Losses Allocated to the Nodes of a Radial Distribution System with Distributed Energy Resources-A Simple and Effective Indicator

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Losses Allocated to the Nodes of a Radial Distribution System with Distributed Energy Resources – A Simple and Effective Indicator

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Abstract—**This paper presents the effectiveness of exploiting the losses allocated to the nodes of a radial distribution system as an indicator of the impact of the diffusion of distributed energy resources in the network. The calculation of the losses allocated to the nodes is not included in the commercial power flow solvers, even though the implementation of this calculation is simple and the results provide meaningful information. The interpretation of the allocated losses is illustrated in this paper, on the basis of the results obtained on a typical test network under different case studies.**

Keywords—branch current decomposition, circuit-based model, distributed generation, distribution network losses, distribution system, loss allocation, power flow.

I. INTRODUCTION

The present evolution of the electric distribution systems is occurring with growing diffusion of distributed energy resources (DERs) connected to the grid. Specific issues appeared recently, such as node voltage increase at the DER connection nodes and possibility of reverse power flows in the network branches, up to the limit case in which the distribution system supplies the transmission system in some periods of the day.

Today, the analysis of distribution systems has to be carried out by considering the dependence on time and the uncertainty of the power provided by generators (especially when this power production depends on weather conditions) and loads. Also the indicators that show the performance of the distribution systems have to be adapted to take into account network losses, branch loading limits and voltage deviations in networks in which the power flows have become largely variable in different periods of the day.

Among the indicators that express the DER impact on the distribution networks, this paper exploits the calculation of the losses allocated to each node as a promising way to understand the effects of the DER diffusion in different parts of the network. The allocation of the system losses to the nodes of a *radial* network can be carried out with different techniques, among which the circuit-based methods [1-3] are particularly interesting, because they do not require additional calculations of matrices not needed in the power flow solution, as it happens in [4]. In particular, the Branch Current Decomposition for Loss Allocation (BCDLA) method [3] is very effective, simple to implement, and the total allocated losses exactly match the total network losses, without requiring the adoption of specific rules to allocate parts of the losses as in [5] and [6]. This paper adopts the BCDLA method, directly included in the power flow analysis tool implemented by the authors. The BCDLA method takes the power flow results as inputs and provides the allocation of the losses to the distribution system nodes with a small addition to the calculation code. This calculation is not implemented yet in the commercial tools for power flow analysis. This paper shows that the information on the losses allocated to the system nodes is particularly effective and provides meaningful information to the decision makers on the possible development of the DER connection to the distribution network.

The next sections of this paper are organised as follows. Section II recalls the mathematical background on the calculation of the allocated losses. Section III presents a number of case study applications to apply a strategy of learning by examples, with which a number of critical aspects are discovered and commented. Section IV provides a systematic discussion on the lessons learned from the cases analysed. The last section contains the conclusions.

II. CALCULATION OF THE LOSSES ALLOCATED TO THE NODES

A. Variables for Distribution Network Analysis

Let us consider a distribution network with slack node denoted as node 0, further $k = 1, \ldots, K$ nodes containing local generation and/or load, and $b = 1,\ldots, B$ branches. The distribution network analysis carried out in this paper is based on partitioning the time period of observation into T successive time steps, denoted as $t = 1, \ldots, T$. In the steady-state analysis illustrated in this paper, within each time step the active and reactive power values are considered to be constant. Thereby, the net average active power load (i.e., the difference between the average power of the local load and the average power of the local generation) at node k and time step t is indicated as $P_{k,t}$. The total losses at time step t are denoted as $L_{\text{tot},t}$.

B. Distribution System Losses and Sensitivity Information

Starting from the base case with total losses $L_{\text{tot},t}\big|_{\text{base}}$ and changing the net average power load at one node from $P_{k,t}\big|_{\text{base}}$ to $P_{k,t}\big|_{\text{new}}$, the total losses become $L_{\text{tot},t}\big|_{\text{new}}$. For a relatively small net average power change, the linearized version of the relation between net average power load and total losses becomes

$$L_{\text{tot},t}\big|_{\text{new}} - L_{\text{tot},t}\big|_{\text{base}} \cong \sigma_{k,t}^{(P)} \left(P_{k,t}\big|_{\text{new}} - P_{k,t}\big|_{\text{base}} \right) \quad (1)$$

where the *sensitivity* coefficient $\sigma_{k,t}^{(P)}$ is calculated by using the results of the two power flows run in the base case and with the new net average power load.

