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ANALYSIS OF TUNABILITY FOR A THREE TERMINAL MICROWAVE NETWORK

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ABSTRACT

Three-terminal microwave networks can be shown to exhibit qualities that would make them desirable to be used as multiple terminal devices. Using microwave theory, where instantaneous voltage is a function of position on the line, this situation will be examined. These devices use the interference principle of propagating waves as the basis of their characteristics. This investigation will probe a three-terminal microwave network with one input and two output. This configuration acts similar to a splitter where the outputs may be controlled. These three terminals, plus manipulation of the lengths of each side, actually makes this a four terminal device, much like a transistor with one source, one gate, and two drains. The three terminal network will be examined in a situation such that the lengths of two of the sides will be varied and the wavelength will be held constant. The power of the signal at each of the output terminals, along with reflected power will be recorded. By using a case where the total length of the ring is an integer number of wavelengths, two cases of behavior arise. One case exhibits qualities much like the Aharanov-Bohm effect. The other case has tendencies to reflect most of the incoming signal, except near equal length sides.

I. INTRODUCTION

The three terminal microwave network that will be discussed consists of three equal lengths of transmission line. First, the total length of the three sides combined will be held constant, and the wavelength adjusted, while measuring the reflected power. The reflected power of the network is then seen as a function of the ratio of total length to wavelength. The function of reflected power is periodic every 3N, N being an integer.

The tunability of this network can be examined by fixing the input wavelength. For instance, the wavelength is set so that the total length of the three sides contains an integer value of wavelengths. The input terminal of the network is adjusted so that the lengths of the two sides adjacent to the input terminal are varied. The total length of the sides remains the same, so one of the adjacent side's length increases by a small amount, and the other adjacent side's length decreases by the same small amount. The power at the output terminals, along with the reflected power, are then seen as a function of the amount of the change in length per side, or Δ. For the integer values of the ratio of total length, or L, to wavelength, or λ, two cases were observed to exist. In one case, Δ can be set so that the power is seen only in one of the output terminals. In the other case, each individual side is an integer number of wavelengths, and the output terminals' powers are both even functions.

The importance of this, however, is that this is a microwave analogy of the three terminal networks proposed by Wu, Javurek, and Bookout[1] [1991]. Thus, the qualities of this three terminal microwave network are similar to the Aharanov-Bohm effect, which may be used for a quantum circulator. The evaluation of the network will be discussed in Section III, including two cases for integer values of L/λ, an d non-integer values. The derivation for the numerical analysis is shown in Section II.

II. THEORETICAL DERIVATION FOR A THREE-TERMINAL MICROWAVE NETWORK

The three terminal ring, with equal length sides, and only one input, is shown in the figure 1.

