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by

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A perusal of the current mathematics education literature reveals that assessment of elementary school mathematics teaching and learning is the dominating topic of a myriad of books, journals, conference proceedings, and commissioned documents from organizations such as the National Council of Teachers of Mathematics (NCTM), the Mathematical Sciences Education Board (MSEB), the National Research Council (NRC), and the National Center for Research in Mathematical Sciences Education (NCRMSE). Mathematics educators at all levels are continuously inundated with suggested techniques of assessment. As a result, assessment has become a nebulous idea for many teachers of mathematics in the elementary school. The purpose of this article is to support elementary mathematics teachers in their specific efforts to implement authentic, effective, and innovative assessment techniques in their mathematics teaching. We will provide a framework that can be used to guide the integration of alternative assessment techniques into curriculum and instruction in elementary school mathematics.

Mathematics Education, 1920-1970

A clear understanding of how and why assessment is such a critical issue in elementary mathematics education can be supported by reviewing the history of mathematics education. Through this review, we gain some perspective on the forces that have contributed to current issues and concerns related to assessment of elementary school mathematics teaching and learning. One of the major forces driving elementary mathematics education throughout this century has been learning theories.

At the beginning of the 20th century, the enrollment in elementary schools rapidly expanded. As a result, there was growing pressure to provide an education for all children. The organizational structure of eight years of elementary school followed by four years of high school was changed to six years of elementary school followed by three years of junior high school and, finally, three years of high school. Arithmetic was incorporated into the elementary school curriculum in order to prepare students for mathematics courses they would take in junior high and high school (Jones & Coxford, 1970a).

During the 1920s and 1930s, mathematics education was influenced by Edward Thorndike's idea of connectionism (Kroll, 1989). The emphasis in elementary mathematics education was on rote memorization of facts and algorithms with speed and accuracy. That is, the sole purpose of elementary mathematics was to develop mechanical facility with computation. Arithmetic was fragmented into small facts and skills that could be easily taught and mastered. The goal was to help students make stimulus-response bonds by exposing them to drill and practice on each skill until the skill was mastered. In an effort to avoid establishing incorrect stimulus-response bonds, even closely related skills were taught in isolation. For example, facts such as $3 + 4 = 7$ and $4 + 3 = 7$ were presented as separate and independent (Brownell, 1970). Since the emphasis was on mechanical proficiency, it was perceived that the relationship between these two facts was not only unimportant but, if emphasized, could lead to the formation of incorrect bonds. Each of these facts would be presented at different times and then mastered by means of drill and practice. However, when implemented, the drill was often disorganized as it did not provide connections between and among arithmetic concepts. Teaching was very prescriptive and closed to innovative solutions to mathematical problems due to the belief that individual bonds existed in isolation (Kroll, 1989).

As progressive education became influential in the United States, parents and educators began questioning the heavy emphasis on drill, as they were concerned that students were not learning mathematics that was of any practical value. New emphasis was placed on practical mathematics, and the focus of mathematics education shifted towards developing meaningful arithmetic.

Thorndike's connectionism theory remained dominant during the beginning of the meaningful mathematics period. During the second part of this period, the meaning theory became the dominant theory of arithmetic instruction. Brownell (1970) described the purpose of the meaning theory as a way to enable students to develop number sense and the ability to understand the mathematical and practical significance of arithmetical situations. The performance of rote or mechanical operations was not indicative of learning. During this period, the emphasis was on students' understanding of arithmetic ideas, real life applications of mathematics, and meaningful arithmetical concepts.

This shift in emphasis necessitated changes in both content and instructional methods. Arithmetic content was organized to effectively promote students' understanding of arithmetical concepts and their inherent relationships. For example, Kroll (1989) reported that when basic subtraction facts were presented to students, the relationship of these facts to previously learned addition facts was emphasized. The relationships between addition facts, such as $3 + 4 = 7$ and $4 + 3 = 7$, and corresponding subtraction facts of $7 - 4 = 3$ and $7 - 3 = 4$ were now pointed out to students. The purpose of organizing arithmetic content in this manner was to enable students to gain insight into arithmetic as a unified whole, or system, rather than a body of unrelated facts. In order to effectively do so, arithmetic content was organized in a spiral format with various arithmetical concepts being developed over a wider span of grades (Brownell, 1970). Less emphasis was placed on rote memorization and an activity based instructional approach was employed (Kroll, 1989).

Conflicting ideas and the widespread dissatisfaction with American education after the Second World War (WWII) gave birth to what has been called the new math era. The majority of service recruits during WWII were found to lack minimal competence in mathematics. This, in combination with the Russian launching of Sputnik, were catalysts of a movement to improve American education. Advances in technology at this time made it abundantly clear that mathematical knowledge was essential for all people who were to understand the world in which they lived (DeVault & Weaver, 1970).