C. Allocated Losses

Let us introduce the set $\mathbb{B}_k$ that contains the branches located in the path between node k and the slack node. Being a radial system, all branches are numbered in a unique way as their receiving nodes. Starting from the power flow solution at time step t, with the BCDLA method the losses allocated to node k are expressed in the following way [3][7]:

$$L_{k,t} = \mathcal{R}e\left\{\bar{I}_{k,t}^{*} \sum_{b\in\mathbb{B}_k}^{B} \left(R^{(b)} \bar{I}_t^{(b)}\right)\right\} \qquad (2)$$

where $R^{(b)}$ is the resistance of branch b, $\bar{I}_{k,t}^{*}$ is the complex conjugate current absorbed by all the shunt elements connected to node k at time step t, and $\bar{I}_t^{(b)}$ is the complex current that flows in the series impedance of the model of branch $b = 1,\ldots, B$. This calculation is straightforward to be implemented in a power flow calculation code, e.g., it requires a single line of code in Matlab®, see Eq. (12) in [7].

The allocated losses $L_{k,t}$ are not sensitivity coefficients by themselves. However, they embed a meaning similar to the one of sensitivity coefficients. Basically, *positive* allocated losses in a node are found when an increase of the net power load in the node leads to *increase* the total system losses. Conversely, *negative* allocated losses in a node correspond to *decrease* the total system losses if the net load power at that node is increased.

III. Understanding the Concepts through Case Study Applications

The conceptual aspects referring to the allocated losses have been tested on the model of a MV rural distribution

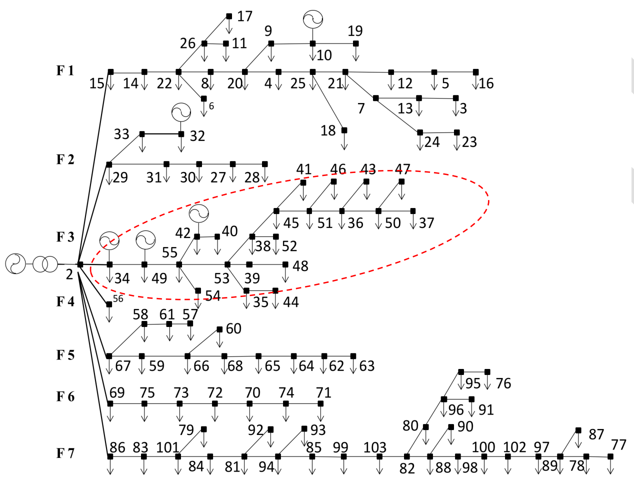


Fig. 1. Rural distribution system defined in the project Atlantide. The encircled nodes and branches of the Feeder F3 are the main subjects of the study.

system (supplied by a 150 kV transmission system) defined in the project Atlantide [8], with base voltage 20 kV and base power 1 MVA. The network is radial, has 102 nodes connected into 7 feeders (indicated as F1-F7 in Fig. 1).

The loads (residential, industrial, and agricultural) and local photovoltaic (PV) generations in the network are defined by using load and generation profiles with average power calculated at successive time steps of 15 minutes.

This paper considers two main cases:

- Case 0: Reference case from [8], with slack voltage set to 1 p.u.
- Case 1: High-DG solution, in which the multiplier of the PV generation is increased until a branch becomes very close to its loading limit (branch loading equal to unity in p.u.). This happens at branch 34 (the starting branch of Feeder F3, Fig. 2), with the multiplier equal to 1.806. In these conditions, even though the network is rural, the voltages are within acceptable ranges at all times (Fig. 3).

The evolution in time of the average active and reactive power taken from the HV grid in Case 0 is shown in Fig. 4. The active power has a valley at mid-day due to the PV generation, but never sends power back to the HV grid. Fig. 5 tracks the minimum losses allocated by the BCDLA method, also showing the corresponding node. It can be seen that for most quarters of hour in the central part of the day the nodes (encircled in Fig. 1) belong to Feeder F3, and change in time.

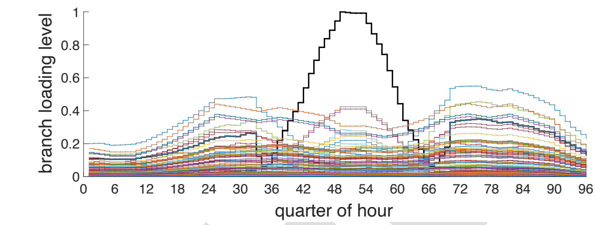


Fig. 2. Case 1 – Branch loading levels (thick line for branch 34).

