Gravitomagnetic resonance in the field of a gravitational wave

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Using the construction of the Fermi frame, the field of a gravitational wave can be described in terms of gravitoelectromagnetic fields that are transverse to the propagation direction and orthogonal to each other. In particular, the gravitomagnetic field acts on spinning particles and we show that, due to the action of the gravitational-wave field, a new phenomenon—which we call gravitomagnetic resonance—may appear. We give both a classical and a quantum description of this phenomenon and suggest that it can be used as the basis for a new type of gravitational-wave detectors. Our results highlight the effectiveness of collective spin excitations, e.g., spin waves in magnetized materials, in detecting high-frequency gravitational waves. Here we suggest that, when gravitational waves induce a precession of the electron spin, power is released in the ferromagnetic resonant mode endowed with quadrupole symmetry of a magnetized sphere. This offers a possible path to the detection of the gravitomagnetic effects of a gravitational wave.

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I. INTRODUCTION

The recent detection of gravitational waves [1,2] by means of the large interferometers confirms once again the predictions of general relativity and the role of Einstein's theory as the best model of gravitational interactions, even though large-scale astronomical observations keep challenging the Einsteinian paradigm with the problems of dark matter and dark energy [3,4]. The measurement process needs to be carefully analyzed in general relativity, since it becomes meaningful only when the observer and the object of observation are clearly identified [5]. This is of uttermost importance when dealing with gravitational waves, since their effects on terrestrial detectors are very small; misconceptions may arise [6], or subtleties need to be properly taken into account [7]. One possible approach is to use a Fermi coordinate system, defined in the vicinity of the world-line of an observer moving arbitrarily in spacetime. Even though the interaction between gravitational waves and detectors is usually studied using the so-called transverse and traceless coordinates, other approaches are viable [8]. In particular, Fermi coordinates have a direct operational meaning, since they are the coordinates an observer would use to perform space and time measurements; indeed, using these coordinates the metric tensor contains (up to the required approximation level) only quantities that are invariant under coordinate transformations internal to the reference frame.

Actually, beyond the aforementioned difficulties for Einstein's theory deriving from cosmological observations, it is a matter of fact that the interplay between general relativity and quantum mechanics is currently unclear. How could we reconcile the Standard Model of particle physics with the geometrical description of the gravitational interaction? There are no conclusive answers from the theoretical point of view, and there are no observations whose explanations require both general relativity and quantum mechanics. In this context, spinning point-like particles have an important role. In fact, while intrinsic spin is a fundamentally quantum property, in Einstein's theory spin is present only at the classical level and derives from the rotation of finite-size bodies [9]. So general relativity, as is, does not explicitly describe the interaction of spacetime with spinning point-like particles; in particular, a relevant question is whether these particles undergo gravitomagnetic effects [10]. As suggested by Mashhoon [11], this question is related to the inertia of intrinsic spin; more generally, the spin-gravity coupling is related to the spin-rotation coupling, on the basis of Einstein's principle of equivalence. In a spacetime that is almost locally flat, around the world-line of an observer this interaction can be described in terms of coupling between the gravitomagnetic field and the intrinsic spin, and it can also be obtained from suitable limits of a Dirac-type equation [12]. Any experiments aimed at testing effects of gravity on spinning particles is indeed a test of the equivalence principle in a new regime [13]. If we go beyond general relativity, the theoretical possibilities increase; for instance, in the Einstein-Cartan theory the role of spin is to generate (nonpropagating) torsion, while mass and energy determine curvature [14,15]. More generally, the spin-gravity interaction is peculiar in extended theories of gravity [16], and there are many experiments of fundamental physics [17] in which various theoretical possibilities are investigated.

