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A probabilistic approach for the evaluation of the stabilizing forces of fully grouted bolts

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Abstract

The essential task of the ground reinforcement techniques is to keep the rock as stable as possible. In particular passive rock bolt should resist the rock movements along its entire length and through the resulting reaction forces, to improve the load-bearing capacity of the rock. Among different calculation techniques, the calculations based on Block Reinforcement Procedure (BRP) was used in this paper, also adopting some simplified equations available in the scientific literature. However, parameters influencing the interaction are difficult to evaluate. Therefore, the problem of the reliable definition of the parameters that most influence the behavior of the bolts and the evaluation of the stabilizing forces of the potentially unstable block of rock remains. A new probabilistic approach is presented in this article, able to appropriately manage the uncertainties present on the fundamental parameters of the bolt-rock interaction and on the mechanical characteristics of the sliding surface of the block. Through the use of a Monte Carlo procedure, in fact, it was possible to obtain different samples of the safety factors of the rock block, one for each diameter of the steel bars used for its stabilization. Finally, the probabilistic management of the safety factor samples allowed the correct design of the steel bars, by evaluating the probability that the safety factor of the block with regard to potential slipping has a value lower than a pre-established limit. The probabilistic approach developed was applied to a real problem of stabilization of a potentially unstable rock block due to planar sliding, present on a municipal road in North Italy.

Keywords: rock bolt; Winkler spring approach; rock block stabilisation; safety factor; Monte Carlo simulation, probabilistic approach,

Abbreviations and nomenclature

A Area of the sliding surface of the block;

A_{bar} Area of the section of the steel bar constituting the bolt

c Cohesion on the natural discontinuity which constitutes the sliding surface;

$(EA)_{bolt}$ Axial stiffness of the bolt

E_{binder} Elastic modulus of the binder surrounding the steel bar in the hole

$(EJ)_{bolt}$ Bending stiffness of the bolt

E_{st} Steel elastic modulus

$f(x)$ Probability density associated with the x value of the geotechnical or geomechanical parameter considered;

F_S Safety factor;

J_{bar} Moment of inertia of the steel bar constituting the bolt

k Ratio between the normal pressure, p, which is applied on the perimeter of the bolt (on the wall of the hole) by the surrounding rock and the normal displacements, y, of the bolt

L_a Bolt length inside the unstable block

L_p Bolt length in the stable rock behind the unstable block

L_{test} Length of the tested bolt

M Bending moment in the bolt

N Axial force in the bolt

$N_{0,\delta_{max}}$ Bolt stabilising force in the direction of the bolt axis

N_{test} Tensile axial force applied at the bolt head from pull-out tests

N_{yield} Force causing the bar failure under tensile stress

N_{slip} Force causing the bolt-rock interface to fail for a unit bolt length

N_0 Value of the tensile force in the axial direction of the bolt on the intersection point between the bolt and a block surface

n Number of fully grouted passive bolts present;

T Shear force in the bolt

t_{binder} Thickness of the binder annulus surrounding the steel bar

T_0 Value of the shear force perpendicular to the axial direction of the bolt on the intersection point between the bolt and a block surface

$T_{0,\delta_{max}}$ Bolt stabilising force in the transverse direction

v_r Value of the relative axial displacement between the bolt and the surrounding rock

W Weight of the potentially unstable rock block;

y Normal displacements of the bolt perpendicular to the axial direction of the bolt

α Parameter characterising the interaction in the axial direction between the bolt and the surrounding rock $\alpha = \sqrt{\frac{\beta_c \cdot P_{hole}}{EA}}$

β Parameter characterizing the interaction in the transverse direction between the bolt and the surrounding rock $\beta = \sqrt[4]{\frac{k \cdot \Phi_{hole}}{4 \cdot EJ}}$

β_c Ratio between the shear stresses, τ, that develop on the perimeter of the bolt and the relative axial displacements, v_r

δ_{max} Maximum displacement component of the block in the axial direction of the bolt

φ Friction angle on the natural discontinuity constituting the sliding surface

Φ_{bar} Diameter of the steel bar

Φ_{hole} Diameter of the hole (of the bolt)

μ	Mean value of the distribution;
χ	Adimensional parameter for the evaluation of the stabilising forces
λ	Adimensional parameter for the evaluation of the stabilising forces
σ	Standard deviation of the distribution.
σ_{yield}	Steel yield stress
τ	Shear stress on the lateral surface of the bolt
τ_{lim}	Ultimate limit shear stress of the rock-bolt interface
ϑ	Inclination of the sliding surface with respect to the horizontal plane

Introduction

During underground construction of different infrastructures, stability is expected, therefore reinforcement is needed to keep the excavation stable (Pelizza et al., 2000). A number of factors affect the underground stability in joint rock masses, e.g. high rock stress, poor rock mechanical properties, excessive ground water pressure (Chen, 1994). Fully grouted passive bolts are widely used in the tunnel and underground caverns as a stabilization intervention. Many studies have been carried out to describe their behavior in rock masses considered to be homogeneous and continuous (Osgoui and Oreste, 2007; Ranjbarbia et al., 2014; 2016; Oreste, 2013). Passive rock bolt elements have a zero initial load and the mobilized stabilizing load increases with the displacement of the potentially unstable rock block. Continuously mechanically coupled (CMC) bolts rely on a curing agent (cementitious of resin grout; i.e. Spagnoli et al., 2021) that fills the annulus between the element and the borehole wall (Bawden, 2011). Rock bolts are primarily stressed by tensile and shear loads, which are caused by rock movements. The stress on the rock bolts depends on the type of rock failure (crack fracture, folding, shear fracture etc.). The essential task of the rock bolt consists in keeping the rock as stable as possible or to increase the shear resistance (Feder, 1980). Especially in tunnel construction, rock-bearing elements are in a statically undetermined system with different rock stiffness values (Blümel, 1996). Ferrero (1995) and Kilic et al. (2002) pointed out that the main factors affecting the shear and bond strength of rock bolts are the rock bolts' materials, the geometry of the bolt (bolt shape, diameter and length), type of binder, type of rock mass and fracture system. Moosavi et al. (2002) proved that a stress decrease in poor-quality rock resulted in completely ineffective bolt behavior. Therefore, any changes occurring at the bolt–grout or grout–rock interface affect the bolt bond strength and bolt load capacity.

Recently Oreste et al. (2020) used the Block Reinforcement Procedure (BRP) (Oreste and Cravero, 2008; Oreste, 2009), to run a parametric analysis considering different diameter of the steel bar, thickness of the binder ring around the bar, length of the bolt in the unstable block, total length of the bolt, elastic modulus of the binder and inclination of the sliding surface of a rock block with respect to the horizontal plane. This model considers a bolt which crosses the potentially unstable block (with a length L_a) and reach the stable rock behind it, where it penetrates for a certain length (L_p), see Fig. 1.

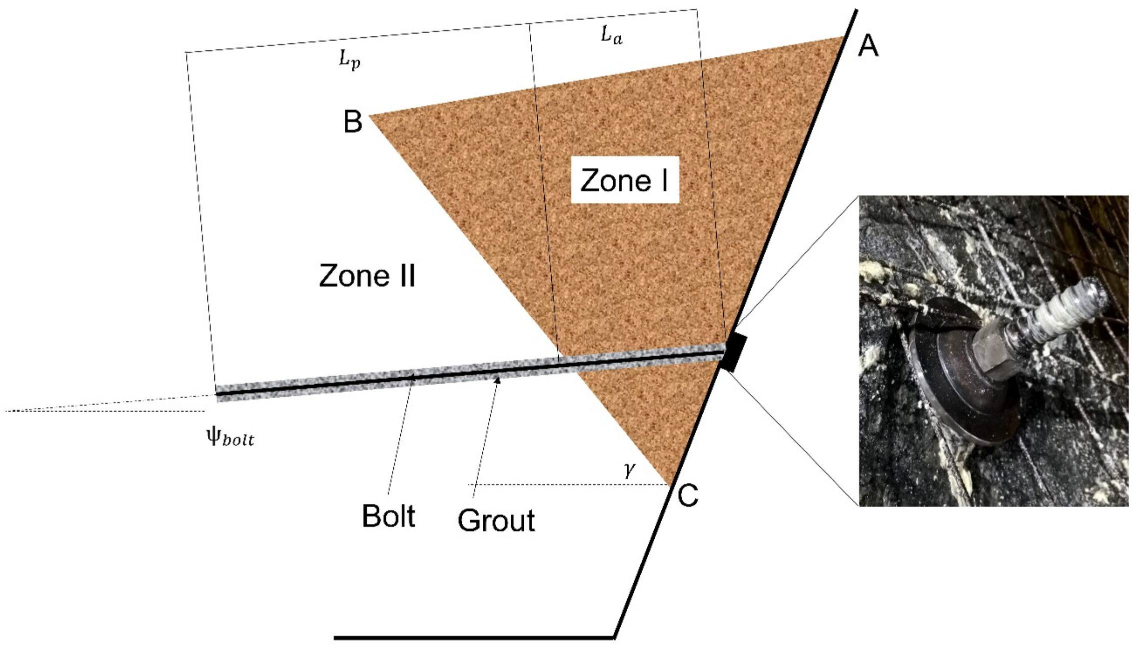

Fig. 1 Schematic representation of the potentially unstable block of rock and the passive bolt crossing it (not to scale).

The method allows to calculate the axial, N, and shear, T, forces, and the bending moments M developing along the bolt, as a linear function of the (very small) displacements of the block. Then the stabilizing forces, applied by the single bolt to the potentially unstable block, are evaluated. The rock-bolt interaction involves the presence of independent springs according to Winkler's approach, both in the transverse and axial direction with respect to the bolt (Figure 2) (Oreste and Cravero, 2008).

