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(Article begins on next page)

# The Wiener-Hopf Theory for the Scattering by an Impenetrable Polygonal Structure

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***Abstract*— The Generalized Wiener-Hopf technique and the associated Fredholm factorization method constitute powerful tools that allow to study in quasi-analytical form the diffraction by complex structures with edges. A characteristic of this technique is the possibility to break down the complexity of the diffraction problem into different homogeneous canonical sub-regions where the WH functional equations and their associated integral representations of Fredholm kind are deduced. The mathematical-physical model is comprehensive and it allows spectral interpretation. In this paper we consider a novel canonical scattering problem: the three face impenetrable polygon.**

***Keywords*— *Wedges, Wiener-Hopf method, Integral equations, Diffraction, Scattering, Near-field interactions, Propagation.***

## I. INTRODUCTION

The accurate and efficient study of scattering from complex wedge structures is of great interest in electromagnetic engineering communities. In this paper we investigate a novel complex canonical electromagnetic problem constituted of impenetrable polygonal shape proposed by Bernard in [1]. This problem enlarges the library of canonical scattering problems and its solution has a great impact in propagation community where ray techniques may integrate this structure in their database avoiding to resort to full local numerical techniques or iterative techniques based on PO/PTD/GTD and localization principle [2].

In particular we consider the three face impenetrable polygon as reported in Fig. 1. Cartesian coordinate system is used to describe the problem. The origin is located on the top edge. Two polar systems are also used: the first one (ρ,φ) is centered in (x=0,y=0) and the second one (ρ',φ') is centered in (x=0,y=-d). For the sake of simplicity we consider an Ez-polarized plane wave incident on the structure with incidence angle φo: $E_z^i(\rho,\varphi)=E_o e^{jk\rho\cos(\varphi-\varphi_o)}$. We define three homogenous regions: a) angular region 0<φ<Φa, b) angular region -Φb <φ'<0, c) half layer region x<0, -d<y<0. The most general case of three face impenetrable polygon is the one with impedance boundary conditions as depicted in Fig. 1. We consider in this preliminary work the case with PEC and PMC faces in any of the possible 8 dispositions.

In this paper we propose a novel and effective technique to study this problem with a formulation in terms of Wiener-Hopf (WH) equations. The proposed WH solution procedure is general and it is called Fredholm factorization [3-4]. This technique has been effectively used to solve diffraction problems in geometries that present angular regions and planar regions filled by arbitrary material, see for instance [5-8]. The method in fact is capable to break down the complexity of the diffraction problem into different homogeneous canonical sub-regions where the WH functional equations and their associated integral representations of Fredholm kind are deduced. We observe that each WH equation and each integral representation (related to a single sub-region) is obtained independently from the geometries of the other sub-regions. The integral representations can be interpreted as equivalent network that is useful to order and systematize the procedure [9].

We assert that our method has the benefit to model the entire structure with a true comprehensive mathematical physical model in spectral domain that avoids multiple steps of interaction among separated objects and it allows semi-analytical solutions with spectral interpretation.

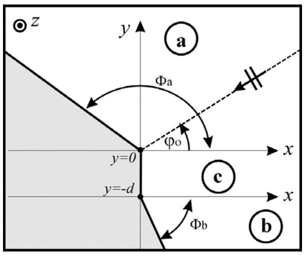


Fig. 1. Scattering by an Impenetrable Polygonal Structure

## II. FROM WH EQUATIONS TO THE SOLUTION

The problem reported in Fig. 1 is modelled in terms of WH equations where the unknowns are the Laplace transforms of the electromagnetic field components:

$$I_{1+}(\eta)=\int_0^\infty H_x(x,0)e^{j\eta x}dx\,,\; V_{1+}(\eta)=\int_0^\infty E_z(x,0)e^{j\eta x}dx \tag{1}$$

$$I_{2+}(\eta)=\int_0^\infty H_x(x,-d)e^{j\eta x}dx\,,\; V_{2+}(\eta)=\int_0^\infty E_z(x,-d)e^{j\eta x}dx \tag{2}$$

In general a plus function $F_+(\eta)$ can be written in the form $F_+(\eta)=F_+^{s.}(\eta)+F_+^{n.s.}(\eta)$ where $F_+^{s.}(\eta)$ constitutes the standard part, i.e. it is regular in the half-plane $\mathrm{Im}[\eta]\geq 0$, and $F_+^{n.s.}(\eta)$ constitutes the not standard part, i.e. it contains the poles of $F_+(\eta)$ located in the half-plane $\mathrm{Im}[\eta]\geq 0$ due to the sources. It is remarkable that the non-standard parts of the WH unknowns are coincident with the non-standard parts of the geometrical optical (GO) contributions $F_+^{G.O.}(\eta)$ known a-priori.

### A. Region a and b

As stated, in this preliminary work, we consider that Region a (Fig. 1) is terminated by PEC or PMC boundary conditions, i.e. Za=0,+∞. Starting from the Generalized Wiener Hopf equation of an angular region [9], by applying Fredholm factorization [3-4] we obtain the following integral representation that relates $I_{1+}(\eta)$ to $V_{1+}(\eta)$:

$$I_{1+}(\eta) = Y_c(\eta)V_{1+}(\eta) + \mathscr{Y}_a[V_{1+}(\eta')] - I_{sca}(\eta), \quad \eta \in \mathbb{R} \qquad (3)$$

where $Y_c(\eta) = \sqrt{k^2-\eta^2}/(kZ_o)$ and $\mathscr{Y}_a[...] = \frac{1}{2\pi j}\int_{-\infty}^{\infty} y_a(\eta,\eta'][...]d\eta$, $k$ and $Z_o$ are respectively propagation constant and the impedance of the free space. For the PEC case we have