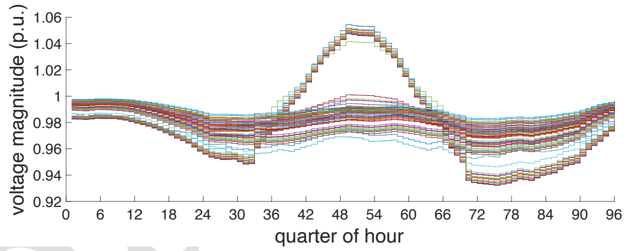


Fig. 3. Case 1 – Voltage magnitudes at the network nodes.

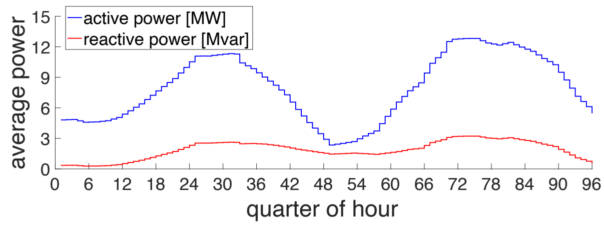


Fig. 4. Case 0 – Power taken from the grid.

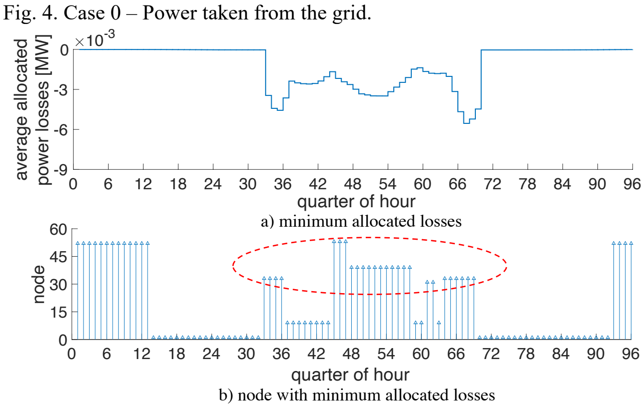


Fig. 5. Case 0 – Evolution of the minimum allocated losses (amount and location).

In the high-DG solution (Case 1) the evolution in time of the average active and reactive power taken from the HV grid in Case 1 is shown in Fig. 6. The reactive power taken from the grid has a very low change with respect to Case 0, as the PV model assumes null reactive power. Reverse power flow conditions (i.e., with active power sent back to the HV system) occur in the central part of the day. The minimum losses allocated by the BCDLA method are found again mainly in some nodes belonging to Feeder F3 (Fig. 7).

The total network losses at each quarter of hour (also equal to the total allocated losses by the BCDLA algorithm) are shown in Fig. 8. It can be seen that the high-DG Case 1 leads to a loss reduction with respect to Case 0 only for two distinct time periods, while in the central part of the day the total losses are much higher in Case 1. Therefore, the interpretation of the DG impact on the total losses is not trivial. The results of the comparison between these two cases at selected quarters of hour are shown below to discuss a number of specific findings.

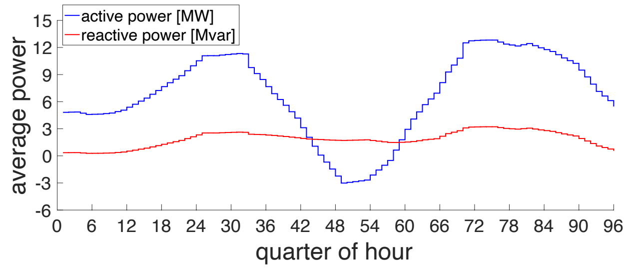
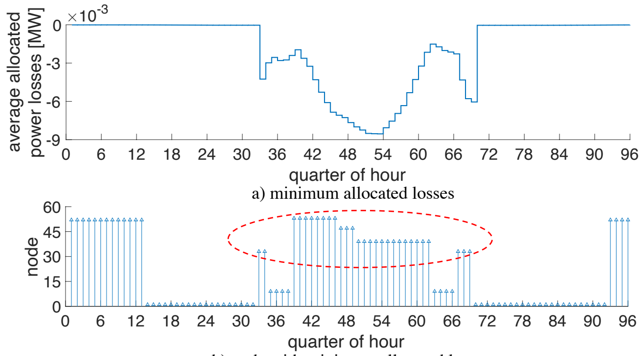


Fig. 6. Case 1 - Power taken from the grid.



a) minimum allocated losses

b) node with minimum allocated losses

Fig. 7. Case 1 – Evolution of the minimum allocated losses (amount and location).