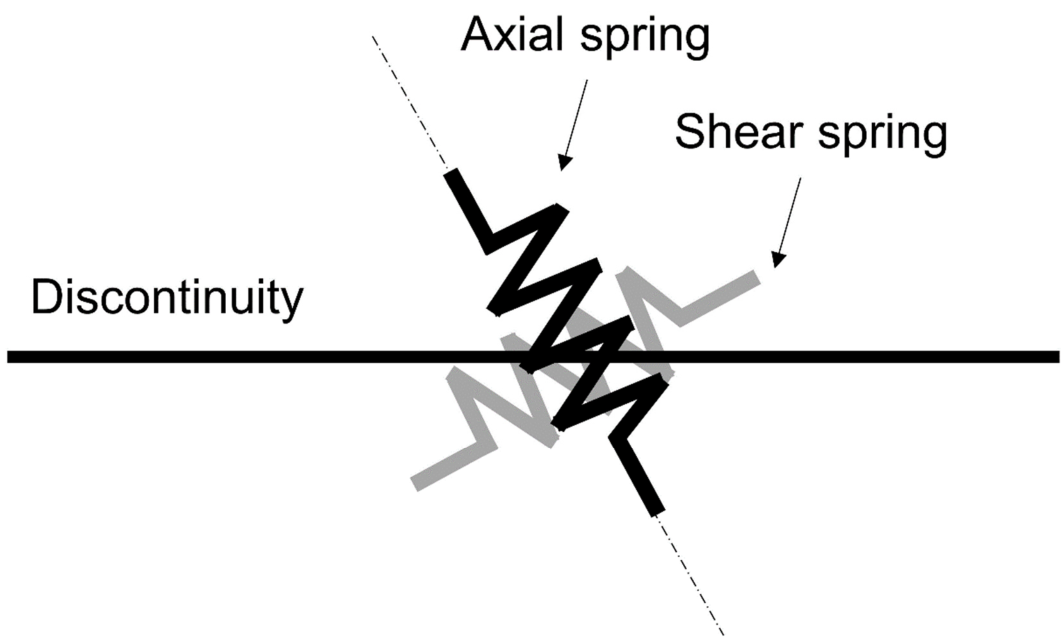

Fig. 2 Model for axial and shear springs at a discontinuity

The parameters influencing the interaction are difficult to evaluate, because they depend on the bending and axial rigidity of the bolts, on the stiffness of the grout surrounding the bar, on the stiffness of the rock at the contour of the bolt. Furthermore, laboratory tests are time-consuming to carry out.

In addition, the forces necessary to simulate the rock-bolt interaction during movements, even very small, of the potentially unstable block of rock can be considerable.

More recently, Oreste and Spagnoli (2020) have proposed simplified equations to simulate the static contribution of fully grouted bolts on potentially unstable (by sliding) rock blocks, in terms of axial and transverse force to the bolt. These equations are reliable even if they are based on simplifying hypotheses: the errors are negligible, considering the typical range of variability of the parameters influencing the rock-bolt interaction. However, the problem of the reliable definition of the parameters influencing the evaluation of the stabilizing forces of the potentially unstable block of rock remains.

This work illustrates a new probabilistic approach able to manage the uncertainty on various parameters that affect the behavior of bolts and to provide the probabilistic distribution of the safety factor of the rock blocks for each different stabilization intervention scheme. supposed. From the results obtained, it is possible to design the stabilization work, for example by defining the diameter of the bolts required, based on a greater knowledge of the effects of the uncertainty of the geotechnical parameters on the degree of stability of the rock blocks. The same probabilistic approach used for this specific stabilization problem can be adopted in other stability problems in the geotechnical field.

After describing the proposed probabilistic procedure in the field of geotechnical engineering and of rock mechanics, the stabilization mechanisms of passive fully grouted bolts on rock blocks showing a potential planar slip are illustrated. Finally, the application of the probabilistic approach to a specific real case will be illustrated.

The authors hope that the use of a probabilistic approach such as the one illustrated in this work, which does not require the use of specific software or complex procedures, will allow a more correct and responsible design of the engineering interventions necessary in stability problems in geotechnical engineering.

Proposed probabilistic approach in the evaluation of the safety factors in geotechnical engineering

The probabilistic analysis in geotechnical engineering evaluates the probability of a certain event occurs considering certain data relating to the geotechnical properties of the system (e.g. Griffiths and Fenton, 2009). Variability of ground properties constitutes a major source of uncertainty when contending with geotechnical problems (Franco et al., 2019). The adoption of probabilistic methods relating the uncertainty of the different geotechnical properties on the final output has proven to be a valuable approach (e.g. Tang et al., 1976; Ronold and Bysveen, 1992; Oreste, 2005; Spagnoli et al., 2018; Spagnoli and Shimobe, 2020).

Several probabilistic techniques are used to account the uncertainty of the geotechnical parameters. General probabilistic methods are used to quantify the probability of occurrence of a single behavior (or property) for rocks and soils (e.g. Schubert and Goricki, 2004; Oggeri and Oreste, 2012; Mollon et al., 2013; Oreste, 2015; Spagnoli et al., 2017). More specifically, Cherubini et al. (2004), Trivedi and Zimmer (2005), Nelsen (2006), to name a few, modelled multivariate data based on the copula theory in which a copula function instead of the correlation matrix is used to represent the dependence relationship among random variables. For instance, Cao and Wang (2014), Ching et al. (2016), Zhang et al. (2014), Contreras et al. (2018), used a Bayesian method to characterize the spatial variability of soil (rock) properties, quantify the model selection uncertainty and to compare the validity of the candidate models. The point estimate method was used in geotechnical reliability analysis by Schweiger et al. (2001) and Christian and Baecher (2002).

Monte Carlo technique, which involves generating a large number of random samples from the input distributions and put into the transfer function, were investigated by Oreste (2005), Sari et al. (2010), Aladejare and Akeju (2020).

In the absence of more detailed information on the probabilistic distributions of the parameters considered uncertain in the calculation, the normal (Gaussian) distribution is used, expressed by the following equation:

$$f(x) = \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{(x-\mu)^2}{2 \cdot \sigma^2}} \qquad (1)$$

Where:

$f(x)$ is the is the probability density associated with the x value of the geotechnical or geomechanical parameter considered;

μ is the mean value of the distribution;

σ is the standard deviation of the distribution.

The probabilistic distribution of Gauss is symmetrical and requires that 69.83% of cases are included within the range [$\mu - \sigma$]- [$\mu + \sigma$], 95.45% of cases in the interval [$\mu - 2\sigma$]- [$\mu + 2\sigma$] and 99.73% of cases in the interval [$\mu - 3\sigma$]- [$\mu + 3\sigma$] (Fig. 3).

More specifically, 95% of cases are included in the range [$\mu - 1.96\sigma$]- [$\mu + 1.96\sigma$] and 99% of the cases in the interval [$\mu - 2.58\sigma$]- [$\mu + 2.58\sigma$].

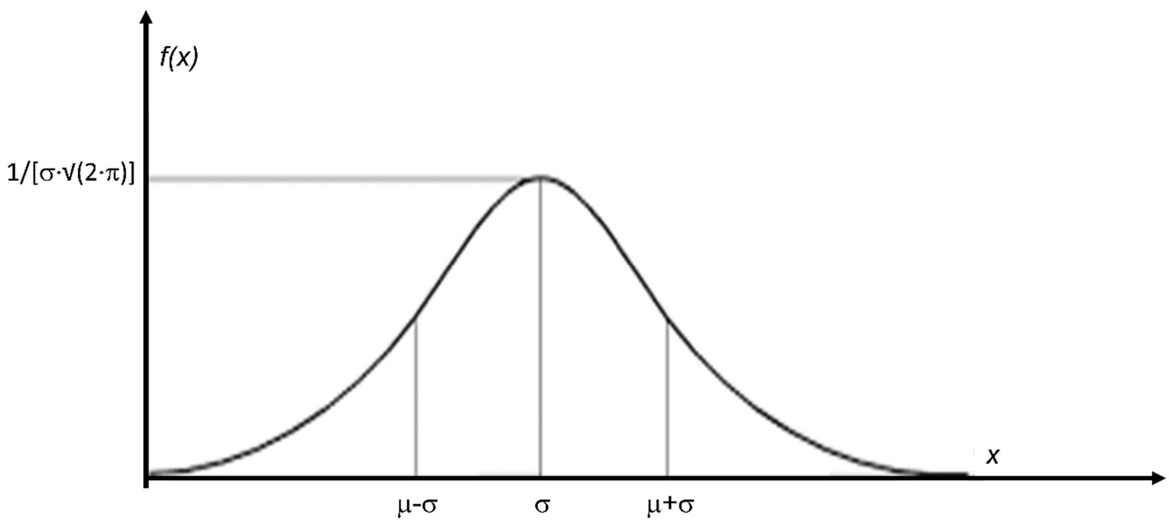

Fig 3. Gauss probabilistic distribution trend used to represent the uncertainty of the parameters in the geotechnical and geomechanical field.

Therefore, starting from *in situ* or laboratory tests, it is possible to obtain samples of measurements for each influential parameter (input data), in order to have an estimate of the average values and standard deviations of the probabilistic distributions to be adopted in the calculation. Alternatively, by identifying the variability interval of a parameter x associated with a certain probability, p, that the real value falls within that interval (for example 95% or 99%), it will be possible to determine the average value μ and the standard deviation σ of the Gaussian distribution to be used in the calculation:

$$\mu = \frac{(x_{max}+x_{min})}{2} \quad (2)$$

$$\sigma(p = 95\ \%) = \frac{(x_{max}-x_{min})}{3.92} \quad (3)$$

$$\sigma(p = 99\ \%) = \frac{(x_{max}-x_{min})}{5.16} \quad (4)$$

Where x_{max} and x_{min} are respectively the minimum and maximum values of the variability interval of x.