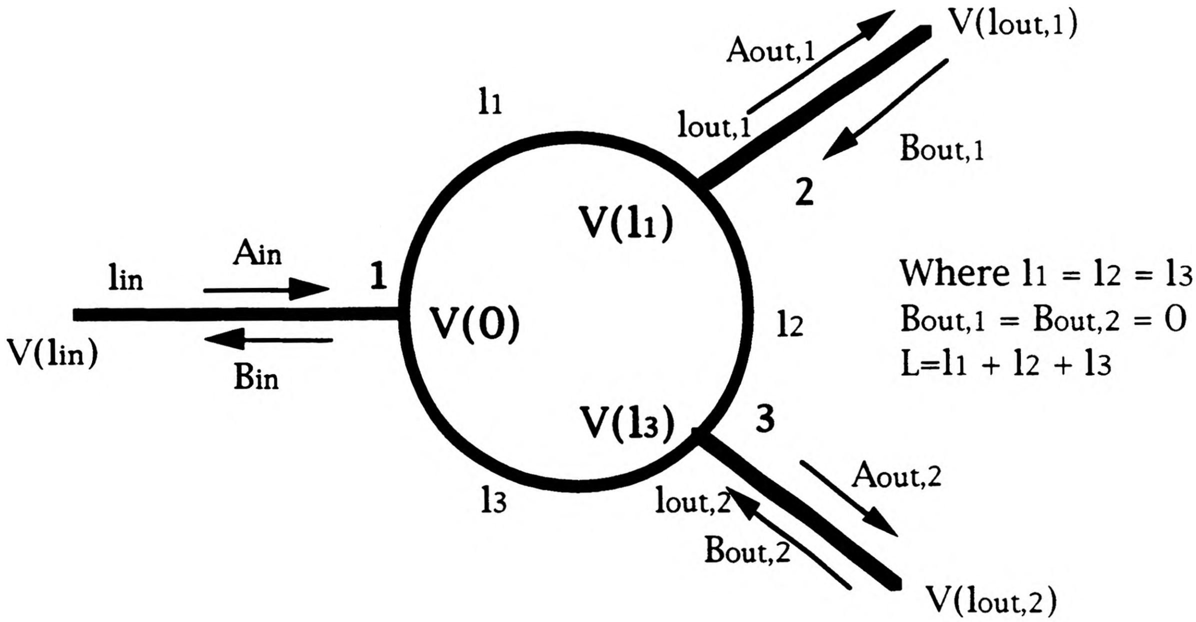

Figure 1 - Three Terminal Microwave Network

On a simple transmission line, where $\gamma = \alpha + j\beta$, (lossy)[2]

$$V(x) = Ae^{\gamma x} + Be^{-\gamma x} = a \cosh(\gamma x + \delta), \tag{1}$$

$$V(l) = V(0) [\cosh(\gamma l) + \sinh(\gamma l) \tanh(\delta)]. \tag{2}$$

Now, in a node with three branches, using Kirchhoff's Current Law,

$$\Sigma I_i = \Sigma \frac{1}{Z} * \frac{dV}{dX} = 0, \text{ or } \Sigma \frac{dV}{dX} = 0. \tag{3}$$

By substitution Eq. (2) into (3), then

$$\tanh(\delta_1) + \tanh(\delta_2) + \tanh(\delta_3) = 0. \tag{4}$$

This gives a phase relation between δ_1, δ_2, and δ_3, the phases of the incoming waves.

If Eq. (2) is applied to the three lines at node 1, then

$$V(l1) = V(0) [\cosh(\gamma l_1) + \sinh(\gamma l_1) \tanh(\delta_{o,l1})], \tag{5}$$

$$V(l3) = V(0) [\cosh(\gamma l_3) + \sinh(\gamma l_3) \tanh(\delta_{o,l3})], \text{ and} \tag{6}$$

$$V_{in} = V(0) [\cosh(\gamma l_{in}) + \sinh(\gamma l_{in}) \tanh(\delta_{o,lin})]. \tag{7}$$

By dividing each Eqs. (5), (6), and (7) by the sinh term, and then summing the three, the node equation may be found:

$$V(0)[\coth(\gamma l1) + \coth(\gamma l3) + \coth(\gamma lin)] - \frac{V(l1)}{\sinh(\gamma l1)} - \frac{V(l3)}{\sinh(\gamma l3)} - \frac{V(lin)}{\sinh(\gamma lin)} = 0. \quad (8)$$

This can be applied to each of the three nodes. The other two nodes give the following

$$V(l1)[\coth(\gamma l1) + \coth(\gamma l2) + \coth(\gamma lout,1)] - \frac{V(0)}{\sinh(\gamma l1)} - \frac{V(l3)}{\sinh(\gamma l2)} - \frac{V(lout,1)}{\sinh(\gamma lout,1)} = 0 \quad (9)$$

and

$$V(l3)[\coth(\gamma l2) + \coth(\gamma l3) + \coth(\gamma lout,2)] - \frac{V(0)}{\sinh(\gamma l3)} - \frac{V(l1)}{\sinh(\gamma l2)} - \frac{V(lout,2)}{\sinh(\gamma lout,2)} = 0 \quad (10)$$

The Eqs. (8), (9), and (10) have three independent unknowns: V(0), $V(l_1)$, and $V(l_3)$; and their three dependent variables $V(l_{in})$, $V(l_{out,1})$, and $V(l_{out,2})$.

