Single Frequency Network Broadcasting with 5GNR Numerology

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***Abstract*—This paper investigates the possibility of using 5G New Radio (5GNR) OFDM numerology in the deployment of efficient Single Frequency Networks (SFNs) for delivering TV services to user devices. SFNs are modeled with an equivalent channel characterized by very large delay spread due to the typical inter-site distance (ISD) of the network. The straightforward approach in the design of the physical layer for broadcasting application is based on the adoption of OFDM signalling with very long OFDM symbol and very low sub-carrier spacing (SCS). This design choice allows to dimension the cyclic prefix length to eliminate ISI and ICI induced by the large delay spread with a consequent overhead reduction.**

The 5GNR numerology, with a minimal SCS of 15 kHz and a maximum CP length of 4.7μs, is designed for unicast transmission and Cyclic Prefix lengths are not compatible with those required for large SFN networks. As a consequence classical OFDM receiver based on single-tap equalizer provides unsatisfactory performance. In this paper we consider a general receiver based on the channel shortening principle, but in the frequency domain. The receiver consists in a bank of per tone time/frequency 2D filters, possibly followed by Maximum-Likelihood (ML) trellis processing on the shortened channel. We provide promising information theoretic bound showing that the extension of 5GNR numerology to SFN is possible with very small performance loss. Even the simplest detector architecture that does not employ trellis processing provides throughput competitive with those that can be obtained with smaller SCS. The possibility of using shorter OFDM symbol for SFN is also a great advantage in mobile channel scenario, where the introduced time selectivity of the channel prevent the use of large symbol duration. We also provide some simulation results confirming that the results predicted by the bounds can be closely matched in practice.

I. Introduction

In a Single Frequency Network (SFN), a signal is transmitted simultaneously through multiple stations over the same frequency channel. Several useful signals are available to the receiver either from multi path echoes or from different transmitters. The receiver in SFN must be able to overcome multi path conditions and its performances are strongly affected by the performance of channel equalizer. In an Orthogonal Frequency Division Multiplexing (OFDM) system by adding a Cyclic Prefix (CP) between the OFDM symbols which is at least the size of the Channel Impulse Response (CIR), the non-constructive combination of signals in the receiver can be prevented and a simple receiver structure can be obtained to equalize the channel with only one complex multiplication for each carrier. The DVB-T2 (2^{nd} Generation Digital Video Broadcasting Terrestrial) [1] as well as the ATSC 3.0 (Advanced Television Systems Committee) [2] standards allow for large inter-site distances covering up to hundreds of kilometers (e.g. 60 km - CP duration of 200 μs - or 120 km - CP duration of 400 µs). In parallel, the Third Generation Partnership Project (3GPP) introduced 5G New Radio (5GNR) air interface from Release 15 [3] offering a more flexible and scalable design than LTE, in order to satisfy a wider range of use cases requirements, frequency bands and deployment options. However, 5GNR only supports user-specific uni-cast transmissions. i.e. transmission modes and core functionality not complying with broadcaster's requirements. A multi-cast mode is currently under development in 3GPP Rel-17 Multimedia Broadcast Service (MBS), but it is limited to supporting general multi-cast and broadcast communication services (e.g. transparent IPv4/IPv6 multi-cast delivery, IPTV, IoT applications, V2X applications, public safety) relevant for distribution over 5G mobile networks. Indeed, 5GNR air interface standardized up to now, is not suitable for delivering media services over stand-alone broadcast down-link network only, employing large SFN areas in a Free-to-Air reception, and receive-only device [3].

It is known that the addition of CP in OFDM broadcasting systems reduces the throughput of the channel as it transports unneeded data. The channel shortening is an alternative solution for OFDM receiver with long CP length to deal with multi paths environment. Channel shortening was first proposed in single carrier systems [4] to reduce complexity of trellis detector in ML receiver. The channel shortening principle was also used in multi carrier system [5] as a time domain equalizer to make the equivalent channel response length smaller that the CP thus allowing single tap equalization in the frequency domain.