The procedure proposed in this article provides that all parameters considered uncertain are described by a probabilistic distribution, while those considered certain are described by a simple representative (deterministic) value.

Once the probabilistic distributions of the uncertain parameters ($x_1, x_2, \dots x_i, \dots x_n$, where n is the total number of parameters considered uncertain) necessary for the calculation are known, it is possible to proceed with the random extraction of the values by adopting the Monte Carlo procedure.

If it can be assumed that these parameters are independent of each other, samples of m values can be created for each parameter x_i, by ordering the values thus obtained. At this point m data vectors are formed $[x_1, x_2, \dots x_i, \dots x_n]_{j=1 \text{ to m}}$ with all the parameters present in the same position j of the extracted sequence, with j varying from 1 to m. m is the number of random extractions that are performed for each of the n uncertain parameters, using the probabilistic distributions of each parameter.

For example, if in the problem under examination there are 5 parameters considered uncertain (n = 5: x_1, x_2, x_3, x_4, x_5) and 1000 extractions are adopted with the Monte Carlo procedure (m = 1000), 1000 vectors can be obtained of input data, as shown below:

$[x_1;\ x_2;\ x_3;\ x_4;\ x_5]_{j=1}$

$[x_1;\ x_2;\ x_3;\ x_4;\ x_5]_{j=2}$

...

$[x_1;\ x_2;\ x_3;\ x_4;\ x_5]_{j=999}$

$[x_1;\ x_2;\ x_3;\ x_4;\ x_5]_{j=1000}$

After having built up a sample of values extracted from the probabilistic distribution of each random variable, it is possible to proceed to the determination of the safety factor of the problem under examination for each series of values obtained from the different extracted samples. In this way it is possible to create a sample of values of the safety factor which can then be statistically treated:

$[x_1;\ x_2;\ x_3;\ x_4;\ x_5]_{j=1}$ F_S (j=1)

$[x_1;\ x_2;\ x_3;\ x_4;\ x_5]_{j=2}$ F_S (j=2)

...

$[x_1;\ x_2;\ x_3;\ x_4;\ x_5]_{j=999}$ F_S (j=999)

$[x_1;\ x_2;\ x_3;\ x_4;\ x_5]_{j=1000}$ F_S (j=1000)

The safety factor is calculated starting from the extracted values of the uncertain parameters and from the representative values for those considered certain (deterministic values, kept constant in the calculation). The next paragraph illustrates how to evaluate the safety factor for the problem under examination: the stability of a block of rock potentially unstable due to sliding on a planar surface, in the presence of a stabilization intervention with fully grouted passive bolts.

The result of the procedure is a sample, with a number m of safety factor values, i.e. $[(F_s)_1\ ;\ (F_s)_2\ ;\ \ldots\ ;\ (F_s)_i\ ;\ \ldots\ ;\ (F_s)_m\ ]$.

In order to have a good representation of the uncertainties present, a value of m of the order of a thousand values is generally required. If there is a degree of correlation between 2 or more parameters, for these we proceed to the extraction of the values considering the multivariate statistics, that is, for the Gauss distribution, in addition to the average value and the standard deviation of each parameter, we also consider the correlation coefficients between the pairs of related parameters.

The analysis of the sample of the m values of the safety factor allows us to understand the nature of its probabilistic distribution. Even if a Gaussian distribution is assumed for each of the input data of the problem, the sample of the safety factor values in general shows a probabilistic distribution different from the Gaussian one, which can also have multimodal trends, such as the one shown in Figure 4.

It is important to check the sample of the safety factors obtained by analyzing the trend of the histogram of the relative frequencies, in order to identify the theoretical distribution that best represents the sample of the safety factors obtained from the calculation.

Any confirmation of the theoretical distribution identified can then be made through the Q-Q plot which compares the quantiles of the identified theoretical distribution with the empirical quantiles on the sample data of the obtained safety factors.

The theoretical distribution that best represents the sample of safety factors allows subsequently to have an indication of the probability that the safety factor is lower than a certain predefined value $\overline{F_S}$. For example, it is very interesting to evaluate the stability limit condition associated with a safety factor of 1 (i.e. $\overline{F_S} = 1$) and to determine the probability that the safety factor is lower than this value $(F_S < \overline{F_S})$. To do this, reference is made to the cumulative distribution of probabilities (Figure 5).

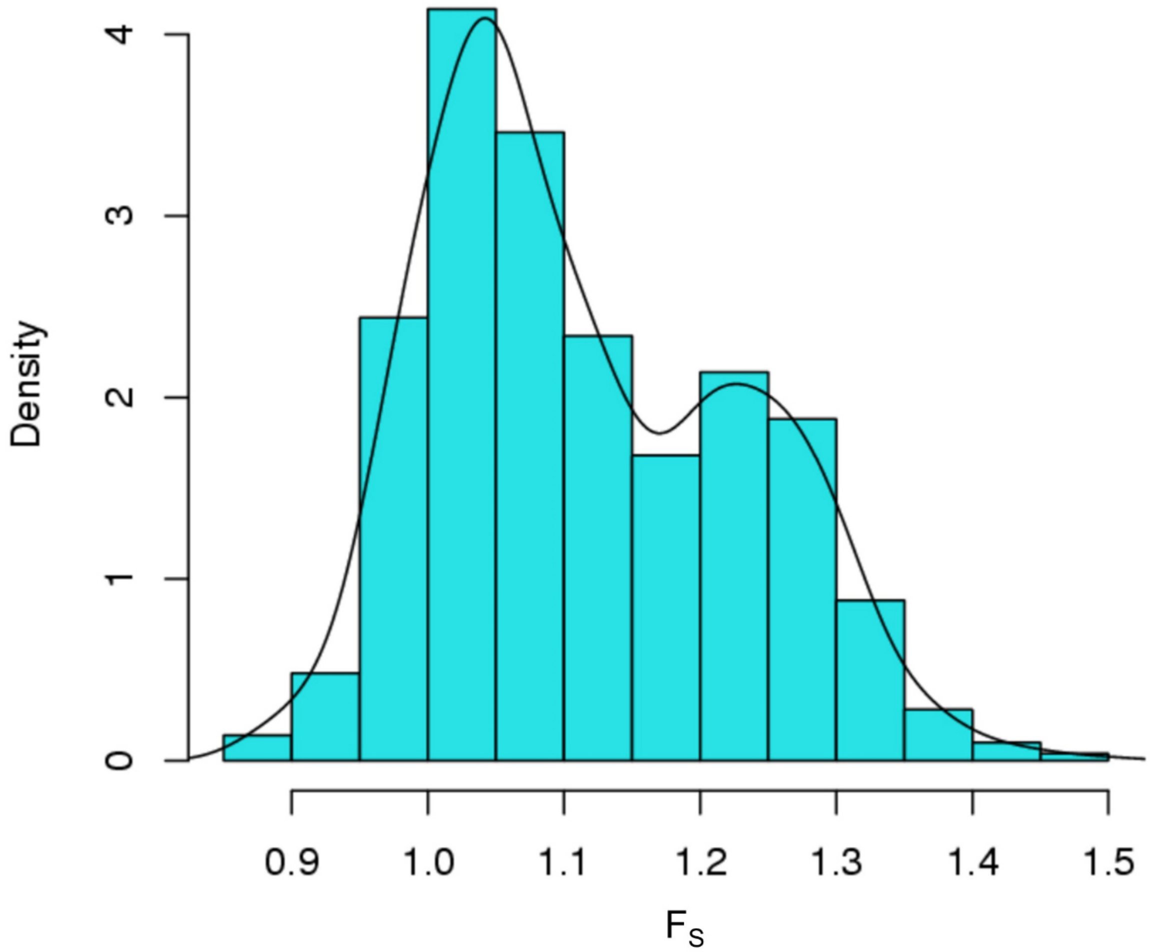

Fig 4. General trend of the distribution of safety factors (F_S) obtained by calculating the relative frequencies through the histogram.