For the transmission line of figure 1,

$$Rin = \frac{B}{A} \quad (11)$$

is defined as the input reflection coefficient. Eq. (2) is applied to the input terminal, and the $\tanh(\delta)$ is given as $\frac{A-B}{A+B}$, so

$$V(lin) = V(0)[\cosh(\gamma lin) + \frac{A-B}{A+B}\sinh(\gamma lin)], \text{ or} \quad (12)$$

$$V(lin) = V(0)[\cosh(\gamma lin) + \frac{1-Rin}{1+Rin}\sinh(\gamma lin)]. \quad (13)$$

Likewise, by using the above equations, and using $Rout = \frac{A}{B}$ for the reflection coefficient of any output loads, the other two equations relating the dependent variable to the independent variables are

$$V(lout,1) = (Vl1)[\cosh(\gamma lout,1) + \frac{Rout,1-1}{Rout,1+1}\sinh(\gamma lout,1)] \text{ and} \quad (14)$$

$$V(lout,2) = (Vl3)[\cosh(\gamma lout,2) + \frac{Rout,2-1}{Rout,2+1}\sinh(\gamma lout,2)]. \quad (15)$$

By inserting Eqs. (13), (14), and (15) into Eqs. (8), (9), and (10), they become three equations, three unknowns:

$$V(0)[\coth(\gamma l1) + \coth(\gamma l3) - \frac{1-Rin}{1+Rin}] + V(l1)[\frac{-1}{\sinh(\gamma l1)}] + V(l3)[\frac{-1}{\sinh(\gamma l3)}] = 0, \quad (16)$$

$$V(0)[\frac{-1}{\sinh(\gamma l1)}] + V(l1)[\coth(\gamma l1) + \coth(\gamma l2) - \frac{Rout,1-1}{Rout,1+1}] + V(l3)[\frac{-1}{\sinh(\gamma l2)}] = 0, \quad (17)$$

and $$V(0)[\frac{-1}{\sinh(\gamma l3)}] + V(l1)[\frac{-1}{\sinh(\gamma l2)}] + V(l3)[\coth(\gamma l2) + \coth(\gamma l3) - \frac{R_{out,2} - 1}{R_{out,2} + 1}] = 0. \quad (18)$$

Since only one input is assumed, then $R_{out,1}$ and $R_{out,2}$ are equal to zero. R_{in} can then be solved for by setting the matrix of the coefficients to zero. From this, R_{in}, and by normalizing the input to one,

$$V(0) = 1 + R_{in}. \quad (19)$$

With V(0) known, $V(l_1)$ and $V(l_3)$ are found by using the Eqs. (17) and (18) and solving them in matrix form.

III. NUMERICAL RESULTS AND DISCUSSION

The first situation that was examined involved the three terminal ring with equal lengths. The power in the reflected signal was examined as a function of the ratio of total length of the ring to the wavelength. This graph is shown in figure 2, which is plotted for L/λ from 3 to 6, which is one period. Since the graph is periodic, the value of $L/\lambda=10$ corresponds to 4 on this graph, 11 corresponds to 5, and 12 corresponds to 6.