Ackerr et al. [6] suggested the usage of per tone equalizer (PTEQ) for ADSL applications. They assigned a specific T-Taps equalizer for each carrier separately to optimize the SNR for each carrier. These technique were based on the minimization of mean square error and this metric does not provide in general the highest throughput. In [7] a general framework for channel shortening for any arbitrary linear

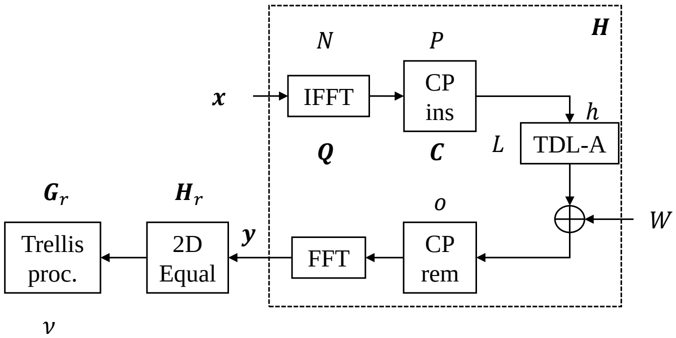

Figure 1. System Model

channel was proposed. The framework is optimized for Gaussian inputs and generalized mutual information. In this paper we start from the framework developed in [7] to derive optimal receiver structures satisfying broadcast transmission requirements using 5GNR numerologies.

The rest of this paper is organised as follows. In Section II, the complete system model for broadcasting transmission in an SFN network is introduced. In particular, the steps describing the construction of equivalent channel matrix are illustrated. In Section III the channel shortening method in [7] is described. Simulation results are illustrated and discussed in Section IV. Conclusions follow under Section V.

II. System Model

The system model is represented in Figure 1. N parallel inputs $\mathbf{x}$ are mapped from the frequency domain to the time domain by means of an Inverse Fast Fourier Transform (IFFT) and CP insertion of size P. The OFDM symbol, of length $N+P$ is transmitted over the physical channel, modeled as a Tapped Delay Line (TDL) 5G channel model [8]. The TDL channel model can characterize a SFN, where all the transmitters use the same frequency and signals copies can reach user from different transmitters and possibly scatterers at the same time. In this case the ISD characterizing the SFN is modeled with a proper Delay Spread (DS) of the TDL channel model. Two type of channels for TDL model are defined, the TDL-A channel profile with Non Line of Sight (NLOS) for handheld/mobile reception environments, and the TDL-E channel profile with Line Of Sight (LOS) for rooftop reception. In Table I we report the model parameters of 4 SFN networks scenarios with different ISD and transmitted power. The 4 scenarios cover some possible configurations for terrestrial networks: High Power High Tower (HPHT) and Medium Power Medium Tower (MPMT) typically based on a limited number of transmitters with large antenna heights and Effective Radiated Power (ERP) values in the range of some kW to many tens of kW. Low Power Low Tower (LPLT) architecture is characterized by a dense network of transmitters, with rather low power levels and antenna heights. For each SFN network we report the delay spread derived from system level simulation to be used in the two TDL channel profiles [8].

Parameter	LPLT	MPMT	HPHT1	HPHT2
ISD [km]	15	50	125	173.2
Transmitted power [dBm]	46	60	70	70
LOS DS [μs]	16	35	45	70
NLOS DS [μs]	20	40	50	75

Table I

Different SFN networks and correspondent Delay Spread (DS) in LOS (TDL-E) and NLOS (TDL-A) channel models.

We consider a TDL channel with channel impulse response (h) with length L, which is known to the receiver. In our setting the length of the impulse response of the channel L can be much larger than an OFDM symbol. So we derive a matrix representation of the system shown in Figure 1 as follows.