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It is then manifest that investigating the interaction between gravity and spin could shed new light on the interplay between our best model of gravitational interaction—general relativity—and the Standard Model of particle physics, which is based on quantum mechanics. In a previous paper [18] we showed that the effects of a plane gravitational wave can be described, in a Fermi coordinate system, in terms of a gravitoelectromagnetic analogy. Namely, we illustrated that the wave field is equivalent to the action of a gravitoelectric and a gravitomagnetic field, which are transverse to the propagation direction and orthogonal to each other. Hence, the interaction with detectors is operationally defined by a (tidal) gravitoelectromagnetic Lorentz force. In particular, all current detectors (such as LIGO and VIRGO) or foreseeable one (like LISA) are indeed aimed at revealing the interaction of a system of masses with the electric-like component of the field. However, due to the magnetic-like component, there is an interaction with moving masses and spinning particles [19]. This is not a surprise, since a gravitational wave transports angular momentum. There have been recent proposals to use spinning particles as a probe for gravitational waves [20]; however, they are based on the tidal gravitoelectric effect and hence are quite similar to the detection methods of the interferometers.

Here we propose a new effect: focusing on the interaction of a gravitational wave with a spinning particle, we show that—in analogy with what happens in electromagnetism—a gravitomagnetic resonance phenomenon may appear, and we suggest that this effect can be exploited, in principle, to design new types of detectors of gravitational waves.

The paper is organized as follows. In Sec. II we introduce the gravitoelectromagnetic approach in the Fermi frame, while in Sec. III we introduce the phenomenon of gravitomagnetic resonance for spinning particles in the field of a gravitational wave. Conclusions are given in Sec. IV.

II. GRAVITOELECTROMAGNETIC FIELDS IN THE FERMI FRAME

Fermi coordinates in the vicinity of an observer's world-line can be defined as follows [21,22]. In the background spacetime describing the gravitational field, we consider a set of coordinates[1] x^μ; accordingly, the world-line $x^\mu(\tau)$ of a reference observer as function of the proper time τ is determined by the equation $\frac{\mathrm{D}x^\mu}{\mathrm{d}\tau} = \ddot{x}^\mu + \Gamma^\mu_{\nu\sigma}\dot{x}^\nu \dot{x}^\sigma = a^\mu$, where D stands for the covariant derivative along the world-line, a dot means a derivative with respect to τ, and a^μ is the four-acceleration. In the tangent space along the world-line $x^\mu(\tau)$ we define the orthonormal tetrad of the observer $e^\mu_{(\alpha)}(\tau)$ such that $e^\mu_{(0)}(\tau)$ is the unit vector tangent to their world-line and $e^\mu_{(i)}(\tau)$ (for $i = 1, 2, 3$) are the spatial vectors orthogonal to each other and orthogonal to $e^\mu_{(0)}(\tau)$. The equation of motion of the tetrad is $\frac{\mathrm{D}e^\mu_{(\alpha)}}{\mathrm{d}\tau} = -\Omega^{\mu\nu} e_{\nu(\alpha)}$, where $\Omega^{\mu\nu} = a^\mu \dot{x}^\nu - a^\nu \dot{x}^\mu + \dot{x}_\alpha \Omega_\beta \epsilon^{\alpha\beta\mu\nu}$. In the latter equation, Ω^α is the four-rotation of the tetrad. In particular, we notice that for a geodesic ($a^\mu = 0$) and nonrotating ($\Omega^\alpha = 0$) tetrad we have $\Omega^{\mu\nu} = 0$; consequently, in this case the tetrad is parallel transported. If $\Omega = 0$ and $a^\mu \neq 0$, the tetrad is Fermi-Walker transported; indeed, Fermi-Walker transport enables to define the natural *nonrotating* moving frame for an accelerated observer [21]. Fermi coordinates are defined within a cylindrical spacetime region of radius $\mathcal{R}$, in the vicinity of the reference world-line, where $\mathcal{R}$ is the spacetime radius of curvature: the observer along the congruence measures time intervals according to the proper time, so the time coordinate is defined by $T = \tau$; the spatial coordinates X, Y, Z are defined by space-like geodesics, with unit tangent vectors n^μ, whose components with respect to the orthonormal tetrad are $n^{(i)} = n_{(i)} = n_\mu e^\mu_{(i)}(\tau)$ and $n^{(0)} = 0$. The reference observer's frame equipped with Fermi coordinates is the *Fermi frame*. Fermi coordinates in the vicinity of the world-line of an observer in accelerated motion with rotating tetrads were studied in Refs. [22–24]. It is possible to show that the spacetime element in Fermi coordinates in the vicinity of the observer's world-line can be recast in terms of the gravitoelectromagnetic potentials $(\Phi, \mathbf{A})$ [11,18],