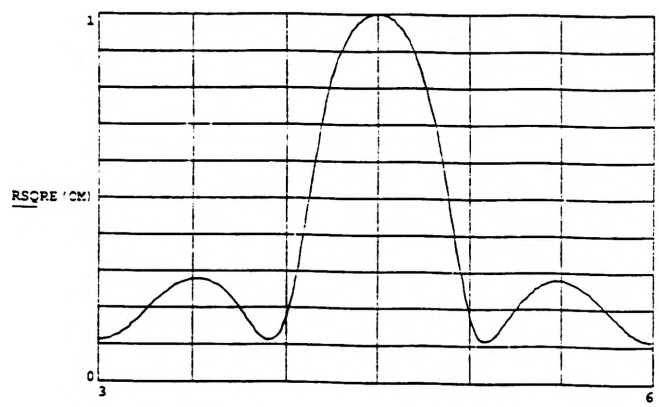

Figure 2 - Reflected Power versus L/λ for One Period

From this graph it is evident that at the integer values of 3N, the power is at a minimum of .111 (lossless). The other integer values of 3N+1 and 3N+2 are also of interest, along with the minimums that occur at non-integer values. Of the integer values, two cases of results arise, with one case having two sub-cases.

Case I: 3N+1, 3N+2

The first case consists of the ratio of total length to wavelength being either 3N+1 or 3N+2. To investigate these, an equivalent length ring was used with a fixed wavelength equal to $\frac{3N+1}{L}$ or $\frac{3N+2}{L}$. The tunability of the ring was examined. The input terminal position was varied by Δ, thus changing l1 by + Δ and l3 by - Δ. The power in the reflected signal, $V(l_1)$, and $V(l_3)$ are shown for $L/\lambda = 10$ in figures 3, 4, and 5 for both lossless and lossy cases. The period for Δ for all three functions was found to be $\lambda/2$. The power of the reflected signal is also an even function. Some points of interest include $\Delta = 0$ and $\Delta = \pm \lambda/6$. At $\Delta = 0$, the power in the reflected signal reaches a maximum of .179 (lossless), and the output powers are equal to .410 (lossless). However, at $\Delta = \pm \lambda/6$ is of particular interest. At the negative location, $V(l_1)$ has no power, whereas $V(l_3)$ has all of the power, with no power reflected. At the positive location, $V(l_3)$ has all of the power, with $V(l_1)$ and the reflected power equal to zero. At $\Delta = \pm \lambda/6$, the lengths of the sides are such that the waves come into one of the output terminals exactly in phase. For example, if $L/\lambda=10$, and L is set to 3, then each individual side is equal to $(10/3) * \lambda$. If Δ is equal to $\lambda/6$, then l_1 is increased to $(21/6) * \lambda$ and l_3 is decreased to $(19/6) * \lambda$. The path length to output terminal 1, through l_1, is then $(21/6) * \lambda$, and the path through l_2 and l_3 is $(39/6) * \lambda$. The path difference is $(18/6) * \lambda$, or 3λ, meaning that the two waves arrive in phase. Thus, it is possible, through tuning of a three terminal ring, to produce a device where all of the input signal could go out one port of all of the signal go out another port. This type of tunability is easily done in microwaves by physically changing the lengths of each of the sides. However, another method to realize this is optically by adjusting the wavelengths of the light by changing the applied electric field. This method would not require any physical adjustments on the length of any part, and could be useful for an optical computer.

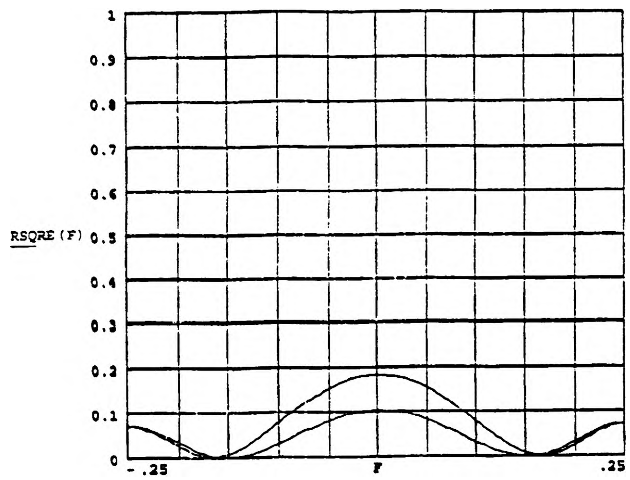

Figure 3 - Reflected Power versus Δ for $L/\lambda=10$. The top curve shows lossless case and the bottom curve shows $\alpha = 0.05$. The graph for Reflected Power versus Δ for $L/\lambda=11$ is identical to this graph.