First define the following matrices, representing the block processing at TX and RX side:

- **IFFT:** $\mathbf{Q}^*_{ij} = \frac{1}{\sqrt{N}} e^{j2\pi\frac{i \cdot j}{N}} \quad \forall i, j \in [0, N-1]$
- **CP prefix insertion:** $\mathbf{C}_I = \begin{bmatrix} \mathbf{0}_{N-P} & \mathbf{I}_P \\ \hline \multicolumn{2}{c}{\mathbf{I}_N} \end{bmatrix}$, where $\mathbf{I}_N$ is the identity matrix of size N, and $\mathbf{0}_{N-P}$ is a zero matrix with P rows and $N-P$ columns.
- **CP removal:** $\mathbf{C}_R^{(o)} = \text{cshift}_o \begin{bmatrix} \mathbf{I}_N & \mathbf{0}_P \end{bmatrix}$, where "cshift$_o$" accounts for a possible circular shift of the columns of the matrix, controlled the parameter o.
- **FFT:** $\mathbf{Q}_{ij} = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{i \cdot j}{N}}$.

We then and use the conventional infinite Toeplitz matrix,

$$\tilde{\mathbf{H}}' = \text{Toep}_\infty[h_{L-1}, h_{L-2}, h_0],$$

to represent the effect of the channel linear convolution, and introduce the following block diagonal infinite matrices

$$\begin{aligned} \tilde{\mathbf{Q}}^* &\triangleq \text{Toep}_\infty[\mathbf{C}_I \mathbf{Q}^*] \\ \tilde{\mathbf{Q}} &\triangleq \text{Toep}_\infty[\mathbf{Q}\mathbf{C}_R^{(o)}], \end{aligned}$$

to represent TX and RX OFDM block processing. The notation $\text{Toep}_\infty(\mathbf{M}_1, \ldots, \mathbf{M}_N)$ represents the infinite (block) Toeplitz matrix with (block) diagonals $\mathbf{M}_1, \ldots, \mathbf{M}_N$. The output sequence $\mathbf{y}$ can now be written as

$$\mathbf{y} = \overbrace{\tilde{\mathbf{Q}}\mathbf{H}'\tilde{\mathbf{Q}}^*}^{\tilde{\mathbf{H}}} \mathbf{x} + \mathbf{w}'.$$

The OFDM processing then transforms the stationary channel in the time domain into a cyclo-stationary channel in the Finite Fourier Transform domain. The structure of channel matrix $\tilde{\mathbf{H}}$ becomes

$$\tilde{\mathbf{H}} = \text{Toep}_\infty(\mathbf{H}_{J-1}, \ldots, \mathbf{H}_0)$$

where $J = \lceil \frac{L}{N+P} \rceil$, and $\mathbf{H}_i$ are $N \times N$ matrices, substituting the original samples of the time domain impulse response h_i in $\mathbf{H}'$. Notice that this representation depends on the choice of the position of the CP removal offset o.

The introduced notation allows to represent OFDM systems in the general case where the length of the impulse response of the channel L take any value, even larger than P and N.

In the special case, where $L < P$, corresponding to the usual setting for OFDM systems, the input-output relationship boils down to

$$\tilde{\mathbf{H}} = \mathrm{Toep}_{\infty}(\mathbf{D_0}),$$

where $\mathbf{D_0}$ is a diagonal matrix carrying the FFT of the channel impulse response.

The derived input-output relationships can be now casted in the general model introduced in [7] to derive the optimal receiver based on channel shortening.