$$ds^2 = -\left(1 - 2\frac{\Phi}{c^2}\right)c^2 dT^2 - \frac{4}{c}(\mathbf{A}\cdot d\mathbf{X})dt + \delta_{ij}dX^i dX^j, \tag{1}$$

and this peculiarity allows us to apply the corresponding formalism to spinning particles.[2] The gravitoelectromagnetic potentials depend on both the inertial features of the Fermi frame through $\mathbf{a}$ and $\mathbf{\Omega}$ (i.e., the projection of the observer's acceleration and tetrad rotation onto the local frame, respectively) and the spacetime curvature through the Riemann curvature tensor. Here we are interested in the gravitomagnetic effects: the gravitomagnetic potential is $A_i(T, \mathbf{X}) = A^I_i(\mathbf{X}) + A^C_i(T, \mathbf{X})$, where the *inertial* contribution is $A^I_i(\mathbf{X}) = -(\frac{\mathbf{\Omega} c}{2} \wedge \mathbf{X})_i$ and the *curvature* contribution is $A^C_i(T, \mathbf{X}) = \frac{1}{3} R_{0jik}(T) X^j X^k$. Accordingly, it is possible to define the gravitomagnetic field $\mathbf{B} = \mathbf{B}^I + \mathbf{B}^C$, where

$$B^I_i = -\Omega_i c, \qquad B^C_i(T, \mathbf{R}) = -\frac{c^2}{2}\epsilon_{ijk} R^{jk}_{0l}(T) X^l. \tag{2}$$

[1]Greek indices refer to spacetime coordinates, and assume the values 0,1,2,3, while latin indices refer to spatial coordinates and assume the values 1,2,3, usually corresponding to the coordinates x, y, z.

[2]Here and henceforth $\mathbf{X}$ is the position vector in the Fermi frame.

III. GRAVITOMAGNETIC RESONANCE

Exploiting the gravitoelectromagnetic analogy, we may say that a test spinning particle with mass m and spin $\mathbf{S}$ has a gravitomagnetic charge $q_B = -2m$ and, as a consequence, it possesses a *gravitomagnetic dipole moment* $\boldsymbol{\mu}_g = -\frac{\mathbf{S}}{c}$ [11]. Hence, in an external gravitomagnetic field $\mathbf{B}$, its evolution equation is

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}T} = \boldsymbol{\mu}_g \times \mathbf{B} = -\frac{1}{c}\mathbf{S}\times\mathbf{B} = \frac{1}{c}\mathbf{B}\times\mathbf{S}. \tag{3}$$

Now, let us consider a spinning particle interacting with the wave gravitomagnetic field. In the Fermi frame, we consider the coordinates T, X, Y, Z defined as above, with a set of unit vectors $\{\mathbf{u}_X, \mathbf{u}_Y, \mathbf{u}_Z\}$, and we suppose that the plane gravitational wave propagates along the X axis (see Fig. 1). As discussed in Ref. [18], the curvature part of the gravitomagnetic field (2) has the following components:

$$B_X^C = 0, \qquad B_Y = -\frac{\omega^2}{2}[-A^\times \cos(\omega T)Y + A^+ \sin(\omega T)Z],$$
$$B_Z = -\frac{\omega^2}{2}[A^+ \sin(\omega T)Y + A^\times \cos(\omega T)Z]. \tag{4}$$

In the above definitions, A^+ and $A^\times$ are the amplitudes of the wave in the two polarization states, while ω is its frequency. In order to evaluate the effects, we consider a circularly polarized wave, so that $A^+ = A^\times = A$ and the field (4) becomes

$$B_X^C = 0, \qquad B_Y = -\frac{A\omega^2}{2}[-\cos(\omega T)Y + \sin(\omega T)Z],$$
$$B_Z = -\frac{A\omega^2}{2}[\sin(\omega T)Y + \cos(\omega T)Z]. \tag{5}$$

If we consider a frame rotating clockwise in the YZ plane with the wave frequency ω, its basis vectors are given by