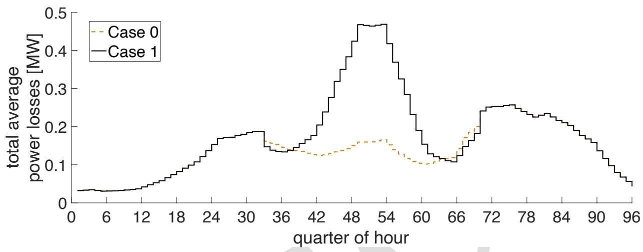


Fig. 8. Evolution of the total losses for Case 0 and Case 1.

The cases studied are based on tracking the allocated losses during the successive time steps, to identify their evolution in time and across the nodes. On the basis of the previous results, the attention is focused on the central part of the day and on Feeder F3.

A. Positive and negative allocated losses

The ranges of variation of the allocated losses in the nodes of Feeder F3 are shown in Fig. 9 for Case 0 and Fig. 10 for Case 1. While the variation of the allocated losses in the three nodes containing the PV generation (nodes 34, 42 and 49) is higher than for the other nodes, it is interesting to notice that negative allocated losses appear also at some quarters of hour for all the other nodes located in Feeder F3.

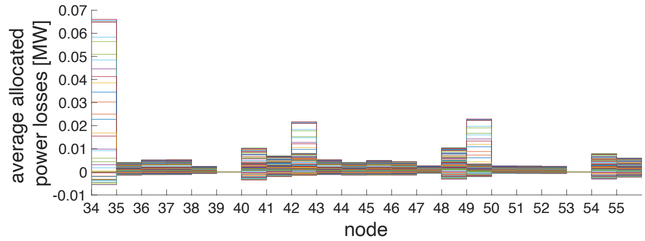


Fig. 9. Case 0 – Allocated losses in the nodes of Feeder F3. Losses shown for all quarters of hour at the right hand-side of the node label.

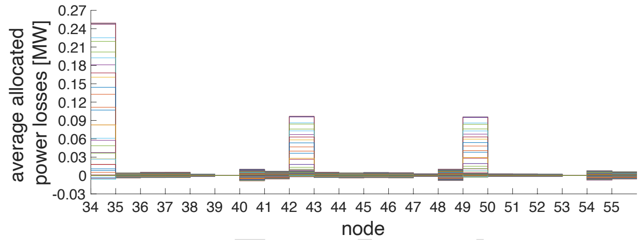


Fig. 10. Case 1 – Allocated losses in the nodes of Feeder F3. Losses shown for all quarters of hour at the right hand-side of the node label.

The presence of negative losses allocated at nodes that contain no local generation could seem counterintuitive. In order to better understand the situation, let us focus in more details on a subset of nodes (40, 42, 53, 54, and 55), and on the corresponding branches. The branches are the ones connected to node 55 and located downward branch 42 (in node 42 there is a local PV generation). Fig. 11 and Fig. 12 show the evolution in time of the average allocated power losses in the subset of nodes, for case 0 and Case 1, respectively. The allocated losses exhibit a quite various evolution, making it possible to analyze different situations.

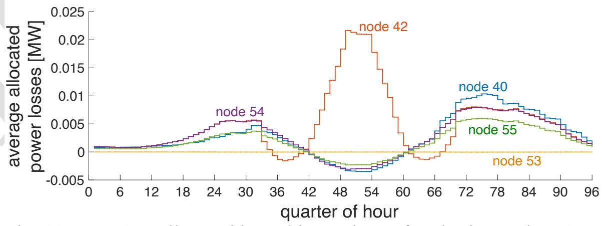


Fig. 11. Case 0 – Allocated lossed in a subset of nodes in Feeder F3.

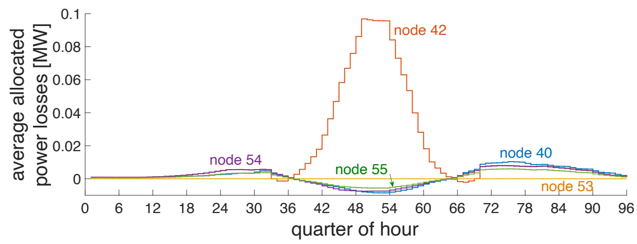


Fig. 12. Case 1 – Allocated lossed in a subset of nodes in Feeder F3.

