R}{t}}\right)\left(1 + C_N\sqrt{\frac{N}{t}}\right)\left(1 - C_h\sqrt{\frac{h}{t}}\right) \qquad \text{(Eq. C3.5.2-1)}$$

where

F_y = 55 ksi.

θ = 90 degrees

R = 0.1875 in.

N = bearing length = 5.0 in.

Exterior span:	Interior Span
h = 8.485 in.	h = 8.507 in.
t = 0.070 in.	t = 0.059 in.

From Table C3.4.1-3, using the coefficients for the case of:

Fastened to Support/One-Flange Loading or Reaction/Interior

$C \quad = 13$

$C_R = 0.23$

$C_N = 0.14$

$C_h = 0.01$

$\phi_w = 0.90$

Check Limits:

Exterior Span t = 0.070 in.	Interior Span t = 0.059 in.
R/t = 0.1875/0.070 = 2.68 < 9 OK	R/t = 0.1875/0.059=3.18 < 9 OK
h/t = 8.485/0.070 = 121 < 200 OK	h/t = 8.507/0.059 = 144 < 200 OK
N/t = 5.0/0.070 = 71.4 < 210 OK	N/t = 5.0/0.059 = 84.7 < 210 OK
N/h = 5.0/8.485 = 0.59 < 2.0 OK	N/h = 5.0/8.507 = 0.59 < 2.0 OK

Exterior Span t = 0.070 in.

$$P_n = (13)(0.070)^2(55)\sin 90\left(1-0.23\sqrt{\frac{0.1875}{0.070}}\right)\left(1+0.14\sqrt{\frac{5.0}{0.070}}\right)\left(1-0.01\sqrt{\frac{8.485}{0.070}}\right) \qquad \text{(Eq. C3.5.2-1)}$$

$= 4.25$ kip

For t = 0.059 in.

$$P_n = (13)(0.059)^2(55)\sin 90\left(1-0.23\sqrt{\frac{0.1875}{0.059}}\right)\left(1+0.14\sqrt{\frac{5.0}{0.059}}\right)\left(1-0.01\sqrt{\frac{8.507}{0.059}}\right) \qquad \text{(Eq. C3.5.2-1)}$$

$= 2.96$ kip

At first interior support,

$\phi_w P_n = (0.90)(4.25+2.96) = 6.49 \text{ kip} > 4.74 \text{ kip}$ OK (Eq. A5.1.1-1)

At center support,

$\phi_w P_n = (0.90)(2.96+2.96) = 5.33 \text{ kip} > 3.66 \text{ kip}$ OK (Eq. A5.1.1-1)

e. Combined Bending and Web Crippling (*Specification* Section C3.5)

$$0.91\left(\frac{\overline{P}}{P_n}\right)+\left(\frac{\overline{M}}{M_{nxo}}\right) \le 1.33\phi \qquad \text{(Eq. C3.5.2-1)}$$

$\phi = 0.90$

At the first interior support,

$$0.91\left(\frac{4.74}{4.25+2.96}\right)+\left(\frac{12.1}{11.3+8.66}\right) = 1.20 \le (1.33)(0.90) = 1.20 \text{ OK} \qquad \text{(Eq. C3.5.1-1)}$$

At second interior support

$$0.91\left(\frac{3.66}{2.96+2.96}\right)+\left(\frac{7.08}{8.66+8.66}\right) = 0.97 < (1.33)(0.90) = 1.20 \text{ OK} \qquad \text{(Eq. C3.5.1-1)}$$

4. Check Uplift Loads

a. Strength for Bending Only (*Specification* Section D6.1)

Required Strength:

By inspection, LRFD load combination $0.9M_D + 1.6M_W$ controls.

$M_u = 0.9M_D + 1.6M_w$

End Span:
Moment near center of span: $M = (0.9)(0.68)+(1.6)(-3.40) = -4.83$ kip-ft.
Interior Span:
Moment near center of span: $M = (0.9)(0.30)+(1.6)(-1.48) = -2.10$ kip-ft.

Design Strength:

$M_n = RS_eF_y$ (Eq. D6.1.2-1)

$R = 0.75$ for both purlin thicknesses

End Span:

For t = 0.070 in.

$M_n = (0.75)(2.47)(55) = 101.9$ kip-in. or 8.49 kip-ft. (Eq. D6.1.2-1)

$\phi_bM_n = (0.90)(8.49) = 7.64$ kip-ft > 4.83 kip-ft OK (Eq. A4.1.1-1) (Eq. A5.1.1-1)

Interior Span:

For t = 0.059 in.

$M_n = (0.75)(1.89)(55) = 78.0$ kip-in. or 6.50 kip-ft. (Eq. D6.1.2-1)

$\phi_bM_n = (0.90)(6.50) = 5.85$ kip-ft > 2.10 kip-ft OK (Eq. A5.1.1-1)

b. Other Comments

Since the magnitude of the shears, moments and reactions are less than the gravity case and the compression flange in all other regions is braced by the sheathing, it can be concluded that the design satisfies the *Specification* criteria for uplift.

Note: Continuous purlin design examples using through-fastened panels are found in the AISI Cold-Formed Steel Design Manual (AISI 2008a).

CHAPTER 4 SYSTEM ANCHORAGE REQUIREMENTS

Symbols and Definitions Used in Chapter 4

A_p	Gross cross-sectional area of roof panel per unit width
b	Flange width of the purlin
B	In-plane depth of diaphragm
B	Width of roof plane
C1 to C6	Coefficients tabulated in *Specification* Tables D6.3.1-1 to D6.3.1-3
D	Dead load
d	Depth of section
$d_{p_{i,j}}$	Distance along roof slope between the ith purlin line and the jth anchorage device
$d_{p_{oj}}$	Distance along roof slope between the ridge and the jth anchorage device
$d_{p_{ip}}$	Distance along roof slope between the ith purlin line and the ridge
E	Modulus of elasticity of steel, 29,500 ksi (203,000 MPa, or 2,070,000 kg/cm^2)
G′	Shear stiffness of the diaphragm
I_x	Moment of inertia of full unreduced section about major centroidal axis
I_{xy}	Product of inertia of full unreduced section about major and minor centroidal axes
i	Index of each purlin line
j	Index for each anchorage device
k	Index for each purlin spacing
K_a	Lateral stiffness of anchorage device
K_{a_req}	Required lateral stiffness of anchorage device
K_{D_k}	Stiffness of spring modeling connection between purlin k and k+1
$K_{eff_{i,j}}$	Effective lateral stiffness of jth anchorage device with respect to ith purlin
K_{req}	Required stiffness
K_{sys}	Lateral stiffness of roof system, neglecting anchorage devices
K^*_{sys}	Lateral stiffness of spring modeling inherent restraint at one purlin
K_{total_i}	Effective lateral stiffness of all elements resisting force P_i
L	Span length
L_{diaph}	Span of diaphragm between lines of anchorage
m	Distance from shear center to mid-plane of web
N_a	Number of anchorage devices along a line of anchorage
N_p	Number of purlin lines on roof slope
P_i	Lateral force introduced into the system at the ith purlin
P_L	Lateral force resisted by the anchorage system
P_{L_j}	Lateral force to be resisted by the jth anchorage device
P_{L-s}	Lateral force to be resisted by each anchorage device

q	Design load in plane of web
q_{drift}	Design drift snow load
S	Typical purlin spacing
S	Snow load
S_k	Distance between purlin k+1 and k
t	Base steel thickness of purlin
t_e	Base steel thickness of eave member
W_{pi}	Total required vertical load supported by ith purlin in a single bay
W_s	Total required vertical load supported by all purlins in a single bay
w_{diaph}	Distributed in-plane load on roof diaphragm
w_i	Required distributed gravity load supported by the ith purlin per unit length
α	Coefficient for purlin directions
Δ_{diaph}	In-plane deflection of roof diaphragm
Δ_S	Simplified approximation of purlin deflection along line of anchorage
Δ_a	Allowable deflection of purlin along line of anchorage
γ	Ratio of lateral anchorage force to the applied gravity load
θ	Angle between vertical and plane of web of Z-section
ϕ	Resistance factor
Ω	Safety factor

4.1 Introduction

The design of purlins, as presented in the *Specification* and illustrated in the previous chapters of this Design Guide, neglects any torsional stresses in the members due to loading that is oblique to the principal axes or eccentric to the shear center. Typically in metal building roof systems, the roof sheathing provides sufficient lateral and some torsional restraint to the purlins to justify the use of the provisions within the *Specification*. The lateral support provided to the purlins produces lateral forces perpendicular to the web of the purlins. These lateral forces accumulate as a shear force across the roof diaphragm that must be transferred from the plane of the sheathing into the primary lateral resistance system, or to a point where it is counteracted by an opposing force.

This Chapter presents background information on the research behind and the development of the provisions presented in the *Specification,* as well as guidance on the application of the provisions to several commonly encountered special conditions. These special procedures are introduced and illustrated with examples. Also both a simplified and a more thorough solution procedure are presented.

In metal building roofs, the anchorage force is commonly transferred to the main frames through clips, typically referred to as anti-roll clips, which prevent the torsional rotation of the purlin at the rafters. This technique requires few additional parts and does not rely on the presence of counteracting forces. Also, lateral braces may be provided at discrete locations along a purlin span. These discrete braces must either be arranged so the anchorage forces counteract or they must be tied to the building's lateral load resisting system. Resisting the anchorage forces with opposing forces is frequently not practical due to asymmetric geometry or loading, such as unbalanced snow loads. The transfer of the anchorage forces from the lines of anchorage to the lateral load resisting system can be accomplished through the addition of diagonal bracing or by connecting the braces to a collector element, such as a spandrel beam. During the development of the latest anchorage provisions it was found that the flexibility of these collector elements can greatly influence the forces and displacements within the system anchorage.

It is important to recognize that a typical metal building structure consists of several interrelated load resisting systems, and that the same mechanisms that provide lateral support to the purlins under flexural loads will also resist lateral movement under other loading conditions. For example, the thermal movement of the roof sheathing relative to the primary framing below will create differential movement between the top and bottom flanges of the purlins. The forces that develop in the system due to this movement can be avoided by concentrating the lateral resistance at a single location and allowing the roof to expand away from this point. However, due to the flexibility of the roof system, this may not provide adequate support for the purlins most remote from this anchorage device. Another approach is to provide several points of anchorage distributed throughout the roof plan, each with enough stiffness to adequately support the purlins, but also flexible enough to allow the roof system to expand and contract under thermal changes without generating excessive lateral forces. Also, in metal building roof systems, it is common to rely on the diaphragm action of the roof sheathing to transfer the longitudinal wind and seismic loads on the building to the sidewalls. Depending on the details used, this diaphragm shear may also be resisted by the purlin system anchorage and must also be addressed.

Alternatively, torsional braces may be used to directly resist the torsional moments developed within the roof system. These will typically take the form of diaphragm members, such as cold formed channels, that connect to the purlin webs. When these braces are used, the roof system may deflect significantly in the plane of the roof sheathing, but since the purlins cannot rotate, they still achieve their full design strength.

4.2 Development of Design Provisions

The provisions presented in *Specification* Section D6.3 Roof System Bracing and Anchorage draw heavily on the testing performed at Virginia Tech by Lee and Murray (2001), and Seek and Murray (2004a), as well as the experience of the members of the AISI Task Group on Anchorage and Bracing. The knowledge base upon which the provisions are built was expanded by the analytical work of Sears (2007), Seek (2007), Sears and Murray (2007), and Seek and Murray (2007), which culminated in the development of the design provisions and analysis models presented in the *Specification* and in this Design Guide.

4.2.1 Provisions of *Specification* Section D6.3.1 Anchorage of Bracing for Purlin Roof Systems under Gravity Load with Top Flange Connected to Sheathing

The provisions presented in the *Specification* have two parts: a semi-empirical force calculation and a simplified relative stiffness analysis. The provisions replicate the results of the 3D computer stiffness model presented in Section 5.2 of this Design Guide. The computer model was developed and validated using the results of laboratory testing, and allowed for the analysis of systems that were not tested. A large matrix of test models was analyzed and the results were used to develop the provisions in the *Specification*.

Modeling Basics

In both the computer model and the manual calculation procedure, uniform roof loads are resolved as line loads at each purlin based on the tributary area. In the physical roof system these loads are transferred from the sheathing to the purlin through bearing on the top flange. This bearing produces an uneven force distribution across the width of the purlin flange that is not well defined. It was found during the development of the computer stiffness model, that the effects of this eccentric load can be approximated by applying the line load at a distance equal to one-fourth of the flange width from the mid-plane of the purlin web.

The anchorage devices are represented by spring restraints at the top of the anchored purlins. The stiffness of the springs must be quantified by testing or detailed analysis. When establishing both the strength and stiffness of the anchorage devices it is important to recognize that the *Specification* provisions assume the anchorage device is connected at the purlin-to-sheathing interface. Since most anchorage devices will connect to the purlin web, the appropriate adjustments must be made to the results of the tests or analyses to yield values for an equivalent anchor at the purlin-to-sheathing interface.

In the computer stiffness model, the purlin-to-rafter and the purlin-to-sheathing connections are modeled as rotational springs. For the manual calculation procedure there is no need to quantify the stiffness of these springs, as this spring stiffness is not explicitly used.

Model Simplifications

Since the intent of these design provisions is to address the lateral behavior, the design model is simplified to a one-dimensional system with only lateral degrees of freedom. The lateral effect of the gravity load is found from the semi-empirical equation for P_i, the lateral force introduced into the system at the ith purlin,

$$P_i = C1 \cdot W_{p_i} \cdot \left[\left(\frac{C2}{1000} \cdot \frac{I_{xy} L}{I_x d} + C3 \cdot \frac{(m + 0.25b)t}{d^2} \right) \alpha \cdot \cos\theta - C4 \cdot \sin\theta \right] \qquad \text{(Eq. D6.3.1-2)}$$

This equation includes three primary parts, where the first represents the effect of the load being applied oblique to the principal axes, the second models the load application eccentric to the shear center while the third applies the downslope component of the applied load. The form of this equation was derived from a combination of basic mechanics and statistical analysis of the results of the computer stiffness model. The force, P_i, represents the anchorage force at a given purlin if every purlin is anchored with a rigid anchorage device.

In this one-dimensional model, the rotational degrees of freedom are not included, so the rotational restraint provided at the purlin to sheathing connections must be replaced with equivalent lateral restraints. The mechanisms involved that contribute to the behavior of this spring are the rotation of the connections and the out of plane bending of the purlin web. It was found, through statistical analysis of the computer stiffness model results, which the stiffness of these lateral springs is found from

$$K^*_{sys} = \frac{C5}{1000} \cdot \frac{ELt^2}{d^2} \qquad (4.2.1)$$

The purlins are linked together by the roof sheathing, which will carry an axial load. The behavior of the sheathing is modeled as axial springs with a stiffness of

$$K_{D_k} = \frac{C6 \cdot LA_p E}{S_k} \qquad (4.2.2)$$

At this point the model can be solved with matrix methods. The applicable solution is presented in Section 4.5.2.

However, there was a desire to simplify the procedure so that it is practical to carry out the calculations with basic algebraic calculations. The model was simplified slightly by grouping the stiffness of all the K^*_{sys} springs into one, globally acting, spring with a stiffness of

$$K_{sys} = \frac{C5}{1000} \cdot N_p \cdot \frac{ELt^2}{d^2} \qquad \text{(Eq. D6.3.1-6)}$$

This may be done without substantially affecting the results because the stiffness of the K_{sys} springs is relatively small compared to the axial behavior of the roof sheathing and the sheathing behaves approximately as a rigid strut.

The forces in the multi-degree-of-freedom system may be found without the use of matrix methods by considering each purlin separately and the elements of the model that contribute to its stability. At each purlin, the force, P_i, is distributed to each of the anchorage devices and to the K_{sys} spring, based on the relative stiffness of each of the elements. Since the anchorage devices are in series with the axial behavior of the roof sheathing, the stiffness of the anchorage devices is effectively reduced due to the axial flexibility of the sheathing. This reduced stiffness is found from

$$K_{eff_{i,j}} = \left[\frac{1}{K_a} + \frac{d_{p_{i,j}}}{C6 \cdot LA_pE} \right]^{-1} \qquad \text{(Eq. D6.3.1-4)}$$

With the stiffness of each supporting element defined, the force, P_i, is distributed to each of these elements based upon their relative stiffness.

This pseudo-single-degree-of-freedom procedure provides nearly the same force, P_L, as the matrix solution. However, the procedure does not provide the lateral displacements of the purlins. To develop an estimate of the lateral deflections, two special cases were considered.

1) For a roof system with anchorage only at one purlin line, the deflection at the anchorage device is the anchorage force divided by the device stiffness, P_L/K_a. The deflection at other purlins is larger due to the flexibility of the sheathing. This increase mimics the decrease in the value of K_{eff} and indicates that the stiffness reduction included in K_{eff} should be included in the deflection calculation.

2) For a roof system with an anchorage device at every purlin, the deflection at each purlin is again the anchorage force divided by the device stiffness, P_L/K_a. Multiplying the numerator and denominator by the number of purlins, N_p, produces an equivalent expression, $N_pP_L/(N_PK_a)$. If the system stiffness is neglected, and the purlins are uniformly spaced, the anchorage force is simply P_i and the numerator of the previous expression is equal to the sum of all P_i terms. If the flexibility of the sheathing is neglected, K_{eff} equals K_a, and the denominator becomes the summation of K_{eff}, or K_{total}. Making these substitutions results in $\Delta_i = \text{sum}(P_i)/K_{total}$.

This approximation works well when anchorage devices are relatively stiff and reasonably spaced. As the distance between anchorage points increases, the approximation will tend to over-predict the displacement.

For the anchorage devices to perform properly they must possess adequate strength and stiffness. Based on the observations of past purlin tests by members of the AISI Task Group on Anchorage and Bracing, a lateral displacement limit of the purlin depth divided by 20 has been established as the critical point where an anchorage device becomes ineffective and the purlin is at risk of a global stability failure. For convenience, this displacement limit has been reformulated as a critical stiffness limit for use in the *Specification*.

For multi-span systems with anchorage at the supports, the anchorage force is transferred partially from each of the adjacent bays. If these two bays have different span lengths or purlin sizes, the following procedure is used to average the effect from each bay.

1) The forces, P_i, are calculated independently using the properties of each of the two bays separately. The resulting force is then averaged for each purlin line.
2) The system stiffness, K_{sys}, is calculated by evaluating the equation with L, t, and d taken as the average of the values from the two bays.
3) The effective lateral stiffness of the anchor, K_{eff}, is calculated using the average of the two span lengths.

This procedure is illustrated in Example 3, in Section 4.4.

For multi-span systems with restraints at 1/3 points or midpoints, analysis of computer stiffness models has shown that the anchorage forces can be reasonably estimated by calculating an average force in a manner similar to that outlined above for restraints at the supports, except the procedure considers three bays: the current bay and one to each side of the current bay.

During the verification of the model and the calibration of the coefficients in the calculation procedure, significant scatter was observed in the results at the end anchor in multi-span systems. This is most likely due to the difference in the lateral stiffness of the purlins at the end frame line and the first interior frame line and how this affects the distribution of the forces between the first few lines of anchorage. To account for this scatter and minimize the chance of calculating significantly unconservative forces, the provisions require that the line of anchorage nearest the end of a multi-span system be designed for the larger of the forces calculated for the line of anchorage, and 80% of the value found using the coefficients C2, C3 and C4 for the typical interior case.

4.2.2 Provisions of *Specification* Section D6.3.2 Alternate Lateral and Stability Bracing for Purlin Roof Systems

The provisions of the *Specification* Section D6.3.2 are intended to cover torsional braces along the span of a C- or Z-section that only resist torsion of the purlin and do not resist the lateral movement of the section. When used in conjunction with lateral restraints at the frame lines, the torsional brace restraints are effective in stabilizing the purlin even as it is permitted to move laterally. For this reason, a more relaxed requirement for the lateral displacement of the C-section or Z-section at midspan is permitted.

Analysis of torsional braces can be first order because if the lateral deflection criteria are met, second order effects should be minimal and easily absorbed by the torsional braces. An analysis of torsional braces should consider the forces generated due to eccentricity of applied loads and the effects of the interaction of the diaphragm restraint with the purlin. Analysis should also include the effects the torsional braces have on the lateral restraints applied at the frame line. The component stiffness method presented in detail in Section 5.1 of this Design Guide provides a method for evaluating torsional braces at one-third points. Torsional braces may also be evaluated using the finite element analysis models outlined in Section 5.3.

4.3 Applications of the Specification Provisions

The provisions of *Specification* Section D6.3 provide design criteria and analysis procedures for the required restraint of purlin systems so that the torsional stresses in the members are negligible and the purlin design provisions in *Specification* Section D6.1 may be utilized. The provisions of *Specification* Section D6.3.1 are applicable to restraint systems that resist the movement of purlins within the plane of the roof sheathing, thus indirectly minimizing the member torsion. *Specification* Section D6.3.2 provides general guidance for the design of systems where lateral displacements are not restrained, but the torsional moments are resisted directly by braces attached to the purlins.

ASD vs. LRFD

Since the procedures are used to calculate forces and/or moments within the bracing systems rather than the resistance provided by an anchorage device, they are equally applicable to ASD and LRFD design methodologies. The only distinction required is the use of nominal loads for ASD and factored loads for LRFD when calculating the force distributed to the anchorage devices. However, for the stability check within the provisions an adjustment is required. The displacement limit of d/20 in the *Specification* was established based on observations of tests. Therefore, the limit is based on the deflection under ultimate loads. However, in this formulation no adjustment has been made in either equation for the likely variability in the provided stiffness. To make this adjustment a resistance factor, ϕ, or the safety factor, Ω, is included in the deflection and/or minimum stiffness limits.

Load Cases and Pattern Loading

It is generally accepted that the lateral effects in metal building roof systems are only of concern for downward loading on the roof, due to the destabilizing effect of the top flange gravity loading. Generally the anchorage force and stiffness requirements need only be checked under the controlling gravity load combination, typically a combination of dead load and roof live or snow load. Since the lateral effects are primarily induced by the application of the load, and not the flexural behavior of the purlin, the effects of pattern loading only need to be considered when the forces tend to counteract, such as back-to-back C-sections with restraints at the supports.

4.3.1 Discrete Bracing

When lateral restraints resist the movement of the purlins within the plane of the roof sheathing and transfer the resulting force to an element of the lateral load resisting system, the provisions of *Specification* Section D6.3.1 provide a means of predicting the forces in the anchorage devices and verifying stability of the purlin system. As presented in the *Specification*, the provisions address the requirements for the majority of metal building roof systems. Various conditions exist where the applicability of the provisions is not perfectly clear. The following provides the authors' recommendations for applying the provisions to several of these conditions.

Non-Uniform Loading

The general presentation of the *Specification* provisions is based on the application of uniform loads. The provisions can be directly applied to systems where the gravity load varies from the eave to ridge (for example, snow drift at the high side) by using the appropriate line load, W_p, at each purlin. However, when the loads vary along the length of the purlin (e.g., snow drift at the end wall) a special approach is needed.

In the *Specification* procedure the force W_p is equal to wL, or twice the simple span end reaction. When the load is non-uniform the force W_p should be taken as twice the end reaction of a simple span beam with the same bay span and loading as the purlin under consideration.

Cantilevers

For purlin spans that cantilever over an end support, the value of W_p when analyzing the line of anchorage nearest the cantilevered end is taken as twice the end reaction for a simple span beam with the same back span, cantilever and loading. Also, the value of L used to calculate K_{sys} and K_{eff} is the back span length. At other lines of anchorage, the effect of the cantilever is neglected. This procedure is illustrated in Example 5.

Flush Mounted Purlins

Where purlins are flush mounted to the web of the supporting rafter, each connection must be designed to resist the moment caused by the force P_i applied at the purlin-to-sheathing interface. This moment will typically be equal to $P_i(d/2)$.

Purlin-to-Rafter Connections

The provisions in the *Specification* are based on tests of purlins where the bottom flanges were bolted directly to the rafter flange. However, it is also common for purlins to be attached through the web to wing plates that are welded to the rafter as shown in Figure 1.6.

If all purlins are connected with identical wing plates, the attachment may simply be designed for the force P_i as described above for flush mounted purlins. If the typical connection does not provide adequate strength or stiffness, then selected locations may be stiffened. For this condition, it is recommended that the provisions be applied by either treating the stiffened connections as anchorage points and assuming the behavior of the other purlins is the same as that for the flange bolted condition, or by treating all of the connections as anchorage points with the appropriate stiffness values, K_a, used at each purlin. In this second condition K_{sys} should be conservatively taken as zero.

Lines of Anchorage Not At Exact 1/3 Points

Where two lines of anchorage are installed at points somewhere between the 1/3 points and the midpoint, the anchorage devices should conservatively be designed for the larger of the forces found for 1/3 point restraints or half the value for midpoint restraints. Using two lines of anchorage outboard of the 1/3 points is not recommended.

Non-Parallel Rafters

In cases where building rafters are not parallel (hips, valleys, skewed end walls etc.) the force P_i is to be calculated using the appropriate span length for each purlin. The values of K_{sys} and K_{eff} may be calculated using the average purlin span.

Eave Struts

When the deflection and rotation of the building eave member is continuously supported by the wall system, the member may be neglected in the anchorage calculations. However, if this is not the case (open sidewall) the eave member needs to be considered.

Opposing Purlins

For low sloped roofs, where the anchorage force is positive, a portion of the purlins can be turned with the top flange facing downslope to counteract the tendency of the roof system to displace upslope. To model this condition, the term α in *Specification* Equation D6.3.1-2 is taken as +1 for those purlins that face upslope and -1 for those that face downslope. If the reversed purlins are not evenly distributed throughout the roof plane, the *Specification* provisions will not adequately address the minimum stiffness requirements. If the reversed purlins are grouped into one area, the matrix solution presented in Section 4.5.2 of this Design Guide is recommended to more completely address the displacement of the purlins.

Diaphragm Load

To ensure that the roof sheathing provides adequate support to the purlins to justify the design of the purlins without considering torsional stresses, the *Specification* limits the lateral deflection of the diaphragm to L/360, at service level loads. The *Specification* is silent as to the force to be used for this check, and the work by Sears and Murray (2007), upon which the *Specification* provisions are based, did not directly address this requirement. However, the component stiffness method (Seek and Murray 2006) does provide a method to check this displacement. By starting with the provisions of the component stiffness method, and identifying those terms that have the greatest influence on the diaphragm load, it can be shown that the line load applied to the diaphragm is very reasonably approximated as follows.

$$w_{diaph} = \sum\left[\frac{w}{L}\left(\frac{I_{xy}}{I_x}\cos\theta - \sin\theta\right)\right]_i \tag{4.3.1}$$

With this value the diaphragm deflection is found from,

$$\Delta_{diaph} = \frac{w_{diaph}L^2}{8G'B} \tag{4.3.2}$$

4.3.2 Torsional Bracing

Torsional bracing (Figure 1.7(a)) can be evaluated using the component stiffness method (Seek and Murray 2006). The components of gravity load normal and parallel to the plane of sheathing on a sloped roof cause torsional moments of a purlin that must be resisted by the torsional restraints. Diaphragm forces generated in the sheathing as a purlin is loaded and deflects laterally produce additional torsional moments. It is typically assumed that torsional restraints completely restrict rotation of the purlin at the restraint because most torsional restraints, if properly connected near the top and bottom flanges, possess considerably more stiffness than the torsional stiffness of a cold-formed purlin. However, the behavior of these systems is very sensitive to the connection details, and typically the behavior of these systems should be verified by tests. The magnitude of the moments at the internal torsional restraints affects the lateral forces anchored at each frame line.

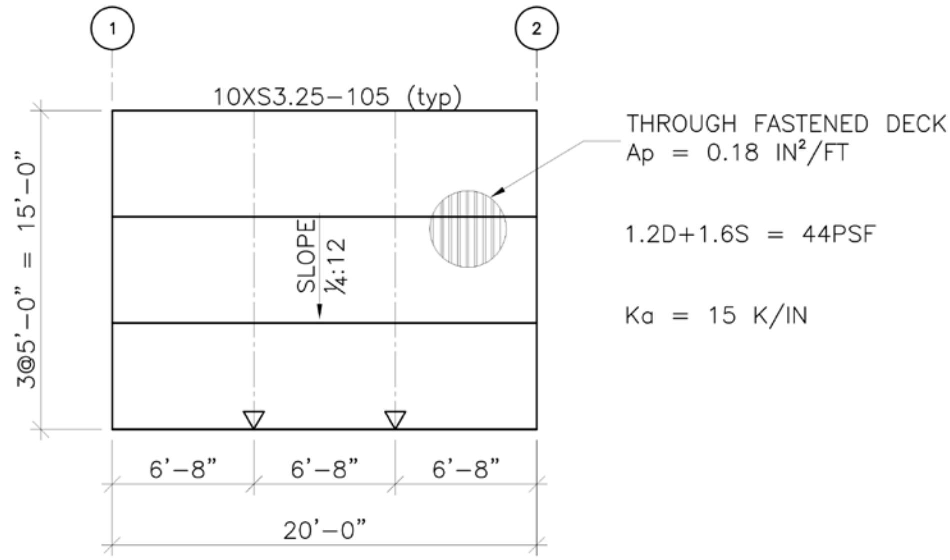
Systems of purlins with torsional braces can be evaluated in pairs. At each end of the torsional brace, a moment is generated. Equilibrium of the brace is maintained through the development of opposing shear forces at each end of the brace applying uplift forces to one purlin and downward forces to the other. These forces should be considered when evaluating the flexural strength of the purlin.

4.4 Examples

In the following examples Eq. numbers referenced are from *Specification* and *Design Manual* refers to the AISI *Cold-Formed Steel Design Manual* (AISI 2008a).

4.4.1 Example 1: Single Bay Z-Purlin Attached to Through-Fastened Panels with One-Third Points Anchorage at Low Eave Purlin -- LRFD

Determine the anchorage force, using LRFD, for a single bay roof with four parallel 10ZS3.25x105 purlin lines spaced at 5 ft-0 in. on center, top purlin flange facing in up-slope direction, a slope of ¼ in. per ft, and a factored uniform load of 44 psf. The roof sheathing is a through-fastened panel with a cross-sectional area of 0.18 in²/ft and shear stiffness of 9000 lb/in. The system is anchored at one-third points of the low eave purlin, and the anchorage devices have a stiffness of 15 kip/in.



System Configuration:

N_p = 4
N_a = 1
K_a = 15 kip/in.
A_p = 0.18 in²/ft.
G' = 9000 lb/in
θ = arctan(0.25/12) = 1.194 degrees

10ZS3.25x105 Purlin Properties from *Design Manual* Table I-4

d = 10 in.
b = 3.25 in.
t = 0.105 in.
I_x = 28.4 in⁴
I_{xy} = 8.41 in⁴

Coefficients from *Specification* Table D6.3.1-3

C1 = 0.5

C2 = 7.8

C3 = 42

C4 = 0.98

C5 = 0.39

C6 = 0.40

Purlin 1 is the eave (anchored) purlin and Purlin 4 is the ridge purlin.

The gravity load tributary to each purlin is:

$$W_{p_1} = W_{p_4} = (2.5)(44)(20) = 2200 \text{ lb}$$

$$W_{p_2} = W_{p_3} = (5)(44)(20) = 4400 \text{ lb}$$

To verify whether the diaphragm provides the required stiffness, the diaphragm deflection is checked. Since this check is performed at the service load level, the loads found above are divided by 1.5 to adjust the forces to approximately the unfactored level

$$w_{diaph} = \sum\left[\frac{w_p}{L}\left(\frac{I_{xy}}{I_x}\cos\theta - \sin\theta\right)\right]_i \tag{4.3.1}$$

$$= \left[2\frac{(2200+4400)/1.5}{20}\left(\frac{8.41}{28.4}\cos(1.194) - \sin(1.194)\right)\right] = 121.1 \text{ plf}$$

With this value the in-plane diaphragm deflection at each frame line is found by treating one-third of the span as a cantilever.

$$\Delta_{diaph} = \frac{w_{diaph} L_{diaph}{}^2}{2G'B} \tag{4.3.2}$$

$$= \frac{(121.1)(20/3)^2}{2(9000)(15)} = 0.020\text{in} < L/360 = (20\text{x}12/3)/360 = 0.22 \text{ in} \quad \text{OK}$$

The in-plane deflection at the midpoint of bay is one fourth of this value.

Specification Equation D6.3.1-2 is used to calculate the lateral load introduced into the system at each purlin:

$$P_i = C1 \cdot W_{p_i} \cdot \left[\left(\frac{C2}{1000} \cdot \frac{I_{xy}L}{I_x d} + C3 \cdot \frac{(m+0.25b)t}{d^2}\right)\alpha \cdot \cos\theta - C4 \cdot \sin\theta\right] \tag{Eq. D6.3.1-2}$$

$$P_1 = P_4 = 0.5(2200)\left[\left(\frac{7.8}{1000}\frac{(8.41)(20\text{x}12)}{(28.4)(10)} + 42\frac{(0+0.25(3.25))(0.105)}{10^2}\right)(1)\cos(1.194) - (0.98)\sin(1.194)\right]$$

$$= 78 \text{ lb}$$

Then P_2 and P_3 is found by scaling the result by the W_p terms,

$$P_2 = P_3 = 78\frac{4400}{2200} = 156 \text{ lb}$$

The system stiffness is found from *Specification* Equation D6.3.1-6:

$$K_{sys} = \frac{C5}{1000} \cdot N_p \frac{ELt^2}{d^2} \tag{Eq. D6.3.1-6}$$

$$= \frac{0.39}{1000} \cdot (4) \frac{(29500)(20x12)(0.105^2)}{10^2} = 1.22 \,{}^{kip}\!/_{in}$$

The effective stiffness of the anchorage device, relative to each purlin is found from *Specification* Equation D6.3.1-4,

$$K_{eff(i,j)} = \left[\frac{1}{K_a} + \frac{d_{p_{i,j}}}{C6 \cdot LA_p E} \right]^{-1} \qquad \text{(Eq. D6.3.1-4)}$$

$$K_{eff(1,1)} = \left[\frac{1}{15} + 0 \right]^{-1} = 15 \,{}^{kip}\!/_{in}$$

$$K_{eff(2,1)} = \left[\frac{1}{15} + \frac{(5x12)\ {}^{1}\!/_{\cos(1.194)}}{0.40(20x12)(0.18/12)(29500)} \right]^{-1} = 14.69 \,{}^{kip}\!/_{in}$$

$$K_{eff(3,1)} = \left[\frac{1}{15} + \frac{(10x12)\ {}^{1}\!/_{\cos(1.194)}}{0.40(20x12)(0.18/12)(29500)} \right]^{-1} = 14.39 \,{}^{kip}\!/_{in}$$

$$K_{eff(4,1)} = \left[\frac{1}{15} + \frac{(15x12)\ {}^{1}\!/_{\cos(1.194)}}{0.40(20x12)(0.18/12)(29500)} \right]^{-1} = 14.10 \,{}^{kip}\!/_{in}$$

The total stiffness at each purlin is calculated from the results above and *Specification* Equation D6.3.1-5.

$$K_{total(i)} = \sum_{j=1}^{N_a} \left(K_{eff_{i,j}} \right) + K_{sys} \qquad \text{(Eq. D6.3.1-5)}$$

$$K_{total(1)} = 15 + 1.22 = 16.22 \,{}^{kip}\!/_{in}$$

$$K_{total(2)} = 14.69 + 1.22 = 15.91 \,{}^{kip}\!/_{in}$$

$$K_{total(3)} = 14.39 + 1.22 = 15.61 \,{}^{kip}\!/_{in}$$

$$K_{total(4)} = 14.10 + 1.22 = 15.32 \,{}^{kip}\!/_{in}$$

The smallest of these stiffness values, which is the stiffness provided to the most remote purlin, is compared to the required minimum stiffness from *Specification* Equation D6.3.1-8b

$$K_{req} = \frac{1}{\phi} \frac{20 \cdot \left| \sum_{i=1}^{N_p} P_i \right|}{d} \qquad \text{(Eq. D6.3.1-8b)}$$

$$= \frac{1}{0.75} \cdot \frac{20(2 \cdot 0.078 + 2 \cdot 0.156)}{10} = 1.25 \,{}^{kip}\!/_{in} < K_{total(4)} = 15.32 \,{}^{kip}\!/_{in} \quad \text{OK}$$

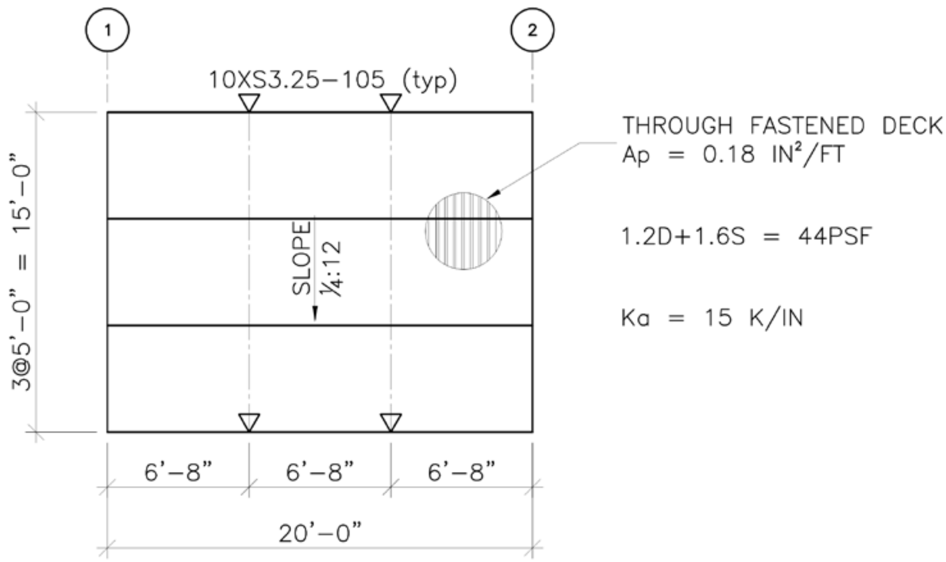
Finally the force at an anchorage device is found from the basic design *Specification*, Equation D6.3.1-1,

$$P_L = \sum_{i=1}^{N_p} \left(P_i \frac{K_{eff_{i,j}}}{K_{total_i}} \right) \qquad \text{(Eq. D6.3.1-1)}$$

$$= 78\frac{15}{16.22} + 156\frac{14.69}{15.91} + 156\frac{14.39}{15.61} + 78\frac{14.10}{15.32} = 431\text{ lb}$$

4.4.2 Example 2: Single Bay Z-Purlin Attached to Through-Fastened Panels with One-Third Points Anchorage at Low and Ridge Purlins -- LRFD

Repeat Example 1, but add anchorage devices to the ridge purlin.



The following values from Example 1 are also applicable to this example:

P_1=P_4 = 78 lb

P_2=P_3 = 156 lb

K_{sys} = 1.22 kip/in

K_{req} = 1.25 kip/in

$K_{eff(1,1)}$ = 15 kip/in

$K_{eff(2,1)}$ = 14.69 kip/in

$K_{eff(3,1)}$ = 14.39 kip/in

$K_{eff(4,1)}$ = 14.10 kip/in

By inspection the effective stiffness values for the second anchor are simply the reverse of those for the first anchorage device.

$K_{eff(1,2)}$ = 14.10 kip/in

$K_{eff(2,2)}$ = 14.39 kip/in

$K_{eff(3,2)}$ = 14.69 kip/in

$K_{eff(4,2)}$ = 15 kip/in

The total stiffness at each purlin is calculated from the results above and the *Specification* Equation D6.3.1-5,

$$K_{total_i} = \sum_{j=1}^{N_a}\left(K_{eff_{i,j}}\right) + K_{sys} \qquad \text{(Eq. D6.3.1-5)}$$

$$K_{totat(1)} = 15 + 14.10 + 1.22 = 30.32 \,{}^{kip}\!/_{in}$$

$$K_{total(2)} = 14.69 + 14.39 + 1.22 = 30.30 \,{}^{kip}\!/_{in}$$

$$K_{total(3)} = 14.39 + 14.69 + 1.22 = 30.30 \,{}^{kip}\!/_{in}$$

$$K_{total(4)} = 14.10 + 15 + 1.22 = 30.32 \,{}^{kip}\!/_{in}$$

The smallest of these stiffness values is compared to the required minimum stiffness from *Specification* Equation D6.3.1-8b, which yields the same value as in Example 1.

K_{req} = 1.25 kip/in < $K_{total(2)}$ = 30.30 kip/in OK

Finally the anchorage force is found from the *Specification* Equation D6.3.1-1.

$$P_{L_j} = \sum_{i=1}^{N_p} \left(P_i \frac{K_{eff\,i,j}}{K_{total\,i}} \right) \qquad \text{(Eq. D6.3.1-1)}$$

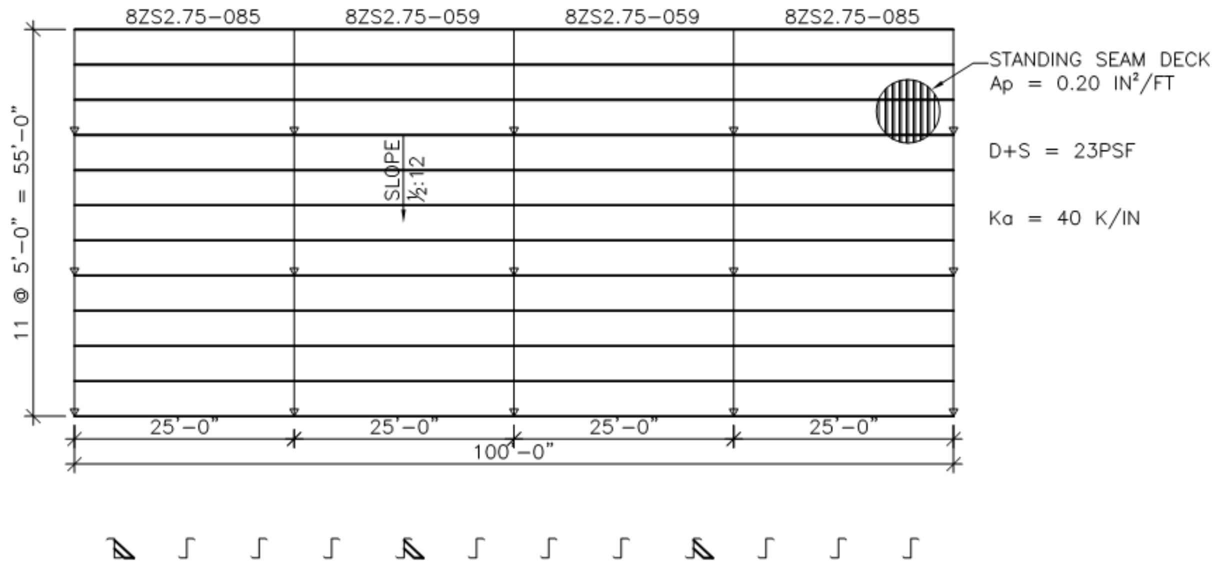
$$P_{L_1} = 78\frac{15}{30.32} + 156\frac{14.69}{30.30} + 156\frac{14.39}{30.30} + 78\frac{14.10}{30.32} = 224\text{ lb}$$

$$P_{L_2} = 78\frac{14.10}{30.32} + 156\frac{14.39}{30.30} + 156\frac{14.69}{30.30} + 78\frac{15}{30.32} = 224\text{ lb}$$

Note that the anchorage forces from this example are slightly greater than half of that found from Example 1. This occurs because as additional stiffness is introduced into the system, less force is taken by the “system effect”.

4.4.3 Example 3: Four Span Continuous Z-Purlin Attached to Standing Seam Panels - ASD

Evaluate the anchorage system for the four continuous spans, standing seam roof system shown. The system consists of twelve parallel Z-purlin lines, with the first purlin reversed (top flange facing downslope) and anchorage devices at the support points of the first, fifth, and ninth purlins. The anchorage devices provide a lateral stiffness of 40 kip/in and the roof sheathing has a cross-sectional area of 0.20 in^2/ft and a shear stiffness value of 1200 lb/in. The roof slope is ½ in. per ft and the dead plus snow service level roof load is 23 psf.



System Properties:

N_p = 12
N_a = 3
A_p = 0.20 in^2/ft.
θ = arctan(0.5/12) = 2.39 degrees
K_a = 40 kip/in
G' = 1200 lb/in.

Purlin Properties from AISI *Design Manual* Table I-4

	End Bays	Interior Bays
	8ZS2.75x085	8ZS2.75x059
d	= 8 in.	8 in.
b	= 2.75 in.	2.75 in.
t	= 0.085 in.	0.059 in.
I_x	= 12.4 in^4	8.69 in^4
I_{xy}	= 4.11 in^4	2.85 in^4

Coefficients from *Specification* Table D6.3.1-1

		Frame Line 1	Frame Line 2	Frame Line 3
C1	=	0.5	1.0	1.0
C2	=	13	1.7	4.3
C3	=	11	69	55
C4	=	0.35	0.77	0.71
C5	=	2.4	1.6	1.4
C6	=	0.25	0.13	0.17

Purlin Layout and Loading

Purlin Number	1	2	3	4	5	6	7	8	9	10	11	12
Location (ft)	0	5	10	15	20	25	30	35	40	45	50	55
Dist From Anch A (ft)	0.00	5.00	10.01	15.01	20.02	25.02	30.03	35.03	40.03	45.04	50.04	55.05
Dist From Anch B (ft)	20.02	15.01	10.01	5.00	0.00	5.00	10.01	15.01	20.02	25.02	30.03	35.03
Dist From Anch C (ft)	40.03	35.03	30.03	25.02	20.02	15.01	10.01	5.00	0.00	5.00	10.01	15.01

Purlin Number, i	1	2	3	4	5	6	7	8	9	10	11	12
Tributary Width (ft)	2.5	5	5	5	5	5	5	5	5	5	5	2.5
W_{p1} (lb)	1438	2875	2875	2875	2875	2875	2875	2875	2875	2875	2875	1438
Purlin Orientation, α	-1	1	1	1	1	1	1	1	1	1	1	1

Frame Line 1 Calculations

Find system stiffness from *Specification* Equation D6.3.1-6

$$K_{sys} = \frac{C5}{1000} \cdot N_p \frac{ELt^2}{d^2} \qquad \text{(Eq. D3.2.1-6)}$$

$$= \frac{2.4}{1000} \cdot (12) \frac{(29500)(25x12)(0.085^2)}{8^2} = 28.77 \, {}^{kip}\!/_{in}$$

The effective stiffness values for each anchor, with respect to each purlin, are calculated from *Specification* Equation D6.3.1-4

$$K_{eff_{i,j}} = \left[\frac{1}{K_a} + \frac{d_{p_{i,j}}}{C6 \cdot LA_p E} \right]^{-1} \qquad \text{(Eq. D6.3.1-4)}$$

$$= \left[\frac{1}{40} + \frac{d_{p_{i,j}}}{0.25(25x12)(0.20/12)(29500)} \right]^{-1}$$

Purlin Number	1	2	3	4	5	6	7	8	9	10	11	12
$K_{eff,A}$ (kip/in)	40.0	37.6	35.4	33.5	31.7	30.2	28.8	27.5	26.3	25.2	24.2	23.3
$K_{eff,B}$ (kip/in)	31.7	33.5	35.4	37.6	40.0	37.6	35.4	33.5	31.7	30.2	28.8	27.5
$K_{eff,C}$ (kip/in)	26.3	27.5	28.8	30.2	31.7	33.5	35.4	37.6	40.0	37.6	35.4	33.5
Sum (kip/in)	98.0	98.5	99.5	101.2	103.5	101.2	99.5	98.5	98.0	92.9	88.4	84.2
K_{total}=S_{um}+K_{sys} (kip/in)	126.8	127.3	128.3	130.0	132.2	130.0	128.3	127.3	126.8	121.7	117.1	113.0

Force distribution factors are calculated by finding the stiffness ratio in *Specification* Equation D6.3.1-1 and dividing the K_{eff} values above by the corresponding K_{total}.

Purlin Number	1	2	3	4	5	6	7	8	9	10	11	12
DF A	0.315	0.295	0.276	0.257	0.240	0.232	0.224	0.216	0.207	0.207	0.207	0.206
DF B	0.250	0.263	0.276	0.289	0.302	0.289	0.276	0.263	0.250	0.248	0.246	0.243
DF C	0.207	0.216	0.224	0.232	0.240	0.257	0.276	0.295	0.315	0.309	0.302	0.296

The individual purlin forces are calculated from *Specification* Equation D6.3.1-2

$$P_i = C1 \cdot W_{p_i} \cdot \left[\left(\frac{C2}{1000} \cdot \frac{I_{xy} L}{I_x d} + C3 \cdot \frac{(m + 0.25b)t}{d^2} \right) \alpha \cdot \cos\theta - C4 \cdot \sin\theta \right] \qquad \text{(Eq. D6.3.1-2)}$$

$$P_1 = 0.5(1438)\left[\left(\frac{13}{1000}\frac{(4.11)(25x12)}{(12.4)(8)} + 11 \cdot \frac{(0 + 0.25(2.75))(0.085)}{8^2}\right)(-1)\cos(2.39) - (0.35)\sin(2.39)\right] = -133.7\text{lb}$$

$$P_2 \text{ to } P_{11} = 0.5(2875)\left[\left(\frac{13}{1000}\frac{(4.11)(25x12)}{(12.4)(8)} + 11 \cdot \frac{(0 + 0.25(2.75))(0.085)}{8^2}\right)(1)\cos(2.39) - (0.35)\sin(2.39)\right] = 225.6\text{lb}$$

$$P_{12} = P_2 / 2 = 112.8\text{lb}$$

These values must not be taken as less than 80% of that found using "All Other Locations" coefficients.

$$P_1 = (0.80)0.5(1438)\left[\left(\frac{4.3}{1000}\frac{(4.11)(25x12)}{(12.4)(8)} + 55 \cdot \frac{(0 + 0.25(2.75))(0.085)}{8^2}\right)(-1)\cos(2.39) - (0.71)\sin(2.39)\right] = -76.6\text{lb}$$

$$P_2 \text{ to } P_{11} = (0.80)0.5(2875)\left[\left(\frac{4.3}{1000}\frac{(4.11)(25x12)}{(12.4)(8)} + 55 \cdot \frac{(0 + 0.25(2.75))(0.085)}{8^2}\right)(1)\cos(2.39) - (0.71)\sin(2.39)\right] = 85.1\text{lb}$$

$$P_{12} = P_2 / 2 = 42.6\text{lb}$$

These forces are then distributed to each anchorage device with the distribution factors found above.