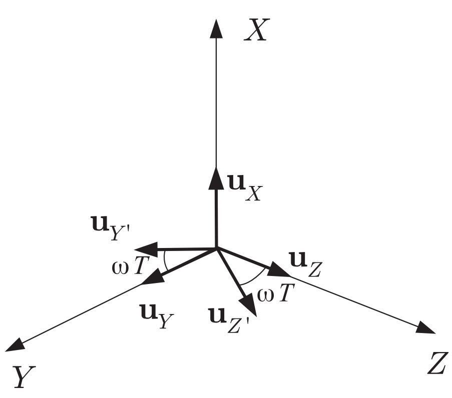

FIG. 1. The Fermi frame is equipped with spatial coordinates X, Y, Z, with unit vectors $\{\mathbf{u}_X, \mathbf{u}_Y, \mathbf{u}_Z\}$; the unit vectors $\{\mathbf{u}'_Y, \mathbf{u}'_Z\}$ are rotating with frequency ω.

$\mathbf{u}_{X'} = \mathbf{u}_X$, $\mathbf{u}_{Y'}(T) = \cos(\omega T)\mathbf{u}_Y - \sin(\omega T)\mathbf{u}_Z$, and $\mathbf{u}_{Z'}(T) = \sin(\omega T)\mathbf{u}_Y + \cos(\omega T)\mathbf{u}_Z$ (see Fig. 1). The above field (5) can be written in the form

$$\mathbf{B}^C(T) = \frac{A\omega^2}{2}[Y\mathbf{u}_{Y'}(T) - Z\mathbf{u}_{Z'}(T)]. \tag{6}$$

The magnitude of this field is $B_C = \frac{A\omega^2}{2}\sqrt{Y^2+Z^2} = \frac{A\omega^2}{2}L$, where L is the distance from the origin of the frame; the gravitomagnetic field $\mathbf{B}^C$ is rotating clockwise with the wave frequency ω, but is a *static field* in the considered rotating frame.

As we have seen, in the Fermi frame the total gravitomagnetic field is $\mathbf{B} = \mathbf{B}^I + \mathbf{B}^C$, where $\mathbf{B}^I = -\boldsymbol{\Omega}c$ and it is simply proportional to the rotation rate $\boldsymbol{\Omega}$ of the frame. We suppose that the latter field is static and that the frame rotates along the direction of propagation of the wave; hence, we may write $\mathbf{B}^I = -B^I\mathbf{u}_X$, where $B^I = \Omega c$. Accordingly, the evolution of a spinning test particle is determined by Eq. (3), which in this case becomes

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}T} = \frac{1}{c}[\mathbf{B}^C(T) + \mathbf{B}^I]\times\mathbf{S}. \tag{7}$$

If we consider the frame corotating with $\mathbf{B}^C(T)$, since $\boldsymbol{\omega} = -\omega\mathbf{u}_X$ is the rotation rate, the time derivatives in the two frames are related by

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}T} = \left(\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}T}\right)_{\mathrm{rot}} + \boldsymbol{\omega}\times\mathbf{S} = \left(\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}T}\right)_{\mathrm{rot}} - \omega\mathbf{u}_{X'}\times\mathbf{S}. \tag{8}$$

Then, if we set $\omega - \frac{1}{c}B^I = \omega - \Omega = \Delta\omega$ the spin evolution equation in the rotating frame turns out to be

$$\left(\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}T}\right)_{\mathrm{rot}} = \left[\Delta\omega\mathbf{u}_{X'} + \frac{1}{c}\mathbf{B}^C\right]\times\mathbf{S} = \frac{1}{c}\mathbf{B}_{\mathrm{eff}}\times\mathbf{S}. \tag{9}$$

In summary, in the rotating frame the spinning particle undergoes a precession around the *static* effective gravitomagnetic field $\mathbf{B}_{\mathrm{eff}} = c[\Delta\omega\mathbf{u}_{X'} + \frac{1}{c}\mathbf{B}^C]$. Let us define $\frac{1}{c}B^C = \omega^*$; then, Eq. (9) suggests that if $\Delta\omega \gg \omega^*$, the precession is in practice around $\mathbf{u}_{X'}$, but if $\Delta\omega \simeq 0$, i.e., if the *resonance* condition is satisfied, the spin precession is around the direction of $\mathbf{B}^C$, which is in any case in the YZ plane, so the precession may flip the spin completely. This condition is obtained when the rotation rate of the frame is equal to the frequency of the gravitational wave. In this case, the spin precesses with frequency ω^*. Notice that all precessions are referred with respect to the reference spinning particle [19] at the origin of the Fermi frame so, in any case, we are talking about a *relative precession.*

