Confinement and Mott Transitions of Dynamical Charges in One-Dimensional Lattice Gauge Theories

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Confinement is an ubiquitous phenomenon when matter couples to gauge fields, which manifests itself in a linear string potential between two static charges. Although gauge fields can be integrated out in one dimension, they can mediate nonlocal interactions which in turn influence the paradigmatic Luttinger liquid properties. However, when the charges become dynamical and their densities finite, understanding confinement becomes challenging. Here we show that confinement in 1D $\mathbb{Z}_2$ lattice gauge theories, with dynamical matter fields and arbitrary densities, is related to translational symmetry breaking in a nonlocal basis. The exact transformation to this string-length basis leads us to an exact mapping of Luttinger parameters reminiscent of a Luther-Emery rescaling. We include the effects of local, but beyond contact, interactions between the matter particles, and show that confined mesons can form a Mott-insulating state when the deconfined charges cannot. While the transition to the Mott state cannot be detected in the Green's function of the charges, we show that the metallic state is characterized by hidden off-diagonal quasi-long-range order. Our predictions provide new insights to the physics of confinement of dynamical charges, and can be experimentally addressed in Rydberg-dressed quantum gases in optical lattices.

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Introduction.—Lattice gauge theories (LGTs), originally introduced to get insights about nonperturbative regimes in particle physics [1,2], have become a powerful tool to tackle many-body problems in condensed matter systems [3–5]. These theories turn out to be particularly rich and interesting when the matter is coupled to dynamical gauge fields: For example, in some cases the confinement-deconfinement transition [1] can be associated with the appearance of topological phases with non-Abelian anyons and charge fractionalization [6]. On the other hand, when the matter acquires its own quantum dynamics the confinement problem is poorly understood and, in this regime, a general physical description of the phenomenon is still lacking. Furthermore, the high level of complexity of LGTs makes theoretical studies based on standard numerical methods [7–10] very challenging.

At the same time, due to their impressive level of control and accuracy, ultracold atomic systems are establishing themselves as a fundamental platform where LGT models can be systematically studied [11–20]. In this context LGTs with an Ising gauge group, i.e., $\mathbb{Z}_2$ LGTs [21–23], are particularly meaningful to explore, allowing, for instance, to study their connections to strongly correlated electronic systems [24–28] including high-T_c superconductivity [29,30]. Recent theoretical studies of two-dimensional $\mathbb{Z}_2$ LGTs with matter-gauge coupling have revealed a wealth of intriguing properties [31–34]. Experimentally, a first instance of a $\mathbb{Z}_2$ LGT with dynamical matter has recently been realized in a mixture of ultracold bosons in a

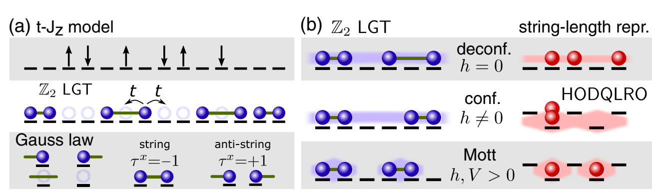

FIG. 1. (a) The 1D $t-J_z$ model (top) maps exactly to 1D $\mathbb{Z}_2$ LGTs (middle); blue full spheres correspond to hard-core bosons or fermions and the empty blue circles denote holes. Pairs of particles are connected with green lines, which correspond to $\mathbb{Z}_2$ electric strings, according to the configurations allowed by the Gauss law (bottom). (b) Comparison between the deconfined, confined, and Mott states, both in the original and in the corresponding string-length representation. As indicated, the confining phases are characterized by a broken translational symmetry in the string-length basis.

double well potential [17] by means of a Floquet scheme [22]. Using an extension of this Floquet scheme [18,22], or coupling superconducting qubits [21,23], allows us to study $\mathbb{Z}_2$ LGTs with dynamical matter in extended geometries and higher dimensions, thus paving the way towards a deeper understanding of such models. Moreover, as it will be discussed below, in one dimension the direct implementation of Hamiltonians with encoded gauge degrees of freedom [35,36] can also be employed to explore $\mathbb{Z}_2$ LGTs, see Fig. 1(a).