Purlin Number	1	2	3	4	5	6	7	8	9	10	11	12
P (lb)	-133.7	225.6	225.6	225.6	225.6	225.6	225.6	225.6	225.6	225.6	225.6	112.8
P to Anch A (lb)	-42.2	66.6	62.2	58.1	54.1	52.4	50.6	48.7	46.8	46.7	46.6	23.3
P to Anch B (lb)	-33.5	59.3	62.2	65.2	68.2	65.2	62.2	59.3	56.4	55.9	55.4	27.4
P to Anch C (lb)	-27.7	48.7	50.6	52.4	54.1	58.1	62.2	66.6	71.2	69.6	68.1	33.4

Then taking the summation across each row yields the final anchorage forces.

P_{LA} (lb)	514
P_{LB} (lb)	603
P_{LC} (lb)	607

The stiffness of the system is checked using *Specification* Equation D6.3.1-8a

$$K_{total} \geq K_{req} \quad \text{(Eq. D6.3.1-7)}$$

$$K_{req} = \Omega \frac{20 \cdot \left| \sum_{i=1}^{N_p} P_i \right|}{d} \quad \text{(Eq. D6.3.1-8a)}$$

$$= 2.0 \frac{20(-0.1337 + 10 \cdot 0.2256 + 0.1128)}{8} = 11.2 \,{}^{kip}\!/_{in} < K_{total(12)} = 113 \,{}^{kip}\!/_{in} \quad \text{OK}$$

Frame Line 2 Calculations

The system stiffness is found from

$$K_{sys} = \frac{C5}{1000} \cdot N_p \frac{ELt^2}{d^2} \quad \text{(Eq. D6.3.1-6)}$$

$$= \frac{1.6}{1000} \cdot (12) \frac{(29500)(25x12)((0.085 + 0.059)/2)^2}{8^2} = 13.76 \,{}^{kip}\!/_{in}$$

The effective stiffness values and distribution factors for each anchor, with respect to each purlin, are calculated from *Specification* Equation D6.3.1-4

$$K_{eff_{i,j}} = \left[\frac{1}{K_a} + \frac{d_{p_{i,j}}}{C6 \cdot LA_p E} \right]^{-1} \quad \text{(Eq. D6.3.1-4)}$$

$$= \left[\frac{1}{40} + \frac{d_{p_{i,j}}}{0.13(25x12)(0.20/12)(29500)} \right]^{-1}$$

Purlin Number	1	2	3	4	5	6	7	8	9	10	11	12
$K_{eff,A}$ (kip/in)	40.0	35.8	32.5	29.7	27.3	25.3	23.6	22.0	20.7	19.5	18.5	17.5
$K_{eff,B}$ (kip/in)	27.3	29.7	32.5	35.8	40.0	35.8	32.5	29.7	27.3	25.3	23.6	22.0
$K_{eff,C}$ (kip/in)	20.7	22.0	23.6	25.3	27.3	29.7	32.5	35.8	40.0	35.8	32.5	29.7
Sum (kip/in)	88.0	87.5	88.5	90.8	94.6	90.8	88.5	87.5	88.0	80.7	74.5	69.2
K_{total}=Sum+K_{sys}(kip/in)	101.8	101.3	102.2	104.5	108.4	104.5	102.2	101.3	101.8	94.4	88.3	83.0
DF A	0.393	0.354	0.317	0.284	0.252	0.242	0.230	0.218	0.204	0.207	0.209	0.211
DF B	0.268	0.293	0.317	0.343	0.369	0.343	0.317	0.293	0.268	0.268	0.267	0.266
DF C	0.204	0.218	0.230	0.242	0.252	0.284	0.317	0.354	0.393	0.379	0.368	0.357

The individual purlin forces are calculated from *Specification* Equation D6.3.1-2 using the properties of the first bay, then the interior bay and the resulting forces are averaged.

$$P_i = C1 \cdot W_{p_i} \cdot \left[\left(\frac{C2}{1000} \cdot \frac{I_{xy}L}{I_x d} + C3 \cdot \frac{(m+0.25b)t}{d^2} \right) \alpha \cdot \cos\theta - C4 \cdot \sin\theta \right] \qquad \text{(Eq. D6.3.1-2)}$$

$$P_{1-Left} = 1.0(1438)\left[\left(\frac{1.7}{1000} \frac{(4.11)(25x12)}{(12.4)(8)} + 69 \cdot \frac{(0+0.25(2.75))(0.085)}{8^2} \right)(-1)\cos(2.39) - (0.77)\sin(2.39) \right] = -167.5\text{lb}$$

$$P_{2-Left} = 1.0(2875)\left[\left(\frac{1.7}{1000} \frac{(4.11)(25x12)}{(12.4)(8)} + 69 \cdot \frac{(0+0.25(2.75))(0.085)}{8^2} \right)(1)\cos(2.39) - (0.77)\sin(2.39) \right] = 148.3\text{lb}$$

$$P_{12-Left} = P_2 / 2 = 74.2\text{lb}$$

$$P_{1-Right} = 1.0(1438)\left[\left(\frac{1.7}{1000} \frac{(2.85)(25x12)}{(8.69)(8)} + 69 \cdot \frac{(0+0.25(2.75))(0.059)}{8^2} \right)(-1)\cos(2.39) - (0.77)\sin(2.39) \right] = -139.5\text{lb}$$

$$P_{2-Right} = 1.0(2875)\left[\left(\frac{1.7}{1000} \frac{(2.85)(25x12)}{(8.69)(8)} + 69 \cdot \frac{(0+0.25(2.75))(0.059)}{8^2} \right)(1)\cos(2.39) - (0.77)\sin(2.39) \right] = 92.3\text{lb}$$

$$P_{12-Right} = P_2 / 2 = 46.2\text{lb}$$

These forces are then distributed to each anchorage device with the distribution factors found above.

Purlin Number	1	2	3	4	5	6	7	8	9	10	11	12
P Left (lb)	-167.5	148.3	148.3	148.3	148.3	148.3	148.3	148.3	148.3	148.3	148.3	74.2
P Right (lb)	-139.5	92.3	92.3	92.3	92.3	92.3	92.3	92.3	92.3	92.3	92.3	46.2
P (lb)	-153.5	120.3	120.3	120.3	120.3	120.3	120.3	120.3	120.3	120.3	120.3	60.2

Force to Anch A (lb)	-60.3	42.6	38.2	34.1	30.3	29.1	27.7	26.2	24.5	24.9	25.2	12.7
Force to Anch B (lb)	-41.2	35.2	38.2	41.2	44.4	41.2	38.2	35.2	32.3	32.2	32.1	16.0
Force to Anch C (lb)	-31.3	26.2	27.7	29.1	30.3	34.1	38.2	42.6	47.3	45.7	44.2	21.5

P_{LA} (lb)	255
P_{LB} (lb)	345
P_{LC} (lb)	356

$$K_{req} = \Omega \frac{20 \cdot \left| \sum_{i=1}^{N_p} P_i \right|}{d} \qquad \text{(Eq. D6.3.1-4)}$$

$$= 2.0 \frac{20(-0.1535 + 10 \cdot 0.1203 + 0.0602)}{8} = 5.5 \,{}^{\text{kip}}\!/_{\text{in}} < K_{total(12)} = 83 \,{}^{\text{kip}}\!/_{\text{in}} \quad \text{OK}$$

Frame Line 3 Calculations

The forces along frame line 3 are found by applying the procedure using the properties of the interior bays. For brevity, the calculations are presented only in tabular form.

$$K_{sys} = 8.09 \,{}^{\text{kip}}\!/_{\text{in}}$$

Purlin Number	1	2	3	4	5	6	7	8	9	10	11	12
$K_{eff,A}$ (kip/in)	40.0	36.5	33.6	31.1	28.9	27.0	25.4	23.9	22.6	21.5	20.4	19.5
$K_{eff,B}$ (kip/in)	28.9	31.1	33.6	36.5	40.0	36.5	33.6	31.1	28.9	27.0	25.4	23.9
$K_{eff,C}$ (kip/in)	22.6	23.9	25.4	27.0	28.9	31.1	33.6	36.5	40.0	36.5	33.6	31.1
Sum (kip/in)	91.6	91.5	92.5	94.6	97.8	94.6	92.5	91.5	91.6	85.0	79.4	74.5
K_{total}=Sum+K_{sys} (kip/in)	99.7	99.6	100.6	102.7	105.9	102.7	100.6	99.6	99.7	93.1	87.5	82.6
DF A	0.401	0.366	0.334	0.303	0.273	0.263	0.252	0.240	0.227	0.231	0.234	0.236
DF B	0.290	0.312	0.334	0.355	0.378	0.355	0.334	0.312	0.290	0.290	0.290	0.290
DF C	0.227	0.240	0.252	0.263	0.273	0.303	0.334	0.366	0.401	0.392	0.384	0.376
P (lb)	-168.5	167.1	167.1	167.1	167.1	167.1	167.1	167.1	167.1	167.1	167.1	83.5
Force to Anch A (lb)	-67.6	61.2	55.7	50.5	45.6	44.0	42.2	40.2	38.0	38.5	39.0	19.7
Force to Anch B (lb)	-48.9	52.1	55.7	59.4	63.1	59.4	55.7	52.1	48.5	48.5	48.5	24.2
Force to Anch B (lb)	-38.3	40.2	42.2	44.0	45.6	50.5	55.7	61.2	67.1	65.5	64.1	31.4

P_{LA} (lb)	407
P_{LB} (lb)	518
P_{LC} (lb)	529

$$K_{req} = 7.9\,{}^{kip}\!/_{in} < 82.6\,{}^{kip}\!/_{in} \quad \text{OK}$$

Finally the diaphragm deflection is checked. Since the loads and spans are the same for all bays, the bay with the largest I_{xy}/I_x ratio will control. If all other dimensions are equal, this will be the thicker purlin.

$$w_{diaph} = \sum\left[\frac{w_p}{L}\left(\frac{I_{xy}}{I_x}\alpha\cos\theta - \sin\theta\right)\right]_i \tag{4.3.1}$$

$$= \left[\frac{-1438 + 10(2875) + 1438}{25}\left(\frac{4.11}{12.4}\cos(2.39) - \sin(2.39)\right)\right] = 333\text{plf}$$

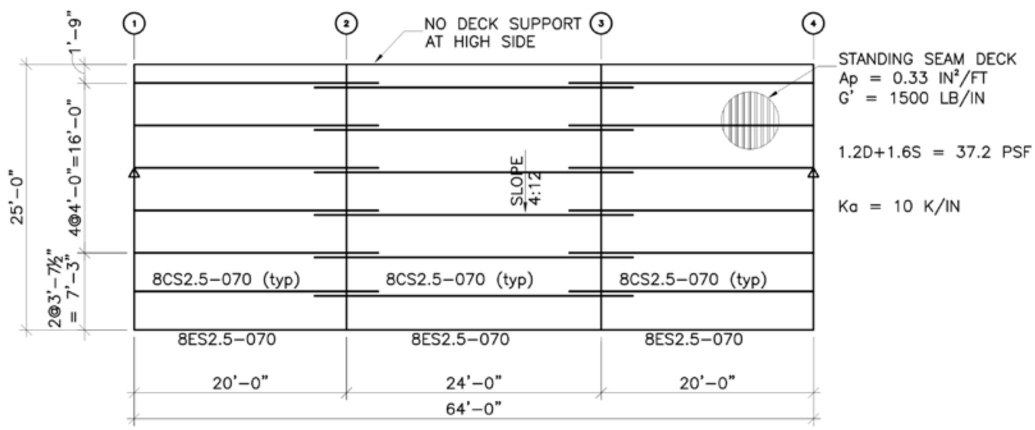
With this value the diaphragm deflection is found from,

$$\Delta_{diaph} = \frac{w_{diaph} L^2_{diaph}}{8G'B} \tag{4.3.2}$$

$$= \frac{(333)(25)^2}{8(1200)(55)} = 0.394\text{in} < L/360 = (25\text{x}12)/360 = 0.83\text{in} \quad \text{OK}$$

4.4.4 Example 4: Three Span Continuous C-Purlins Supporting Standing Seam Panels -- LRFD

Evaluate the interior frame lines of the three span, standing seam roof system shown and determine whether anchorage devices are needed. The system consists of seven parallel lines of C-shaped purlins. The purlins in the end bays are oriented with the flanges facing upslope, while the center bay has flanges facing downslope, except the low eave members, which are simple span and all three bays are oriented with flanges facing upslope. No external anchorage devices are provided at the interior frame lines. The roof sheathing is a standing seam panel with a cross-sectional area of 0.33 in²/ft and the roof slope is 1 in./ft. The purlins are 8C2.5x070. The roof load consists of a uniform dead load (panel and purlin self weight) of 3 psf and roof snow load of 21 psf. Use LRFD.



System Properties

N_p = 7

A_p = 0.33 in²/ft.

θ = arctan(1/12) = 4.76 degrees

q = 1.2D+1.6S = 1.2(3)+1.6(21)=37.2 psf

Purlin Properties from AISI Design Manual Table I-4

d = 8 in.

b = 2.5 in.

t = 0.070 in.

m = 1.09 in.

Coefficients from *Specification* Table D6.3.1-1

C1 = 1.0

C2 = 1.7

C3 = 69

C4 = 0.77

C5 = 1.6

C6 = 0.13

The system stiffness is found from *Specification* Equation D6.3.1-6:

$$K_{sys} = \frac{C5}{1000} \cdot N_p \frac{ELt^2}{d^2} \qquad \text{(Eq. D6.3.1-6)}$$

$$= \frac{1.6}{1000} \cdot (7) \frac{(29500)((20+24)/2)(12)(0.070^2)}{8^2} = 6.68 \,{}^{kip}\!/_{in}$$

The purlin arrangement and tributary widths are as follows where purlins are numbered from the low eave and only the first interior frame line is considered.

Purlin Number	1	2	3	4	5	6	7
Location (ft)	0	3.625	7.25	11.25	15.25	19.25	23.25
α End Bay	1.00	1.00	1.00	1.00	1.00	1.00	1.00
α Interior Bay	1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Tributary Width (ft)	1.8125	3.625	3.8125	4.0	4.0	4.0	3.75

Specification Equation D6.3.1-2 is used to calculate the load introduced into the system at each purlin. All terms except W_{pi} are dependent only on the purlin section and span, not the load or location within the roof plane. By recognizing this, a purlin load ratio, $\gamma = P_i/W_{pi}$, can be calculated for each purlin section.

Purlin Load Ratios:

$$\gamma = C1\left[\left(\frac{C2}{1000} \cdot \frac{I_{xy}L}{I_x d} + C3 \cdot \frac{(m+0.25b)t}{d^2}\right)\alpha \cdot \cos\theta - C4 \cdot \sin\theta\right] \qquad \text{(Eq. D6.3.1-2)}$$

$$\gamma_{(\alpha=1)} = 1.0\left[\left(0 + 69 \cdot \frac{(1.09+0.25(2.5))(0.070)}{8^2}\right)(1)\cos(4.76) - 0.77 \cdot \sin(4.76)\right] = -0.0650$$

$$\gamma_{(\alpha=-1)} = 1.0\left[\left(0 + 69 \cdot \frac{(1.09+0.25(2.5))(0.070)}{8^2}\right)(-1)\cos(4.76) - 0.77 \cdot \sin(4.76)\right] = -0.1929$$

Since the loads will be partially counteracting at the frame lines, the three possible cases for pattern snow loading must be considered.

Load Case 1: Full Snow on End Bay, Half on Interior Bay

Purlin Number	1	2	3	4	5	6	7
W_p Left (lb)	1348.5	2697.0	2836.5	2976.0	2976.0	2976.0	2790.0
P Left (lb)	87.7	175.4	184.5	193.5	193.5	193.5	181.5
W_p Right (lb)	887.4	1774.8	1866.6	1958.4	1958.4	1958.4	1836.0
P Right (lb)	57.7	-342.4	-360.1	-377.8	-377.8	-377.8	-354.2
P (lb)	72.7	-83.5	-87.8	-92.1	-92.1	-92.1	-86.4

K_{req}	1.54
K_{req}/K_{sys}	23%

Load Case 2: Full Snow on Interior Bay, Half on End Bay

Purlin Number	1	2	3	4	5	6	7
W_p Left (lb)	739.5	1479.0	1555.5	1632.0	1632.0	1632.0	1530.0
P Left (lb)	48.1	96.2	101.2	106.1	106.1	106.1	99.5
W_p Right (lb)	1373.2	3236.4	3403.8	3571.2	3571.2	3571.2	3348.0
P Right (lb)	89.3	-624.4	-656.7	-689.0	-689.0	-689.0	-645.9
P (lb)	68.7	-264.1	-277.8	-291.4	-291.4	-291.4	-273.2

K_{req}	5.40
K_{req}/K_{sys}	81%

Load Case 3: Full Snow on Both Bays

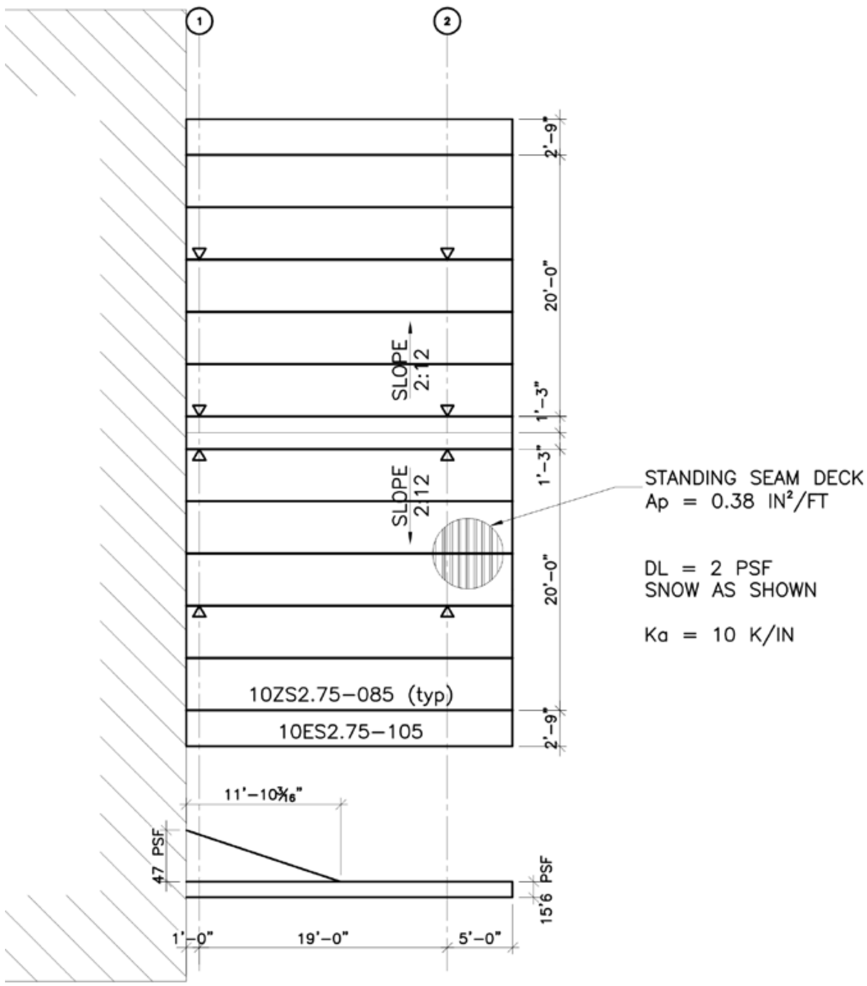
Purlin Number	1	2	3	4	5	6	7
Wp Left (lb)	1348.5	2697.0	2836.5	2976.0	2976.0	2976.0	2790.0
P Left (lb)	87.7	175.4	184.5	193.5	193.5	193.5	181.5
Wp Right (lb)	1373.2	3236.4	3403.8	3571.2	3571.2	3571.2	3348.0
P Right (lb)	89.3	-624.4	-656.7	-689.0	-689.0	-689.0	-645.9
P (lb)	88.5	-224.5	-236.1	-247.7	-247.7	-247.7	-232.2

K_{req}	4.49
K_{req}/K_{sys}	67%

Since K_{sys} is greater than K_{req}, these calculations indicate that the system is adequate without the addition of discrete points of anchorage. It must also be shown that each purlin to rafter connection is capable of resisting the lateral force P_i; in this case the maximum P_i is 294 lb. Also, it should be noted that there remains a significant amount of uncertainty in the behavior of purlin anchorage systems, especially in terms of lateral stiffness and displacements. Therefore this technique of eliminating external anchorage requirements should be used with caution.

4.4.5 Example 5: Cantilever Z-Purlin System - ASD

Evaluate the roof system with the snow drift and end cantilevers shown. The system consists of seven parallel purlin lines on each side of the ridge, but no continuity is provided between the opposing slopes. The purlins have a 19 ft main span with 1 ft left cantilever (frame setback) and a 5 ft right overhang (gable overhang). Anchorage devices are provided at the supports at the 4th and 7th purlin lines. The roof sheathing is standing seam panels with a cross-sectional area of 0.38 in^2/ft and the roof slope is 4 in./ft. The purlins are 10ZS2.75x085 and 105 (eave). The roof loading consists of a uniform dead load of 2 psf, a uniform snow load of 13.6 psf, and a snow drift against the adjacent structure. The snow drift load tapers from 47 psf over a distance of 11 ft – 10 3/16 in. Use ASD



System Properties:

N_p = 7

N_a = 2

K_a = 20 kip/in.

A_p = 0.38 in^2/ft.

θ = arctan(4/12) = 18.43 degrees

Purlin Properties from AISI *Design Manual* Table I-4

d = 10 in.
b = 2.75 in.
t = 0.085 in.
I_x = 21.0 in^4
I_{xy} = 5.20 in^4

Eave Member Properties from AISI *Design Manual* Table I-4

d = 10 in.
b = 2.5 in.
t = 0.105 in.
I_x = 23.3 in^4
I_{xy} = 0.098 in^4
m = 1.56 in.

Coefficients from *Specification* Table D6.3.1-1

C1 = 0.5
C2 = 8.3
C3 = 28
C4 = 0.61
C5 = 0.29
C6 = 0.051

To calculate the system stiffness *Specification* Equation D6.3.1-6 is modified slightly to account for the difference in thickness of the eave strut and the typical purlins. Also, L is taken as the length of the main span without the cantilevers.

$$K_{sys} = \frac{C5}{1000} \cdot N_p \cdot \frac{ELt^2}{d^2} \qquad \text{(Eq. D6.3.1-6)}$$

$$= \frac{0.29(29500)(19x12)}{1000(10^2)} \cdot \left[(6)(0.085)^2 + (1)(0.105)^2\right] = 1.06\text{kip/in}$$

The effective stiffness of each anchorage device, relative to each purlin is calculated from *Specification* Eq. D6.3.1-4. The Equation is evaluated in the table below for each combination of purlin and anchor. The only changing variable is the distance between the purlin and the anchorage device, $d_{p(i,j)}$. Also in this table is the total stiffness supporting each purlin, $K_{total(i)}$.

$$K_{eff_{i,j}} = \left[\frac{1}{K_a} + \frac{d_{p_{i,j}}}{C6 \cdot LA_pE}\right]^{-1} \qquad \text{(Eq. D6.3.1-4)}$$

$$K_{total_i} = \sum_{j=1}^{N_a}\left(K_{eff_{i,j}}\right) + K_{sys} \qquad \text{(Eq. D6.3.1-5)}$$

Purlin Number	1	2	3	4	5	6	7
Location (ft)	0	2.75	6.75	10.75	14.75	18.75	22.75
Dist From Anch A (ft)	11.33	8.43	4.22	0.00	4.22	8.43	12.65
Dist From Anch B (ft)	23.98	21.08	16.87	12.65	8.43	4.22	0.00

$K_{eff,A}$ (kip/in)	16.0	16.9	18.3	20.0	18.3	16.9	15.6
$K_{eff,B}$ (kip/in)	13.1	13.6	14.6	15.6	16.9	18.3	20.0
Sum (kip/in)	29.1	30.5	32.9	35.6	35.2	35.2	35.6
K_{total}=Sum+K_{sys} (kip/in)	30.1	31.6	33.9	36.7	36.2	36.2	36.7

With these values the stiffness ratio in *Specification* Equation D6.3.1-1 is evaluated to find a distribution factor representing the portion of the load introduced at each purlin that is distributed to each anchorage device. The sum of the distribution factors at each purlin is less than unity due to the portion of the load that is resisted by the system effect.

Purlin Number	1	2	3	4	5	6	7
DF A	0.531	0.534	0.539	0.545	0.505	0.466	0.426
DF B	0.434	0.432	0.429	0.426	0.466	0.505	0.545

To address the effects of the snow drift and the cantilever, the load on each frame is calculated taking into account the snow drift and the adjacent cantilever, but neglecting the other cantilever.

By taking moments about frame line 2, the load on frame line 1 is:

$$R_1 = \frac{0.5(47)(11.85)\left(20 - {}^{11.85}\!/_{3}\right) + 15.6(20)(10)}{19} = 399.4\text{plf}$$

For the load on frame line 2 the snow drift is truncated at frame line 1.

$$q'_{drift} = 47\frac{10.85}{11.85} = 43.03\text{psf}$$

Then taking moments about frame line 1 yields the load on frame line 2 of:

$$R_2 = \frac{0.5(43.03)(10.85)\left({}^{10.85}\!/_{3}\right) + 15.6(24)(12)}{19} = 280.9\text{plf}$$

Specification Equation D6.3.1-2 is used to calculate the load introduced into the system at each purlin. All terms except W_{pi} are dependent only on the purlin section and span, not the load or location within the roof plane. By recognizing this, a purlin load ratio, $\gamma = P_i/W_{pi}$, can be calculated for each purlin section.

Purlin Load Ratios:

$$\gamma = C1\left[\left(\frac{C2}{1000}\cdot\frac{I_{xy}L}{I_x d} + C3\cdot\frac{(m+0.25b)t}{d^2}\right)\alpha\cdot\cos\theta - C4\cdot\sin\theta\right] \qquad \text{(From Eq. D6.3.1-2)}$$

$$\gamma_p = 0.5\left[\left(\frac{8.3}{1000}\cdot\frac{(5.2)(19x12)}{(21)(10)} + 28\cdot\frac{0.25(2.75)(0.085)}{10^2}\right)1\cdot\cos(18.43) - 0.61\cdot\sin(18.43)\right] = -0.0665$$

$$\gamma_e = 0.5\left[\left(\frac{8.3}{1000}\cdot\frac{(0.098)(19x12)}{(23.3)(10)} + 28\cdot\frac{(1.56+0.25(2.5))(0.105)}{10^2}\right)1\cdot\cos(18.43) - 0.61\cdot\sin(18.43)\right] = -0.0656$$

These load ratios are then multiplied by the force W_p, which due to the cantilevers and non-uniform load, is taken as two times the frame loads above times the tributary width at each purlin. Then the forces P_i are multiplied by the distribution factors found above to find the force distributed to each anchorage device.

For anchorage along frame line 1:

Purlin Number	1	2	3	4	5	6	7
Tributary Width (ft)	1.375	3.375	4	4	4	4	3.25
Load Factor	-0.0656	-0.0665	-0.0665	-0.0665	-0.0665	-0.0665	-0.0665

W_{p1} (lb)	1098.5	2696.3	3195.6	3195.6	3195.6	3195.6	2596.4
P_1 (lb)	-72.1	-179.2	-212.4	-212.4	-212.4	-212.4	-172.6

Load to Anch A (lb)	-38.3	-95.7	-114.5	-115.8	-107.3	-98.9	-73.5
Load to Anch B (lb)	-31.3	-77.5	-91.2	-90.5	-98.9	-107.3	-94.1
Load to System Effect (lb)	-2.4	-5.6	-6.2	-5.7	-5.8	-5.8	-4.6

For anchorage along frame line 2:

Purlin Number	1	2	3	4	5	6	7
W_{p2} (lb)	772.5	1896.1	2247.2	2247.2	2247.2	2247.2	1825.9
P_2 (lb)	-50.7	-126.0	-149.4	-149.4	-149.4	-149.4	-121.3

Load to Anch A (lb)	-26.9	-67.3	-80.5	-81.4	-75.5	-69.5	-51.7
Load to Anch B (lb)	-22.0	-54.5	-64.1	-63.6	-69.5	-75.5	-66.1
Load to System Effect (lb)	-1.7	-3.9	-4.3	-4.0	-4.1	-4.1	-3.3

The summation in *Specification* Equation D6.3.1-1 is carried out by taking the sum of the values in the rows in the tables above. This yields the anchorage force at each purlin. The portion of the force carried by the system effect is also calculated in a similar fashion.

	FL 1	FL 2
P_{LA} (lb)	-644	-453
P_{LB} (lb)	-591	-415
P_L System (lb)	-36	-25
Sum	-1271	-894

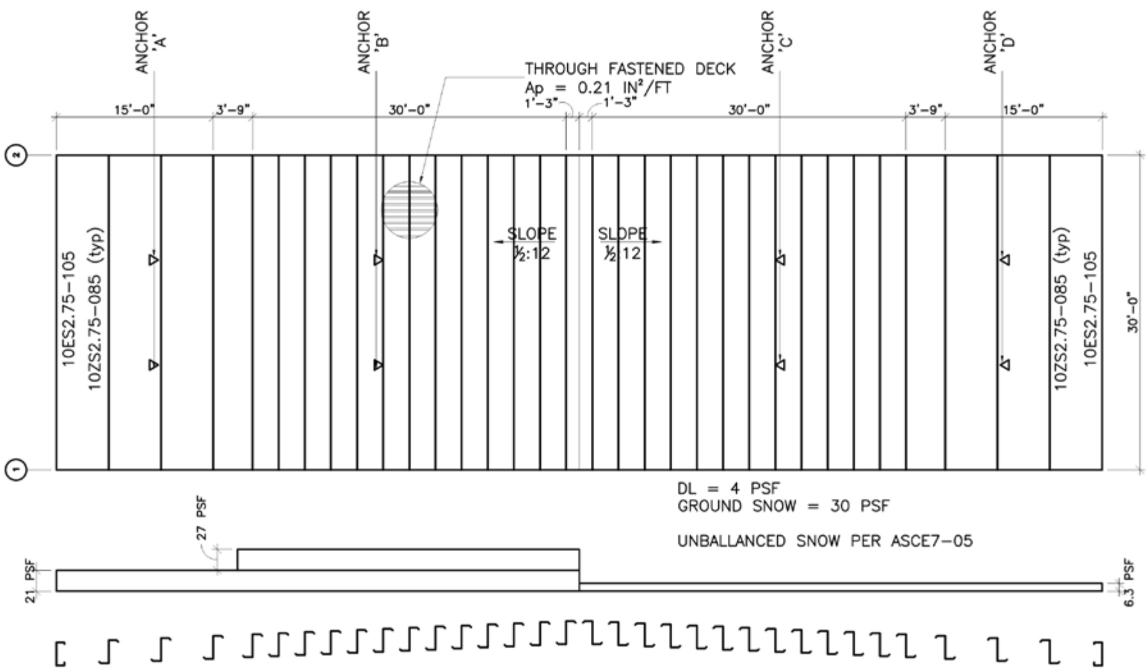
The "Sum" values in the table above are the total of all the anchorage forces along each frame line, and are equivalent to the sum of the P_i values. Therefore these values are used in *Specification* D6.3.1-8a for the calculation of the required minimum effective stiffness.

$$K_{req} = \Omega\cdot\frac{20\cdot\left|\sum_{i=1}^{N_p} P_i\right|}{d} \qquad \text{(Eq. D3.2.1-8a)}$$

$$= 2.0\frac{20(1271)}{10} = 5.09\,{}^{kip}\!/_{in} < K_{total(1)} = 30.1\,{}^{kip}\!/_{in} \quad \text{OK}$$

4.4.6 Example 6: Single Span Z-Purlin Attached to Through-Fastened Panels with One-Third Points Anchorage - ASD

The system shown consists of 17 parallel purlin lines on each side of the ridge, and continuity is provided between the opposing slopes. Anchorage devices are provided at the one-third points at the 3rd and 10th purlin lines on each side of the ridge and have a lateral stiffness of 10 kip/in. The roof sheathing is a through-fastened panel with a cross-sectional area of 0.21 in²/ft and the roof slope is ½ in./ft. The purlins are 10ZS2.5x085 and the strut purlins are 10ZS2.5x105. Evaluate the anchorage system for a dead load of 4 psf and the effects of a 30 psf ground snow load.



System Properties

$N_{p1} = N_{p2} = 17$

$N_a = 4$

$K_a = 10$ kip/in.

$A_p = 0.21$ in²/ft.

$\theta = \arctan(0.5/12) = 2.39$ degrees

Purlin Properties from *Design Manual* Table I-4

d = 10 in.

b = 2.75 in.

t = 0.085 in.

$I_x = 21.0$ in⁴

$I_{xy} = 5.20$ in⁴

Eave Strut Properties from *Design Manual* Table I-4

d = 10 in.
b = 2.5 in.
t = 0.105 in.
I_x = 23.3 in^4
I_{xy} = 0.098 in^4
m = 1.56 in.

Coefficients from *Specification* Table D6.3.1-3
C1 = 0.5
C2 = 7.8
C3 = 42
C4 = 0.98
C5 = 0.39
C6 = 0.40

To calculate the system stiffness *Specification* Equation D6.3.1-6 is modified to account for the difference in thickness of the eave strut and the typical purlins and the reduced effectiveness of elements on the far side of the ridge.

$$K_{sys} = \frac{C5}{1000} \cdot \left[\left(N_{p1} - 1\right)\left(\frac{ELt^2}{d^2}\right) + \left(\frac{ELt_e^2}{d^2}\right) \right] \cdot \left[1 + (\cos(2\theta))^2\right] \tag{4.4.1}$$

$$= \frac{0.39}{1000} \cdot \left[\left(\frac{(29500)(30\times 12)}{10^2}\right)\left((17-1)0.085^2 + 0.105^2\right) \right] \cdot \left[1 + \left(\cos\left(2(2.39)\right)\right)^2\right] = 10.45 \, \text{kip}/\text{in}$$

The effective stiffness of each anchorage device, relative to each purlin is calculated from *Specification* Equation D6.3.1-5. The Equation is evaluated in the table below for each combination of purlin and anchor with the only changing variable being the distance between the purlin and the anchorage device, $d_{p(i,j)}$. Also in this table is the total stiffness supporting each purlin, $K_{total(i)}$.

If purlin "i" and anchor "j" are on the same side of the ridge then:

$$K_{eff_{i,j}} = \left[\frac{1}{K_a} + \frac{d_{p_{i,j}}}{C6 \cdot LA_p E} \right]^{-1} \tag{Eq. D6.3.1-4}$$

If purlin "i" and anchor "j" are on the opposite sides of the ridge then:

$$K_{eff_{i,j}} = \left[\left(\frac{1}{K_a} + \frac{d_{p_{o,j}}}{C6 \cdot LA_p E} \right) \frac{1}{\cos^2(2\theta)} + \frac{d_{p_{i,o}}}{C6 \cdot LA_p E} \right]^{-1} \tag{4.4.2}$$

Where $d_{po,j}$ is the distance from the ridge to the anchor, and $d_{pi,o}$ is the distance from the purlin to the ridge.

$$K_{total_i} = \sum_{j=1}^{N_a} \left(K_{eff_{i,j}}\right) + K_{sys} \tag{Eq. D6.3.1-5}$$

Purlin Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Location (ft)	0	5	10	15	18.75	21.25	23.75	26.25	28.75	31.25	33.75	36.25	38.75	41.25	43.75	46.25	48.75
Dist From Anch A (ft)	10.01	5.00	0.00	5.00	8.76	11.26	13.76	16.26	18.77	21.27	23.77	26.27	28.77	31.28	33.78	36.28	38.78
Dist From Anch B (ft)	31.28	26.27	21.27	16.26	12.51	10.01	7.51	5.00	2.50	0.00	2.50	5.00	7.51	10.01	12.51	15.01	17.52
Dist From Anch C (ft	68.81	63.81	58.80	53.80	50.04	47.54	45.04	42.54	40.03	37.53	35.03	32.53	30.03	27.52	25.02	22.52	20.02
Dist From Anch D (ft)	90.08	85.07	80.07	75.07	71.31	68.81	66.31	63.81	61.30	58.80	56.30	53.80	51.29	48.79	46.29	43.79	41.29
Alpha	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$K_{eff,A}$ (kip/in)	9.81	9.90	10.00	9.90	9.83	9.79	9.74	9.70	9.65	9.60	9.56	9.52	9.47	9.43	9.39	9.34	9.30
$K_{eff,B}$ (kip/in)	9.43	9.52	9.60	9.70	9.76	9.81	9.86	9.90	9.95	10.00	9.95	9.90	9.86	9.81	9.76	9.72	9.67
$K_{eff,C}$ (kip/in)	8.77	8.84	8.92	9.00	9.06	9.10	9.14	9.18	9.22	9.26	9.30	9.34	9.39	9.43	9.47	9.52	9.56
$K_{eff,D}$ (kip/in)	8.46	8.53	8.60	8.67	8.73	8.77	8.80	8.84	8.88	8.92	8.96	9.00	9.03	9.07	9.11	9.16	9.20
Sum (kip/in)	36.5	36.8	37.1	37.3	37.4	37.5	37.5	37.6	37.7	37.8	37.8	37.8	37.7	37.7	37.7	37.7	37.7
K_{total}=Sum+K_{sys} (kip/in)	45.2	45.5	45.8	46.0	46.1	46.2	46.2	46.3	46.4	46.5	46.5	46.5	46.5	46.5	46.4	46.4	46.4

Purlin Number	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Location (ft)	51.25	53.75	56.25	58.75	61.25	63.75	66.25	68.75	71.25	73.75	76.25	78.75	81.25	85	90	95	100
Dist From Anch A (ft)	41.29	43.79	46.29	48.79	51.29	53.80	56.30	58.80	61.30	63.81	66.31	68.81	71.31	75.07	80.07	85.07	90.08
Dist From Anch B (ft)	20.02	22.52	25.02	27.52	30.03	32.53	35.03	37.53	40.03	42.54	45.04	47.54	50.04	53.80	58.80	63.81	68.81
Dist From Anch C (ft	17.52	15.01	12.51	10.01	7.51	5.00	2.50	0.00	2.50	5.00	7.51	10.01	12.51	16.26	21.27	26.27	31.28
Dist From Anch D (ft)	38.78	36.28	33.78	31.28	28.77	26.27	23.77	21.27	18.77	16.26	13.76	11.26	8.76	5.00	0.00	5.00	10.01
Alpha	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
$K_{eff,A}$ (kip/in)	9.20	9.16	9.11	9.07	9.03	9.00	8.96	8.92	8.88	8.84	8.80	8.77	8.73	8.67	8.60	8.53	8.46
$K_{eff,B}$ (kip/in)	9.56	9.52	9.47	9.43	9.39	9.34	9.30	9.26	9.22	9.18	9.14	9.10	9.06	9.00	8.92	8.84	8.77
$K_{eff,C}$ (kip/in)	9.67	9.72	9.76	9.81	9.86	9.90	9.95	10.00	9.95	9.90	9.86	9.81	9.76	9.70	9.60	9.52	9.43
$K_{eff,D}$ (kip/in)	9.30	9.34	9.39	9.43	9.47	9.52	9.56	9.60	9.65	9.70	9.74	9.79	9.83	9.90	10.00	9.90	9.81
Sum (kip/in)	37.7	37.7	37.7	37.7	37.7	37.8	37.8	37.8	37.7	37.6	37.5	37.5	37.4	37.3	37.1	36.8	36.5
K_{total}=Sum+K_{sys} (kip/in)	46.4	46.4	46.4	46.5	46.5	46.5	46.5	46.5	46.4	46.3	46.2	46.2	46.1	46.0	45.8	45.5	45.2

With these values the stiffness ratio in *Specification* D6.3.1-1 is evaluated to find a distribution factor representing the portion of the load introduced at each purlin that is distributed to each anchorage device.

Purlin Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
DF A	0.22	0.22	0.22	0.22	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.20	0.20	0.20	0.20	0.20	0.20
DF B	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.22	0.21	0.21	0.21	0.21	0.21	0.21	0.21
DF C	0.19	0.19	0.19	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.21
DF D	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.20	0.20	0.20	0.20
DF Sys	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19

Purlin Number	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
DF A	0.20	0.20	0.20	0.20	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19
DF B	0.21	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.19	0.19	0.19
DF C	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.22	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
DF D	0.20	0.20	0.20	0.20	0.20	0.20	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.22	0.22	0.22	0.22
DF Sys	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19

Specification Equation D6.3.1-2 is used to calculate the load introduced into the system at each purlin. All terms except W_{pi} are dependent only on the purlin section and span, not the load or location within the roof plane. By recognizing this, a purlin load ratio, $\gamma = P_i/W_{pi}$, can be calculated for each purlin section.

Purlin Load Ratios:

$$\gamma = C1\left[\left(\frac{C2}{1000}\cdot\frac{I_{xy}L}{I_x d} + C3\cdot\frac{(m+0.25b)t}{d^2}\right)\alpha\cdot\cos\theta - C4\cdot\sin\theta\right] \quad \text{(D6.3.1-2)}$$

$$\gamma_p = 0.5\left[\left(\frac{7.8}{1000}\cdot\frac{(5.2)(19x12)}{(21)(10)} + 42\cdot\frac{0.25(2.75)(0.085)}{10^2}\right)1\cdot\cos(2.39) - 0.98\cdot\sin(2.39)\right] = 0.0208$$

$$\gamma_e = 0.5\left[\left(\frac{7.8}{1000}\cdot\frac{(0.098)(19x12)}{(23.3)(10)} + 42\cdot\frac{(1.56+0.25(2.5))(0.105)}{10^2}\right)1\cdot\cos(2.39) - 0.98\cdot\sin(2.39)\right] = 0.0282$$

These load factors are then multiplied by the force W_p and the direction factor α. Then the force P is multiplied by the distribution factors found above to find the force distributed to each anchorage device.

Purlin Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Tributary Width (ft)	2.5	5	5	4.375	3.125	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
Load Factor	0.0282	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208
W_{p1} (lb)	1312.5	2625.0	2625.0	2437.5	3187.5	3000.0	3000.0	3000.0	3000.0	3000.0	3000.0	3000.0	3000.0	3000.0	3000.0	3000.0	3000.0
P_1 (lb)	37.1	54.6	54.6	50.7	66.3	62.4	62.4	62.4	62.4	62.4	62.4	62.4	62.4	62.4	62.4	62.4	62.4
Load to Anch A (lb)	8.0	11.9	11.9	10.9	14.1	13.2	13.1	13.1	13.0	12.9	12.8	12.8	12.7	12.7	12.6	12.6	12.5
Load to Anch B (lb)	7.7	11.4	11.4	10.7	14.0	13.3	13.3	13.3	13.4	13.4	13.4	13.3	13.2	13.2	13.1	13.1	13.0
Load to Anch C (lb)	7.2	10.6	10.6	9.9	13.0	12.3	12.3	12.4	12.4	12.4	12.5	12.6	12.6	12.7	12.7	12.8	12.9
Load to Anch D (lb)	6.9	10.2	10.3	9.6	12.6	11.9	11.9	11.9	11.9	12.0	12.0	12.1	12.1	12.2	12.2	12.3	12.4
Load to System Effect (lb)	7.1	10.5	10.4	9.6	12.5	11.8	11.8	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7
Purlin Number	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Tributary Width (ft)	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	3.125	4.375	5	5	2.5
Load Factor	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0208	0.0282
W_{p1} (lb)	393.8	393.8	393.8	393.8	393.8	393.8	393.8	393.8	393.8	393.8	393.8	393.8	492.2	689.1	787.5	787.5	393.8
P_1 (lb)	-8.2	-8.2	-8.2	-8.2	-8.2	-8.2	-8.2	-8.2	-8.2	-8.2	-8.2	-8.2	-10.2	-14.3	-16.4	-16.4	-11.1
Load to Anch A (lb)	-1.6	-1.6	-1.6	-1.6	-1.6	-1.6	-1.6	-1.6	-1.6	-1.6	-1.6	-1.6	-1.9	-2.7	-3.1	-3.1	-2.1
Load to Anch B (lb)	-1.7	-1.7	-1.7	-1.7	-1.7	-1.6	-1.6	-1.6	-1.6	-1.6	-1.6	-1.6	-2.0	-2.8	-3.2	-3.2	-2.2
Load to Anch C (lb)	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7	-1.8	-1.8	-1.8	-1.8	-1.7	-1.7	-2.2	-3.0	-3.4	-3.4	-2.3
Load to Anch D (lb)	-1.6	-1.6	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7	-2.2	-3.1	-3.6	-3.6	-2.4
Load to System Effect (lb)	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.9	-2.7	-3.1	-3.1	-2.1

The summation in *Specification* Equation D6.3.1-1 is carried out by taking the sum of the values in the rows in the tables above. This yields the anchorage force at each purlin. Also, the portion of the force carried by the system effect is calculated in a similar fashion.

P_{LA} (lb)	179
P_{LB} (lb)	181
P_{LC} (lb)	167
P_{LD} (lb)	159
P_L System (lb)	159
Sum	846

The resulting forces indicate that the entire roof system translates in the upwind direction (to the right in the figure above). With anchorage points at one-third points it is common that the anchorage system can only resist forces in one direction. If this is the case, then the anchorage points on the windward side of the roof (C&D) will be ineffective. Therefore the system would need to be reanalyzed neglecting the ineffective anchors. For this analysis, most of the values above are still applicable. The total stiffness is computed neglecting the two ineffective anchors, and different distribution factors are calculated and applied to the purlin forces.

Purlin Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$K_{eff,A}$ (kip/in)	9.8	9.9	10.0	9.9	9.8	9.8	9.7	9.7	9.6	9.6	9.6	9.5	9.5	9.4	9.4	9.3	9.3
$K_{eff,B}$ (kip/in)	9.4	9.5	9.6	9.7	9.8	9.8	9.9	9.9	10.0	10.0	10.0	9.9	9.9	9.8	9.8	9.7	9.7
Sum (kip/in)	19.2	19.4	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.5	19.4	19.3	19.2	19.1	19.1	19.0
K_{total}=Sum+K_{sys} (kip/in)	27.9	28.1	28.3	28.3	28.3	28.3	28.3	28.3	28.3	28.3	28.2	28.1	28.0	27.9	27.9	27.8	27.7
DF A	0.351	0.352	0.353	0.350	0.347	0.346	0.344	0.342	0.341	0.339	0.339	0.338	0.338	0.337	0.337	0.336	0.336
DF B	0.337	0.338	0.339	0.342	0.345	0.347	0.348	0.350	0.352	0.353	0.353	0.352	0.352	0.351	0.350	0.350	0.349
DF Sys	0.312	0.310	0.308	0.308	0.308	0.308	0.308	0.308	0.308	0.308	0.309	0.310	0.311	0.312	0.313	0.314	0.315
Load to Anch A (lb)	13.0	19.2	19.3	17.7	23.0	21.6	21.5	21.4	21.3	21.2	21.1	21.1	21.1	21.1	21.0	21.0	21.0
Load to Anch B (lb)	12.5	18.5	18.5	17.4	22.9	21.6	21.7	21.8	21.9	22.0	22.0	22.0	21.9	21.9	21.9	21.8	21.8
Load to System Effect (lb)	11.5	16.9	16.8	15.6	20.4	19.2	19.2	19.2	19.2	19.2	19.3	19.3	19.4	19.5	19.5	19.6	19.6

Purlin Number	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
$K_{eff,A}$ (kip/in)	9.2	9.2	9.1	9.1	9.0	9.0	9.0	8.9	8.9	8.8	8.8	8.8	8.7	8.7	8.6	8.5	8.5
$K_{eff,B}$ (kip/in)	9.6	9.5	9.5	9.4	9.4	9.3	9.3	9.3	9.2	9.2	9.1	9.1	9.1	9.0	8.9	8.8	8.8
Sum (kip/in)	18.8	18.7	18.6	18.5	18.4	18.3	18.3	18.2	18.1	18.0	17.9	17.9	17.8	17.7	17.5	17.4	17.2
K_{total}=Sum+K_{sys} (kip/in)	27.5	27.4	27.3	27.2	27.1	27.0	27.0	26.9	26.8	26.7	26.7	26.6	26.5	26.4	26.2	26.1	25.9
DF A	0.335	0.334	0.334	0.333	0.333	0.333	0.332	0.332	0.331	0.331	0.330	0.330	0.329	0.329	0.328	0.327	0.326
DF B	0.348	0.348	0.347	0.346	0.346	0.345	0.345	0.344	0.344	0.343	0.343	0.342	0.342	0.341	0.340	0.339	0.338
DF Sys	0.317	0.318	0.319	0.320	0.321	0.322	0.323	0.324	0.325	0.326	0.327	0.328	0.329	0.330	0.332	0.334	0.336
Load to Anch A (lb)	-2.7	-2.7	-2.7	-2.7	-2.7	-2.7	-2.7	-2.7	-2.7	-2.7	-2.7	-2.7	-3.4	-4.7	-5.4	-5.4	-3.6
Load to Anch B (lb)	-2.9	-2.8	-2.8	-2.8	-2.8	-2.8	-2.8	-2.8	-2.8	-2.8	-2.8	-2.8	-3.5	-4.9	-5.6	-5.6	-3.8
Load to System Effect (lb)	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.7	-2.7	-2.7	-2.7	-2.7	-3.4	-4.7	-5.4	-5.5	-3.7

P_{LA} (lb)	292
P_{LB} (lb)	295
P_L System (lb)	259
Sum	846

The simplified model that forms the basis for the minimum stiffness requirement in *Specification* Equation D6.3.1-8 does not correctly represent a system where large groups of purlins have opposing flanges. This can be seen in the special case of a flat roof where all purlins top flanges are oriented toward the centerline. *Specification* Equation D6.3.1-8 would yield a required stiffness of zero (also implying zero lateral displacement) since the forces P_i cancel out.