In particular, during the night branch 42 supplies only the loads at node 40 and node 42 (the PV generation is null). Then, the growing PV generation initially reduces the current magnitude in branch 42, then reverts (at quarter of hour 39) the current flowing in branch 42, up to conditions in the mid part of the day in which the branch current reaches high magnitudes. Fig. 13 and Fig. 14 show the branch currents for Case 0 and Case 1, respectively.

When dealing with the currents, the results depend on the current phasors, not only on the current magnitude. For example, the node current balance for Case 1, node 55, quarter of hour 48 (noon) is expressed by the equation $\bar{I}_{B55} = \bar{I}_{B42} + \bar{I}_{B53} + \bar{I}_{B54} - \bar{I}_{C55}$, in which the terms are

$\bar{I}_{B42}$ = 1.2355+j0.1371 p.u., $\bar{I}_{B53}$ = −0.7759+j0.1127 p.u., $\bar{I}_{B54}$ = −0.1286+j0.0228 p.u., $-\bar{I}_{S55}$ = −0.0914+j0.0129 p.u., and $\bar{I}_{B55}$ = 0.2396 + j0.2855 p.u. (Fig. 15, where, according with the initial hypotheses, the positive load currents are injected in the nodes, and the positive branch currents flow from the nodes to the root). The PV generation at node 42 affects the current flowing in branch 55, provided by load+PV with a strong PV contribution.

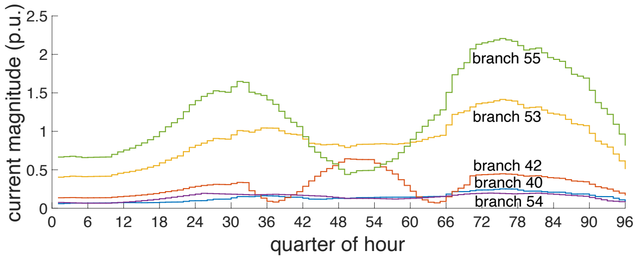


Fig. 13. Case 0 – Branch currents in a subset of branches in Feeder F3.

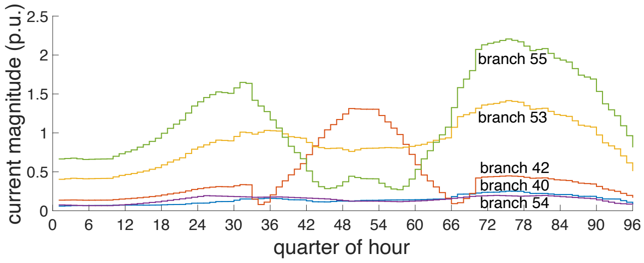


Fig. 14. Case 1 – Branch currents in a subset of branches in Feeder F3.

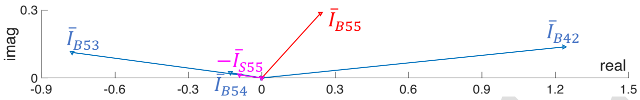


Fig. 15. Case 1 – Node current balance (node 55, quarter of hour 48).

Three particular situations are considered:

a) A situation with no PV contribution (in the example considered): quarter of hour 24, in which the solutions are the same for Case 0 and Case 1.

b) A situation in which the average allocated power losses are negative at node 42 and positive at the other nodes: quarter of hours 38 for Case 0 and 34 for Case 1.

c) A situation in which the average allocated power losses are positive at node 42 and negative at the other nodes: quarter of hour 48 for Case 0 and Case 1.

Table I reports the active power load in the situations considered. Table II shows the average power losses allocated to the selected nodes, as well as the total allocated losses (equal to the total network losses). The very small negative values that appear for node 53 (with null load) are commented in Section IV.B.