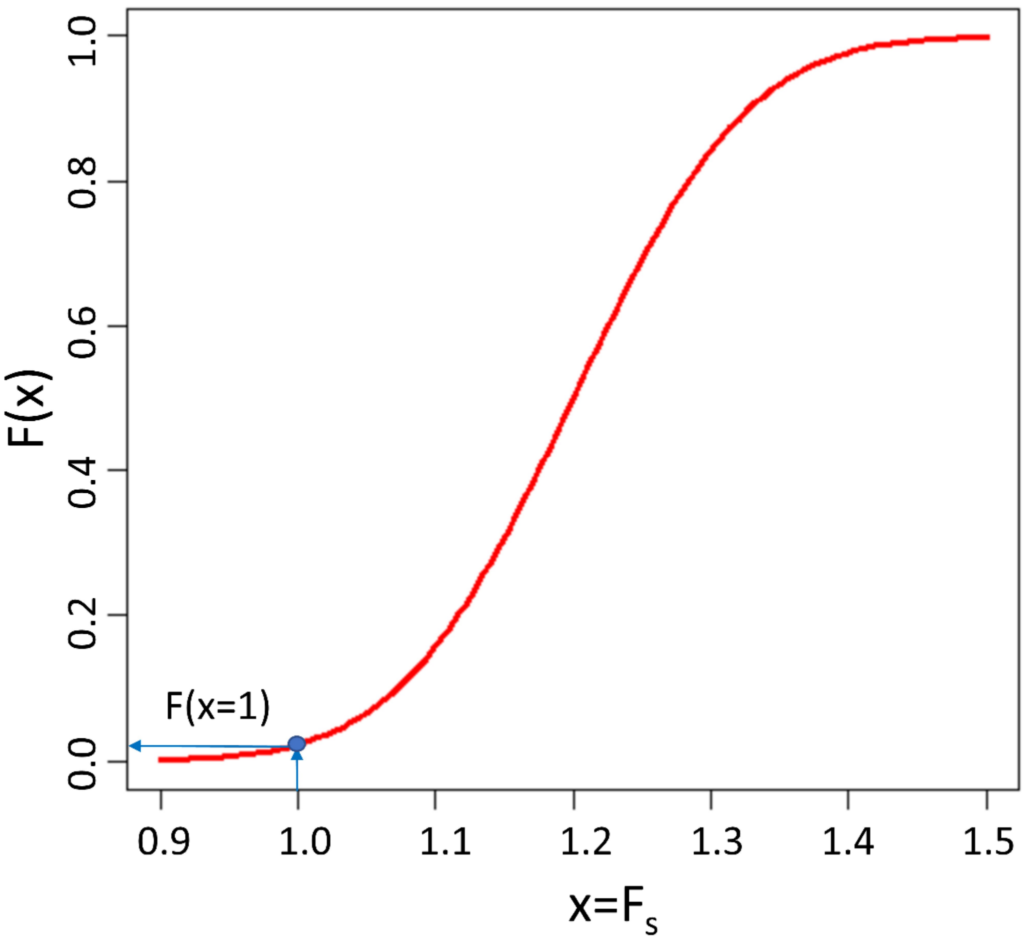

Fig 5. Cumulative distribution of the probabilities (Gauss curve) of the safety factor, with indication of the probability F (x = 1) that the safety factor is less than unity.

This procedure can be carried out for the geotechnical problem under examination to ensure the stability of the soil or rock in the absence and in the presence of the supports and reinforcements to be designed.

A modern approach to the design of the interventions can therefore be conducted by checking whether the probability of instability is significantly reduced to very low and acceptable values in the presence of the supports and reinforcements of the soil or rock assumed in the project.

Figure 6 shows a summary flow chart of the proposed procedure in order to to perform the probabilistic analysis of the stability of a rock block in a simple and fast way in the presence of the planned stabilization interventions.

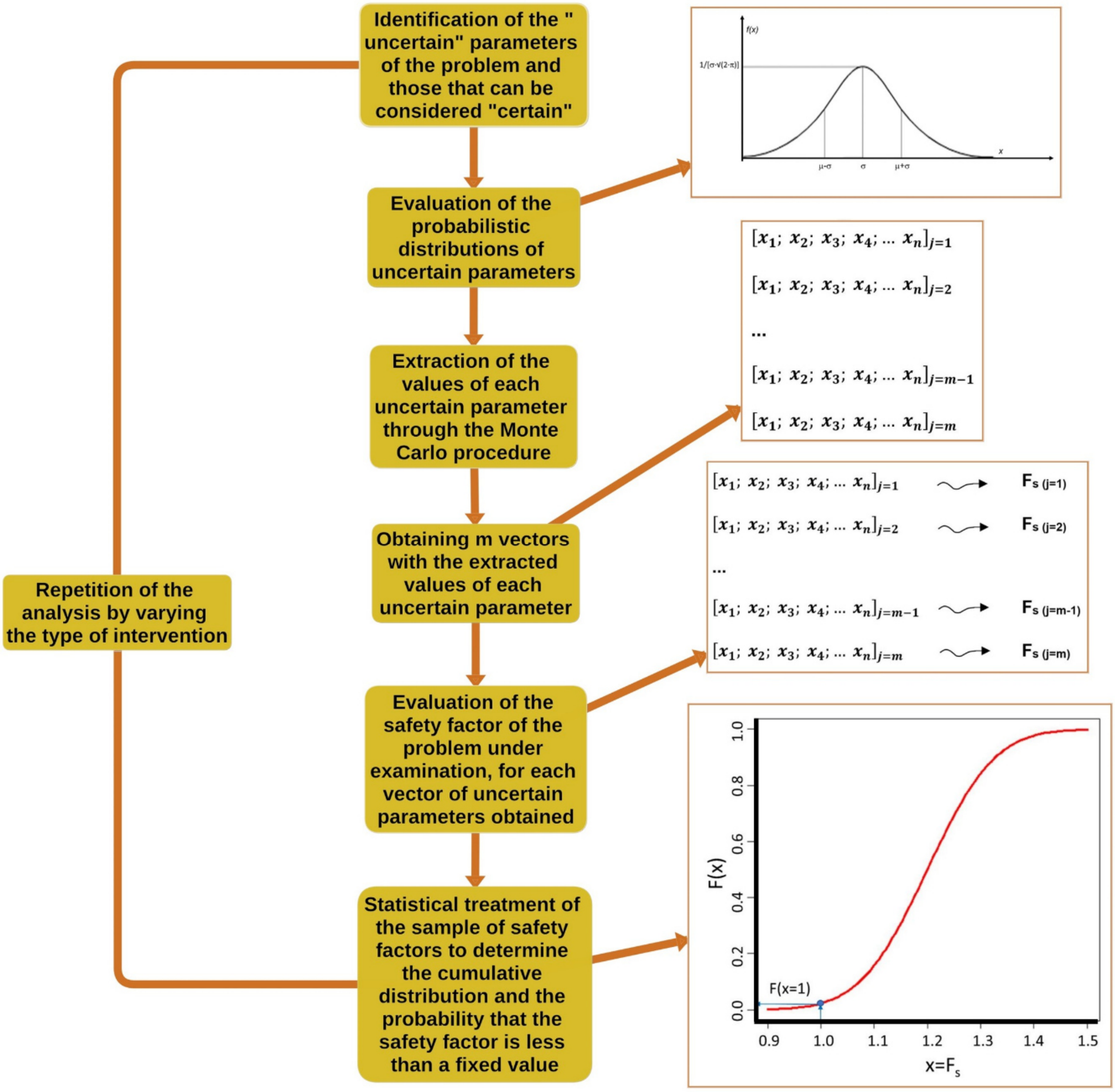

Fig 6. Flow chart of the procedure proposed for the evaluation of the stability conditions (through the evaluation of the safety factor) of a potentially unstable rock block, in the presence of stabilization interventions with fully grouted passive bolts.

Model description considering the stability of a two-dimensional block of rock with regard to sliding

Fully grouted passive bolts develop internal forces linearly dependent on the displacements of the rock block that they must stabilize (Oreste and Cravero, 2008). The internal forces developed can be analyzed by referring to the interaction mechanism in the two directions perpendicular and parallel to the lateral interface of the bolt, in contact with the rock. There is a maximum displacement δ_{max} of the block for which the internal forces induce safety factors at breakage and extraction equal to the minimum ones considered admissible. The displacement δ_{max} is, therefore, to be considered as the maximum displacement of the block still compatible with the stability and efficiency of the bolt.

The shear T_0 and axial forces N_0 developing in the bolt at the point of intersection with an external surface of the block (which isolates the block from the stable rock behind) are also the stabilizing forces that the single bolt applies to the potentially unstable block. The maximum values of these forces that the bolt is able to offer to the block are obtained in correspondence with the displacement δ_{max} and are, therefore, indicated as $T_{0,\delta_{max}}$, and $N_{0,\delta_{max}}$ (Fig. 7).

Following a detailed parametric study within the typical variability ranges of the parameters influencing the bolt-rock interaction, it was possible to obtain the evaluation of the forces $T_{0,\delta_{max}}$, and $N_{0,\delta_{max}}$ that each single bolt potentially applies to the unstable block (Oreste and Spagnoli, 2020):

$$T_{0,\delta_{max}} = min\left(\frac{N_{yield}}{F_{s,adm,yield}} \cdot \frac{2}{\sqrt{\frac{\lambda^2 \cdot \chi^2}{tan^2(\vartheta)} + \frac{64}{3}}} \; ; \; \frac{N_{slip}}{F_{s,adm,slip}} \cdot \frac{2 \cdot tan(\vartheta)}{\lambda \cdot \psi \cdot \alpha}\right) \qquad (5)$$

$$N_{0,\delta_{max}} = min\left(\frac{N_{yield}}{F_{s,adm,yield}} \cdot \frac{1}{\sqrt{1 + \frac{64}{3} \cdot \frac{tan^2(\vartheta)}{\lambda^2 \cdot \chi^2}}} \; ; \; \frac{N_{slip}}{F_{s,adm,slip}} \cdot \frac{\omega}{\alpha}\right) \qquad (6)$$

Where:

$$\lambda = \left[\frac{(EA)_{bolt} \cdot \alpha}{(EJ)_{bolt} \cdot \beta^3}\right] \quad (7)$$

$$\chi = \left[\frac{\left(1+e^{-2\alpha L_a}\right) \cdot \left(1-e^{-2\alpha L_p}\right)}{\left(1+e^{-2\alpha (L_a+L_p)}\right)}\right] \quad (8)$$