III. Optimal Channel Shortening

The received signal can be represented as a complex-valued discrete-time model as follows:

$$\mathbf{y} = \tilde{\mathbf{H}}\mathbf{x} + \mathbf{w}', \quad (1)$$

where $\mathbf{y}$ is the received signal, $\tilde{\mathbf{H}}$ is ISI/ICI channel matrix of dimension $T \times T$ which is perfectly known to receiver, $\mathbf{x}$ is the input data which is circularly-symmetric complex Gaussian distributed and $\mathbf{w}'$ Additive Gaussian Noise with variance N_0. The channel matrix $\tilde{\mathbf{H}}$ includes IFFT, TDL channel (h) and FFT. The absolute limit of achievable rate (I_R) over TDL channel (h) with length L is

$$I_R = \log_2\left(1 + \frac{||h||^2}{N_0}\right), \quad (2)$$

where N_0 is the noise power spectral density. The Equation (2) assumes uniform power allocation over all carrier. Now based on the approach in [7], the optimal receiver is characterized by an optimal $\tilde{\mathbf{H}}^{\mathbf{r}}$ filter given by:

$$\tilde{\mathbf{H}}^r = \left[\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\dagger} + N_0\mathbf{I}\right]^{-1}\tilde{\mathbf{H}}^{\dagger}(\tilde{\mathbf{G}}^r + \mathbf{I}), \quad (3)$$

which is a standard MMSE/Wiener filter compensated by trellis processor represented by matrix $\tilde{\mathbf{G}}^{\mathbf{r}}$. The $\tilde{\mathbf{G}}^r$ is a suitably designed matrix that satisfy the following property:

$$(G^r)_{mn} = 0 \quad \text{if} \quad |m-n| > \nu,$$

where $(G^r)_{mn}$ define elements of matrix $\tilde{\mathbf{G}}^{\mathbf{r}}$ and ν denotes memory of reduced trellis memory.

According to [7] the lower bound of theoretical achievable rate of optimal filter $\tilde{\mathbf{H}}^{\mathbf{r}}$ becomes:

$$\begin{aligned} I_{LB}(\nu) = &\log\left(\det\left(\mathbf{I} + \tilde{\mathbf{G}}^{\mathbf{r}}\right)\right) \\ &+ \mathrm{Tr}\left\{[\tilde{\mathbf{G}}^{\mathbf{r}} + \mathbf{I}]\tilde{\mathbf{H}}^{\mathbf{r}}[\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\dagger} + \mathbf{N_0 I}]^{-\mathbf{1}}\tilde{\mathbf{H}}\right\}, \\ &- \mathrm{Tr}\left\{\tilde{\mathbf{G}}^{\mathbf{r}}\right\}, \end{aligned} \quad (4)$$

where $I_{LB}(\nu) < I_R$.

We can define the following throughput efficiency metric as performance metric of channel shortening receiver:

$$T_E(\nu) = \frac{I_{LB}(\nu)}{I_R} \times \frac{N}{N+P}, \quad (5)$$

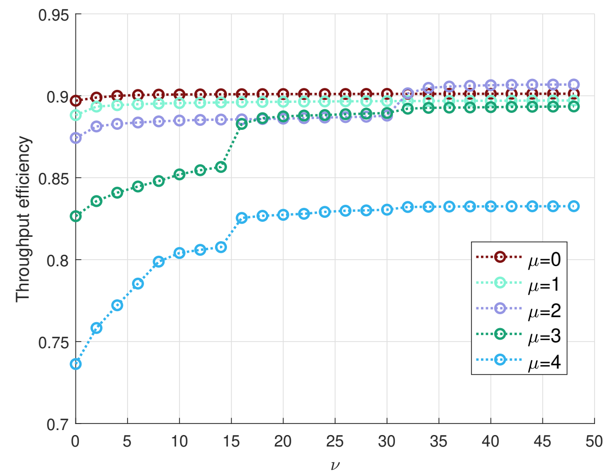

Figure 2. Throughput efficiency versus ν for 5GNR numerologies with fixed SNR=5dB and $SCS = 2^{\mu} \times 15$kHz

with $0 < T_E \leq 1$. Notice that we included in the definition the correction coefficient $\frac{N}{N+P}$ due to insertion of CP.

