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A Development of a Monte Carlo Method for the Determination of the Gamma Ray Detecting Efficiency of a Germanium Detector

Wayne T. Greene

A DEVELOPMENT OF A MONTE CARLO METHOD FOR THE DETERMINATION OF THE GAMMA RAY DETECTING EFFICIENCY OF A GERMANIUM DETECTOR

Author: Wayne T. Greene
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The determination of gamma ray transport has proven difficult to handle by ordinary methods. The use of the Monte Carlo method for the transport of various gamma rays impinging upon a detector has provided an accurate way to describe the gamma ray detection phenomenon. Using the Monte Carlo method, a procedure has been developed to describe all gamma ray interactions with the material of which the detector is made. The gamma ray detection efficiency is determined by computing the ratio of the gamma particles that interact and leave a certain minimum energy in the detector divided by the number of gamma particles emitted by the source.

INTRODUCTION:

The detection efficiency of a detector is defined as the ratio of the number of photons detected divided by the number of photons emitted by the source. The number of photons detected depends on the gammas that interact in the detector and deposit a certain minimum energy in it. This minimum energy is related to the minimum electronic pulse that can be recorded.

To date, there has not been developed a good method to determine very accurately the efficiency of a gamma ray detector. This is a major concern to fields such as health physics and if left unaddressed this problem may lead to incorrect radiation doses for radiation workers and members of the public.

The basis of the Monte Carlo method is the use of the Random Walk procedure. Random Walk is an expression used for the random implementation of various factors in the calculation of the possible events describing a particle's motion. To accomplish this end, a computer simulation of the history of the incident particle which is in our case a gamma ray (or photon) was developed in relation to another particle which the photon will then interact with. A random number generation scheme is utilized to determine a random number between the values zero and one. This number, once generated, will then be used in the decision making process regarding all events describing the photon position, energy, direction of motion, and type of interaction.

1.0. THE DETECTOR:

A Geranium (Ge) crystal was assumed to be used as the detector. The size of the crystal is variable such that it may be used as an input parameter for the calculations within the program.

2.0. THE MONTE CARLO SIMULATION:

2.1. The Types of Interactions:

In following the history of each photon as it moves through the detector there are many types of interactions that need to be addressed. In this work, the following three were considered:

A. Photoelectric Effect
B. Compton Effect
C. Pair Production.

A. Photoelectric Effect: As illustrated in **Figure 1**, a photon is traveling and soon collides with an atom in its path. The atom recoils from the impact and emits an electron from either its outer or inner shell. If this electron is emitted from the outer shell, we will simply have an electron flying off in one direction with the atom continuing to proceed in the direction of its recoil. This is just a simple case of Ionization of an atom and our interest in this matter is terminated. However, if the electron emitted from the collision originates from the atoms inner shell, then a hole is left in its wake within the inner shell where the electron originally occupied. Since this is an unstable condition for an atom, another electron will fall from an outer shell to fill the void left in the inner shell. This collapse of an electron to an inner shell will produce an x-ray which will then be emitted from the atom. This x-ray is very important to our calculation since it carries part of the gamma ray energy. Photoelectric interaction is dominant for photon energies less than 1 Mev.

B. The Compton Effect: As illustrated in **Figure 1**, a photon is traveling and collides with a free electron. From the collision with the electron, the photon will lose some of its energy in the form of kinetic energy to the electron and will have a lower amount of energy than it had prior to the collision. This type of interaction is dominant for photon energies from 0.5 Mev to 5 Mev..

C. The Pair Production Effect: As illustrated in **Figure 1:** a photon collides with a nucleus. This causes the nucleus to totally absorb the energy of the photon but being unable to contain this abundant energy, the nucleus breaks apart and forms an electron - positron pair. The positron proceeds only a short distance when Annihilation occurs. In this process, the positron collides with another free electron, and the two particles are both converted into two separate photons with an energy of .511 Mev each. This type of interaction occurs at energies above 1.02 Mev..