In this Letter, we solve the confinement problem in a class of 1D $\mathbb{Z}_2$ LGTs with dynamical charges [37] at arbitrary densities. This is achieved by representing the $\mathbb{Z}_2$ LGT model in the nonlocal basis of string lengths, where we prove that confinement is equivalent to a broken translational symmetry. Our argument applies for a larger class of 1D LGTs. We also study the Mott transition of $\mathbb{Z}_2$ charges, which defies conventional wisdom for at least two reasons.

First we show that an exponentially decaying $\mathbb{Z}_2$ invariant Green's function no longer provides a unique signature of the Mott state. Instead, we show that the confined Luttinger liquid [37] is characterized by hidden off-diagonal quasi-long-range order (HODQLRO) in the string-length basis. This quasicondensate of string excitations is destroyed at the Mott transition, see Fig. 1(b). More formally, we derive a Luther-Emery-like relation between the Luttinger parameters in the original $\mathbb{Z}_2$ LGT and the effective model in the string-length basis.

Second we show that the Mott insulator occurring at the specific filling $n^a = 2/3$ is stabilized by a combination of the attractive confining potential and a nearest-neighbor (NN) repulsion. If either of those terms is absent, a gapless liquid is obtained; on the other hand, when both are sizable our numerical simulations, based on density-matrix-renormalization-group (DMRG) algorithm [38–41], yield a significant charge gap. Our predictions can be tested in Rydberg-dressed atomic gases in optical lattices, where site-resolved quantum projective measurements provide direct access to the nonlocal string-length basis.

Model.—We consider a 1D $\mathbb{Z}_2$ LGT Hamiltonian where N hard-core bosons in a lattice with L sites are coupled to $\mathbb{Z}_2$ gauge fields,

$$\hat{\mathcal{H}} = -t\sum_{\langle i,j\rangle}(\hat{a}_i^\dagger \hat{\tau}^z_{\langle i,j\rangle}\hat{a}_j + \mathrm{H.c.}) - h\sum_{\langle i,j\rangle}\hat{\tau}^x_{\langle i,j\rangle} + V\sum_{\langle i,j\rangle}\hat{n}_i\hat{n}_j. \quad (1)$$

Here $\hat{a}_i^\dagger$ denotes the hard-core bosonic creation operator, t describes NN tunneling processes mediated by the $\mathbb{Z}_2$ gauge field $\hat{\tau}^z_{\langle i,j\rangle}$ defined on the links $\langle i,j\rangle$ between NN sites, and $V > 0$ represents a NN repulsion between bosons. The gauge-invariant $\mathbb{Z}_2$ electric-field term $\hat{\tau}^x_{\langle i,j\rangle}$ with strength h introduces quantum fluctuations of $\hat{\tau}^z_{\langle i,j\rangle}$. The physics remains unchanged if $\hat{a}$'s are replaced by fermionic operators $\hat{c}$, as can be shown by a Jordan-Wigner transformation.

The $\mathbb{Z}_2$ electric field is subject to a Gauss law that ensures that the former changes sign across a particle [37]; i.e., pairs of particles are connected by $\mathbb{Z}_2$ electric fields of the same sign, which we denote as $\mathbb{Z}_2$ electric strings and antistrings, see Fig. 1(a). For concreteness, we assume open boundary conditions with $\tau^x_{\langle 0,1\rangle} = 1$ (no $\mathbb{Z}_2$ electric string entering from the left). The corresponding $\mathbb{Z}_2$ gauge group is defined by the operator

$$\hat{G}_i = \hat{\tau}^x_{\langle i-1,i\rangle}\hat{\tau}^x_{\langle i,i+1\rangle}(-1)^{\hat{n}_i}, \quad (2)$$

which commutes with the Hamiltonian $[\hat{\mathcal{H}}, \hat{G}_i] = 0$ and itself $[\hat{G}_i, \hat{G}_j] = 0$. As a consequence, the effective Hilbert space of Eq. (1) is split into different sectors $\hat{G}_i = \pm 1$ [37,42]. The Gauss law we choose corresponds to the sector where $\hat{G}_i = 1,\ \forall\ i$; see Fig. 1.