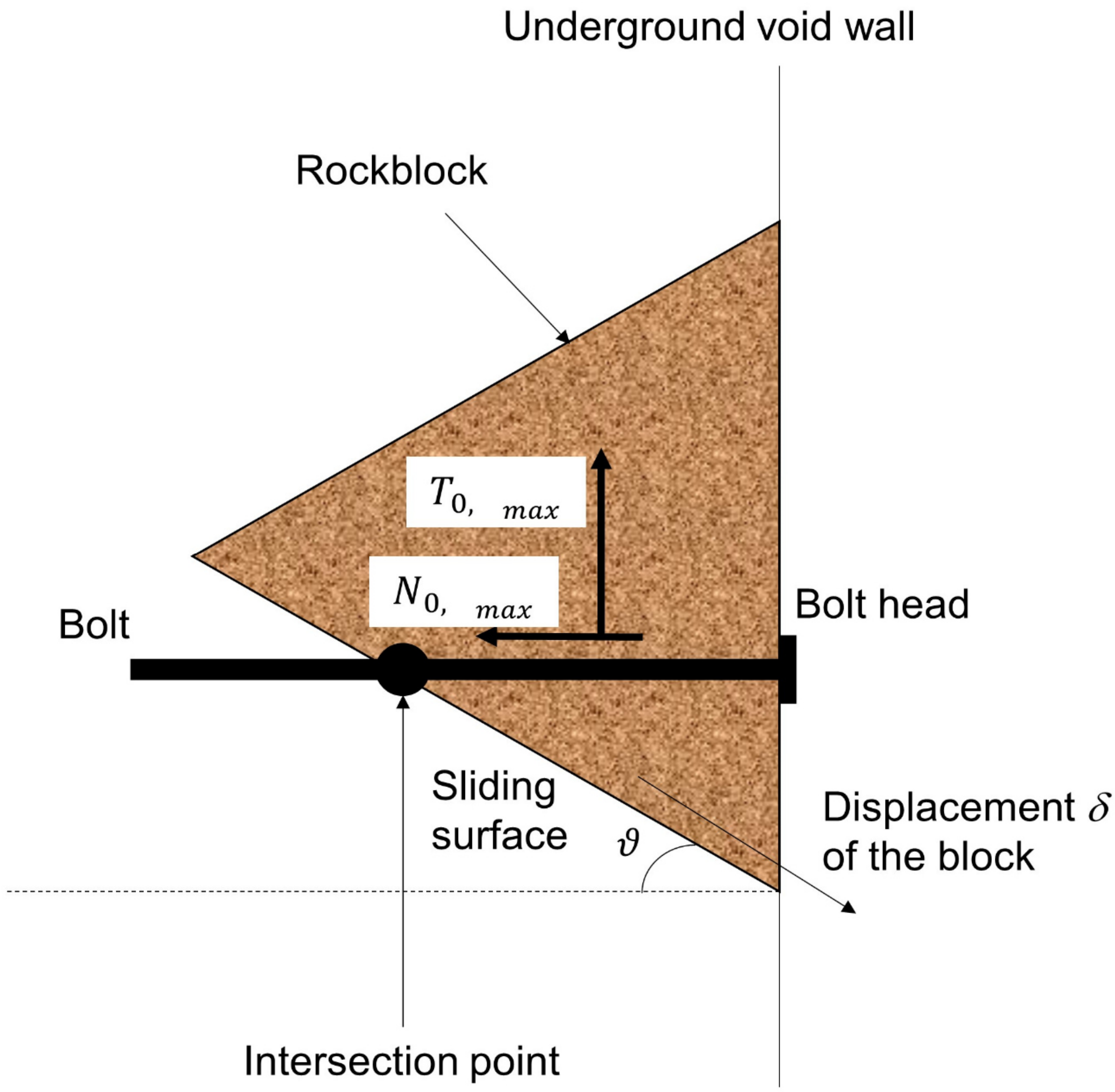

Fig. 7 Sketch of the stabilizing forces applied by the fully grouted passive bolt to the potentially unstable rock block on the walls of an underground cavity (not to scale).

N_{slip} is the force which causes the bolt-rock interface to fail for a unit bolt length $N_{slip} = \tau_{lim} \cdot \pi \cdot \Phi_{hole}$;

τ_{lim} is ultimate limit shear stress of the interface rock-bolt;

N_{yield} is the force causing the bar failure under a tensile stress $N_{yield} = \sigma_{yield} \cdot A_{bar}$;

σ_{yield} is the yield stress of steel;

A_{bar} is the area of the section of the steel bar constituting the bolt $A_{bar} = \pi \cdot \frac{\Phi_{bar}^2}{4}$;

J_{bar} is the moment of inertia of the steel bar constituting the bolt, $J_{bar} = \pi \cdot \frac{\Phi_{bar}^4}{64}$

β is the parameter that characterizes the interaction in the transversal direction between bolt and rock:

$$\beta = \sqrt[4]{\frac{k \cdot \Phi_{hole}}{4 \cdot (EJ)_{bolt}}} \qquad (9)$$

k is the ratio between the normal pressure, p, which is applied on the perimeter of the bolt by the surrounding rock, and the transversal displacement, y, of the bolt;

α is a parameter characterizing the interaction in the axial direction between bolt and rock as:

$$\alpha = \sqrt{\frac{\beta_c \cdot \pi \cdot \Phi_{hole}}{(EA)_{bolt}}} \qquad (10)$$

$(EA)_{bolt}$ is the axial stiffness of the bolt, evaluated as:

$$(EA)_{bolt} = E_{st} \cdot \left(\frac{\pi}{4} \cdot {\Phi_{bar}}^2\right) + E_{binder} \cdot \left[\frac{\pi}{4} \cdot \left({\Phi_{hole}}^2 - {\Phi_{bar}}^2\right)\right] \qquad (11)$$

$(EJ)_{bolt}$ is the bending stiffness of the bolt, evaluated on the basis of the following equation:

$$(EJ)_{bolt} = E_{st} \cdot \left(\frac{\pi}{64} \cdot {\Phi_{bar}}^4\right) + E_{binder} \cdot \left[\frac{\pi}{64} \left({\Phi_{hole}}^4 - {\Phi_{bar}}^4\right)\right] \qquad (12)$$

Φ_{bar} is the bar diameter;

Φ_{hole} is the diameter of the hole where the bolt is inserted as $\Phi_{hole} = \Phi_{bar} + 2 \cdot t_{binder}$;

t_{binder} is the thickness of the binder annulus around the steel bar;

E_{st} is the steel elastic modulus;

E_{binder} is the elastic modulus of the binder surrounding the steel bar in the hole.

β_c is the ratio between the shear stresses developing on the perimeter of the bolt (on the wall of the hole), τ, and the relative axial displacements, v_r. β_c depends in general on the characteristics of the material surrounding the steel bar and on the elastic modulus of the rock;

L_a and L_p are respectively the lengths of the bolt inside the potentially unstable rock block (zone I) and in the stable rock (zone II); their sum is the total length of the bolt.

The normal (k) and tangential (β_c) stiffness parameters describe the bolt-rock interaction and significantly affect the behavior of the bolt. Other fundamental parameters are the values of the ultimate breaking stress of the bolt-rock interface (τ_{lim}) and the strength of the steel (σ_{yield}).

ϑ is the angle that the sliding surface of the block forms with the horizontal plane. In the typical case of horizontal bolts and perpendicular to the vertical rock wall, this angle is also the angle that the sliding surface forms with the direction of the bolts.

A detailed parametric analysis, considering the typical variability of the parameters that affect the bolt-rock interaction and, therefore, the behavior of the bolts, allowed to evaluate the points where the bolt can fail.

Thanks to the evaluation of these points (Oreste and Spagnoli, 2020), it was possible to identify simple summary equations of the maximum values of the two forces ($T_{0,\delta_{max}}$ and $N_{0,\delta_{max}}$) which still guarantee a certain safety margin with regard to the failure of the bolt in the critical points identified by the parametric analysis. The parameters falling within the equations are obtained by the *in situ* tests described in some detail in the next paragraph.

The safety factor of the block, evaluated as the ratio between the resisting forces and the unstable forces, is expressed by the following equation in the presence of the stabilizing forces of the bolts (in the case of horizontal bolts):

$$F_s = \frac{c \cdot A + [(W - n \cdot T_{0,max}) \cdot cos\vartheta + n \cdot N_{0,max} \cdot sin\vartheta] \cdot tan\varphi}{(W - n \cdot T_{0,max}) \cdot sin\vartheta - n \cdot N_{0,max} \cdot cos\vartheta} \quad (13)$$

Where:

c is the cohesion on the natural discontinuity which constitutes the sliding surface;

A is the area of the sliding surface of the block;

W is the weight of the potentially unstable rock block;

n is the number of fully grouted passive bolts present;

φ is the friction angle on the natural discontinuity constituting the sliding surface.

This equation reports at the numerator the stabilizing forces, which oppose the movement of the block, evaluated in the direction of sliding (i.e. the line of maximum slope on the sliding surface); the denominator includes the unstable forces, those that tend to move the block, also evaluated in the direction of sliding of the block.
This equation permits to proceed with the design of the bolts through the choice of the solution that allows to obtain the safety factor of the desired rock block. It is possible to proceed by trial and error, changing the geometric characteristics of the bolt (dimensions of the steel bar and of the entire bolt, length of the bolt) and the number of bolts, until the block is stabilized, with a safety margin considered acceptable.

Application of the probabilistic approach to a real case

There are several parameters influencing the safety factor of a rock block considering fully grouted passive bolts:

- cohesion and friction angle of the discontinuity representing the sliding surface;
- weight of the rock block, which is function of the volume and the specific weight of the rock;
- geometry of the bolts (of the steel bar and of the grout surrounding it);

- stiffness parameters of the bolt-rock interaction on the interface at the lateral surface of the bolt;
- strength of the bolt-rock interface to bolt extraction;
- tensile strength of the steel constituting the bolt bar;
- elastic modulus of the steel and of the grout around the bar.

Several of these parameters are usually known only with some accuracy. In particular, there are often large uncertainties on the stiffness parameters characterizing the interaction between bolts and rock (k and β_c), on the strength of the bolt-rock interface to bolt extraction (τ_{lim}), as well as on the cohesion (c) and friction angle (φ) of the natural discontinuity representing the sliding surface. Specific laboratory tests are carried out in order to evaluate these parameters, but from the tests it is possible to obtain values that are often not very representative because the results can be dispersive and the number of tests is generally limited.