TABLE I. ACTIVE POWER LOAD [KW]

	Case 0			Case 1		
	quarter of hour *t*			quarter of hour *t*		
node *k*	24	38	48	24	34	48
40	91.51	144.34	126.96	91.51	146.65	126.96
42	174.21	**-210.26**	**-731.04**	174.21	**-123.47**	**-1430.80**
53	0	0	0	0	0	0
54	171.10	168.94	132.92	171.10	164.43	132.92
55	105.93	116.62	94.74	105.93	114.88	94.74

TABLE II. AVERAGE POWER LOSSES ALLOCATED TO NODE *k* AT QUARTER OF HOUR *t* [KW]

	Case 0			Case 1		
node *k*	quarter of hour *t*			quarter of hour *t*		
	24	38	48	24	34	48
40	2.628	1.404	-2.983	2.628	2.213	-7.483
42	4.977	**-1.363**	**18.364**	4.977	**-1.352**	**86.198**
53	-0.017	-0.021	-0.017	-0.017	-0.021	-0.016
54	4.983	1.813	-2.882	4.983	2.608	-7.442
55	2.939	1.097	-2.147	2.939	1.672	-5.388
total allocated losses	152.495	137.096	150.092	152.495	138.299	424.183

To establish a relation between the allocated losses and the variation of the total network losses, a net power load reduction is applied to each selected node and quarter of hour. On the basis of the active power load indicated in Table I, the net power load reduction is 10 kW, not too small to get an impact on the total network losses, and not too large with respect to the node power load. The reduction is applied node by node by starting each time from the solutions of Case 0 or Case 1. Table III contains the reduction of the total network losses obtained (negative when the losses increased). It can be clearly seen that:

- The amounts of loss reduction at the same quarter of hour are relatively similar at the different nodes, regardless of the fact that there is a positive or negative net load in the node.
- In both situations b) and c) node 42 has a negative net power load, but Table III shows a total loss reduction in situation b) and a loss increase in situation c). The *sign* of the total loss reduction is *opposite* with respect to the sign of the losses allocated to node 42.

TABLE III. TOTAL LOSS REDUCTION AFTER A 10 KW REDUCTION OF THE ACTIVE POWER LOAD TO NODE *k* AT QUARTER OF HOUR *t* [KW]

	Case 0			Case 1		
	quarter of hour *t*			quarter of hour *t*		
node *k*	24	38	48	24	34	48
40	0.563	0.168	-0.481	0.563	0.286	-1.143
42	0.559	**0.161**	**-0.486**	0.559	**0.280**	**-1.147**
53	0.580	0.205	-0.432	0.580	0.320	-1.076
54	0.569	0.186	-0.447	0.569	0.300	-1.089
55	0.546	0.165	-0.461	0.546	0.279	-1.101

IV. DISCUSSION ON THE LOSSES ALLOCATED TO THE DISTRIBUTION NETWORK NODES

A. *The incontrovertible importance of the net load power*

It has to be stressed that the BCDLA algorithm allocates the losses to the nodes with respect to the *net* load power, without providing an individual loss allocation to the power generation $P_{k,t}^{(g)}$ and the power load $P_{k,t}^{(d)}$ if both are connected to the same node. Any attempt to allocate the losses to the generation(s) and load(s) connected to the same node is totally arbitrary and not justified. Starting from this concept, practical sensitivity calculations can be carried out by changing the net power in both directions, namely, a net load power increase can be obtained by either increasing $P_{k,t}^{(d)}$ with constant $P_{k,t}^{(g)}$, or by decreasing $P_{k,t}^{(g)}$ with constant $P_{k,t}^{(d)}$. If the average power variation is the same, the results of the practical sensitivity calculations do not change.

B. *Interpretation of the positive/negative allocated losses*

From the results shown in Tables I to III, some key results can be highlighted:

- The losses allocated to the nodes embed a *zonal* meaning rather than a nodal meaning. This is evident by merging the results showing that (i) negative allocated losses can be found also in nodes with no local generation, (ii) similar values of the total loss reduction found in neighboring nodes, and (iii) negative allocated losses appear in the nodes belonging to zones with excess of local generation (and reverse current flows).

In order to understand the possibility of obtaining reductions in the total losses, it is important to consider the sign of the losses allocated to the nodes together with the sign of the active power load. This is important to get an indication on the possible reduction of the losses in cases of reverse power flows, in which the total loss reduction can be obtained by reducing the local generation. The complete picture of this aspect does no depend on the active power only, as what matters is the location of the current phasors. However, an indicative rule to assess the possible total loss reduction on the basis of the active power load $P_{k,t}$ and the allocated losses $L_{k,t}$ at node k and time step t is:

⇒ If the product $\text{sign}\{P_{k,t}\}\cdot\text{sign}\{L_{k,t}\}$ is positive, there is a total loss reduction by reducing the net load power at node k.

⇒ If the product $\text{sign}\{P_{k,t}\}\cdot\text{sign}\{L_{k,t}\}$ is negative, there is a total loss increase by reducing the net load power at node k.