Implementation.—In order to implement the LGT Hamiltonian Eq. (1), we propose a Rydberg dressing scheme in a spin-dependent superlattice potential with period $2a$, a being the lattice spacing. We require the following potential,

$$V_{\sigma,j} = (-1)^\sigma \frac{\omega_0}{2} - (-1)^\sigma \frac{\delta}{2}[1 - (-1)^j], \quad (3)$$

where the first term describes the splitting ω_0 between the two spin states. The second term realizes a staggered magnetic Zeeman field and can be realized by an antimagic superlattice, e.g., using ytterbium atoms [43–45]. We propose to realize the required NN Ising interactions by dressing the spin states independently by two Rydberg dressing lasers $\Omega_\uparrow$ and $\Omega_\downarrow$—see Ref. [46] for details.

This scheme gives an effective $t - J_z$ Hamiltonian [55,56], which is best written in a rotating frame [46],

$$\hat{\mathcal{H}}_{t-J_z} = \hat{\mathcal{H}}_t + \sum_{\langle i,j\rangle}[J_{\uparrow\uparrow}\hat{n}_i^\uparrow\hat{n}_j^\uparrow + J_{\uparrow\downarrow}(\hat{n}_i^\uparrow\hat{n}_j^\downarrow + \hat{n}_i^\downarrow\hat{n}_j^\uparrow) + J_{\downarrow\downarrow}\hat{n}_i^\downarrow\hat{n}_j^\downarrow] + \delta\sum_j(-1)^j\hat{S}_j^z, \quad (4)$$

where $\hat{\mathcal{H}}_t = -t\sum_{\langle i,j\rangle}\sum_\sigma \hat{P}(\hat{a}_{i,\sigma}^\dagger\hat{a}_{j,\sigma} + \mathrm{H.c.})\hat{P}$ is the hopping term, projected into the subspace without double occupancies. This model maps to the LGT model Eq. (1) by introducing a constraint on our Hilbert space where opposite spins appear in alternating fashion, leading to the $\mathbb{Z}_2$ Gauss law $\hat{G}_i = +1$. In addition, the parameters in the LGT model (1) are directly related to the parameters in the effective $t - J_z$ Hamiltonian (4) since $h = 2\delta$ and $V = J_{\uparrow\downarrow}$ (for details see Ref. [46]).

Confinement in $\mathbb{Z}_2$ LGTs.—In order to observe the confined and deconfined phases the $\mathbb{Z}_2$ gauge-invariant equal-time Green's function is considered,

$$g^{(1)}(i-j) = \left\langle \hat{a}_i^\dagger \prod_{i\leq \ell \leq j} \hat{\tau}_\ell^z \hat{a}_j \right\rangle. \quad (5)$$

An algebraic decay of the correlator Eq. (5) signals a deconfined phase where the charges can move around freely. An exponential decay, on the other hand, signals a confined phase where the particles are bound in pairs [37].

The Gauss law, $\hat{\tau}^x_{\langle i-1,i\rangle} = \hat{\tau}^x_{\langle i,i+1\rangle}(-1)^{\hat{n}_i}$, can be successively applied to express the $\mathbb{Z}_2$ electric field as [37]

$$\hat{\tau}^x_{\langle i,i+1\rangle} = \cos\left(\pi \sum_{j<i} \hat{n}_j\right), \quad (6)$$

which leads to a nonlocal term $-h\sum_i \cos(\pi \sum_{j<i} \hat{n}_j)$ in the Hamiltonian. By rewriting the density as $\hat{n}_j \to n(x) = n^a - \partial_x \phi(x)/\pi$, where n^a is the average density, the $\mathbb{Z}_2$ electric field term after bosonization becomes

$$-h\int \mathrm{d}x \cos[\pi n^a x - \phi(x)]. \quad (7)$$

Such oscillatory integrals should vanish, which means that the term is RG irrelevant [37]. By this appealing but naive argument, the field term would be negligible and the system should behave like free fermions [42]. Hence, from this standard bosonization argument, one expects the correlator Eq. (5) to have an algebraic decay $g^{(1)}(d) \simeq |d|^{-\alpha}$ for nonzero values of $h \neq 0$ [57]. However, as shown in Ref. [37] the decay is exponential—and the charges confined—for any $h \neq 0$. We attribute this failure of naive bosonization arguments to the nonlocal nature of the field term, Eq. (6), emphasized above.