For this example the stiffness requirement can be conservatively evaluated by taking only the leeward side of the roof alone.

$$K_{req} = \Omega \cdot \frac{20\left|\sum_{i=1}^{N_p} P_i\right|}{d} \qquad \text{(Eq. D3.2.1-8a)}$$

$$= 2.0\frac{20(1.013)}{10} = 4.05\,{}^{kip}\!/_{in} < K_{total(17)} = 25.9\,{}^{kip}\!/_{in} \quad \text{OK}$$

4.5 Alternate Solution Procedures

4.5.1 Simplified Procedure

The solution procedure in the *Specification* can be conservatively simplified if the roof system under consideration has nominally uniform purlin spaces, uniform loads, approximately uniformly distributed anchorage devices, and the majority of the purlin top flanges facing upslope. For this simplification the total load supported by all the purlins within a bay is found.

$$W_s = qLB \qquad (4.5.1)$$

The approximate and conservative anchorage force, $P_{L\text{-}s}$, is then found from a modified form of *Specification* Equation D3.6.2-2:

$$P_{L-s} = C1 \cdot \left[\left(\frac{C2}{1000} \cdot \frac{I_{xy}L}{I_x d} + C3 \cdot \frac{(m+0.25b)t}{d^2}\right)\cos\theta - C4 \cdot \sin\theta\right]\frac{W_s}{N_a} \qquad (4.5.2)$$

Or the equation may be simplified even further and more conservatively to:

$$P_{L-s} = C1 \cdot \left[\left(\frac{I_{xy}}{2I_x} + \frac{(m+0.25b)}{d}\right)\cos\theta\right]\frac{W_s}{N_a} > C1 \cdot (\sin\theta)\frac{W_s}{N_a} \qquad (4.5.3)$$

For the stiffness requirement, one may either compare the estimated purlin deflection to the allowable, or the minimum required anchor stiffness to the actual stiffness. The purlin deflection is conservatively estimated from:

$$\Delta_S = P_{L-s}\left[\frac{1}{K_a} + \frac{(N_p - N_a)S}{C6 \cdot LA_pE}\right] \qquad (4.5.4)$$

The resulting estimated deflection is then compared to the allowable deflection:

$$\Delta_a = \frac{d}{\Omega \cdot 20} \text{ or } \phi\frac{d}{20} \qquad \text{(Eq. D6.3.1-9)}$$

Alternatively the required anchor stiffness, K_a, can be found from one of the following:

$$K_{a\_req} = \frac{20 \cdot C6 \cdot LA_p EP_{L-s}}{\frac{1}{\Omega} C6 \cdot LA_p Ed - 20P_{L-s} S(N_p - N_a)}, \text{ for ASD} \tag{4.5.5a}$$

$$K_{a\_req} = \frac{20 \cdot C6 \cdot LA_p EP_{L-s}}{\phi C6 \cdot LA_p Ed - 20P_{L-s} S(N_p - N_a)}, \text{ for LRFD or LSD} \tag{4.5.5b}$$

Unlike *Specification* Equation D6.3.1-8, this yields the required anchor stiffness directly, not the required total effective stiffness. The solution of this equation can result in negative values, meaning that the stiffness criteria, as evaluated with the simplified procedure, can not be satisfied with the current number of anchorage devices. It is noted that this stiffness evaluation can be very conservative, especially for large roof systems.

4.5.2 Matrix-Based Solution

The design procedure presented in the *Specification* utilizes a relative stiffness technique to distribute anchorage forces. To develop the manual procedure, the stiffness analysis was simplified slightly and presented in a revised, single-degree-of-freedom format. The same underlying stiffness model can also be solved using matrix methods. This allows for the direct calculation of the displacements and potentially a better evaluation of the minimum stiffness requirements.

To formulate the stiffness model, the forces P_i are applied to nodes representing each purlin. Linear springs connect each adjacent node and model the axial behavior of the roof sheathing. The stiffness of these springs is related to K_{eff} and is found from,

$$K_{D_k} = \frac{C6 \cdot LA_p E}{S_k} \tag{4.2.2}$$

where k varies from one to the number of purlin spaces and S_k is the panel span between purlin k and k+1. To simplify the calculations in the manual procedure, the stiffness of all the purlins in the absence of the anchorage devices was collected into a single term K_{sys}. For the matrix solution K_{sys} is found for each purlin individually and applied as a spring support at the corresponding node. The stiffness of the spring is found by removing the number of purlins, N_p, from *Specification* Equation D6.3.1-6, yielding

$$K^*_{sys} = \frac{C5}{1000} \cdot \frac{ELt^2}{d^2} \tag{4.2.1}$$

The resulting one-dimensional model can be solved with a relatively simple system of equations as shown in Example 7.

4.5.3 Example 7: Example 3 Using the Simplified Method and Matrix-Based Solution

Re-solve the frame line 3 case from Example 3 using the simplified method and the matrix based solution. Note the diaphragm deflection check is the same as that shown in Example 3.

Simplified Method

$$W_s = qLB \tag{4.5.1}$$

$$W_s = (23\text{psf})(55\text{ft})(25\text{ft}) = 31.625\text{kip}$$

$$P_{L-s} = C1 \cdot \left[\left(\frac{C2}{1000} \cdot \frac{I_{xy}L}{I_x d} + C3 \cdot \frac{(m + 0.25b)t}{d^2} \right) \cos\theta - C4 \cdot \sin\theta \right] \frac{W_s}{N_a} \tag{4.5.2}$$

$$P_{L-s} = 1.0 \cdot \left[\left(\frac{4.3}{1000} \frac{(2.85)(25x12)}{(8.69)(8)} + 55 \cdot \frac{(0 + 0.25(2.75))(0.059)}{8^2} \right) \cos(2.39) - 0.71 \sin(2.39) \right] \frac{31.625}{3} = 612\text{lb}$$

$$S = 5\text{ft}$$

$$\Delta_S = P_{L-s} \left[\frac{1}{K_a} + \frac{(N_p - N_a)S}{C6 \cdot LA_p E} \right] \tag{4.5.4}$$

$$= 0.612 \left[\frac{1}{40} + \frac{(12 - 3)(5x12)}{0.17(25x12)(0.20/12)(29500)} \right] = 0.028\text{in}$$

$$\Delta_a = \frac{d}{\Omega \cdot 20} \tag{Eq. D6.3.1-9a}$$

$$= \frac{8}{2(20)} = 0.2\text{in}$$

$$K_{a\_req} = \frac{20 \cdot C6 \cdot LA_p E P_{L-s}}{\frac{1}{\Omega} C6 \cdot LA_p Ed - 20 P_{L-s} S (N_p - N_a)} \tag{4.5.5a}$$

$$= \frac{20(0.17)(25x12)(0.2/12)(29500)(0.612)}{\frac{1}{2.0}(0.17)(25x12)(0.2/12)(29500)(8) - 20(0.612)(5x12)(12-3)} = 3.28 \, {}^{\text{kip}}\!/_{\text{in}}$$

Matrix Solution

$$K_{Dk} = \frac{C6 \cdot LA_p E}{S_k} \tag{4.2.2}$$

$$K_{Dk} = \frac{0.17(25)(0.20)(29500)}{(5x12)/\cos(2.39)} = 417.6 \, {}^{\text{kip}}\!/_{\text{in}}$$

$$K^*_{sys} = \frac{C5}{1000} \cdot \frac{ELt^2}{d^2} \tag{4.2.1}$$

$$K^*_{sys} = \frac{1.4}{1000} \cdot \frac{(29500)(25x12)(0.059^2)}{8^2} = 0.674 \, {}^{\text{kip}}\!/_{\text{in}}$$

Stiffness Matrix Representing System with No Anchorage Devices:

$$K_1 = \begin{bmatrix} 418 & -418 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -418 & 835 & -418 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -418 & 835 & -418 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -418 & 835 & -418 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -418 & 835 & -418 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -418 & 835 & -418 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -418 & 835 & -418 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -418 & 835 & -418 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -418 & 835 & -418 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -418 & 835 & -418 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -418 & 835 & -418 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -418 & 418 \end{bmatrix}$$

Stiffness Matrix Representing System Effect:

$$K_2 = \begin{bmatrix} 0.674 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.674 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.674 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.674 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.674 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.674 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.674 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.674 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.674 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.674 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.674 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.674 \end{bmatrix}$$

Stiffness Matrix Representing Anchorage Devices:

$$K_3 = \begin{bmatrix} 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Complete Stiffness Matrix, $K=K_1+K_2+K_3$:

$$K = \begin{bmatrix} 458 & -418 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -418 & 836 & -418 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -418 & 836 & -418 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -418 & 836 & -418 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -418 & 876 & -418 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -418 & 836 & -418 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -418 & 836 & -418 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -418 & 836 & -418 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -418 & 876 & -418 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -418 & 836 & -418 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -418 & 836 & -418 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -418 & 418 \end{bmatrix}$$

Applied Force Vector:

P = {-169 167 167 167 167 167 167 167 167 167 167 74}

Solve for Nodal Displacements:

$\Delta = K^{-1}P$ = {9.6 11.0 11.9 12.5 12.7 13.7 14.4 14.7 14.6 15.5 16.0 16.2} x 10^{-3} in. (4.5.5)

Solve for Anchorage Forces:

$P_L = K_3\Delta$ = {385 0 0 0 508 0 0 0 583 0 0 0} (4.5.6)

Comparison of Methods

The anchorage force from the three solution methods are presented below. It can be seen that the main *Specification* method and the matrix solution method provide similar results. As discussed above the Simplified Procedure provides conservative forces as long as the anchorage devices are evenly distributed along the roof slope.

Method	P_{LA}, lb	P_{LB}, lb	P_{LC}, lb
Main *Specification*	407	518	529
Simplified Procedure	612	612	612
Matrix Based Method	385	508	583

4.5.4 Component Stiffness Method

The component stiffness method is an alternative approach to the provisions set forth in the *Specification*. It is based upon the same principles as the provisions. The component stiffness method takes a more in depth approach to determine the forces generated by the system of purlins and the lateral stiffness of the system, but then uses a more simplified approach to the distribution of the forces between anchorage devices.

The roof system is therefore divided into three types of "components": the external restraint, the connection between the purlin and rafter, and the connection between the sheathing and purlin. The roof system is treated as a single degree of freedom system and the contributions of each component are related by stiffness. The stiffness of each of the components is defined as the force or moment developed in the component per unit lateral displacement of the top flange at the restraint.

There are three main steps to determining the restraint forces in a purlin supported roof system using the component stiffness method. First it is necessary to determine the forces generated in the roof system. The lateral forces generated are primarily a function of the geometry of the system, but the purlin also interacts with the sheathing to contribute to the lateral forces that must be restrained. Using a displacement compatibility approach and assuming rigid restraints, the additional contribution of the sheathing/purlin interaction to the lateral force in the system is determined. These sheathing/purlin interaction forces are dependent upon the restraint configuration.

Similar to the provisions of the *Specification*, once the forces generated by the system have been determined, the stiffness of the system is determined and forces are distributed through the system according to the stiffness of each of the components. Each purlin in the bay contributes a rafter stiffness and a sheathing stiffness to the overall stiffness of the system. The rafter stiffness is defined as the moment developed in the connection between the purlin and the rafter per unit displacement of the top flange of the Z- or C- section at the rafter. The sheathing stiffness is defined as the total moment generated between the purlin and sheathing along the span of the purlin per unit lateral displacement of the top flange. The final component contributing to the stiffness of the system is the restraint. The restraint stiffness is the force resisted at the top of the purlin at the restraint per unit displacement of the top flange of the purlin at the restraint. The total stiffness of the bay of purlins then is the sum of the stiffness of each of the components: the sum of the sheathing and rafter stiffness for each purlin and the sum of each restraint applied in the bay. Once the total stiffness of the system has been determined, the force in each component of the system is determined by distributing the total force developed by the system of purlins according to the relative stiffness of the components.

The component stiffness method is presented in detail in Chapter 5. Equations are provided to predict anchorage forces for supports, third points, supports plus third point lateral restraints and supports plus third point torsional restraints.

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CHAPTER 5 ALTERNATE ANALYSIS PROCEDURES

Symbols and Definitions used in Section 5.1.

Note: Other variables in this Chapter have been defined as they are introduced.

A	Full unreduced cross-sectional area of member (in.²) (mm²)
a	Torsional constant $\sqrt{\frac{EC_w}{GJ}}$
b	Width of C- or Z-section top flange (in.) (mm)
b_{pl}	Width of anti-roll anchorage device or web bolted rafter plate (in.) (mm)
Bay	Total width of diaphragm perpendicular to span (ft) (m)
C_w	Torsional warping constant of cross-section (in.⁶) (mm⁶)
C1	Flexural deflection constant
d	Depth of C- or Z-section (in.) (mm)
E	Modulus of elasticity (29,500,000 psi) (203,400 MPa)
G	Shear modulus (11,300,00 psi) (78,000 MPa)
G′	Diaphragm shear stiffness. Ratio of shear per foot to the deflection per unit width of diaphragm assembly. (lb/in.) (N/m)
h	Height of applied restraint measured from base of purlin parallel to web (in.) (mm)
I_{my}	Modified moment of inertia $\frac{I_x I_y - I_{xy}^2}{I_x}$
I_x	Moment of inertia of full unreduced section about axis perpendicular to the plane of the web (in.⁴) (mm⁴)
I_{xy}	Product of inertia of full unreduced section about major and minor centroidal axes (in.⁴) (mm⁴)
I_{panel}	Gross moment of inertia of sheathing panel
J	Saint-Venant torsion constant (in.⁴) (mm⁴)
k_{conn}	Rotational stiffness of connection between the purlin and sheathing per unit length along span of purlin (lb-in./ft) (N-m/m)
k_{mclip}	Combined rotational stiffness of sheathing and connection between the purlin and sheathing per unit length along span of purlin (lb-in./ft) (N-m/m)
K_{3rd}	Total stiffness of anchors at third point (lb/in.) (N/m)
K_{config}	Stiffness of anchorage configuration (lb/in.) (N/m)
K_{device}	Stiffness of anchorage device at height along the web of purlin restraint is applied (lb/in.) (N/m)
K_{rafter}	Moment developed in connection between purlin and rafter per unit lateral displacement of top flange of purlin at restraint (lb-in./in.) (N-m/m)
K_{rest}	Force restrained at top flange of purlin per unit lateral displacement of top flange at restraint location (lb/in.) (N/m)
K_{shtg}	Moment developed in connection between purlin and sheathing per unit lateral displacement of top flange of purlin at restraint (lb-in./in.) (N-m/m)
K_{Spt}	Combined anchor and rafter stiffness at the frame line (lb/in.) (N/m)
K_{total}	Total stiffness of system at anchor location.
K_{trib}	Combined anchor and rafter stiffness at frame line tributary to half span (lb/in.) (N/m)

L	Span of purlin (ft) (m)
L_B	Purlin segment length for test for determining rotational stiffness of connection between purlin and sheathing (ft) (m)
m	Horizontal distance from shear center of C-section to mid-plane of web ($m = 0$ for Z-sections) (in.) (mm)
M_{3rd}	Moment in third point torsional brace (lb-in.) (N-m)
M_{local}	Moment developed in sheathing due to cross sectional deformation of purlin (lb-in.) (N-m)
M_{rafter}	Moment developed in connection between rafter and purlin due to lateral movement of top flange relative to base (lb-in.) (N-m)
M_{shtg}	Moment developed in sheathing along the span of the purlin due to lateral movement of top flange relative to base (lb-in.) (N-m)
$M_{torsion}$	Moment developed in sheathing due to twist of purlin relative to sheathing (lb-in.) (N-m)
P_h	Anchorage force per anchorage device at height of restraint (lb) (N)
P_i	Overturning force generated per purlin per half span (lb) (N)
P_L	Anchorage force per anchorage device at top of purlin (lb) (N)
P_N	Nominal test load for determining rotational stiffness of connection between purlin and sheathing.
P_{sc}	Shear force in connection between purlin and sheathing at anchorage location (lb) (N)
N_a	Number of anchors at a particular restraint location
$n_{downslope}$	Number of purlins with flanges facing down slope in a bay
N_p	Number of purlins in a bay
$n_{upslope}$	Number of purlins with flanges facing up slope in a bay
R_{local}	Coefficient to account for purlin cross sectional deformations
R_{sys}	Reduction factor to account for sheathing system effects
span	Span of sheathing panels between adjacent purlins (ft) (m)
t	Thickness of C- or Z-section (in.) (mm)
t_{lap}	Effective plate bending thickness of purlins at lap (in.) (mm)
t_{pl}	Thickness of web bolted rafter plate (in.) (mm)
w	Uniform loading on purlin (lb/ft) (N/m)
w_{rest}	Uniform diaphragm restraint force provided by sheathing (lb/ft) (N/m)
Width	Tributary width of diaphragm (perpendicular to purlin span) per purlin (in.) (mm)
α	Coefficient for purlin direction
β	Torsional constant for beam subjected to uniform torsion (rad.)
β_{3rd}	Torsional constant for beam subjected to concentrated torque at third points (rad/lb/in.) (rad/N/m)
δ	Coefficient for determining load eccentricity on purlin top flange (1/3)
Δ_{diaph}	Horizontal deflection of the diaphragm (in.) (mm)
Δ_{rest}	Horizontal deflection of the top flange of purlin at restraint (in.) (mm)
η	Number of up slope facing purlins minus the number of down slope facing purlins ($n_{upslope}$ - $n_{downslope}$)
κ	Torsional constant for beam subjected to parabolically varying

torsion (rad·in.^{2}) (rad·mm^2)

σ Proportion of uniformly applied load transferred to a uniform restraint force in the sheathing

θ Angle between the vertical and the plane of the purlin web (degrees)

τ Torsional constant for beam subjected to uniform torsion with uniformly distributed rotational springs resistance (rad/lb) (rad/N)

Φ Rotation of Z purlin at mid-span without torsional resistance (rad.)

Φ_{net} Rotation of Z purlin at mid-span with torsional restraint from sheathing (rad.)

$\Phi_{Mtorsion}$ Rotation of purlin at mid-span due to torsional restraint from sheathing (rad.)

5. Alternate Analysis Procedures

The *Specification* allows for rational analysis to predict anchorage forces to be used in lieu of the prediction method outlined in Section D6.3.1. In the following section, three alternative analysis procedures are presented: the component stiffness method, a frame element finite element model and a shell element finite element model.

5.1 Component Stiffness Method

The component stiffness method is an alternative manual calculation procedure to Section D6.3.1 in the *Specification*. The two procedures are based on the same principal – forces generated by a system of purlins are distributed to the external anchorage devices according to relative stiffness. In each procedure, however, the forces generated by the system of purlins and the distribution of the forces are calculated differently. The component stiffness method can be used to determine anchorage forces for supports, third points, midpoints, supports plus third point torsional brace and supports plus third point lateral bracing configurations.

Gravity loads that are applied to the sheathing of a roof system must be transferred through the purlins to the rafters and out of the system. Because the plane of the sheathing is typically separated from the rafters by the depth of the purlin to facilitate lapping and form a continuous system, as the gravity forces are transferred through the purlin, overturning moments are developed. To maintain stability of the roof system and restrict lateral movement of the top flange of the purlin, external anchors are required.

A three step process is used to determine the magnitude of the anchorage force using the component stiffness method. First, the magnitude of the overturning force, P_i, generated by each purlin as the purlin transfers the gravity loads from the sheathing to the frame lines is determined. Calculations of the overturning force consider the eccentricity of the applied loads and the effects of the resistance provided by the sheathing. Next, the resistance of the system to the overturning forces is determined. The anchorage device provides most of the resistance to the overturning forces, however the system of purlins has some inherent resistance to the overturning forces in the connection between the purlin and the frame line and the connection between the purlin and sheathing. The stiffness of each "component" of the system, the anchorage device, the purlin-rafter connection and the purlin-sheathing connection, is calculated. In the third step, the stiffness of each "component" is compared to determine the distribution of the overturning force between the anchorage device, the purlin-rafter connection and purlin-sheathing connection. The anchorage force, P_L, is magnitude of the overturning force distributed to each anchorage device.

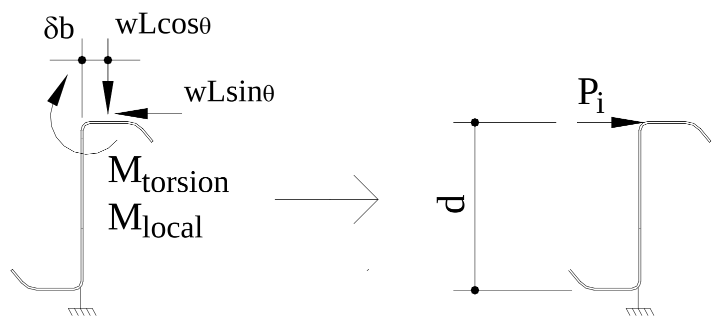
5.1.1 General Method

The basis for the component stiffness method is discussed in the following section. Solutions to different bracing configurations are discussed in Section 5.1.4 and equations specific to the different bracing configurations are presented in Section 5.1.6.

Generation of Overturning Moments. The component stiffness method is based upon the free body diagrams shown in Figures 5.1.1(a) and 5.1.2(a). Figure 5.1.1(a) displays the overturning forces and moments developed in the roof system and Figure 5.1.2(a) shows the forces restraining the system. The gravity load applied to the top flange of the purlin as shown in Figure 5.1.1(a) is divided into a normal component, $w \cdot L \cdot \cos\theta$, perpendicular to the plane of the sheathing and a down slope component, $w \cdot L \cdot \sin\theta$, in the plane of the sheathing, where w is the uniformly applied gravity load along the span of the purlin, L is the span of the purlin and θ is the angle of the roof plane relative to the horizontal. The normal component of the gravity load is approximated to act at some eccentricity, δb, along the top flange of the purlin.

As the gravity loads are applied, lateral deformation of the purlin is restrained through the development of diaphragm forces in the sheathing. Moments are developed in the connection between the purlin and sheathing as the sheathing resists the torsion of the purlin and the local rotation of the top flange of the purlin. Twisting of the purlin results from the gravity load applied eccentrically to the shear center of the purlin and the eccentricity of the diaphragm restraint provided by the sheathing. The sheathing resists the torsional rotations through the development of moment, $M_{torsion}$. Additional moments in the sheathing, M_{local}, are developed by the resistance of the sheathing to cross sectional deformation of the purlin. The forces and moments shown in Figure 5.1.1(a) have a net overturning effect on the purlin about its base at the rafter location , $P_i \cdot d$, shown in Figure 5.1.1(b) .

$$\sum M_{Base} = P_i d = wL(\delta b \cos\theta - d \sin\theta) + M_{torsion} + M_{local} \tag{5.1.1}$$



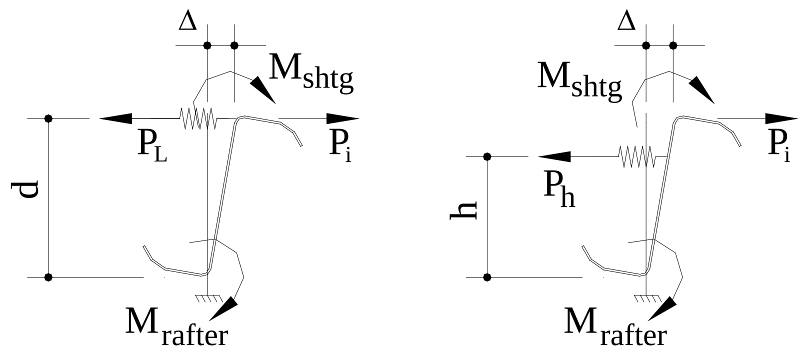
(a) Forces Generated **(b) Net Overturning**

Figure 5.1.1 Free Body Diagram of Forces Generated

The overturning moment due to the normal component of gravity load, the torsional moment and the local bending moment are all a function of the eccentricity of the gravity load, δb. Ghazanfari and Murray (1983) proposed that the normal component of the gravity load acts at an eccentricity of δb = b/3. Comparison of the component stiffness method to test results by Lee and Murray (2001) and Seek and Murray (2004) show good correlation for an eccentricity of one third of the flange width so it is recommended that this value be used. Generally, for a low slope roof (roof slope < 1:12), it is typically conservative to overestimate eccentricity while for steeper slope roofs, underestimating eccentricity is conservative.

Resistance to Overturning Moments. Figure 5.1.2 (a) shows the resistance of the system of purlins to these overturning moments. The majority of the resistance is provided by the external anchorage, P_L. However an anchorage device has a finite stiffness, and as the anchorage device permits displacement of the top flange, additional resistance is provided by the inherent lateral stiffness of the system, "system effects". As the top of the purlin moves laterally relative to the base, the purlin rotates about the longitudinal axis relative to the sheathing and a resisting moment is developed in the sheathing, M_{shtg}. Similarly, the connection between the rafter and the purlin resists the rotation of the purlin through the development of a moment at the connection. Summing moments about the base of the purlin, the anchorage force, P_L, at the top of the purlin is

$$P_L = P_i + \frac{M_{shtg} + M_{rafter}}{d} \tag{5.1.2}$$



(a) Anchorage At Top Flange **(b) Anchorage at Web**

Figure 5.1.2 Free Body Diagram of Resisting Forces

Note that in Figure 5.1.2, P_i, P_L, M_{shtg}, and M_{rafter}, and Δ are shown in the positive direction. For a positive applied force, P_i, the anchorage force, P_L, and the lateral displacement will be positive while the resisting moments of the sheathing and rafter, M_{shtg}, and M_{rafter} respectively, will be negative.

The forces resisted by each "component", the external anchorage device, the sheathing, and the connection of the purlin to the rafter, are directly related to the displacement of the top flange at the anchor. The component stiffness method defines the stiffness of each of these components as the force or moment generated in the component per unit displacement of the top flange of the purlin at the anchor location. Therefore, the anchorage stiffness, K_{rest}, is the force in the anchorage device at the top flange of the sheathing relative to the displacement of the top flange at the anchor location. The sheathing stiffness, K_{shtg}, is the moment generated in the connection between the purlin and the sheathing per unit displacement of the purlin top flange at the anchor location. Similarly, the rafter stiffness, K_{rafter}, is the moment generated at the rafter location per unit displacement of the top flange at the anchor location. By defining the stiffness of each of the components relative to the displacement of the top flange of the purlin at the anchorage device, the purlin is treated as a single degree of freedom system. The force resisted by the anchorage device is the product of the net overturning forces in the system and the relative stiffness of the restraint to the total stiffness of the system, or

$$P_L = P_i \cdot \frac{K_{rest}}{K_{rest} + K_{shtg} + K_{rafter}} \tag{5.1.3}$$

Locating the anchor below the top flange of the purlin will reduce the anchor stiffness because of the additional flexibility introduced through the bending of the web, and increase the anchorage force because of the reduced moment arm, h, as shown in Figure 5.1.2 (b). The reduced stiffness is accounted for in equations for the anchorage stiffness. The increase in the anchorage force is calculated

$$P_h = P_L \cdot \frac{d}{h} \qquad (5.1.4)$$

Note the difference between the moments developed in the sheathing. The torsional moment, $M_{torsion}$, and the local bending moment, M_{local}, are generated along the span of the purlin as the top flange of the purlin is rigidly restrained at the anchor location. Therefore, the torsional moment and local bending moment are considered part of the overturning forces that must be restrained and are embedded in the overturning force equation, P_i. As the flexibility of the anchorage device allows lateral displacement of the top flange, the sheathing moment, M_{shtg} is developed to resist this lateral movement. It is dependent upon the lateral displacement of the top flange at the anchorage device and therefore is considered part of the stiffness of the system.

Torsional Moment-General. The torsional moment is developed from the resistance applied by the sheathing to the torsion of the purlin along its span. To determine the torsional moment, the purlin, subjected to a uniform gravity load, is rigidly restrained at the top flange at the anchorage location. The lateral deflection and rotation of the purlin along its span is restrained by the sheathing through the development of shear forces in the diaphragm and moments due to the torsional resistance of the sheathing. Compatibility between the displaced shape of the purlin and the sheathing is used to determine the restraining forces in the sheathing. Because the displaced shape of the purlin is dependent upon the anchor location, an individual set of equations must be derived for each anchorage configuration. To illustrate how the torsional moment and diaphragm forces are determined, the two quantities will be derived for a supports configuration for both a Z-section and a C-section.

Torsional Moment – Z-sections. To determine the interacting forces between the purlin and the sheathing for a supports configuration, compatibility of the displaced shape is determined at the mid-span of a single span Z-section. The Z-section is restrained laterally at the top and bottom flanges at the frame line and subjected to external gravity loads applied to the top flange. In the absence of sheathing and neglecting second order effects, as the component of the gravity load normal to the plane of the sheathing, $w \cdot L \cdot \cos\theta$, acts at an eccentricity, δb, on the top flange of the Z-section, as shown in Figure 5.1.1(a), the Z-section will deflect laterally ($\Delta_{x,cen}$) and twist clockwise about its longitudinal axis ($\Delta_{x,torsion}$) as shown in Figure 5.1.3. Sheathing attached to the top flange of the Z-section resists this deformation through the development of a uniform horizontal force along the length of the Z-section, w_{rest}, as shown in Figure 5.1.4. Application of this uniform force to the top flange of the Z-section results in a restoring lateral deflection ($\Delta_{x,restoring,center}$) and a counterclockwise twist of the Z-section ($\Delta_{x,restoring,torsion}$). The uniform restraint force in the sheathing, w_{rest}, and the down slope component of the gravity load, $w \cdot L \cdot \sin\theta$, result in additional deformation of the top flange of the Z-section at mid-span due to the diaphragm flexibility of the sheathing. Equating the unrestrained displacements to the restoring displacements, the uniform restraint force in the sheathing is determined by

$$w_{rest} = w \cdot \sigma \qquad (5.1.5)$$

where

$$\sigma = \frac{\dfrac{5\left(\dfrac{I_{xy}}{I_x}\cos\theta\right)L^4}{384EI_{my}} + \dfrac{[(\delta b + m)\cos\theta]d}{2}\tau + \dfrac{L^2\sin\theta}{8G'Width}}{\dfrac{5L^4}{384EI_{my}} + \dfrac{d^2}{4}\tau + \dfrac{L^2}{8G'Width}} \tag{5.1.6}$$

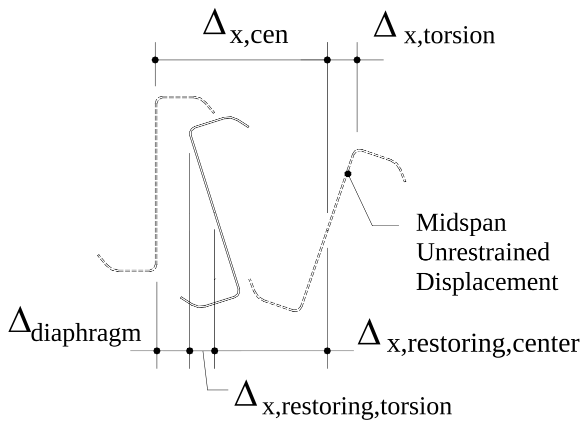


Figure 5.1.3 Midspan Displacement Compatibility

For definitions of the terms used in Equation (5.1.6) refer to the nomenclature in the beginning of this chapter. The distance from the shear center to the center of the web, m, is zero for Z-sections. The term σ is used for convenience of calculation and is the proportion of the uniformly applied vertical force, w, transferred to a horizontal force in the sheathing, w_{rest}. This proportion can typically be approximated as $\sigma \approx I_{xy}/I_x$ for Z-sections. As the combined torsional stiffness of the Z-section and sheathing increases, the second terms in the numerator and denominator will approach zero. Similarly, as the diaphragm stiffness increases, the third terms in the numerator and denominator will approach zero. Therefore, for a perfectly rigid diaphragm and torsionally rigid Z-section-sheathing connection, constrained bending is achieved and σ will reduce to I_{xy}/I_x. For low slope roofs, reducing the diaphragm stiffness will reduce the uniform restraint force in the sheathing which will result in $\sigma < I_{xy}/I_x$. For roofs with steeper slopes (typically greater than a 1:12 pitch) reducing the diaphragm stiffness will increase the uniform restraint force in the sheathing relative to the rigid case, or $\sigma > I_{xy}/I_x$. Unlike with the diaphragm stiffness, no simple trend was observed with respect to the torsional stiffness of the Z-section but in general, the torsional stiffness has a small effect on σ.

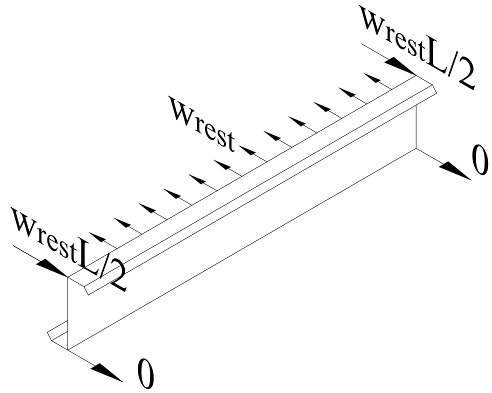


Figure 5.1.4 Uniform Restraint Force in Sheathing

Once the uniform restraint force in the sheathing has been determined, the mid-span torsional rotation of the Z-section and corresponding moment generated in the sheathing is determined. For a Z-section with anchorage at the supports, the rotation of a Z-section about its longitudinal axis is restricted at its ends and increases to maximum at mid-span, Φ, as shown in Figure 5.1.5(a). The variation of the torsional rotation is approximately parabolic along the length of the Z-section. The connection between the sheathing and the Z-section resists this torsion through the development of a moment along the length of the Z-section, $M_{torsion}$, as shown in Figure 5.1.5(b). The moment in the sheathing is proportional to the rotation of the Z-section about its longitudinal axis. The stiffness of the connection between the sheathing and Z-section, k_{mclip}, is defined as the moment developed in the connection per unit rotation of the Z-section per unit length along the span. The moment caused by the resistance of the sheathing results in an additional torsional rotation of the Z-section, $\Phi_{Mtorsion}$, as shown in Figure 5.1.5(b). The net torsional rotation of the Z-section at mid-span, Φ_{net}, is the sum of the rotation caused by the eccentrically applied gravity load, the rotation caused by the uniform lateral resistance of the sheathing at the top flange, and the rotation due to the sheathing moment, or

$$\Phi_{net} = \left(w((\delta b + m)\cos\theta) - w_{rest}\frac{d}{2} \right)\frac{a^2\beta}{GJ} - \phi_{net}\cdot k_{mclip}\left(\frac{\kappa}{GJ}\right) \tag{5.1.7}$$

Equation (5.1.7) is simplified to yield

$$\Phi_{net} = w\left(\delta b\cos\theta - \sigma\frac{d}{2}\right)\tau \tag{5.1.8}$$

where

$$\tau = \frac{\dfrac{a^2\beta}{GJ}}{1 + k_{mclip}\dfrac{\kappa}{GJ}} \tag{5.1.9}$$

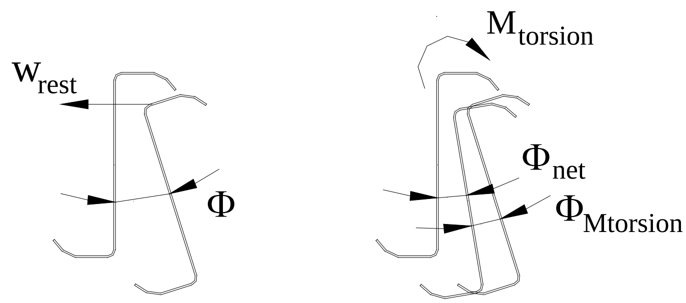
The net torsional rotation of the Z-section in Equation (5.1.8) at mid-span is the peak rotation in the parabolic distribution. Relating the moment in the sheathing to the rotation by $M=\Phi_{net}\cdot k_{mclip}$ and integrating along the length of the Z-section, the total moment in the connection between the sheathing and Z-section generated along the length of the Z-section for a supports restraint configuration is

$$M_{torsion} = \tfrac{2}{3} k_{mclip} \cdot wL\left(\sigma\frac{d}{2} - ((\delta b)\cos\theta)\right)\tau \qquad (5.1.10a)$$

For C-Section purlins, with consideration of the shear center location, $M_{torsion}$ can be expressed as

$$M_{torsion} = \tfrac{2}{3} k_{mclip} \cdot wL\left(\sigma\frac{d}{2} - ((\delta b + m)\cos\theta)\right)\tau \qquad (5.1.10b)$$

As the Z-section is oriented in Figure 5.1.6, with the top flange facing to the right, the positive direction for the torsional rotation and the torsional moment is clockwise. Therefore, as the Z-section undergoes positive torsion, a negative moment is developed in the sheathing.



(a) Rotation without Torsional Resistance **(b) Net Rotation with Torsional Resistance**

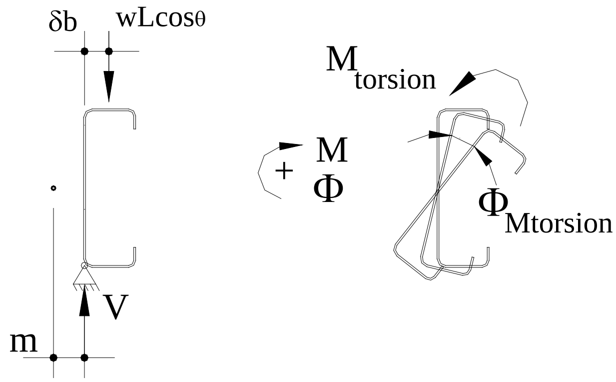
Figure 5.1.5 Rotation of Z-Section at Mid-Span

Torsional Moment – C-sections. For determining anchorage forces, Z-sections and C-sections have two main differences. First, Z-sections have principal axes that are oblique to the centroidal axes while, for a C-section, the principal axes correspond with the centroidal axes. A C-section does not deflect laterally when subjected to loads in the plane of the purlin web as a Z-section does. As a result, the uniform restraint force in the sheathing is much less for a C-section than for a similar configuration with a Z-section. Secondly, the shear center of a C-section is located at some distance, m, from the web while for a Z-section, m = 0. Because of this additional eccentricity, torsion of the C-section plays a larger role in the development of the torsional moment.

Like for a Z-section, the first step in determining the anchorage forces for a C-section is to calculate the uniform restraint force in the sheathing. For a single span C-section with supports anchorage, Equation 5.1.6 is used to calculate the uniform restraint force in the sheathing. In the numerator of the equation to calculate the uniform restraint force, the term including I_{xy} is eliminated and the eccentricity of the shear center, m, is included in the torsional term. The equation reduces to

$$\sigma = \frac{\dfrac{((\delta b + m)\cos\theta)d}{2}\tau + \dfrac{\alpha \cdot N_p \cdot L^2 \sin\theta}{8G'Width}}{\dfrac{5L^4}{384EI_{my}} + \dfrac{d^2}{4}\tau + \dfrac{\alpha \cdot \eta \cdot L^2}{8G'Width}} \qquad (5.1.11)$$

The uniform restraint force along the length of the purlin is small and will typically be on the order of 1% to 5% of the applied uniform load.



(a) Moment due to Applied Gravity Load **(b) Torsional Moment**

Figure 5.1.6. Moments Acting on C-Section

The torsional moment is calculated by Equation 5.1.10. Because the uniform restraint force in the sheathing is relatively small, the torsion of the C-section is dominated by the overturning caused by the component of gravity load normal to the sheathing (Refer to Figure 5.1.6). This torsion results in a positive (clockwise) rotation of the C-section about its longitudinal axis. The sheathing resists this positive rotation with the development of a negative torsional moment.

It is important to note that vertical reaction at the base of the purlin is assumed to act in the plane of the web (refer to Figure 5.1.6(a)). Thus for a C-section, the overturning moment due to the normal component of the gravity load ($wL \cdot \delta b \cos\theta$) is independent of the location of the shear center. The location of the shear center only affects the torsion of the C-section along its span (and the corresponding torsional moment). As a result, for a flat slope roof, the negative torsional moment can exceed the positive gravity moment, resulting in a negative net overturning moment.

Local Bending Moment. The equations for the torsional moment assume that plane sections always remain plane. Because purlins are manufactured from relatively thin material, the purlin cross section can deform without fully transferring the moments predicted in the torsion moment equations (Equation 5.1.10). This additional deformation is approximated by the model shown in Figure 5.1.7. As the component of the gravity load normal to the plane of the sheathing acts eccentrically on the top flange of the purlin, the flange deflects causing a local rotation of the purlin relative to the sheathing. Due to the rotational resistance of the connection between the purlin and sheathing, a moment is developed in the sheathing. The magnitude of this moment, referred to as the local bending moment, is

$$M_{local} = -wL \cdot \delta b \cos\theta \cdot \frac{k_{mclip}}{k_{mclip} + \frac{Et^3}{3d}} \tag{5.1.12}$$

In the component stiffness method, the local bending moment is incorporated into the equations as a reduction factor, R_{local}, where

$$R_{local} = \frac{k_{mclip}}{k_{mclip} + \frac{Et^3}{3d}} \tag{5.1.13}$$

The overturning force due to the eccentricity of the normal component of the gravity load is reduced by a factor of (1-R_{local})

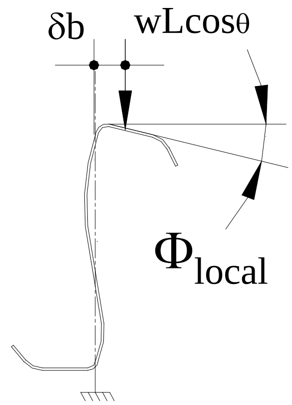


Figure 5.1.7 Local Deformation of Z-Section

5.1.2 Stiffness of Components

Anchor Stiffness. For the purposes of determining stiffness, anchors are divided into two categories: support and interior. A support anchor is applied along the frame line while an interior anchor is applied along the interior of the span.

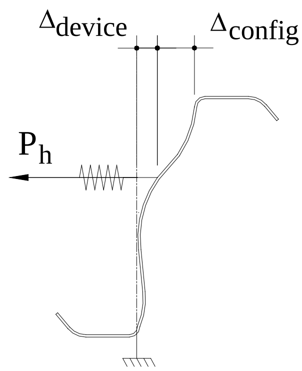
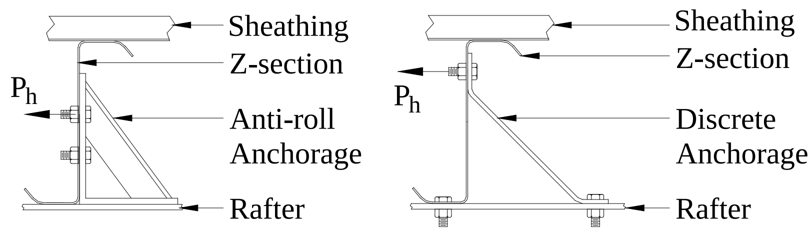


Figure 5.1.8 Combined Displacement of Device and Configuration

Anchor Stiffness – Support Anchor. The stiffness of each anchor is defined as the force developed at the top flange of the purlin at the anchor per unit displacement of the top flange at the anchor location. As shown in Figure 5.1.8, the deformation at the anchor is the combination of the deformation of the anchorage device, Δ_{device}, and the deformation of the web of the purlin relative to the height the applied restraint, Δ_{config}. The total stiffness at the anchor, K_{rest}, provided in Equation 5.1.14 is the combination of the stiffness of the device at the height at which the restraint is applied, K_{device}, and the stiffness of the web of the purlin as the force is transferred from the top flange of the purlin to the height of the anchor, K_{config}. The net stiffness of the anchor, K_{rest}, is defined as the anchorage force at the top flange of the purlin per unit displacement of the top flange.

$$K_{rest} = \frac{\left(\frac{h}{d}\right)^2 K_{device} \cdot K_{config}}{\frac{h}{d} K_{device} + K_{config}} \tag{5.1.14}$$

For a support configuration, the stiffness of the device is generally very high relative to the configuration stiffness. The device stiffness will typically have a negligible effect on the anchor stiffness and can be considered rigid in many cases. For determination of the anchorage force, this approximation will be conservative, although the predicted deformation of the system will be unconservative.



(a) Anti-roll Anchorage Device **(b) Discrete Anchorage Device**

Figure 5.1.9 Supports Anchorage Configurations

Support anchors are divided into two categories – discrete and anti-roll anchorage devices. A discrete anchorage device provides lateral resistance at a discrete location along the height of the purlin as shown in Figure 5.1.9(b) while an anti-roll anchorage device clamps to the web of the purlin at multiple locations along the depth as shown in Figure 5.1.9(a). The anti-roll anchorage device prevents deformation of the purlin web below the anchorage location while a discrete anchor permits some deformation, resulting in a variation in stiffness. An equation to predict the stiffness of each type of configuration is provided due to this variation in stiffness.

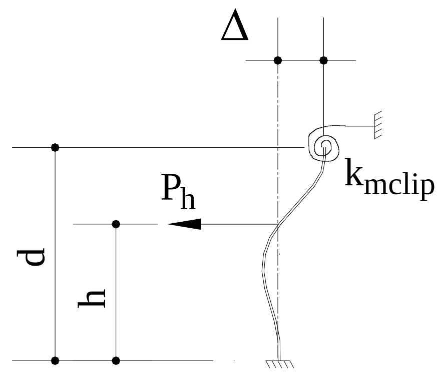


Figure 5.1.10 Stiffness Model – Discrete Anchorage

The equation to predict the configuration stiffness of a discrete anchorage configuration is based on a two dimensional beam model bent about the thickness of the web as shown in Figure 5.1.10. To account for the third dimension, the effective width of the web and sheathing, the representative equation was calibrated to the results of finite element models as described by Seek and Murray (2004). The resulting configuration stiffness for a discrete anchor is

$$K_{config} = \frac{\frac{1}{15} d \cdot (3Et^3)}{h(d-h)^2} \left[\frac{\frac{1}{15} d \cdot 2Et^3 (3d-h) + \frac{L}{80} \cdot k_{mclip} \cdot d(3d-2h)}{\frac{1}{15} d \cdot Et^3 (4d-h) + \frac{L}{80} \cdot k_{mclip} \cdot d(d-h)} \right] \tag{5.1.15}$$

where d is the depth of the section, h is the height of the applied restraint, L is the span length of the purlin and k_{mclip} is the rotational stiffness of the connection between the purlin and sheathing.

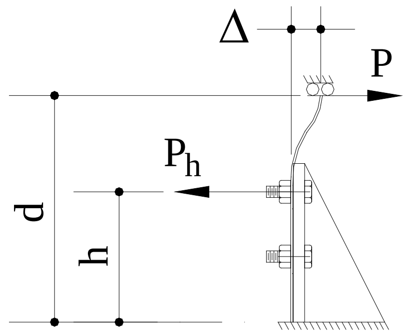


Figure 5.1.11 Stiffness Model – Anti-Roll Anchorage Device

For an anti-roll anchorage device, the configuration stiffness is based upon the two-dimensional line element model shown in Figure 5.1.11. The model assumes that restraint is applied at the top row of bolts and the web of the purlin is rotationally fixed at this point. The effective width of the web is assumed to be the width of the anti-roll clip, b_{pl} and the top of the purlin is assumed to be fixed to the sheathing. The configuration stiffness equation given by Equation 5.1.16 is the familiar formula for a fixed-fixed cantilever beam multiplied by d/h to transfer the stiffness to the restraint height, h.

$$K_{config} = \frac{Eb_{pl}t^3}{(d-h)^3} \left(\frac{d}{h} \right) \tag{5.1.16}$$

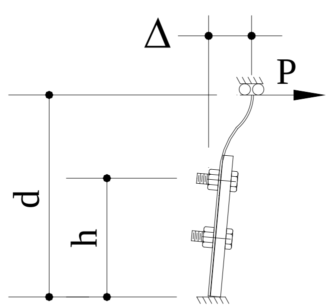


Figure 5.1.12 Stiffness Model – Rafter Plate

A type of anti-roll anchorage device is a single web plate, sometimes referred to as a wing plate. The device stiffness and the configuration stiffness of the web plate is calculated directly by modeling the web plate and purlin web as a two dimensional cantilever beam model as shown in Figure 5.1.12. The beam, modeled as a prismatic section, has a width equal to width of the web plate. For a distance from the top of rafter elevation to the top row of bolts, the beam has a thickness equal to that of the web plate. Above the top row of bolts the beam model has the thickness equal to purlin web thickness. The model incorporates both the device and configuration stiffness, and the resulting net stiffness of the anchor for a web bolted rafter plate is

$$K_{rest,webplate} = \frac{E \cdot b_{pl} \cdot t_{pl}{}^3 \cdot t^3\left(t^3 h + t_{pl}^3 (d-h)\right)}{\left(t^3 h^2 - t_{pl}^3 (d-h)^2\right)^2 + 4t^3 t_{pl}{}^3 d^2 h(d-h)} \quad (5.1.17)$$

The provided equations for the configuration stiffness for a discrete anchorage device will typically overestimate the stiffness of the configuration, which will lead to a conservative approximation of anchorage force but may underestimate the amount of deflection in the system. Conversely, for an anti-roll anchorage device, the provided equation will typically underestimate the stiffness which will result in an overestimation of deformation at the anchor location. Most anti-roll anchorage devices have substantial strength and the design of such systems will typically be deflection controlled. Because no testing or finite element modeling was performed with anti-roll anchorage devices, it is conservative to underestimate the stiffness of the anti-roll anchorage device.

Anchor Stiffness – Interior Anchor. Interior anchors are considered to be attached as close as possible to the top flange of the purlin to minimize the deformation of the purlin web as the anchorage force is transferred out of the sheathing and into the anchor. Flexibility therefore is introduced only through the deformation of the anchorage. From Equation 5.1.14, the configuration stiffness is considered infinite and the net anchor stiffness reduces to

$$K_{rest} = \left(\frac{h}{d}\right)^2 K_{device} \quad (5.1.18)$$

Stiffness of System. The system of purlins has an inherent resistance to lateral forces through the connection of the purlin to the rafter and through the connection of the sheathing to the purlin, referred to as the rafter stiffness, K_{rafter} and sheathing stiffness, K_{shtg}, respectively. For determining both anchorage forces and the deformation of the system, it is conservative to underestimate the rafter stiffness and sheathing stiffness. A low estimate of the rafter stiffness and sheathing stiffness will result in the prediction of a larger anchorage force than actual and will result in a larger displacement. For simplicity, the rafter stiffness and sheathing stiffness can conservatively be eliminated. However, the contribution of the sheathing and the rafter connection to the resistance of overturning forces can be significant and economically advantageous to include.