These three effects represent the major interactions the photon will encounter as it moves through the detector. To determine which type of effect will occur one uses the individuals cross sections (probabilities) along with a random number. Assume the following:

$$\mu_1 = \textit{probability of photoelectric} \qquad (1)$$

$$\mu_2 = \text{probability of Compton} \quad (2)$$

$$\mu_3 = \text{probability of pair production} \quad (3)$$

$$\mu_t = \text{total interaction probability} \quad (4)$$

$$\mu_t = \mu_1 + \mu_2 + \mu_3 \quad (5)$$

The following algorithm is then used:

$RN <= \mu_1 / \mu_t$, *Photoelectric effect occurred*

$\mu_1/\mu_t < RN < (\mu_1+\mu_2)/\mu_t$, *Compton effect occurred*

$RN > (\mu_1 + \mu_2)/\mu_t$, *Pair Production occurred*

Depending on the interaction, the program branches off and records energy deposited, and particles still alive along with their energy, direction of motion, and position. The attenuation coefficients (probabilities) μ are functions of energy. They are tabulated at certain fixed energy values. To obtain the values of the coefficients for energies not on the table, an interpolation scheme is used which may be linear or more elaborate like cubic spline interpolation.

2.2. The Location Of The Photon:

It is extremely important to know the location of the photon relative to the detector boundaries. To achieve this, two coordinate systems are set up. The first is a fixed coordinate system with the z axis placed at the center of the cylindrical detector (see **Figure 2**). The second is a frame of reference moving along with the particle and having the z axis always coinciding with the direction of motion of the photon. To obtain the coordinates of the position of the particle in the fixed system, a transformation is employed as shown in **Figure 3**. If a photon has an interaction at point 2, to transform this location to the fixed reference frame, the angle position 2 makes with the z-axis of the fixed reference frame is given the value $\theta 1$ and the projection of point 2 onto the x-y plane with the angle between the x-axis and the projection is known as $\phi 1$. Using the following equations a determination for the direction cosines α, β, and γ may be found:

$$\mu = \cos\theta 2 \quad (6)$$

$$\alpha = \sin\theta 1 * \cos\phi 1 \quad (7)$$

$$\beta = \sin\theta 1 * \sin\phi 1 \quad (8)$$

$$\gamma = \cos\theta 1 \quad (9)$$

After the above values have been found, the initial values of the x, y, and z coordinates may be located using the following equations:

$$x = r_{12}*\sin\theta 1 * \cos\phi 1 = r_{12} * \alpha \tag{10}$$

$$y = r_{12}*\sin\theta 1 * \sin\phi 1 = r_{12} * \beta \tag{11}$$

$$z = r_{12} * \cos\theta 1 = r_{12} * \gamma \tag{12}$$

This calculation determines point 2, the first point away from the z-axis. The determination of point 3 as illustrated in the **Figure 3** is handled in a similar manner, except that the equations are slightly changed. The direction cosines α', β', γ' of the direction r_{23} are now determined and the new coordinates x_3, y_3, z_3 are obtained as shown at the bottom of **Figure 3**.

2.3. Methods For Killing the Photon:

To avoid going into an infinite "loop", a decision must be made to place limits upon the life of each particle. These limits are I Absorption, II. Failure To Reach Detector, III. Energy Falls Below a Set Limit, IV. Leaving the Detector Volume.

The decision for absorption is based on the type of interaction. Photoelectric and pair production lead to photon absorption, while Compton does not.

After a Compton interaction, the photon survives with reduced energy. Clearly, there is a possibility for a photon to have numerous Compton interactions with a corresponding energy reduction after every such collision. Since in practice there is always some energy below which the photon cannot produce a pulse that can be recorded, it is a waste of effort to follow such a particle forever. A decision is made to consider the particle lost (dead) after its energy falls below a certain limit which is taken to be in the range of 10 - 1 KeV.

The present status of this work is this: All the parts of the program have been written but no results for the detector efficiency have been obtained. This work will be continued until such time as the detector's efficiency may be determined.