However, this rule is not general, because it does not account for cases with null net power in a node, for which the $\text{sign}\{P_{k,t}\}$ is undefined. This is for example the case of node 53, for which (from Table II) the allocated losses are not zero, but are slightly negative in both Case 0 and Case 1 at any time step. This condition becomes undetermined by looking only at the single node, as can be seen also from Table III, in which the effect of a net power reduction is different in the three situations (the total losses are reduced in the first two situations, but are increased in the third situation), and depends on the behaviour of the network in the neighbouring zone. The results are confirmed by applying a net load increase at node 53, with opposite outcomes for the total loss reductions (Table IV).

TABLE IV. TOTAL LOSS REDUCTION AFTER A 10 KW INCREASE OF THE ACTIVE POWER LOAD TO NODE k AT QUARTER OF HOUR t [KW]

	Case 0			Case 1		
	quarter of hour t			quarter of hour t		
node k	24	38	48	24	34	48
53	-0.584	-0.210	0.429	-0.584	-0.306	1.072

C. *Variations of the allocated losses*

The allocated losses change during time. Interesting insights are found from looking at the evolution in time of the losses allocated to the same node. This evolution is somehow particular. First of all, the numerical amount of the losses allocated to the same node during time cannot be compared, because at each time step the total allocated losses are equal to the total network losses, and the total network losses change in time. As such, a comparison among the numerical amounts of the allocated losses at the same node makes sense at a given time step in the hypothesis that the net power load varies. In this case, it may happen that the allocated losses pass from positive to negative values in a node because of the changes in load and local generation power occurring in other nodes. On the other side, each time the net power changes in the same node the allocated losses change as well, as they are calculated each time starting from the power flow solution.

Two types of results are presented here, with reference to Case 1 at quarter of hour 48. The net power is changed at node 53, which has a null net power in the initial conditions. The results are given considering:

1. Variation of the losses allocated to node 53 due to different variations of the net power in the same node.
2. Variation of the allocated losses at node 34 (the first node belonging to Feeder F3, Fig. 1) and node 42, due to the variation of the net power in another node (node 53).

In these cases, both the losses allocated and the total losses change in a nearly linear way with respect to the variation of the net load power a node 53 on a relatively significant range (Fig. 16).

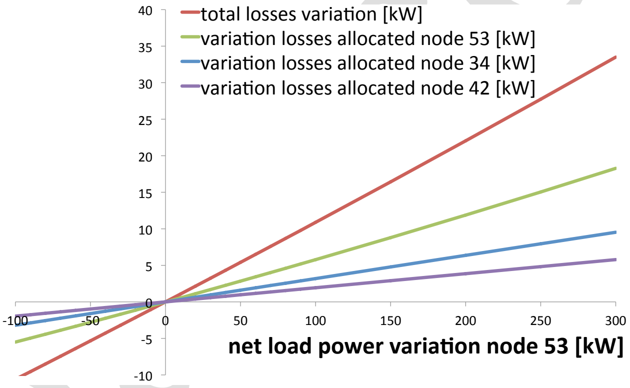


Fig. 16. Case 1 – Net load power variation at node 53 (quarter of hour 48).

D. *Generation of benchmark cases with high penetration of distributed energy resources*

The information on the losses allocated to the nodes can be used with the objective to construct specific cases with high penetration of DER [7]. In this case, on the basis of the results discussed in Section IV.B, the nodes with positive product $\text{sign}\{P_{k,t}\}\cdot\text{sign}\{L_{k,t}\}$ and the highest allocated losses are the most suitable candidates to host additional generation. In this way, it is possible to identify viable scenarios with high DER diffusion. Starting from Case 0, the active power load at node 42 and quarter of hour 48 is changed in such a way to reach null net active power (the reactive power is maintained). The total losses change from 150.1 kW to 123.9 kW, and the average power losses allocated to the selected nodes become -1.337 kW at node 40, -0.007 kW at node 42, -0.015 kW at node 53, -1.356 kW at node 54, and -1.060 kW at node 55, with the expected strong reduction at node 42 and a reduction also at the other nodes, mainly due to lower total network losses.