Stiffness of System – Rafter Stiffness. The connection of the purlin to the rafter provides resistance to overturning forces through the development of a moment at the base of the purlin. The rafter stiffness is defined as the moment generated at the base of the purlin per unit lateral displacement of the top flange of the purlin. Two basic connection configurations are considered, a flange-bolted connection, shown in Figure 5.1.13(a) and a web bolted rafter plate connection, shown in Figure 5.1.13(b). In a flange-bolted configuration, the bottom flange of the purlin is through-bolted to the top flange of the rafter with two bolts in line with the web of the purlin. The clamping action of the bolts permits the development of a moment, M_{rafter}, of the base of the purlin as the top flange of the purlin moves laterally. The stiffness of a flange bolted connection, derived from two dimensional beam element model and calibrated to the results of finite element models as described in Seek and Murray (2006), is

$$K_{rafter} = 0.45\frac{Et^3}{2d} \tag{5.1.19}$$

where E is the modulus of elasticity of the purlin, t is the thickness of the purlin and d is the depth of the purlin.

(a) Flange Bolted **(b) Web Bolted Rafter Plate**

Figure 5.1.13 Typical Rafter to Purlin Connections

The second rafter connection configuration considered is a web bolted rafter plate. In this connection configuration, a plate, typically welded to the rafter, is bolted to the web of the purlin. Like the flange bolted connection, as the top flange of purlin moves laterally, a moment is generated at the base of the purlin to resist this movement. Because of the added stiffness of the rafter plate, the web-bolted rafter plate configuration has considerably more stiffness than the flange bolted configuration. The rafter stiffness for a web plate shown in Equation 5.1.20 is the same as Equation 5.1.17 for a supports restraint except the stiffness is multiplied by the depth of the purlin, d, to convert it to moment at the base of the purlin per unit displacement of the top of the purlin.

$$K_{rafter} = \frac{E \cdot b_{pl} \cdot t_{pl}{}^3 \cdot t^3\left(t^3 h + t_{pl}^3(d-h)\right)\cdot d}{\left(t^3h^2 - t_{pl}^3(d-h)^2\right)^2 + 4t^3 t_{pl}{}^3 d^2 h(d-h)} \tag{5.1.20}$$

It is important not to count the stiffness of a rafter plate twice when using the component stiffness method. When considered a restraint, the stiffness of the rafter plate should only be considered in the restraint stiffness. When the rafter plate is considered a typical rafter connection and used in conjunction with stiffer anti-roll anchorage devices, the stiffness of the rafter plate should be considered part of the rafter stiffness.

Stiffness of System – Sheathing Stiffness. The second inherent contribution of the system to the lateral resistance comes from the connection between the purlin and the sheathing. As the top flange of the purlin moves laterally, the purlin is approximated to rotate about its base as shown in Figure 5.1.14. As the purlin rotates relative to the plane of the sheathing, a moment is developed in the connection between the sheathing and purlin. The rotation of the purlin about its base is approximated to be uniform and thus generates a uniform moment in the connection between the purlin and sheathing along the length of the purlin. As the uniform moment is applied to the purlin, additional torsional rotations are generated in the purlin. These torsional rotations are approximated to vary parabolically along the length of the purlin and are accounted for by the $(1\text{-}2/3k_{mclip}\tau)$ term in Equation (5.1.21). The theoretical equation for the sheathing stiffness was further modified through comparison of the equation to the results of finite element models as described in Seek and Murray (2004; 2006). The resulting equation for the stiffness of the sheathing is

$$K_{shtg} = \frac{k_{mclip} L}{d}\left(\frac{\frac{1}{4}Et^3}{0.38k_{mclip}d + 0.71\frac{Et^3}{4}}\right)\left(1 - \tfrac{2}{3}k_{mclip}\tau\right) \tag{5.1.21}$$

Where L is the purlin span, d is the depth of the purlin, t is the thickness of the purlin, k_{mclip} is the rotational stiffness of the connection between the purlin and the sheathing and τ is the torsional term defined by Equation 5.1.9.

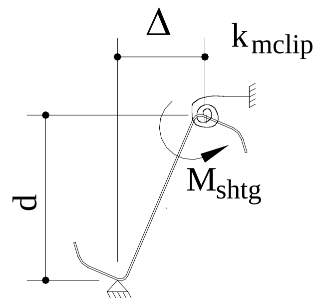


Figure 5.1.14 Sheathing Moment Stiffness

5.1.3 Anchorage Effectiveness

System Deformation. Lateral deflection should be checked at the anchor location as excessive deformation undermines the intent of anchors to prevent overturning of the purlin. In the event that adequate stiffness is not provided to limit deflection, the stiffness of the anchors can be increased by adding anchors or increasing the stiffness of the existing anchors. Lateral deflection also should be checked at the extremes of the system to ensure that the diaphragm has sufficient stiffness to transfer the forces along the length of the purlin to the restraints. The lateral deflection of the top flange of the purlin at the anchor location can be approximated by

$$\Delta_{rest} = \frac{P_L}{K_{rest}} \tag{5.1.22}$$

In general, as a purlin is allowed to deflect laterally, the calculated anchorage force decreases. The method does not account for any second order effects, therefore displacements should be minimized, particularly at the anchor location. The *Specification* limits the lateral displacement of the top flange of the purlin, Δ_{tf}, calculated at factored load levels for LRFD and at nominal load levels for ASD to a deflection

$$\Delta_{tf} \le \frac{1}{\Omega}\frac{d}{20} \quad \text{(ASD)} \tag{D6.3.1-9a}$$

$$\Delta_{tf} \le \phi\frac{d}{20} \quad \text{(LRFD)} \tag{D6.3.1-9b}$$

With a flexible diaphragm, lateral deflection of the purlin mid-span relative to the anchors is expected. For a supports anchorage and supports plus third points torsion restraint configuration, the lateral displacement of the diaphragm at the mid-span of the purlin relative to the anchor location is

$$\Delta_{diaph} = \sum_{N_p} \left(w(\alpha\sigma - \sin\theta) \right)_i \frac{L^2}{8G'Bay} \tag{5.1.23}$$

For a third points configuration, the deformation of the diaphragm at the frame line relative to the third points is

$$\Delta_{diaph} = \sum_{N_p} \left(-w\left(\alpha\sigma - \sin\theta \frac{\sum K_{rest}}{K_{total}} \right) \right)_i \frac{L^2}{9G'Bay} + \frac{\sum P_L L}{3G'Bay} \tag{5.1.24}$$

For a midpoint anchorage configuration, the diaphragm displacement at the frame line relative to the midpoint is

$$\Delta_{diaph} = \sum_{N_p} \left(-w\left(\alpha\sigma - \sin\theta \frac{\sum K_{rest}}{K_{total}} \right) \right)_i \frac{L^2}{8G'Bay} + \frac{\sum P_L L}{4G'Bay} \tag{5.1.25}$$

In the above equations, a positive deflection indicates upslope translation.

Shear Transfer from Sheathing to Purlin. An aspect critical to the performance of the restraints that must be considered is the shear force transferred from the sheathing to the purlin at the anchor location, P_{sc}. This force can be significant and must be transferred over a small width of panel approximately 12 in. (300 mm) to either side of the anchor location along the length of the purlin. For configurations with anchors at the frame lines, the magnitude of the shear force transferred from the sheathing to the purlin may actually exceed the anchorage force and is calculated by

$$P_{sc} = P_L + \frac{wL}{2}(0.9\sigma\alpha - \sin\theta) - P_i \tag{5.1.26}$$

For a midpoint or third point configuration, the shear force between the purlin and sheathing may be conservatively approximated as the anchorage force. A reduction in this force can typically be achieved using Equation (5.1.27).

$$P_{sc} = P_L + \frac{wL}{20}\left(-0.9\sigma + \frac{\delta b \cos\theta}{d} \right)\alpha \tag{5.1.27}$$

5.1.4 Anchorage Configurations

The component stiffness method has been applied to five anchorage configurations: supports, third points, midpoint, support plus third point torsion braces, and supports plus third point lateral anchors. In the following section, a brief summary of each configuration is given. A summary of equations applicable to each configuration is given in Section 5.1.6.

To determine anchorage forces for all configurations except midpoints, roof systems are evaluated per half span (from the frame line to the center of the span). A single span system has two half spans but because of symmetry, only one half span must be evaluated. In multiple span systems, each half span must be evaluated separately although symmetry and repetition are used wherever possible. For midpoint configurations, each span is evaluated individually.

The overturning force, P_i, for each purlin must be determined. Fundamental to the overturning force is the uniform restraint force, $w_{rest} = w \cdot \sigma$, in the sheathing due to diaphragm action in the sheathing. Both P_i and σ must be calculated for each purlin in the half span. For repetitive members, w_{rest} and P_i will be proportional to the applied load and only need to be recalculated for varying cross sections and orientations.

After the overturning forces are determined, they are distributed according to relative stiffness of each of the components. The stiffness of each anchor is calculated and the total stiffness at an anchor location is the sum of the individual stiffness of each anchor. The stiffness of the system is the inherent resistance of the system to lateral movement. Each purlin contributes a rafter and sheathing stiffness to the system stiffness. The sheathing stiffness for the half-bay is one half the stiffness of the full span. The rafter stiffness is dependent upon the connection between the rafter and the purlin at the frame line. The total stiffness is the sum of the anchor stiffness and the system stiffness. The force in each component is the sum of the all of the overturning forces in the half span multiplied by the stiffness of the component relative to the total stiffness of the half span. *Supports Anchorage*. A supports anchorage configuration is the most common anchorage configuration and fortunately the simplest. As the purlin is supported vertically and anchored laterally at the frame lines, maximum vertical and lateral displacements as well as torsional rotations occur at the mid-span (or close to the mid-span for the exterior span of a multi-span system) relative to a fixed location at the frame lines.

Supports Anchorage – Single Span. A single span system is evaluated for the half-span from the frame line to the center of the span. The uniform restraint force in the sheathing (Equation (5.1.51)) and the overturning force, P_i (Equation (5.1.49)), are calculated for each purlin in the bay. Note that for repetitive members, the overturning force is proportional to the applied load which simplifies calculations. The total overturning force is the sum of overturning forces for each individual purlin. The total stiffness, K_{total} (Equation (5.1.48)), is the sum of the stiffness of the half span: all of the anchors along the frame line, K_{rest} (Equations (5.1.30), (5.1.33)), the rafter stiffness of all of the purlins without anchors, K_{Rafter} (Equations (5.1.35), (5.1.36)), and the sheathing stiffness of each purlin in the bay, K_{shtg} (Equation (5.1.37)). The force at each anchor location at the top of the purlin, P_L (Equation (5.1.46)), is determined by multiplying the total overturning force by the ratio of the stiffness of the anchor to the total stiffness of the half span. If the anchorage device provides restraint at a height below the top of the purlin, the force at the height anchorage is provided, P_h, is calculated by (Equation (5.1.47)).

Supports Anchorage – Multiple Span Systems. Multiple span systems are categorized whether it is an exterior or interior frame line. The exterior frame line for a multiple span system is treated similar to a single span system. The total force is the sum of the overturning forces for each purlin, P_i (Equation (5.1.49)), for the half span adjacent to the exterior frame line. The total stiffness, K_{total} (Equation (5.1.48)), is the sum of the anchor stiffness along the exterior frame line, K_{rest} (Equations (5.1.30) and (5.1.33)), the rafter stiffness of the purlins not directly anchored along the exterior frame line, K_{Rafter} (Equations (5.1.35), (5.1.36)), and the sheathing stiffness, K_{shtg} (Equation (5.1.37)), of the half span adjacent to the exterior frame line. The force at each anchor location at the top of the purlin, P_L (Equation (5.1.46)), is the total force multiplied by the proportion of the stiffness of the anchor to the total stiffness.

At an interior frame line, the anchorage force must consider the effects of both half spans adjacent to the frame line. The overturning forces, P_i (Equation (5.1.49)), must be determined for both half spans adjacent to the frame line taking into account the torsional and flexural differences whether the adjoining bay is a typical interior or exterior bay. The total overturning force along the frame line is the sum of overturning forces for all purlins along the frame line for both half spans adjacent to the frame line. The total stiffness, K_{total} (Equation (5.1.48)), is the stiffness of all of the anchors along the frame line, K_{rest} (Equations (5.1.30), (5.1.33)), the rafter stiffness of each purlin not directly attached to an anchor, K_{Rafter} (Equations (5.1.35), (5.1.36)), and the sheathing stiffness, K_{shtg} (Equation (5.1.37)), of each purlin for both half spans adjacent to the frame line. The anchorage force at the top of the purlin for an individual anchor, P_L (Equation (5.1.46)), is the total overturning force multiplied by the proportion of the stiffness of the anchor to the total stiffness.

Supports Anchorage - Deflection and Sheathing Connection Shear. Displacement of the top flange of the purlin at the frame lines is checked to ensure that the anchor has sufficient stiffness. The displacement of the mid-span of the purlin relative to the frame line, Δ_{diaph} (Equation (5.1.53)), is checked from the displacement of the diaphragm. The mid-span diaphragm displacement is compared to the lateral deflection limits of the *Specification*.

For a supports anchorage configuration, the force transfer between the sheathing and purlin, P_{sc} (Equation (5.1.54)), is significant along the frame lines. The connection between the purlin and sheathing should be checked at each restraint as the shear force in the connection will typically exceed the anchorage force. For an exterior frame line, the force in the connection between the sheathing and purlin includes the restraint force and the uniform restraint force in the sheathing for half the span. At an interior frame line, it is important to include the restraint force in the sheathing from both half spans adjacent to the interior frame line.

Third Point Anchorage. For both single and multiple span systems, each half span between the centerline of the span and the frame line is analyzed independently. The total stiffness, K_{total} (Equation (5.1.57)) is the sum of anchor stiffness of all third point anchors, K_{rest} (Equation (5.1.34)), and the sheathing stiffness (K_{shtg} Equation (5.1.37)) of all of the purlins in the half span. The stiffness of the rafter connection is assumed to be small and is ignored in the development of equations for third point anchors. If the connection between the rafter and purlin has considerable stiffness such as a welded web plate, the bracing configuration should be considered as support plus third point anchorage configuration. For single span and multiple span interior systems, the anchorage force at both third points will be equal. For the exterior span of multiple span system, the third point force for each half span must be calculated separately. The forces are then summed and distributed to both third points in the exterior span according to the relative stiffness of each anchorage device.

For a third point configuration, the deformation of the system must be checked at the third points, Δ_{rest} (Equation (5.1.61)) and along the frame lines, Δ_{diaph} (Equation (5.1.62)). At the anchor points, the displacement is compared to the $\phi d/20$ (LRFD) or $d/(20\Omega)$ (ASD) limits specified in the *Specification*. The difference in the displacement between the third points and the frame lines is compared to the L/360 limit in the *Specification*. For a low slope roof, it is common to get a negative (down slope) displacement of the top flange of the purlin at the frame line.

Midpoint Anchorage. For a midpoint anchorage configuration, overturning forces for each purlin, P_i (Equation (5.1.67)), are determined for a full span. The total stiffness, K_{total} (Equation (5.1.66)) is the sum of anchor stiffness, K_{rest} (Equation (5.1.34)), of all midpoint anchors along the line of anchorage and the sheathing stiffness, K_{shtg} (Equation (5.1.37)), of all of the purlins in the bay. Like for a third point configuration, the stiffness of the rafter connection is assumed to be small (as for a flange bolted configuration) and is ignored. For a rafter connection that has considerable stiffness (as with a web plate) the equations provided are invalid.

The deformation of the system must be checked at the midpoints, Δ_{rest} (Equation (5.1.70)) and along the frame lines, Δ_{diaph} (Equation (5.1.71)). At the anchor points, the displacement is compared to the $\phi d/20$ (LRFD) or $d/(20\Omega)$ (ASD) limits specified in the *Specification*. The displacement of the diaphragm between the midpoint and the frame lines is compared to the L/360 limit in the *Specification*. For a low slope roof, it is common to get a negative (down slope) displacement of the top flange of the purlin at the frame line.

Supports Plus Third Point Torsion Restraints. For a supports plus third point torsion restraint configuration, lateral anchorage is provided along the frame lines and torsional restraints that resist rotation of the purlin are connected at third points between pairs of purlins. The behavior of a third point plus torsional restraint configuration is similar to that of a supports anchorage configuration. For a supports configuration, as the purlin twists, moments are developed in sheathing to resist this torsion. For a third point torsional restraint, the torsional restraints resist the twisting through the development of moments at each end of the torsional brace. The braces are considered to have a much greater torsional stiffness than the sheathing, so all of the torsion is resisted by the third point braces. This approximation will lead to conservative anchorage forces.

Some moment is developed in the sheathing near the frame lines as the supports anchors allow the top flange of the purlin to move laterally, causing rotation of the purlin. The inherent stiffness of the system, therefore, is a combination of this moment in the sheathing near the frame lines and the moments developed in the third point torsion braces. These system effects are included in the calculation of the moments at the third point torsion braces, M_{3rd}, and the overturning forces at the frame line, P_i, so there is no need to further reduce the anchorage forces for system effects.

The system of purlins is evaluated per half span from the frame line to the centerline of the span. The first thing that must be determined is the stiffness along the frame line tributary to the half span analyzed, K_{trib}. Because the sheathing stiffness is embedded in the equations, the total stiffness of the system, K_{total} (Equation (5.1.76)), is the sum the stiffness of all the anchors and the connections of the purlin to the rafter for all purlins not directly restrained along the frame line. At an interior frame line, the tributary stiffness to each of the half spans adjacent to the frame line is half of the total stiffness. At an exterior frame line or for a single span system, the tributary stiffness is the total stiffness along the frame line. Next, for each purlin along the length of the bay, the uniform restraint force is calculated. Once the uniform restraint force is calculated, the moment in the third point torsion brace, M_{3rd} (Equation (5.1.73)), and the overturning force at the frame line, P_i (Equation (5.1.77)), is calculated for each purlin. The anchorage force at the top of the purlin at each anchor along the frame line, P_L (Equation (5.1.74)) is determined by summing the overturning forces for all purlins and multiplying by the proportion of the anchor stiffness to the total stiffness along the frame line.

Lateral deflection of the system is checked at the frame lines, Δ_{rest} (Equation (5.1.81)) and at the mid-span of the bay, Δ_{diaph} (Equation (5.1.82)). The lateral displacement of the top flange at the frame line is limited to $\phi d/20$ (LRFD) or $d/(20\Omega)$ (ASD) as specified in Section D6.3.2 in the *Specification*. The mid-span displacement is calculated from the deformation of the diaphragm from the uniform restraint force in the sheathing. For an exterior span in a multiple span system, the average uniform restraint force for each half span in the exterior bay is used to determine the deformation of the diaphragm. The mid-span deflection is compared to the L/180 limit specified in Section D6.3.2 of the *Specification*. This limit is less stringent than the limit for other restraint configurations because the *Specification* recognizes that as the purlins deflect laterally, torsional rotation is limited by the torsional restraints allowing the purlins to maintain their strength. Second order overturning moments are also easily absorbed by the torsional braces.

Like a supports anchorage configuration, the force in the connection between the sheathing and purlin, P_{sc} (Equation (5.1.83)), is critical along the frame lines. At an exterior frame line at an anchor, P_{sc} includes the anchorage force and the uniform restraint force from the half span. At an interior frame line, P_{sc} includes the anchorage force and the uniform restraint force in the sheathing for both spans adjacent to the frame line.

Supports Plus Third Points Lateral Anchorage. The supports plus third points lateral anchorage configuration is solved in a slightly different manner than the other anchorage configurations. Because the system is restrained at the frame lines and at the interior, both the anchors and the diaphragm restrain movement of the purlin relative to the frame lines. The distribution of anchorage forces between the third points and supports is a function of the relative stiffness of the restraints to the stiffness of the diaphragm. For a stiff diaphragm and flexible restraints, most of the mid-span lateral deflection of the purlin is restrained by diaphragm action in the sheathing and a large uniform restraint force is developed in the sheathing. Most of the overturning force is resisted by the anchors along the frame line. As the stiffness of the diaphragm decreases relative to the stiffness of the restraints, the uniform restraint force in the sheathing decreases and more of the overturning force is resisted by the third point anchors. The net restraint force, the sum of forces between the supports and third point anchors, will remain relatively constant as the relative stiffness between the diaphragm and anchors changes. Only the distribution of forces between the third points and supports changes significantly.

The first step is to calculate the stiffness of the anchors at each anchorage location. At the frame line, the total stiffness, K_{spt} (Equation (5.1.88)) is the sum of the anchorage stiffness, $(K_{rest})_{spt}$ (Equations (5.1.30), (5.1.33)), and the stiffness of the purlin to rafter connection, K_{rafter} (Equations (5.1.35), (5.1.36)), for the purlins not directly connected to an anchor. For an interior frame line, the tributary stiffness, K_{trib}, to each half span adjacent to the frame line is half the total stiffness at the frame line. At an exterior frame line or in a single span configuration, the tributary stiffness to the half span adjacent to the frame line is the total stiffness. The total stiffness at the third point, K_{3rd} (Equation (5.1.87)) is the sum of the stiffness of all the anchors along the line of anchorage $(K_{rest})_{3rd}$ (Equation (5.1.34).

The equation for the uniform restraint force in the sheathing for a supports plus third points configuration is based on the equation for a third points configuration with additional factors applied to account for the stiffness of the anchors, local bending effects and system effects. The uniform restraint force in the sheathing for a supports plus third points configuration should always be less than the same system with a third point only anchorage configuration.

The system of purlins is divided into half spans. For each purlin, the ratio of uniform restraint force in the sheathing to the uniformly applied load, σ (Equation (5.1.93)), is calculated. The overturning forces at the third point, $(P_{3rd})_i$ (Equation (5.1.89)), and the frame line, $(P_{spt})_i$ (Equation (5.1.90)) are calculated directly for each purlin. Purlin system effects are included in the calculation for $(P_{3rd})_i$ and $(P_{spt})_i$. The anchorage force for each anchor at the top of the purlin, $(P_L)_{3rd}$ (Equation (5.1.84)) or $((P_L)_{spt}$ Equation (5.1.85)) is the sum of all the overturning forces at that location, ($(P_{spt})_i$ or $(P_{3rd})_i$), multiplied by the ratio of the stiffness of the individual anchor, (K_{rest}), to the total stiffness at the location, (K_{spt} or K_{3rd}).

For supports plus third points anchorage configuration, lateral displacements tend to be small. Lateral displacement, Δ_{rest} (Equation (5.1.104)), is checked at the frame line and third points and compared to the allowable anchor deflection in the *Specification*. Because there is little diaphragm displacement between the third points, the deflection of the interior of the span is approximated as the displacement at the third points. The difference between the third point and support displacement at service load levels is compared to the limit of L/360 in the *Specification.*

Forces in the connection between the purlin and the sheathing at the anchorage location, P_{sc}, are calculated the same as for a supports anchorage configuration for anchors along the frame line (Equation (5.1.105)) and for a third points configuration for the anchors at the third points (Equation (5.1.106)). Because the uniform restraint force in the sheathing is typically less for a supports plus third points configuration relative to a supports configuration, the uniform restraint force along the frame line will typically be less for a supports plus third points configuration than a similar system with supports only anchors. Conversely, the force in the connection between the sheathing and purlin will typically be larger at the third points in a supports plus third points configuration than a similar system with only third point restraints.

5.1.5 Tests to Determine Stiffness of Components

Anchor stiffness. The anchor stiffness at the frame line may also be determined by a simple test procedure with the apparatus shown in Figure 5.1.15. The apparatus consists of a segment of purlin approximately 2 ft. long anchored in the manner representing the typical anchorage device connection. For a typical through fastened rib style sheathing, a total of 3 fasteners at 12 in. intervals should be used to connect the purlin to the sheathing, with the center fastener located directly over the centerline of the restraint. For a standing seam profile deck, the seam of the deck with a single clip should be centered directly over the restraint. The stiffness of the sheathing connection is affected by the presence of insulation so if insulation is to be incorporated into the actual roof system, it should be included in the test as well. Displacement should be recorded as close as possible to the top flange of the purlin, Δ_1, and if it is desirable to capture the relative slip between the Z-section and sheathing, the deflection of the sheathing, Δ_2. The sheathing is permitted to move laterally but prevented from moving vertically at a distance of span/2 from the purlin where "span" is the purlin spacing.

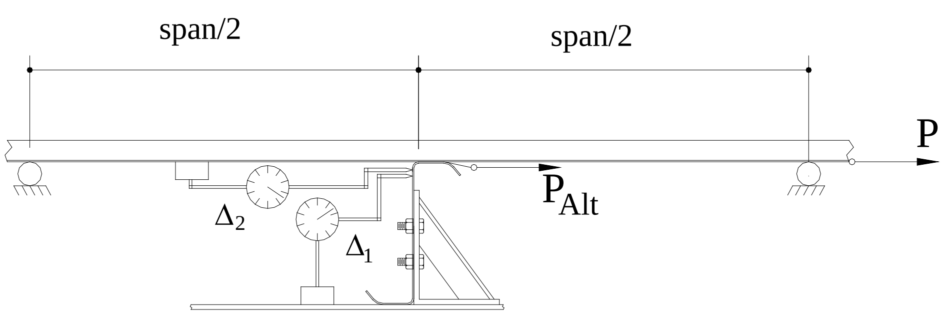


Figure 5.1.15 Test to Determine Stiffness of Support Restraint

Horizontal load, P, is applied through the sheathing parallel to the original plane of the sheathing. Applying the load through the sheathing provides verification of the strength of the sheathing-to-purlin connection as well. Alternatively, the horizontal load, P_{Alt}, can be applied directly to the top flange of the purlin if the connection between the purlin and sheathing possesses considerable slip. The restraint stiffness is defined as the load applied at the top flange, P, per unit displacement at the top flange, Δ_1. For a non-linear relationship between displacement and applied load, a criterion for determining the nominal stiffness similar to that of AISI Test S901 in the *Cold-Formed Steel Design Manual* (2008a) should be used. This test procedure captures the net restraint stiffness, that is, the combined effect of the device stiffness and the configuration stiffness. The component stiffness method does not make accommodations for slip between the Z-section and sheathing so if excessive slip between the Z-section and sheathing is observed through this test procedure, some mechanism for transferring forces through the system of Z-sections should be considered.

Stiffness of purlin – sheathing connection. The rotational stiffness of the connection between the purlin and the sheathing, k_{mclip}, is defined as the moment generated per unit rotation of the purlin per unit length of the purlin. This moment is developed as a result of prying action. For a through fastened system, the connection is made with a single screw placed near a rib of the sheathing and into the top flange of the purlin. As the purlin rotates, a compressive force is developed between the sheathing and the tip of the flange, and tension is developed in the fastener. The stiffness of the connection is a function of many factors: the purlin thickness, flange width and spacing, the deck thickness and moment of inertia, the fastener spacing, position of the fastener relative to the web of the purlin and relative to the sheathing rib and the presence of insulation.

For a standing seam clip, as the purlin rotates, compression is developed in the shoulders of the clip and tension is developed in the tab as it pulls from the seam. Because it is connected directly to the seam it can possess considerable stiffness. For a standing seam system, the stiffness of the sheathing-to-purlin connection is a function of the clip material and geometry, the "tightness" of clip tab in the panel seam, the purlin thickness and the presence of insulation.

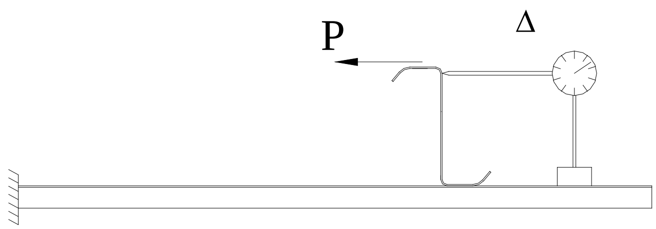


Figure 5.1.16 Test Set-up for Determining Stiffness of Sheathing Z-Section Connection

With so many factors involved, the stiffness of the connection between the purlin and sheathing, k_{mclip}, cannot easily be determined analytically but can be readily determined by test. The test procedure is outlined in the *Cold-Formed Steel Design Manual* (2008a)*:* AISI S901-*Rotational Lateral Test Method for Beam-to-Panel Assemblies.* The basic test assembly is shown in Figure 5.1.16. A panel segment with a span representative of the purlin spacing in a roof system is attached to a segment of purlin and a load is applied to the free flange of the purlin. As the lateral load, P, is applied to the free flange of the purlin, the lateral displacement, Δ, of the free flange is measured. By relating the displacement to the applied load the rotational-lateral stiffness is determined. Some modifications to the results of the test procedure are required to determine the rotational stiffness of the sheathing-to-purlin connection, k_{mclip}.

The displacement of the free flange measured according to AISI S901 is the combined displacement of the flexure in the web of the purlin and the rotation of the sheathing-to-purlin connection. Provided the apparatus is set up as prescribed in AISI S901, additional deformation due to flexure of the sheathing is eliminated. To determine the sheathing-to-purlin connection stiffness, k_{mclip}, the displacement due to the flexibility of the purlin web must be eliminated as described by Heinz (1994). The displacement of the web is approximated from theory by treating the purlin web as fixed-free cantilever beam element, and the resulting stiffness of the connection between the sheathing and the deck is

$$k_{conn} = \frac{P_N d^2}{L_B}\left(\frac{1}{\Delta_N - \dfrac{4P_N d^3}{EL_B t^3}}\right) \tag{5.1.28}$$

where P_N is the nominal test load, Δ_N is the nominal test displacement, L_B is the purlin length, d is the depth of the purlin, and t is the thickness of the purlin. Equation 5.1.28 thus provides the stiffness of the connection between the purlin and the sheathing in terms of moment per unit displacement of the free flange per unit length of the purlin. The net rotational stiffness, k_{mclip}, must also include the rotational flexibility of the sheathing spanning between purlin lines. This flexibility, derived from theory, is added to the flexibility of the sheathing-to-purlin connection and the net stiffness of the purlin-sheathing connection is

$$k_{mclip} = \frac{12E \cdot I_{panel} \cdot k_{conn}}{span \cdot k_{conn} + 12E \cdot I_{panel}} \tag{5.1.29}$$

where I_{panel} = the moment of inertia of the sheathing panel; and span = the distance between the centerline of each span of the sheathing.

5.1.6 Equation Summary

Summary of Stiffness Equations

Restraint Stiffness – Support Restraints

$$K_{rest} = \frac{\left(\frac{h}{d}\right)^2 K_{device} \cdot K_{config}}{\frac{h}{d} K_{device} + K_{config}} \tag{5.1.30}$$

with

K_{device} = stiffness of anchorage device at height along web of purlin restraint is applied (lb/in)

K_{config} = stiffness of anchorage configuration (lb/in)

Discrete Brace

$$K_{config} = \frac{\frac{1}{15} d(3Et^3)}{h(d-h)^2}\left[\frac{\frac{1}{15} d \cdot 2Et^3(3d-h) + \frac{L}{80} k_{mclip} \cdot d(3d-2h)}{\frac{1}{15} d \cdot Et^3(4d-h) + \frac{L}{80} k_{mclip} \cdot d(d-h)}\right] \tag{5.1.31}$$

Anti-roll Clip

$$K_{config} = \frac{Eb_{pl} t^3}{(d-h)^3}\left(\frac{d}{h}\right) \tag{5.1.32}$$

Total Stiffness of rafter web plate (welded wing plate)

$$K_{rest,webplate} = \frac{E \cdot b_{pl} \cdot t_{pl}{}^3 \cdot t^3\left(t^3 h + t_{pl}^3 (d-h)\right)}{\left(t^3 h^2 - t_{pl}^3 (d-h)^2\right)^2 + 4t^3 t_{pl}{}^3 d^2 h(d-h)} \tag{5.1.33}$$

Restraint Stiffness – Interior Restraint

$$K_{rest} = \left(\frac{h}{d}\right)^2 K_{device} \tag{5.1.34}$$

with

K_{device} = stiffness of restraining device at restraint height (lb/in)

Rafter Stiffness

Web Bolted to Rafter Clip

$$K_{Rafter} = \frac{E \cdot b_{pl} \cdot t_{pl}{}^3 \cdot t^3}{2t^3\left(h^2(3d-h)\right) + 4t_{pl}{}^3 (d-h)^3} \tag{5.1.35}$$

Flange Bolted

$$K_{Rafter} = 0.45\frac{Et^3}{2d} \tag{5.1.36}$$

Sheathing Stiffness

$$K_{shtg} = \frac{k_{mclip} L}{d}\left(\frac{\frac{1}{4} Et^3}{0.38 k_{mclip} d + 0.71\frac{Et^3}{4}}\right)\left(1 - \frac{2}{3} k_{mclip} \tau\right) \tag{5.1.37}$$

Summary of Torsion Equations

$$a = \sqrt{\frac{EC_W}{GJ}} \tag{5.1.38}$$

$$\tau = \frac{\dfrac{a^2\beta}{GJ}}{1 + \dfrac{k_{mclip}}{GJ}\kappa} \tag{5.1.39}$$

Single span and multi-span exterior half span adjacent to exterior frame line (Warping "Free") From Seaburg and Carter (1997).

$$\beta = \frac{L^2}{8a^2} + \frac{1}{\cosh\left(\dfrac{L}{2a}\right)} - 1 \tag{5.1.40}$$

$$\kappa = \frac{8a^4}{L^2}\left(\frac{\cosh\left(\dfrac{L}{2a}\right) - 1}{\cosh\left(\dfrac{L}{2a}\right)}\right) + \frac{5L^2}{48} - a^2 \tag{5.1.41}$$

$$\beta_{3rd} = \frac{L}{GJ}\left(\frac{1}{3} - \frac{a}{L}\frac{\sinh\left(\dfrac{L}{3a}\right)}{\cosh\left(\dfrac{L}{2a}\right)}\right) \tag{5.1.42}$$

Multi-span interior and multi-span exterior half span adjacent to interior frame line (Warping "Fixed") From Seaburg and Carter (1997)

$$\beta = \frac{L^2}{8a^2} + \frac{L}{2a}\frac{1 - \cosh\left(\dfrac{L}{2a}\right)}{\sinh\left(\dfrac{L}{2a}\right)} \tag{5.1.43}$$

$$\kappa = \left(\frac{aL}{3} - \frac{4a^3}{L}\right)\left(\frac{1 - \cosh\left(\dfrac{L}{2a}\right)}{\sinh\left(\dfrac{L}{2a}\right)}\right) + \frac{5L^2}{48} - a^2 \tag{5.1.44}$$

$$\beta_{3rd} = \frac{L}{GJ}\left(\frac{1}{3} + \frac{a}{L}\left(\frac{\left(1 - \cosh\left(\dfrac{L}{3a}\right)\right)\left(1 - \cosh\left(\dfrac{L}{2a}\right)\right)}{\sinh\left(\dfrac{L}{2a}\right)} - \sinh\left(\frac{L}{3a}\right)\right)\right) \tag{5.1.45}$$

Supports Anchorage Configuration

Anchorage force per anchorage device

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} \quad \text{(force at top of purlin)} \tag{5.1.46}$$

$$P_h = P_L \frac{d}{h} \qquad \text{(force at height of anchor)} \tag{5.1.47}$$

Where

$$K_{total} = \sum_{N_a} K_{rest} + \frac{\sum_{N_p} K_{shtg} + \sum_{N_p - N_a} K_{rafter}}{d} \tag{5.1.48}$$

with

K_{rest} = stiffness of externally applied restraint (lb/in)

K_{rafter} = rotational stiffness of the purlin to rafter connection (lb-in/in)

K_{shtg} = rotational stiffness provided by the sheathing (lb-in/in)

Total force generated per purlin per half span

$$P_i = \frac{wL}{2d} \cdot \left[\left(\delta b \cos\theta (1 - R_{local}) + \tfrac{2}{3} k_{mclip} \left(\sigma \frac{d}{2} - (\delta b + m)\cos\theta \right) \tau \right) \alpha - d \sin\theta \right] \tag{5.1.49}$$

$$R_{local} = \frac{k_{mclip}}{k_{mclip} + \dfrac{Et^3}{3d}} \tag{5.1.50}$$

$$\sigma = \frac{C1 \dfrac{\left(\dfrac{I_{xy}}{I_x} \cos\theta \right) L^4}{EI_{my}} + \dfrac{((\delta b + m)\cos\theta)d}{2} \tau + \dfrac{\alpha \cdot n_p \cdot L^2 \sin\theta}{8G'Bay}}{C1 \dfrac{L^4}{EI_{my}} + \dfrac{d^2}{4} \tau + \dfrac{\alpha \cdot \eta \cdot L^2}{8G'Bay}} \tag{5.1.51}$$

where C1 = 5/384 Single Span
= 1/185 Multi-Span Exterior
= 1/384 Multi-Span Interior

Single Span

Total stiffness is the sum of restraints at the frame line, sum of rafter stiffness at the frame line and sum of half the sheathing stiffness for all purlins in the bay.

Total force is sum of P_i for all purlins in the bay.

Multi-Span Half Span Adjacent to Exterior Frame Line

Total stiffness is the sum of restraints at the exterior frame line, sum of rafter stiffness at exterior frame line and half the sum of sheathing stiffness for purlins in exterior bay.

Total force is sum of P_i for all purlins for the half bay closest to the exterior frame line.

Multi-Span Interior

Total stiffness is the sum of restraints at the interior frame line, sum of rafter stiffness at the frame line, half the sum of sheathing stiffness for purlins in each bay adjacent to the frame line.

Total force is sum of P_i for all purlins in each half bay adjacent to the frame line.

Lateral Displacement of top flange of purlin at frame line.

$$\Delta_{rest} = \frac{P_L}{K_{rest}} \tag{5.1.52}$$

Deformation of diaphragm at mid-span relative to restraint

$$\Delta_{diaph} = \sum_{N_p} \left(w(\alpha\sigma - \sin\theta) \right)_i \frac{L^2}{8G'Bay} \tag{5.1.53}$$

Shear force in purlin sheathing connection at restraint

$$P_{sc} = P_L + \frac{wL}{2}(0.9\sigma\alpha + \sin\theta) - P_i \tag{5.1.54}$$

Third Point Anchorage

Anchorage force per anchorage device

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} \quad \text{(force at top of purlin)} \tag{5.1.55}$$

$$P_h = P_L \frac{d}{h} \quad \text{(force at height of anchor)} \tag{5.1.56}$$

where

$$K_{total} = \sum K_{rest} + \frac{\sum K_{shtg}}{d} \tag{5.1.57}$$

with

K_{rest} = stiffness of externally applied restraint (lb/in)

K_{shtg} = rotational stiffness provided by the sheathing (lb-in/in)

Total overturning force generated per purlin per half span

$$P_i = \frac{wL}{2d} \cdot \left[\begin{array}{l} \left(\delta b \cos\theta (1 - R_{local}) + \frac{2}{3} k_{mclip} \left(\sigma \frac{d}{2} - (\delta b + m)\cos\theta \right) \tau \right) \alpha - d \sin\theta \\ + \dfrac{L^2 k_{mclip}}{3G'Bay \cdot d} \left(1 - \frac{2}{3} k_{mclip}\right) \left(\dfrac{\eta\sigma}{3} + \dfrac{n_p \sin\theta}{6} - \dfrac{\eta \cdot \delta b \cos\theta}{2d} \right) \end{array} \right] \tag{5.1.58}$$

$$R_{local} = \frac{k_{mclip}}{k_{mclip} + \frac{Et^3}{3d}} \tag{5.1.59}$$

$$\sigma = \frac{C1\frac{\left(\frac{I_{xy}}{I_x}\cos\theta\right)L^4}{EI_{my}} + \frac{((\delta b + m)\cos\theta)d}{2}\tau - \frac{\alpha \cdot n_p \cdot L^2 \sin\theta}{18G'Bay}}{C1\frac{L^4}{EI_{my}} + \frac{d^2}{4}\tau + \frac{\alpha \cdot \eta \cdot L^2}{9G'Bay}} \tag{5.1.60}$$

where C1 = 11/972 Single Span
= 5/972 Multi-Span Exterior – outer half span
= 7/1944 Multi-Span Exterior – inner half span
= 1/486 Multi-Span Interior

Distribution of Forces - all conditions (single and multiple span)

Total stiffness is the sum of restraints along third point and half the sum of the sheathing stiffness for all purlins in the bay

Total force is the sum of P_i for each half span for all purlins in the bay.

Lateral Displacement of top flange of purlin at frame line.

$$\Delta_{rest} = \frac{P_L}{K_{rest}} \tag{5.1.61}$$

Deformation of diaphragm at frame lines relative to restraint

$$\Delta_{diaph} = \sum_{N_p}\left(-w\left(\alpha\sigma - \sin\theta\frac{\sum K_{rest}}{K_{total}}\right)\right)_i \frac{L^2}{9G'Bay} + \frac{\sum P_L L}{3G'Bay} \tag{5.1.62}$$

Shear force in purlin - sheathing connection at restraint

$$P_{sc} = P_L + \frac{wL}{20}\left(-0.9\sigma + \frac{\delta b\cos\theta}{d}\right)\alpha \tag{5.1.63}$$

Midpoint Point Anchorage

Anchorage force per anchorage device

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} \quad \text{(force at top of purlin)} \tag{5.1.64}$$

$$P_h = P_L\frac{d}{h} \quad \text{(force at height of anchor)} \tag{5.1.65}$$

where

$$K_{total} = \sum K_{rest} + \frac{\sum K_{shtg}}{d} \tag{5.1.66}$$

with

K_{rest} = stiffness of externally applied restraint (lb/in)

K_{shtg} = rotational stiffness provided by the sheathing (lb-in/in)

Total overturning force generated per purlin per span

$$P_i = \frac{wL}{d} \cdot \left[\begin{array}{l} \left(\delta b \cos\theta (1 - R_{local}) + \tfrac{2}{3} k_{mclip} \left(\sigma \frac{d}{2} - (\delta b + m) \cos\theta \right) \tau \right) \alpha - d \sin\theta \\ + \frac{L^2 k_{mclip}}{4G'Bay \cdot d} \left(1 - \tfrac{2}{3} k_{mclip} \right) \left(\frac{\eta\sigma}{2} + \frac{N_p \sin\theta}{2} - \frac{\eta \cdot \delta b \cos\theta}{d} \right) \end{array} \right] \tag{5.1.67}$$

$$R_{local} = \frac{k_{mclip}}{k_{mclip} + \frac{Et^3}{3d}} \tag{5.1.68}$$

$$\sigma = \frac{C1 \frac{\left(\frac{I_{xy}}{I_x} \cos\theta \right) L^4}{EI_{my}} + \frac{((\delta b + m)\cos\theta) d}{2} \tau - \frac{\alpha \cdot N_p \cdot L^2 \sin\theta}{8G'Bay}}{C1 \frac{L^4}{EI_{my}} + \frac{d^2}{4} \tau + \frac{\alpha \cdot \eta \cdot L^2}{8G'Bay}} \tag{5.1.69}$$

where C1 = 5/384 Single Span
= 1/185 Multi-Span Exterior
= 1/384 Multi-Span Interior

Distribution of Forces - all conditions (single and multiple span)

Total stiffness is the sum of restraints along midpoint and the sum of the sheathing stiffness for all purlins in the bay

Total force is the sum of P_i for all purlins in the bay.

Lateral Displacement of top flange of purlin at frame line.

$$\Delta_{rest} = \frac{P_L}{K_{rest}} \tag{5.1.70}$$

Deformation of diaphragm at frame lines relative to restraint

$$\Delta_{diaph} = \sum_{N_p} \left(-w \left(\alpha\sigma - \sin\theta \frac{\sum K_{rest}}{K_{total}} \right) \right)_i \frac{L^2}{8G'Bay} + \frac{\sum P_L L}{4G'Bay} \tag{5.1.71}$$

Shear force in purlin - sheathing connection at restraint

$$P_{sc} = P_L + \frac{wL}{20} \left(-0.9\sigma + \frac{\delta b \cos\theta}{d} \right) \alpha \tag{5.1.72}$$

Supports Plus Third Point Torsional Restraint

Moment in each third point torsion restraint

$$M_{3rd} = w\left(\alpha\left(\sigma\frac{d}{2}-(\delta b+m)\cos\theta\right)\frac{a^2\beta}{GJ}\left(d^2\frac{K_{trib}}{n_p}+\frac{L}{9}k_{mclip}\right)+\left(d\sin\theta-(\alpha)\delta b\cos\theta(1-R_{local})\right)\frac{L}{2}\right)\xi \tag{5.1.73}$$

Anchorage force per anchor along frame line (at the top of the purlin)

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} \quad \text{(force at top of purlin)} \tag{5.1.74}$$

$$P_h = P_L\frac{d}{h} \quad \text{(force at height of anchor)} \tag{5.1.75}$$

where

$$K_{total} = \sum_{N_a} K_{rest} + \frac{\sum\limits_{n_p-n_a} K_{rafter}}{d} \tag{5.1.76}$$

with

K_{rest} = stiffness of externally applied restraint (lb/in)

K_{shtg} = rotational stiffness provided by the sheathing (lb-in/in)

K_{trib} = stiffness at frame line tributary to each half span
= C3 K_{total}

C3 = 1.0 single span and multi-span exterior frame line
= 0.5 multi-span interior frame line

Overturning force generated per purlin

$$P_i = w\left(\alpha\left(\sigma\frac{d}{2}-(\delta b+m)\cos\theta\right)\frac{a^2\beta}{GJ}+\left(d\sin\theta-(\alpha)\delta b\cos\theta(1-R_{local})\right)\frac{\beta_{3rd}L}{2}\right)d\cdot\frac{K_{trib}}{N_p}\xi \tag{5.1.77}$$

Where

$$\xi = \frac{1}{1+\left(d^2\frac{K_{trib}}{n_p}+\frac{L}{9}k_{mclip}\right)\beta_{3rd}} \tag{5.1.78}$$

with

$$R_{local} = \frac{k_{mclip}}{k_{mclip}+\frac{Et^3}{3d}} \tag{5.1.79}$$

$$\sigma = \frac{C1\dfrac{\left(\dfrac{I_{xy}}{I_x}\cos\theta\right)L^4}{EI_{my}} + \dfrac{((\delta b + m)\cos\theta)d}{2}\left(\dfrac{a^2\beta}{GJ} - \dfrac{L}{2}\cdot\beta_{3rd}\right)\xi + \dfrac{\alpha\cdot n_p\cdot L^2\sin\theta}{8G'Bay}}{C1\dfrac{L^4}{EI_{my}} + \dfrac{d^2}{4}\dfrac{a^2\beta}{GJ}\xi + \dfrac{\alpha\cdot\eta L^2}{8G'Bay}} \qquad (5.1.80)$$

where C1 = 5/384 Single Span
= 1/185 Multi-Span Exterior
= 1/384 Multi-Span Interior

Single Span

Total stiffness is the sum of restraints at one frame line, sum of rafter stiffness at one frame line and sum of half the sheathing stiffness for all purlins in the bay.

Total force is sum of P_i for all purlins in the bay.

Multi-Span Half Span Adjacent to Exterior Frame Line

Total stiffness is the sum of restraints at the exterior frame line, sum of rafter stiffness at exterior frame line and half the sum of sheathing stiffness for purlins in exterior bay.

Total force is sum of P_i for all purlins for the half span closest to the exterior frame line.

Multi-Span Interior

Total stiffness is the sum of restraints at the interior frame line, sum of rafter stiffness at the frame line, half the sum of sheathing stiffness for purlins in each half span adjacent to the frame line.

Total force is sum of P_i for all purlins in each half span adjacent to the frame line.