To obtain the estimate of the cohesion and the angle of friction of the sliding surface, shear tests are carried out on rock samples at the laboratory scale; to evaluate the stiffness parameters of the bolt-rock interaction, specific load tests are prepared *in situ* both in the axial and transverse direction of the bolt (Oreste and Spagnoli, 2020). To determine the shear strength of the bolt-rock interface, we use the results pull-out tests of a *in situ* test bolt with the application of a force in the axial direction.

The uncertainty about the evaluation of these parameters cannot be represented simply by an average value of the results of *in situ* and laboratory tests. It is more appropriate to consider a range of variability for uncertain parameters and associate it to a certain probability that the real value falls within this range.

Such an approach was adopted to study the stabilization intervention of a block of limestone potentially unstable due to flat sliding on a natural discontinuity with an inclination of 35° with the horizontal plane (Oreste and Spagnoli, 2020). This potentially unstable block, located on a municipal road in the northern part of Piedmont (Italy), was analyzed to verify the need for a stabilization intervention with fully grouted passive bolts and to design the intervention. Specific *in situ* and laboratory tests have been developed.

The *in situ* tests on test bolts made it possible to obtain the stiffness coefficients of the normal (k) and transverse (β_c) interaction of 8.90 ± 1.20 MPa/mm and 1.18 ± 0.38 MPa/mm respectively, with a confidence level of each variability interval greater than 99%.

The pull-out tests provided a stress limit of 2.08 ± 0.73 MPa. The shear tests developed in the laboratory on samples including natural discontinuity, allowed to determine the values of cohesion and friction angle of the sliding surface: c = 8.0 ± 2.3 kPa and φ = 23.0 ° ± 1.40 °.

Assuming the mutual independence of the identified random variables and also assuming a normal distribution (Gaussian probabilistic distribution), it is possible to obtain the probabilistic distribution of each uncertain parameter and in particular the standard deviation as well as the average value already known:

- cohesion c of the sliding surface: $\bar{x}_c$=8.0 kPa; σ_c=0.89147 kPa
- friction angle φ of the sliding surface: $\bar{x}_\varphi$=23.0 °; σ_φ=0.54264 °
- stiffness parameter β_c in the bolt-rock shear interaction: $\bar{x}_{\beta c}$=1.18 MPa/mm; $\sigma_{\beta c}$=0.14729 MPa/mm
- stiffness parameter k in the normal bolt-rock interaction: $\bar{x}_k$=8.90 MPa/mm; σ_k=0.46512 MPa/mm
- limit shear stress on the bolt-rock interface: $\bar{x}_{\tau lim}$=2.08 MPa; $\sigma_{\tau lim}$=0.28295 MPa**.**

The standard deviation was assumed to be $\frac{1}{5.16}$ of the width of the variability interval, associating the latter with a confidence level of 99%.

The calculation of the safety factor F_S in the presence of some random variables can be performed with the Monte Carlo method. After having built up a sample of values extracted from the probabilistic distribution of each random variable, it is possible to proceed to the determination of the safety factor for each series of values obtained from the different extracted samples. In this way it is possible to create a sample of values of the safety factor which can then be statistically treated.

If, for example, the number of the values of each sample obtained is m, it will be possible to constitute m data series of the random variables, represented as follows:

[$(c)_1$; $(\varphi)_1$; $(\beta_c)_1$; $(k)_1$; $(\tau_{lim})_1$]

[$(c)_i$; $(\varphi)_i$; $(\beta_c)_i$; $(k)_i$; $(\tau_{lim})_i$]

[$(c)_m$; $(\varphi)_m$; $(\beta_c)_m$; $(k)_m$; $(\tau_{lim})_m$]

The result is a sample, with a number m, of safety factor values:

[$(F_s)_1$; $(F_s)_2$; ... ; $(F_s)_i$; ... ; $(F_s)_m$]

The remaining parameters affecting the calculation of the safety factor were considered known with an acceptable accuracy and remain constant during the calculation:

- inclination ϑ of the sliding surface with respect to the horizontal plane: 35°;
- sliding surface area A: 10 m^2;
- weight W of the block: 1080 kN;
- thickness of the grout ring t_{binder} around the steel bar: 0.01 m;
- length of the bolt inside the rock block L_a: 1.5 m;
- length of the bolt in the stable rock L_p: 2.5 m;

- elastic modulus of steel E_{st}: 210000 MPa;
- elastic modulus of the cementitious grout E_{binder}: 25000 MPa;
- yield strength of steel σ_{yield}: 400 MPa;
- safety factors required with regard to the failure of the bar $F_{s,adm,yield}$ and the pull-out strength of the bolt-rock interface $F_{s,adm,slip}$: 1.25.

These parameters are considered for the evaluation of the safety factor of the block in a deterministic way, with a value remaining constant in the calculation.

In natural conditions, without the effect of the stabilization intervention, the safety factor was found to be 0.735 and, therefore, not sufficient to guarantee the stability of the block.

To stabilize the rock block, it was decided to adopt two fully grouted passive bolts (n = 2) with steel bars of ranging from diameter Φ_{bar} = 20 mm to diameter Φ_{bar}= 26 mm.

From the simple deterministic analysis considering the midpoint of the intervals of the uncertain parameters it is possible to obtain the safety factor trend shown in Figure 7. According to this approach, all the parameters present in the calculation take on a constant (deterministic) value.

The traditional approach should involve setting the desired safety factor and obtaining the smallest diameter of the bar able to achieve this value, using the graph in Fig. 8.

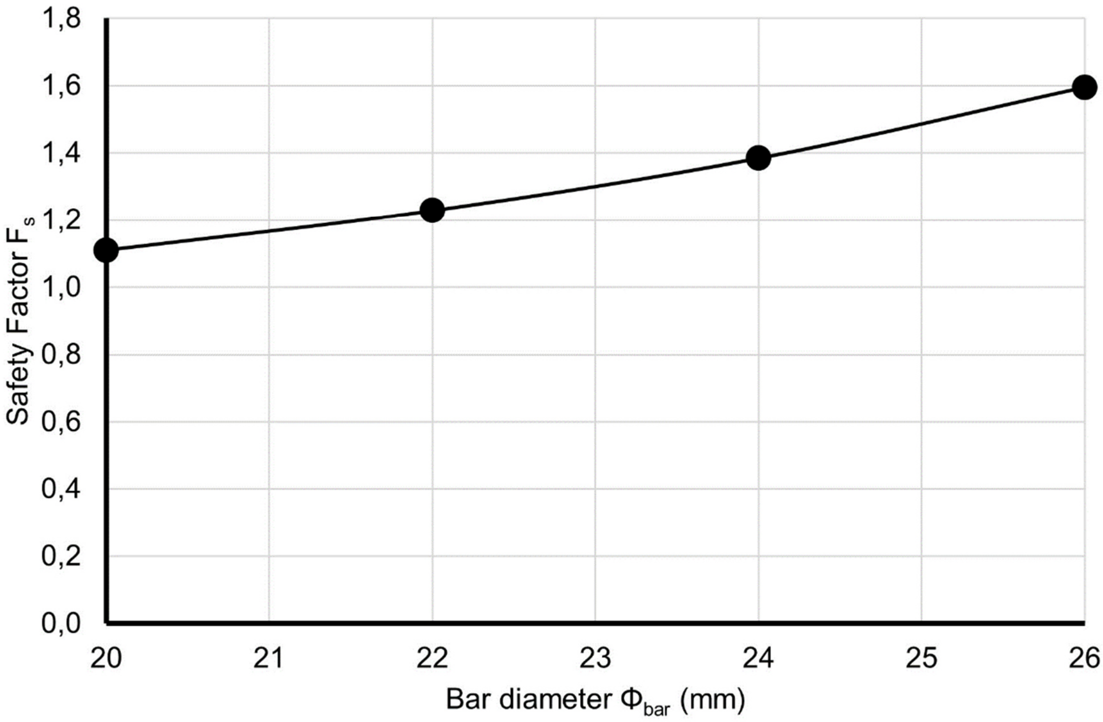

Fig. 8. Trend of the safety factor of the block as the diameter of the steel bars change, based on a deterministic analysis of the safety factors, assuming the average value of the variability intervals of each as a representative value of the parameters considered uncertain.

A subsequent and more in-depth probabilistic analysis using the Monte Carlo simulation allowed to obtain a different sample of the safety factors for each of the considered diameters. In total, therefore, 4 different samples, each consisting of thousand values of safety factors (m = 1000).

The Monte Carlo simulation is time consuming, even if for the proposed procedure the calculations proceeds rapidly, and the final solution is reached in a very limited time. The m value to be adopted depends on the stabilization of the safety factor sample. It is necessary to continuously analyze the mean and standard deviation values of this sample until these values change significantly as the number of extractions increases and, therefore, as m increases.

Figure 9 shows the cumulative frequencies values for each of the 4 safety factor samples obtained with the Monte Carlo simulation. After the verification of the probabilistic distribution closest to the obtained samples, developed through traditional analysis systems, it was possible to adopt the normal distribution of Gauss (Figures 10-12).

The comparison between the sample of safety factors and the theoretical distributions available was made in relation to the cumulative frequencies, the Q-Q plot and the Box Plot. Figures 10-12 show the comparisons adopting the normal distribution. Since the comparison gave positive results, no further comparisons were made with other different theoretical probabilistic distributions.