CONCLUSION:

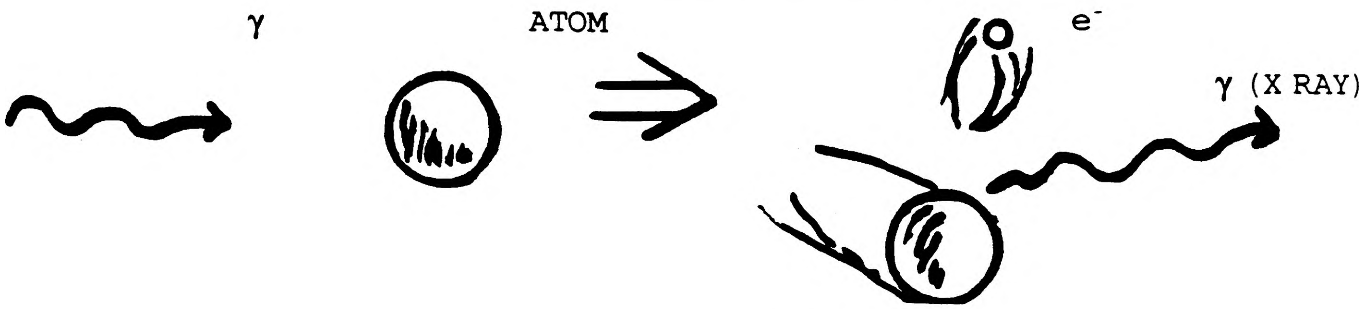
The use of the Monte Carlo method is a good way to accurately take into account all the possible interactions as well as complex geometries which may be encountered when one studies particles. For the calculation of the gamma ray detection efficiency, the Monte Carlo method may be applied successfully for any size and detector shape and for any gamma energy range.