The theories of Jerome Bruner, Jean Piaget, and Robert Gagne impacted elementary school programs during this era. Bruner supported the importance of teaching the unifying structure of all subjects and stated that learning fundamental principles of a subject could allow one to reconstruct details of the subject if these details were forgotten. He further supported the ideas of a spiral curriculum and discovery learning, both of which were incorporated into the elementary mathematics programs of the new math period (Bruner, 1960). The work of Gagne and other psychologists on task-analysis also influenced elementary mathematics programs during this time, particularly with regard to computer-assisted instruction (Gagne, Wager, & Rojas, 1981). Piaget's developmental stages were of keen interest to researchers and curriculum developers due to the implications of these stages for the organization of mathematics curricula (Piaget, 1977).

During the new math period, the emphasis was on the understanding of the fundamental principles or structure of mathematics (Vogeli, 1976). This focus on the structure of mathematics led to the introduction of more abstract mathematical concepts into the elementary mathematics curriculum. Topics such as number theory, numeration, set theory, negative integers, and intuitive geometry were brought down from the secondary to the primary level (Jones & Coxford, 1970b; Kroll, 1989). In order to do so, content was organized in a spiral curriculum format. That is, these unifying mathematical concepts were introduced earlier in the curriculum and then continually returned to in subsequent lessons and grades. DeVault and Weaver (1970) reported that changes were also made in the context in which these concepts were taught. These included the use of team teaching, teacher assistants, nongraded primary schools, content specialists, and technology ranging from manipulatives to computers.

Vogeli (1976) reported that despite the strong support from psychologists and learning theorists, mathematics achievement attained during the new math period was disappointing. Computation scores declined in many schools, while there was only slight improvement in concept scores; problem-solving scores were inconclusive. Some proponents of new math explained the poor test results as a natural consequence of curricular change. Others argued that the instruments used to obtain these scores did not measure what the new math curricula were teaching. These arguments did little to appease concerned parents, teachers, and administrators. Confidence in the new math was further shaken by the publication of *Why Johnny Can't Add* (Kline, 1973). As a result, the emphasis of elementary mathematics returned back to basic skills.

The influence of each of the learning theories mentioned earlier was evident in mathematics classrooms during the back-to-basics movement. Drill and practice continued to be a focus of mathematics education, but there was also an interest in helping students develop problem-solving skills by providing them with an opportunity to use arithmetic skills in real-life situations. The idea of helping children understand the structure of mathematics remained an important goal during the back-to-basics movement.

Mathematics Education, 1980-Present

The emphasis on basic skills helped children improve their computational abilities but did little to enable them to apply these skills to solving problems. In its report, *A Nation At Risk: The Imperative for Educational Reform*, the National Commission on Excellence in Education (NCEE) (1983) reported that teenage children were not able to implement higher order thinking skills or solve multiple-step problems. There were numerous calls for reform in mathematics education during this time period. The National Science Board Commission on Precollege Education in Mathematics, Science and Technology (1983) recommended that standardized tests be modified to support the belief that as long as these tests stressed computation students would continue to be drilled in computation and little improvement in mathematics education would be achieved.

In *An Agenda for Action: Recommendations for School Mathematics of the 1980s,* NCTM (1980) made eight recommendations for school mathematics. Among these were the recommendations that "problem solving be the focus of school mathematics in the 1980s; basic skills in mathematics be defined to encompass more than computational facility; mathematics programs take full advantage of the power of calculators and computers at all grade levels [and] the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing" (p. 1).

The Educational Testing Service (ETS) (1988) chronicled four national mathematics assessments conducted by the National Assessment of Educational Progress (NAEP). It reported slight improvements in the mathematics achievement levels of students during the 1980s, but noted that these gains were made primarily in low-level skills. ETS further reported that far too many high school graduates were not prepared to participate productively in a modern society. The organization maintained that if students in the United States were to achieve in mathematics, changes were needed in the curriculum, in instruction, and in evaluation procedures.

In an effort to reform school mathematics to reflect societal and educational changes, NCTM published its *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), *Professional Standards for Teaching Mathematics* (NCTM, 1991), and *Assessment Standards for School Mathematics* (NCTM, 1995). An underlying assumption of the NCTM publications and the philosophy of the organization is that learning mathematics requires doing mathematics (NCTM, 1989). The vision of mathematics education advocated by NCTM implies that students acquire new knowledge primarily through cognitive activity. Presently, there is a strong effort being made to reform elementary mathematics education to reflect these changes.

Failing to Address the Assessment Issue

During both the drill and practice and the meaningful mathematics periods, mathematics achievement was evaluated by traditional assessment methods. For the most part, these methods examined students' ability to obtain the correct result. However, achieving the goals set forth by new math required drastic changes in curriculum, instruction, and assessment. It is evident that a major portion of the problems of the reforms called for during the new math era were the result of failure to implement changes in assessment along with changes in the mathematics curriculum and instruction.

When NCTM recommended changes be made in the elementary mathematics curriculum in the 1980s, the required use of a wider range of assessment methods presented a major conflict. The proposed curriculum focused on problem solving and heavily emphasized relationships within mathematics. Achievement could not always be easily measured, as problem solving is process-oriented as well as product-oriented.

