Erratum: Fluctuation relations for systems in constant magnetic field

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After the publication of the paper [1] we found an inconsequential mistake in the derivation of the dissipation function for the Nosé-Hoover thermostatted system Eq. (17) of the original manuscript. A complete and correct derivation for $\Omega^{(0)}(X)$ is now reported in the Appendix B of Ref. [2], where, in particular, it is shown that

$$\nabla_X \ln f_0 \cdot \dot{X} = \beta 2K(\Gamma)\xi - \beta \sum_{i=1}^{N} q_i \dot{\boldsymbol{r}}_i \cdot \boldsymbol{E} - 2\beta K^* \xi \delta K(\Gamma)$$

while the compressibility of the (extended) phase space is given by $\Lambda = -\beta 2K^*\xi$. Therefore, Eq. (20) in Ref. [1] should have been written as:

$$\Omega^{(0)}(X) = \beta \mathcal{V} \boldsymbol{J}(\Gamma) \cdot \boldsymbol{E} \tag{1}$$

where $\boldsymbol{J}(\Gamma) = \mathcal{V}^{-1} \sum_{i=1}^{N} q_i \dot{\boldsymbol{r}}_i$ is the microscopic estimator of the current and $\mathcal{V}$ is the volume of the system.

The expression of $\Omega^{(0)}$ for isokinetic systems in a magnetic field, also discussed in Ref. [1], equals Eq. (1). Due to the incorrect expression originally presented, Ref. [1] argued that averages taken over long times, which are conceptually acceptable, would be needed to make the dissipation function of isokinetic and Nosé-Hoover systems agree. The correct calculation reported in Ref. [2] shows, instead, that the expressions for the dissipation functions for the two thermostatted systems are equal not only on average and for $\tau \gg \tau_{\rm NH}$, but also instantaneously. The new expression for the dissipation function given in Eq. (1) does not change the behavior, and in particular the odd signature, of $\Omega^{(0)}(X)$ under the time-reversal operations mentioned in the original manuscript, as shown in Figure 1. This shows that the mistake does not modify any of the conclusions discussed in Ref. [1], with the exception of the already mentioned need of analyzing the long time properties of $\Omega^{(0)}(X)$ to interpret the physical origin and consequences of the two terms dissipation. In fact, this result strengthens the view that $\Omega^{(0)}$, and not other quantities, plays the role of the energy dissipation of nonequilibrium particle systems.

[1] A. Coretti, L. Rondoni, and S. Bonella, Physical Review E **102**, 030101 (2020), ISSN 2470-0045, 2470-0053.
[2] A. Coretti, L. Rondoni, and S. Bonella, Entropy **23** (2021), ISSN 1099-4300, URL `https://www.mdpi.com/1099-4300/23/2/146`.

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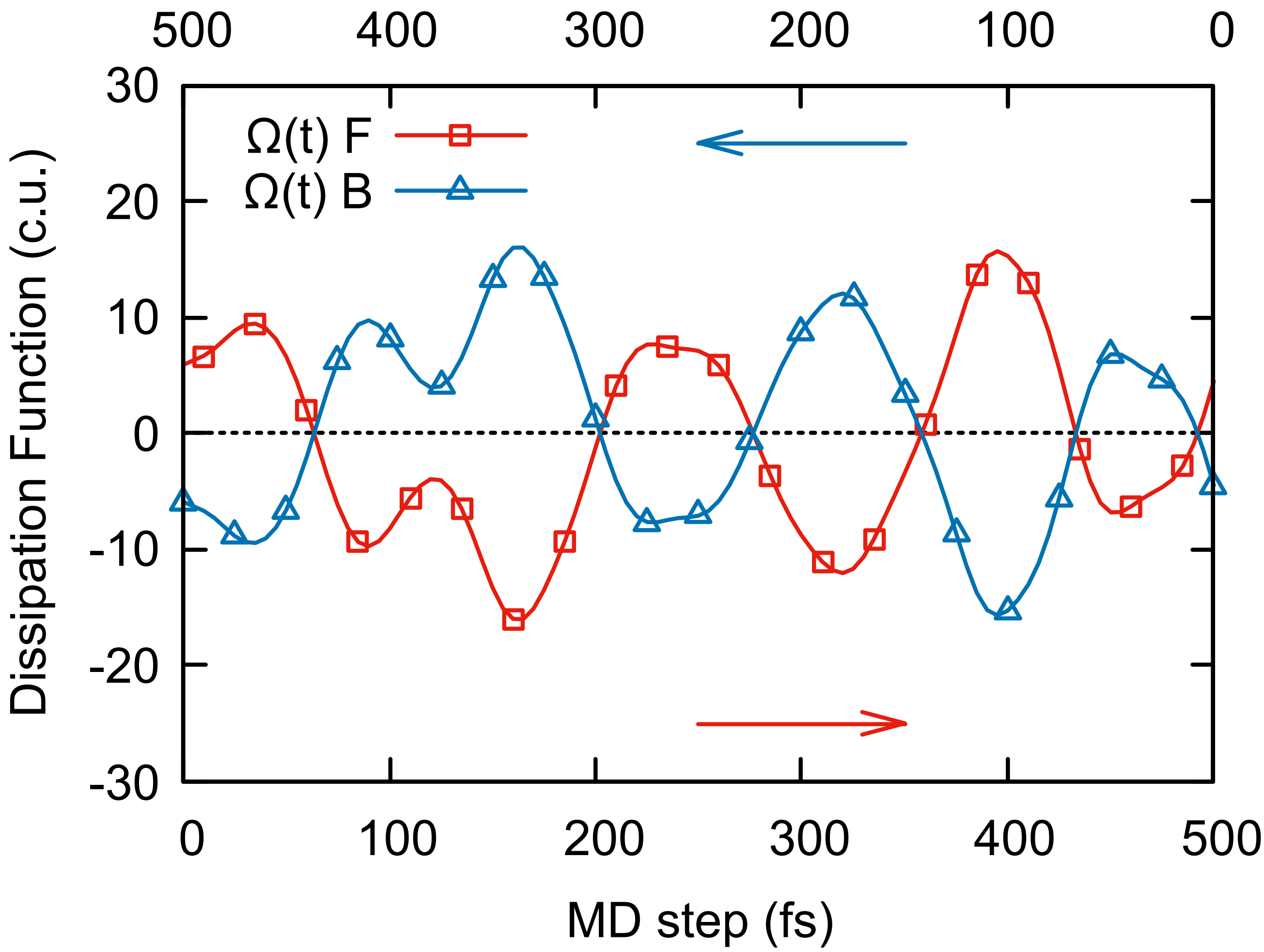

FIG. 1: Same as Figure 1 of the original manuscript, now produced with the corrected expression for $\Omega^{(0)}(X)$ from Eq. (1).