In Figure 2 we report the $T_E(\nu)$ versus ν for different 5GNR numerologies and HPHT1 NLOS channel scenario of Table I. All plots show increasing the performance with ν. In particular for $\mu = 0$ (15 kHz) we notice that the improvement obtained by increasing ν is marginal and the system reach rapidly the ultimate limit corresponding to CP overhead, that is 6% of 5GNR. So the solution with $\nu = \mu = 0$, corresponding to absence of trellis processor is very promising. This receiver can be constructed by simple 2D-MMSE equalizer without adding complexity of trellis processor.

On the other hand a full complexity detector can be obtained by setting $\nu = T-1$ and the optimal theoretical achievable rate becomes $I_{LB}(\nu) = I_R$. As previously mentioned the design of optimal delay offset is crucial for the final performance as it affects the structure of $\tilde{\mathbf{H}}^r$. In all reported performance this parameter was preliminary optimised by maximising the energy of received signal after cyclic prefix removal.

IV. Simulation and Results

In this section, the performance of optimal $\tilde{\mathbf{H}}^{\mathbf{r}}$ filter is presented. The TDL-A channel profile with delay spreads in table I is used for evaluating SFNs networks. The bandwidth used in all tests is 9.6 MHz. The velocity of receiver is equal to zero. The channel is assumed to be known at the receiver. The time synchronization and computation of optimal offset is performed as described in the previous section.

A. Theoretical and Pragmatic achievable rate by channel shortening

In Figure 3 we report the throughput efficiency (5) of channel shortening receiver for $\nu = 0, 7$ and Gaussian inputs with $50\mu s$ delay spread. The standard 5GNR numerologies

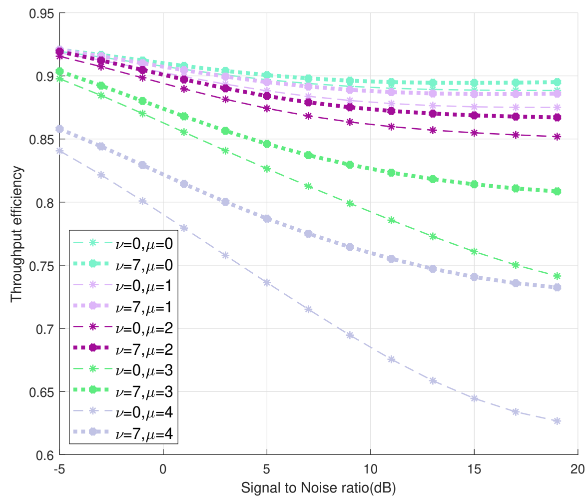

Figure 3. Throughput efficiency T_E versus receiver complexity (ν) for 5GNR numerologies: TDL-A DS=50μs (HPHT1, NLOS, 125km ISD)

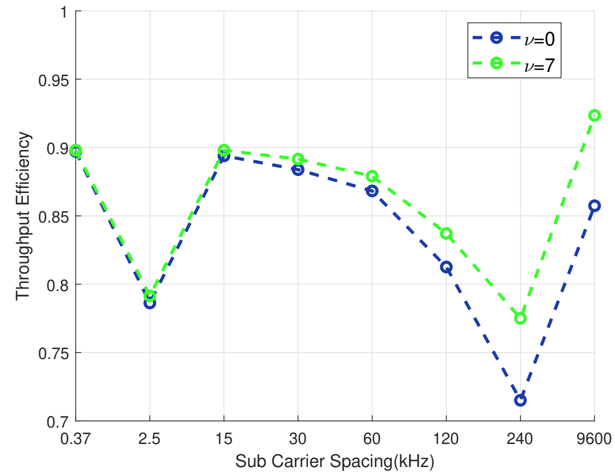

Figure 4. Throughput efficiency versus the sub-carrier spacing for 5G broadcasting, 5GNR numerologies and single carrier spacing with SNR=5dB; TDL-A DS=50μs (HPHT1, NLOS, 125km ISD)