Lateral Displacement of top flange of purlin at frame line.

$$\Delta_{rest} = \frac{P_L}{K_{rest}} \qquad (5.1.81)$$

Deformation of diaphragm at mid-span relative to restraint

$$\Delta_{diaph} = \sum_{N_p}(w(\alpha\sigma - \sin\theta))_i\frac{L^2}{8G'Bay} \qquad (5.1.82)$$

Shear force in purlin - sheathing connection at restraint

$$P_{sc} = P_L + \frac{wL}{2}(0.9\sigma\alpha + \sin\theta) - P_i \qquad (5.1.83)$$

Supports Plus Third Point Lateral Anchorage

Anchorage force per anchorage device

$$(P_L)_{3rd} = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{3rd}} \quad \text{(force at top of purlin at 3rd point anchor)} \tag{5.1.84}$$

$$(P_L)_{spt} = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{spt}} \quad \text{(force at top of purlin at frame line anchor)} \tag{5.1.85}$$

$$P_h = P_L \frac{d}{h} \qquad \text{(force at height of anchor)} \tag{5.1.86}$$

where

$$K_{3rd} = \sum_{N_a} (K_{rest})_{3rd} \tag{5.1.87}$$

$$K_{spt} = \left(\sum_{N_a} (K_{rest})_{spt} + \frac{\sum_{N_p - N_a} K_{rafter}}{d} \right) \tag{5.1.88}$$

with

$(K_{rest})_{3rd}$ = stiffness of anchor at purlin third point (lb/in)

$(K_{rest})_{spt}$ = stiffness of anchor at frame line (lb/in)

K_{rafter} = rotational stiffness of the Z-section to rafter connection (lb-in/in)

K_{trib} = stiffness at frame line tributary to each half span

= C3 K_{spt}

C3 = 1.0 single span and multi-span exterior frame line

= 0.5 multi-span interior frame line

Total force generated per purlin per half span distributed between third point and frame line anchors

$$(P_{3rd})_i = \frac{wL}{2} \cdot \left[\begin{array}{l} \left(\frac{\delta b \cos\theta}{d}(1 - R_{local}) + \frac{2}{3} k_{mclip} \left(\frac{\sigma}{2} - \frac{(\delta b + m)}{d} \cos\theta \right) \tau \right) \left(\frac{2d^2 K_{trib}}{2d^2 K_{trib} + n_p k_{mclip} L R_{sys}} \right) \psi\alpha \\ + \frac{2K_{trib} \sigma L}{9G'Bay} \psi\alpha - d \sin\theta D_{3rd} \end{array} \right] \tag{5.1.89}$$

$$(P_{spt})_i = \frac{wL}{2} \cdot \left[\begin{array}{l} \left(\frac{\delta b \cos\theta}{d}(1 - R_{local}) + \frac{2}{3} k_{mclip} \left(\frac{\sigma}{2} - \frac{(\delta b + m)}{d} \cos\theta \right) \tau \right) \left(\frac{2d^2 K_{trib}}{2d^2 K_{trib} + n_p k_{mclip} L R_{sys}} \right) \alpha(1 + F) \\ + \frac{2K_{trib} \sigma L}{9G'Bay} \alpha F - d \sin\theta D_{spt} \end{array} \right] \tag{5.1.90}$$

where

$$R_{sys} = \left(\frac{\frac{1}{4} Et^3}{0.38 k_{mclip} d + 0.71 \frac{1}{4} Et^3} \right) \left(1 - \frac{2}{3} k_{mclip} \tau \right) \tag{5.1.91}$$

$$R_{local} = \frac{k_{mclip}}{k_{mclip} + \frac{Et^3}{3d}} \tag{5.1.92}$$

$$\sigma = \frac{C1\frac{\left(\frac{I_{xy}}{I_x}\cos\theta\right)L^4}{EI_{my}} + A \cdot \frac{((\delta b)\cos\theta)d}{2}\tau + B \cdot \frac{N_p \alpha L^2 \sin\theta}{18G'Bay}}{C1\frac{L^4}{EI_{my}} + C \cdot \frac{d^2}{4}\tau + \frac{\eta\alpha L^2}{9G'Bay}(1 + K_{trib}\psi\Gamma)} \tag{5.1.93}$$

where C1 = 11/972 Single Span
= 5/972 Multi-Span Exterior – outer half span
= 7/1944 Multi-Span Exterior – inner half span
= 1/486 Multi-Span Interior

$$\psi = \frac{(3G'BayK_{3rd})\left(1 + k_{mclip}\frac{\kappa}{GJ}\right)(N_p k_{mclip} LR_{sys} + 2d^2 K_{trib})}{X1 + X2} \tag{5.1.94}$$

$$X1 = (3G'BayK_{trib} + LK_{trib}K_{3rd})\left(1 + k_{mclip}\frac{\kappa}{GJ}\right)(N_p k_{mclip} LR_{sys} + 2d^2 K_{trib}) \tag{5.1.95}$$

$$X2 = (3G'BayK_{3rd})(2d^2 K_{trib})\left(1 + k_{mclip}\frac{\kappa}{GJ} - \frac{\beta_{3rd}}{4}\frac{2}{3}k_{mclip}L\right) \tag{5.1.96}$$

$$\Gamma = \left(C2\frac{\alpha L^3}{EI_{my}} + \frac{\alpha\frac{d^2}{4}\beta_{3rd}}{\left(1 + k_{mclip}\frac{\kappa}{GJ}\right)} - \frac{N_p \cdot L}{3G'Bay}\right) \tag{5.1.97}$$

where C2 = 5/162 Single Span
= 31/1458 Multi-Span Exterior – outer half span
= 15/1458 Multi-Span Exterior – inner half span
= 1/162 Multi-Span Interior

$$A = \left(\frac{\delta b + m}{\delta b}\right) + \eta\alpha\frac{L}{d^2}\left(\frac{2}{3}k_{mclip}\left(\frac{\delta b + m}{\delta b}\right) - \frac{(1 - R_{Local})}{\tau}\right)\left(\frac{2d^2K_{trib}}{2d^2K_{trib} + N_p k_{mclip} LR_{sys}}\right)\Gamma\psi \tag{5.1.98}$$

$$B = \frac{(2K_{trib} - K_{3rd})3G'Bay}{LK_{trib}K_{3rd} + 3G'Bay(K_{trib} + K_{3rd})} \tag{5.1.99}$$

$$C = 1 + \alpha\eta\frac{\frac{2}{3}k_{mclip}L}{d^2}\left(\frac{2d^2K_{trib}}{2d^2K_{trib} + N_p k_{mclip} LR_{sys}}\right)\Gamma\psi \tag{5.1.100}$$

$$D_{3rd} = \frac{K_{3rd}(9G'Bay + 2LK_{trib})}{3[LK_{trib}K_{3rd} + 3G'Bay(K_{trib} + K_{3rd})]}\left(\frac{2d^2(K_{trib} + K_{3rd})}{2d^2(K_{trib} + K_{3rd}) + N_p k_{mclip} LR_{sys}}\right) \quad (5.1.101)$$

$$D_{Spt} = \frac{K_{trib}(9G'Bay + LK_{3rd})}{3[LK_{trib}K_{3rd} + 3G'Bay(K_{trib} + K_{3rd})]}\left(\frac{2d^2(K_{trib} + K_{3rd})}{2d^2(K_{trib} + K_{3rd}) + N_p k_{mclip} LR_{sys}}\right) \quad (5.1.102)$$

$$F = \frac{\left(\frac{\beta_{3rd}}{4}\frac{2}{3}k_{mclip}L - \left(1 + k_{mclip}\frac{\kappa}{GJ}\right)\right)}{\left(1 + k_{mclip}\frac{\kappa}{GJ}\right)}\left(\frac{2d^2K_{trib}}{2d^2K_{trib} + N_p k_{mclip} LR_{sys}}\right)\psi \quad (5.1.103)$$

Single Span

Total force at each frame line is the sum of $(P_{spt})_i$ for the half span adjacent to the frame line for all purlins in the bay. Total force at each third point is the sum of $(P_{3rd})_i$ for the half span containing the third point for all purlins in the bay. Forces must be distributed along each anchor point (support or third point) according to the relative stiffness of all of the anchors at all purlins at that point.

Multi-Span Half Span Adjacent to Exterior Frame Line

Total force at exterior frame line is the sum of $(P_{spt})_i$ for the half span adjacent to the frame line for all purlins in the bay. Total force at the exterior third point is the sum of $(P_{3rd})_i$ for the half span containing the third point for all purlins in the bay. Forces must be distributed along each anchor point (support or third point) according to the relative stiffness of all of the anchors at all purlins at that point.

Multi-Span Interior

Total force at interior frame line is the sum of $(P_{spt})_i$ for each half span adjacent to the frame line for all purlins in the bay. Total force at the exterior third point is the sum of $(P_{3rd})_i$ for the half span containing the third point for all purlins in the bay. Forces must be distributed along each anchor point (support or third point) according to the relative stiffness of all of the anchors at all purlins at that point.

Lateral displacement of top flange of purlin at frame line or third point anchor.

$$\Delta_{rest} = \frac{P_L}{K_{rest}} \quad (5.1.104)$$

Shear force in purlin - sheathing connection at support anchor

$$P_{sc} = P_L + \frac{wL}{2}(0.9\sigma\alpha + \sin\theta) - P_i \quad (5.1.105)$$

Shear force in purlin - sheathing connection at third point anchor

$$P_{sc} = P_L + \frac{wL}{20}\left(-0.9\sigma + \frac{\delta b\cos\theta}{d}\right)\alpha \quad (5.1.106)$$

5.1.7 Z-Section Examples

Four examples using the component stiffness method to predict anchorage forces are provided based on the roof system from the continuous purlin design example in Section 3.2. The roof system has four 25 ft spans with purlins lapped over the interior supports. The purlins in the exterior spans are 8Z2.75x085 and the interior spans are 8Z2.75x059. There are a total of 12 purlin lines spaced at 5 ft-0 in. on center with the top flange of the purlin closest to the eave turned down slope while the top flanges of the remaining purlins face upslope. Roof slope is a ½ on 12 pitch and the gravity loads are 3 psf dead and 20 psf live. Roof covering is attached with standing seam panel clips along entire length of purlins. The sheathing has a diaphragm stiffness G′ = 1000 lb/in. and the rotational stiffness of the standing seam panel clips, k_{mclip} = 2500 lb-in./(rad.-ft).

In example 8, the anchorage forces are calculated for the anti-roll anchorage devices applied along the frame line at every fourth purlin. In the second example, anti-roll anchorage devices are replaced by anchorage applied at the third points of each span. The third example demonstrates anchorage forces for lateral restraint applied along the frame line in conjunction with torsional restraints applied at the third point of each purlin. In the fourth example, third point anchorage devices are combined with anchorage along the frame lines in the form of wing plates (b_{pl}=5 in.) attached to the rafters.

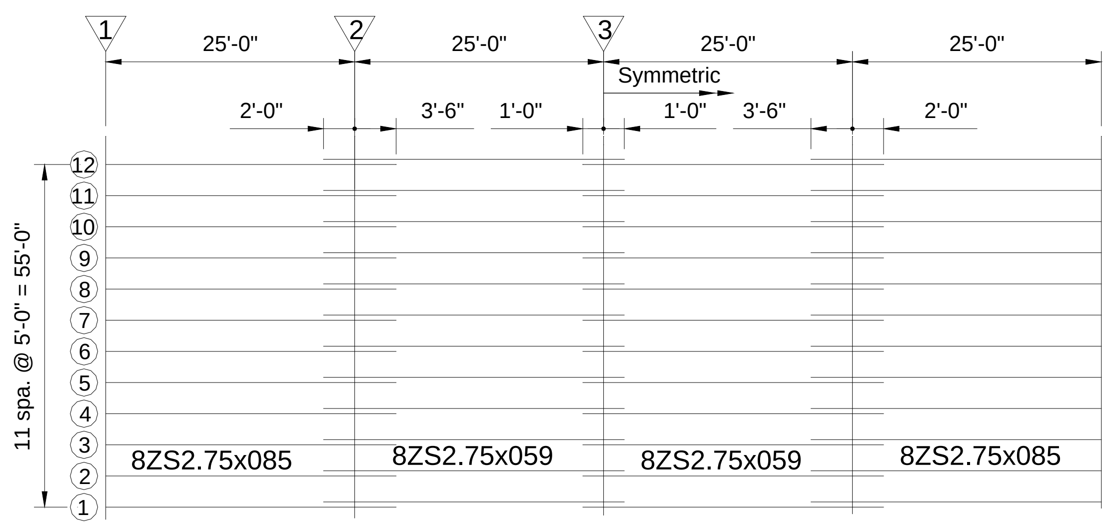


Figure 5.1.17 Roof Layout for Anchorage Examples

The following properties are used in each example.

System Properties

L = 25 ft

Bay = 55 ft

N_p = 12

$\eta = n_{upslope} - n_{downslope} = 11 - 1 = 10$

Uniform load

Dead = 3 psf

Live = 20 psf

$w = (3+20)\text{psf}\cdot 5\text{ft} = 115\text{plf}$

Roof Slope, θ = 2.4 degrees (1/2:12)

G′ = 1000 lb/in

k_{mclip} = 2500 lb-in/rad/ft

E = 29500000 psi

G = 11300000 psi

Section Properties

The following sections properties are used for the two Z-sections:

INTERIOR BAYS	END BAYS
For: 8ZS2.75x059	For: 8ZS2.75x085
t = 0.059 in.	t = 0.085 in.
d = 8.0 in.	d = 8.0 in.
b = 2.75 in.	b = 2.75 in.
I_x = 8.69 in.^{4}	I_x = 12.40 in.^{4}
I_y = 1.72 in.^{4}	I_y = 2.51 in.^{4}
I_{xy} = 2.85 in.^{4}	I_{xy} = 4.11 in.^{4}
J = 0.00102 in.^{4}	J = 0.00306 in.^{4}
C_w = 19.3 in.^{6}	C_w = 28.0 in.^{6}
$I_{my} = \dfrac{I_X I_Y - I_{XY}^2}{I_X}$ = 0.79 in.^{4}	$I_{my} = \dfrac{I_X I_Y - I_{XY}^2}{I_X}$ = 1.15 in.^{4}

Purlin Torsional Properties

Exterior Span 8ZS2.75x085. Outside half span torsionally approximated with both ends "warping free"

$$a = \sqrt{\frac{EC_W}{GJ}} = \sqrt{\frac{E\cdot 28.0\text{in}^6}{G\cdot 0.00306\text{in}^4}} = 154.6\text{ in} \tag{5.1.38}$$

$$\beta = \frac{L^2}{8a^2} + \frac{1}{\cosh\left(\frac{L}{2a}\right)} - 1 = \frac{(300\text{ in})^2}{8\cdot(154.6\text{ in})^2} + \frac{1}{\cosh\left(\frac{300\text{ in}}{2\cdot 154.6\text{ in}}\right)} - 1 = 0.132\text{ rad} \tag{5.1.40}$$

$$\kappa = \frac{8a^4}{L^2}\left(\frac{\cosh\left(\frac{L}{2a}\right)-1}{\cosh\left(\frac{L}{2a}\right)}\right) + \frac{5L^2}{48} - a^2 \tag{5.1.41}$$

$$\kappa = \frac{8(154.6\text{in})^4}{(300\text{in})^2}\left(\frac{\cosh\left(\frac{300\text{in}}{2\cdot 154.6\text{in}}\right)-1}{\cosh\left(\frac{300\text{in}}{2\cdot 154.6\text{in}}\right)}\right) + \frac{5(300\text{in})^2}{48} - (154.6\text{in})^2 = 2582\text{ rad}\cdot\text{in}^2$$

$$\tau = \frac{\frac{a^2\beta}{GJ}}{1+\frac{k_{mclip}}{GJ}\kappa} = \frac{\frac{(154.6\text{ in})^2 \cdot 0.132\text{rad}}{G\cdot 0.00306\text{in}^4}}{1+\frac{2500\,{}^{\text{lb}\cdot\text{in}}\!/_{\text{rad}\cdot\text{ft}}}{G\cdot 0.00306\text{in}^4}\cdot 2582\text{rad}\cdot\text{in}^2} = 0.0055\,\frac{\text{rad}}{\text{lb}} \qquad (5.1.39)$$

$$\beta_{3rd} = \frac{L}{GJ}\left(\frac{1}{3}-\frac{a}{L}\frac{\sinh\left(\frac{L}{3a}\right)}{\cosh\left(\frac{L}{2a}\right)}\right) = \frac{(300\text{in})}{G\cdot 0.00306\text{in}^4}\left(\frac{1}{3}-\frac{154.6\text{ in}}{300\text{in}}\frac{\sinh\left(\frac{300\text{in}}{3\cdot 154.6\text{ in}}\right)}{\cosh\left(\frac{300\text{in}}{2\cdot 154.6\text{ in}}\right)}\right) = 0.000841\,{}^{1}\!/_{\text{lb}\cdot\text{in}}$$

(5.1.42)

Exterior Span 8ZS2.75x085. Inside half span torsionally approximated with both ends "warping fixed"

$$a = \sqrt{\frac{EC_W}{GJ}} = \sqrt{\frac{E\cdot 28.0\text{in}^6}{G\cdot 0.00306\text{in}^4}} = 154.6\text{ in} \qquad (5.1.38)$$

$$\beta = \frac{L^2}{8a^2}+\frac{L}{2a}\frac{1-\cosh\left(\frac{L}{2a}\right)}{\sinh\left(\frac{L}{2a}\right)} = \frac{(300\text{in})^2}{8\cdot(154.6\text{in})^2}+\frac{300\text{in}}{(2)(154.6\text{in})}\frac{1-\cosh\left(\frac{300\text{in}}{2\cdot 154.6\text{in}}\right)}{\sinh\left(\frac{300\text{in}}{2\cdot 154.6\text{in}}\right)} = 0.033\text{ rad}$$

(5.1.43)

$$\kappa = \left(\frac{aL}{3}-\frac{4a^3}{L}\right)\left(\frac{1-\cosh\left(\frac{L}{2a}\right)}{\sinh\left(\frac{L}{2a}\right)}\right)+\frac{5L^2}{48}-a^2 \qquad (5.1.44)$$

$$\kappa = \left(\frac{(154.6\text{in})(300\text{in})}{3}-\frac{4(154.6\text{in})^3}{300\text{in}}\right)\left(\frac{1-\cosh\left(\frac{300\text{in}}{2(154.6\text{in})}\right)}{\sinh\left(\frac{300\text{in}}{2(154.6\text{in})}\right)}\right)+\frac{5(300\text{in.})^2}{48}-(154.6\text{in})^2 = 693.6\text{ rad}\cdot\text{in}^2$$

$$\tau = \frac{\frac{a^2\beta}{GJ}}{1+\frac{k_{mclip}}{GJ}\kappa} = \frac{\frac{(154.6\text{in})^2 \cdot 0.0033\text{rad}}{G\cdot 0.00306\text{in}^4}}{1+\frac{2500\,{}^{\text{lb}\cdot\text{in}}\!/_{\text{rad}\cdot\text{ft}}}{G\cdot 0.00306\text{in}^4}\cdot 693.6\text{rad}\cdot\text{in}^2} = 0.0045\,\frac{\text{rad}}{\text{lb}} \qquad (5.1.39)$$

$$\beta_{3rd} = \frac{L}{GJ}\left[\frac{1}{3}+\frac{a}{L}\left(\frac{\left(1-\cosh\left(\frac{L}{3a}\right)\right)\left(1-\cosh\left(\frac{L}{2a}\right)\right)}{\sinh\left(\frac{L}{2a}\right)}-\sinh\left(\frac{L}{3a}\right)\right)\right] \qquad (5.1.45)$$

$$\beta_{3rd} = \frac{(300\text{in})}{G \cdot 0.00306\text{in}^4}\left[\frac{1}{3} + \frac{154.6\text{in}}{300\text{in}}\left(\frac{\left(1-\cosh\left(\frac{300\text{in}}{3\cdot 154.6\text{in}}\right)\right)\left(1-\cosh\left(\frac{300\text{in}}{2\cdot 154.6\text{in}}\right)\right)}{\sinh\left(\frac{300\text{in}}{2\cdot 154.6\text{in}}\right)} - \sinh\left(\frac{L}{3\cdot 154.6\text{in}}\right)\right)\right] = 0.000231\,{}^{1}\!/_{\text{lb}\cdot\text{in}}$$

Interior Span 8ZS2.75x059. Torsionally approximated with both ends "warping fixed"

$$a = \sqrt{\frac{EC_W}{GJ}} = \sqrt{\frac{E\cdot 19.3\text{in}^6}{G\cdot 0.00102\text{in}^4}} = 222.3\text{ in} \tag{5.1.38}$$

$$\beta = \frac{L^2}{8a^2} + \frac{L}{2a}\frac{1-\cosh\left(\frac{L}{2a}\right)}{\sinh\left(\frac{L}{2a}\right)} = \frac{(300\text{in})^2}{8\cdot(222.3\text{in})^2} + \frac{300\text{in}}{(2)(222.3\text{in})}\frac{1-\cosh\left(\frac{300\text{in}}{2\cdot 222.3\text{in}}\right)}{\sinh\left(\frac{300\text{in}}{2\cdot 222.3\text{in}}\right)} = 0.0081\text{ rad}$$

(5.1.43)

$$\kappa = \left(\frac{aL}{3} - \frac{4a^3}{L}\right)\left(\frac{1-\cosh\left(\frac{L}{2a}\right)}{\sinh\left(\frac{L}{2a}\right)}\right) + \frac{5L^2}{48} - a^2 \tag{5.1.44}$$

$$\kappa = \left(\frac{(222.3\text{in})(300\text{ in})}{3} - \frac{4(222.3\text{ in})^3}{300\text{ in}}\right)\left(\frac{1-\cosh\left(\frac{300\text{ in}}{2(222.3\text{ in})}\right)}{\sinh\left(\frac{300\text{ in}}{2(222.3\text{ in})}\right)}\right) + \frac{5(300\text{ in})^2}{48} - (222.3\text{ in})^2 = 350.9\text{ rad}\cdot\text{in}^2$$

$$\tau = \frac{\frac{a^2\beta}{GJ}}{1+\frac{k_{mclip}}{GJ}\kappa} = \frac{\frac{(222.3\text{in})^2\cdot 0.0081\text{ rad}}{G\cdot 0.00102\text{in}^4}}{1+\frac{2500\,{}^{\text{lb}\cdot\text{in}}\!/_{\text{rad}\cdot\text{ft}}}{G\cdot 0.00102\text{in}^4}\cdot 350.9\text{rad}\cdot\text{in}^2} = 0.0048\,\frac{\text{rad}}{\text{lb}} \tag{5.1.39}$$

$$\beta_{3rd} = \frac{L}{GJ}\left[\frac{1}{3} + \frac{a}{L}\left(\frac{\left(1-\cosh\left(\frac{L}{3a}\right)\right)\left(1-\cosh\left(\frac{L}{2a}\right)\right)}{\sinh\left(\frac{L}{2a}\right)} - \sinh\left(\frac{L}{3a}\right)\right)\right] \tag{5.1.45}$$

$$\beta_{3rd} = \frac{(300\text{in})}{G \cdot 0.00102\text{in}^4}\left[\frac{1}{3} + \frac{222.3\text{in}}{300\text{in}}\left(\frac{\left(1-\cosh\left(\frac{300\text{in}}{3\cdot 222.3\text{in}}\right)\right)\left(1-\cosh\left(\frac{300\text{in}}{2\cdot 222.3\text{in}}\right)\right)}{\sinh\left(\frac{300\text{in}}{2\cdot 222.3\text{in}}\right)} - \sinh\left(\frac{L}{3\cdot 222.3\text{in}}\right)\right)\right] = 0.00035\,{}^{1}\!/_{\text{lb}\cdot\text{in}}$$

Example 8: Anchorage Forces for Anti-Roll Anchorage Device

Given:

1. No discrete bracing lines; anti-roll clips provided at each support at every fourth purlin line (lines 1, 5 and 9 from the eave). Each anti-roll anchorage device is attached to the web of the Z-section with two rows of two ½ in. diameter A307 bolts. The bottom row of bolts is 3 in. from the bottom flange and the top row is 6 in. from the bottom flange. The stiffness of each anti-roll anchorage device, K_{device} = 40 k/in. The width of the anti-roll anchorage device is b_{pl} = 5.0 in.

2. Purlin flanges are bolted to the support member with two ½ in. diameter A307 bolts through the bottom flange.

Required:
1. Compute the anchorage forces along each frame line due to gravity loads.
2. Compute the lateral deflection of the top flange of the Z-section along each frame line and at the purlin mid-span.
3. Compute the shear force in the standing seam panel clips at each anchorage device.

Solutions:

Assumptions for Analysis
1. Since the loading, geometry and materials are symmetric, check the first two spans only.
2. Each restraint location is considered to have a single degree of freedom along the run of purlins. It is assumed that there is some mechanism to rigidly transfer forces from the remote purlins to the anchorage device. The sheathing provides the mechanism to transfer the force as long as the connection between the purlin and sheathing has sufficient strength and stiffness to transfer the force.
3. It is assumed that the total stiffness of adjacent frame lines is approximately the same

Procedure
1. Calculate uniform restraint provided by sheathing, w_{rest}, expressed as a proportion of the applied uniform load.

   $w_{rest} = w \cdot \sigma$

   where

$$\sigma = \frac{C1\dfrac{\left(\dfrac{I_{xy}}{I_x}\cos\theta\right)L^4}{EI_{my}} + \dfrac{((\delta b + m)\cos\theta)d}{2}\tau + \dfrac{\alpha \cdot n_p \cdot L^2 \sin\theta}{8G'Bay}}{C1\dfrac{L^4}{EI_{my}} + \dfrac{d^2}{4}\tau + \dfrac{\alpha \cdot \eta \cdot L^2}{8G'Bay}} \qquad (5.1.51)$$

The uniform restraint force provided by the sheathing must be calculated separately for the up slope and down slope facing purlins.

a. Exterior Span – half span adjacent to frame line 1 (approximated as a simple-fixed beam with warping free ends).

Down slope facing purlin. C1 = 1/185 α = -1

$$\sigma = \frac{\dfrac{\left(\dfrac{4.11\text{in}^4}{12.4\text{in}^4}\cos(2.4°)\right)(300\text{in})^4}{185 \cdot E \cdot 1.15\text{in}^4} + \dfrac{\left(\left(\frac{2.75\text{in}}{3}+0\right)\cos(2.4°)\right)8.0\text{in}}{2}0.0055\dfrac{\text{rad}}{\text{lb}} + \dfrac{(-1)(12)(300\text{in})^2\sin(2.4°)}{8\cdot\left(1000\,{}^{\text{lb}}\!/_{\text{in}}\right)(660\text{in})}}{\dfrac{(300\text{in})^4}{185 \cdot E \cdot 1.15\text{in}^4} + \dfrac{(8.0\text{in})^2}{4}0.0055\dfrac{\text{rad}}{\text{lb}} + \dfrac{(-1)(10)(300\text{in})^2}{8\cdot\left(1000\,{}^{\text{lb}}\!/_{\text{in}}\right)(660\text{in})}}$$

$\sigma = 0.363$

Up slope facing purlin. All terms same as above except with α =1

$\sigma = 0.294$

b. Exterior Span – half span adjacent to frame line 2 (approximated as a simple-fixed beam with warping fixed ends).

Down slope facing purlin. C1 = 1/185 α = -1

$$\sigma = \frac{\dfrac{\left(\dfrac{4.11\text{in}^4}{12.4\text{in}^4}\cos(2.4°)\right)(300\text{in})^4}{185 \cdot E \cdot 1.15\text{in}^4} + \dfrac{\left(\frac{2.75\text{in}}{3}\cos(2.4°)\right)8.0\text{in}}{2}0.0045\dfrac{\text{rad}}{\text{lb}} + \dfrac{(-1)(12)(300\text{in})^2\sin(2.4°)}{8\cdot\left(1000\,{}^{\text{lb}}\!/_{\text{in}}\right)(660\text{in})}}{\dfrac{(300\text{in})^4}{185 \cdot E \cdot 1.15\text{in}^4} + \dfrac{(8.0\text{in})^2}{4}0.0045\dfrac{\text{rad}}{\text{lb}} + \dfrac{(-1)(10)(300\text{in})^2}{8\cdot\left(1000\,{}^{\text{lb}}\!/_{\text{in}}\right)(660\text{in})}}$$

$\sigma = 0.365$

Up slope facing purlin. All terms same as above except with α = 1

$\sigma = 0.295$

c. Interior Span

Interior Span approximated as a fixed-fixed beam with warping restrained at each end.

Down slope facing purlin. C1 = 1/384 α = -1

$$\sigma = \frac{\dfrac{\left(\dfrac{2.85\text{in}^4}{8.69\text{in}^4}\cos(2.4°)\right)(300\text{in})^4}{384 \cdot E \cdot 0.79\text{in}^4} + \dfrac{\left(\frac{2.75\text{in}}{3}\cos(2.4°)\right)8.0\text{in}}{2}0.0048\dfrac{\text{rad}}{\text{lb}} + \dfrac{(-1)(12)(300\text{in})^2\sin(2.4°)}{8\cdot\left(1000\,{}^{\text{lb}}\!/_{\text{in}}\right)(660\text{in})}}{\dfrac{(300\text{in})^4}{384 \cdot E \cdot 0.79\text{in}^4} + \dfrac{(8.0\text{in})^2}{4}0.0048\dfrac{\text{rad}}{\text{lb}} + \dfrac{(-1)(10)(300\text{in})^2}{8\cdot\left(1000\,{}^{\text{lb}}\!/_{\text{in}}\right)(660\text{in})}}$$

$\sigma = 0.376$

Up slope facing purlin all terms same as above except with α =1

$\sigma = 0.28$

2. Calculate the overturning forces generated by each purlin
a. Exterior Span – half-span adjacent to frame line 1 (torsionally approximated as each end warping free)

Local deformation reduction factor

$$R_{local} = \frac{k_{mclip}}{k_{mclip} + \frac{Et^3}{3d}} = \frac{2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}}{2500\,{}^{lb\cdot in}\!/_{rad\cdot ft} + \frac{E\cdot(0.085in)^3}{3\cdot 8.0in}\left(12\,{}^{in}\!/_{ft}\right)} = 0.216 \qquad (5.1.50)$$

Purlin 1 w = 57.5 plf α = -1

$$P_i = \frac{wL}{2d}\cdot\left[\left(\delta b\cos\theta(1-R_{local}) + \tfrac{2}{3}k_{mclip}\left(\sigma\frac{d}{2} - \delta b\cos\theta\right)\tau\right)\alpha - d\sin\theta\right] \qquad (5.1.49)$$

$$P_1 = \frac{(57.5plf)(25ft)}{(2)(8in)}\cdot\left[\begin{array}{l}\left(\begin{array}{l}\frac{2.75in}{3}\cos(2.4°)(1-0.216) + \frac{2}{3}\left(2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}\right)\left((0.363)\frac{8.0in}{2} - \frac{2.75in}{3}\cos(2.4°)\right)x\\ \left(0.0055\frac{1}{lb}\right)\end{array}\right)(-1)\\ -(8.0in)\sin(2.4°)\end{array}\right]$$

$P_1 = -132\text{ lb}$

Purlins 2-11 w = 115 plf α = 1

$P_{2-11} = 106\text{ lb}$

Purlin 12 w = 57.5 plf α = 1

$P_{12} = 53\text{ lb}$

b. Exterior Span – half-span adjacent to frame line 2 (torsionally approximated as each end warping fixed)

Local deformation reduction factor

$R_{local} = 0.216$

Purlin 1 w = 57.5 plf α = -1

$$P_1 = \frac{(57.5plf)(25ft)}{2(8in).}\cdot\left[\begin{array}{l}\left(\begin{array}{l}\frac{2.75in}{3}\cos(2.4°)(1-0.216) + \frac{2}{3}\left(2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}\right)\left((0.365)\frac{8.0in}{2} - \frac{2.75in}{3}\cos(2.4°)\right)x\\ \left(0.0045\frac{1}{lb}\right)\end{array}\right)(-1)\\ -(8.0in)\sin(2.4°)\end{array}\right]$$

$P_1 = -132\text{ lb}$

Purlins 2-11 w = 115 plf α = 1

$P_{2-11} = 99 \text{ lb}$

Purlin 12 w = 57.5 plf α = 1

$P_{12} = 50 \text{ lb}$

b. Interior Span

Local deformation reduction factor

$$R_{local} = \frac{k_{mclip}}{k_{mclip} + \frac{Et^3}{3d}} = \frac{2500 \,{}^{lb \cdot in}/_{rad \cdot ft}}{2500 \,{}^{lb \cdot in}/_{rad \cdot ft} + \frac{E \cdot (0.059in)^3}{3 \cdot 8.0in}\left(12 \,{}^{in}/_{ft}\right)} = 0.45 \quad (5.1.50)$$

Purlin 1 w = 57.5 plf α = -1

$$P_1 = \frac{(57.5plf)(25ft)}{(2)(8in)} \cdot \left[\begin{array}{l} \left(\begin{array}{l} \frac{2.75in}{3}\cos(2.4°)(1-0.45) + \frac{2}{3}\left(2500 \,{}^{lb \cdot in}/_{rad \cdot ft}\right)\left((0.376)\frac{8.0in}{2} - \frac{2.75in}{3}\cos(2.4°)\right) x \\ \left(0.0048 \frac{1}{lb}\right) \end{array} \right)(-1) \\ -(8.0in)\sin(2.4°) \end{array} \right]$$

$P_1 = -110 \text{ lb}$

Purlins 2-11 w = 115 plf α = 1

$P_{2-11} = 55 \text{ lb}$

Purlin 12 w = 57.5 plf α = 1

$P_{12} = 28 \text{ lb}$

3. Calculate the stiffness of the restraints.

The stiffness of each restraint device is

$K_{device} = 40 \,{}^{kip}/_{in}$

The net stiffness of the restraint must include the configuration stiffness which accounts for the flexibility of the web of the purlin between the top of the restraint and the top flange of the purlin.

a. Frame line 1

Configuration stiffness

$$K_{config} = \frac{Eb_{pl}t^3}{(d-h)^3} \cdot \frac{d}{h} = \frac{E(5in)(0.085in)^3}{(8in - 6in)^3} \cdot \frac{8in}{6in} = 15.1 \,{}^{kip}/_{in} \quad (5.1.32)$$

Net restraint stiffness

$$K_{rest} = \frac{\left(\frac{h}{d}\right)^2 K_{device}K_{config}}{\frac{h}{d}K_{device} + K_{config}} = \frac{\left(\frac{6in}{8in}\right)^2 \left(40 \,{}^{kip}/_{in}\right)\left(15.1 \,{}^{kip}/_{in}\right)}{\frac{6in}{8in}\left(40 \,{}^{kip}/_{in}\right) + 15.1 \,{}^{kip}/_{in}} = 7.5 \,{}^{kip}/_{in} \quad (5.1.30)$$

b. Frame line 2

To account for the purlins at the lap, the combined purlins are given an equivalent thickness.

$$t_{lap} = \sqrt[3]{t_1^3 + t_2^3} = \sqrt[3]{(0.085\text{in})^3 + (0.059\text{in})^3} = 0.094\,\text{in}$$

Configuration stiffness

$$K_{config} = \frac{Eb_{pl}t^3}{(d-h)^3} \cdot \frac{d}{h} = \frac{E(5\text{in})(0.094\text{in})^3}{(8\text{in} - 6\text{in})^3} \cdot \frac{8\text{in}}{6\text{in}} = 20.1\,{}^{\text{kip}}\!/_{\text{in}} \qquad (5.1.32)$$

Net restraint stiffness

$$K_{rest} = 9.0\,{}^{\text{kip}}\!/_{\text{in}}$$

c. Frame line 3

Equivalent thickness at lap

$$t_{lap} = \sqrt[3]{t_2^3 + t_2^3} = \sqrt[3]{(0.059\text{in})^3 + (0.059\text{in})^3} = 0.074\text{in}$$

Configuration stiffness

$$K_{config} = \frac{Eb_{pl}t^3}{(d-h)^3} \cdot \frac{d}{h} = \frac{E(5\text{in})(0.074\text{in})^3}{(8\text{in} - 6\text{in})^3} \cdot \frac{8\text{in}}{6\text{in}} = 10.1\,{}^{\text{kip}}\!/_{\text{in}} \qquad (5.1.32)$$

Net restraint Stiffness

$$K_{rest} = 5.7\,{}^{\text{kip}}\!/_{\text{in}}$$

4. Calculate the stiffness of the system

a. Calculate the stiffness of the sheathing.

$$K_{shtg} = \frac{k_{mclip}L}{d}\left(\frac{{}^1\!/_4\,Et^3}{0.38k_{mclip}d + 0.71\,{}^1\!/_4\,Et^3}\right)\left(1 - {}^2\!/_3\,k_{mclip}\tau\right) \qquad (5.1.37)$$

i. Exterior Span. It is conservative to use the torsional coefficient, τ, for a warping free ends.

$$K_{shtg} = \frac{2500\,{}^{\text{lb}\cdot\text{in}}\!/_{\text{rad}\cdot\text{ft}}\,25\text{ft}}{8.0\text{in}}\left(\frac{{}^1\!/_4\,E(0.085\text{in})^3}{0.38\cdot 2500\,{}^{\text{lb}\cdot\text{in}}\!/_{\text{rad}\cdot\text{ft}}\cdot 8.0\text{in} + 0.71\cdot{}^1\!/_4\,E(0.085\text{in})^3}\right)\left(1 - {}^2\!/_3\,2500\,{}^{\text{lb}\cdot\text{in}}\!/_{\text{rad}\cdot\text{ft}}\cdot 0.0055\frac{\text{rad}}{\text{lb}}\right)$$

$$K_{shtg} = 2117\,{}^{\text{lb}\cdot\text{in}}\!/_{\text{in}}$$

ii. Interior Span, t = 0.059 in. τ = 0.0048 rad/lb

$$K_{shtg} = 2317\,{}^{\text{lb}\cdot\text{in}}\!/_{\text{in}}$$

b. Calculate stiffness of connection between rafter and Z-section (flange bolted connection)

$$K_{rafter} = 0.45\frac{Et^3}{2d} \qquad (5.1.36)$$

Exterior Frame Line

$$K_{rafter} = 0.45\frac{E(0.085\text{in})^3}{2\cdot 8\text{in}} = 510\,{}^{\text{lb}\cdot\text{in}}\!/_{\text{in}}$$

At the interior frame lines, the equivalent thickness of the laps is used.

First Interior Frame Line

$$K_{rafter} = 0.45\frac{E(0.094in)^3}{2 \cdot 8in} = 680\,{}^{lb\cdot in}\!/_{in}$$

Second Interior Frame Line

$$K_{rafter} = 0.45\frac{E(0.074in)^3}{2 \cdot 8in} = 341\,{}^{lb\cdot in}\!/_{in}$$

5. Calculate the total stiffness of the system attributed to each restraint location (frame line)

$$K_{total} = \sum K_{rest} + \frac{\sum(K_{shtg} + K_{rafter})}{d} \tag{5.1.48}$$

a. At frame line 1, the stiffness includes three anchorage devices, the rafter stiffness of nine purlins flange bolted to the rafters and the sheathing stiffness of half of the exterior bay for twelve purlins.

$$K_{total} = 3\left(7.5\,{}^{kip}\!/_{in}\right) + 9\frac{510\,{}^{lb\cdot in}\!/_{in}}{8.0in} + 12\frac{2117\,{}^{lb\cdot in}\!/_{in}}{(2)(8in)} = 24.8\,{}^{kip}\!/_{in}$$

b. At frame line 2, the stiffness includes three anchorage devices, the rafter stiffness of nine purlins flange bolted to the rafter, and the sheathing stiffness of half of the exterior bay and half of the interior bay for twelve purlins

$$K_{total} = 3\left(9.0\,{}^{kip}\!/_{in}\right) + 9\frac{680\,{}^{lb\cdot in}\!/_{in}}{8.0in} + 12\frac{2117\,{}^{lb\cdot in}\!/_{in}}{(2)(8in)} + 12\frac{2317\,{}^{lb\cdot in}\!/_{in}}{(2)(8in)} = 31.1\,{}^{kip}\!/_{in}$$

c. At frame line 3, the stiffness includes three anchorage devices, the rafter stiffness of nine purlins flange bolted to the rafter, and two times the sheathing stiffness of half of the interior bay for twelve purlins

$$K_{total} = 3\left(9.0\,{}^{kip}\!/_{in}\right) + 9\frac{680\,{}^{lb\cdot in}\!/_{in}}{8.0in} + 12\frac{2317\,{}^{lb\cdot in}\!/_{in}}{(8in)} = 21.3\,{}^{kip}\!/_{in}$$

6. Distribute forces to each restraint

a. Frame line 1

The total load generated by the exterior half span adjacent to frame line 1 is

$$\sum_{N_p} P_i = (P_1 + 10P_{2-11} + P_{12}) = (-132\text{ lb}) + 10(106\text{ lb}) + 53\text{ lb} = 981\text{ lb}$$

Distribution to each anti-roll anchorage device along frame line 1

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (981\text{ lb})\frac{7.5\,{}^{kip}\!/_{in}}{24.8\,{}^{kip}\!/_{in}} = 297\text{lb} \tag{5.1.46}$$

Anchorage force at the height of restraint

$$P_h = P_L\frac{d}{h} = (297\text{ lb})\frac{8\text{ in}}{6\text{ in}} = 396\text{ lb} \tag{5.1.47}$$

b. Frame line 2

The total load generated by each half span adjacent to frame line 2 is

$$\sum_{N_p} P_i = (P_1 + 10P_{2-11} + P_{12})_{Left} + (P_1 + 10P_{2-11} + P_{12})_{Right}$$

$$\sum_{N_p} P_i = (-132\text{ lb}) + 10(99\text{ lb}) + (50\text{ lb}) + ((-110\text{ lb}) + 10(55\text{ lb}) + (28\text{ lb})) = 1376\text{ lb}$$

Distribution to each anti-roll anchorage device along frame line 2

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (1376\text{ lb})\frac{9.0\,{}^{kip}/_{in}}{31.2\,{}^{kip}/_{in}} = 397\text{ lb} \quad (5.1.46)$$

Anchorage force at the height of restraint

$$P_h = P_L\frac{d}{h} = (397\text{ lb})\frac{8\text{ in}}{6\text{ in}} = 529\text{ lb} \quad (5.1.47)$$

c. Frame line 3

At frame line 3, it is assumed that half of the force generated at each bay adjacent to the frame line is distributed to the interior frame line.

$$\sum_{N_p} P_i = 2((P_1 + 10P_{2-11} + P_{12})_{Int}) = 2((-110\text{ lb}) + 10(55\text{ lb}) + (28\text{ lb})) = 936\text{ lb}$$

Distribution to each anti-roll anchorage device along frame line 3

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (936\text{ lb})\frac{5.7\,{}^{kip}/_{in}}{21.2\,{}^{kip}/_{in}} = 252\text{ lb} \quad (5.1.46)$$

Anchorage force at the height of restraint

$$P_h = P_L\frac{d}{h} = (252\text{ lb})\frac{8\text{ in}}{6\text{ in}} = 336\text{ lb} \quad (5.1.47)$$

7. Check deformation of the system and compare to limits specified in Section D6.3.1

a. Lateral displacement of purlin top flange

Allowable deflection limit (ASD)

$$\Delta_{tf} \le \frac{1}{\Omega}\frac{d}{20} = \frac{1}{2.00}\frac{8\text{ in}}{20} = 0.20\text{ in} \quad (D6.3.1\text{-}9a)$$

Frame Line 1

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(297\text{lb})}{7.5\,{}^{kip}/_{in}} = 0.040\text{ in} \le 0.20\text{ in OK} \quad (5.1.52)$$

Frame Line 2

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(397\text{ lb})}{9.0\,{}^{kip}/_{in}} = 0.044\text{ in} \le 0.20\text{ in OK} \quad (5.1.52)$$

Frame Line 3

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(252\text{ lb})}{5.7\,{}^{kip}/_{in}} = 0.044\text{ in} \le 0.20\text{ in OK} \quad (5.1.52)$$

b. Mid-span displacement of diaphragm relative to restraint

Allowable deflection limit

$$\Delta_{ms} \le \frac{L}{360} = \frac{25\text{ft}\left(12\ {}^{\text{in}}\!/_{\text{ft}}\right)}{360} = 0.83\text{ in}$$

$$\Delta_{diaph} = \Sigma\left(w\left(\alpha\sigma - \sin\theta\right)\right)_i \frac{L^2}{8G'Bay} \tag{5.1.53}$$

Exterior Span

Use average uniform diaphragm force between the two half spans.

$$\overline{\sigma}_1 = \tfrac{1}{2}\left(0.363 + 0.365\right) = 0.364$$

$$\overline{\sigma}_{2-12} = \tfrac{1}{2}\left(0.294 + 0.295\right) = 0.295$$

$$\Delta_{diaph} = \left[57.5\text{plf}\left((-1)0.364 - \sin(2.4°)\right) + (10.5)(115\text{plf})\left((1)0.295 - \sin(2.4°)\right)\right]\frac{(25\text{ft})^2}{8\left(1000\ {}^{\text{lb}}\!/_{\text{in}}\right)(55\text{ft})} = 0.40\text{ in}$$

$$\Delta_{diaph} = 0.40\text{ in} \le 0.83\text{ in} \quad \text{OK}$$

Interior Span

$$\Delta_{diaph} = \left[57.5\text{plf}\left((-1)0.376 - \sin(2.4°)\right) + (10.5)(115\text{plf})\left((1)0.28 - \sin(2.4°)\right)\right]\frac{(25\text{ft})^2}{8\left(1000\ {}^{\text{lb}}\!/_{\text{in}}\right)(55\text{ft})} = 0.375\text{ in}$$

$$\Delta_{diaph} = 0.375\text{ in} \le 0.83\text{ in} \quad \text{OK}$$

8. Calculate force in connection between the sheathing and purlin at anchor location.

Frame line 1

$$P_{sc} = P_L + \frac{wL}{2}\left(0.9\sigma\alpha + \sin\theta\right) - P_i \tag{5.1.54}$$

Down slope purlin 1

$$P_{sc} = 297\text{ lb} + \frac{(57.7\text{plf})(25\text{ft})}{2}\left(0.9(0.363)(-1) - \sin(2.4°)\right) - \left(-132\text{ lb}\right) = 163\text{ lb}$$

Up slope purlin 5, 9

$$P_{sc} = 297\text{ lb} + \frac{(115\text{plf})(25\text{ft})}{2}\left(0.9(0.294)(1) - \sin(2.4°)\right) - \left(106\text{ lb}\right) = 512\text{ lb}$$

Frame line 2

Down slope purlin 1

$$P_{sc} = 417\text{ lb} + \frac{(57.7\text{plf})(25\text{ft})}{2}\left(0.9(0.365)(-1) - \sin(2.4°)\right) - \left(-132\text{ lb}\right)$$
$$+ \frac{(57.7\text{plf})(25\text{ft})}{2}\left(0.9(0.376)(-1) - \sin(2.4°)\right) - \left(-111\text{ lb}\right) = 120\text{ lb}$$

Up slope purlin 5, 9

$$P_{sc} = 417\text{ lb} + \frac{(115\text{plf})(25\text{ft})}{2}\left(0.9(0.295)(1) - \sin(2.4°)\right) - \left(99\text{ lb}\right)$$
$$+ \frac{(115\text{plf})(25\text{ft})}{2}\left(0.9(0.28)(1) - \sin(2.4°)\right) - \left(55\text{ lb}\right) = 888\text{ lb}$$

Frame line 3

Down slope purlin 1

$$P_{sc} = 248\text{ lb} + 2 \cdot \left[\frac{(57.7\text{plf})(25\text{ft})}{2} (0.9(0.376)(-1) - \sin(2.4°)) - (-110\text{ lb}) \right] = -78\text{ lb}$$

Up slope purlin 5, 9

$$P_{sc} = 248\text{ lb} + 2 \cdot \left[\frac{(115\text{plf})(25\text{ft})}{2} (0.9(0.28)(1) - \sin(2.4°)) - (55\text{ lb}) \right] = 744\text{ lb}$$

Example 9: Third Points Anchorage

Given:

1. From the previous example, the anti-roll anchorage devices are replaced by third point anchors along the eave of the system (at purlin line 1). Each third point brace is attached to the top flange and has a stiffness of 15 k/in.
2. Purlin flanges are bolted to the support member with two ½ in. diameter A307 bolts through the bottom flange.

Required:

1. Compute the anchorage forces at each third point restraint due to gravity loads.
2. Compute the lateral deflection of the top flange of the Z-section at each third point and along each frame line.
3. Compute the shear force in the standing seam panel clips at each anchorage device.

Solutions:

Assumptions for Analysis

a. Since the loading, geometry and materials are symmetric, check the first two spans only.
b. Each restraint location is considered to have a single degree of freedom along the run of purlins. It is assumed that there is some mechanism to rigidly transfer forces from the remote purlins to the anchorage device. The sheathing provides the mechanism to transfer the force as long as the connection between the purlin and sheathing has sufficient strength and stiffness to transfer the force.
c. It is assumed that the total stiffness of the system in each bay is approximately the same.

Procedure

1. Calculate uniform restraint provided by sheathing, w_{rest}, expressed as a proportion of the applied uniform load.

$$w_{rest} = w \cdot \sigma$$

where

$$\sigma = \frac{C1\dfrac{\left(\dfrac{I_{xy}}{I_x}\cos\theta\right)L^4}{EI_{my}} + \dfrac{((\delta b + m)\cos\theta)d}{2}\tau - \dfrac{\alpha \cdot n_p \cdot L^2 \sin\theta}{18G'Bay}}{C1\dfrac{L^4}{EI_{my}} + \dfrac{d^2}{4}\tau + \dfrac{\alpha \cdot \eta \cdot L^2}{9G'Bay}} \qquad (5.1.60)$$

The uniform restraint force in the sheathing must be calculated separately for the up slope and down slope facing purlins.

Exterior Span – half span adjacent to frame line 1.

The exterior span is approximated as a simple-fixed beam with warping free ends. Lateral displacements are considered at each third points while torsion is considered at mid-span for simplicity

Purlin 1 C1 = 5/972 α = -1

$$\sigma = \frac{\dfrac{5\left(\dfrac{4.11\text{in}^4}{12.4\text{in}^4}\cos(2.4°)\right)(300\text{in})^4}{972 \cdot E \cdot 1.15\text{in}^4} + \dfrac{\left(\frac{2.75\text{in}}{3}\cos(2.4°)\right)8.0\text{in}}{2 \cdot \text{rad}} 0.0055\dfrac{\text{rad}}{\text{lb}} - \dfrac{(-1)(12)(300\text{in})^2 \sin(2.4°)}{18 \cdot \left(1000\,{}^{\text{lb}}\!/\!_{\text{in}}\right)(660\text{in})}}{\dfrac{5(300\text{in})^4}{972 \cdot E \cdot 1.15\text{in}^4} + \dfrac{(8.0\text{in})^2}{4 \cdot \text{rad}} 0.0055\dfrac{\text{rad}}{\text{lb}} + \dfrac{(-1)(10)(300\text{in})^2}{9 \cdot \left(1000\,{}^{\text{lb}}\!/\!_{\text{in}}\right)(660\text{in})}}$$

$\sigma = 0.370$

Purlins 2-12 same as above except with α = 1

$\sigma = 0.288$

b. Exterior Span – half span adjacent to frame line 2.

Purlin 1 C1 = 7/1944 α = -1

$\sigma = 0.391$

Purlins 2- 12 C1 = 7/1944 α = 1

$\sigma = 0.275$

c. Interior span. Approximated as a fixed-fixed beam with warping restrained at each end.