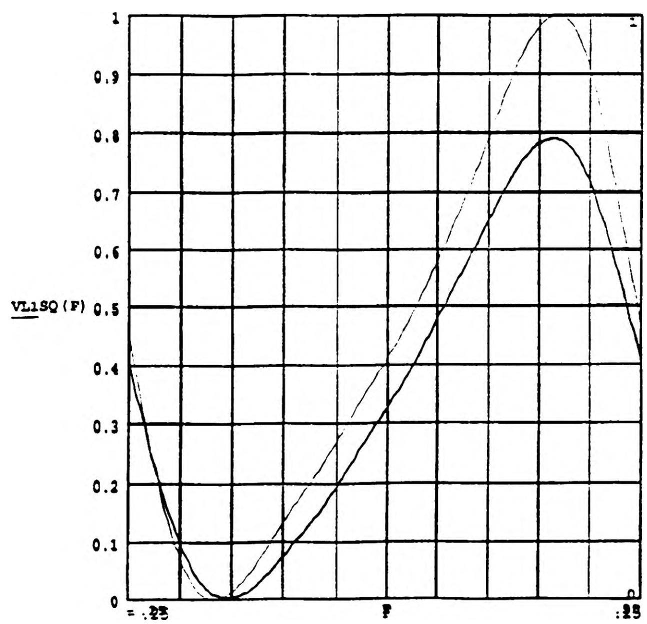

Figure 4 - Power in Output Terminal 1 versus Δ for $L/\lambda=10$. The top curve shows lossless case and the bottom curve shows $\alpha = 0.05$. The Output Terminal 2 versus Δ for $L/\lambda=11$ is identical to this graph.

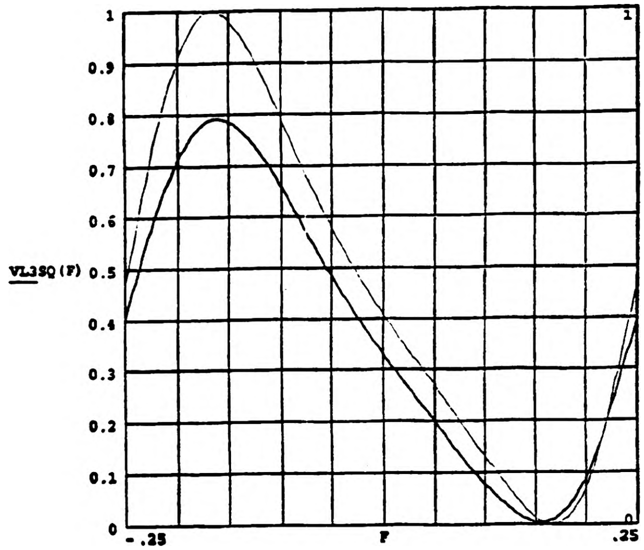

Figure 5 - Power in Output Terminal 2 versus Δ for L/λ=10. The top curve shows lossless case and the bottom curve shows α = 0.05. The Output Terminal 1 versus Δ for L/λ=11 is identical to this graph.

The lossy case of L/λ is also shown in figures 3, 4, and 5. Notice that the minimum of the reflected power, and the extrema of the output power curves are at a slightly lower value of Δ. The peaks of the output powers are scaled down from the lossless values. This may be because the attenuation in the line affects the strength of the traveling wave. If the waves do not have the same amplitude, as they do in the lossless case, then other factors, such as losses, are as important as path difference when determining signal strength. Since the path lengths differ, the two waves arriving at an output terminal are no longer the same strength.