with normal CPs length (1/15 of useful signal) is used. These numerologies are given by:

$$SCS(\mu) = 15 \times 2^{\mu} \ \text{kHz} \quad \mu = 0, \dots, 4,$$

For each of these numerologies and the DS=15μs, the maximum value for J is 7, 14, 28, 55 and 109 OFDM symbols, respectively. The case $\mu = 0$ shows the higher throughput efficiency for any signal to noise ratio, while the throughput efficiency decreases using larger 5GNR numerologies (e.g. $\mu = 4$). The cases with $\nu = 0$ (no trellis processor) and $\nu = 7$ provide similar T_E for small 5GNR numerologies ($\mu = 0, 1, 2$). On the other hand using trellis processing ($\nu = 7$) can provide significant gains for higher numerologies, especially at high signal to noise ratio. In Figure 4 we fixed the SNR to 5dB and reported the throughput efficiencies versus the sub-carrier spacing for the five 5GNR numerologies, two non-standard smaller carrier spacing (0.37, 2.5 kHz) and the single carrier case (9600 kHz), with $\nu = 0, 7$. The CP overhead of the first two non-standard cases is the one specifically designed to allow to deal with large delay spread and with a single tap equalization. The CP overhead is $\frac{1}{9}$ for 0.37 kHz and $\frac{1}{4}$ for 2.5 kHz. On the other hand no CP overhead is associated to the single carrier case.

For $\nu = 0$ (blue line), the best solution is with 15 kHz and provide almost 90% of T_E. For 5GNR carrier spacing around 6% of T_E loss is due to insertion of CP overhead and the remaining is associated to receiver loss. Smaller carrier spacing (0.37,0.25 kHz) provide around 90% and 79% T_E. This loss is almost totally associated to the larger CP overhead associated to them. Notice that with $\nu = 0$ the 15 kHz also outperforms the single carrier case. This can be a motivation for using multi carrier OFDM system with 2D-MMSE $\tilde{\mathbf{H}}^{\mathbf{r}}$ filter for broadcasting in a SFN network.

The single carrier on the other hand performs better by increasing receiver complexity ($\nu = 7$). In fact we can expect that by increasing ν, the T_E converge to the CP correction term in Equation (5) (see Figure 3), which is 1 in this case.

Based on results in Figure 4, 5GNR numerology with 15 kHz with a properly designed 2D-MMSE equalizer ($\nu = 0$) can be a competitive alternative to the 0.37 kHz carrier spacing (3000μs OFDM symbol length) and the need to use trellis processor ($\nu > 0$) is not required.

Previous bounds were obtained assuming an optimal Gaussian input distribution. A more accurate prediction of the system performance can be obtained by computing the mutual information associated to the typical BICM receiver structure. This performance metric, usually referred to as the "pragmatic" capacity, includes the losses due the adoption of a particular constellation and those due to the marginalization to the bit LLR that is performed in the receiver before the channel decoder.

The pragmatic capacities for the practical modulations QPSK (nbits=2), 16QAM (nbits=4) and 64QAM (nbits=6) using 15 kHz and 0.37 kHz carrier spacing are shown in Figure 5. For 15 kHz, we considered both the 2D-MMSE (solid line) and the single tap equalizer (dash-dotted line) receiver, for 0.37 kHz we considered only the single tap receiver (dashed line). As a reference, we reported the theoretical information lower bound (I_{LB}) using 15 kHz carrier spacing with Gaussian inputs.