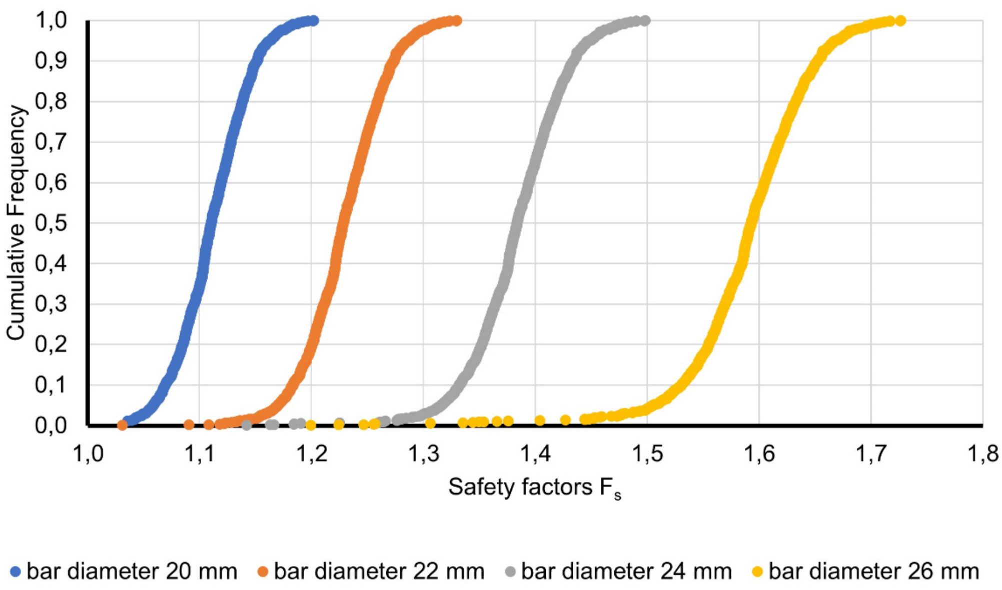

Fig. 9. Trends of the cumulative frequencies of the safety factor samples obtained from the calculation with the Monte Carlo procedure for the 4 diameters of the bars considered: 20 mm (blue), 22 mm (orange), 24 mm (grey) and 26 mm (yellow).

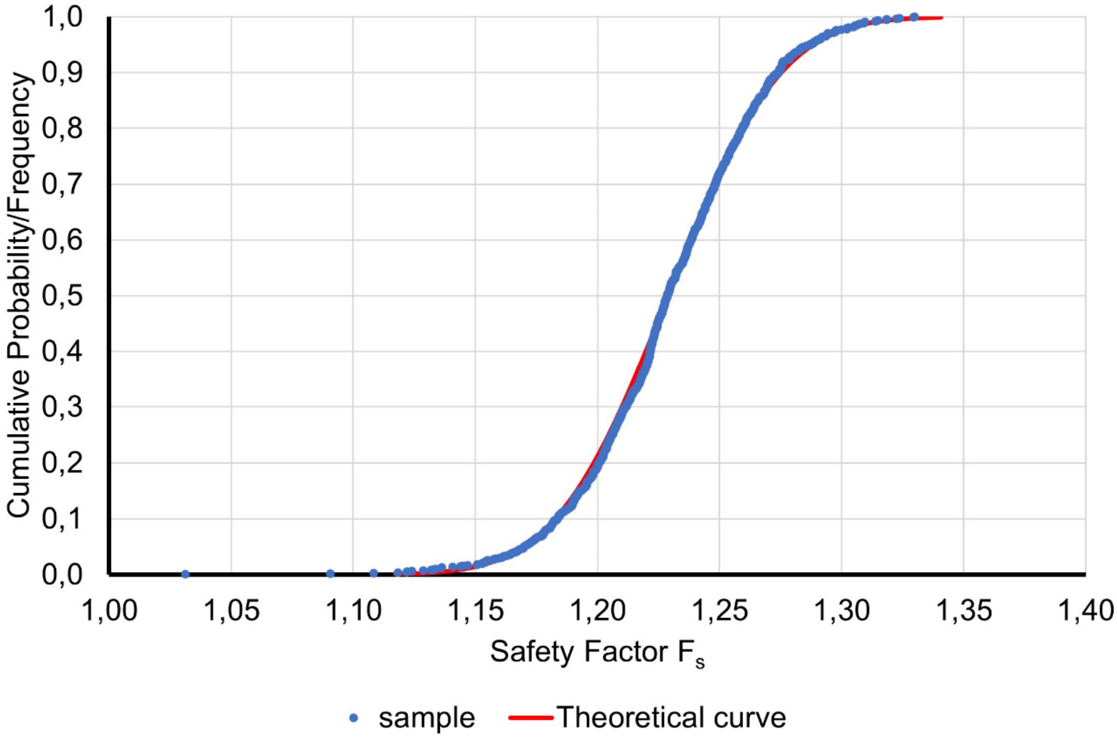

Fig. 10. Verification of the distribution of safety factors for the case of a bar diameter of 22 mm (Φ_{bar} = 22 mm) by comparing the cumulative sample frequencies and the theoretical curve of the cumulative Gaussian distribution.

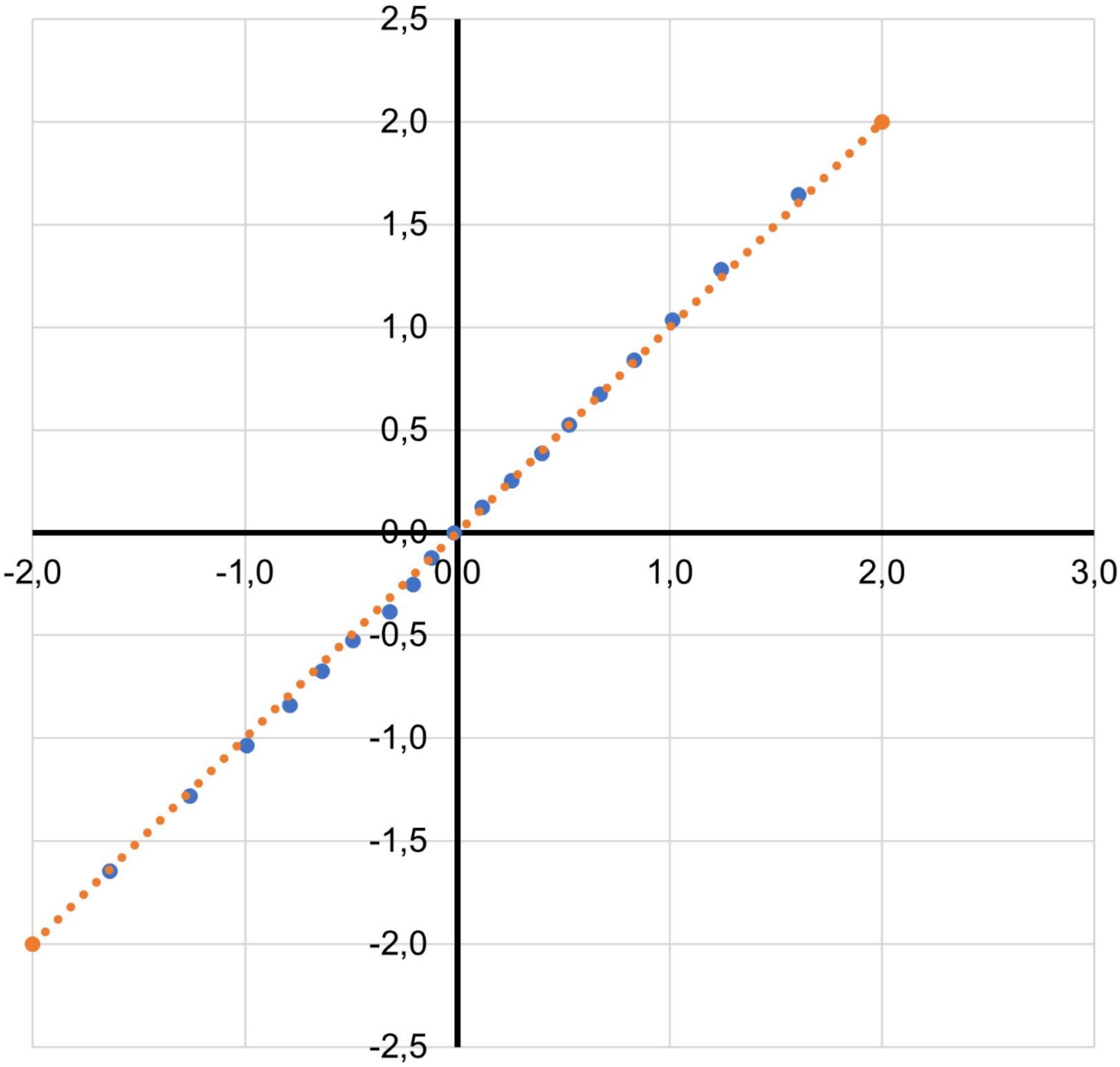

Fig. 11. Verification of the distribution of safety factors for the case of a bar diameter of 22 mm (Φ_{bar} = 22 mm) by examining the Q-Q plot relative to the comparison of the sample distribution with the theoretical curve of the cumulative distribution of Gauss .