Likewise, when L/λ = 11 was examined, the results were similar to when the L/λ = 10, but the results for when the L/λ= 11 for a given Δ were identical to the results of L/λ = 10 for the negative value of that Δ. The graphs corresponding to this situation are similar to what is shown in figures 3, 4, and 5. Thus, the periods of Δ are equal to λ/2. and the points of total transmission to one output remain at ± λ/6. The only difference between the two results is the fact that they are flipped, i.e. $V(l_1(\Delta))$ for L/λ = 11 is equivalent to $V(l_1(-\Delta))$ for L/λ = 10. For this reason, this case is divided into two sub-cases, one for 3N+1 and one for 3N+2. Generally, if both devices were placed in a box and the characteristics measured, it would be difficult to distinguish between the two, since they have identical periods, maximum, and minimum values. But since two different integers can give this result, a distinction between the two must be mentioned. A lossy version of L/λ is also shown in figures 3, 4, and 5.

Case II: 3N

The second case is the situation where the ratio of L/λ = 3N. This situation is different mainly because, for Δ = 0, the three terminals are all on the same point of the wave of the signal. The ratio of each individual side to wavelength is N. By graphing the power in the reflected signal and each output, shown here in figures 6, and 7, it is evident that the output power reaches a maximum at Δ = 0, and is negligible for any value of Δ that is not very close to zero. Also, the reflected signal contains nearly all of the power at for Δ not close to zero. At Δ = 0, the power in the reflected signal minimizes at 0.111, and the output signal power for both output terminals is equal to 0.444. The period of these three functions is also λ/2. All three of the signals are also even functions. With this situation, it is not possible to obtain a result where all or most of the signal is present at only one of the output terminals, since $V(l_1)$ is equal to $V(l_3)$.

By examining the path differences in this case, it is evident that since each side is an integer multiple of wavelength, they will arrive in phase at all three terminals. However, when the input terminal is adjusted by Δ, the difference in path at the output terminals is such that the waves tend to cancel each other. The reflected power is still in phase, though.

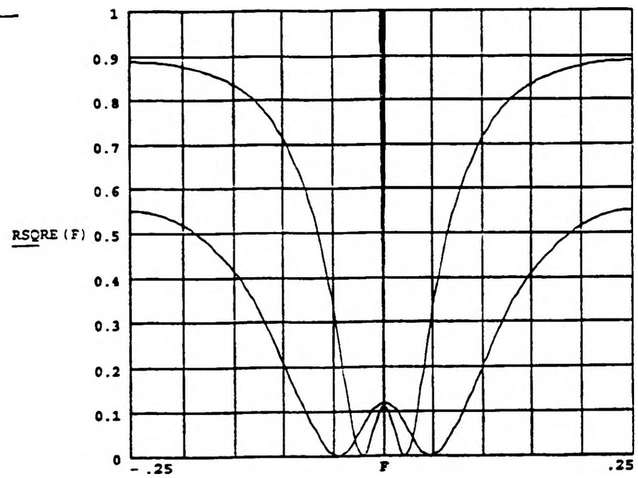

Figure 6 - Reflected Power versus Δ for L/λ=12. The top curve shows lossless case, the middle curve shows for α = 0.01, and the wide curve shows for α = 0.05.

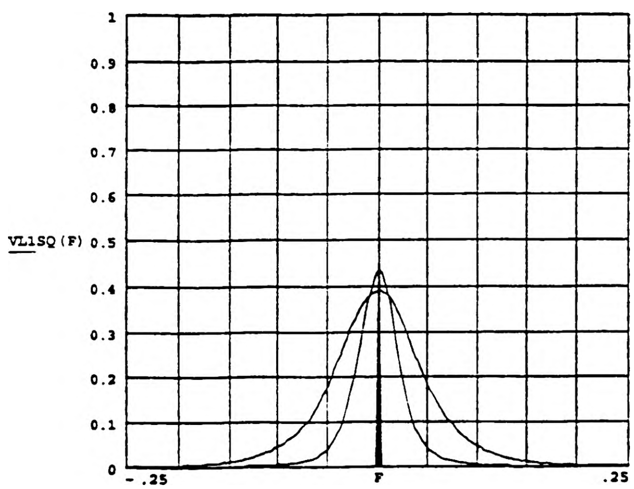

Figure 7 - Power in both Output Terminal 1 and Output Terminal 2 versus Δ for L/λ=12. The top curve shows lossless case, the middle curve shows for α = 0.01, and the wide curve shows for α = 0.05.