The 2D-MMSE equalizer with 15 kHz carrier spacing and single tap equalizer with 0.37 kHz carrier spacing, provide similar pragmatic capacities, with the first slightly better. Increasing the signal to noise ratio the pragmatic capacity of 2D-MMSE and single tap equalizer (0.37 kHz) converges as expected to the modulation efficiency (2, 4 and 6 bits). In low signal to noise ratio the pragmatic capacity provided by 2D-MMSE is close to the theoretical limit with Gaussian inputs (I_{LB}). The single tap equalizer with 15 kHz carrier spacing on the other hand can not compensate ISI/ICI interference and

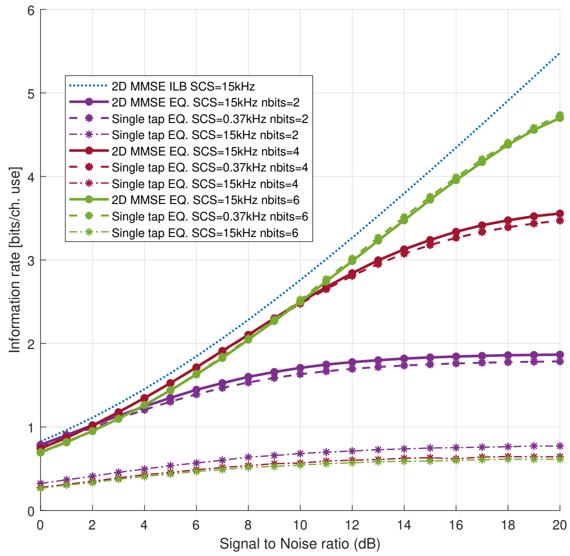

Figure 5. Pragmatic capacity of QPSK, 16QAM and 64QAM inputs for single tap and 2D-MMSE equalizer

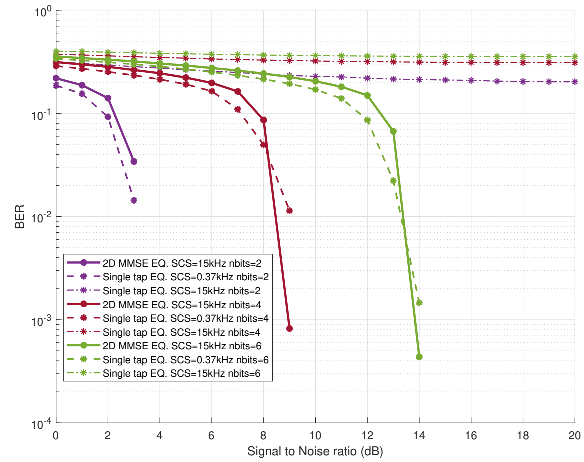

Figure 6. Simulated BER of the three considered realistic systems over TDL-A DS=50μs (HPHT1, NLOS, 125km ISD)

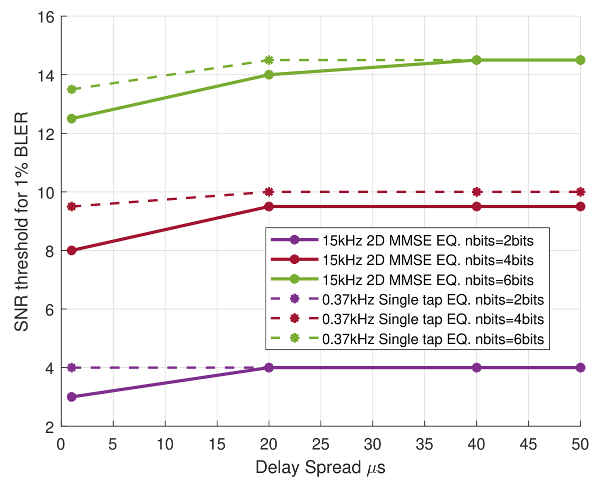

Figure 7. SNR thresholds (dB) at 1% BLER versus Delay spread of TDL-A channel model. Single Tap EQ. vs 2D-MMSE EQ. Code rate 0.53.

increasing the signal to noise ratio can not improve achievable information rate above one. Base on result in Figure 5 the 15 kHz with 2D-MMSE equalizer has similar or even better performance w.r.t. single tap equalizer with long CP length (0.37 kHz carrier spacing).