Conversely, it is also possible to create worst cases in which the uncontrolled DER diffusion could lead to higher network losses. A dedicated procedure has been outlined in [7], considering that the worst cases are created only for simulation purposes, without limiting the analysis to the present DER locations. In this case, the criterion to choose the DER to be added is now updated by taking into account the findings discussed in Section IV.B: the nodes with negative product $\text{sign}\{P_{k,t}\}\cdot\text{sign}\{L_{k,t}\}$ and the maximum (positive) allocated losses are the most suitable candidates to host additional generation. In this way, it is possible to construct critical scenarios with high DER diffusion, taking into account that no network constraints is violated. Further actions can be incorporated in the analysis, to exploit further capabilities of voltage control [9] or energy storage [10][11]. From the applications shown in this paper starting from Case 0, Case 1 is already an example in which the increase in the PV generation has increased considerably the total losses in the central part of the day.

E. Study of reverse power flows to the transmission network

The scenarios of high DER penetration can also be adapted to address how to reduce the reverse power flow occurring when the active power (and the reactive power, in case) are injected in the transmission system. With respect to the results shown above, Case 1 has a consistent reverse power flow in the central part of the day, due to the strong PV production. Changing the net load power at the nodes where the PV systems are installed can reduce the reverse power flow. Under the hypothesis of changing the net load power by curtailing the PV generation at some nodes, the allocated losses can be used to select the zones in which the reduction in the PV generation can be more effective.

In Case 1, the minimum average active power supplied by the transmission system is -3013.8 kW, occurring at quarter of hour 49. The negative sign indicates that there is a reverse power flow from the distribution network to the slack node. From the power flow solution of Case 1, the losses allocated to the five nodes that contain PV generation (that is, nodes 10, 32, 34, 42 and 49) are indicated in the second column of Table V, while the net load power in the five nodes is reported in column 3 (being negative, it is a generation). If the net load power is increased (by curtailing PV generation) at a single node, only at node 34 it is possible to obtain null reverse power flow before curtailing all the PV generation (with a curtailment of 3259 kW). To check the effect of curtaining the same PV generation from the five nodes, a 1000 kW curtailment is applied to each node separately. The total losses change from 467.41 kW (Case 1) to the values indicated in Table V. There is a more remarkable reduction in the total losses when the curtailment is applied to the PV generators located in the same zone (nodes 34, 42, and 49), while when the PV curtailment occurs at nodes 10 and 32 the reverse power flow at the slack node becomes lower (at the expense of higher total losses, because the other PV generators serve the loads in the network to a larger extent than before).

TABLE V. POWER AND LOSSES (KW) AFTER A PV GENERATION CURTAILMENT TO ELIMINATE THE REVERSE POWER FLOW TO THE SLACK NODE AT QUARTER OF HOUR 49, STARTING FROM CASE 1.

	Case 1		After 1000 kW of PV curtailment		
node *k*	Losses allocated	Net load power	Power from slack node	Total losses	Total loss reduction
10	17.494	-1587.8	-2024.2	457.01	10.40
32	22.331	-2779.4	-2025.4	455.81	11.60
34	249.15	-4911.6	-2104.1	377.12	90.29
42	96.861	-1509.7	-2121.4	359.86	107.55
49	95.855	-1575.9	-2118.7	362.58	104.83

V. CONCLUSION

This paper has presented a number of detailed aspects referring to the loss allocation in radial distribution systems. The simple and efficient BCDLA method has been used, as the total allocated losses are by definition equal to the total network losses, and its implementation after the power flow calculation is straightforward. The study has highlighted that, even for radial distribution systems, the loss allocation results need to be interpreted to fully understand the strength of the information provided by the allocated losses in the presence of DG and more generally of DER. The losses are allocated to the net node power, and not to the individual loads and DERs. This encompasses the presence of all types of DERs, including storage in either charging or discharging modes.

An extensive interpretation of the negative allocated losses has been provided, highlighting some situations in which the information provided could seem counterintuitive. The zonal nature of the allocated losses has been highlighted, also providing an indicative rule based on the signs of the allocated losses and of the active power load in a node, to understand the possible total loss reduction when the net load power at that node is changed.

The information provided by the allocated losses is effective to understand whether it would be better to increase or decrease the net load power at any node and time step. This information is important for operational purposes, in the light of providing specific compensations, for example by exploiting local energy storage systems.

Concerning distribution system planning, the use of the allocated losses can be effective if consistent information is provided for certain nodes of the network in many periods of time. In this case, there is room to install new assets. Otherwise, if there are variable results, it is not easy to provide an adequate justification to the addition of new assets. More generally, the allocated losses can be used to create meaningful scenarios of DER evolution, some of which (more critical than others) can be created as intentional worst cases by installing DERs in the direction of total loss increase rather than total loss reduction.

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