If the attenuation in the ring is increased, then the output terminals begin to pass signals for greater values of delta. Graphs for two values of loss are found in figures 6 and 7. The loss in the system also appears to create a small hump around Δ=0 in the reflected power, which appears to have two minima about Δ=0. As the attenuation factor is increased, all three curves appear to widen. The peak value for the output value becomes lower because of the attenuation factor. With loss, both output terminals are equal to each other. Thus, even with a loss, the signal cannot be routed to a certain direction with this case, as both output terminal powers remain an even function.

Discussion of non-integer values of L/λ

Another situation that is addressed is where the curve of reflected power versus the ratio of L/λ becomes a minimum at non-integer values. These values are at 3N+2.093 and 3N+0.907. These functions are also periodic every λ/2. They differ, however, from Case I and Case II in that the outputs peak at Δ = ±.1575 λ. Typical values of power at those points (lossless) are reflected = 0.0814, $V(l_1)$ = 0.0927, and $V(l_3)$

= 0.8259. Even at the peak values, it is still not possible to route all of the signal to one terminal without any leakage to the other terminal. By examining the reflected signal at $\Delta = 0$, the same value for the power is achieved as in Case II (0.111), which was expected, since the wavelength was chosen at the minimum of the L/λ graph. This case, however, differs starkly from the Case II mentioned above, when Δ is varied.

Another point of interest on figure 2 is at the point where $L/\lambda=4.5$. At this point, the network reflects the signal totally. Since $L/\lambda=4.5$, then each individual side is one and one-half wavelength. The output terminals have no power due to path differences of $\lambda/2$, so it is all reflected.

Similarity to the Aharanov-Bohm Network

The network described here has certain similarities to the three terminal quantum resistor networks proposed by Wu, Javurek, and Bookout [1991]. For example, figure 3, showing reflected power is similar to the reflected power in Wu's network. The general shape of the curves are comparable, and the maximum values of the ordinate for zero are identical. The x-axis, however, is not identical. The reflected power of the quantum network displays double periodicity, and is shown versus normalized magnetic flux. The microwave network is plotted against Δ. This does not show double periodicity. It appears that the values correspond to each other, but the abscissas are not related linearly. The horizontal axis is stretched in some places and shrunk in other places. One possible reason for the non-linear relation is that the magnetic flux affects the traveling wave in the entire ring, while the changes in Δ only affect two of the three lengths of the network. The output power in figure 4 is also similar to the output power in the quantum resistor network. Like the reflected power, the output power takes the same maximum and minimum power for both of them. The horizontal axis corresponds in a manner like the previous example.

IV. CONCLUSIONS

Using numerical analysis of a three terminal microwave network, the tunability of the ring was examined. The interference of the propagating wave along the transmission line affected the behavior of the network at the terminals. A graph of reflected power versus the ratio of total length of the ring to wavelength was used to pick values of wavelength to investigate. It was shown that for the three terminal ring, using only integer values of L/λ, there exist two cases of behavior by the power of the signal at the three terminals.

In one case, it is possible to adjust the lengths in such a way that the power in the signal is present only at one terminal. This case could be useful where one would want to use multiplexing. In the other case, it is not possible to have a situation where only one terminal has power present, due to the symmetry in the ring and wavelength being an integer value of each individual side. Another situation that was examined was when the L/λ curve was at a minimum at non-integer values. When the wavelength was fixed at that value, the ring could not be manipulated so that the signal passes only through one of the output terminals.

ACKNOWLEDGMENTS

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