Purlin 1 C1 = 1/486 α = -1

$\sigma = 0.399$

Purlins 2-12 C1 = 1/486 α = 1

$\sigma = 0.263$

2. Calculate the overturning forces generated by each purlin

a. Exterior Span – half span adjacent to frame line 1

R_{local} = 0.216 (from previous example)

Purlin 1 w = 57.5 plf α = -1

$$P_i = \frac{wL}{2d} \cdot \left[\begin{array}{l} \left(\delta b \cos\theta \left(1 - R_{local}\right) + \tfrac{2}{3} k_{mclip} \left(\sigma \dfrac{d}{2} - \delta b \cos\theta \right) \tau \right) \alpha - d \sin\theta \\ + \dfrac{L^2 k_{mclip}}{3G' Bay \cdot d} \left(1 - \tfrac{2}{3} k_{mclip} \tau\right) \left(\dfrac{\eta\sigma}{3} + \dfrac{N_p \sin\theta}{6} - \dfrac{\eta \cdot \delta b \cos\theta}{2d} \right) \end{array} \right] \tag{5.1.58}$$

$$P_1 = \frac{(57.5\text{plf})(25\text{ft})}{(2)(8\text{in})} \cdot \left[\begin{array}{l} \left(\dfrac{2.75\text{in}}{3} \cos(2.4°)(1 - 0.216) + \tfrac{2}{3}\left(2500^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}\right)\left((0.363)\dfrac{8.0\text{in}}{2} - \dfrac{2.75\text{in}}{3}\cos(2.4°)\right)\left(0.0055\dfrac{1}{\text{lb}}\right)\right)(-1) \\ -(8.0\text{in})\sin(2.4°) \\ + \dfrac{(25\text{ft})^2\, 2500^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}}{3\left(1000^{\text{lb}}/_{\text{in}}\right)(55\text{ft})(10\text{in})} \left(1 - \tfrac{2}{3} 2500^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}} \left(0.0055\dfrac{1}{\text{lb}}\right)\right)\left(\dfrac{(10)(0.363)}{3} + \dfrac{12\sin(2.4°)}{6} - \dfrac{(10)(2.75\text{in})\cos(2.4°)}{(2)(3)(8\text{in})}\right) \end{array} \right]$$

$P_1 = -115 \text{ lb}$

Purlins 2-11 w = 115 plf $\alpha = 1$

$P_{2-11} = 125 \text{ lb}$

Purlin 12

Since the load on purlin 12 is half that of purlins 2-11, the overturning force is half that of purlins 2-12, or

$P_{12} = 63 \text{ lb}$

b. Exterior Span – half span adjacent to frame line 2

R_{local} = 0.216 (from previous example)

Purlin 1 w = 57.5 plf α = -1

$P_1 = -98 \text{ lb}$

Purlins 2-11 w = 115 plf $\alpha = 1$

$P_{2-11} = 124 \text{ lb}$

Purlin 12

$P_{12} = 62 \cdot \text{lb}$ (half of purlins 2-11)

c. Interior Span

R_{local} = 0.45 (from previous example)

Purlin 1 w = 57.5 plf α = -1

$P_1 = -86 \text{ lb}$

Purlins 2-11 w = 115 plf $\alpha = 1$

$P_{2-11} = 66\,\text{lb}$

Purlin 12

$P_{12} = 33\,\text{lb}$ (half of purlins 2-11)

3. Calculate the stiffness of the restraints

The stiffness of each restraint device is given

$K_{Device} = 15\,{}^{kip}\!/_{in}$

The restraint is applied to the top of the Z-section and it is assumed that this connection is rigid, i.e., the configuration stiffness is essentially rigid. Therefore, the restraint stiffness becomes

$$K_{rest} = \left(\frac{h}{d}\right)^2 K_{device} = \left(\frac{8\,in}{8\,in}\right)^2 15\,{}^{kip}\!/_{in} = 15\,{}^{kip}\!/_{in} \tag{5.1.34}$$

4. Calculate the stiffness of the system

a. Sheathing stiffness

The sheathing stiffness is the same as was calculated in the previous example

Exterior Span $K_{shtg} = 2117\,{}^{lb\cdot in}\!/_{in}$

Interior Span $K_{shtg} = 2317\,{}^{lb\cdot in}\!/_{in}$

b. Rafter connection stiffness

The connection between the rafter and the Z-section is a flange bolted connection. Because the stiffness of this connection is relatively small, it is ignored for a third points restraint configuration. If the connection between the Z-section and rafter has significant stiffness, such as with a rafter web plate (wing plate), the restraint configuration must be considered a third points plus supports configuration (see the fourth example).

5. Calculate the total stiffness of the system attributed to each restraint location

$$K_{total} = \Sigma K_{rest} + \frac{\Sigma(K_{shtg} + K_{rafter})}{d} \tag{5.1.57}$$

a. Exterior bay. The total stiffness attributed to each restraint is the stiffness of the restraint plus one half of the sheathing stiffness of the exterior bay for 12 purlins.

$$K_{total} = 15\,{}^{kip}\!/_{in} + 12 \cdot \frac{2117\,{}^{lb\cdot in}\!/_{in}}{(2)(8in)} = 16.6\,{}^{kip}\!/_{in}$$

b. Interior bay. The total stiffness attributed to each restraint is the stiffness of the restraint plus one half of the sheathing stiffness of the interior bay for 12 purlins.

$$K_{total} = 15\,{}^{kip}\!/_{in} + 12 \cdot \frac{2317\,{}^{lb\cdot in}\!/_{in}}{(2)(8in)} = 16.7\,{}^{kip}\!/_{in}$$

6. Distribute forces to each restraint

The total load generated by the half the span (tributary to each restraint) is

$$\sum_{N_p} P_i = (P_1 + 10P_{2-11} + P_{12})$$

a. Exterior Span – half span adjacent to frame line 1

$$\sum_{N_p} P_i = (-115 \text{ lb}) + 10(125 \text{ lb}) + (63 \text{ lb}) = 1198 \text{ lb}$$

Force in each anchor closest to frame line 1

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (1198 \text{ lb}) \frac{15 \text{ kip/in}}{16.6 \text{ kip/in}} = 1083 \text{ lb} \qquad (5.1.55)$$

b. Exterior Span – half span adjacent to frame line 2

$$\sum_{N_p} P_i = (-98 \text{ lb}) + 10(124 \text{ lb}) + (62 \text{ lb}) = 1204 \text{ lb}$$

Force in each anchor closest to frame line 2

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (1204 \text{lb}) \frac{15 \text{ kip/in}}{16.6 \text{ kip/in}} = 1088 \text{ lb}$$

c. Interior Span

$$\sum_{N_p} P_i = (-86 \text{ lb}) + 10(66 \text{ lb}) + (33 \text{ lb}) = 607 \text{ lb}$$

Force in each anchor closest to frame line

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (607 \text{ lb}) \frac{15 \text{ kip/in}}{16.7 \text{ kip/in}} = 545 \text{ lb}$$

7. Check deformation of the system and compare to the limits specified in Section D6.3.1

a. Lateral displacement of purlin top flange at anchor

Allowable deflection limit (ASD)

$$\Delta_{tf} = \frac{1}{\Omega}\frac{d}{20} = \frac{1}{2.00}\frac{8 \text{ in}}{20} = 0.20 \text{ in} \qquad (D6.3.1\text{-}9a)$$

Exterior third point adjacent to frame line 1

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(1083 \text{lb})}{15 \text{ kip/in}} = 0.072 \text{ in} \le 0.20 \text{ in OK} \qquad (5.1.61)$$

Exterior third point adjacent to frame line 2

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(1088 \text{ lb})}{15 \text{ kip/in}} = 0.073 \text{ in} \le 0.20 \text{ in OK} \qquad (5.1.61)$$

Interior third points

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(545 \text{ lb})}{15 \text{ kip/in}} = 0.036 \text{ in} \le 0.20 \text{ in OK} \qquad (5.1.61)$$

b. Displacement of diaphragm relative to restraint (displacement along frame lines)

Allowable deflection limit

$$\Delta_{ms} \le \frac{L}{360} = \frac{25\text{ft}\ ^{12\text{in}}\!/\!_{\text{ft}}}{360} = 0.83\text{ in}$$

$$\Delta_{diaph} = \Sigma\left(-w\left(\alpha\sigma - \sin\theta\frac{K_{rest}}{K_{total}}\right)\right)_i \frac{L^2}{9G'Bay} + \frac{\Sigma P_L L}{3G'Bay} \qquad (5.1.62)$$

Exterior Span – half span adjacent to frame line 1

$$\Delta_{diaph} = \begin{bmatrix} -57.5\text{plf}\left((-1)0.370 - \sin(2.4°)\dfrac{15\ ^{\text{kip}}\!/\!_{\text{in}}}{16.6\ ^{\text{kip}}\!/\!_{\text{in}}}\right) \\ -(10.5)(115\text{plf})\left((1)0.288 - \sin(2.4°)\dfrac{15\ ^{\text{kip}}\!/\!_{\text{in}}}{16.6\ ^{\text{kip}}\!/\!_{\text{in}}}\right) \end{bmatrix} \frac{(25\text{ft})^2}{9\left(1000\ ^{\text{lb}}\!/\!_{\text{in}}\right)(55\text{ft})} +$$

$$+\frac{(1120\text{lb})(25\text{ft})}{3(1000\text{lb}/\text{in})(55\text{ft})} = -0.22\text{ in}$$

$$\Delta_{diaph} = -0.22\text{ in} \le 0.83\text{ in} \quad \text{OK}$$

Exterior Span – half span adjacent to frame line 2

$$\Delta_{diaph} = -0.20\text{ in} \le 0.83\text{ in} \quad \text{OK}$$

Interior Span

$$\Delta_{diaph} = -0.26\text{ in} \le 0.83\text{ in} \quad \text{OK}$$

8. Calculate shear force in connection between the sheathing and purlin at anchor location.

$$P_{sc} = P_L + \frac{wL}{20}\left(-0.9\sigma + \frac{\delta b\cos\theta}{d}\right)\alpha \qquad (5.1.63)$$

At exterior third point adjacent to frame line 1

$$P_{sc} = 1119 + \frac{(57.7\text{plf})(25\text{ft})}{20}\left(-0.9(0.370) + \frac{(2.75\text{in})\cos(2.4°)}{(3)(8.0\text{in})}\right)(-1) = 1116\text{ lb}$$

At exterior third point adjacent to frame line 2

$$P_{sc} = 1135 + \frac{(57.7\text{plf})(25\text{ft})}{20}\left(-0.9(0.391) + \frac{(2.75\text{in})\cos(2.4°)}{(3)(8.0\text{in})}\right)(-1) = 1121\text{ lb}$$

At Interior third Points

$$P_{sc} = 592 + \frac{(57.7\text{plf})(25\text{ft})}{20}\left(-0.9(0.399) + \frac{(2.75\text{in})\cos(2.4°)}{(3)(8.0\text{in})}\right)(-1) = 574\text{ lb}$$

Example 10: Supports Plus Third Point Torsional Bracing

Given:

1. Four span continuous Z-purlin system from Example in Section 5.1.7.
2. Torsional braces are applied at the third points of purlins. Each purlin is attached to rafters with rafter web plates (wing plates). Web plates are ¼ in. thick by 5 in. wide (b_{pl}) by 7 in. tall. Web plates are attached to the web of the Z-section with two rows of two ½ in. diameter A307 bolts. The bottom row of bolts is 3 in. from the bottom flange and the top row is 6 in. from the bottom flange.

Required:

1. Compute the anchorage forces along each frame line due to gravity loads.
2. Compute the end moments of the torsional braces.
3. Compute the lateral deflection of the top flange of the Z-section along each frame line and at the purlin mid-span.
4. Compute the shear force in the standing seam panel clips at each anchorage device.

Solutions:

Assumptions for Analysis

a. Since the loading, geometry and materials are symmetrical, check the first two spans only.
b. For simplicity, each half span of the continuous system of purlins is analyzed individually.
c. It is assumed that the stiffness of adjacent frame lines is approximately the same.

Procedure:

1. Calculate the restraint stiffness of the rafter web plates.

$$K_{rest} = \frac{E \cdot b_{pl} \cdot t_{pl}{}^3 \cdot t^3\left(t^3 h + t_{pl}^3 (d-h)\right)}{\left(t^3 h^2 - t_{pl}^3 (d-h)^2\right)^2 + 4t^3 t_{pl}{}^3 d^2 h (d-h)} \qquad (5.1.33)$$

a. Frame line 1

$$K_{rest} = \frac{E \cdot 5in \cdot (0.25in)^3 (0.085in)^3 \left((0.085in)^3 6.0in + (0.25in)^3 (8.0in - 6.0in)\right)}{\left((0.085in)^3 (6.0in)^2 - (0.25in)^3 (8.0in - 6.0in)^2\right)^2 + 4(0.085in)^3 (0.25in)^3 (8.0in)^2 6.0in(8.0in - 6.0in)}$$

$$K_{rest} = 1589\,\mathrm{lb/in}$$

Total stiffness along exterior frame line

$$K_{total} = \Sigma K_{rest} + \frac{\Sigma K_{rafter}}{d} = (12)\left(1589\,{}^{lb}\!/\!_{in}\right) = 19.07\,{}^{kip}\!/\!_{in} \qquad (5.1.76)$$

b. Frame Line 2

Equivalent purlin thickness at the lap

$$t_{lap} = \sqrt[3]{t_1^3 + t_2^3} = \sqrt[3]{(0.085\text{in})^3 + (0.059\text{in})^3} = 0.094\text{in}$$

$$K_{rest} = 1690\,{}^{\text{lb}}\!/\!{}_{\text{in}}$$

$$K_{total} = \sum K_{rest} + \frac{\sum K_{rafter}}{d} = (12)\left(1690\,{}^{\text{lb}}\!/\!{}_{\text{in}}\right) = 20.28\,{}^{\text{kip}}\!/\!{}_{\text{in}} \tag{5.1.76}$$

c. Frame line 3

Equivalent purlin thickness at the lap

$$t_{lap} = \sqrt[3]{t_2^3 + t_2^3} = \sqrt[3]{(0.059\text{in})^3 + (0.059\text{in})^3} = 0.074\text{in}$$

$$K_{rest} = 1451\,{}^{\text{lb}}\!/\!{}_{\text{in}}$$

$$K_{total} = \sum K_{rest} + \frac{\sum K_{rafter}}{d} = (12)\left(1451\,{}^{\text{lb}}\!/\!{}_{\text{in}}\right) = 17.41\,{}^{\text{kip}}\!/\!{}_{\text{in}} \tag{5.1.76}$$

2. Calculate moments generated in each third point torsional restraint and the overturning force along the frame line. Analyze each half span individually

a. Exterior span – half span adjacent to frame line 1.

Calculate uniform restraint provided by sheathing, w_{rest}, expressed as a proportion of the applied uniform load.

$$w_{rest} = w \cdot \sigma$$

where

$$\sigma = \frac{C1\dfrac{\left(\dfrac{I_{xy}}{I_x}\cos\theta\right)L^4}{EI_{my}} + \dfrac{(\delta b\cos\theta)d}{2}\left(\dfrac{a^2\beta}{GJ} - \dfrac{L}{2}\cdot\beta_{3rd}\right)\xi + \dfrac{(\alpha)(N_p)L^2\sin\theta}{8G'Bay}}{C1\dfrac{L^4}{EI_{my}} + \dfrac{d^2}{4}\dfrac{a^2\beta}{GJ}\xi + \dfrac{(\alpha)(\eta)L^2}{8G'Bay}} \tag{5.1.80}$$

The uniform restraint force provide by the sheathing must be calculated separately for the up slope and down slope facing purlins.

Exterior frame line $K_{trib} = 1.0K_{total}$ = 19.07 kip/in = 1907 lb/in.

$$\xi = \frac{1}{1 + \left(d^2\dfrac{K_{trib}}{N_p} + \dfrac{L}{9}k_{mclip}\right)\beta_{3rd}} \tag{5.1.78}$$

$$\xi = \frac{1}{1 + \left((8.0\text{in})^2\dfrac{1907\,{}^{\text{lb}}\!/\!{}_{\text{in}}}{12} + \dfrac{25\text{ft}}{9}2500\,{}^{\text{lb}\cdot\text{in}}\!/\!{}_{\text{rad}\cdot\text{ft}}\right)0.000841\,{}^{1}\!/\!{}_{\text{lb}\cdot\text{in}}} = 0.011$$

Purlin 1 α=-1

$$\sigma=\frac{\left[\frac{\left(\frac{4.11in^4}{12.4in^4}\cos(2.4°)\right)(300in)^4}{185\cdot E\cdot 1.148in^4}+\frac{\left(\frac{2.75in}{3}\cos(2.4°)\right)8.0in}{2\cdot rad}\left(\frac{(154.6in)^2 0.132rad}{G\cdot 0.00306in^4}-\frac{300in\cdot 0.000841\,{}^{1}\!/\!_{lb\cdot in}}{2}\right)0.011+\frac{(-1)(12)(300in)^2\sin(2.4°)}{8\cdot\left(1000\,{}^{lb}\!/\!_{in}\right)(660in)}\right]}{\frac{(300in)^4}{185\cdot E\cdot 1.148in^4}+\frac{(8.0in)^2}{4\cdot rad}\left(\frac{(154.6in)^2 0.132rad}{G\cdot 0.00306in^4}\right)0.011+\frac{(-1)(10)(300in)^2}{8\cdot\left(1000\,{}^{lb}\!/\!_{in}\right)(660in)}}$$

$\sigma = 0.367$

Purlins 2-12. Same as above except α=1

$\sigma = 0.294$

Calculate the moment generated at each torsional restraint

$$M_{3rd}=w\left(\alpha\left(\sigma\frac{d}{2}-\delta b\cos\theta\right)\frac{a^2\beta}{GJ}\left(d^2\frac{K_{trib}}{N_p}+\frac{L}{9}k_{mclip}\right)+\left(d\sin\theta-(\alpha)\delta b\cos\theta\left(1-R_{local}\right)\right)\frac{L}{2}\right)\xi \qquad (5.1.73)$$

where

$$R_{local}=\frac{k_{mclip}}{k_{mclip}+\frac{Et^3}{3d}}=\frac{2500\,{}^{lb\cdot in}\!/\!_{rad\cdot ft}}{2500\,{}^{lb\cdot in}\!/\!_{rad\cdot ft}+\frac{E\cdot(0.085in)^3}{3\cdot 8.0in}}=0.216 \qquad (5.1.79)$$

Purlin 1. $\alpha=-1$ and $w=57.5plf$

$$M_{3rd}=57.5plf\left(\begin{array}{l}(-1)\left(0.367\frac{8.0in}{2}-\frac{2.75in}{3}\cos(2.4°)\right)\frac{(154.6in)^2 0.132rad}{G\cdot 0.00306in^4}\\ \left((8.0in)^2\frac{19{,}070\,{}^{lb}\!/\!_{in}}{12}+\frac{25ft}{9}2500\,{}^{lb\cdot in}\!/\!_{rad\cdot ft}\right)\cdot\frac{1ft}{12in}\\ +\left(8.0in\cdot\sin(2.4°)-(-1)\frac{2.75in}{3}\cos(2.4°)(1-0.216)\right)\frac{25ft}{2}\end{array}\right)0.011$$

$M_{3rd}=-281\,lb\cdot in$

Calculate overturning force at frame line 1

$$P_i=w\left(\alpha\left(\sigma\frac{d}{2}-\delta b\cos\theta\right)\frac{a^2\beta}{GJ}+\left(d\sin\theta-(\alpha)\delta b\cos\theta\left(1-R_{local}\right)\right)\frac{\beta_{3rd}L}{2}\right)d\cdot\frac{K_{trib}}{n_p}\xi \qquad (5.1.77)$$

$$P_i=57.5plf\left(\begin{array}{l}(-1)\left(0.367\frac{8.0in}{2}-\frac{2.75in}{3}\cos(2.4°)\right)\frac{(154.6in)^2 0.132rad}{G\cdot 0.00306in^4}\cdot\frac{1ft}{12in}\\ +\left(8.0in\cdot\sin(2.4°)-(-1)\frac{2.75in}{3}\cos(2.4°)(1-0.216)\right)\frac{0.000841\,{}^{1}\!/\!_{lb\cdot in}\cdot 25ft}{2}\end{array}\right)(8.0in)\left(\frac{19{,}070\,{}^{lb}\!/\!_{in}}{12}\right)(0.011)$$

$P_i=-121\,lb$

Purlins 2-11. Same as Purlin 1 except $\alpha=1 \quad w=115plf \quad \sigma=0.294$

$M_{3rd}=266\,lb\cdot in$

$P_i = 96\ \text{lb}$

Upslope facing Purlin 12. Same as Purlins 2-11 except w = 57.5plf

$M_{3rd} = 133\text{lb} \cdot \text{in}$

$P_i = 48\ \text{lb}$

b. Exterior span – half span adjacent to frame line 2

$K_{trib} = 0.5\ K_{total} = 0.5\left(20.28\ \text{kip}/\text{in}\right) = 10.14\ \text{kip}/\text{in}$

$\xi = 0.066$

Down slope facing purlin 1

$\sigma = 0.363$

Up slope facing purlins 2-12

$\sigma = 0.292$

Calculate the moment generated at each torsional restraint

Purlin 1. $\alpha = -1$ and $w = 57.5\text{plf}$

$M_{3rd} = -194\ \text{lb} \cdot \text{in}$

Calculate overturning force at frame line 2

$P_i = -105\ \text{lb}$

Upslope facing Purlins 2-11. $\alpha = 1$ $w = 115\text{plf}$ $\sigma = 0.292$

$M_{3rd} = 190\text{lb} \cdot \text{in}$

$P_i = 82\ \text{lb}$

Purlins 12. (half of Purlins 2-11)

$M_{3rd} = 95\ \text{lb} \cdot \text{in}$

$P_i = 41\ \text{lb}$

c. Interior span - half span adjacent to frame line 2.

Same as previous section only half of the total stiffness along the frame line is considered tributary to each half span adjacent to the frame line.

$K_{trib} = 0.5\ K_{total} = 0.5\left(20.28\ \text{kip}/\text{in}\right) = 10.14\ \text{kip}/\text{in}$

$\xi = 0.045$

Down slope facing purlin 1 α = -1

$\sigma = 0.375$

Up slope facing purlins 2-12 α = 1

$\sigma = 0.275$

Moment generated at each torsional restraint

Purlin 1. $\alpha = -1$ and $w = 57.5\text{plf}$

$M_{3rd} = -244\ \text{lb} \cdot \text{in}$

Overturning force at frame line 2

$P_i = -94\ \text{lb}$

Upslope facing purlins 2-11 $\alpha = 1 \quad w = 115\text{plf} \quad \sigma = 0.275$

$M_{3rd} = 159\ \text{lb} \cdot \text{in}$

$P_i = 45\ \text{lb}$

Purlin 12. (half of purlins 2-11)

$M_{3rd} = 80\ \text{lb} \cdot \text{in}$

$P_i = 22\ \text{lb}$

d. Interior span - half span adjacent to frame line 3.

$K_{trib} = 0.5\, K_{total} = 0.5\left(17.41\ {}^{kip}\!/_{in}\right) = 18.70\ {}^{kip}\!/_{in}$

$\xi = 0.051$

Uniform restraint force, σ, is calculated the same as previous except ξ = 0.051

Down-slope facing purlin 1, α = -1

$\sigma = 0.373$

Up slope facing purlins 2-12, α=1

$\sigma = 0.275$

Moment generated at each torsional restraint

Purlin 1. $\alpha = -1$ and $w = 57.5\text{plf}$

$M_{3rd} = -220\ \text{lb} \cdot \text{in}$

Overturning force at frame line 3.

$P_i = -91\ \text{lb}$

Upslope facing Purlins 2-11. Same as Purlin 1 except $\alpha = 1 \quad w = 115\text{plf} \quad \sigma = 0.274$

$M_{3rd} = 122\ \text{lb} \cdot \text{in}$

$P_i = 43\ \text{lb}$

Purlin 12. Same as Purlins 2-12 except $w = 57.5\ \text{plf}$

$M_{3rd} = 61\ \text{lb} \cdot \text{in}$

$P_i = 22\ \text{lb}$

3. Calculate anchorage forces along each frame line.

   The forces, P_i, calculated above include system effects (the inherent stiffness of the system of purlins), therefore it is not necessary to reduce for system effects.

a. Frame line 1

   The total force generated along frame line 1 is the sum of forces at each purlin.

$$\sum_{N_p} P_i = P_1 + 10P_{2-11} + P_{12} = -121\text{ lb} + (10)96\text{ lb} + 48\text{ lb} = 887\text{ lb}$$

   The force along the frame line is distributed according to relative stiffness of each restraint along the frame line.

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (887\text{ lb})\frac{1.59\,{}^{kip}\!/_{in}}{19.1\,{}^{kip}\!/_{in}} = 74\text{ lb} \qquad (5.1.74)$$

b. Frame line 2.

   The total force generated is the sum of forces at each purlin in each half span adjacent to the frame line.

$$\sum_{N_p} P_i = (P_1 + 10P_{2-11} + P_{12})_{Left} + (P_1 + 10P_{2-11} + P_{12})_{Right}$$

$$\sum_{N_p} P_i = (-105\text{ lb} + (10)82\text{ lb} + 41\text{ lb}) + (-93\text{ lb} + (10)45\text{ lb} + 23\text{ lb}) = 1136\text{ lb}$$

   The force along the frame line is distributed according to relative stiffness of each restraint along the frame line.

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (1136\text{ lb})\frac{1.69\,{}^{kip}\!/_{in}}{20.3\,{}^{kip}\!/_{in}} = 95\text{ lb} \qquad (5.1.74)$$

c. Frame line 3 Since the system is symmetric, the total force is two times the sum of forces for one half bay.

$$\sum_{N_p} P_i = 2(P_1 + 10P_{2-11} + P_{12})$$

$$\sum_{N_p} P_i = 2(P_1 + 10P_{2-11} + P_{12}) = 2(-93\text{ lb} + (10)45\text{ lb} + 23\text{ lb}) = 760\text{ lb}$$

   The force along the frame line is distributed according to relative stiffness of each restraint along the frame line.

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (760\text{ lb})\frac{1.45\,{}^{kip}\!/_{in}}{17.40\,{}^{kip}\!/_{in}} = 63\text{ lb} \qquad (5.1.74)$$

4. Check deformation of the system and compare to the limits specified in Section D6.3.2

a. Lateral displacement of purlin top flange

   Allowable deflection limit (ASD)

$$\Delta_{tf} = \frac{1}{\Omega}\frac{d}{20} = \frac{1}{2.00}\frac{8\text{ in}}{20} = 0.20\text{ in} \qquad (D6.3.1\text{-}9a)$$

Frame line 1

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(74\ \text{lb})}{1.59\ ^{\text{kip}}\!/_{\text{in}}} = 0.046\ \text{in} \le 0.20\ \text{in}\ \ \text{OK} \tag{5.1.81}$$

Frame line 2

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(95\ \text{lb})}{1.69\ ^{\text{kip}}\!/_{\text{in}}} = 0.056\ \text{in} \le 0.20\ \text{in}\ \ \text{OK} \tag{5.1.81}$$

Frame line 3

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(63\ \text{lb})}{1.45\ ^{\text{kip}}\!/_{\text{in}}} = 0.043\ \text{in} \le 0.20\ \text{in}\ \ \text{OK} \tag{5.1.81}$$

b. Mid-span displacement of diaphragm relative to restraint

Allowable deflection limit

$$\Delta_{ms} \le \frac{L}{180} = \frac{25\text{ft}(^{12\text{in}}\!/_{\text{ft}})}{180} = 1.67\text{in}$$

$$\Delta_{diaph} = \Sigma(w(\alpha\sigma - \sin\theta))_i \frac{L^2}{8G'Bay} \tag{5.1.82}$$

Exterior Span

The uniform restraint force for each half span in the bay is averaged. Typically, there should not but substantial difference between the two.

$$(\sigma_1)_{ave} = \tfrac{1}{2}(0.367 + 0.363) = 0.365$$

$$(\sigma_{2-12})_{ave} = \tfrac{1}{2}(0.294 + 0.292) = 0.293$$

$$\Delta_{diaph} = [57.5\text{plf}((-1)0.365 - \sin(2.4°)) + (10.5)(115\text{plf})((1)0.293 - \sin(2.4°))]\frac{(25\text{ft})^2}{8(1000\ ^{\text{lb}}\!/_{\text{in}})(55\text{ft})} = 0.40\,\text{in}$$

$$\Delta_{diaph} = 0.40\,\text{in} \le 1.67\,\text{in}\quad \text{OK}$$

Interior Span

$$(\sigma_1)_{ave} = \tfrac{1}{2}(0.375 + 0.373) = 0.374$$

$$(\sigma_{2-12})_{ave} = \tfrac{1}{2}(0.274 + 0.275) = 0.275$$

$$\Delta_{diaph} = [57.5\text{plf}((-1)0.374 - \sin(2.4°)) + (10.5)(115\text{plf})((1)0.275 - \sin(2.4°))]\frac{(25\text{ft})^2}{8(1000\ ^{\text{lb}}\!/_{\text{in}})(55\text{ft})} = 0.37\ \text{in}$$

$$\Delta_{diaph} = 0.40\ \text{in} \le 1.66\,\text{in}\quad \text{OK}$$

5. Calculate shear force in connection between sheathing and purlin at anchor location

$$P_{sc} = P_L + \Sigma\left(\frac{wL}{2}(0.9\sigma\alpha + \sin\theta) - P_i\right) \tag{5.1.83}$$

a. Frame line 1 – typical purlin

$$P_{sc} = 74\ \text{lb} + \frac{(115\text{plf})(25\text{ft})}{2}(0.9(0.294)(1) + \sin(2.4°)) - (96\text{lb}) = 419\ \text{lb}$$

b. Frame line 2 – typical purlin

$$P_{sc} = 95 + \frac{(115\text{plf})(25\text{ft})}{2}(0.9(0.292)(1) + \sin(2.4°)) - 82 \text{ lb}$$
$$+ \frac{(115\text{plf})(25\text{ft})}{2}(0.9(0.275)(1) + \sin(2.4°)) - 45 \text{ lb} = 822 \text{ lb}$$

c. Frame line 3 – typical purlin

$$P_{sc} = 63 + 2 \cdot \left[\frac{(115\text{plf})(25\text{ft})}{2}(0.9(0.274)(1) + \sin(2.4°)) - 43\text{lb}\right] = 566 \text{ lb}$$

Example 11: Supports Plus Third Point Lateral Anchorage

Given:

1. Four span continuous Z-purlin system from Example in Section 5.1.7.
2. Third point braces are applied at the third points of the eave purlin (Purlin 1). Each lateral brace is applied at the purlin top flange and has a stiffness of 15.0 kip/in.
3. Each purlin is attached to rafters with rafter web plates (wing plates). Web plates are ¼ in. thick by 5 in. wide (b_{pl} = 5 in.) by 7 in. tall. Web plates are attached to the web of the Z-section with two rows of two ½ in. diameter A307 bolts. The bottom row of bolts is 3 in. from the bottom flange and the top row is 6 in. from the bottom flange.

Required:

1. Compute the anchorage forces along each frame line and each third point due to gravity loads.
2. Compute the lateral deflection of the top flange of the Z-section along each frame line and at the purlin third points.
3. Compute the shear force in the standing seam panel clips at the frame line for a typical upslope facing purlin and at the third points at the restrained purlin.

Solutions:

Assumptions for Analysis

a. Since the loading, geometry and materials are symmetrical, check the first two spans only.
b. For simplicity, each half span of the continuous system of purlins is analyzed individually.
c. It is assumed that the stiffness of adjacent frame lines is approximately the same, i.e., at an interior frame line, half of the total stiffness along the frame line is considered tributary to each adjacent bay.

Procedure:

1. Calculate the restraint stiffness of the rafter web plates.

From previous example

a. Frame line 1

Stiffness of web bolted plate connection $K_{rest} = 1589\,\mathrm{lb/in}$

Total stiffness along frame line 1

$$K_{spt} = \left(\sum_{N_a} (K_{rest})_{spt} + \frac{\sum_{N_p - N_a} K_{rafter}}{d} \right) = (12)(1589\,\mathrm{lb/in}) = 19.07\,\mathrm{kip/in} \qquad (5.1.88)$$

b. Frame line 2

Equivalent purlin thickness at the lap $t_{lap} = 0.094$ in

Stiffness of web bolted plate connection $K_{rest} = 1690\,\mathrm{lb/in}$

Total stiffness along frame line 2

$$K_{spt} = \left(\sum_{N_a} (K_{rest})_{spt} + \frac{\sum_{N_p - N_a} K_{rafter}}{d} \right) = (12)(1690\,{}^{lb}\!/\!_{in}) = 20.28\,{}^{kip}\!/\!_{in} \tag{5.1.88}$$

c. Frame line 3

Equivalent purlin thickness at the lap $t_{lap} = 0.074\text{in}$

Stiffness of web bolted plate connection $K_{rest} = 1451\,{}^{lb}\!/\!_{in}$

Total stiffness along frame line 3

$$K_{spt} = \left(\sum_{N_a} (K_{rest})_{spt} + \frac{\sum_{N_p - N_a} K_{rafter}}{d} \right) = (12)(1451\,{}^{lb}\!/\!_{in}) = 17.41\,{}^{kip}\!/\!_{in}$$

2. Calculate overturning forces and distribute between third points and frame lines.

a. Exterior span – half span adjacent to frame line 1

$K_{trib} = C3(K_{spt}) = 1.0(19.07\,{}^{kip}\!/\!_{in}) \approx 19.1\,{}^{kip}\!/\!_{in}$

$K_{3rd} = 15.0\,{}^{kip}\!/\!_{in}$

Reduction factor to account for sheathing system effects

$$R_{sys} = \frac{1}{2}\left(\frac{{}^1\!/\!_4 Et^3}{0.38k_{mclip}d + 0.71\dfrac{Et^3}{4}} \right)\left(1 - {}^2\!/\!_3 k_{mclip}\tau\right) \tag{5.1.91}$$

Exterior Span

$$R_{sys} = \frac{1}{2}\left(\frac{{}^1\!/\!_4 E(0.085\text{in})^3}{0.38 \cdot 2500\,{}^{lb\cdot in}\!/\!_{rad\cdot ft} \cdot 8.0\text{in} + 0.71 \cdot {}^1\!/\!_4 E(0.085\text{in})^3} \right)\left(1 - {}^2\!/\!_3 2500\,{}^{lb\cdot in}\!/\!_{rad\cdot ft} \cdot 0.0055\frac{rad}{lb}\right) = 0.135$$

Reduction to account for local deformation

$$R_{local} = \frac{k_{mclip}}{k_{mclip} + \dfrac{Et^3}{3d}} = \frac{2500\,{}^{lb\cdot in}\!/\!_{rad\cdot ft}}{2500\,{}^{lb\cdot in}\!/\!_{rad\cdot ft} + \dfrac{E \cdot (0.085\text{in})^3}{3 \cdot 8.0\text{in}}} = 0.216 \tag{5.1.92}$$

Calculate uniform restraint force in sheathing. $w_{rest} = w \cdot \sigma$

$$\psi = \frac{(3G'BayK_{3rd})\left(1 + k_{mclip}\dfrac{\kappa}{GJ}\right)\left(N_p k_{mclip} LR_{sys} + 2d^2 K_{trib}\right)}{X1 + X2} \tag{5.1.94}$$

Where

$$X1 = (3G'BayK_{trib} + LK_{trib}K_{3rd})\left(1 + k_{mclip}\frac{\kappa}{GJ}\right)\left(N_p k_{mclip} LR_{sys} + 2d^2 K_{trib}\right) \tag{5.1.95}$$

$$X2 = \left(3G'BayK_{3rd}\right)\left(2d^2K_{trib}\right)\left(1 + k_{mclip}\frac{\kappa}{GJ} - \frac{\beta_{3rd}}{4}\frac{2}{3}k_{mclip}L\right) \tag{5.1.96}$$

$$X1 = \begin{bmatrix} \left[3\left(1000\,{}^{lb}\!/_{in}\right)\left(660in\right)\left(19.1\,{}^{kip}\!/_{in}\right) + \left(300in\right)\left(19.1\,{}^{kip}\!/_{in}\right)\left(15\,{}^{kip}\!/_{in}\right)\right]\times \\ \left(1 + \dfrac{2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}}{12\,{}^{in}\!/_{ft}}\dfrac{2582rad\cdot in^2}{G\cdot 0.00306in^4}\right)\times \\ \times\left(\left(12\right)\left(2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}\right)\left(300in\right)\left(0.135\right) + \left(2\right)\left(8in\right)^2\left(19.1\,{}^{kip}\!/_{in}\right)\right) \end{bmatrix} = 5.247\cdot 10^{18}\,lb^3$$

$$X2 = \begin{bmatrix} \left[3\left(1000\,{}^{lb}\!/_{in}\right)\left(660in\right)\left(15\,{}^{kip}\!/_{in}\right)\right]\left[\left(2\right)\left(8in\right)^2\left(19.1\,{}^{kip}\!/_{in}\right)\right]\times \\ \times\left(1 + \dfrac{2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}}{12\,{}^{in}\!/_{ft}}\dfrac{2582rad\cdot in^2}{G\cdot 0.00306in^4} - \dfrac{0.000841\,{}^{rad}\!/_{lb\cdot in}}{4}\dfrac{2}{3}\cdot 2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}\cdot 25ft\right) \end{bmatrix} = 5.752\cdot 10^{17}\,lb^3$$

$$\psi = \frac{\left(3\left(1000\,{}^{lb}\!/_{in}\right)\left(660in\right)\left(15\,{}^{kip}\!/_{in}\right)\right)\left(1 + \dfrac{2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}}{12\,{}^{in}\!/_{ft}}\dfrac{2582rad\cdot in^2}{G\cdot 0.00306in^4}\right)\begin{bmatrix}\left(12\right)\left(2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}\right)\left(300in\right)\left(0.135\right) + \\ \left(2\right)\left(8in\right)^2\left(19.1\,{}^{kip}\!/_{in}\right)\end{bmatrix}}{5.247\cdot 10^{18}\,lb^3 + 5.752\cdot 10^{17}\,lb^3} = 0.217$$

$$\Gamma = \frac{1}{N_p}\left(C2\frac{\alpha L^3}{EI_{my}} + \frac{\alpha\dfrac{d^2}{4}\beta_{3rd}}{\left(1 + k_{mclip}\dfrac{\kappa}{GJ}\right)} - \frac{N_p\cdot L}{3G'Bay}\right) \tag{5.1.97}$$

$$\Gamma = \left(\frac{31(-1)(300in)^3}{1458\cdot E\cdot 1.148in^4} + \frac{(-1)\dfrac{(8.0in)^2}{4}0.000841\,{}^{rad}\!/_{lb\cdot in}}{1 + \dfrac{2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}}{12\,{}^{in}\!/_{ft}}\dfrac{2582rad\cdot in^2}{G\cdot 0.00306in^4}} - \frac{(12)25ft}{3\left(1000\,{}^{lb}\!/_{in}\right)\left(55ft\right)}\right)\cdot\frac{1}{12} = -0.00163\,{}^{in}\!/_{lb}$$

$$A = \left(\frac{\delta b + m}{\delta b}\right) + \eta\alpha\frac{L}{d^2}\left(\frac{2}{3}k_{mclip}\left(\frac{\delta b + m}{\delta b}\right) - \frac{\left(1 - R_{Local}\right)}{\tau}\right)\left(\frac{2d^2K_{trib}}{2d^2K_{trib} + N_pk_{mclip}LR_{sys}}\right)\Gamma\psi \tag{5.1.98}$$

$$A = 1 + \begin{bmatrix} (10)(-1)\dfrac{300in}{(8.0in)^2}\left(\dfrac{2}{3}\left(2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}\right)(1) - \dfrac{(1 - 0.216)}{0.0055\,{}^{1}\!/_{lb}}\right)\times \\ \times\left(\dfrac{2(8.0in)^2\,19.1\,{}^{kip}\!/_{in}}{2(8.0in)^2\,19.1\,{}^{kip}\!/_{in} + (12)\left(2500\,{}^{lb\cdot in}\!/_{rad\cdot ft}\right)(25ft)(0.135)}\right)\cdot\left(-0.00163\,{}^{in}\!/_{lb}\right)(0.217) \end{bmatrix} = 0.96$$

$$B = \frac{\left(2K_{trib} - K_{3rd}\right)3G'Bay}{LK_{trib}K_{3rd} + 3G'Bay\left(K_{trib} + K_{3rd}\right)} \tag{5.1.99}$$

$$B = \frac{\left(2\cdot 19.1\,{}^{kip}\!/_{in} - 15\,{}^{kip}\!/_{in}\right)3\cdot 1000\,{}^{lb}\!/_{in}\cdot 55ft}{25ft\left(19.1\,{}^{kip}\!/_{in}\right)\left(15\,{}^{kip}\!/_{in}\right) + 3\left(1000\,{}^{lb}\!/_{in}\right)\left(55ft\right)\left(19.1\,{}^{kip}\!/_{in} + 15\,{}^{kip}\!/_{in}\right)} = 0.30$$

$$C = 1 + \alpha\eta\frac{\frac{2}{3}k_{mclip}L}{d^2}\left(\frac{2d^2K_{trib}}{2d^2K_{trib} + N_pk_{mclip}LR_{sys}}\right)\Gamma\psi \tag{5.1.100}$$

$$C = 1 + (10)(-1)\frac{\frac{2}{3}\left(2500\,{}^{lb\cdot in}\!/\!_{rad\cdot ft}\right)300in}{(8.0in)^2}\left(\frac{2(8.0in)^2\,19.1\,{}^{kip}\!/\!_{in}}{2(8.0in)^2\,19.1\,{}^{kip}\!/\!_{in} + (12)\left(2500\,{}^{lb\cdot in}\!/\!_{rad\cdot ft}\right)(25ft)(0.135)}\right)x$$

$$x\left(0.00163\,{}^{in}\!/\!_{lb}\right)(0.217)$$

$$C = 3.21$$

$$\sigma = \frac{C1\dfrac{\left(\dfrac{I_{xy}}{I_x}\cos\theta\right)L^4}{EI_{my}} + A\cdot\dfrac{((\delta b)\cos\theta)d}{2}\tau + B\cdot\dfrac{n_p\alpha L^2\sin\theta}{18G'Bay}}{C1\dfrac{L^4}{EI_{my}} + C\cdot\dfrac{d^2}{4}\tau + \dfrac{\eta\alpha L^2}{9G'Bay}\left(1 + K_{trib}\psi\Gamma\right)} \tag{5.1.93}$$

$$\sigma = \frac{\dfrac{5\left(\dfrac{4.11in^4}{12.4in^4}\cos(2.4°)\right)(300in)^4}{972\cdot E\cdot 1.15in^4} + (0.96)\dfrac{\left(\frac{2.75in}{3}\cos(2.4°)\right)8.0in}{2\cdot rad}0.0055\dfrac{rad}{lb} + (0.30)\dfrac{(12)(-1)(300in)^2\sin(2.4°)}{18\cdot\left(1000\,{}^{lb}\!/\!_{in}\right)(660in)}}{\dfrac{5(300in)^4}{972\cdot E\cdot 1.15in^4} + (3.21)\dfrac{(8.0in)^2}{4\cdot rad}0.0055\dfrac{rad}{lb} + \dfrac{(10)(-1)(300in)^2}{9\cdot\left(1000\,{}^{lb}\!/\!_{in}\right)(660in)}\left(1 + \left(-0.00163\,{}^{in}\!/\!_{lb}\right)(0.217)\left(19.1\,{}^{kip}\!/\!_{in}\right)\right)} = 0.179$$

$$w_{rest} = w\cdot\sigma = 57.5\,plf\cdot 0.179 = 10.3\,plf$$

Distribution of down slope force between third point and frame line restraint

$$D_{3rd} = \frac{K_{3rd}\left(9G'Bay + 2LK_{trib}\right)}{3\left[LK_{trib}K_{3rd} + 3G'Bay\left(K_{trib} + K_{3rd}\right)\right]}\left(\frac{2d^2\left(K_{trib} + K_{3rd}\right)}{2d^2\left(K_{trib} + K_{3rd}\right) + n_p k_{mclip} LR_{sys}}\right) \tag{5.1.101}$$

$$D_{Spt} = \frac{K_{trib}\left(9G'Bay + LK_{3rd}\right)}{3\left[LK_{trib}K_{3rd} + 3G'Bay\left(K_{trib} + K_{3rd}\right)\right]}\left(\frac{2d^2\left(K_{trib} + K_{3rd}\right)}{2d^2\left(K_{trib} + K_{3rd}\right) + N_p k_{mclip} LR_{sys}}\right) \tag{5.1.102}$$

$$D_{3rd} = \left[\begin{array}{l}\dfrac{15\,{}^{kip}\!/\!_{in}\left(9\left(1000\,{}^{lb}\!/\!_{in}\right)(55ft) + 2\left(19.1\,{}^{kip}\!/\!_{in}\right)25ft\right)}{3\left[(25ft)\left(19.1\,{}^{kip}\!/\!_{in}\right)\left(15\,{}^{kip}\!/\!_{in}\right) + 3\left(1000\,{}^{lb}\!/\!_{in}\right)(55ft)\left(19.1\,{}^{kip}\!/\!_{in} + 15\,{}^{kip}\!/\!_{in}\right)\right]}\times \\ \times\left(\dfrac{2(8.0in)^2\left(19.1\,{}^{kip}\!/\!_{in} + 15\,{}^{kip}\!/\!_{in}\right)}{2(8.0in)^2\left(19.1\,{}^{kip}\!/\!_{in} + 15\,{}^{kip}\!/\!_{in}\right) + (12)\left(2500\,{}^{lb\cdot in}\!/\!_{rad\cdot ft}\right)(25ft)(0.135)}\right)\end{array}\right] = 0.55$$

$$D_{spt} = \left[\begin{array}{l}\dfrac{19.1\,{}^{kip}\!/\!_{in}\left(9\left(1000\,{}^{lb}\!/\!_{in}\right)(55ft) + \left(15\,{}^{kip}\!/\!_{in}\right)25ft\right)}{3\left[(25ft)\left(19.1\,{}^{kip}\!/\!_{in}\right)\left(15\,{}^{kip}\!/\!_{in}\right) + 3\left(1000\,{}^{lb}\!/\!_{in}\right)(55ft)\left(19.1\,{}^{kip}\!/\!_{in} + 15\,{}^{kip}\!/\!_{in}\right)\right]}\times \\ \times\left(\dfrac{2(8.0in)^2\left(19.1\,{}^{kip}\!/\!_{in} + 15\,{}^{kip}\!/\!_{in}\right)}{2(8.0in)^2\left(19.1\,{}^{kip}\!/\!_{in} + 15\,{}^{kip}\!/\!_{in}\right) + (12)\left(2500\,{}^{lb\cdot in}\!/\!_{rad\cdot ft}\right)(25ft)(0.135)}\right)\end{array}\right] = 0.42$$

$$\left(P_{3rd}\right)_i = \frac{wL}{2}\cdot\left[\begin{array}{l}\left(\dfrac{\delta b\cos\theta}{d}\left(1 - R_{local}\right) + \frac{2}{3}k_{mclip}\left(\dfrac{\sigma}{2} - \dfrac{(\delta b + m)}{d}\cos\theta\right)\tau\right)\left(\dfrac{2d^2K_{trib}}{2d^2K_{trib} + n_p k_{mclip} LR_{sys}}\right)\psi\alpha \\ + \dfrac{2K_{trib}\sigma L}{9G'Bay}\psi\alpha - \sin\theta D_{3rd}\end{array}\right] \tag{5.1.89}$$

$$\left(P_{3rd}\right)_1 = \frac{(57.5\text{plf})(25\text{ft})}{2}\cdot\left(\begin{array}{l}\left(\frac{\frac{2.75\text{in}}{3}\cos(2.4°)}{8.0\text{in}}(1-0.216)-\frac{2}{3}\frac{2500\,{}^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}}{12\,{}^{\text{in}}/_{\text{ft}}}\left(\frac{\frac{2.75\text{in}}{3}\cos(2.4°)}{8.0\text{in}}-\frac{0.179}{2}\right)0.0055\,{}^{\text{rad}}/_{\text{lb}}\right)\times \\ \times\left(\frac{2(8.0\text{in})^2\left(19.1\,{}^{\text{kip}}/_{\text{in}}\right)}{2(8.0\text{in})^2\left(19.1\,{}^{\text{kip}}/_{\text{in}}\right)+(12)\left(2500\,{}^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}\right)(25\text{ft})(0.135)}\right)(-1)(0.217) \\ +\frac{(2)\left(19.1\,{}^{\text{kip}}/_{\text{in}}\right)(0.179)(\cdot 25\text{ft})}{9\left(1000\,{}^{\text{lb}}/_{\text{in}}\right)(55\text{ft})}(-1)(0.217)-\sin(2.4°)(0.55)\end{array}\right)$$

$$\left(P_{3rd}\right)_1 = -81\text{ lb}$$

$$F = \frac{\left(\frac{\beta_{3rd}}{4}\frac{2}{3}k_{mclip}L-\left(1+k_{mclip}\frac{\kappa}{GJ}\right)\right)}{\left(1+k_{mclip}\frac{\kappa}{GJ}\right)}\left(\frac{2d^2K_{trib}}{2d^2K_{trib}+N_pk_{mclip}LR_{sys}}\right)\psi \qquad (5.1.103)$$

$$F = \left[\begin{array}{l}\left(\frac{\left(\frac{0.000841\,{}^{1}/_{\text{lb}\cdot\text{in}}}{4}\frac{2}{3}\left(2500\,{}^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}\right)(25\text{ft})-\left(1+\frac{2500\,{}^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}}{12\,{}^{\text{in}}/_{\text{ft}}}\frac{2582\text{rad}\cdot\text{in}^2}{G\cdot 0.00306\text{in}^4}\right)\right)}{\left(1+\frac{2500\,{}^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}}{12\,{}^{\text{in}}/_{\text{ft}}}\frac{2582\text{rad}\cdot\text{in}^2}{G\cdot 0.00306\text{in}^4}\right)}\right)\times \\ \times\left(\frac{2(8.0\text{in})^2\left(19.1\,{}^{\text{kip}}/_{\text{in}}\right)}{2(8.0\text{in})^2\left(19.1\,{}^{\text{kip}}/_{\text{in}}\right)+(12)\left(2500\,{}^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}\right)(25\text{ft})(0.135)}\right)(0.217)\end{array}\right] = -0.10$$

$$\left(P_{spt}\right)_i = \frac{wL}{2}\cdot\left[\begin{array}{l}\left(\frac{\delta b\cos\theta}{d}\left(1-R_{local}\right)+\frac{2}{3}k_{mclip}\left(\frac{\sigma}{2}-\frac{(\delta b+m)}{d}\cos\theta\right)\tau\right)\left(\frac{2d^2K_{trib}}{2d^2K_{trib}+N_pk_{mclip}LR_{sys}}\right)\alpha(1+F) \\ +\frac{2K_{trib}\sigma L}{9G'Bay}\alpha F-\sin\theta D_{spt}\end{array}\right]$$

(5.1.90)

$$\left(P_{spt}\right)_1 = \frac{(57.5\text{plf})(25\text{ft})}{2}\cdot\left(\begin{array}{l}\left(\frac{\frac{2.75\text{in}}{3}\cos(2.4°)}{8.0\text{in}}(1-0.216)-\frac{2}{3}\frac{2500\,{}^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}}{12\,{}^{\text{in}}/_{\text{ft}}}\left(\frac{\frac{2.75\text{in}}{3}\cos(2.4°)}{8.0\text{in}}-\frac{0.179}{2}\right)0.0055\,{}^{\text{rad}}/_{\text{lb}}\right)\times \\ \times\left(\frac{2(8.0\text{in})^2\left(19.1\,{}^{\text{kip}}/_{\text{in}}\right)}{2(8.0\text{in})^2\left(19.1\,{}^{\text{kip}}/_{\text{in}}\right)+(12)\left(2500\,{}^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}\right)(25\text{ft})(0.135)}\right)(-1)(1+(-0.10)) \\ +\frac{(-1)(2)\left(19.1\,{}^{\text{kip}}/_{\text{in}}\right)(0.179)(\cdot 25\text{ft})}{9\left(1000\,{}^{\text{lb}}/_{\text{in}}\right)(55\text{ft})}(-1)(-0.10)-\sin(2.4°)(0.42)\end{array}\right)$$

$$\left(P_{spt}\right)_1 = -32\text{ lb}$$

Up slope Purlins 2-11 $\alpha = 1 \quad w = 115\text{ lb}$

$\psi = 0.217$ (same as purlin 1)

$\Gamma = 0.00111\ {}^{in}\!/_{lb}$

$A = 0.967$

$B = 0.299$ (same as purlin 1)

$C = 2.80$

$\sigma = 0.174$

$D_{3rd} = 0.55$ (same as purlin 1)

$D_{spt} = 0.42$ (same as purlin 1)

Forces generated

$(P_{3rd})_{2-11} = 92\ lb$

$F = -0.10$ (same as purlin 1)

$(P_{spt})_{2-11} = 12\ lb$

Purlin 12 (since the uniform loading is half that of purlins 2-11, the brace forces are half that for purlins 2-11.