The above description is analogous to the classical dynamics of a magnetic moment $\boldsymbol{\mu}$ in a magnetic field $\mathbf{B}(t) = \mathbf{B}_0 + \mathbf{B}_1(t)$ that is the sum of a static field $\mathbf{B}_0$ and a field $\mathbf{B}_1(t)$ rotating with frequency ω in a plane perpendicular to $\mathbf{B}_0$. The description of this electromagnetic interaction can be formulated in quantum terms (see, e.g., Ref. [25]) as follows. Let us suppose that $|g>$ and $|e>$ are the two eigenvectors, respectively, of the ground and excited states of the projection of the spinning particle along the X axis; the eigenvalues of the spin operator S_X are $+\hbar/2$ and $-\hbar/2$, respectively. The Hamiltonian operator is $H = -\boldsymbol{\mu} \cdot [\mathbf{B}_0 + \mathbf{B}_1(t)] = -\gamma \mathbf{S} \cdot [\mathbf{B}_0 + \mathbf{B}_1(t)]$, where γ is the gyromagnetic ratio. We set $\omega_0 = -\gamma B_0$, $\omega_1 = -\gamma B_1$. If we suppose that a spin is, at $t = 0$, in the ground state $|g\rangle$, the probability of transition to the excited state $|e\rangle$ at time t is given by Rabi's formula

$$P_{g\to e}(t) = \frac{(\omega_1)^2}{(\omega_1)^2 + (\omega - \omega_0)^2} \sin^2\left(\sqrt{(\omega_1)^2 + (\omega - \omega_0)^2}\,\frac{t}{2}\right). \tag{10}$$

We see that for $|\omega - \omega_0| \gg |\omega_1|$ the probability is almost equal to zero; however, when $\omega = \omega_0$ we have the magnetic resonance phenomenon, since $P_{g\to e} = 1$ for $t = \frac{2n+1}{(\omega_1)}\pi$. In other words, at resonance the oscillation probability does not depend on the rotating magnetic field $\mathbf{B}_1(t)$ and even a weak field can provoke the flip of the spin direction. We emphasize that in this approach the magnetic field acts as a purely classical quantity.

According to our approach, in the gravitational-wave spacetime there is a gravitomagnetic field $\mathbf{B} = \mathbf{B}^I + \mathbf{B}^C(T)$. It is possible to show [11,26] that the interaction Hamiltonian of a spin $\mathbf{S}$ with the gravitomagnetic field $\mathbf{B}$ is $H = \frac{1}{c}\mathbf{S} \cdot \mathbf{B}$, and this result can also be extended to the intrinsic spin of particles [12,27]. As a consequence, we may introduce a probability transition for spinning particles in the field of a gravitational wave. Accordingly, we obtain

$$P_{g\to e}(T) = \frac{(\omega^*)^2}{(\omega^*)^2 + \Delta\omega^2} \sin^2\left(\sqrt{(\omega^*)^2 + \Delta\omega^2}\,\frac{T}{2}\right). \tag{11}$$

Again, at resonance, i.e., when $\Delta\omega = 0$ or $\omega = \Omega$, even a weak gravitational field can reverse the direction of the spin: the probability of transition is equal to 1 independently of the strength of the gravitomagnetic field for $T = \frac{2n+1}{(\omega^*)}\pi$.

The resonance condition is obtained by combining the gravitomagnetic field of the wave with a rotation field with the same frequency. Since rotations of the frame cannot be obtained for arbitrary frequencies (it is quite impossible to exceed 10^3 Hz for macroscopic systems), if we are dealing with *charged spinning particles* we may get an equivalent situation by using a true magnetic field, on the basis of the Larmor theorem, which states the equivalence between a system of electric charges in a magnetic field, and the same system rotating with the Larmor frequency. If B denotes the magnitude of the magnetic field, the Larmor frequency for electrons is $\omega_L = \frac{\mu_B}{\hbar} B$, where μ_B is the Bohr magneton. Hence, a true magnetic field can be used to produce the gravitomagnetic field $\mathbf{B}^I$.