APPENDIX

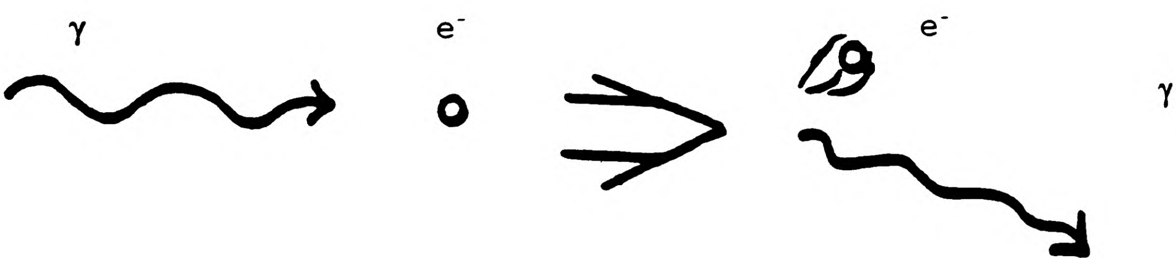
LISTED FIGURES CITED IN DISCUSSION

FIGURE 1: INTERACTIONS

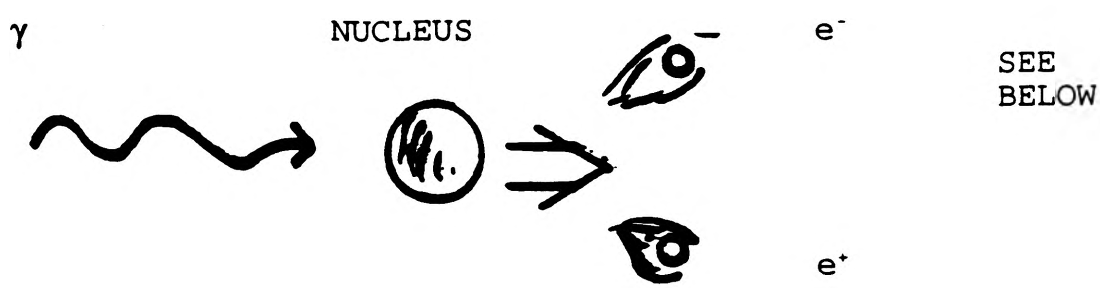
PHOTOELECTRIC EFFECT



COMPTON EFFECT



PAIR PRODUCTION



ANNIHILATION

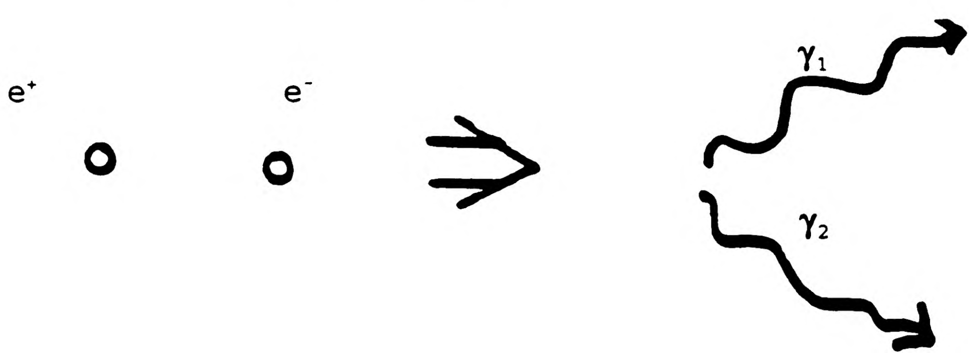


FIGURE 2: SOURCE PRODUCTION VS GAMMA DETECTION

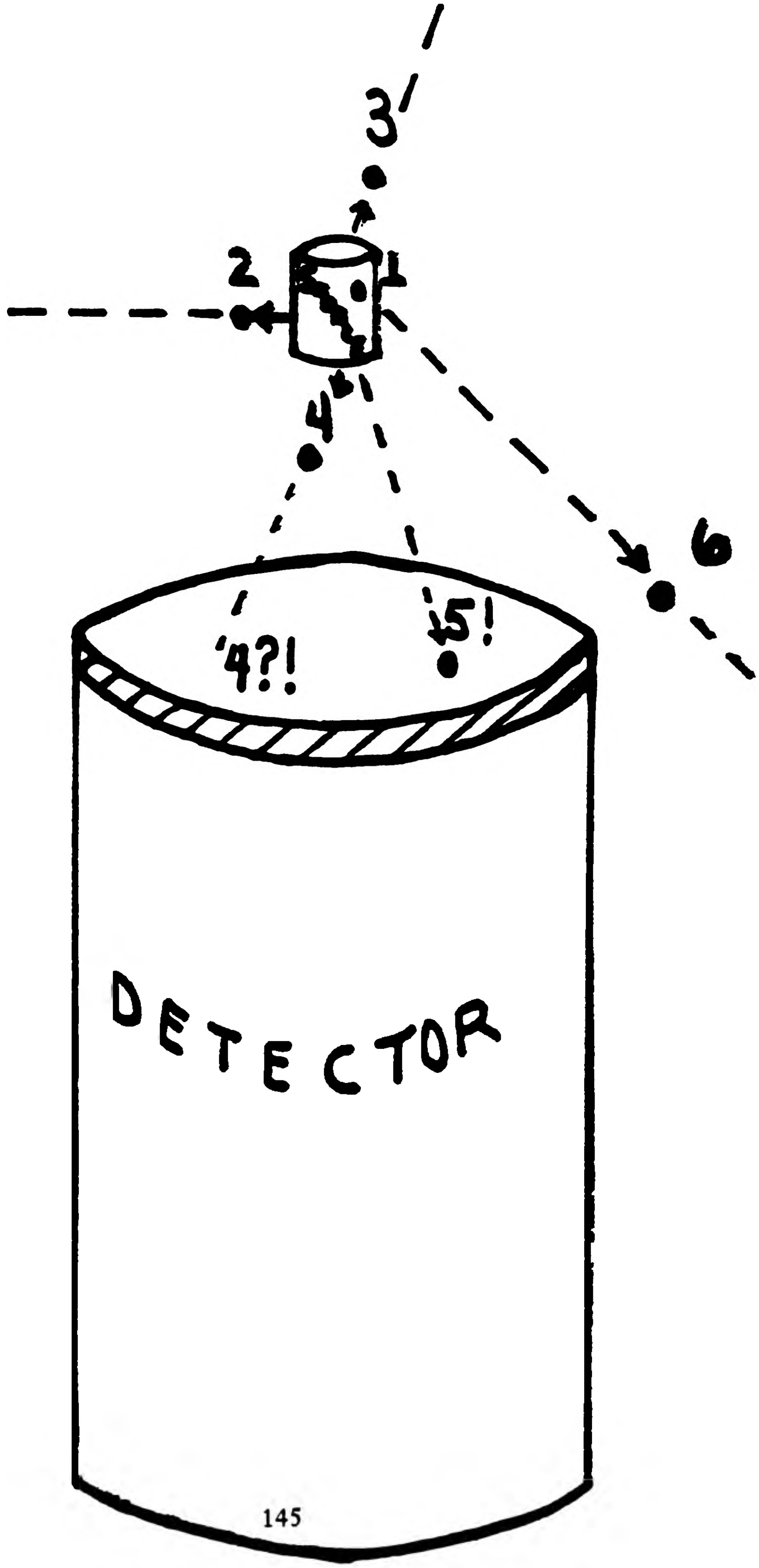
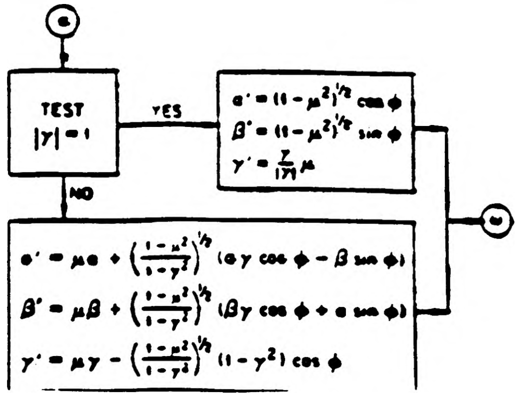
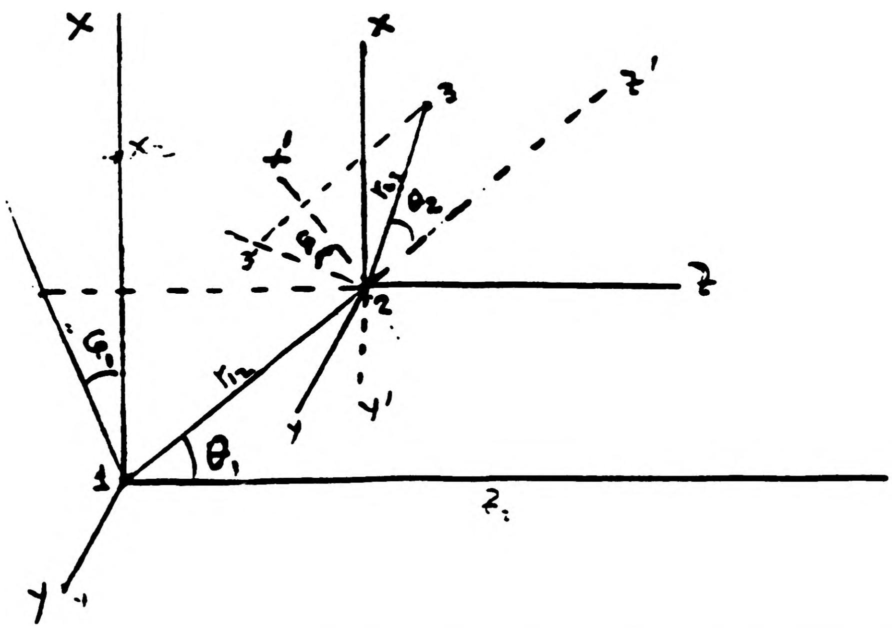


FIGURE 3: ROTATION OF COORDINATES

Assuming that the cosine of the polar angle of scattering is μ, the azimuthal angle is ϕ, and the direction cosines of the initial direction are α, β, and γ, then the direction cosines of the scattered photon, which are designated by α', β', and γ', are calculated as shown in Fig. 13.



α', β', γ' are direction cosines with respect to the fixed coodinate system



$\mu = \cos\theta_2$

$\alpha = \sin\theta_1 * \cos\phi_1$

$\beta = \sin\theta_1 * \sin\phi_1$

$\gamma = \cos\theta_1$

If $\gamma = 1$, this means $\theta_1 = 0$ and the equations for α', β', γ' are simpler to solve.

<u>NEW COORDINATES</u>

$x_3 = x_2 + r_{23}\alpha'$ $\qquad$ $x_2 = r_{12}\sin\theta_1\cos\phi_1 = r_{12}\alpha$

$y_3 = y_2 + r_{23}\beta'$ $\qquad$ $y_2 = r_{12}\sin\theta_1\sin\phi_1 = r_{12}\beta$

$z_3 = z_2 + r_{23}\gamma'$ $\qquad$ $z_2 = r_{12}\cos\theta_1 = r_{12}\gamma$