B. Performances of realistic system

In this section we present our results of practical full link which comprises of a standard 5GNR LDPC encoder with code rate 0.53, a Mapper to 4QAM, 16QAM or 64QAM modulation, an OFDM modulator and the TDL-A channel. The considered target spectral efficiencies are then 1.06, 2.12, 3.18 bit/s/Hz, respectively.

The Bit Error Rate (BER) for the three considered receiver schemes is reported in Figure 6 with the same convention used in Figure 5. The realistic link results confirm the pragmatic capacity results in Figure 5. The performance of 2D-MMSE equalizer with 15 kHz carrier spacing is similar to that of single tap equalizer with 0.37 kHz carrier spacing and with long CP length. On the other hand single tap equalizer with 15 kHz carrier spacing and short CP length (less than 5μs) can not compensate channel effect. Since the achievable information rate in this case is below one and target spectral efficiency equal to 1.06 bit/s/Hz, increasing signal to noise ratio can not improve BER.

The SNR thresholds at 1% Block Error Rate (BLER) for LPLT, MPMT and HPHT SFNs networks are shown in Figure 7 for the 2D-MMSE and single tap receiver system with 15 and 0.37 kHz carrier spacing, respectively. In all SFN network scenarios, the 2D-MMSE equalizer outperforms single tap equalizer with long CP length (300μs). The 2D-MMSE performance is uniform in the considered delay spread range and doesn't degrade significantly by increasing the delay spread, so that it may be used also in more challenging scenarios HPHT2 with 75μs delay spread. The 2D-MMSE thus provides an attractive and simple single solution for all SFN network using 5GNR numerologies.

V. Conclusions and Future Work

In this paper we demonstrated the feasibility of using 5GNR numerologies in the deployment of efficient SFN networks for delivering TV broadcasting services.

In order to achieve this goal, We equalized the ISI/ICI channel using a properly designed 2D-MMSE filter (per tone time/frequency filter) instead of typical single tap equalizer that can be used only with long CP overhead.

The design of the optimal 2D-MMSE filter has been obtained along the procedure outlined in [7], which is valid for any linear channel. The procedure is based on the channel shortening principle and allows to optimally design, assuming Gaussian inputs, a receiver where a suitable filter precedes a trellis processor with bounded state complexity. We provided a general procedure for building the ISI/ICI channel matrix correspondent to the equivalent channel that includes OFDM processing at both TX and RX and use it in the framework of [7] to derive the optimal receiver structure.

The theoretical result of 2D-MMSE filter with Gaussian inputs showed that with 15 kHz carrier spacing the information rate get close to maximum channel capacity even with the simplest low complexity receiver that does not require the adoption of an outer trellis processor. The low complexity 2D-MMSE with 15 kHz carrier spacing provided higher throughput efficiency versus single carrier and the other 5GNR numerologies. The pragmatic capacity associated to practical modulation confirmed the theoretical results.

For the considered SFN network scenarios, single tap equalization with 5GNR numerologies provides very poor performances due to the unacceptable ISI/ICI conditions. On the other hand the adoption of 2D-MMSE filter allows to completely recover the performance losses and provides performances even better than those that can be obtained with OFDM parameters specifically designed for SFN networks [8], requiring much lower carrier spacing and longer CP length.

The presented result are promising but based on the very strong assumption of perfect channel knowledge at the receiver. In practice is well known that channel estimation is a very crucial function for the receiver performances, especially in mobile environment.

Notice that the adoption of the shorter OFDM symbol associated to 5GNR numerologies is also expected to be more suitable in mobile scenario, where the coherence time of channel may becomes too short wrt the OFDM symbol length.

Our future work will then be devoted to the design of a low complexity and adaptive 2D channel equalizer which can acquire and track the ISI/ICI channel also in highly mobile environments. The crucial parameter that will be considered for complexity will be the number and positions of the required active taps in both dimensions and its trade off with performance.

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