$(P_{3rd})_{12} = 46\ lb$

$(P_{spt})_{12} = 6\ lb$

b. Exterior span – half span adjacent to frame line 2.

Half of the stiffness along interior frame line is considered tributary to each adjacent half span.

$K_{trib} = C3(K_{spt}) = 0.5(20.2\ {}^{kip}\!/_{in}) = 10.1\ {}^{kip}\!/_{in}$

$K_{3rd} = 15.0\ {}^{kip}\!/_{in}$

Reduction factor to account for sheathing system effects

$R_{sys} = 0.222$

Reduction to account for local deformation.

$R_{local} = 0.216$ (same as previous)

Calculate uniform restraint force in sheathing. $w_{rest} = w \cdot \sigma$

$X1 = 5.02 \cdot 10^{17}\ lb^3$

$X2 = 1.085 \cdot 10^{17}\ lb^3$

$\psi = 0.372$

$\Gamma = -0.000894\ {}^{in}\!/_{lb}$

$A = 0.50$

$B = 0.30$

$C = 2.92$

$\sigma = 0.205$

$w_{rest} = w \cdot \sigma = 57.5\,\text{plf} \cdot 0.205 = 11.8\,\text{plf}$

Distribution of down slope force between third point and frame line restraint

$D_{3rd} = 0.60$

$D_{spt} = 0.35$

$(P_{3rd})_1 = -94\text{ lb}$

$F = -0.18$

$(P_{spt})_1 = -27\text{ lb}$

Upslope Purlins 2-11 $\alpha = -1 \quad w = 115\text{ lb}$

$\psi = 0.372$ (same as purlin 1)

$\Gamma = 0.00059\,\text{in}/\text{lb}$

$A = 0.67$

$B = 0.11$ (same as purlin 1)

$C = 2.27$

$\sigma = 0.196$

$D_{3rd} = 0.60$ (same as purlin 1)

$D_{spt} = 0.35$ (same as purlin 1)

Restraint Forces

$(P_{3rd})_{2-11} = 109\text{ lb}$

$F = -0.10$ (same as purlin 1)

$(P_{spt})_{2-11} = 11\text{ lb}$

Purlin 12 (since the uniform loading is half that of purlins 2-11, the forces generated are half that for purlins 2-11.

$(P_{3rd})_{12} = 55\text{ lb}$

$(P_{spt})_{12} = 6\text{ lb}$

c. Interior span – half span adjacent to frame line 2.

Half of the stiffness along interior frame line is considered tributary to each adjacent half span.

$K_{trib} = 0.5(K_{spt}) = 0.5(20.2\,\text{kip}/\text{in}) = 10.1\,\text{kip}/\text{in}$

$K_{3rd} = 15.0\,\text{kip}/\text{in}$

Reduction factor to account for sheathing system effects

$R_{sys} = 0.148$

Reduction to account for local deformation

$R_{local} = 0.452$

Calculate uniform restraint force in sheathing. $w_{rest} = w \cdot \sigma$

$X1 = 6.85 \cdot 10^{17}\,lb^3$

$X2 = 1.447 \cdot 10^{17}\,lb^3$

$\psi = 0.373$

$\Gamma = -0.000814\ {}^{in}\!/\!_{lb}$

$A = 1.322$

$B = 0.11$

$C = 2.82$

$\sigma = 0.207$

$w_{rest} = w \cdot \sigma = 57.5\,plf \cdot 0.207 = 11.9\,plf$

Distribution of down slope force between third point and frame line restraint

$D_{3rd} = 0.61$

$D_{spt} = 0.36$

$(P_{3rd})_1 = -89\,lb$

$F = -0.17$

$(P_{spt})_1 = -14\,lb$

Upslope Purlins 2-11 $\alpha = -1$ $w = 115\,lb$

$\psi = 0.373$ (same as purlin 1)

$\Gamma = 0.00051\ {}^{in}\!/\!_{lb}$

$A = 1.20$

$B = 0.11$ (same as purlin 1)

$C = 2.14$

$\sigma = 0.194$

$D_{3rd} = 0.61$ (same as purlin 1)

$D_{spt} = 0.36$ (same as purlin 1)

Restraint Forces

$(P_{3rd})_{2-11} = 95\,lb$

$F = -0.17$ (same as purlin 1)

$(P_{spt})_{2-11} = -15\,lb$

Purlin 12 (since the uniform loading is half that of purlins 2-11, the brace forces are half that for purlins 2-11.

$(P_{3rd})_{12} = 48\,lb$

$(P_{spt})_{12} = -8\,lb$

d. Interior span – half span adjacent to frame line 3.

The stiffness of the restraints along frame line 3 is slightly less than along frame line 2. In lieu of performing additional calculations, the restraint forces determined above can be conservatively used for the half span adjacent to frame line 3.

Restraint Stiffness

$K_{trib} = 0.5(K_{spt}) = 0.5(17.4\,{}^{kip}\!/\!_{in}) = 8.7\,{}^{kip}\!/\!_{in}$

$K_{3rd} = 15.0\,{}^{kip}\!/\!_{in}$

Downslope Purlin 1

$(P_{3rd})_1 = -89 \text{ lb}$

$(P_{spt})_1 = -14 \text{ lb}$

Upslope Purlins 2-11 $\alpha = -1 \quad w = 115 \text{ lb}$

Forces Generated

$(P_{3rd})_{2-11} = 95 \text{ lb}$

$(P_{spt})_{2-11} = -15 \text{ lb}$

Upslope Purlin 12

$(P_{3rd})_{12} = 48 \text{ lb}$

$(P_{spt})_{12} = -8 \text{ lb}$

4. Calculate forces in anchors along each frame line.

The forces, P_i, calculated above include system effects (the inherent stiffness of the system of purlins), therefore it is not necessary to reduce for system effects.

The total force generated along the exterior frame line is the sum of forces at each purlin.

$$\sum_{N_p} P_i = P_1 + 10P_{2-11} + P_{12} = -32 \text{ lb} + (10)12.3 \text{ lb} + 6 \text{ lb} = 97 \text{ lb}$$

The force along the frame line is distributed according to relative stiffness of each restraint along the frame line.

$$(P_L)_{spt} = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{spt}} = (97 \text{ lb}) \frac{1.59\,{}^{kip}\!/\!_{in}}{19.1\,{}^{kip}\!/\!_{in}} = 8 \text{ lb} \qquad (5.1.85)$$

At the first interior frame line, the total force generated is the sum of forces at each purlin in each half span adjacent to the frame line.

$$\sum_{N_p} P_i = (P_1 + 10P_{2-11} + P_{12})_{Left} + (P_1 + 10P_{2-11} + P_{12})_{Right}$$

$$\sum_{N_p} P_i = (-27\text{ lb} + (10)10.9\text{ lb} + 5.5\text{ lb}) + (-14\text{ lb} + (10)(-15.4\text{ lb}) + (-8\text{ lb})) = -89\text{ lb}$$

The force along the frame line is distributed according to relative stiffness of each restraint along the frame line.

$$(P_L)_{spt} = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{spt}} = (-89\text{ lb})\frac{1.69\,{}^{kip}/_{in}}{20.3\,{}^{kip}/_{in}} = -8\text{ lb} \qquad (5.1.85)$$

At the middle frame line, because the system is symmetric, the total force is two times the sum of forces for one half bay.

$$\sum_{N_p} P_i = 2(P_1 + 10P_{2-11} + P_{12}) = 2(-14\text{ lb} + (10)(-14.6\text{ lb}) + (-8\text{ lb})) = -336\text{ lb}$$

The force along the frame line is distributed according to relative stiffness of each restraint along the frame line.

$$(P_L)_{spt} = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{spt}} = (-336\text{ lb})\frac{1.45\,{}^{kip}/_{in}}{17.40\,{}^{kip}/_{in}} = -28\text{ lb} \qquad (5.1.85)$$

5. Forces at third point anchorages

The forces generated in each span are averaged (assuming that both third points have approximately equivalent stiffness).

a. Exterior Span

$$\sum_{N_p} P_i = \frac{1}{2}\left[(P_1 + 10P_{2-11} + P_{12})_{outside} + (P_1 + 10P_{2-11} + P_{12})_{inside}\right]$$

$$\sum_{N_p} P_i = \frac{1}{2}\left[(-81\text{ lb} + (10)91.6\text{ lb} + 46\text{ lb}) + (-94\text{ lb} + (10)109\text{ lb} + 55\text{ lb})\right] = 966\text{ lb}$$

$$(P_L)_{3rd} = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{3rd}} = (966\text{ lb})\frac{15\,{}^{kip}/_{in}}{15\,{}^{kip}/_{in}} = 966\text{ lb} \qquad (5.1.84)$$

Interior Span

$$\sum_{N_p} P_i = \frac{1}{2}\left[2\cdot(P_1 + 10P_{2-11} + P_{12}) = 2\cdot(-89\text{ lb} + (10)(95.1\text{ lb}) + 48\text{ lb})\right] = 910\text{ lb}$$

$$(P_L)_{3rd} = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{3rd}} = (910\text{ lb})\frac{15\,{}^{kip}/_{in}}{15\,{}^{kip}/_{in}} = 910\text{ lb} \qquad (5.1.84)$$

6. Check deformation of the system and compare to limits specified in Section D6.3.1

a. Lateral displacement of purlin top flange at frame line

Allowable deflection limit (ASD)

$$\Delta_{tf} = \frac{1}{\Omega}\frac{d}{20} = \frac{1}{2.00}\frac{8\text{in}}{20} = 0.20\text{ in} \qquad (D6.3.1\text{-}9a)$$

Frame Line 1

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(8 \cdot lb)}{1.59\,{}^{kip}\!/_{in}} = 0.005in \le 0.20in \;\; OK \tag{5.1.104}$$

Frame Line 2

$$\Delta_{spt2} = \frac{P_L}{K_{rest}} = \frac{(-8lb)}{1.69\,{}^{kip}\!/_{in}} = -0.005in \le 0.20in \;\; OK \tag{5.1.104}$$

Frame Line 3

$$\Delta_{spt3} = \frac{P_L}{K_{rest}} = \frac{(-28lb)}{1.45\,{}^{kip}\!/_{in}} = -0.019in \le 0.20\,in \;\; OK \tag{5.1.104}$$

b. Deformation of the interior of the span is checked at the third points. Diaphragm deformation between anchorage locations is typically minimal.

Allowable deflection limit

$$\Delta_{ms} \le \frac{L}{360} = \frac{25ft\,{}^{12in}\!/_{ft}}{360} = 0.83\,in$$

Exterior Span

$$\Delta_{3rdExt} = \frac{P_L}{K_{rest}} = \frac{(966 \cdot lb)}{15\,{}^{kip}\!/_{in}} = 0.064\,in \le 0.83\,in \;\;\; OK \tag{5.1.104}$$

Interior Span

$$\Delta_{3rdInt} = \frac{P_L}{K_{rest}} = \frac{(910 \cdot lb)}{15\,{}^{kip}\!/_{in}} = 0.061\,in \le 0.83\,in \;\; OK \tag{5.1.104}$$

7. Calculate force in connection between the sheathing and purlin at anchor locations.

a. At frame line

$$P_{sc} = P_L + \frac{wL}{2}(0.9\sigma\alpha + \sin\theta) - P_i \tag{5.1.105}$$

Frame line 1 - typical purlin

$$P_{sc} = 8\,lb + \frac{(115\,plf)(25\,ft)}{2}(0.9(0.174)(1) + \sin(2.4°)) - (12\,lb) = 281\,lb$$

Frame line 2 - typical purlin

$$P_{sc} = -8 + \frac{(115\,plf)(25\,ft)}{2}(0.9(0.196)(1) + \sin(2.4°)) - 11\,lb$$
$$+ \frac{(115\,plf)(25\,ft)}{2}(0.9(0.194)(1) + \sin(2.4°)) - (-15) = 621\,lb$$

Frame line 3 - typical purlin

$$P_{sc} = -28 + 2 \cdot \left[\frac{(115\,plf)(25\,ft)}{2}(0.9(0.193)(1) + \sin(2.4°)) - 15\,lb\right] = 562\,lb$$

b. Third point restraints

$$P_{sc} = P_L + \frac{wL}{20}\left(-0.9\sigma + \frac{\delta b \cos\theta}{d}\right)\alpha \tag{5.1.106}$$

Exterior 3rd Points

$$P_{sc} = 966 + \frac{(57.5\,\text{plf})(25\,\text{ft})}{20}\left(-0.9(0.205) + \frac{(2.75\text{in})\cos(2.4°)}{(3)(8.0\text{in})}\right)(-1) = 971\text{ lb}$$

At Interior 3rd Points

$$P_{sc} = 910 + \frac{(57.5\,\text{plf})(25\,\text{ft})}{20}\left(-0.9(0.207) + \frac{(2.75\text{in})\cos(2.4°)}{(3)(8.0\text{in})}\right)(-1) = 915\text{ lb}$$

5.1.8 C-Section Example

An example using the component stiffness method to predict anchorage forces is provided based on the roof system from the continuous purlin design example in Section 3.3.

Given:

1. The roof system has four 25 ft spans with purlins lapped over the interior supports. The purlins in the exterior spans are 9CS2.5x070 and the interior spans are 9CS2.5x059. There is a total of 12 purlin lines spaced at 5 ft-0 in. on center. To facilitate lapping of the purlins, webs of purlins in adjacent spans are placed back to back. Referring to the roof plan below in Figure 5.1.18, the top flanges of the purlins in the exterior span on the left are facing downslope. The direction of the top flanges of the purlins alternate moving from left to right on the plan. Roof slope is a ½ on 12 pitch.
2. Gravity loads are 3 psf dead and 20 psf live.
3. Roof covering is attached with through fasteners along entire length of the purlins. The sheathing has a diaphragm stiffness G′ = 2500 lb/in. and the rotational stiffness of the panel to purlin connection is k_{mclip} = 3600 lb-in./rad./ft.
4. No discrete bracing lines; anti-roll clips are provided at each support at every fifth purlin line. Each anti-roll anchorage device is attached to the web of the C-section with two rows of two ½ in. diameter A307 bolts. The bottom row of bolts is 3 in. from the bottom flange and the top row is 7 in. from the bottom flange. The stiffness of each anti-roll anchorage device, k_{device} = 40 k/in. The width of the anti-roll anchorage device is b_{pl} = 5.0 in.
5. Purlin flanges are bolted to the support member with two ½ in. diameter A307 bolts through the bottom flange.

Required:

1. Compute the anchorage forces along each frame line due to gravity loads.
2. Compute the lateral deflection of the top flange of the C-section along each frame line and at the purlin mid-span.
3. Compute the shear force in the standing seam panel clips at each anchorage device.

Solutions:

Assumptions for Analysis

1. Because the direction of the top flanges of the C-sections alternates, symmetry cannot be used.
2. Each anchor location is considered to have a single degree of freedom along line of anchorage. It is assumed that there is some mechanism to rigidly transfer forces from the remote purlins to the anchorage device. The sheathing provides the mechanism to transfer the force as long as the connection between the purlin and sheathing has sufficient strength and stiffness to transfer the force.
3. It is assumed that the stiffness of adjacent frame lines is approximately the same.

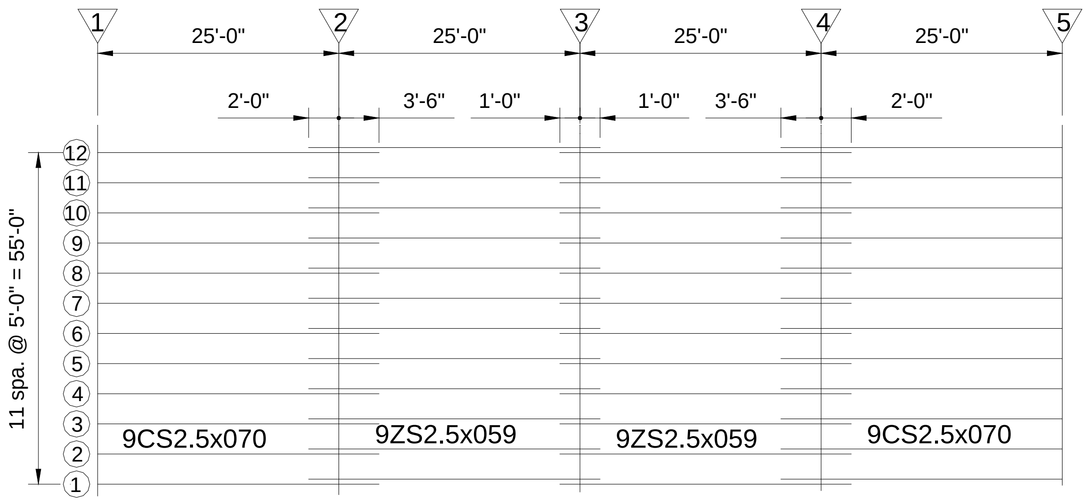


Figure 5.1.18 Roof Layout for Anchorage Examples

System Properties

L = 25 ft

Bay = 55 ft

Width = 55 ft/12 purlins = 55.0 in

Uniform load

Dead = 3 psf

Live = 20 psf

Roof Slope, θ = 2.4 degrees (1/2:12)

G′ = 2500 lb/in

k_{mclip} = 3600 lb-in/rad/ft

E = 29500000 psi

G = 11300000 psi

Section Properties

The following sections properties are used for the two C-sections:

INTERIOR BAYS	END BAYS
For: 9CS2.5x059	For: 9CS2.5x070
t = 0.059 in.	t = 0.070 in.
d = 9.0 in.	d = 9.0 in.
b = 2.50 in.	b = 2.50 in.
I_x = 10.3 in.^{4}	I_x = 12.20 in.^{4}
I_y = 0.698 in.^{4}	I_y = 0.890 in.^{4}
I_{xy} = 0.0 in.^{4}	I_{xy} = 0.0 in.^{4}
m = 1.05 in.	m = 1.05 in.

$J = 0.00102$ in.^{4} | $J = 0.00171$ in.^{4}
$C_w = 11.9$ in.^{6} | $C_w = 14.2$ in.^{6}
$I_{my} = \dfrac{I_X I_Y - I_{XY}^2}{I_X} = 0.698$ in.4 | $I_{my} = \dfrac{I_X I_Y - I_{XY}^2}{I_X} = 0.89$ in.^{4}

Torsional Properties

Exterior Span 9CS2.5x070. Outside half span torsionally approximated with both ends "warping free"

$$a = \sqrt{\frac{EC_W}{GJ}} = \sqrt{\frac{E \cdot 14.2\text{in}^6}{G \cdot 0.00171\text{in}^4}} = 147.2\text{ in} \tag{5.1.38}$$

$$\beta = \frac{L^2}{8a^2} + \frac{1}{\cosh\left(\frac{L}{2a}\right)} - 1 = \frac{(300\text{in})^2}{8 \cdot (147.2\text{in})^2} + \frac{1}{\cosh\left(\frac{300\text{in}}{2 \cdot 147.2\text{in}}\right)} - 1 = 0.158\text{ rad} \tag{5.1.40}$$

$$\kappa = \frac{8a^4}{L^2}\left(\frac{\cosh\left(\frac{L}{2a}\right) - 1}{\cosh\left(\frac{L}{2a}\right)}\right) + \frac{5L^2}{48} - a^2 \tag{5.1.41}$$

$$\kappa = \frac{8(147.2\text{in})^4}{(300\text{in})^2}\left(\frac{\cosh\left(\frac{300\text{in}}{2 \cdot 147.2\text{in}}\right) - 1}{\cosh\left(\frac{300\text{in}}{2 \cdot 147.2\text{in}}\right)}\right) + \frac{5(300\text{in})^2}{48} - (147.2\text{in})^2 = 2785\,\text{rad} \cdot \text{in}^2$$

$$\tau = \frac{\frac{a^2\beta}{GJ}}{1 + \frac{k_{mclip}}{GJ}\kappa} = \frac{\frac{(147.2\text{in})^2 \cdot 0.158\text{rad}}{G \cdot 0.00171\text{in}^4}}{1 + \frac{3600\,{}^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}}{G \cdot 0.00171\text{in}^4} \cdot 2785\text{rad} \cdot \text{in}^2} = 0.0040\,\frac{\text{rad}}{\text{lb}} \tag{5.1.39}$$

Exterior Span 9CS2.5x070. Inside half span torsionally approximated with both ends "warping fixed"

$$\beta = \frac{L^2}{8a^2} + \frac{L}{2a}\frac{1 - \cosh\left(\frac{L}{2a}\right)}{\sinh\left(\frac{L}{2a}\right)} = \frac{(300\text{in})^2}{8 \cdot (147.2\text{in})^2} + \frac{300\text{in}}{(2)(147.2\text{in})}\frac{1 - \cosh\left(\frac{300\text{in}}{2 \cdot 147.2\text{in}}\right)}{\sinh\left(\frac{300\text{in}}{2 \cdot 147.2\text{in}}\right)} = 0.041\text{ rad} \tag{5.1.43}$$

$$\kappa = \left(\frac{aL}{3} - \frac{4a^3}{L}\right)\left(\frac{1 - \cosh\left(\frac{L}{2a}\right)}{\sinh\left(\frac{L}{2a}\right)}\right) + \frac{5L^2}{48} - a^2 \tag{5.1.44}$$

$$\kappa = \left(\frac{(147.2\text{in})(300\text{in})}{3} - \frac{4(147.2\text{in})^3}{300\text{in}}\right)\left(\frac{1-\cosh\left(\frac{300\text{in}}{2(147.2\text{in})}\right)}{\sinh\left(\frac{300\text{in}}{2(147.2\text{in})}\right)}\right) + \frac{5(300\text{in.})^2}{48} - (147.2\text{in})^2$$

$$\kappa = 763.9\ \text{rad}\cdot\text{in}^2$$

$$\tau = 0.0036\cdot\frac{\text{rad}}{\text{lb}}$$

Interior Span 9CS2.5x059 Torsionally approximated with both ends "warping fixed"

$$a = 174.5\ \text{in}$$

$$\beta = 0.021\ \text{rad}$$

$$\kappa = 558.9\ \text{rad}\cdot\text{in}^2$$

$$\tau = 0.0036\ \frac{\text{rad}}{\text{lb}}$$

Procedure

1. Calculate uniform restraint provided by sheathing, w_{rest}, expressed as a proportion of the applied uniform load.

   $w_{rest} = w\cdot\sigma$

   where

$$\sigma = \frac{C1\frac{\left(\frac{I_{xy}}{I_x}\cos\theta\right)L^4}{EI_{my}} + \frac{((\delta b+m)\cos\theta)d}{2}\tau + \frac{\alpha\cdot Np\cdot L^2\sin\theta}{8G'Bay}}{C1\frac{L^4}{EI_{my}} + \frac{d^2}{4}\tau + \frac{\alpha\cdot\eta\cdot L^2}{8G'Bay}} \qquad (5.1.51)$$

   Since I_{xy} = 0, for C-sections, the above equation reduces to

$$\sigma = \frac{\frac{((\delta b+m)\cos\theta)d}{2}\tau + \frac{\alpha\cdot Np\cdot L^2\sin\theta}{8G'Bay}}{C1\frac{L^4}{EI_{my}} + \frac{d^2}{4}\tau + \frac{\alpha\cdot\eta\cdot L^2}{8G'Bay}} \qquad (5.1.11)$$

a. Left Exterior Span – half span adjacent to frame line 1 (approximated as a simple-fixed beam with warping free ends).

   C1 = 1/185, α = -1, η = -12 (purlins facing downslope)

$$\sigma = \frac{\frac{\left(\left(\frac{2.5\text{in}}{3}+1.05\text{in}\right)\cos(2.4°)\right)9.0\text{in}}{2\cdot\text{rad}}0.0040\frac{\text{rad}}{\text{lb}} + \frac{(-1)(12)(300\text{in})^2\sin(2.4°)}{8\cdot\left(2500\,{}^{\text{lb}}/_{\text{in}}\right)(660\text{in})}}{\frac{(300\text{in})^4}{185\cdot E\cdot 0.89\text{in}^4} + \frac{(9.0\text{in})^2}{4\cdot\text{rad}}0.0040\frac{\text{rad}}{\text{lb}} + \frac{(-1)(-12)(300\text{in})^2}{8\cdot\left(2500\,{}^{\text{lb}}/_{\text{in}}\right)(660\text{in})}} = 0.017$$

b. Left Exterior Span – half span adjacent to frame line 2 (approximated as a simple-fixed beam with warping fixed ends).

C1 = 1/185, α = -1, η = -12 (purlins facing down slope)

$$\sigma = \frac{\dfrac{\left(\left(\frac{2.5\text{in}}{3}+1.05\text{in}\right)\cos(2.4°)\right)9.0\text{in}}{2\cdot\text{rad}}0.0036\dfrac{\text{rad}}{\text{lb}}+\dfrac{(-1)(12)(300\text{in})^2\sin(2.4°)}{8\cdot\left(2500\,{}^{\text{lb}}\!/\!_{\text{in}}\right)(660\text{in})}}{\dfrac{(300\text{in})^4}{185\cdot\text{E}\cdot0.89\text{in}^4}+\dfrac{(9.0\text{in})^2}{4\cdot\text{rad}}0.0036\dfrac{\text{rad}}{\text{lb}}+\dfrac{(-1)(-12)(300\text{in})^2}{8\cdot\left(2500\,{}^{\text{lb}}\!/\!_{\text{in}}\right)(660\text{in})}} = 0.015$$

c. Interior Span – between frame lines 2 and 3

Interior Span approximated as a fixed-fixed beam with warping restrained at each end.

C1 = 1/384, α = 1, η = 12 (purlins facing upslope)

$$\sigma = \frac{\dfrac{\left(\left(\frac{2.5\text{in}}{3}+1.05\text{in}\right)\cos(2.4°)\right)9.0\text{in}}{2\cdot\text{rad}}0.0036\dfrac{\text{rad}}{\text{lb}}+\dfrac{(1)(12)(300\text{in})^2\sin(2.4°)}{8\cdot\left(2500\,{}^{\text{lb}}\!/\!_{\text{in}}\right)(660\text{in})}}{\dfrac{(300\text{in})^4}{384\cdot\text{E}\cdot0.70\text{in}^4}+\dfrac{(9.0\text{in})^2}{4\cdot\text{rad}}0.0036\dfrac{\text{rad}}{\text{lb}}+\dfrac{(1)(12)(300\text{in})^2}{8\cdot\left(2500\,{}^{\text{lb}}\!/\!_{\text{in}}\right)(660\text{in})}} = 0.029$$

d. Interior Span – between frame lines 3 and 4

C1 = 1/384 α = -1, η = -12 (purlins facing downslope)

$\sigma = 0.023$

e. Right Exterior Span – half span adjacent to frame line 4 (approximated as a simple-fixed beam with warping fixed ends).

C1 = 1/185 α = 1, η = 12 (purlins facing up slope)

$\sigma = 0.018$

f. Right Exterior Span – half span adjacent to frame line 5 (approximated as a simple-fixed beam with warping free ends).

C1 = 1/185 α = 1 (purlins facing up slope)

$\sigma = 0.02$

2. Calculate the overturning forces generated by each purlin

a. Exterior Span – half-span adjacent to frame line 1 (torsionally approximated as each end warping free)

Local deformation reduction factor

$$R_{\text{local}} = \frac{k_{\text{mclip}}}{k_{\text{mclip}}+\dfrac{Et^3}{3d}} = \frac{3600\,{}^{\text{lb}\cdot\text{in}}\!/\!_{\text{rad}\cdot\text{ft}}}{3600\,{}^{\text{lb}\cdot\text{in}}\!/\!_{\text{rad}\cdot\text{ft}}+\dfrac{\text{E}\cdot(0.070\text{in})^3}{3\cdot9.0\text{in}}\left(12\,{}^{\text{in}}\!/\!_{\text{ft}}\right)} = 0.445 \qquad (5.1.50)$$

Typical purlins (purlins 2-11) α = -1 and w = 115 plf

$$P_i = \frac{wL}{2d} \cdot \left[\left(\delta b \cos\theta (1 - R_{local}) + \tfrac{2}{3} k_{mclip} \left(\sigma \frac{d}{2} - (\delta b + m) \cos\theta \right) \tau \right) \alpha - d \sin\theta \right] \quad (5.1.49)$$

$$P_{2-11} = \frac{(115\text{plf})(25\text{ft})}{(2)(9\text{in})} \cdot \left[\begin{array}{l} \left(\begin{array}{l} \frac{2.5\text{in}}{3} \cos(2.4°)(1 - 0.445) \\ + \frac{2}{3}\left(3600 \,{}^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}\right) \left((0.017) \frac{9.0\text{in}}{2} - \left(\frac{2.5\text{in}}{3} + 1.05\text{in} \right) \cos(2.4°) \right) \left(0.0040 \frac{1}{\text{lb}} \right) \end{array} \right) (-1) \\ -(9.0\text{in}) \sin(2.4°) \end{array} \right]$$

$$P_{2-11} = 178 \text{ lb}$$

Purlins 1 and 12. The load on purlins 1 and 12 is half that of purlins 2-11, the overturning force is half that of purlins 2-12, or

$$P_{1,12} = 89 \text{ lb}$$

b. Exterior Span – half-span adjacent to frame line 2 (torsionally approximated as each end warping fixed)

Local deformation reduction factor

$$R_{local} = 0.445$$

Purlins 2-11 α = -1 and w = 115 plf

$$P_{2-11} = \frac{(115\text{plf})(25\text{ft})}{(2)(9\text{in})} \cdot \left[\begin{array}{l} \left(\begin{array}{l} \frac{2.5\text{in}}{3} \cos(2.4°)(1 - 0.445) \\ + \frac{2}{3}\left(3600 \,{}^{\text{lb}\cdot\text{in}}/_{\text{rad}\cdot\text{ft}}\right) \left((0.015) \frac{9.0\text{in}}{2} - \left(\frac{2.5\text{in}}{3} + 1.05\text{in} \right) \cos(2.4°) \right) \left(0.0036 \frac{1}{\text{lb}} \right) \end{array} \right) (-1) \\ -(9.0\text{in}) \sin(2.4°) \end{array} \right]$$

$$P_{2-11} = 132 \text{ lb}$$

Purlins 1 and 12. The load on purlins 1 and 12 is half that of purlins 2-11, the overturning force is half that of purlins 2-12, or

$$P_{1,12} = 66 \text{ lb}$$

c. Interior Span between frame lines 2 and 3

Local deformation reduction factor

$$R_{local} = 0.572$$

Purlins 2-11 α = 1 and w = 115 plf

$$P_{2-11} = -373 \text{ lb}$$

Purlins 1 and 12. The load on purlins 1 and 12 is half that of purlins 2-11, the overturning force is half that of purlins 2-12, or

$$P_{1,12} = -187 \text{ lb}$$

d. Interior span between frame lines 3 and 4

Local deformation reduction factor

$$R_{local} = 0.572$$

Purlins 2-11 α = -1 and w = 115 plf

$$P_{2-11} = 160 \text{ lb}$$

Purlins 1 and 12. The load on purlins 1 and 12 is half that of purlins 2-11, the overturning force is half that of purlins 2-12, or

$$P_{1,12} = 80 \text{ lb}$$

e. Exterior span – half span closest to frame line 4

Local deformation reduction factor

$$R_{local} = 0.445$$

Purlins 2-11 α = 1 and w = 115 plf

$$P_{2-11} = -346 \text{ lb}$$

Purlins 1 and 12. The load on purlins 1 and 12 is half that of purlins 2-11, the overturning force is half that of purlins 2-12, or

$$P_{1,12} = -173 \text{ lb}$$

f. Exterior span – half span closest to frame line 5

Local deformation reduction factor

$$R_{local} = 0.445$$

Purlins 2-11 α = 1 and w = 115 plf

$$P_{2-11} = -392 \text{ lb}$$

Purlins 1 and 12. The load on purlins 1 and 12 is half that of purlins 2-11, the overturning force is half that of purlins 2-12, or

$$P_{1,12} = -196 \text{ lb}$$

3. Calculate the stiffness of the restraints.

The stiffness of each restraint device is

$$K_{device} = 40 \, {}^{\text{kip}}\!/_{\text{in}}$$

The net stiffness of the restraint must include the configuration stiffness which accounts for the flexibility of the web of the purlin between the top of the restraint and the top flange of the purlin.

a. Frame line 1

Configuration stiffness

$$K_{config} = \frac{Eb_{ar}t^3}{(d-h)^3} \cdot \frac{d}{h} = \frac{E(5\text{ in})(0.070\text{ in})^3}{(9\text{ in} - 7\text{ in})^3} \cdot \frac{9\text{ in}}{7\text{ in}} = 8.1\,\text{kip}/\text{in} \qquad (5.1.32)$$

Net restraint stiffness

$$K_{rest} = \frac{\left(\frac{h}{d}\right)^2 K_{device}K_{config}}{\frac{h}{d}K_{device} + K_{config}} = \frac{\left(\frac{7\text{ in}}{9\text{ in}}\right)^2 \left(40\,\text{kip}/\text{in}\right)\left(8.1\,\text{kip}/\text{in}\right)}{\frac{7\text{ in}}{9\text{ in}}\left(40\,\text{kip}/\text{in}\right) + 8.1\,\text{kip}/\text{in}} = 5.0\,\text{kip}/\text{in} \qquad (5.1.30)$$

b. Frame line 2

To account for the purlins at the lap, the combined purlins are given an equivalent thickness.

$$t_{lap} = \sqrt[3]{t_1^3 + t_2^3} = \sqrt[3]{(0.070\text{ in})^3 + (0.059\text{ in})^3} = 0.082\text{ in}$$

Configuration stiffness

$$K_{config} = \frac{E(5\text{ in})(0.082\text{ in})^3}{(9\text{ in} - 7\text{ in})^3} \cdot \frac{9\text{ in}}{7\text{ in}} = 13.0\,\text{kip}/\text{in}$$

Net restraint stiffness

$$K_{rest} = \frac{\left(\frac{7\text{ in}}{9\text{ in}}\right)^2 \left(40\,\text{kip}/\text{in}\right)\left(13.0\,\text{kip}/\text{in}\right)}{\frac{7\text{ in}}{9\text{ in}}\left(40\,\text{kip}/\text{in}\right) + 13.0\,\text{kip}/\text{in}} = 7.1\,\text{kip}/\text{in}$$

c. Frame line 3.

Equivalent thickness at lap

$$t_{lap} = \sqrt[3]{t_2^3 + t_2^3} = \sqrt[3]{(0.059\text{in})^3 + (0.059\text{in})^3} = 0.074\text{ in}$$

Configuration stiffness

$$K_{config} = \frac{E(5\text{ in})(0.074\text{ in})^3}{(9\text{ in} - 7\text{ in})^3} \cdot \frac{9\text{ in}}{7\text{ in}} = 9.7\,\text{kip}/\text{in}$$

Net restraint Stiffness

$$K_{rest} = \frac{\left(\frac{7\text{ in}}{9\text{ in}}\right)^2 \left(40\,\text{kip}/\text{in}\right)\left(9.7\,\text{kip}/\text{in}\right)}{\frac{7\text{ in}}{9\text{ in}}\left(40\,\text{kip}/\text{in}\right) + 9.7\,\text{kip}/\text{in}} = 5.8\,\text{kip}/\text{in}$$

4. Calculate the stiffness of the system

a. Calculate the stiffness of the sheathing.

$$K_{shtg} = \frac{k_{mclip} L}{d}\left(\frac{\frac{1}{4} E t^3}{0.38 k_{mclip} d + 0.71 \frac{E t^3}{4}}\right)\left(1 - \frac{2}{3} k_{mclip} \tau\right) \qquad (5.1.37)$$

i. Exterior Span. It is conservative to use the torsional coefficient, τ, for a warping free ends.

$$K_{shtg} = \frac{3600 \,{}^{lb\cdot in}/_{rad\cdot ft}\, 25\,ft}{9.0\,in}\left(\frac{\frac{1}{4} E (0.070\,in)^3}{0.38 \cdot 3600 \,{}^{lb\cdot in}/_{rad\cdot ft} \cdot 9.0\,in + 0.71 \cdot \frac{1}{4} E (0.070\,in)^3}\right) x$$

$$\left(1 - \frac{2}{3} 3600 \,{}^{lb\cdot in}/_{rad\cdot ft} \cdot 0.0040 \frac{rad}{lb}\right)$$

$$K_{shtg} = 1791 \,{}^{lb\cdot in}/_{in}$$

ii. Interior Span

$$K_{shtg} = \frac{3600 \,{}^{lb\cdot in}/_{rad\cdot ft}\, 25\,ft}{9.0\,in}\left(\frac{\frac{1}{4} E (0.059\,in)^3}{0.38 \cdot 3600 \,{}^{lb\cdot in}/_{rad\cdot ft} \cdot 9.0\,in + 0.71 \cdot \frac{1}{4} E (0.059\,in)^3}\right) x$$

$$\left(1 - \frac{2}{3} 3600 \,{}^{lb\cdot in}/_{rad\cdot ft} \cdot 0.0036 \frac{rad}{lb}\right)$$

$$K_{shtg} = 2019 \,{}^{lb\cdot in}/_{in}$$

b. Calculate stiffness of connection between rafter and Z-section (flange bolted connection)

$$K_{rafter} = 0.45 \frac{E t^3}{2d} \qquad (5.1.36)$$

Exterior Frame Line

$$K_{rafter} = 0.45 \frac{E (0.070\,in)^3}{2 \cdot 9\,in} = 253 \,{}^{lb\cdot in}/_{in}$$

At the interior frame lines, the equivalent thickness of the laps is used.

First Interior Frame Line

$$K_{rafter} = 0.45 \frac{E (0.082\,in)^3}{2 \cdot 9\,in} = 404 \,{}^{lb\cdot in}/_{in}$$

Second Interior Frame Line

$$K_{rafter} = 0.45 \frac{E (0.074\,in)^3}{2 \cdot 9\,in} = 303 \,{}^{lb\cdot in}/_{in}$$

5. Calculate the total stiffness of the system attributed to each restraint location (frame line)

$$K_{total} = \Sigma K_{rest} + \frac{\Sigma (K_{shtg} + K_{rafter})}{d} \qquad (5.1.48)$$

a. At frame line 1, the stiffness includes three restraints, the rafter stiffness of nine purlins and the sheathing stiffness of half of the exterior bay for twelve purlins.

$$K_{total} = 3\left(5.0 \,{}^{kip}/_{in}\right) + 9 \frac{\cdot 253 \,{}^{lb\cdot in}/_{in}}{9.0\,in} + 12 \cdot \frac{1791 \,{}^{lb\cdot in}/_{in}}{(2)(9\,in)} = 17.0 \,{}^{kip}/_{in}$$

b. At frame line 2, the stiffness includes three restraints, the rafter stiffness of nine purlins, and the sheathing stiffness of half of the exterior bay and half of the interior bay for twelve purlins

$$K_{total} = 3\left(7.1\,{}^{kip}\!/\!_{in}\right) + 9\frac{\cdot\,404\,{}^{lb\cdot in}\!/\!_{in}}{9\text{ in}} + 12\cdot\frac{1791\,{}^{lb\cdot in}\!/\!_{in}}{(2)(9\text{ in})} + 12\cdot\frac{2019\,{}^{lb\cdot in}\!/\!_{in}}{(2)(9\text{ in})} = 24.9\,{}^{kip}\!/\!_{in}$$

At frame line 3, the stiffness includes three restraints, the rafter stiffness of nine purlins, and two times the sheathing stiffness of half of the interior bay for twelve purlins

$$K_{total} = 3\left(5.8\,{}^{kip}\!/\!_{in}\right) + 9\frac{\cdot\,303\,{}^{lb\cdot in}\!/\!_{in}}{9\text{ in}} + 12\cdot\frac{2019\,{}^{lb\cdot in}\!/\!_{in}}{9\text{ in}} = 20.4\,{}^{kip}\!/\!_{in}$$

6. Distribute forces to each restraint

a. Frame line 1

The total load generated by the exterior half span adjacent to frame line 1 is

$$\sum_{N_p} P_i = \left(P_1 + 10P_{2-11} + P_{12}\right) = (89\text{ lb}) + 10(178\text{ lb}) + (89\text{ lb}) = 1958\text{ lb}$$

Distribution to each anti-roll anchorage device along frame line 1

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (1958\text{ lb})\frac{5.0\,{}^{kip}\!/\!_{in}}{17.0\,{}^{kip}\!/\!_{in}} = 576\text{ lb} \qquad (5.1.46)$$

Anchorage force at the height of restraint

$$P_h = P_L\frac{d}{h} = (576\text{ lb})\frac{9\text{ in}}{7\text{ in}} = 740\text{ lb} \qquad (5.1.47)$$

b. Frame line 2

The total load generated by each half span adjacent to frame line 2 is

$$\sum_{N_p} P_i = \left(P_1 + 10P_{2-11} + P_{12}\right)_{Left} + \left(P_1 + 10P_{2-11} + P_{12}\right)_{Right}$$

$$\sum_{N_p} P_i = (66\text{ lb}) + 10(132\text{ lb}) + (66\text{ lb}) + ((-187\text{ lb}) + 10(-373\text{ lb}) + (-187\text{ lb})) = -2652\text{ lb}$$

Distribution to each anti-roll anchorage device along frame line 2

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (-2652\text{ lb})\frac{7.1\,{}^{kip}\!/\!_{in}}{24.9\,{}^{kip}\!/\!_{in}} = -756\text{ lb} \qquad (5.1.46)$$

Anchorage force at the height of restraint

$$P_h = P_L\frac{d}{h} = (-756\text{ lb})\frac{9\text{ in}}{7\text{ in}} = -972\text{ lb}$$

c. Frame line 3

The total load generated by each half span adjacent to frame line 3 is

$$\sum_{N_p} P_i = ((-187\text{ lb}) + 10(-373\text{ lb}) + (-187\text{ lb})) + (80\text{ lb}) + 10(160\text{ lb}) + (80\text{ lb}) = -2344\text{ lb}$$

Distribution to each anti-roll anchorage device along frame line 3

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (-2344\ \text{lb})\frac{5.8\ \text{kip}/\text{in}}{20.4\ \text{kip}/\text{in}} = -666\ \text{lb} \quad (5.1.46)$$

Anchorage force at the height of restraint

$$P_h = P_L \frac{d}{h} = (-666\ \text{lb})\frac{9\ \text{in}}{7\ \text{in}} = 857\ \text{lb} \quad (5.1.47)$$

d. Frame line 4

The total load generated by each half span adjacent to frame line 4 is

$$\sum_{N_p} P_i = ((80\ \text{lb}) + 10(160\ \text{lb}) + (80\ \text{lb})) + ((-173\ \text{lb}) + 10(-346\ \text{lb}) + (-173\ \text{lb})) = -2046\ \text{lb}$$

Distribution to each anti-roll anchorage device along frame line 4

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (-2046\ \text{lb})\frac{7.1\ \text{kip}/\text{in}}{24.9\ \text{kip}/\text{in}} = -583\ \text{lb} \quad (5.1.46)$$

Anchorage force at the height of restraint

$$P_h = P_L \frac{d}{h} = (-583\ \text{lb})\frac{9\ \text{in}}{7\ \text{in}} = 750\ \text{lb} \quad (5.1.47)$$

e. Frame line 5

The total load generated by the exterior half span adjacent to frame line 5 is

$$\sum_{N_p} P_i = (-196\ \text{lb}) + 10(-392\ \text{lb}) + (-196\ \text{lb}) = -4312\ \text{lb}$$

Distribution to each anti-roll anchorage device along frame line 5

$$P_L = \sum_{N_p} P_i \cdot \frac{K_{rest}}{K_{total}} = (-4312\ \text{lb})\frac{5.0\ \text{kip}/\text{in}}{17.0\ \text{kip}/\text{in}} = -1268\ \text{lb} \quad (5.1.46)$$

Anchorage force at the height of restraint

$$P_h = P_L \frac{d}{h} = (-1268\ \text{lb})\frac{9\ \text{in}}{7\ \text{in}} = -1630\ \text{lb} \quad (5.1.47)$$

7. Check deformation of the system and compare to limits specified in Section D6.3.1

a. Lateral displacement of purlin top flange

Allowable deflection limit (ASD)

$$\Delta_{tf} = \frac{1}{\Omega}\frac{d}{20} = \frac{1}{2.00}\frac{9\ \text{in}}{20} = 0.23\ \text{in} \quad (D6.3.1\text{-}9a)$$

Frame Line 1

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{576\ \text{lb}}{5.0\ \text{kip}/\text{in}} = 0.12\ \text{in} \le 0.23\ \text{in} \ \ \text{OK} \quad (5.1.52)$$

Frame Line 2

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(-756\ \text{lb})}{7.1\ \text{kip}/\text{in}} = -0.11\ \text{in} \le 0.23\ \text{in} \ \ \text{OK} \quad (5.1.52)$$

Frame Line 3

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(-666\text{ lb})}{5.8\,{}^{kip}\!/_{in}} = -0.11\text{ in} \le 0.23\text{ in OK} \qquad (5.1.52)$$

Frame Line 4

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(-583\text{ lb})}{7.1\,{}^{kip}\!/_{in}} = -0.08\text{ in} \le 0.23\text{ in OK} \qquad (5.1.52)$$

Frame Line 5

$$\Delta_{rest} = \frac{P_L}{K_{rest}} = \frac{(-1268\text{ lb})}{5.0\,{}^{kip}\!/_{in}} = -0.25\text{ in} > 0.23\text{ in N.G.} \qquad (5.1.52)$$

The top flange deflection at frame line 5 exceeds the allowable deflection. Displacement of the top flange may be reduced by adding an additional anchorage device at frame line 5.

b. Mid-Span displacement of diaphragm relative to frame line

Allowable deflection limit

$$\Delta_{ms} \le \frac{L}{360} = \frac{25\text{ft}\,{}^{12in}\!/_{ft}}{360} = 0.83\text{ in}$$

$$\Delta_{diaph} = \Sigma(w(\alpha\sigma - \sin\theta))_i \frac{L^2}{8G'Bay} \qquad (5.1.53)$$

Exterior Span (between frame line 1 and 2)

Use average uniform diaphragm force between the two half spans.

$$\overline{\sigma} = {}^1\!/_2(0.017 + 0.015) = 0.016$$

$$\Delta_{diaph} = [(11)(115\text{ plf})((-1)0.016 - \sin(2.4°))]\frac{(25\text{ ft})^2}{8(2500\,{}^{lb}\!/_{in})(55\text{ ft})} = -0.04\text{ in} \le 0.83\text{ in OK}$$

Interior Span (between frame line 2 and 3)

$$\Delta_{diaph} = [(11)(115\text{ plf})((1)0.029 - \sin(2.4°))]\frac{(25\text{ ft})^2}{8(2500\,{}^{lb}\!/_{in})(55\text{ ft})} = -0.01\text{ in} \le 0.83\text{ in OK}$$

Interior Span (between frame line 3 and 4)

$$\Delta_{diaph} = [(11)(115\text{ plf})((-1)0.023 - \sin(2.4°))]\frac{(25\text{ ft})^2}{8(2500\,{}^{lb}\!/_{in})(55\text{ ft})} = -0.05\text{ in} \le 0.83\text{ in OK}$$

Exterior Span (between frame line 4 and 5)

Use average uniform diaphragm force between the two half spans.

$$\overline{\sigma} = {}^1\!/_2(0.018 + 0.020) = 0.019$$

$$\Delta_{diaph} = [(11)(115\text{ plf})((1)0.019 - \sin(2.4°))]\frac{(25\text{ ft})^2}{8(2500\,{}^{lb}\!/_{in})(55\text{ ft})} = -0.02\text{ in} \le 0.83\text{ in OK}$$

8. Calculate shear force in connection between the sheathing and purlin at anchor location.

At frame line 1

$$P_{sc} = P_L + \frac{wL}{2}(0.9\sigma\alpha + \sin\theta) - P_i \qquad (5.1.54)$$

$$P_{sc} = 576 + \frac{(115\,\text{plf})(25\,\text{ft})}{2}(0.9(0.017)(-1) + \sin(2.4°)) - (178\,\text{lb}) = 436\,\text{lb}$$

At frame line 2

$$P_{sc} = -756 + \frac{(115\,\text{plf})(25\,\text{ft})}{2}(0.9(0.015)(-1) + \sin(2.4°)) - (132\,\text{lb})$$
$$+ \frac{(115\,\text{plf})(25\,\text{ft})}{2}(0.9(0.029)(1) + \sin(2.4°)) - (-373\,\text{lb}) = -376\,\text{lb}$$

At frame line 3

$$P_{sc} = -666 + \frac{(115\,\text{plf})(25\,\text{ft})}{2}(0.9(0.029)(1) + \sin(2.4°)) - (-373\,\text{lb})$$
$$+ \frac{(115\,\text{plf})(25\,\text{ft})}{2}(0.9(0.023)(-1) + \sin(2.4°)) - (160\,\text{lb}) = -325\,\text{lb}$$

At frame line 4

$$P_{sc} = -583 + \frac{(115\,\text{plf})(25\,\text{ft})}{2}(0.9(0.023)(-1) + \sin(2.4°)) - (160\,\text{lb})$$
$$+ \frac{(115\,\text{plf})(25\,\text{ft})}{2}(0.9(0.018)(1) + \sin(2.4°)) - (-347\,\text{lb}) = -282\,\text{lb}$$

At frame line 5

$$P_{sc} = -1268 + \frac{(115\,\text{plf})(25\,\text{ft})}{2}(0.9(0.02)(1) + \sin(2.4°)) - (-392\,\text{lb}) = -790\,\text{lb}$$

5.2 Frame Element Stiffness Model

The computer model presented here was used to develop and calibrate the calculation procedure presented in the *Specification*. It can be used to analyze conditions that are beyond the scope of the available manual procedures or when a better understanding of the behavior is needed. The stiffness mode replicates the physical geometry of the roof system with simple frame elements and has been validated by comparing the resulting forces to test results.