Next we provide a general argument under which conditions the model in Eq. (1) is confining. To this end, we introduce a new nonlocal basis in which the Hamiltonian becomes local, meaning that conventional bosonization arguments can be safely applied.

String-length representation.—So far we represented basis states in our model by hard-core boson occupation numbers $n_j = 0, 1$, and the $\mathbb{Z}_2$ electric strings $\tau^x_{\langle i,j\rangle} = \pm 1$; as shown in Eq. (6) the latter can be expressed by the former. Now we introduce new bosonic occupation numbers $\ell_1, \ldots, \ell_{N+1} \geq 0$ to label our basis states, where $N = \sum_j n_j$ is the total conserved boson number.

If $x_1, \ldots, x_N$ denote the positions ($x_j = 1, \ldots, L$) of hard-core bosons, we define

$$\ell_1 = x_1 - 1, \qquad \ell_n = x_n - x_{n-1} - 1, \qquad \ell_{N+1} = L - x_N. \quad (8)$$

This allows us to identify the corresponding Fock configuration $|n_1, \ldots, n_L\rangle$ with a bosonic Fock configuration:

$$|n_1, \ldots, n_L\rangle = |\ell_1, \ldots, \ell_{N+1}\rangle \equiv \prod_{n=1}^{N+1} \frac{(\hat{\Psi}_n^\dagger)^{\ell_n}}{\sqrt{\ell_n!}} |0\rangle. \quad (9)$$

In the last step we introduced bosonic operators $\hat{\Psi}_n^\dagger$ acting on the string-length vacuum $|0\rangle$.

Physically, the integers $\ell_n \in \mathbb{Z}_{\geq 0}$ describe the length of the $\mathbb{Z}_2$ (anti-) strings connecting pairs of consecutive $\mathbb{Z}_2$ charges, up to a shift of one: the shortest possible string connecting charges on NN sites is counted as having no excitation, $\ell = 0$. The total number of string excitations, $\tilde{N} \equiv \sum_{\ell=1}^{N+1} \ell_n = L - N$, is conserved.

In the new string-length basis, we can express the $\mathbb{Z}_2$ LGT Hamiltonian as

$$\hat{\mathcal{H}} = -t\sum_{\langle m,n\rangle} (\hat{\rho}_m^{-1/2} \hat{\Psi}_m^\dagger \hat{\Psi}_n \hat{\rho}_n^{-1/2} + \mathrm{H.c.}) - h\sum_n (-1)^n \hat{\rho}_n + V\sum_n \delta_{\hat{\rho}_n,0}. \quad (10)$$

Here $\delta_{a,b}$ denotes the Kronecker delta and $\hat{\rho}_n = \hat{\Psi}_n^\dagger \hat{\Psi}_n$ is the string-length density operator. The transformed Hamiltonian (10) is purely local. It is defined on a lattice of size $\tilde{L} = N + 1$ with $\tilde{N}$ excitations; i.e., the average boson density in this model is given by

$$\rho^\Psi = \frac{\tilde{N}}{\tilde{L}} = \frac{L - N}{N + 1} = \frac{1}{n^a} - 1 + \mathcal{O}(1/N). \quad (11)$$

It is worthwhile to underline that the amplitude of the hopping in this new basis does not carry the usual Bose-enhancement factors, however, it requires extra factors $\hat{\rho}_n^{-1/2}$ in the Hamiltonian. Since the latter only show up in combination with $\hat{\Psi}_n$, the expression vanishes and remains well defined when the bosonic occupation numbers become zero.