Mitchell (1992) reported that traditional assessment methods were further strengthened during the 1960s and 1970s by the increased use of technology by the testing industry. She maintained that, despite protests from some education psychologists, the test publishers had a captive market and our nation's main concern was in seeing test scores rise. Very little attention was given to evaluating exactly what these tests measured. The consequences of the failure to evaluate what tests measure are far reaching and extremely detrimental, as test results continue to influence educational decisions.

Current Reform of Assessment in Mathematics Education

During most of this century, student achievement has been measured by means of quantitative data, collected most often by subjective tests. This supported the belief that knowing mathematics meant being able to perform computational procedures and identify certain concepts. The need for more effective assessment has evolved as a new definition of what it means to know mathematics is evolving. Kieran (1994) reported that learning cannot be separated from understanding. Understanding is an ongoing process rather than an achievement. Thus the emphasis on assessment in mathematics has shifted towards the process of learning and understanding and away from the product of learning and test scores.

If the current reform movement in mathematics education is to be successful, more attention must be given to the assessment models that are used by teachers of mathematics. Mathematics educators are challenged to develop and implement assessment techniques that will effectively evaluate the processes employed by students in their efforts to develop an understanding of mathematics. If we fail to do so, the current elementary mathematics education reform movement, like the new math era, will be thwarted by the use of inappropriate assessment techniques and results. Given the amount of current literature dealing with assessment of elementary mathematics, it is evident that mathematics educators are cognizant of this critical need to develop new and more effective assessment techniques. An attempt to align assessment with curriculum and instruction presents mathematics educators with an extremely important and difficult job, as assessment of learning processes is a considerably complex task.

Assessment in Elementary Mathematics Education

Numerous assessment ideas, strategies, and techniques are being developed and recommended for implementation in elementary mathematics classrooms. The following framework is intended to serve as a guide for elementary mathematics educators as they decide which assessment procedures are best suited for implementation in their classrooms. Regardless of the assessment technique in question, the framework can be used to guide decisions about the purposes and appropriateness of the said technique. All questions are not answered by this framework, but the framework does initiate a critical analysis of assessment techniques. This framework reflects the basic belief that authentic assessment considers each individual learner and informs teachers and learners about instruction and curriculum, learners' thinking processes, learners' levels of understanding, and the effectiveness of the curriculum and instruction.

A Framework for Assessment

1. Assessment techniques must be slowly incorporated into mathematics teaching. Time is needed for the evaluation of the effectiveness and usefulness of chosen techniques. In addition, children need time to also become familiar with various techniques. To attempt implementation of too many assessment techniques at one time will not provide a clear picture of the worthiness of the different and individual techniques. Instead take the time to become comfortable with and to properly evaluate each new technique employed.

2. Assessment methods should provide a thorough view of the information collected. Before implementing assessment, choose the specific criteria by which the children's learning and performance are to be measured. In doing so, characteristics of individual children should be carefully considered. Assessment techniques should reflect the learning styles and academic needs of children.

3. Children should be active participants in assessment procedures. They are capable of providing a wealth of assessment information about themselves when given the opportunity to do so. As children participate in the assessment of their own learning, they will be able to use the assessment process as a learning process. Assessment should be designed so that it is a positive experience for all children and encourages improvement.

4. Design assessment models in conjunction with designing the curriculum and instruction program. Assessment models should accommodate changes in the curriculum and in instruction as needed. Individual assessment tasks should model the purposes of assessment, the curriculum presented, the instructional methods employed, and the academic needs of the children.

5. In order for assessment to be an integral part of instruction, implementation of assessment techniques should not always require a complete interruption of instruction. Assessment is a component of instruction and instruction is a component of assessment. When assessment is an integral part of instruction, children continue to learn during the assessment process. They will have opportunity to develop and enhance their metacognitive skills and use the assessment opportunity as an opportunity for learning about themselves as learners.

6. Assessment results should be utilized to set goals for learning rather than simply report current achievement levels of children. These results must be interpreted and analyzed so they positively impact student learning and improve mathematics curriculum and instruction. Assessment should be consistent with the goals of the curriculum and instruction.

7. Model the assessment environment after the learning environment; children should be assessed under conditions similar to those in which they learn. Learning tools, concrete and abstract, can also be assessment tools. The use of learning tools during the assessment process provides a natural initiative for making assessment an integral part of instruction.

8. No single assessment technique provides a complete picture of the child as a learner. Combine techniques as needed to gather a wider, most useful range of information which can be used to improve mathematics teaching and learning. When a technique does not work to certain, needed specifications, eliminate that technique from the assessment model until the point when the technique may be useful.

Conclusion

Assessment should inform mathematics teaching and reveal what mathematics children have learned as well as what mathematics they are still in the process of learning or have not learned. Assessment is an asset for the improvement of mathematics teaching and learning. Mathematics education history supports the role of assessment in the current reform efforts. The framework provided can guide assessment decision that promote authentic and effective mathematics teaching and learning. With appropriate implementation of assessment techniques, mathematics educators can make spectacular advances in the current mathematics education reform efforts.

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