This interesting resonance condition between Larmor and gravitational-wave frequencies occurring for spinning particles can be easily translated into a resonance of the magnetization of a ferro-/ferrimagnetic material. In fact, if the sample is a sphere of a cubic crystal magnetized along its symmetry axis, then its magnetization exhibits magnetostatic modes (MSMs) with fundamental frequency $\omega_0(B_0) = \gamma B_0$, with $\gamma = 2\pi 28$ GHz/T. The fundamental mode is also known as the *Kittel mode* and corresponds to ferromagnetic resonance with a uniform magnetization. Clearly, this spin-0 mode does not couple to the gravitomagnetic field of the wave. However, higher MSM modes known as spin waves have nonuniform magnetization. The MSM resonant frequencies can be derived from the solution of the magnetostatic equation [28] of a sphere in terms of the associated Legendre functions $P_n^m(\omega, B_0)$. In particular, for modes with $n = |m|$ the relation between the resonant frequency of MSMs and the external magnetic field is linear and reads [29]

$$\omega_{m,m,0} = \omega_H + \frac{m}{2m+1}\omega_M \equiv \gamma H_{e0} + \gamma \frac{4\pi}{3} \frac{m-1}{2m+1} M_0 \tag{12}$$

in terms of the steady field H_{e0} and magnetization M_0, which is supposed to be along the X axis. Accordingly, for $n = m = 2$ the spatial dependence of the magnetization components turns out to be [29]

$$m_Y = Y - iZ, \qquad m_Z = iY + Z. \tag{13}$$

We notice the quadrupolar-like behavior for this mode and the correspondence with the precession determined by the quadrupolar gravitomagnetic field of the wave (5). A possible approach to the detection of these effects could be obtained by considering the hybridization of microwave-frequency cavity modes with collective spin excitations, such as the interaction among the magnetization precession modes in a small magnetically saturated YIG (Yttrium Iron Garnet) sphere and the microwave electromagnetic modes resonating in a radio frequency cavity [30].

IV. CONCLUSIONS

The construction of the Fermi frame enables to describe the field of a gravitational wave in terms of a gravitoelectromagnetic analogy; in other words, the wave field is equivalent to the action of tidal gravitoelectric and gravitomagnetic fields, which are transverse to the propagation direction and orthogonal to each other. As for the gravitomagnetic part of the wave field, it acts on moving or spinning particles; in particular, we have shown that, in analogy with what happens in electromagnetism, a gravitational magnetic resonance phenomenon may appear.

Namely, in the Fermi frame the total gravitomagnetic field is made of a curvature contribution (due to the gravitational wave) and an inertial contribution (due to the rotation rate of the frame). The gravitomagnetic resonance is produced when the frame rotates along the direction of propagation of the wave and the rotation rate is equal to the wave frequency. Since the precession frequency is proportional to the square of the wave frequency, high-frequency waves (of the order of GHz) are favored. If we are dealing with a quantum description of spinning particles, the resonance phenomenon means that the transition probability reaches the value 1 at suitable times; moreover, this probability does not depend on the gravitomagnetic field, and even a weak field can provoke the flip of the spin direction. However, since it is not possible to have physical rotations for arbitrary frequencies, we suggested that an equivalent situation can be obtained by using a true magnetic field, on the basis of the Larmor theorem. Hence, a static magnetic field acting on the probe can mimic the action of a rotating frame. Just like in a magnetic resonance phenomenon, it is not the spin of a single particle that can be observed, but that of a great number of identical particles. For instance, the precession induced by the gravitational wave can modify the magnetization of a sample. We suggested that the hybridization of microwave-frequency cavity modes with collective spin excitations could be used to measure these effects. However, such an analysis is beyond the scope of this paper, whose aim was just to suggest the possibility of considering the phenomenon of gravitomagnetic resonance as the basis of new gravitational-wave detection techniques.

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