Field theory analysis.—Now we analyze the model (10) from a field-theoretic perspective. By construction these models are connected by a unitary transformation (the nonlocal basis change), ensuring their spectra to coincide. At long wavelengths, distances are related as follows: x in the $\mathbb{Z}_2$ LGT corresponds to a "distance" (particle number) in the string-length basis $\tilde{x} = n^a x$. As a result we can directly relate coarse-grained densities in the two models.

This allows us to directly relate their Luttinger parameters $\tilde{K}$ and K, which can be defined via the compressibility [57]. An explicit calculation [46] yields

$$K = (n^a)^2 \tilde{K}, \quad (12)$$

reminiscent of the Luther-Emery rescaling solution [57,58], except for a factor of 2.

Alternatively, we can relate density-density correlations at long distances in the two models: We start from $\langle \delta\hat{n}(x)\delta\hat{n}(0)\rangle$, where $\delta\hat{n}(x) = \hat{n}(x) - n^a$ denotes local density fluctuations. At long wavelengths, the density of hard-core bosons is $\hat{n}(x) \approx \Delta\hat{N}_a/\Delta x$, when $\Delta\hat{N}_a$ particles are found per coarse-grained distance Δx. In the string-length basis, $\hat{\rho}(\tilde{x})$ describes the distance $\Delta\hat{x}$ between two hard-core bosons, minus one unit per particle [Eq. (8)], per coarse-grained number of particles $\Delta\tilde{x}$; i.e., $\hat{\rho}(\tilde{x}) \approx (\Delta\hat{x} - \Delta\tilde{x})/\Delta\tilde{x}$. This leads to

$$\hat{n}(x) \approx \{1 + \hat{\rho}[\tilde{x}(x)]\}^{-1}, \tag{13}$$

which allows us to calculate density fluctuations at long distances, $\delta\hat{n}(x) = -(n^a)^2\delta\hat{\rho}(\tilde{x}) + \mathcal{O}(\delta\hat{\rho}^2)$. Hence both models share the same long-wavelength correlations:

$$\langle\delta\hat{n}(x)\delta\hat{n}(0)\rangle \simeq (n^a)^4\langle\delta\hat{\rho}(n^a x)\delta\hat{\rho}(0)\rangle. \tag{14}$$

For the local Hamiltonian (10) we can safely apply Luttinger-liquid theory, which yields [57]

$$\langle\delta\hat{\rho}(\tilde{x})\delta\hat{\rho}(0)\rangle \simeq \frac{\tilde{K}}{2\pi^2}\frac{1}{\tilde{x}^2} + \frac{(\rho^\Psi)^2}{2}\left(\frac{\tilde{\alpha}}{\tilde{x}}\right)^{2\tilde{K}}\cos(2\pi\rho^\Psi\tilde{x}) + \cdots, \tag{15}$$

where $\tilde{\alpha}$ is a nonuniversal short-distance cutoff. From Eq. (14) we thus predict in the original model:

$$\begin{aligned}\langle\delta\hat{n}(x)\delta\hat{n}(0)\rangle &\simeq \frac{\tilde{K}(n^a)^2}{2\pi^2 x^2} + \cdots \\ &= \frac{K}{2\pi^2 x^2} + \frac{(n^a)^2}{2}\left(\frac{\alpha}{x}\right)^{2K}\cos(2\pi n^a x) + \cdots\end{aligned} \tag{16}$$

This result confirms the relation between Luttinger parameters stated earlier; see Eq. (12).

Note however that the relation (14) does *not* correctly predict the power law of the oscillatory part in the correlations, which involves large wave vectors $2\tilde{k}_F = 2\pi\rho^\Psi$. We believe this is directly related to the failure of naive bosonization arguments in predicting the correct long-wavelength behavior of the Green's function. Since $\cos(2\pi\rho^\Psi\tilde{x}) = \cos[2\pi(x - n^a x)] \equiv \cos(2\pi n^a x)$, the period of the oscillations is correctly captured however. As shown in Ref. [46] our field-theoretic arguments are supported by the behavior of the density-density correlations which, for $h = V = 0$, we calculate by Monte Carlo sampling of the resulting free fermion theory [42] in the string-length representation and by DMRG calculations for finite h and V. The resulting fits for the DMRG data for $h = V = 0$ confirm the universal Luttinger liquid behaviors (15), (16), which together with a good agreement of the DMRG results for $h > 0$ according to Eq. (14), confirm the predicted relation between the Luttinger parameters.