5.2.1 Source of Test Data

The computer model was built in a way that closely mimics the physical properties of the actual system, so it was expected that the model behavior should mimic the behavior of the physical system. To verify the model results and to calibrate some of the model properties, the model results were compared to the available test results. Previously proposed calculation procedures were calibrated to tests of flat roof systems performed at the University of Oklahoma by Curtis and Murray (1983) and Seshappa and Murray (1985). Since the development of the previously proposed procedures, additional tests including sloped roofs have been performed at Virginia Tech by Lee and Murray (2001) and Seek and Murray (2004a). These later tests were used as the primary source of data when verifying the calculation procedure.

5.2.2 Selection of Computer Model

The work by Elhouar and Murray (1985), which formed the basis of the procedure in previous editions of the *Specification*, utilized a first order elastic stiffness model with a combination of frame and truss elements to model the roof system. This model was later modified slightly by Neubert and Murray (2000) and was further updated as part of recent research at Virginia Tech (Seek 2007 and Sears 2007). Seek also developed a separate computer model that utilizes finite elements and agrees very well with the results of tests, this model is summarized in section 5.3.

5.2.3 Development of Stiffness Model

The computer stiffness model utilizes linear frame elements to model the purlins and a combination of frame and truss elements to model the sheathing. The material properties of all elements in the model is taken as isotropic steel with a modulus of elasticity of 29,500 ksi. Shear deformations and the effects of warping under torsion are neglected. The analysis solution is strictly linear-elastic and neglects all material and geometric non-linearity.

5.2.3.1 Local and Global Axes

For defining the attributes of the model, a global coordinate system is defined, and a local system of axes is defined for each element type. The global Y-axis is aligned normal to the plane of the roof sheathing, the global Z-axis parallel to the purlin span, and the global X-axis up the slope of the roof, perpendicular to the purlin web. The local axes of each element are oriented so that the local x-axis lies along the length of the element. The y axis and z axis are as shown in Figure 5.2.1.

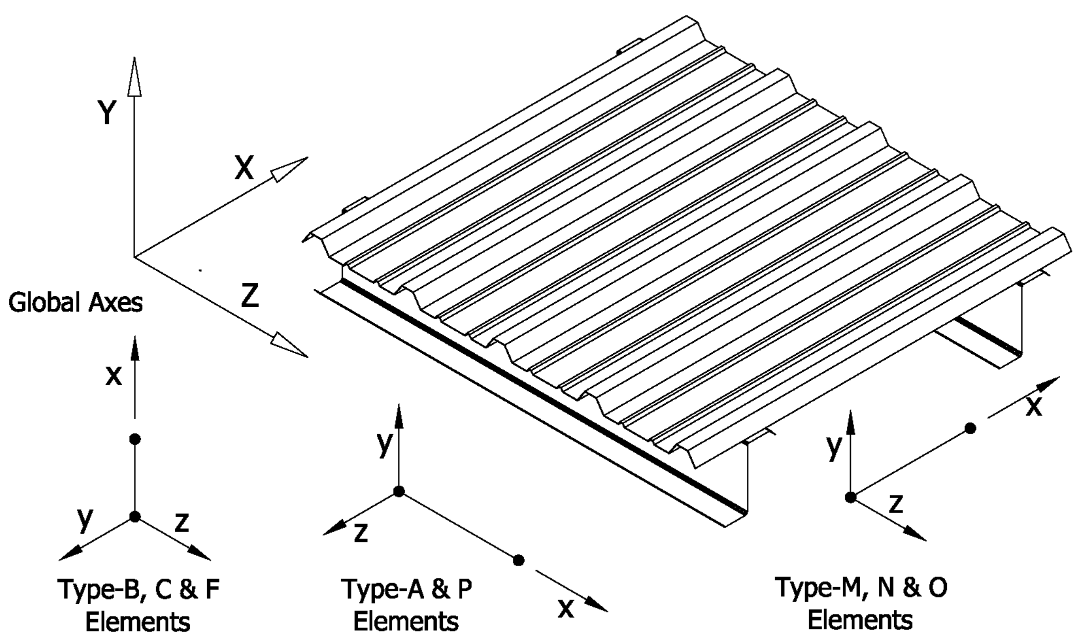


Figure 5.2.1 Local and Global Axes Orientations

5.2.3.2 Modeling of Purlins

In the computer model, the purlins are represented by a series of frame elements along the axis of the purlin in the plane of the web. The length of the purlin is divided into twelve equal segments to provide nodes for discretizing the roof diaphragm and for providing nodes at one-third points or one-quarter points for the attachment of anchorage devices. The geometry of a purlin is represented by four element types as shown in Figure 5.2.2.

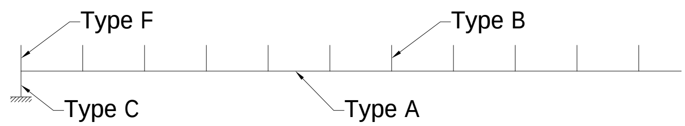


Figure 5.2.2 Purling Frame Elements

The longitudinal, Type A, elements are assigned the gross area and principal bending moments of inertia of the purlin section being modeled. Table 5.2.1 shows how the purlin properties given in the *Cold Formed Steel Design Manual* (2008a) correlate to the properties used in the stiffness model. Also the axes of the Type A elements are rotated by the principal axis angle, θ_p. The torsional constant, J, is assigned an arbitrarily high value of 10 in.^{4} because the torsional flexibility of the purlin is modeled by the Type B, Type C and Type F elements. Because the purlin cross-section may vary between bays, the element property input must typically include a definition for each bay (e.g. A1, A2, A3…).

Table 5.2.1 Type A Element Properties

Stiffness Model Property	Purlin Property Assigned to Type A Element
Area	Area
I_{yy}	I_{x2}
I_{zz}	I_{y2}
J	10 in^4
x-axis rotation	θ

For roof systems with multiple spans, the purlins from adjacent bays are typically lapped. To simplify the modeling and the user input, the lapped sections are assumed to extend into each bay for one-twelfth of the bay span. Within this region the area and the moments of inertia of the Type A elements are taken as the sum of the values for the two adjacent bays. The principal axis angle, θ_p, is taken as the average of the two values.

The Type B and Type F elements are included to provide the link between the plane of the roof sheathing and the neutral axis of the purlin and to model the deformations of the purlin web. A moment release for the moments about the y-y axis is added to the element end at the connection between the Type A elements and the vertical elements. This eliminates the Vierendeel truss action that would artificially stiffen the system. The properties of the Type B elements are assigned to be consistent with a flat plate with a width equal to one-twelfth of the span and a thickness equal to the purlin thickness (see Table 5.2.2). For simple span purlins and end bays, the Type F elements have properties equal to one-half of the Type B elements. At interior supports in multi-span systems, the purlins are assumed to extend into the adjacent bays. Therefore the properties of the Type F elements are found by the same principles as the Type B elements with the two purlins assumed to act as two non-composite sections. The resulting properties are summarized in Table 5.2.3.

Table 5.2.2 Type B Element Properties

Stiffness Model Property	Purlin Property Assigned to Type B Element
Area	L/12 x t
I_{yy}	0.0001
I_{zz}	$(L/12 \times t^3)/12$
J	I_{x2}
x-axis rotation	Zero

Table 5.2.3 Type F Element Properties

Stiffness Model Property	Purlin Property Assigned to Type F Element at Ends	Purlin Property Assigned to Type F Element at Laps
Area	L/24 x t	$(L_1+L_2)/12 \times (t_1 + t_2)$
I_{yy}	0.0001	0.0001
I_{zz}	$(L/24 \times t^3)/12$	$((L_1+L_2)/12 \times (t_1^3+t_2^3))/12$
J	I_{x2}	$(I_{x2})_1 + (I_{x2})_2$
x-axis rotation	Zero	Zero

At the purlin end, the Type C elements provided the connection to the support and modeled the behavior of the lower half of the purlin web in the vicinity of the support. A moment release is assigned to the end of the element at the connection to the Type A and Type F elements to eliminate bending in the plane of the purlin web (M_{ZZ}-moment). The properties of the Type C elements are formulated in a similar fashion as the Type F elements and have properties associated with the end one-twelfth of the span. The resulting properties for the Type C elements are summarized in Table 5.2.4.

Table 5.2.4 Type C Element Properties

Stiffness Model Property	Purlin Property Assigned to Type C Element at Ends	Purlin Property Assigned to Type C Element at Laps
Area	L/24 x t	$(L_1+L_2)/12 \times (t_1 + t_2)$
I_{yy}	0.0001	0.0001
I_{zz}	$(L/24 \times t^3)/12$	$((L_1+L_2)/12 \times (t_1^3+t_2^3))/12$
J	I_{x2}	$(I_{x2})_1 + (I_{x2})_2$
x-axis rotation	Zero	Zero

5.2.3.3 Modeling of Roof Sheathing

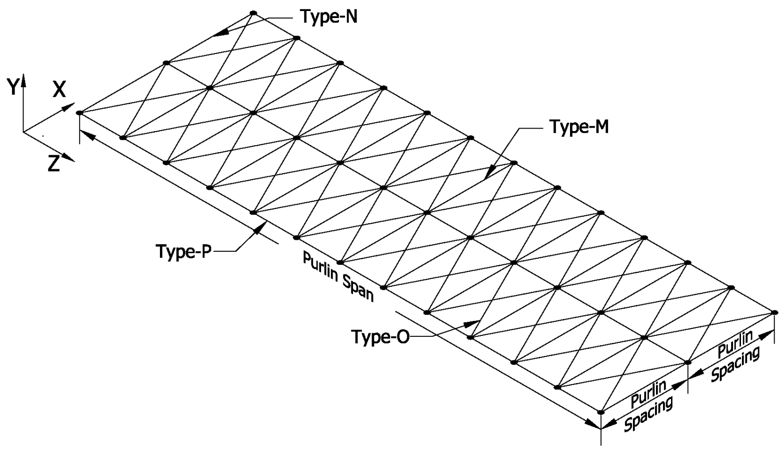


Figure 5.2.3 Panel Truss Elements

The model developed by Seek (2007), which accurately models the axial, shear and flexural stiffness of the roof diaphragm, is used to model the roof sheathing. This formulation, shown in Figure 5.2.3, uses four element types. The diagonal Type O members are modeled with pinned end truss elements which provide the shear stiffness of the diaphragm. The cross sectional area of the Type O elements is taken as

$$A_O = \frac{G'z(\alpha^2+1)^{1.5}}{2E\alpha^2} \tag{5.2.1}$$

where G' is the shear stiffness of the sheathing, z is the purlin spacing and α is the module aspect ratio, z/(L/12). The "posts" of the truss are modeled with Type M and Type N elements. The cross sectional area of these elements is calculated to yield the appropriate axial stiffness using the following.

$$A_N = \frac{\sqrt{b^2+4ac}-b}{2a} \tag{5.2.2}$$

$$A_M = 2A_N \tag{5.2.3}$$

where

$$a = 2zE\alpha(\alpha^2+1)^{1.5} \tag{5.2.4}$$

$$b = 2A_O zE(\alpha^4+1) - K_{axial} z^2 \alpha(\alpha^2+1)^{1.5} \tag{5.2.5}$$

$$c = K_{axial} A_O z^2 \tag{5.2.6}$$

$$K_{axial} = \frac{A_p E}{z} \tag{5.2.7}$$

and A_p is the cross sectional area of the roof sheathing per unit width. To model the bending stiffness of the sheathing, the Type M and Type N elements are assigned a moment of inertia, I_{zz}, equal to the moment of inertia of the sheathing within the width tributary to the element. Moment releases are added at both ends of the Type M and Type N to eliminate bending about the y-y axis and torsion. The longitudinal Type P "chords" of the truss are modeled as axial only truss elements with a cross sectional area of,

$$A_p = \alpha A_N \tag{5.2.8}$$

The above formulation works well for systems with through-fastened sheathing. The test results (Lee and Murray 2001; Seek and Murray 2004) for standing seam systems show a significant reduction in anchorage force when compared to through-fastened systems. This reduction is not seen in the computer model using the above diaphragm model. The transfer of shear forces in a standing seam system is fundamentally different from that of a through fastened system due to slip between the individual panels. To represent this in the model, a hybrid treatment of the panel truss is used. For the effects of the load that acts in the plane of the purlin web, the sheathing is modeled as described above. Then a separate analysis is executed with the Type O elements removed and the torsional and down slope loads applied. The results of these two analyses are then superimposed.

5.2.3.4 Modeling of Loads

The loads applied to the model are calculated based an input uniform total roof load distributed with a tributary area approach. In the physical roof system, the gravity loads are applied to the roof sheathing. In the computer stiffness model, the loads are represented by a series of distributed line loads and torsional moments. Typically, the roof system will have some slope; however, the geometry in the computer model is constructed with the plane of the roof parallel to the X-Z plane. To account for the slope, the applied gravity load is separated into vector components acting normal to and in the plane of the roof sheathing, resulting in

$$w_{normal} = w \cos\theta \tag{5.2.9}$$

$$w_{ds} = w \sin\theta \tag{5.2.10}$$

The load in the plane of the sheathing, w_{ds}, is applied as a uniform line load in the acting in the negative X direction along the Type P elements.

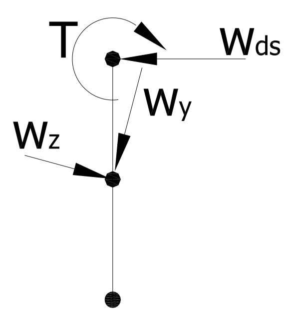


FIGURE 5.2.4 Summary of Loads

The component of the load that acts normal to the sheathing acts in a plane eccentric to the shear center of the purlin and causes torsion in the purlin. The gravity loads are transferred from the sheathing to the purlin by bearing on the purlin top flange. The true load distribution across the width of the flange is not known. Previous models have assumed a triangular distribution, and therefore a resultant force a distance of b/3 from the purlin web, where b is the width of the purlin flange. The latest research in the application of this model (Sears 2007) found that an eccentricity of b/4 agreed better with tests. For sections, such as channels, where the shear center is not located in the plane of the purlin web, the eccentricity is m+b/4, where m is the distance between the shear center and the plane of the web. To model this torsion in the computer stiffness model a uniform torsion is applied along the length of the Type P elements. The magnitude of this moment is taken as,

$$T = w_{normal}\frac{b}{4} \qquad \text{For Z-Section purlins} \tag{5.2.11a}$$

$$= w_{normal}\left(m+\frac{b}{4}\right) \qquad \text{For C-Section purlins} \tag{5.2.11b}$$

For Z-sections the principal axes are inclined with respect to the geometric axes. Therefore, the applied load must be translated into vector components that act in the planes of the principal axes.

$$w_y = w_{normal}\cos\theta_p \tag{5.2.12}$$

$$w_z = w_{normal}\sin\theta_p \tag{5.2.13}$$

5.2.3.5 Modeling of the Purlin-to-Sheathing Connection

With the direct consideration of the axial and flexural stiffness of the sheathing included in the model, it is also important to represent the connection between the sheathing and the purlin. Therefore, linear springs in the local y-axis at the top of the Type B and Type F elements are added and assigned a stiffness of 5000 lb/in. for standing seam systems and 100,000 lb/in. for through-fastened systems. Rotational springs are placed at the ends of the Type M and Type N elements and have a stiffness of 1500 in.-lb/radian per foot of width for both roofing systems.

5.2.3.6 Modeling of Anchorage Devices

Spring supports are used at the top of the Type B or Type F elements at user selected locations in the model. By using spring supports, the finite stiffness of various anchorage devices can be accurately represented. Due to the indeterminate nature of the roof system, reduction in device stiffness can greatly affect the predicted anchorage forces. Modeling the points of anchorage with discrete nodal supports accurately represents typical construction details for anchorage devices at the frame lines, and for certain cases when anchorage devices are located along the purlin span. If lines of anchorage are constructed, such as ¼ point anchors connected to a beam at the eave, so that displacement at a line of anchorage is coupled with the displacement of other lines of anchorage, this method of modeling may not correctly represent the constructed system.

5.3 Shell Finite Element Models to Predict Anchorage Forces

5.3.1 Components of Finite Element Model

A finite element model was developed for the prediction of anchorage forces. The model is the most complete representation of a purlin supported roof system for the prediction of anchorage forces. The model has been validated by comparisons to the test results of Lee and Murray (2001) and Seek and Murray (2004) and was used in the development of the component stiffness method.

The model is composed of four basic elements. Shell elements are used to represent the purlin and the sheathing. Frame elements are used to represent the anchorage devices and strap bracing. Connection between the sheathing and the purlin is made using a two node link element. A representation of the elements comprising the model and the global axes are shown in Figure 5.3.1.

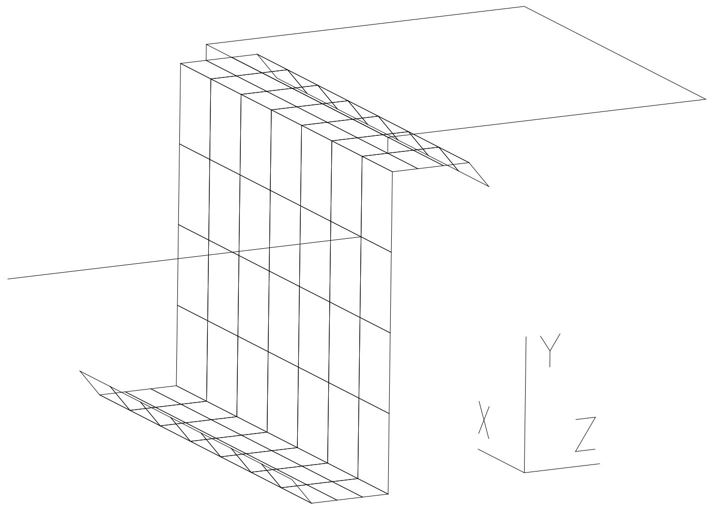


Figure 5.3.1 Representative elements of Finite Element Model

5.3.1.1 Finite Element Representation of Purlin

To represent the purlin as finite elements, the web is discretized into four elements, the flanges into three elements, and the lips into single elements. Discretization along the length of the purlin should be chosen to maintain a maximum aspect ratio of 4:1.

The elements representing the purlins are assigned a membrane thickness and a bending thickness equal to the nominal thickness of the purlin. In the case of a multi-span system in which the purlins are lapped, the modeled purlin is given a membrane thickness equivalent to the sum of the thicknesses of the two purlins at the lap. The bending thickness of the element at the lap is equivalent to the combined moment of inertia of the two purlins comprising the lap. That is,

$$t_{lap,bending} = \sqrt[3]{t_1^3 + t_2^3} \qquad (5.3.1)$$

where t_1 and t_2 are the thicknesses of each purlin at the lap.

5.3.1.2 Finite Element Representation of Sheathing

The sheathing is represented in the finite element model by a shell element discretized into 12 in. segments along the length of the purlin and divided into five equal segments between the purlins. The elements representing the sheathing are given a membrane thickness equal to the material thickness of the sheathing. To account for the bending stiffness provided by the sheathing ribs, the bending thickness of the element equivalent to the gross moment of inertia of the deck is calculated by:

$$t_{sheathing,bending} = \sqrt[3]{12 \cdot I_{panel}} \qquad (5.3.2)$$

To allow for variations in sheathing diaphragm stiffness, the sheathing elements are designated as orthotropic material and the shear modulus is adjusted. For the two material directions in the plane of the sheathing, the panel shear modulus, G, for a desired diaphragm shear stiffness, G′, is

$$G = \frac{G'}{t} \tag{5.3.3}$$

For the material direction perpendicular to the plane of the sheathing, the shear modulus of steel (11,300 ksi) is used.

5.3.1.3 Link Connection Between Sheathing and Purlin

The connection between the sheathing and purlin is made by a 2-node link element at 1 ft. intervals along the length of the purlin and at an eccentricity of 1/3 of the flange width. The link element allows for the translational and rotational stiffness between two joints to be defined about three axes. The link element also provides an efficient means to track forces transferred between the sheathing and purlin. Because there is some rotational flexibility in the connection between the purlin and sheathing about the axis parallel to the length of the purlin, the link rotational stiffness about this axis will typically range between 500 lb-in./radian and 10000 lb-in./radian. To prevent the purlin and sheathing from behaving like a composite section, the translational stiffness of the link element about the axis parallel to the length of the purlin is released.

In standing seam systems, the connection between the purlin and sheathing is made by a clip screwed to the purlin and sandwiched in the seam between two adjacent panels. There is some translational slip in this connection parallel to the seam, whether it is intentional in a sliding clip or inadvertent due to a loose seam. Although the stiffness of this connection is nonlinear, it can be approximated by assigning the link element a linear stiffness in the axis perpendicular to the web of the purlin. The stiffness of this connection is assumed to range between 250 lb/in. and 5000 lb/in for most standing seam systems. The flexibility of this connection has the effect of reducing the diaphragm stiffness of the system.

5.3.1.4 External Restraints

An external restraint representing the connection to the rafter is applied at a single node at the base of the purlin at the intersection between the web and bottom flange of the purlin. Translational restraint is applied in the global Y and Z directions and rotational restraint is applied about the global X axis.

External anchors are modeled as axial loaded frame elements between the web of the purlin and an external support. The location of the anchor along the height of the web should reflect the actual anchor modeled. The stiffness of the anchor is the combined stiffness of the anchorage device and the stiffness of the web transferring this force to the anchor. The stiffness of the anchor can be modeled in one of two ways. The first is to model the anchor with the combined device and configuration stiffness. Using the stiffness derived from the test in Section 5.3.1, restraint is applied at the top flange of the purlin and assigned the linear spring stiffness of the test specimen. If test data is unavailable, the configuration and device stiffness are treated separately. The restraint is applied in the model at the same height along the web as the actual specimen. For example, the top row of bolts in an anti roll anchorage device is considered the anchor height. The restraint is assigned a linear spring stiffness equivalent to that of the device. The flexibility of the web between the top of the restraint and the top flange of the purlin when modeled in this way will typically underestimate the configuration stiffness.

5.3.2 Model Loading

Load is applied directly to the sheathing in the model as a uniformly applied area loading. To account for roof slope, a uniform load is applied both normal and parallel to the sheathing (downslope). The vertical gravity load, W, is then divided into normal, W_{normal}, and downslope, $W_{downslope}$, components according to roof slope, or:

$$W_{normal} = W\cos\theta \tag{5.3.4}$$

$$W_{downslope} = W\sin\theta \tag{5.3.5}$$

where θ is the angle of the roof with respect to the horizontal.

5.3.3 Finite Element Model Example

The following example shows the development of a shell finite element model to predict anchorage forces based on the roof system of Example 3.2.

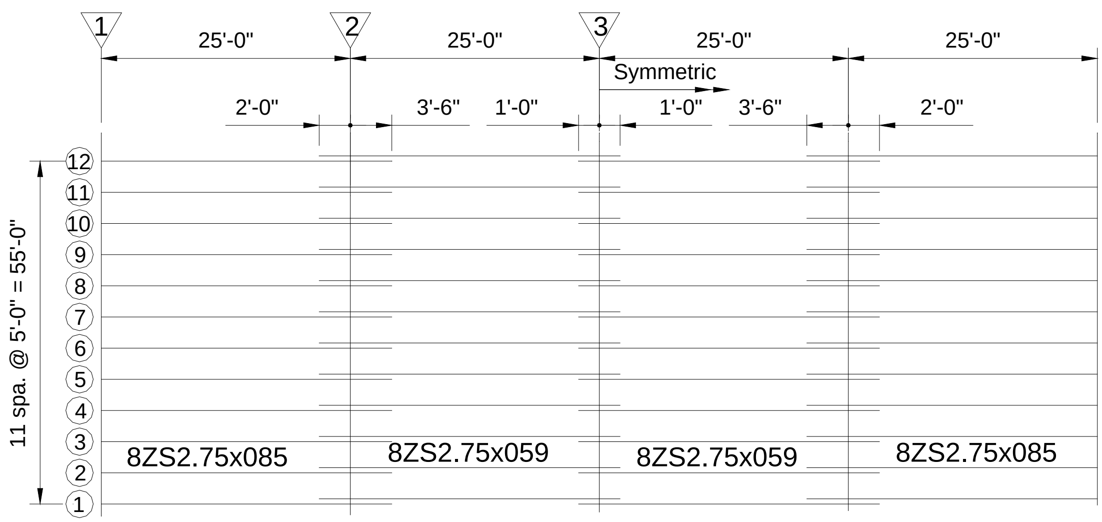


Figure 5.3.2 Roof Layout For Finite Element Model

Given:

1. Twelve purlin lines spaced at 5 feet. The top flange of the first purlin closest to the eave faces down slope. The top flanges of the remaining purlins face upslope. Roof slope is a ½ on 12 pitch and the gravity loads are 3 psf dead and 20 psf live.
2. The system of purlins is a four span continuous system symmetric about the center frame line. Each span is 25 ft. In the exterior bays, the purlins are 8ZS2.75x085. In the interior bays the purlins are 8ZS2.75x059. Laps are as shown in Figure 5.3.2.
3. Roof covering is attached with standing seam panel clips along entire length of purlins. The panel is a 26 gage (0.0179 in.) rib type panel profile with fixed clips and a mechanical seam. The gross moment of inertia of the panel is 0.254 in^4/ft. The sheathing has a diaphragm stiffness G′ = 1000 lb/in. and the rotational stiffness of the standing seam panel clips, k_{mclip} = 2500 lb-in./(rad.-ft).

4. No discrete bracing lines; anti-roll clips provided at each support at every fourth purlin line. Each anti-roll anchorage device is attached to the web of the Z-section with two rows of two ½ in. diameter A307 bolts. The bottom row of bolts is 3 in. from the bottom flange and the top row is 6 in. from the bottom flange. The stiffness of each anti-roll anchorage device, k_{device} = 40 k/in. The width of the anti-roll anchorage device is b_{pl} = 5.0 in.
5. Purlin flanges are bolted to the support member with two ½ in. diameter A307 bolts through the bottom flange.

Required:

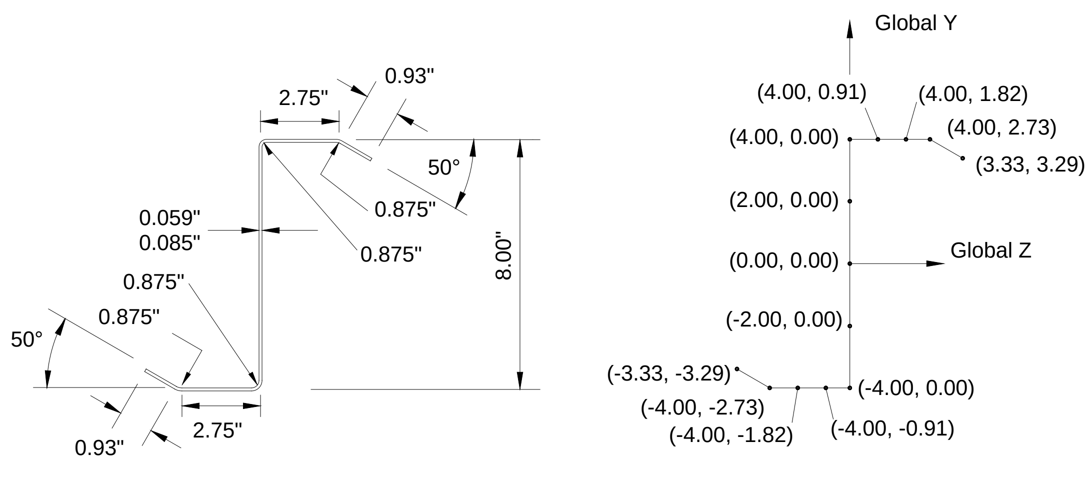
1. Anchorage forces along each frame line due to gravity loads at the top of the anchorage device.
2. Lateral deflection of the top flange of the Z-section along each frame line and at the purlin mid-span.
3. Shear force in the standing seam panel clips at each anchorage device.

Solutions:

Assumptions for Analysis

a. The model is first order linear elastic.
b. Purlin and sheathing are modeled as shell elements with thin plate behavior. Purlins are modeled with a zero bend radius.
c. Connection between the sheathing and purlin is made through a single spring connection
d. Connections to rafters are made at a single node at the junction of the purlin web and bottom flange.

1. Model Properties:



a) Cross Section Dimensions **b) FE Model Discretization**

Figure 5.3.3 Purlin Cross Section

a. Purlins

The purlin cross section is discretized as shown in Figure 5.3.3 b). Along the span of the purlin, the purlin is discretized in 2 in. increments. The purlin is modeled as a shell element with thin plate behavior. Along the interior of the span, the nominal thickness of the purlin is assigned to the membrane and bending thickness of the elements. At the lap, a single element is used to approximate the two purlins in the lapped region. In the lapped region, the membrane thickness is the sum of the thicknesses of the two purlins, and the bending thickness is equivalent thickness such that the single plate thickness has the same moment of inertia of the sum of the moments of inertia of the individual plate thicknesses.

Exterior span

$t_{membrane}$ = 0.085 in.

$t_{bending}$ = 0.085 in.

First interior lap

$t_{membrane}$ = 0.085 in. + 0.059 in. = 0.144 in.

$t_{bending} = \sqrt[3]{(0.085\text{in})^3 + (0.059\text{in})^3}$ = 0.094 in.

Interior span

$t_{membrane}$ = 0.059 in.

$t_{bending}$ = 0.059 in.

Second interior lap

$t_{membrane}$ = 0.059 in. + 0.059 in. = 0.118 in.

$t_{bending} = \sqrt[3]{(0.059\text{in})^3 + (0.059\text{in})^3}$ = 0.074 in.

b. Diaphragm Elements

The diaphragm is discretized into 12 in. by 12 in. elements. Each element is modeled as a shell element with thin plate behavior. The membrane thickness of the panel elements is the nominal thickness of the panel (26 ga. panel, t = 0.0179 in.). To account for the diaphragm stiffness, the shear modulus of the material is adjusted.

$t_{membrane}$ = 0.0179 in.

$$G = \frac{G'}{t} = \frac{1000\,{}^{\text{lb}}\!/\!{}_{\text{in}}}{0.179\text{in}} = 55{,}866 \text{ psi.}$$

The panel has a gross moment of inertia of I_{panel} = 0.254 in^4/ft. The bending thickness of the sheathing is adjusted to give an equivalent moment of inertia.

$$t_{bending} = \sqrt[3]{12 I_{panel}} = \sqrt[3]{12\left(0.254\,{}^{\text{in}^4}\!/\!{}_{\text{ft}}\right)\left(\frac{1\text{ft}}{12\text{in}}\right)} = 0.634 \text{ in.}$$

c. Link Elements

The diaphragm is located 0.1 in. above the top flange of the purlin. Link elements provide the connection between the panel elements and purlin elements at 12 in. increments along the length of the purlin at the panel element joints. The link elements are attached to the purlin at the node at 1/3 the distance from the purlin web to model the eccentricity of the gravity loads acting on the purlin top flange. Link elements are convenient because they allow for the stiffness of the connection between the purlin and sheathing to be specified directly and quickly adjusted. The link element local axes are shown in Figure 5.3.4. The rotational stiffness of the connection between the sheathing and purlin is assigned to the no. 3 axis. The connection is considered translationally rigid in the 1 and 2 directions and rotationally rigid about the 2 axis. To prevent the purlin from acting like a composite member, translational stiffness in the 3 direction is reduced to a negligible value. Rotational stiffness about the 1 axis is also reduced to a negligible value. The link stiffness values are tabulated below. Because the links are located at 12 in intervals along the span of the purlin, tabulated stiffness values are considered per foot along the length of the purlin

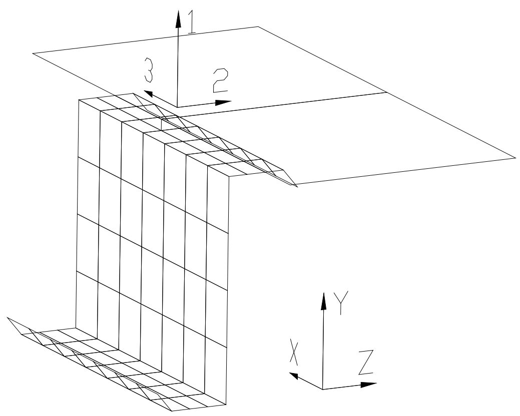


Figure 5.3.4 Link Element Local Axes

Summary of link element properties.

Translation	Rotation
U1 = 1×10^7 lb/in	R1 = 1.0 lb-in/rad
U2 = 1×10^7 lb/in	R2 = 1×10^7 lb-in/rad
U3 = 0.1 lb/in	R3 = 2500 lb-in/rad

d. Connection to the rafter

The connection to the rafter is modeled as a single node joint restraint at the junction of the bottom flange and web at the centerline of the frame line. The joint is restrained from translation in the global Y and Z axes (refer to Figure 5.3.4) and restrained from rotation about the global X axis. The remaining degrees of freedom are released.

e. Anchorage Device

The anchorage device has a stiffness of 40 kip/in. This stiffness is considered at the top row of bolts of the anchorage device (6 in. from the bottom flange). Therefore, a spring restraint with a stiffness of 40 kip/in in the global Z direction is applied to a node at 6 in. from the base of the purlin. To account for the width of the anchorage device and the stiffening effect it has on the purlin web, frame elements were added to the web of the purlin as shown in Figure 5.3.5. Frame 1 was modeled as a bar 1/2 in. x 4 in and frame 2 was modeled as a bar 1/4 x 2 in. The thickness of the elements was oriented in the same direction as the thickness of the purlin. To prevent moment transfer at the base of the purlin, the rotational stiffness of the frame elements was released at the connection at the base of the purlin web.

Note, if the stiffness of the anchorage device includes the deformation of the purlin web, as can be determined by the test discussed in Section 5.1.6, a spring restraint with the stiffness determined from the test should be applied at the top flange of the purlin.

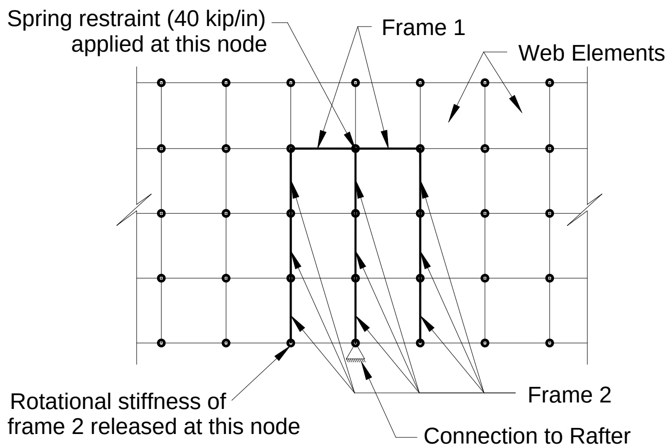


Figure 5.3.5 Frame elements to represent anti-roll anchorage

2. Loading

Gravity loads are applied as uniform area loads on the sheathing. The total gravity load, dead plus live, is 23 psf. To account for the slope of the roof, the gravity load is broken into components normal to the sheathing and in the plane of the sheathing.

$U_{normal} = U \cos\theta = (23 \text{ psf}) \cos(2.4°) = 22.98 \text{ psf}$

$U_{downslope} = U \sin\theta = (23 \text{ psf}) \sin(2.4°) = 0.958 \text{ psf}$

3. Model Solution

a. Anchorage Forces

Frame Line 1

Purlin 4 $P_h = 432$ lb

Purlin 8 $P_h = 410$ lb

Purlin 12 P_h = 406 lb

Frame Line 2
Purlin 4 P_h = 415 lb
Purlin 8 P_h = 391 lb
Purlin 12 P_h = 431 lb

Frame Line 3
Purlin 4 P_h = 380 lb
Purlin 8 P_h = 367 lb
Purlin 12 P_h = 435 lb

b. Displacement of Z-section
Positive values indicate upslope translation.
Lateral displacements extracted from Purlin 4 from model.

Top flange at Frame Line 1 $\Delta = 0.111$ in.

Specification limit $$\Delta_{tf} = \frac{1}{\Omega}\frac{d}{20} = \frac{1}{2.00}\frac{8in}{20} = 0.20 \text{ in} > 0.14 \text{ in OK}$$

Top flange at centerline of exterior span $\Delta = 0.382$ in.
Bottom flange at centerline of exterior span $\Delta = 0.629$ in.

Specification limit $$\Delta_{ms} \leq \frac{L}{360} = \frac{25ft\ {}^{12in}\!/\!_{ft}}{360} = 0.83 \text{ in} > 0.382 \text{ in. OK}$$

Approximate rotation at centerline of exterior span $\Phi \approx (0.382in\text{-}0.629in)/8in = -0.031$

Top flange at Frame Line 2 $\Delta = 0.14$ in.

Specification limit $$\Delta_{tf} = \frac{1}{\Omega}\frac{d}{20} = \frac{1}{2.00}\frac{8in}{20} = 0.20 \text{ in} > 0.14 \text{ in OK}$$

Top flange at centerline of interior span $\Delta = 0.310$ in.
Bottom flange at centerline of interior span $\Delta = 0.649$ in.

Specification limit $$\Delta_{ms} \leq \frac{L}{360} = \frac{25ft\ {}^{12in}\!/\!_{ft}}{360} = 0.83 \text{ in} > 0.310 \text{ in. OK}$$

Approximate rotation at centerline of exterior span $\Phi \approx (0.310in\text{-}0.649in)/8in = -0.042$

Top flange at Frame Line 3 $\Delta = 0.110$ in.

Specification limit $$\Delta_{tf} = \frac{1}{\Omega}\frac{d}{20} = \frac{1}{2.00}\frac{8in}{20} = 0.20 \text{ in} > 0.110 \text{ in OK}$$

Vertical displacements extracted from Purlin 4 from model.

Top flange at centerline of exterior span Δ = -1.189 in.
Top flange at centerline of interior span Δ = -0.515 in.

c. Shear Forces in the purlin to sheathing connection

The shear forces in the link connection are plotted in Figure 5.3.6 from the exterior frame line to the centerline of the system (frame line 3) for both purlin 3 and purlin 4. The spikes in fastener forces occur at the frame lines. For comparison to the calculation in the component stiffness method of the shear force in the purlin to sheathing connection, P_{sc}, the total fastener force is the sum of the forces at 12 in to either side of the frame line. This total fastener force can be considered to be distributed evenly between each of the fasteners within 12 in. of the frame line. The forces along the length of the span represent the uniform restraint force in the sheathing, w_{rest}, calculated in the component stiffness method.

Fastener forces for purlin 3 (typical purlin).

Frame Line 1 P_{sc} = 148 lb + (-19.0 lb) = 129 lb
Frame Line 2 P_{sc} = 68.6 lb + 93.0 lb + 69.9 lb = 232 lb
Frame Line 2 P_{sc} = 55.5 lb + 58.2 lb + 55.5 lb = 169 lb

Along the interior of the span, the uniform restraint force in the sheathing is the average of forces along the length.

Exterior span w_{rest} = 29.6 lb/ft
Interior span w_{rest} = 25.2 lb/ft

Fastener forces for purlin 4 (directly anchored).

Frame Line 1 P_{sc} = 453 lb + 44.0 lb = 497 lb
Frame Line 2 P_{sc} = 130 lb + 305 lb + 131 lb = 566 lb
Frame Line 2 P_{sc} = 107 lb + 270 lb + 107 lb = 484 lb

Along the interior of the span, the uniform restraint force in the sheathing is the average of forces along the length.

Exterior span w_{rest} = 29.1 lb/ft
Interior span w_{rest} = 24.4 lb/ft

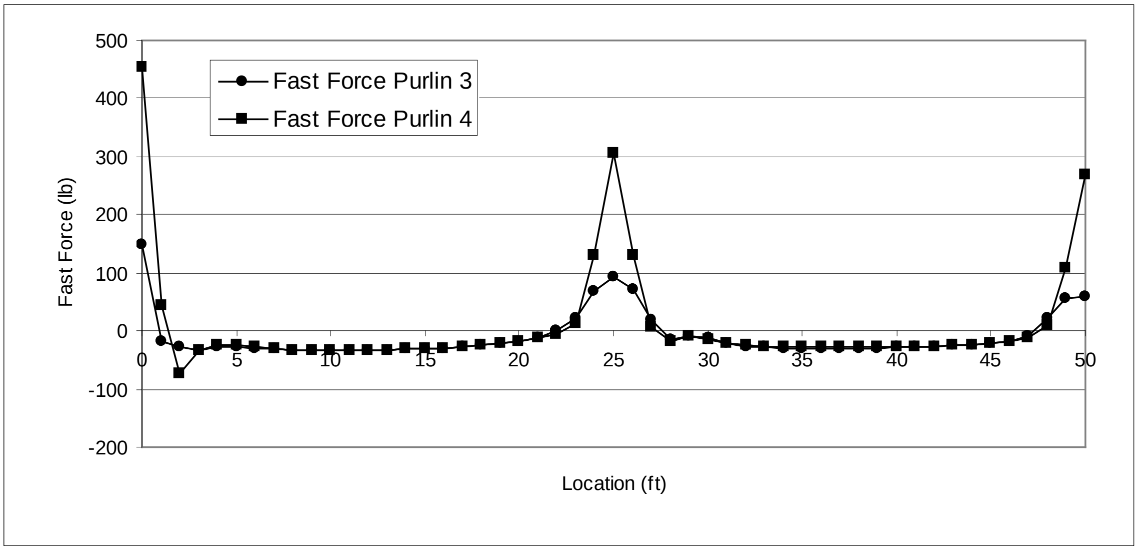


Figure 5.3.6 Shear Force Transfer Between Purlin and Sheathing

d. Sheathing Moment

The total moment in the connection between the sheathing and the purlin is plotted from frame line 1 to the centerline of the system (frame line 3) for purlins 3 and 4 in Figure 5.3.7. This total moment includes the parabolic moment in the sheathing due to torsion effects, $M_{torsion}$, the moment due to local deformations, M_{local}, and the moments due to the deformation of the anchorage, M_{shtg}.

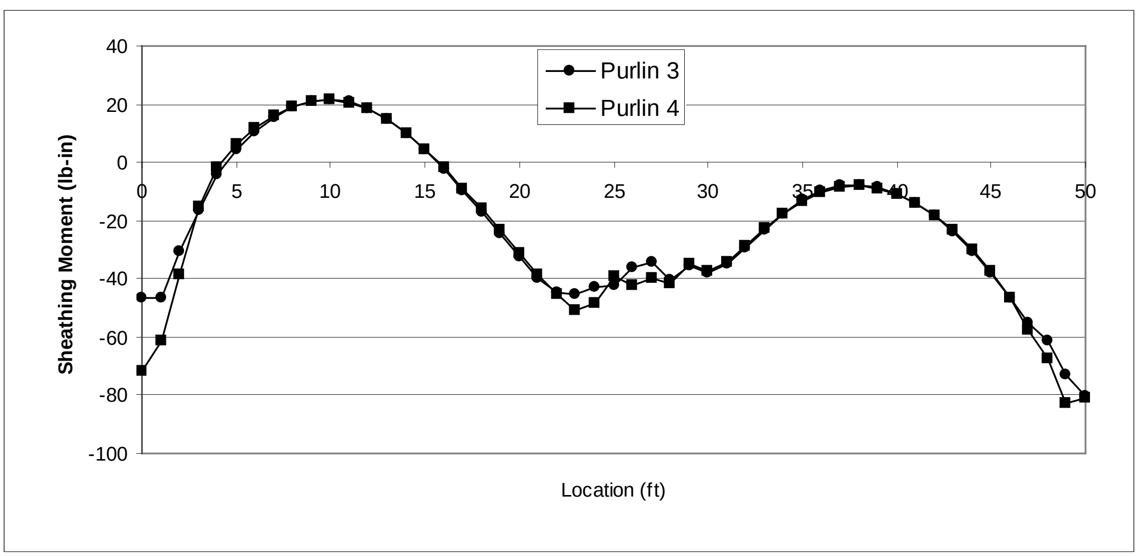


Figure 5.3.7 Moments in Connection Between Purlin and Sheathing

REFERENCES

AISC (2005), *Specification for Structural Steel Buildings*, American Institute of Steel Construction, Chicago, Illinois.

AISI (1997), *A Guide for Designing with Standing Seam Roof Panels, Design Guide 97-1*, American Iron and Steel Institute, Washington, D.C.

AISI (2007), *North American Specification for the Design of Cold-Formed Steel Structural Members, 2007 Edition*, American Iron and Steel Institute, Washington, D.C.

AISI (2008), *Base Test Method for Purlins Supporting a Standing Seam Roof Systems and the Commentary S908-08*, American Iron and Steel Institute, Washington, D.C.

AISI (2008a), *Cold-Formed Steel Design Manual*, American Iron and Steel Institute, Washington, D.C.

ASCE (2005), *Minimum Design Loads for Buildings and other Structures,* ASCE Standard 7-98, American Society of Civil Engineers, Ralston, Virginia.

Bryant, M. R. and T. M. Murray (2000), "Investigation of Inflection Points as Brace Points in Multi-Span Purlin Roof Systems", Research Report No. CE/VPI-ST-00/11, Department of Civil and Environmental Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

Curtis, L. and T. M. Murray (1983), "Simple Span Z-Purlin Tests to Determine Brace Force Accumulation", Fears Structural Engineering Laboratory Research Report FSEL/MBMA 83-02, University of Oklahoma, Norman, Oklahoma.

Center for Cold-Formed Steel Structures (1992), Bulletin, Vol. 1, No. 2, August 1992.

Elhouar, S. and T. M. Murray (1985), "Stability Requirements of Z-Purlin Supported Conventional Metal Building Roof Systems," *Annual Technical Session Proceedings*, Structural Stability Research Council, Bethlehem, PA.

Ghazanfari, A., and T. M. Murray. (1983). "Prediction of the Lateral Restraint Forces of Single Span Z-purlins with Experimental Verification." Fears Structural Engineering Laboratory Research Report No. FSEL/MBMA 83-04, University of Oklahoma, Norman, Oklahoma.

Heinz, D. A. (1994). "Application of Generalized Beam Theory to the Design of Thin-Walled Purlins", *Thin Walled Structures*, 19, 311-335.

Lee, S. and T. M. Murray (2001), "Experimental Determination of Required Lateral Restraint Forces for Z-Purlin Supported, Sloped Metal Roof Systems." Research Report CE/VPI-ST 01/09, Department of Civil and Environmental Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

Murray, T. M. and S. Elhouar (1994), North American Approach to the Design of Continuous Z- and C-Purlins for Gravity Loading with Experimental Verification, *Engineering Structures*, Elserier,Vol. 16, No. 5, pp. 337-341.

Neubert, M. C. and T. M. Murray (2000), "Estimation of Restraint Forces for Z-Purlin Roofs under Gravity Loads," *15th International Specialty Conference on Cold-Formed Steel Structures*, St. Louis, Missouri.

Seaburg, Paul A. and Carter, Charles J. (1997) *Steel Design Guide #9, Torsional Analysis of Structural Steel Members.* American Institute of Steel Construction, Inc. Chicago, IL.

Sears, J. M. (2007), "Development of Prediction Methods for the Lateral Anchorage Requirements in Metal Building Roof Systems", MS Thesis, Department of Civil and Environmental Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

Sears, J. M. and T. M. Murray (2007), "Proposed Method for the Prediction of Lateral Restraint Forces in Metal Building Roof Systems," *Annual Stability Conference Proceedings*, Structural Stability Research Council, New Orleans, Louisiana, April 19-21 .

Seek, M. W. (2007), "Prediction of Lateral Restraint Forces in Sloped Z-Section Supported Roof Systems Using the Component Stiffness Method" PhD Dissertation, Department of Civil and Environmental Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

Seek, M. W. and T. M. Murray (2004), "Testing of the Lateral Restraint Force Requirements of Sloped Z-Purlin Supported Standing Seam and Through-Fastened Roof Systems with Two, Four and Six Purlin Lines." Research Report CE/VPI-ST-04/01, Department of Civil and Environmental Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

Seek, M. W., and T. M. Murray (2004a), "Computer Modeling of Sloped Z-Purlin Supported Roof Systems to Predict Lateral Restraint Force Requirements." *Conference Proceedings, 17th International Specialty Conference on Cold-Formed Steel Structures*, Department of Civil Engineering, University of Missouri-Rolla. Rolla, Missouri.

Seek, M. W., and T. M. Murray. (2006). "Component Stiffness Method to Predict Lateral Restraint Forces in End Restrained Single Span Z-Section Supported Roof Systems with One Flange Attached to Sheathing." *Conference Proceedings, 18th International Specialty Conference on Cold-Formed Steel Structures*, Department of Civil Engineering, University of Missouri-Rolla. Rolla, Missouri.

Seek, M. W. and T. M. Murray (2007), "Component Stiffness Approach to Determine Lateral Forces in Z-Section Roof Systems," *Annual Stability Conference Proceedings*, Structural Stability Research Council, New Orleans, Louisiana, April 19-21.

Seshappa, V. and T. M. Murray, "Study of Thin-Walled Metal Building Roof Systems Using Scale Models", *Proceedings of the IABSE Colloquium on Thin-Walled Metal Structures in Buildings*, IABSE, Zurich, Switzerland, 1986, pp. 177-183.

Trout, A. M. and T. M. Murray (2000), "Reduced Number of Base Tests," Research Report CE/VPI-ST00/17, Department of Civil and Environmental Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.