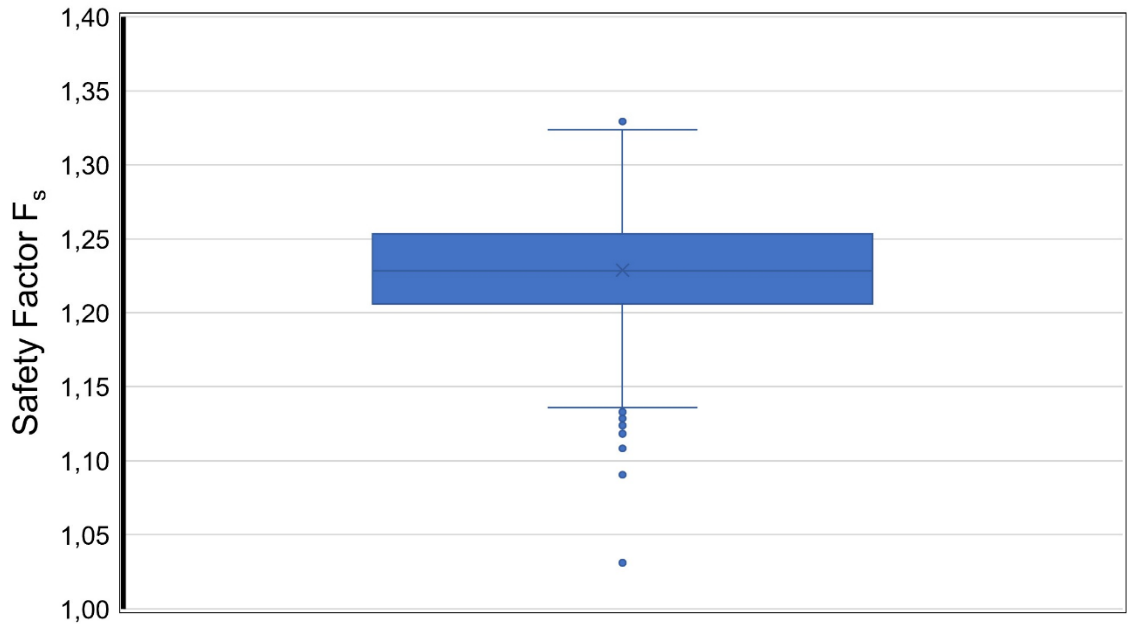

Fig. 12. Verification of the characteristics of the sample of safety factors for the case of a bar diameter of 22 mm (Φ_{bar} = 22 mm) by examining the Box Plot.

After the verification of the samples allowed to consider the Gaussian normal probabilistic distribution as representative, it was possible to plot the cumulative probability curves for each diameter of the bar considered, referring to the mean values and standard deviations obtained for each sample:

Φ_{bar}=20 mm: $\bar{x}_{Fs}$=1.111 ; σ_{Fs}=0.032

Φ_{bar}=22 mm: $\bar{x}_{Fs}$=1.229 ; σ_{Fs}=0.036

Φ_{bar}=24 mm: $\bar{x}_{Fs}$=1.383 ; σ_{Fs}=0.043

Φ_{bar}=26 mm: $\bar{x}_{Fs}$=1.590 ; σ_{Fs}=0.057

Thanks to the cumulative probability curves adopting the normal distribution, it is possible to evaluate, for each bar diameter, what is the probability that the safety factor is lower than a predetermined value. These probability values can be summarized in a graph as the one shown in Fig. 13. It is, therefore, possible to obtain, for example, the indication to use a bar diameter equal to 24 mm to have a probability of only $1 \cdot 10^{-5}$ that the safety factor of the block is lower to 1.2.

It is possible therefore to obtain information of a probabilistic nature on the safety factors of a block of rock starting from the degree of uncertainty on the fundamental parameters of the problem under consideration, for each different diameter of the bar used. Such an approach, therefore, permits to consciously design the extent of the instability risks of a block of rock even in the presence of the stabilization intervention to be adopted, allowing for a modern and effective design of the interventions.

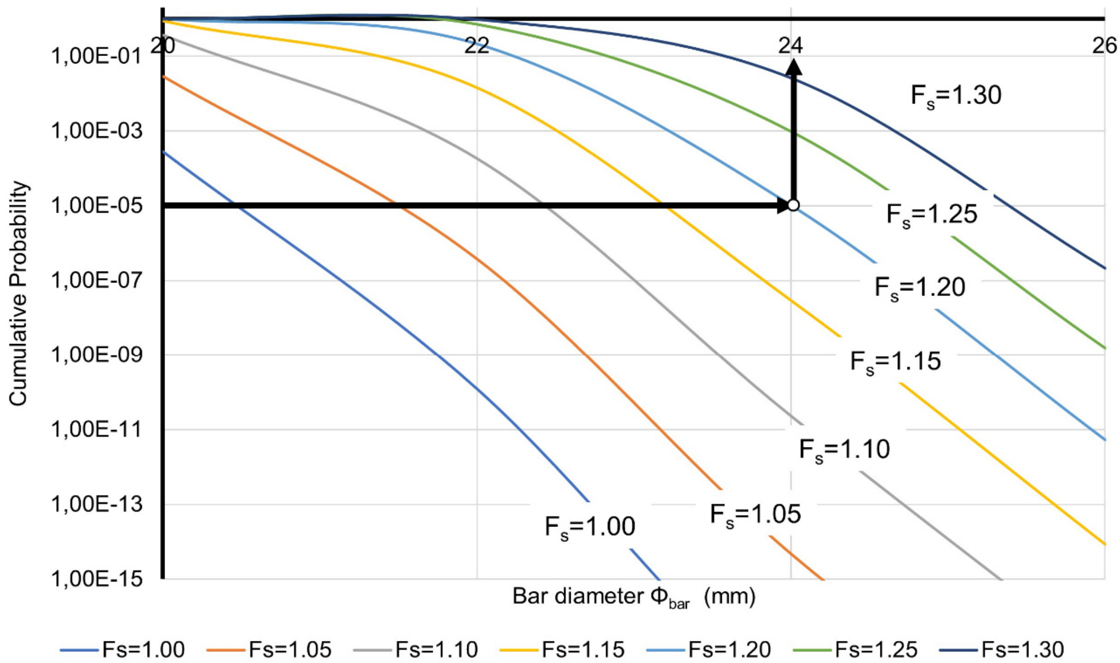

Fig. 13. Cumulative probability as the diameter of the bar varies for different values of the rock block safety factor. The graph permits to appropriately define the diameter of the bar necessary to allow having a predetermined probability that the safety factor is lower than a given value. In the example shown, it is possible to identify a bar diameter of 24 mm in order to limit the probability that the safety factor is less than 1.2 to 1 case in 100 thousand ($1 \cdot 10^{-5}$).

The probability values were obtained with reference to a normal probabilistic distribution, having the value of the mean and standard deviation equal to those of the sample of safety factors obtained from the Monte Carlo simulation.

Conclusions

The stabilization of a block of rock on the walls of an underground cavity is usually done through fully grouted passive bolts. The interaction mechanism between these bolts and the surrounding rock is complex and is established only with a movement, even if imperceptible, of the rock block.

Oreste and Cravero (2008) developed a calculation procedure for the evaluation of the stabilization forces of passive bolts fully grouted on potentially unstable rock blocks. More recently Oreste and Spagnoli (2020) identified some simplified formulas for the definition of the stabilization forces of grouted passive bolts. However, the parameters influencing the behavior of fully grouted passive bolts and leading to identify the extent of the stabilization forces applied to potentially unstable blocks, are difficult to evaluate and require specific laboratory tests or tests *in situ*. Such tests can be carried out only in a limited number and often the results obtained are dispersive. Rather than precise deterministic values, it is possible to estimate ranges of variability of the parameters involved in the calculation.

A probabilistic approach is therefore necessary, which, starting from the uncertainties of the parameters governing the bolt-rock interaction problem, leads to an assessment of the possible variability of the safety factor of the rock block in the presence of stabilization interventions.

In this article, a probabilistic approach has been proposed which is able to allow the correct design of the passive fully grout bolts starting from the uncertainties on the fundamental parameters of the bolt-rock interaction and on the resistance parameters of the sliding surface of the block consisting of a natural discontinuity. This approach is based on the Monte Carlo procedure and allows to obtain samples of the safety factors for each different diameter of the steel bars of the bolts.

From the probabilistic analysis of these samples it was, therefore, possible to design the steel bars considering the probability that the safety factor of the block with regard to instability due to slipping is lower than a predetermined limit. In this way it is possible to design a stabilization intervention by exploiting all the knowledge available on the physical-mechanical phenomenon studied, including those relating to the uncertainty of the fundamental parameters of the problem.

The proposed approach was applied with reference to a real case of a potentially unstable rock block due to planar sliding on a natural discontinuity. The definition of the diameter of the steel bars used for the stabilization intervention was obtained by imposing that the block of rock may have a safety factor lower than 1.2 only for one case out of one hundred thousand ($1 \cdot 10^{-5}$).

Conflict of interests

Authors declare they have no conflict of interest.

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FIGURE CAPTION

Fig. 1 Schematic representation of the potentially unstable block of rock and the passive bolt crossing it (not to scale).

Fig. 2 Model for axial and shear springs at a discontinuity

Fig 3. Gauss probabilistic distribution trend used to represent the uncertainty of the parameters in the geotechnical and geomechanical field.

Fig 4. General trend of the distribution of safety factors (F_S) obtained by calculating the relative frequencies through the histogram.

Fig 5. Cumulative distribution of the probabilities (Gauss curve) of the safety factor, with indication of the probability F (x = 1) that the safety factor is less than unity.

Fig 6. Flow chart of the procedure proposed for the evaluation of the stability conditions (through the evaluation of the safety factor) of a potentially unstable rock block, in the presence of stabilization interventions with fully grouted passive bolts.

Fig. 7 Sketch of the stabilizing forces applied by the fully grouted passive bolt to the potentially unstable rock block on the walls of an underground cavity (not to scale).

Fig. 8. Trend of the safety factor of the block as the diameter of the steel bars change, based on a deterministic analysis of the safety factors, assuming the average value of the variability intervals of each as a representative value of the parameters considered uncertain.

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