Confinement as translational symmetry breaking.—In the string-length basis, the gauge invariant Green's function $g^{(1)}(x)$ translates to a highly nonlocal operator. Its most important effect is to shift string-length labels $\ell_m \to \ell_{m+1}$ for particle numbers m between $\tilde{x}_1 < m < \tilde{x}_2$, where $\tilde{x}_2 - \tilde{x}_1 = \tilde{x} = n^a x$, i.e.,

$$g^{(1)}(x) \simeq \langle\hat{T}(0,\tilde{x})\rangle, \tag{17}$$

where we define the partial translation operator:

$$\begin{aligned}&\hat{T}(\tilde{x}_1,\tilde{x}_2)|\ldots\ell_{\tilde{x}_1-1}\ell_{\tilde{x}_1}\ldots\ell_{\tilde{x}_2-1}\ell_{\tilde{x}_2}\ell_{\tilde{x}_2+1}\ldots\rangle \\ &\quad = |\ldots\ell_{\tilde{x}_1-1}\ell_{\tilde{x}_2}\ell_{\tilde{x}_1}\ldots\ell_{\tilde{x}_2-1}\ell_{\tilde{x}_2+1}\ldots\rangle,\end{aligned} \tag{18}$$

which cyclically shifts all string occupations by one unit between "sites" $\tilde{x}_1$ and $\tilde{x}_2$.

Aside from local terms around $\tilde{x}_1$ and $\tilde{x}_2$, which can be assumed to yield nonzero additional factors and were thus neglected in Eq. (17), the $g^{(1)}(x)$ function essentially probes translational invariance of the eigenstates in the string-length basis. Whenever the lattice translation symmetry $\tilde{x} \to \tilde{x} + 1$ is broken throughout the system [spontaneously, or as in Eq. (10) by a nonzero field $h \neq 0$], it follows that

$$g^{(1)}(x) \simeq \langle\hat{T}(0,\tilde{x})\rangle \simeq e^{-\tilde{\kappa}\tilde{x}} = e^{-\tilde{\kappa}n^a x}, \tag{19}$$

i.e., the corresponding $\mathbb{Z}_2$ LGT is confining.

Using this argument, it is now easy to see that the original model in Eq. (1) must be confining for any $h \neq 0$. Random $h_{\langle i,j\rangle}$ would similarly lead to confinement. Even for $h = 0$ it can become confining if translational symmetry is spontaneously broken by additional interactions: this case corresponds to a Mott insulating phase.

Mott transition and HODQLRO.—Earlier studies of the model (1) have revealed no Mott insulating states in the absence of the repulsive NN interaction, $V = 0$ [37]. There, the model maps to free fermions for $h = 0$ [42] and the field $h \neq 0$ inducing confinement is not sufficient to reach the insulating state. In the limit where $h \to \infty$ and $V = 0$ the particles are bound in dimers and the effective model maps exactly to a 1D Heisenberg antiferromagnet. Further analysis showed that at a special filling $n^a = 2/3$ the system is at the critical point with $K = 8/9$, right at the transition from the Luttinger liquid to a Mott insulating phase [37]. In this regime the string-length model features HODQLRO since $\tilde{K} = 2 > 1$.

In the following we will focus on the filling $n^a = 2/3$, which corresponds to half-filing in the string-length basis, $\rho^\Psi = 1/2$. We consider the repulsive interaction $V \geq 0$ and show that it can stabilize the Mott insulator. Two limits are analytically tractable: For $h \to \infty$, infinitesimal $V > 0$ opens a Mott gap. This is a BKT transition, as can be understood from the aforementioned mapping of the effective dimer model to the SU(2) invariant Heisenberg model following Ref. [37]. On the other hand, for $h = 0$ even $V \to \infty$ is insufficient to obtain the gapped state.