$$y_a(\eta,\eta'] = \frac{Y_c(\eta')}{\alpha(\eta')-\alpha(\eta)}\frac{d\alpha}{d\eta'} - \frac{Y_c(\eta)}{\eta'-\eta} + \sum_{n=1}^{n_{Max}} \frac{q_n^{\Phi_a}(\eta)}{\eta'-p_n^{\Phi_a}(\eta)} u(\frac{\pi}{2}-n\Phi_a) \qquad (4)$$

$$\alpha(\eta) = -k\cos\left[\frac{\pi}{\Phi_a}\arccos(-\frac{\alpha}{k})\right] \qquad (5)$$

$$q_n^{\Phi_a}[\eta] = \frac{1}{\mathrm{k}\,\mathrm{Z_o}}(\eta\sin 2n\Phi_a + \sqrt{k^2-\eta^2}\cos 2n\Phi_a) \qquad (6)$$

$$p_n^{\Phi}[\eta] = \eta\cos(2\Phi_a) - \sqrt{k^2-\eta^2}\sin 2\Phi_a,$$

The known function $I_{sca}$ in (3) depends on the non-standard GO contributions and for space reasons it is not reported here. The finite sum in (4) is vanishing for obtuse angular regions. It is remarkable that when the sub-region a is inserted in a complex structure different from of Fig. 1, only the source $I_{sca}$ changes because it depends on the contributions of GO field of the whole structure. Representations similar to (5) hold for region a and b terminated by PMC or PEC.

### B. Region c

For region c (Fig. 1) we introduce directly the general case of surface impedance $Z_c = Z_o/\sin\theta$ at x=0, -d<y<0. In order to get WH equations for the semi-layer region we resort to a generalization of the technique proposed in [8] where the characteristic Green's function procedure is used starting from the wave equation in Laplace domain. In this case the semi-layer region is terminated with impenetrable material. It yields

$$-j(\eta-k\sin\theta)p_o(\eta)Z_o/\sin\theta - I_{1+}(\eta) = Y_{11}(\eta)V_{1+}(\eta) + Y_{12}(\eta)V_{2+}(\eta) \qquad (7)$$

$$-j(\eta-k\sin\theta)r_o(\eta)Z_o/\sin\theta - I_{2+}(\eta) = -Y_{21}(\eta)V_{1+}(\eta) - Y_{22}(\eta)V_{2+}(\eta) \qquad (8)$$

where $Y_c(\eta) = \sqrt{k^2-\eta^2}/kZ_o$ , $\xi(\eta) = \sqrt{k^2-\eta^2}$ and with $Y_{ii}(\eta) = -jY_c(\eta)\cot[\xi(\eta)d]$ , $Y_{ij}(\eta) = jY_c(\eta)/\sin[\xi(\eta)d]$ (i,j=1,2). The functions $p_o$ and $r_o$ reported on the left hand side of (7) and (8) are due to the boundary conditions at x=0.

$$p_o(\eta) = \frac{\int_{-d}^{0}\sin(\xi(y'+d))H_y(y',0)dy'}{-jkZ_o\sin(\xi d)}, r_o(\eta) = \frac{\int_{-d}^{0}\sin(\xi y')H_y(y',0)dy'}{-jkZ_o\sin(\xi d)} \qquad (9)$$

By resorting to the property that both $p_o$ and $r_o$ are even functions of η and applying the Fredholm factorization [3-4] in a generalized form we get from (7)

$$I_{1+}(\eta) = -Y_{11}(\eta)V_{1+}(\eta) + Y_{12}(\eta)V_{2+}(\eta) + \frac{1}{2\pi j}\int_{-\infty}^{\infty} y(\eta,\eta)\frac{\eta-k\sin\theta}{\eta'-k\sin\theta}V_{1+}(\eta')d\eta' + \qquad (10)$$

$$+\frac{1}{2\pi j}\int_{-\infty}^{\infty} y_m(\eta,\eta)\frac{\eta-k\sin\theta}{\eta'-k\sin\theta}V_{2+}(\eta')d\eta'$$

$$y(\eta,\eta) = \frac{Y_{11}(\eta')-Y_{11}(\eta)}{\eta'-\eta} - \frac{Y_{11}(\eta)-Y_{11}(\eta')}{\eta'+\eta}, \; y_m(\eta,\eta) = \frac{Y_{12}(\eta')-Y_{12}(\eta)}{\eta'-\eta} - \frac{Y_{12}(\eta)-Y_{12}(\eta')}{\eta'+\eta}.$$

A similar equation holds for $I_{2+}(\eta)$ starting from (8). Again when the region c is inserted in a structure different from one reported in Fig. 1, the kernels do not change.

### C. The complete problem

The problem is formulated using the four integral representations reported above (1 each for regions a,b and 2 for c). By substitution in the system of equations we obtain the following vector Fredholm integral equation of second kind:

$$\mathbf{V}_+(\eta) + \frac{1}{2\pi j}\int_{-\infty}^{+\infty}\mathbf{M}(\eta,\eta')\mathbf{V}_+(\eta')d\eta' = \mathbf{N}(\eta), \eta\in\mathbb{R} \qquad (11)$$

where $\mathbf{V}_+(\eta) = |V_{1+}(\eta), V_{2+}(\eta)|^t$ and $\mathbf{M}(\eta,\eta')$ is the integral kernel, $\mathbf{N}(\eta)$ the source term constituted of the not standard part of the Geometrical Optics field present in the whole structure of Fig.1. Approximate semi-analytical solutions of (19) and asymptotics for the computation of total uniform field will be presented during the Symposium.

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