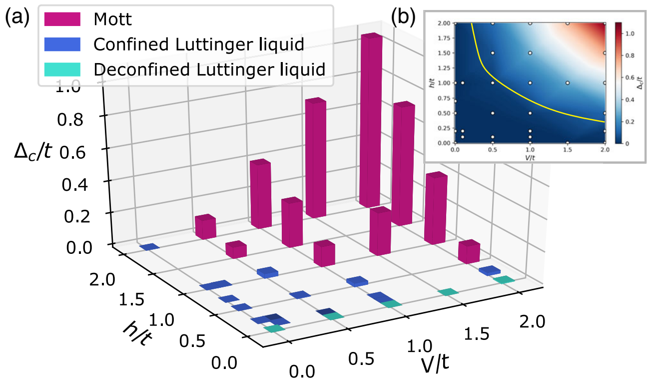

FIG. 2. Charge gap extrapolated in the thermodynamic limit, Δ_c, at filling $n^a = 2/3$ as a function of h and V. (a) As a guide for the eye the violet (blue) bars denote $\Delta_c/t > 0.05$ ($\Delta_c/t \le 0.05$) where we expect a Mott insulator (Luttinger liquid). (b) Heatmap of the gap diagram where the yellow curve, added by hand, corresponds to the approximate phase boundary.

However, as we show by an explicit calculation in Ref. [46], an infinitesimal $h \neq 0$ is sufficient to obtain a gapped phase when $V \to \infty$.

For generic nonzero values h, $V > 0$ we performed DMRG [39–41] calculations to extract the charge gap

$$\Delta_c(L, N) = \frac{1}{2}[(E^L_{N+2} - E^L_N) - (E^L_N - E^L_{N-2})], \qquad (20)$$

where E^L_N is the ground state energy of the original $\mathbb{Z}_2$ LGT model with chain length L and boson number N. We fixed the ratio of N and L at $n^a = (N/L) = 2/3$ and extrapolated the gap Δ_c in the thermodynamic limit by considering $L \to \infty$, see Ref. [46]. As can be seen in Fig. 2 the Mott insulating state is reached only in the case when both parameters are nonzero h, $V \neq 0$, and large enough. For a fixed value of V we observe an exponential opening of the gap as a function of h. The precise value of the transition point h_c is difficult to extract, but the exponential behavior of the gap opening points to a BKT nature of the transition, see Ref. [46].

Discussion and outlook.—We have solved the confinement problem of dynamical charges in a class of 1D $\mathbb{Z}_2$ LGTs by means of a nonlocal string-length representation, which has revealed an unexpected relation to translational symmetry breaking. Our arguments should apply equally for other gauge groups in one dimension. We found that, while the gauge symmetry keeps the Luttinger-liquid paradigm valid, the nonlocal interactions mediated by the gauge field must be treated with care. In particular, the confined gapless phase is characterized by an exponentially decaying $\mathbb{Z}_2$ invariant Green's function, but we found that it features HODQLRO in the string-length basis before a confined Mott state is realized.

In addition, we propose experimental realization of our model with Rydberg-dressed atomic gases in optical lattices. There, confinement can be studied by looking at the signatures of the translational symmetry breaking, by studying snapshots in the Fock basis and translating them into the string-length basis.

We have analyzed the Mott insulating state at filling $n^a = 2/3$ and showed that it is stabilized by a combination of h, $V \neq 0$. An interesting future extension would be to consider the filling $n^a = 1/2$, where repulsive NN interactions $V \neq 0$ can readily stabilize a Mott insulator when $h = 0$. On the other hand, one still finds a gapless system for $h \neq 0$ [37] and a large $|h| \gg t$ is expected to destabilize the Mott insulator. Other extensions of our work include generalization to spin-full systems, higher dimensions, and more complicated gauge groups. A particularly interesting and natural extension to our work would be to analyze confinement in more realistic models such as QED_2 [59].

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